Designing master surgery schedules with downstream unit integration via stochastic optimization

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Introduction

A master surgery schedule is a timetable for the operating theater.

- strategic decisions define the operating theater time blocks;
- surgical specialties/groups or surgeons are allocated to those blocks;
- tactical decision and cyclical.

	Mon		Tue		Wed		Thu		Fri	
Room	1	2	1	2	1	2	1	2	1	2
M	SS_4	SS_2	SS_1	SS_3	SS_4	SS_2	SS_1	SS_3	SS_4	SS_4
A	SS_1	SS_2	SS_1	SS_3	SS_1	SS_1	SS_1	SS_3	SS_4	SS_3

Table: Example of a master surgery schedule - one week, two ORs, two shifts.

Introduction

Tactical decision:

- demand and availability for each specialty is known;
- surgeries are not scheduled at this point, however
- predicted impact should be taken into account.

Downstream unit integration:

- wards, ICU, ...;
- overutilization leads to cancellations or early discharges;
- strong impact on patient safety;
- historical data can be used to predict utilization rates.

The master surgery scheduling problem

Formal definition

Master surgery (block) scheduling problem:

- assign surgical specialties to operating theater time blocks;
- minimize or maximize some objective;
- subject to demand and availability;
- subject to cyclical nature;
- subject to ...

Sets and indices	
$t \in T$	days in the master surgery schedule cycle
$s \in S$	surgical specialties
$b \in B$	operating theater time blocks (day, room and shift)
Parameters	
p_s	expected benefit for assigning each surgical specialty
d_s	demand of each surgical specialty
a_s	availability of each surgical specialty
Decision variables	
x_{sb}	1 if surgical specialty s is assigned to block b; 0 otherwise

Table: Notation (part 1)

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{s \in S} \sum_{b \in B} p_s x_{sb} \\
\text{subject to:} \quad & \sum_{s \in S} x_{sb} \leq 1 && \forall b \in B, \\
& \sum_{b \in B} x_{sb} \geq d_s && \forall s \in S, \\
& \sum_{b \in B} x_{sb} \leq a_s && \forall s \in S, \\
& x_{sb} \in \{0, 1\} && \forall s \in S,\ \forall b \in B.
\end{aligned}
$$

Downstream unit integration

Surgical specialty assigned to a block $\Rightarrow$ patients requiring beds. Instead of "blindly" designing the master surgery schedule, we can take this certain uncertainty into account.

Sets and indices	
$l \in L$	bed types
$\theta \in \{0, 1, ..., \Theta\}$	"days ago"

Table: Notation (part 2)

We define $r_{lts\theta}$ as the number of beds of type l required in day t by surgical specialty s assigned to a block θ days ago. These parameters are random variables with a probability distribution that may be estimated using historical data.

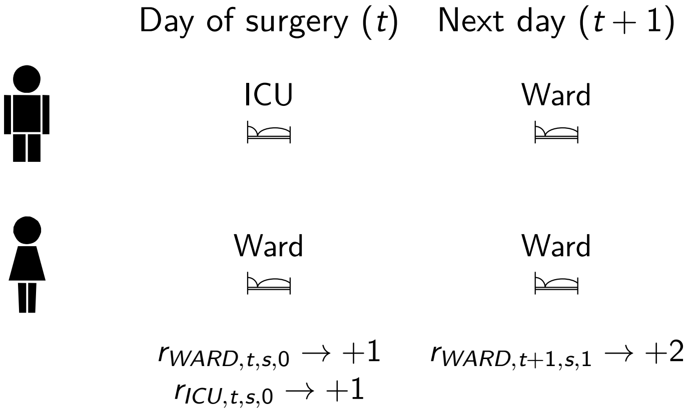

Figure: Example

The master surgery schedule is cyclical, but bed requirements are not:

- two cycles: T and $U = kT$, where $k \in \mathbb{N}$;
- $k = +\infty$ would be more realistic (non-cyclical);
- "sufficiently large" k should be enough.

For every $i = 1, \ldots, k$, let U_i be the i-th time period of length T.

- for each time period we have bed requirements $r^i_{lts\theta}$;
- identically distributed to $r_{lts\theta}$.

