# Lecture 5: Recurrent Neural Networks

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# Getting targets when modeling sequences

- When applying machine learning to sequences, we often want to turn an input sequence into an output sequence that lives in a different domain.
  - *E. g.* turn a sequence of sound pressures into a sequence of word identities.
- When there is no separate target sequence, we can get a teaching signal by trying to predict the next term in the input sequence.
- The target output sequence is the input sequence with an advance of 1 step.
  - For temporal sequences there is a natural order for the predictions.

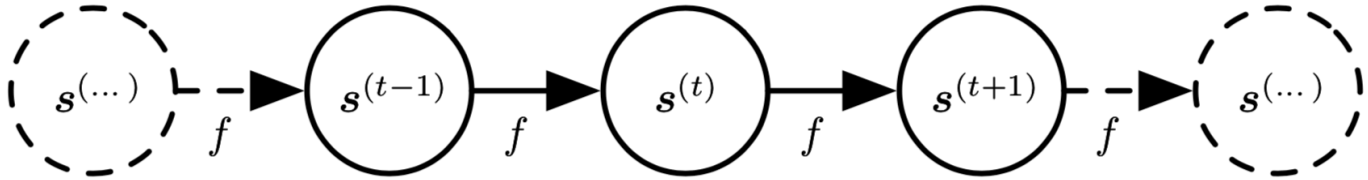
# Recurrent Networks

- Recurrent neural networks or RNNs are a family of neural networks for processing sequential data
- Recurrent neural network is a neural network that is specialised for processing a sequence of values $x^{(1)}, \cdots, x^{(\tau)}$
- Recurrent networks can scale to much longer sequences than would be practical for networks without sequence-based specialisation.
- Most recurrent networks can also process sequences of variable length.

- A traditional fully connected feedforward network would have separate parameters for each input feature
  - It would need to learn for example all of the rules of the language separately at each position in the sentence.
- By comparison, a recurrent neural network shares the same weights across several time steps.
- Each member of the output is a function of the previous members of the output.
- Each member of the output is produced using the same update rule applied to the previous outputs.

# Classical Dynamical System

- Unfolding a recursive or recurrent computation into a computational graph that has a repetitive structure, typically corresponding to a chain of events.
- Unfolding this graph results in the sharing of parameters across a deep network structure.



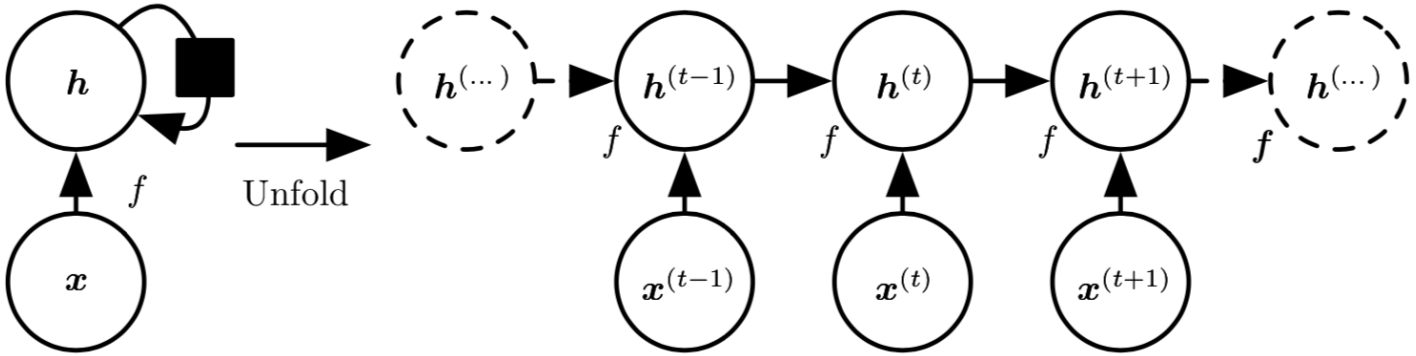

Dynamical system

$$\mathbf{s}^{(t)} = f(\mathbf{s}^{(t-1)}; \mathbf{w}),$$

with $t = 3$

$$\mathbf{s}^{(3)} = f(\mathbf{s}^{(2)}; \mathbf{w}) = f(f(\mathbf{s}^{(1)}; \mathbf{w}); \mathbf{w})$$

# Unfolding Computational Graphs




Variable $\mathbf{h}$ represents the state

$$\mathbf{h}^{(t)} = f(\mathbf{h}^{(t-1)}; \mathbf{w}),$$

It maps an arbitrary length sequence

$$(\mathbf{x}^{(t)}, \mathbf{x}^{(t-1)}, \mathbf{x}^{(t-2)}, \cdots, \mathbf{x}^{(2)}, \mathbf{x}^{(1)})$$

to a fixed length vector $\mathbf{h}^{(t)}$

# Statistical Language Modeling

- For example, if the RNN is used in statistical language modeling, typically to predict the next word given previous words, it may not be necessary to store all of the information in the input sequence up to time *t*,

- .. but rather only enough information to predict the rest of the sentence.

