Mecânica Computacional — PB12

→ Aplicação de condições de fronteira naturais em elementos quadráticos.

Matriz Rigidez Global

$$K_{ij}^G = \int_\Omega \left(\frac{\partial \Phi_i}{\partial x}\frac{\partial \Phi_j}{\partial x} + \frac{\partial \Phi_i}{\partial y}\frac{\partial \Phi_j}{\partial y} \right) dx\, dy + \int_\Gamma p \Phi_i \Phi_j \, ds$$

→ matriz simétrica H^G

$$H_{ij}^G = \int_\Gamma p(s) \Phi_i \Phi_j \, ds$$

Vector de forças global

$$f_i^G = \int_\Omega f(x,y) \Phi_i \, dx\, dy + \int_\Gamma \gamma(s) \Phi_i \, ds$$

→ Vector p^G

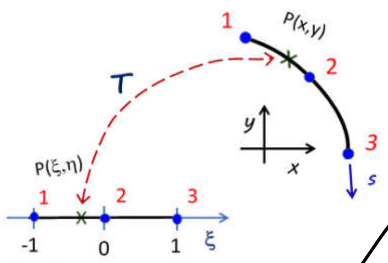
$$P_i^G = \int_\Gamma \gamma(s) \Phi_i \, ds$$

Utilizando coordenadas locais:

"determinante" da matriz Jacobiana

$$H_{ij}^e = \int_{\Gamma_e} p \Psi_i \Psi_j \, ds = \int_{-1}^{1} p \Psi_i(\xi) \Psi_j(\xi) \frac{ds}{d\xi} d\xi \qquad P_i^e = \int_{\Gamma_e} \gamma(s) \Psi_i \, ds = \int_{-1}^{1} \gamma \Psi_i(\xi) \frac{ds}{d\xi} d\xi$$

Ou seja, envolve uma Transformação de coordenadas:



↳ Elemento de referência

Funções de forma

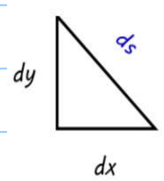
$$\Psi_1(\xi) = \frac{1}{2}\xi(\xi - 1) \qquad \Psi_2(\xi) = 1 - \xi^2 \qquad \Psi_3(\xi) = \frac{1}{2}\xi(\xi + 1)$$

$\xrightarrow{d/d\xi}$

$$\Psi_1'(\xi) = \xi - \frac{1}{2} \qquad \Psi_2'(\xi) = -2\xi \qquad \Psi_3'(\xi) = \xi + \frac{1}{2}$$

$$\begin{cases} x = x(\xi) = \sum_{i=1}^{3} \Psi_i(\xi) x_i \\ y = y(\xi) = \sum_{i=1}^{3} \Psi_i(\xi) y_i \end{cases} \xrightarrow{d/d\xi} \begin{cases} x'(\xi) = \frac{dx}{d\xi} = \sum_{i=1}^{3} \Psi_i'(\xi) x_i \\ y'(\xi) = \frac{dy}{d\xi} = \sum_{i=1}^{3} \Psi_i'(\xi) y_i \end{cases}$$

$$(ds)^2 = (dx)^2 + (dy)^2 \Rightarrow \frac{ds}{d\xi} = \sqrt{\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dy}{d\xi}\right)^2}$$



→ "determinante" da matriz Jacobiana.

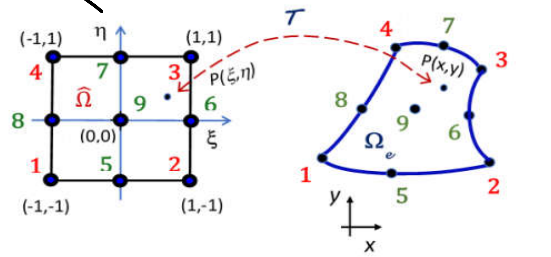
Number of points, n	Points, x_i		Weights, w_i	
1	0		2	
2	$\pm\frac{1}{\sqrt{3}}$	±0.57735...	1	
3	0		$\frac{8}{9}$	0.888889...
	$\pm\sqrt{\frac{3}{5}}$	±0.774597...	$\frac{5}{9}$	0.555556...