Stochastic optimization model

Additional notation

Subsets	
$B(u-\theta)$	blocks of day $(u-\theta) \mod T$, with $u \in U$
Parameters	
C_l	number of beds of type l available
α	penalty for every bed used above capacity
γ	maximum overutilization
Decision variables	
y_{lu}	number of extra beds of type l used in day u

Table: Notation (part 3)

Recourse objective function and bed capacity constraints

The benefit parameter "naturally" guarantees that operating rooms are not underutilized ⇒ recourse objective function penalizes overutilization:

$$\text{Minimize} \sum_{l \in L} \sum_{u \in U} \alpha \, y_{lu}$$

Bed requirements roll over to the following days ⇒ add them all up and limit them by the corresponding capacity:

$$\sum_{s \in S} \sum_{\theta=0}^{\Theta} \sum_{b \in B(u-\theta)} r^{i}_{lus\theta} x_{sb} \leq C_l + y_{lu} \qquad \forall l \in L, \, \forall i \in \{1, \ldots, k\}, \, \forall u \in U_i$$

Most hospitals have dedicated operating room time for non-elective patients:

- whole operating rooms or divided among several;
- pre-planned in the master surgery schedule;
- random variables $\overline{r}^{i}_{lt\theta}$.

Assuming that every urgent patient is admitted:

$$\sum_{s\in S}\sum_{\theta=0}^{\Theta}\sum_{b\in B(u-\theta)} r^{i}_{lus\theta}x_{sb} + \sum_{\theta=0}^{\Theta}\overline{r}^{i}_{lu\theta} \leq C_l + y_{lu} \quad \forall l \in L,$$

$$\forall i \in \{1,\ldots,k\},\ \forall u \in U_i$$

$$\begin{aligned}
\text{Maximize} \quad & \sum_{s\in S}\sum_{b\in B} p_s x_{sb} - E[Q(x,r)] \\
\text{subject to:} \quad & \sum_{s\in S} x_{sb} \leq 1 && \forall b \in B, \\
& \sum_{b\in B} x_{sb} \geq d_s && \forall s \in S, \\
& \sum_{b\in B} x_{sb} \leq a_s && \forall s \in S, \\
& x_{sb} \in \{0,1\} && \forall s \in S,\ \forall b \in B.
\end{aligned}$$

where $Q(x, r)$ is the optimal value of the second stage model

$$\begin{aligned}
\text{Minimize} \quad & \sum_{l\in L}\sum_{u\in U} \alpha\, y_{lu} \\
\text{subject to:} \quad & \sum_{s\in S}\sum_{\theta=0}^{\Theta}\sum_{b\in B(u-\theta)} r^i_{lus\theta} x_{sb} + \sum_{\theta=0}^{\Theta} \bar{r}^i_{lu\theta} \leq C_l + y_{lu} && \forall l \in L, \forall i \in \{1,\dots,k\},\ \forall u \in U_i \\
& 0 \leq y_{lu} \leq \gamma && \forall l \in L,\ \forall u \in U
\end{aligned}$$

Preliminary (proof of concept) results

Generating patients

For every i-th time period of length T and for every surgical specialty we generate patients (elective and non-elective) which will require a bed:

- the number of patients;
- their paths (ICU only, ICU then ward, ward only);
- their length of stay in the ICU and/or ward;

$\Rightarrow$ coefficients of the x variables in the bed requirement constraints.

Historical data is fundamental to estimate probabilities.

- results so far are based on randomly generated data;
- we are now preparing real data.

Summary of the results

				Ward OvUt		ICU OvUt	
α	γ	Time (s)	Benefit	Max	Avg	Max	Avg
1	0	6	4523	0	0	0	0
1	1	374	4635	1	0.0000412	1	0.000412
1	2	546	4848	2	0.000343	2	0.000962
1	3	49	5143	3	0.000989	3	0.001786
3	0	5	4523	0	0	0	0
3	1	308	4632	1	0.0000412	1	0.000275
3	2	3058	4830	2	0.000247	2	0.000824
3	3	42	5129	3	0.000907	3	0.001374

Table: Preliminary results ($T = 7$, $|S| = 10$, $|B| = 10 \times 2$, $k = 52 \times 20$)

Solutions for one sample only $\Rightarrow$ use Sample Average Approximation.

Conclusion

Conclusions and future work

We present a stochastic optimization model to integrate downstream units in the design of master surgery schedules.

- improve patient safety;
- improve the efficiency of the operating theater;
- different approach compared to the literature;
- solvable sample size can be improved;
- requires real (good) data.

Future work:

- subdividing surgical specialties for a more accurate prediction;
- solving bigger sample sizes and faster (e.g. decomposition);
- real data.

Thank you for your attention!

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