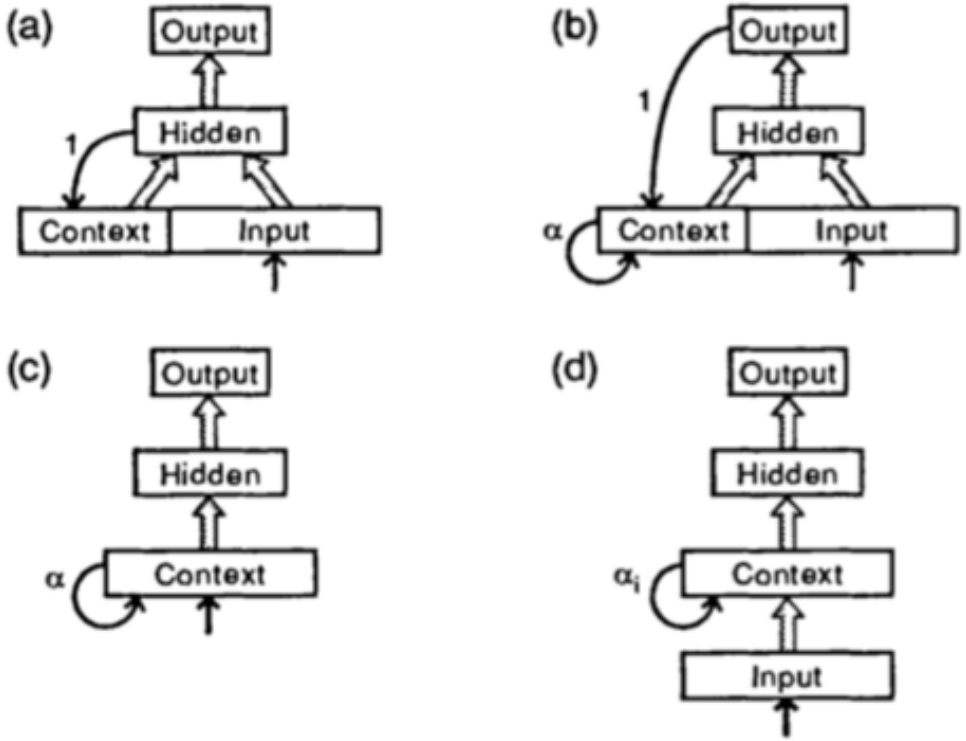
- We can represent the unfolded recurrence after $t$ steps with a function $g^t$

$$\boldsymbol{h}^{(t)} = g^{(t)}(\boldsymbol{x}^{(t)}, \boldsymbol{x}^{(t-1)}, \boldsymbol{x}^{(t-2)}, \cdots, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(1)}) = f(\boldsymbol{h}^{(t-1)}, \boldsymbol{x}^{(t)}; \boldsymbol{w})$$

- Regardless of the sequence length, the learned model always has the same input size, because it is specified in terms of transition from one state to another state, rather than specified in terms of a variable-length history of states.

- It is possible to use the same transition function f with the same parameters at every time step

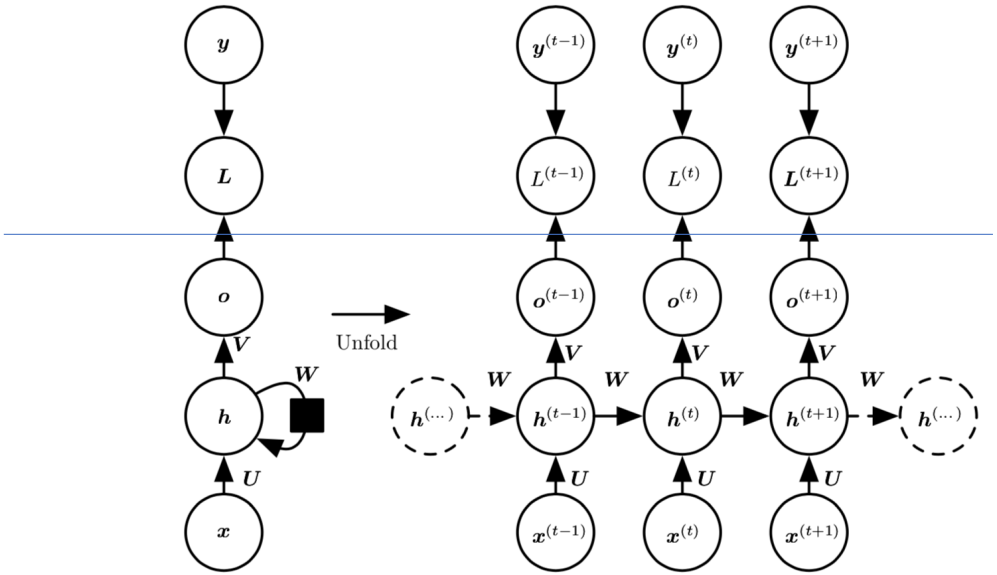
- These two factors make it possible to learn a single model *f* that operates on all time steps and all sequence lengths, rather than needing to learn a separate model $g^{(t)}$ for all possible time steps.



