Distributed Predictive Control and Estimation

–File 6–

João Miranda Lemos
jlml@inesc-id.pt

Instituto Superior Técnico

2022

8
Distributed LQ control

Infinite horizon Linear Quadratic optimal control

plant

$$x(k+1) = Ax(k) + bu(k),$$

in which u is scalar. The control that results from minimizing the infinite horizon cost

$$J_\infty = \sum_{k=0}^{\infty} x^T(k)Qx(k) + \rho u^2(k),$$

with $Q = Q^T \succ 0$ and $\rho > 0$, is given by the state feedback control law

$$u(k) = -Fx(k).$$

The vector gain F is given by

$$F = (\rho + b^T S b)^{-1} b^T S A,$$

where $S = S^T$ is the positive definite solution of the discrete-time algebraic Riccati equation (ARE)

$$S = (A - bF)^T S(A - bF) + Q + \rho F^T F.$$

Infinite horizon Linear control: properties

Can be obtained either by

- Pontryagin's Maximum Principle in discrete time
- Dynamic Programming (studied in *Control of Cyber-physical Systems*

The performance may not be good (sometime too much overshoot), but always stabilizing, provided (A, b) controllable and $(A, \sqrt{Q})$ observable, even when $Q \succeq 0$.

See problem 20 for a stability proof using Lyapunov's direct method and the candidate Lyapunov function $V(x) = x^\top Sx$.

A machine of producing stabilizing controllers that can be used to initialize more powerful control design methods.

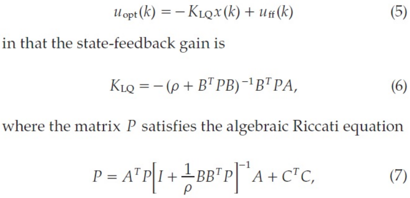
LQ control with accessible disturbances

A step towards distributed LQ control.

For the plant $x(k+1) = Ax(k) + Bu(k) + \Gamma d \quad y(k) = Cx(k)$
with d accessible for measure,
minimize the infinite horizon cost

$$J(u) = \sum_{k=1}^{\infty} [(r - y(k))^2 + \rho u^2(k)$$

Solution:



$$u_{\mathrm{opt}}(k) = -K_{\mathrm{LQ}}x(k) + u_{\mathrm{ff}}(k) \tag{5}$$

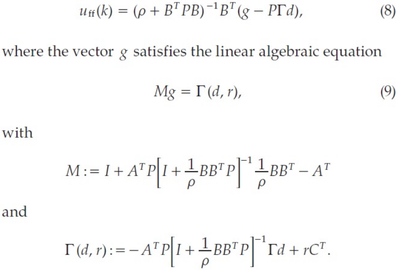
in that the state-feedback gain is

$$K_{\mathrm{LQ}} = -(\rho + B^T PB)^{-1} B^T PA, \tag{6}$$

where the matrix P satisfies the algebraic Riccati equation

$$P = A^T P \left[I + \frac{1}{\rho} BB^T P \right]^{-1} A + C^T C, \tag{7}$$

The feedforward term is



$$u_{\mathrm{ff}}(k) = (\rho + B^T PB)^{-1} B^T (g - P\Gamma d), \tag{8}$$

where the vector g satisfies the linear algebraic equation

$$Mg = \Gamma(d, r), \tag{9}$$

with

$$M := I + A^T P \left[I + \frac{1}{\rho} BB^T P \right]^{-1} \frac{1}{\rho} BB^T - A^T$$

and

$$\Gamma(d, r) := -A^T P \left[I + \frac{1}{\rho} BB^T P \right]^{-1} \Gamma d + rC^T.$$

Plant decomposition

$$x(k+1) = Ax(k) + Bu(k) + \Gamma d(k) \qquad y(k) = Cx(k)$$

Matrix A is imposed to be block diagonal:

$$A = \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 & 0 \\ B_{21} & B_{22} & B_{23} & 0 \\ 0 & B_{32} & B_{33} & B_{34} \\ 0 & 0 & B_{43} & B_{44} \end{bmatrix}$$