Para a integração numérica, utiliza-se a regra de Gauss-Legendre a 1D:
(admitindo que p e γ são constantes no elemento)

$$H_{ij}^e = \sum_{k=1}^{nip} p . \Psi_i(\xi_k) \Psi_j(\xi_k) \frac{ds}{d\xi}(\xi_k)\, w_k \qquad P_i^e = \sum_{k=1}^{nip} \gamma . \Psi_i(\xi_k) \frac{ds}{d\xi}(\xi_k)\, w_k$$

→ Elemento quadrangular de 9 nós

Elemento de referência



Funções de forma

$$\Psi_1(\xi,\eta)=\xi(1-\xi)\eta(1-\eta)/4$$
$$\Psi_2(\xi,\eta)=\xi(1+\xi)\eta(1-\eta)/4$$
$$\Psi_3(\xi,\eta)=\xi(1+\xi)\eta(1+\eta)/4$$
$$\Psi_4(\xi,\eta)=\xi(1-\xi)\eta(1+\eta)/4$$
$$\Psi_5(\xi,\eta)=\left(1-\xi^2\right)\eta(\eta-1)/2$$
$$\Psi_6(\xi,\eta)=\xi(1+\xi)\left(1-\eta^2\right)/2$$
$$\Psi_7(\xi,\eta)=\left(1-\xi^2\right)\eta(\eta+1)/2$$
$$\Psi_8(\xi,\eta)=\xi(\xi-1)\left(1-\eta^2\right)/2$$
$$\Psi_9(\xi,\eta)=\left(1-\xi^2\right)\left(1-\eta^2\right)$$

$\partial/\partial\xi$

$\partial/\partial\eta$

$$\frac{\partial\Psi_1}{\partial\xi}=(2\xi-1)\eta(1-\eta)/4$$
$$\frac{\partial\Psi_2}{\partial\xi}=(2\xi+1)\eta(1-\eta)/4$$
$$\frac{\partial\Psi_3}{\partial\xi}=(2\xi+1)\eta(1+\eta)/4$$
$$\frac{\partial\Psi_4}{\partial\xi}=(2\xi-1)\eta(1+\eta)/4$$
$$\frac{\partial\Psi_5}{\partial\xi}=\xi\eta(1-\eta)$$
$$\frac{\partial\Psi_6}{\partial\xi}=(2\xi+1)\left(1-\eta^2\right)/2$$
$$\frac{\partial\Psi_7}{\partial\xi}=-\xi\eta(\eta+1)$$
$$\frac{\partial\Psi_8}{\partial\xi}=(2\xi-1)\left(1-\eta^2\right)/2$$
$$\frac{\partial\Psi_9}{\partial\xi}=-2\xi\left(1-\eta^2\right)$$

$$\frac{\partial\Psi_1}{\partial\eta}=\xi(1-\xi)(1-2\eta)/4$$
$$\frac{\partial\Psi_2}{\partial\eta}=\xi(1+\xi)(2\eta-1)/4$$
$$\frac{\partial\Psi_3}{\partial\eta}=\xi(1+\xi)(2\eta+1)/4$$
$$\frac{\partial\Psi_4}{\partial\eta}=\xi(1-\xi)(2\eta+1)/4$$
$$\frac{\partial\Psi_5}{\partial\eta}=\left(1-\xi^2\right)(2\eta-1)/2$$
$$\frac{\partial\Psi_6}{\partial\eta}=-\xi(1+\xi)\eta$$
$$\frac{\partial\Psi_7}{\partial\eta}=\left(1-\xi^2\right)(2\eta+1)/2$$
$$\frac{\partial\Psi_8}{\partial\eta}=\xi(1-\xi)\eta$$
$$\frac{\partial\Psi_9}{\partial\eta}=-2\eta\left(1-\xi^2\right)$$