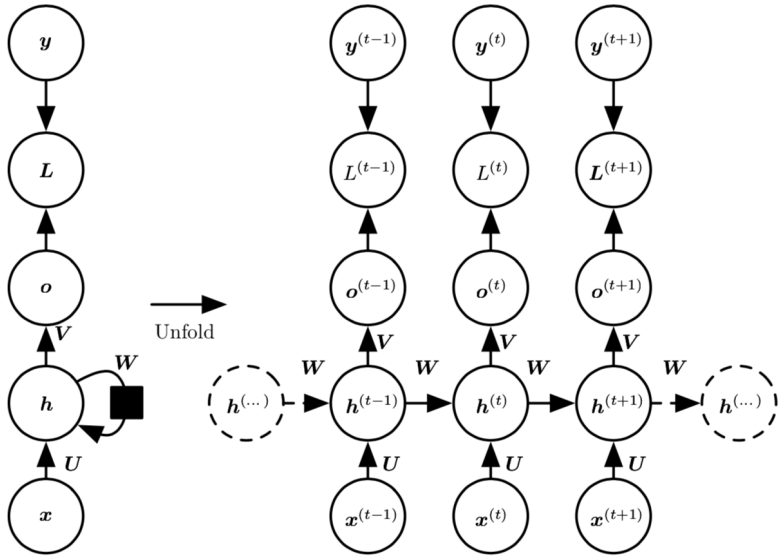

(a) Elman Network, (b) Jordan Network

# Recurrent Neural Networks

- Recurrent networks that produce an output at each time step and have recurrent connections between hidden units, Vanilla RNN or Elman RNN



# Recurrent Hidden Units




With specified

$$\mathbf{h}^{(0)} = initial\ state$$

For each time step $t = 1$ to $t = \tau$

$$\mathbf{a}^{(t)} = \mathbf{b} + W \cdot \mathbf{h}^{(t-1)} + U \cdot \mathbf{x}^{(t)}$$

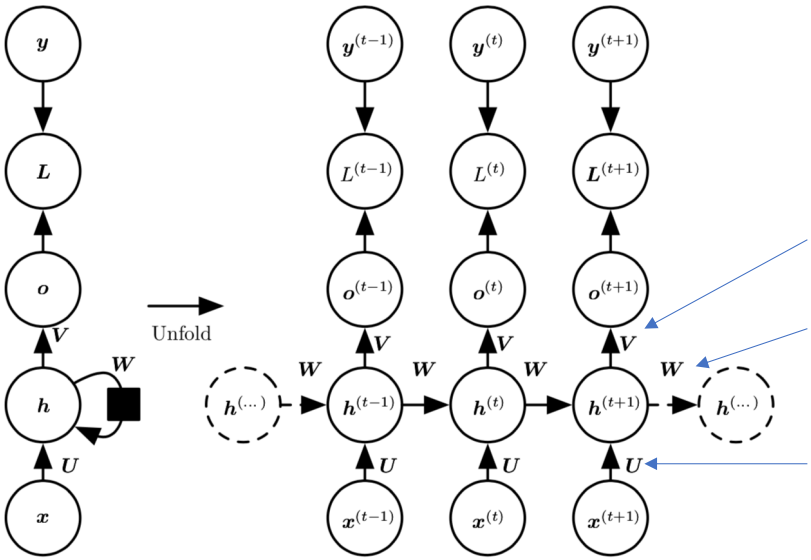
$$\mathbf{h}^{(t)} = tanh(\mathbf{a}^{(t)})$$

$$\mathbf{o}^{(t)} = \mathbf{c} + V \cdot \mathbf{h}^{(t)}$$

$$\mathbf{y}^{(t)} = softmax(\mathbf{o}^{(t)})$$

with $\mathbf{b}$ and $\mathbf{c}$ being the bias vectors and $U, V, W$ the weight matrices