Local models

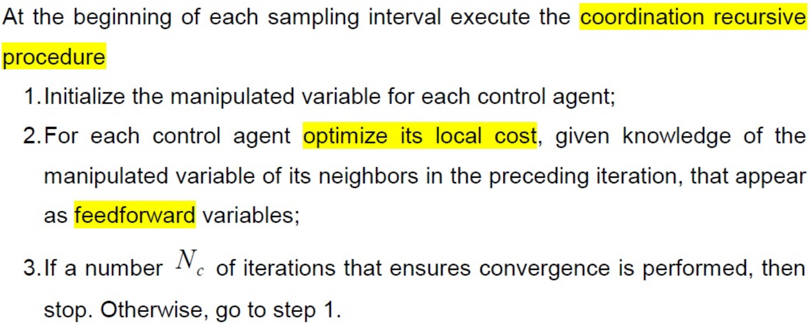
Subsystems are assumed to interact only through their manipulated inputs

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \Gamma_i \delta_i(k)$$

$$\delta_i(k) = \begin{bmatrix} d_i(k) & u_{i-1}(k) & u_{i+1}(k) \end{bmatrix}^T$$

$$y_i(k) = C_i x_i(k)$$

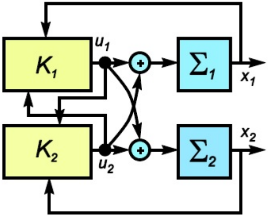
Coordination procedure

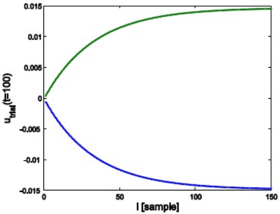


At the beginning of each sampling interval execute the coordination recursive procedure

1. Initialize the manipulated variable for each control agent;
2. For each control agent optimize its local cost, given knowledge of the manipulated variable of its neighbors in the preceding iteration, that appear as feedforward variables;
3. If a number N_c of iterations that ensures convergence is performed, then stop. Otherwise, go to step 1.

Example: distributed control of 2 double integrators



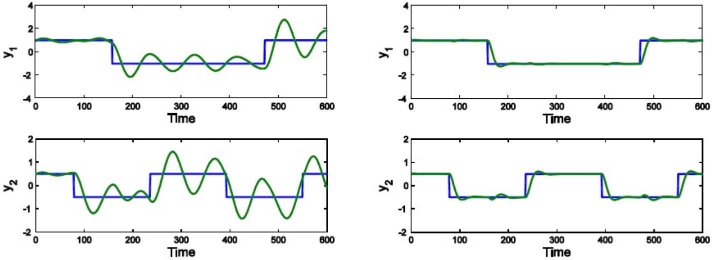


The input of each integrator is a disturbance to the other.

Each local control agent controls one of the integrators.

During one sampling interval, the coordination procedure adjusts the manipulated variables as indicated.

Example: distributed control of 2 double integrators (cont.)

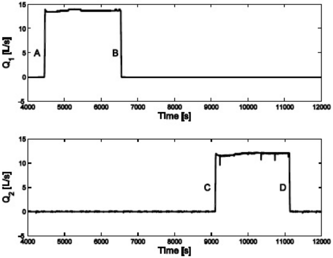


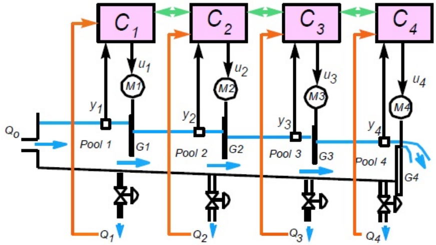
$N_c = 10$

$N_c = 100$

Increasing the number of coordination steps N_c , the performance improves.

