Instituto Superior Técnico

Calculus of variations and Optimal Control

Problem Series nº 5

P1. An air balloon is to be operated between $t = 0$ and $t = 100$ (minutes) and its motion satisfies

$$\dot{x} = -0.01x + u$$

with the initial condition $x(0) = 0$, where x (scalar) is the balloon height.

At each time t, the manipulated variable is subject to the constraint

$$0 \leq u \leq 5$$

By the application of Pontryagin's principle, find the control that maximizes

$$J(u) = x(100) - \int_0^{100} 0{,}5u(t)dt$$

Make a sketch of the optimal profiles of λ (co-state), u and x.

Hints: In this problem what is the shape of the Hamiltonian as a function of u? This shape defines the way in which you find the optimum.

P2. Consider the scalar Riccati differential equation

$$\dot{w} = q_1 + q_2 w + q_3 w^2, \qquad \text{(P2-1)}$$

where q_1, q_2, and q_3 are time functions. Assume that you know a solution w_1.

a) Write an equation verified by the function v such that

$$w = w_1 + \frac{1}{v}$$

is as a solution of the scalar Riccati equation (P2-1).

b) Given the scalar system

$$\dot{x} = -x + u,$$

The solution of the problem that consists in minimizing

$$J(u) = \frac{1}{2}\int_0^6 [x^2(t) + \rho u^2(t)]dt$$

leads to the scalar Riccati differential equation

$$\dot{p} = -1 + 2p + \frac{1}{\rho}p^2, \qquad p(6) = 0.$$

Solve this equation using the technique suggested in a).

Hint: The technique assumes that you know one solution. Is there a solution that is easy to obtain?