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Advanced Plasma Physics

MEFT 2021/22

Problem Class 1

Clearly present your approximations and enclose all pertinent calculations. Try to solve the problems yourself. Follow the instructions of the Lecturer.

Problem 1. Vlasov equation. As derived in the theory class, the Klimontovich equation for the α-species of the plasma reads

$$\frac{\partial N_\alpha}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} N_\alpha + \frac{\mathbf{F}_\alpha^{(m)}}{m_\alpha} \cdot \boldsymbol{\nabla}_{\mathbf{v}} N_\alpha = 0, \tag{1}$$

where $\mathbf{F}_\alpha^{(m)}$ is the microscopic force (due to the microscopic fields $\mathbf{E}^{(m)}$ and $\mathbf{B}^{(m)}$). Define the *smooth distribution function* $f_\alpha(\mathbf{r}, \mathbf{v}, t)$, in terms of which we may rewrite the microscopic distribution function as

$$N_\alpha(\mathbf{r}, \mathbf{v}, t) = f_\alpha(\mathbf{r}, \mathbf{v}, t) + \delta N_\alpha(\mathbf{r}, \mathbf{v}, t).$$

a) Discuss in class with your colleagues the physical meaning of both f_α and δN_α.

b) Average out the Klimontovich equation and show that the equation for the smooth function f_α now reads

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f_\alpha + \frac{\mathbf{F}_\alpha}{m_\alpha} \cdot \boldsymbol{\nabla}_{\mathbf{v}} f_\alpha = \mathcal{C}[\delta N_\alpha],$$

specifying the form (and the physical meaning) of $\mathcal{C}[\delta N_\alpha]$. Is the latter equation more or less accurate than Eq. (1)?

c) Consider the case of a fully ionized, dilute plasma, for which the free mean path is sufficiently large, i.e. under the condition $n_\alpha \ell_\alpha^3 \gg 1$ (understand the physical meaning of this approximation). Show that the plasma is appropriately described by the Vlasov equation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f_\alpha + \frac{\mathbf{F}_\alpha}{m_\alpha} \cdot \boldsymbol{\nabla}_{\mathbf{v}} f_\alpha \simeq 0.$$

Discuss how you would relate the mean-field force $\mathbf{F}_\alpha$ to the EM-fields and the distribution function f_α, when dealing with both electrostatic and electromagnetic phenomena.

Problem 2. Electrostatic waves. Let us consider small fluctuations around a certain initial distribution (that we here assume to be the thermal equilibrium) as $f_\alpha = f_{0,\alpha} + f_{1,\alpha}$, where $f_{1,\alpha} \ll f_{0,\alpha}$ is a small perturbation.

c) Show that the dielectric function reads

$$\epsilon(k, \omega) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{k} \int_{-\infty}^{+\infty} \frac{g'_{0,\alpha}(v)}{\omega - kv} dv, \tag{2}$$

where $v = \boldsymbol{v} \cdot \hat{\boldsymbol{k}}$ and $f_{0,\alpha}(\boldsymbol{v}) = n_0 g_{0,\alpha}(\boldsymbol{v})$ for homogeneous plasmas (quasi-neutrality is assumed here, so $n_{0,e} = n_{0,i} \equiv n_0$.

b) Let us focus on the case of electronic waves only. As such, we take the limit in which ions are inertia-less, $m_i \to \infty$. Assuming that electrons follow the Maxwell-Boltzmann distribution,

$$g_{0,e} = \frac{1}{\sqrt{2\pi} v_e} e^{-v^2/(2v_e^2)},$$

where $v_e = \sqrt{k_B T_e / m_e}$ is the electron thermal speed. Moreover, it is expected for electron plasma waves to feature very large phase speeds in the long-wavelength limit $k \to 0$ (why?), i.e. they satisfy the condition $\omega/k \gg v_e$. Obtain the dispersion relation for the Langmuir waves,

$$\omega = \sqrt{\omega_{pe}^2 + 3v_e^2 k^2}.$$

Discuss this result in the light of what you have learned from the hydrodynamic formulation of plasmas, with Prof. Jorge Vieira.

c) Consider now oscillation taking place in the ion sector. For that task, we may anticipate that some of the previous considerations for the electrons remain valid. However, we can no longer assume the electrons to be inertialess (why?). On the contrary, we assume that electrons follow the motion of the ions adiabatically, therefore remaining in thermal equilibrium at all times. Make the proper adjustments to Eq. (2) to show that the dispersion relation of ion-acoustic waves is given by

$$\omega \simeq \frac{c_s k}{\sqrt{1 + k^2 \lambda_D^2}}.$$

Obtain explicit expressions for c_s and λ_D in terms of the basic parameters of the system and discuss their physical meaning.

d) Plot the dispersion relation ω vs k and digress over its features in both limits $k\lambda_D \ll 1$ and $k\lambda_D \gg 1$. Vividly discuss your conclusions with your colleagues.