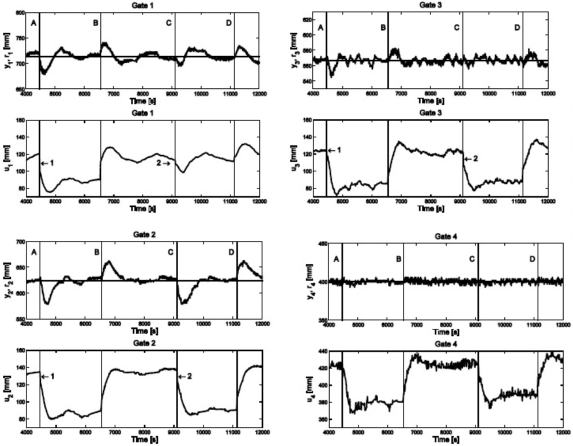
Experimental results with the water canal (1)





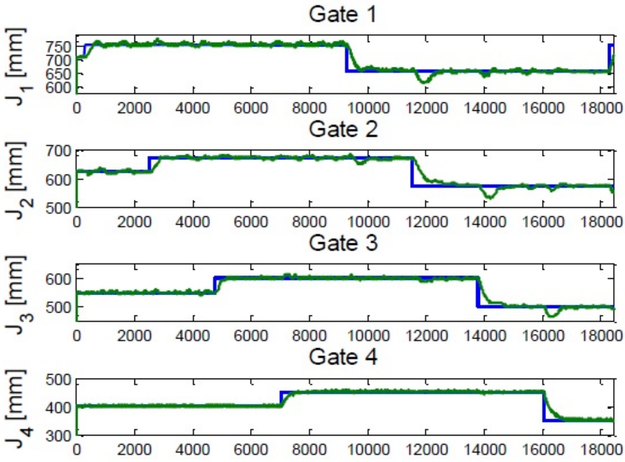
Control objective: Keep the downstream level of each pool close to a reference level when an oftake valve is open.

Experimental results with the water canal (2)



Remark: The propagation from local controller to local controller of the feedforward action.

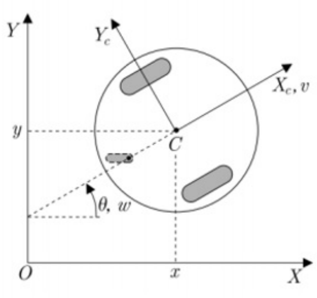
Experimental results with the water canal (3)

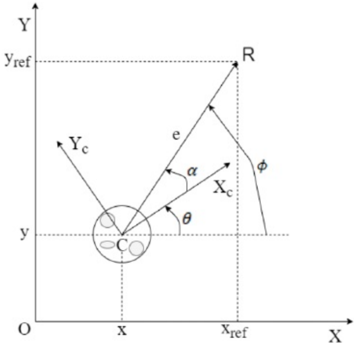


8
Robotic soccer

M. Sc. thesis of André Menezes

Unicycle robot and polar coordinates





Unicycle model

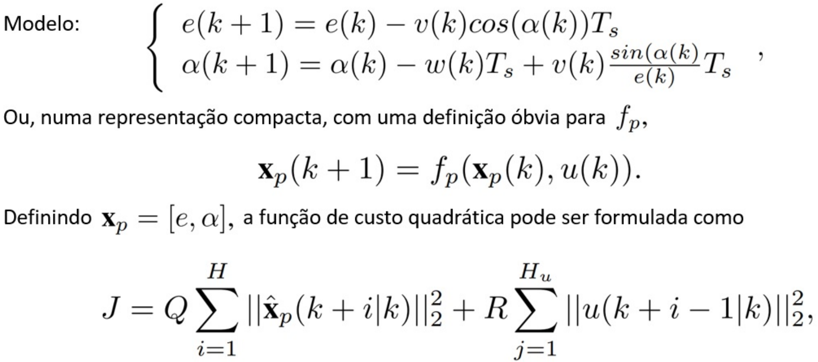
$$\left\{\begin{array}{l} x(k+1) = x(k) + v(k)cos(\theta(k))T_s \\ y(k+1) = y(k) + v(k)sin(\theta(k))T_s \\ \theta(k+1) = \theta(k) + w(k)T_s \end{array}\right. ,$$