# Recurrent Hidden Units

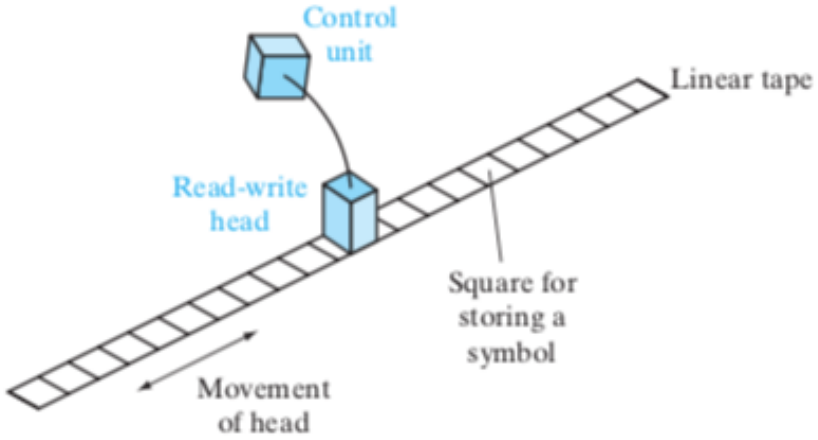



*V* weight matrix hidden to output layer

*W* weight matrix hidden to hidden layer *(new)*. Input is the preceding state.

*U* weight matrix input to hidden layer

# Computational Power




- The recurrent neural network is universal in the sense that any function computable by a Turing machine can be computed by such a recurrent network of a finite size.

Eduardo Daniel Sontag is an American mathematician, and Distinguished University Professor at Northeastern University,

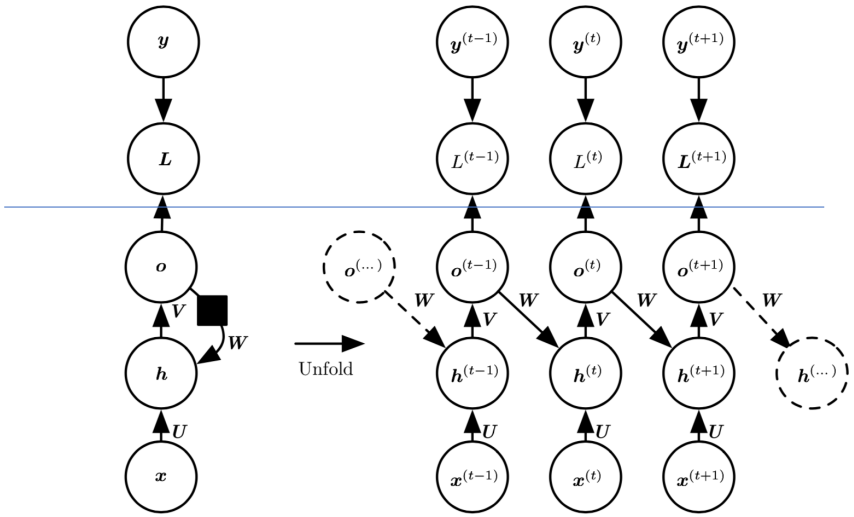
Hava Siegelmann is a professor of computer science, and a world leader in the fields of Lifelong Learning, Artificial Intelligence, Machine Learning, Neural Networks, and Computational Neuroscience.

- Siegelmann and Sontag, 1991

**Theorem:**

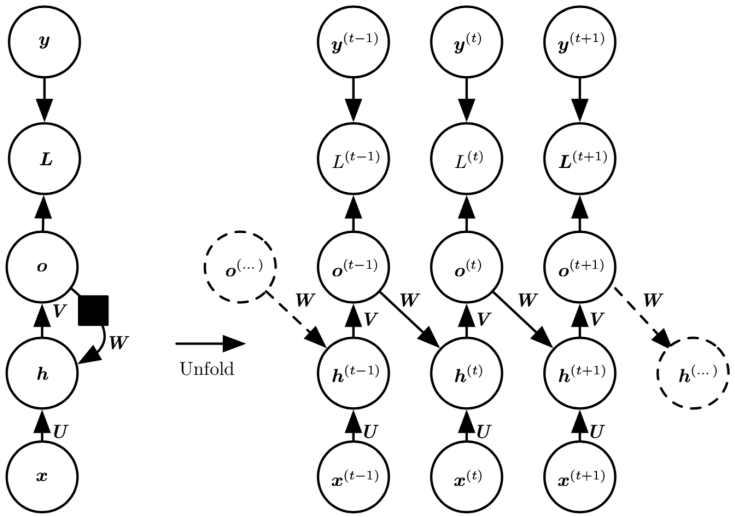
- *All Turing machines may be simulated by fully connected recurrent networks built on neurons with sigmoidal activation functions.*

