Advanced Plasma Physics

Computational Projects

In these notes, we enclose the basic ideas for some interesting computational problems for those students who have not yet decided what project to tackle. This is a proposal that we may discuss in the classes and adapt to your own interests. Be aware that the most fruitful projects are those which we have fun with.

1 Ion-solitonic turbulence

As we have seen, ion waves deviate from their acoustic behaviour for short enough wavelengths, i.e. $k\lambda_{De} \sim 1$. This is a manifestation of the violation of the quasi-neutrality condition $n_e \simeq n_i$, and the ion waves become dispersive. When taking into account the non-linearity in the convection term in the fluid equations, dispersion competes with the non-linearity and gives origin to solitons, as described by the Kortweg-de Vries (KdV) equation. Solitons are non-linear waves that behave as particles, as they do not deform much.

In strong turbulence situations, we may expect that an array of solitons may be produced. As such, we could describe a turbulent ion plasma waves as a "gas" of solitons. As such, if you promote solitons to particles, they must admit their own kinetic description. This project aims at constructing a kinetic (Klimontovich) equation for KdV solitons. For that, you should start by writting the Lagrangian from which KdV solitons arise and then construct the two-soliton interaction. Once you have done that, you can use the variational method to express the Hamiltonian of each (x_i, v_i) pair of solitons and consider that the gas interacts via two-body potentials, then culminating in a Vlasov-like equation for solitons. You final task will be comparing your model with the numerical solutions of the KdV equations for a gas of solitons. A similar approach to dark-solitons in Bose-Einstein condensates can be found here, while a discussion for KdV solitons can be checked here.

2 Vlasvov-Poisson systems with particle-in-cell (PIC) methods

As we have seen in class, Vlasov equations can be obtained from the smoothening of the Klimontovich equation. One very simple and efficient approach to the Klimontovich equation (and eventually Vlasov) is based on the particle-in-cell (PIC) method. The latter consists in drawing randomly a finite number of particles and evolving them in time according to the Lorentz force. The phase-space is divided into cells, composed of a number or particles N_c, for which the fields are computed. Then, each "superparticle" (in a total of N_p) is evolved in agreement with the field resulting from the averaging inside each cell. Considering a 1d problem, $f = f(x, v, t) = \sum_k \delta(x - x_k(t))\delta(v - v_k(t))$, the relevant equations to be solved are

$$\dot{x}_k = v_k, \quad \dot{v}_k = \frac{q}{m} E(x_k, t).$$

Discretizing time in intervals of $t\Delta t$, the leap-frog method can be used to compute the quantities $t_n = \Delta t n$

$$v_k^{n+1/2} = v_k^n + \frac{q\Delta t}{2m} E_k^n(x_k^n), \quad x_k^{n+1} = x_k^n + \Delta t v_k^{n+1/2}, \quad v_k^{n+1} = v_k^{n+1/2} + \frac{q\Delta t}{2m} E_k^{n+1}(x_k^{n+1}).$$

You may use develop your own PIC code to investigate the validity of some of the features investigate in classes

a) Dispersion relation of Langmuir waves

b) Non-linear regimes of the Landau Damping

c) Saturation of two stream instabilities.

A nice starting point for the numerical implementation of PICs can be found here. A home-made PIC cold developed by our colleague Ricardo Fonseca at IST is the ZPIC, that you can download and run here.

3 Plasmon instabilities in dual-gate field-effect transistors (FETs)

As we have seen in class, a gated two-dimensional plasma - a field-effect transistor (FET) - can display instabilities due to the presence of asymmetric boundary conditions between the source and the drain (the celebrated Dyakonov-Shur instability).

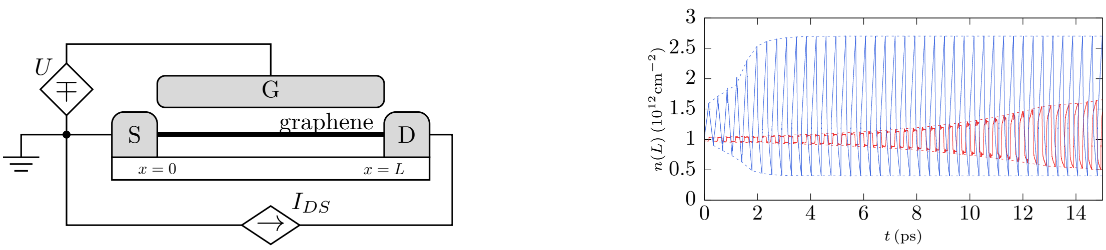

Figure 1: Left panel: the schematics of a field-effect transistor (FET) configuration. The asymmetric boundary conditions lead to the Dyakonov-Shur instability. Right panel: the 2D electron density at the drain as a function of time, displaying an initial linear growth followed by nonlinear saturation.

You could try to investigate another instability mechanism taking place in dual-gated FETs, i.e. in a configuration where the gate voltages vary periodically between two values, U_1 and U_2. We may expect that the successive wave reflections may be amplified, leading to the "plasma bloom" instability. You could modify TETHYS - the 2D hydrodynamical code developed by Pedro Cosme - and investigate the dynamical aspects of the plasma bloom instability. You may also develop an analytical model to predict some features, such as the instability growth rate, and compare with your code.

4 Magnetized plasmonic instabilities in Corbino FETS

Another interesting configuration to investigate instabilities in 2DEGs is the so-called *Corbino geometry*, consisting of a FET of cylindrical geometry. These configuration is particularly interesting for investigations of the viscosity of electrons in graphene, for example. Following the lines of the

Figure 2: Left panel: A field-effect transistor in the Corbino geometry. The scheme of Phys. Rev. Lett. 113, 235901 (2014) used to investigate the viscosity of graphene electrons. (Copyright)

authors M. Khavronin and collaborators in Phys. Rev. Applied 13, 064072, you could investigate the dynamics of the plasma in the presence of magnetic field. For that, you could be interested in adapting the 2D version of TETHYS.