Ou, numa representação compacta, com uma definição óbvia para f_d,

$$\mathbf{x}(k+1) = f_d(\mathbf{x}(k), \mathbf{u}(k)).$$

Tracking a reference

Error model



Modelo: $$\begin{cases} e(k+1) = e(k) - v(k)cos(\alpha(k))T_s \\ \alpha(k+1) = \alpha(k) - w(k)T_s + v(k)\frac{sin(\alpha(k)}{e(k)}T_s \end{cases} ,$$

Ou, numa representação compacta, com uma definição óbvia para f_p,

$$\mathbf{x}_p(k+1) = f_p(\mathbf{x}_p(k), u(k)).$$

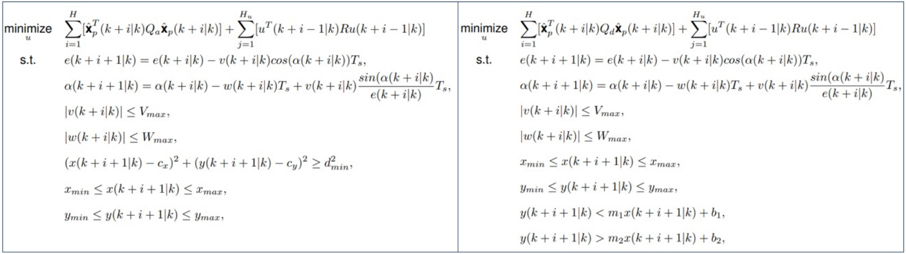
Definindo $\mathbf{x}_p = [e, \alpha]$, a função de custo quadrática pode ser formulada como

$$J = Q\sum_{i=1}^{H} ||\hat{\mathbf{x}}_p(k+i|k)||_2^2 + R\sum_{j=1}^{H_u} ||u(k+i-1|k)||_2^2,$$

Ball intersection

$$
\begin{aligned}
\underset{u}{\text{minimize}} \quad & J \\
\text{s.t.} \quad & e(k+i+1|k) = e(k+i|k) - v(k+i|k)cos(\alpha(k+i|k))T_s, \\
& \alpha(k+i+1|k) = \alpha(k+i|k) - w(k+i|k)T_s + v(k+i|k)\frac{sin(\alpha(k+i|k)}{e(k+i|k)}T_s, \\
& |v(k+i|k)| \leq V_{max}, \\
& |w(k+i|k)| \leq W_{max}, \\
& (x(k+i+1|k) - c_x)^2 + (y(k+i+1|k) - c_y)^2 \geq d_{min}^2, \\
& x_{min} \leq x(k+i+1|k) \leq x_{max}, \\
& y_{min} \leq y(k+i+1|k) \leq y_{max},
\end{aligned}
$$

Difference game: striker vs. defender



striker

$$
\begin{aligned}
\underset{u}{\text{minimize}} \quad & \sum_{i=1}^{H}[\hat{\mathbf{x}}_p^T(k+i|k)Q_a\hat{\mathbf{x}}_p(k+i|k)] + \sum_{j=1}^{H_u}[u^T(k+i-1|k)Ru(k+i-1|k)] \\
\text{s.t.} \quad & e(k+i+1|k) = e(k+i|k) - v(k+i|k)cos(\alpha(k+i|k))T_s, \\
& \alpha(k+i+1|k) = \alpha(k+i|k) - w(k+i|k)T_s + v(k+i|k)\frac{sin(\alpha(k+i|k)}{e(k+i|k)}T_s, \\
& |v(k+i|k)| \leq V_{max}, \\
& |w(k+i|k)| \leq W_{max}, \\
& (x(k+i+1|k) - c_x)^2 + (y(k+i+1|k) - c_y)^2 \geq d_{min}^2, \\
& x_{min} \leq x(k+i+1|k) \leq x_{max}, \\
& y_{min} \leq y(k+i+1|k) \leq y_{max},
\end{aligned}
$$