# Recurrence through only the Output



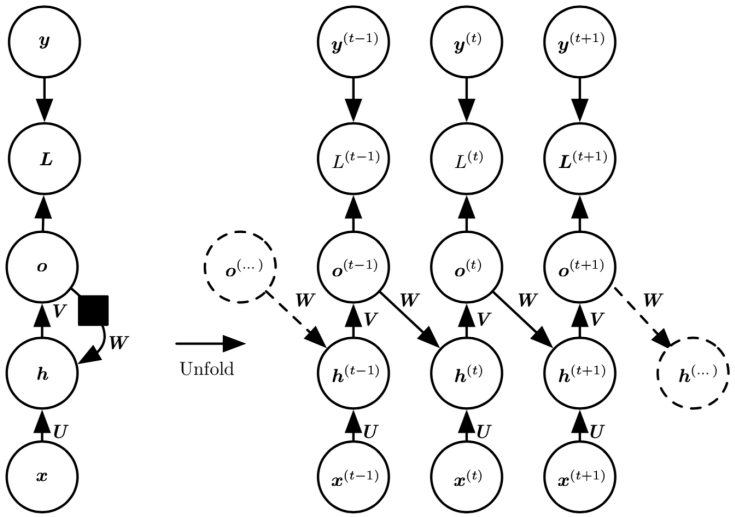

- Recurrent networks that produce an output at each time step and have recurrent connections only from the output at one time step to the hidden units at the next time step

# Recurrence through only the Output



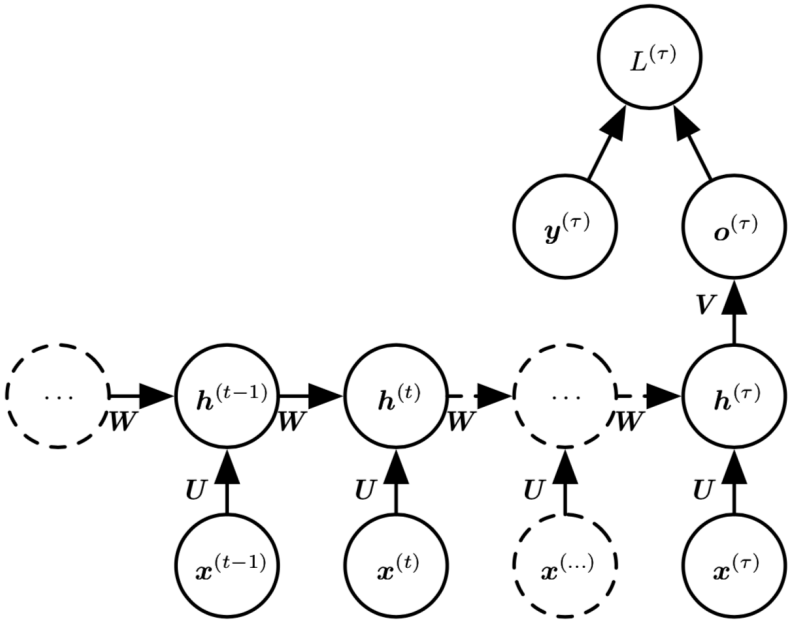

- Less powerful because it lacks hidden-to-hidden recurrent connections.
- For example, it cannot simulate a universal Turing machine.

# Recurrence through only the Output




- It requires that the output units capture all of the information about the past that the network will use to predict the future.

# Sequence Input, Single Output




Recurrent networks with recurrent connections between hidden units, that read an entire sequence and then produce a single output