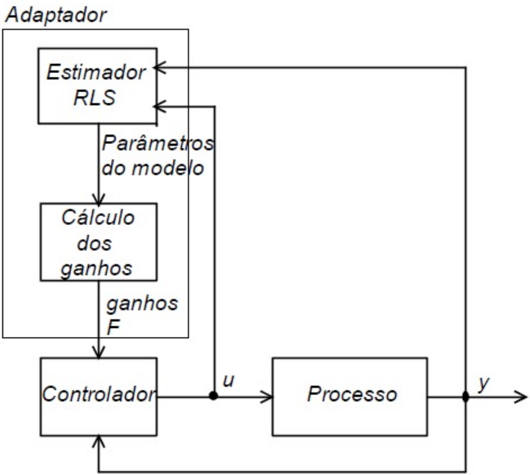
defender

$$
\begin{aligned}
\underset{u}{\text{minimize}} \quad & \sum_{i=1}^{H}[\hat{\mathbf{x}}_p^T(k+i|k)Q_d\hat{\mathbf{x}}_p(k+i|k)] + \sum_{j=1}^{H_u}[u^T(k+i-1|k)Ru(k+i-1|k)] \\
\text{s.t.} \quad & e(k+i+1|k) = e(k+i|k) - v(k+i|k)cos(\alpha(k+i|k))T_s, \\
& \alpha(k+i+1|k) = \alpha(k+i|k) - w(k+i|k)T_s + v(k+i|k)\frac{sin(\alpha(k+i|k)}{e(k+i|k)}T_s, \\
& |v(k+i|k)| \leq V_{max}, \\
& |w(k+i|k)| \leq W_{max}, \\
& x_{min} \leq x(k+i+1|k) \leq x_{max}, \\
& y_{min} \leq y(k+i+1|k) \leq y_{max}, \\
& y(k+i+1|k) < m_1x(k+i+1|k) + b_1, \\
& y(k+i+1|k) > m_2x(k+i+1|k) + b_2,
\end{aligned}
$$

8

Adaptive MPC

Structure of an adaptive controller



The GPC algorithm

$$U_t^{t+T-1} = -M^{-1} W' \Pi' s(t)$$

According to a receding horizon strategy, only the first element of this sequence is actually applied to the plant

$$u(t) = F' s(t)'$$

$$F' = -\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} M^{-1} W' \Pi'$$

this formula for computing the gains is the basis for a version of the position version of GPC - Generalized Predictive Control *(Somewhat simplified)*.

Predictors with constant feedback control

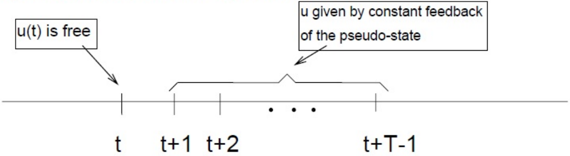
Assume a constant feedback law

$$u(t+k) = F_0^{'} s(t+k) + \eta(t+k)$$

constant feedback

white dither noise uncorrelated with {e(t)}

is acting on the plant from time *t+1* up to time *t+T-1*



u(t) is free

u given by constant feedback of the pseudo-state

Predictors with constant feedback control (cont.)

Assuming a constant feedback F_0 is acting on the plant from *t+1* up to *t+T-1*

$$\hat{y}(t+i|t) = \theta_i u(t) + \psi_i' s(t)$$

$$\hat{u}(t+i-1|t) = \mu_{i-1} u(t) + \phi_{i-1}' s(t)$$

$$i = 1, \ldots, T$$

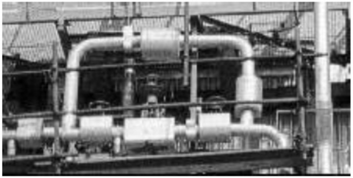
Note that: $\mu_0 = 1 \phi_0 = \bar{0}$

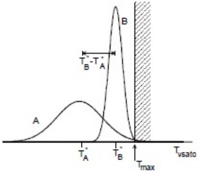
All the predictors depend on the feedback gain F_0, except θ_1 and ψ_1 which depend only on the parameters of the ARX plant model:

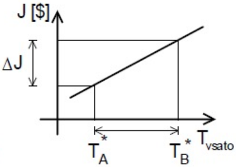
$$\theta_1 = b_0; \psi_1' = \begin{bmatrix} -a_1 & \ldots & -a_n & b_1 & \ldots & b_m \end{bmatrix}$$

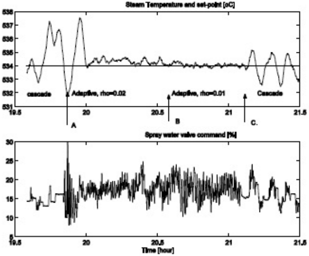
Example: superheatee steam temperature control

MUSMAR = Multistep, Multivariable, Adaptive Regulator

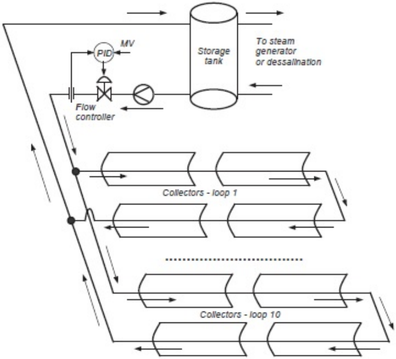


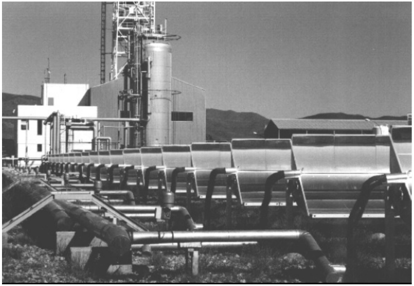






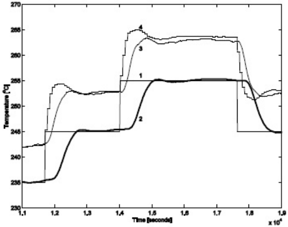
Example: Solar thermal plant

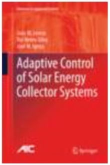




Example: solar thermal plant

Cascade adaptive MPC control

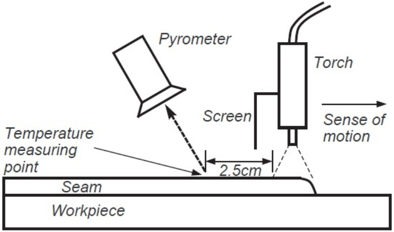


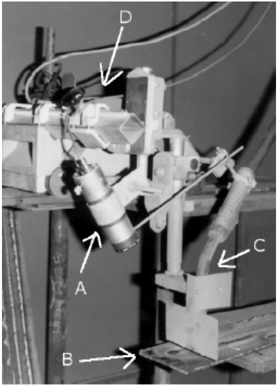


Adaptive Control
of Solar Energy
Collector Systems

J. M. Lemos, R. Neves-Silva and J. M. Igreja.
Adaptive Control of Solar Energy Collector Systems,
Springer (Advances in Industrial Control), 2014
(*Copy available at the IST library - DEEC*)

Example: Arc welding - plant





Example: Arc welding - experimental results