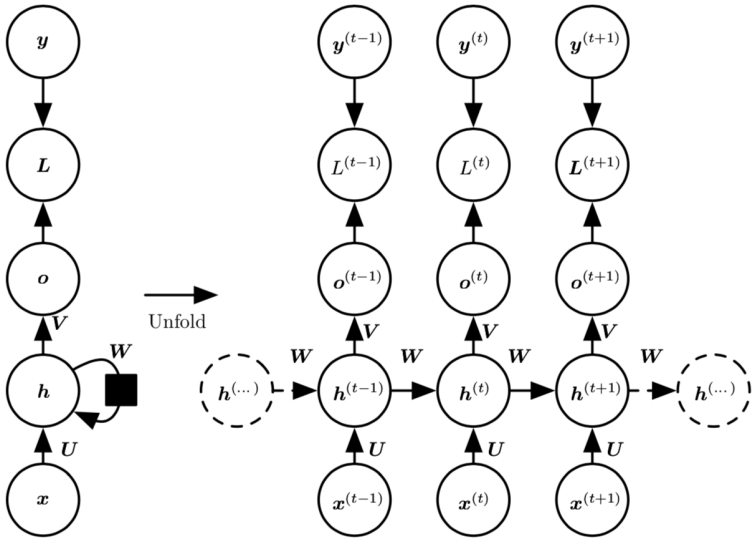
# Backpropagation Trough Time

- The back-propagation algorithm applied to the unrolled graph with $O(\tau)$ cost is called back-propagation through time
- The training data for a recurrent neural network is an ordered sequence of τ input-output pairs
- Same idea as training over training set $D$, now the training set is $D \cdot \tau$ with an additional input is the preceding state.

# Backpropagation Trough Time

- The runtime is $O(\tau)$ and cannot be reduced by parallelization because the forward propagation graph is inherently sequential; each time step may only be computed after the previous one.
- BPTT begins by unfolding a recurrent neural network in time. The unfolded network contains $\tau$ inputs and outputs, but every copy of the network shares the **same** parameters.
- Then the backpropagation algorithm is used to find the gradient of the cost with respect to all the network parameters

# Backpropagation Trough Time is THE usual Backpropagation!




With specified

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For each time step $t = 1$ to $t = \tau$

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$$\mathbf{y}^{(t)} = softmax(\mathbf{o}^{(t)})$$

- The back-propagation algorithm applied to the unrolled graph with *O(τ)* cost is called back-propagation through time

# The Challenge

- BPTT has difficulty with local optima.
- With recurrent neural networks, local optima are a much more significant problem than with feed-forward neural networks
- The recurrent feedback in such networks tends to create chaotic responses in the error surface which cause local optima to occur frequently, and in poor locations on the error surface.
- The basic problem is that gradients propagated over many stages tend to either vanish (most of the time) or explode (rarely, but with much damage to the optimization).

# The Challenge

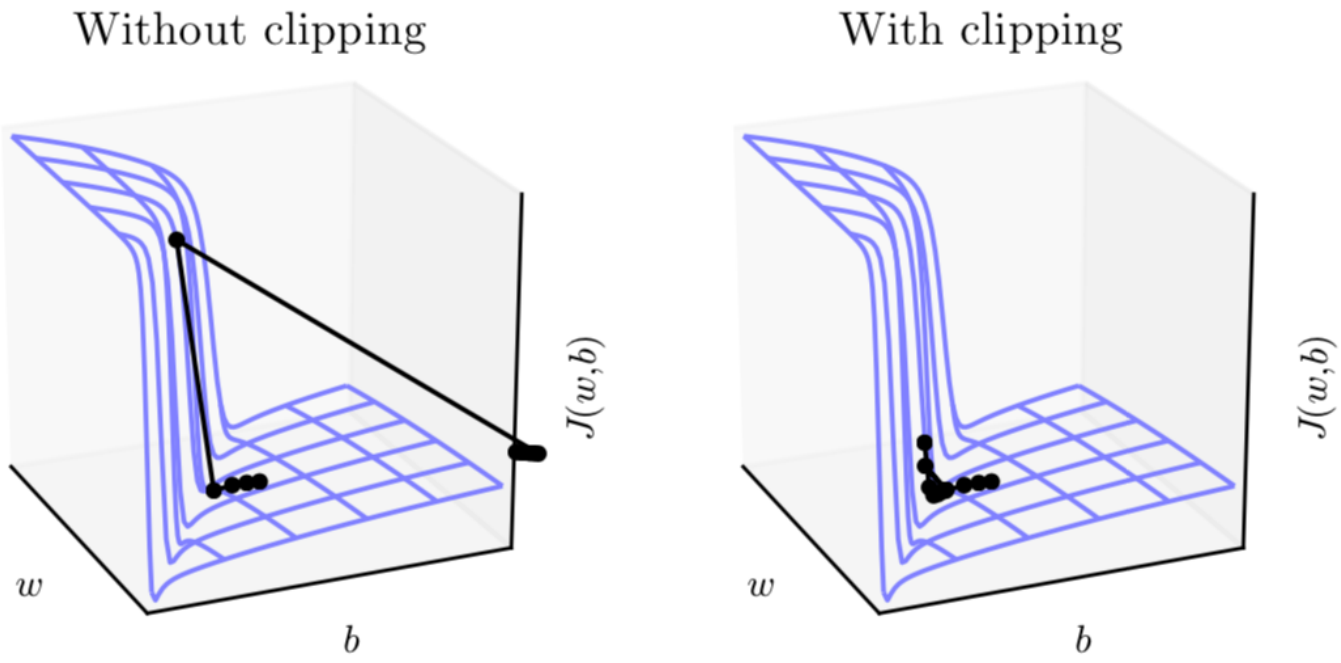
- Even if we assume that the parameters are such that the recurrent network is stable (can store memories, with gradients not exploding), the difficulty with long-term dependencies arises from the exponentially smaller weights given to long-term interactions

- Clip the norm $\|\boldsymbol{g}\|$ of the gradient $\boldsymbol{g}$ with $v$ is the norm threshold. The gradient is normalised and multiplied with the scalar $v$

# Gradient Clipping

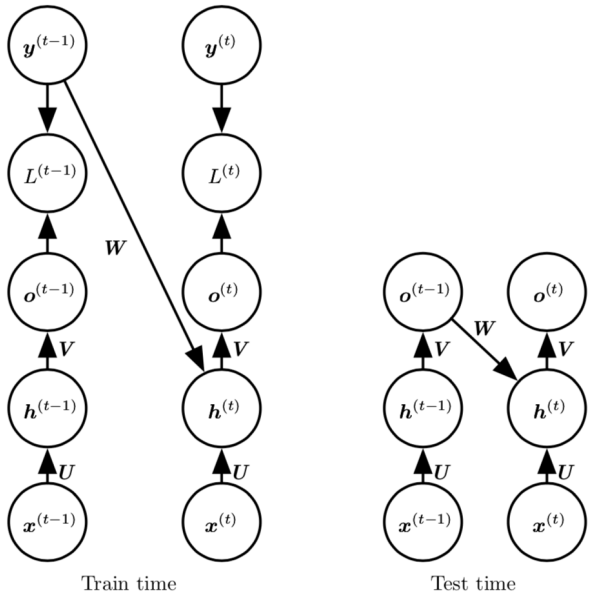
$$\text{if } ||\boldsymbol{g}|| > v$$

$$\boldsymbol{g} \leftarrow \frac{\boldsymbol{g}v}{||\boldsymbol{g}||}$$



# Teacher Forcing and Networks with Output Recurrence

- Teacher Forcing is applicable to RNNs that have connections from their out-put to their hidden states at the next time step




$$\log p\left(\boldsymbol{y}^{(1)}, \boldsymbol{y}^{(2)} \mid \boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}\right)$$
$$= \log p\left(\boldsymbol{y}^{(2)} \mid \boldsymbol{y}^{(1)}, \boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}\right) + \log p\left(\boldsymbol{y}^{(1)} \mid \boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}\right)$$

# Deep Recurrent Networks

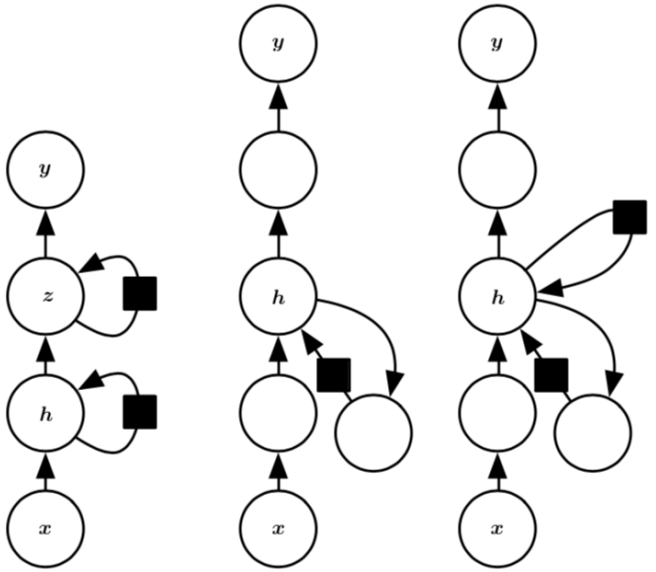
- The computation in most RNNs can be decomposed into three blocks of parameters and associated transformations:
  - from the input to the hidden state,
  - from the previous hidden state to the next hidden state,
  - from the hidden state to the output.

- Would it be advantageous to introduce depth in each of these operations?
  - *The experimental evidence is in agreement with the idea that we need enough depth in order to perform the required mappings.*

# Deep RNNs

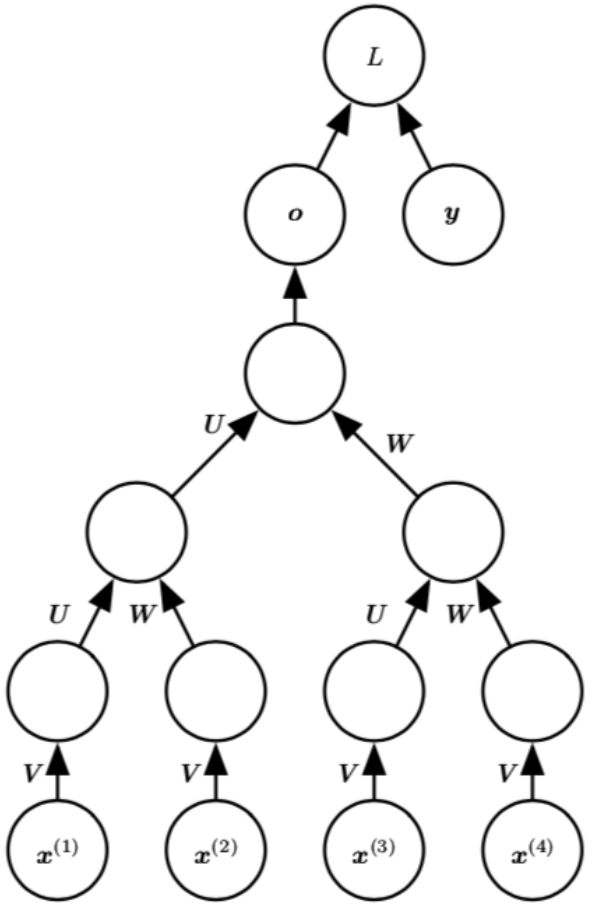
A recurrent neural network can be made deep in many ways

- The hidden recurrent state can be broken down into groups organized hierarchically.
- Deeper computation (e.g., an MLP) can be introduced in the input-to-hidden, hidden-to-hidden and hidden-to-output parts
- This may lengthen the shortest path linking different time steps
  The path-lengthening effect can be mitigated by introducing skip connec- tions.

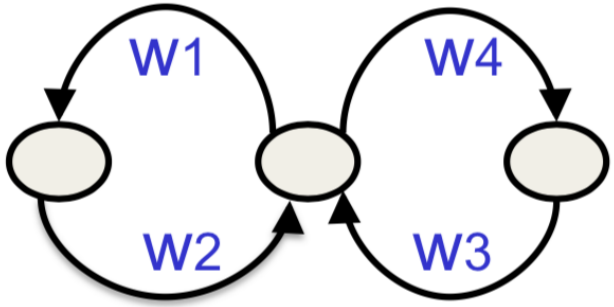


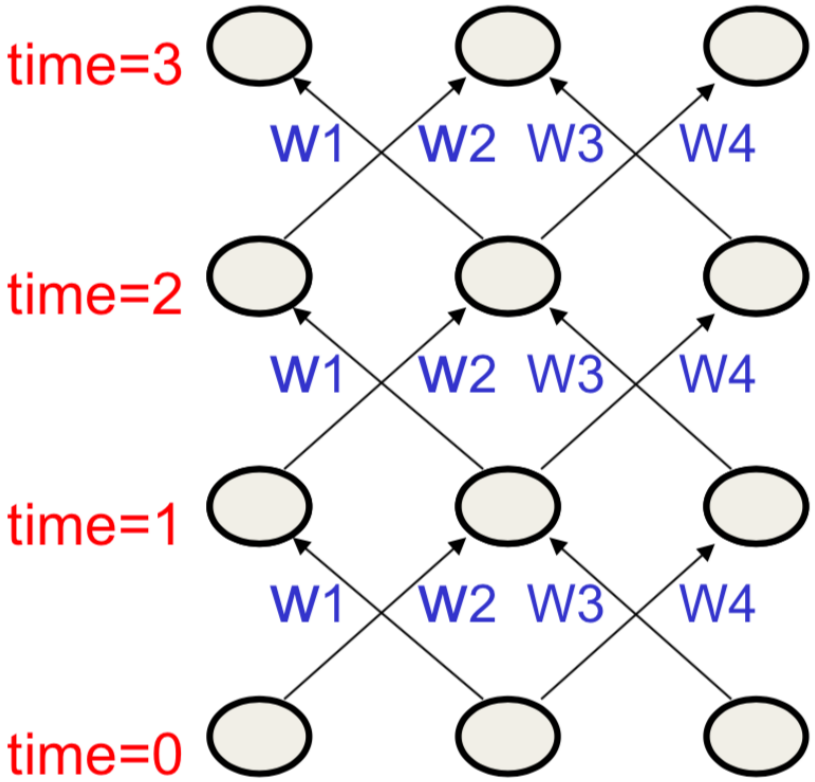
# Recursive Network

- A recursive network has a computational graph that generalizes that of the recurrent network from a chain to a tree



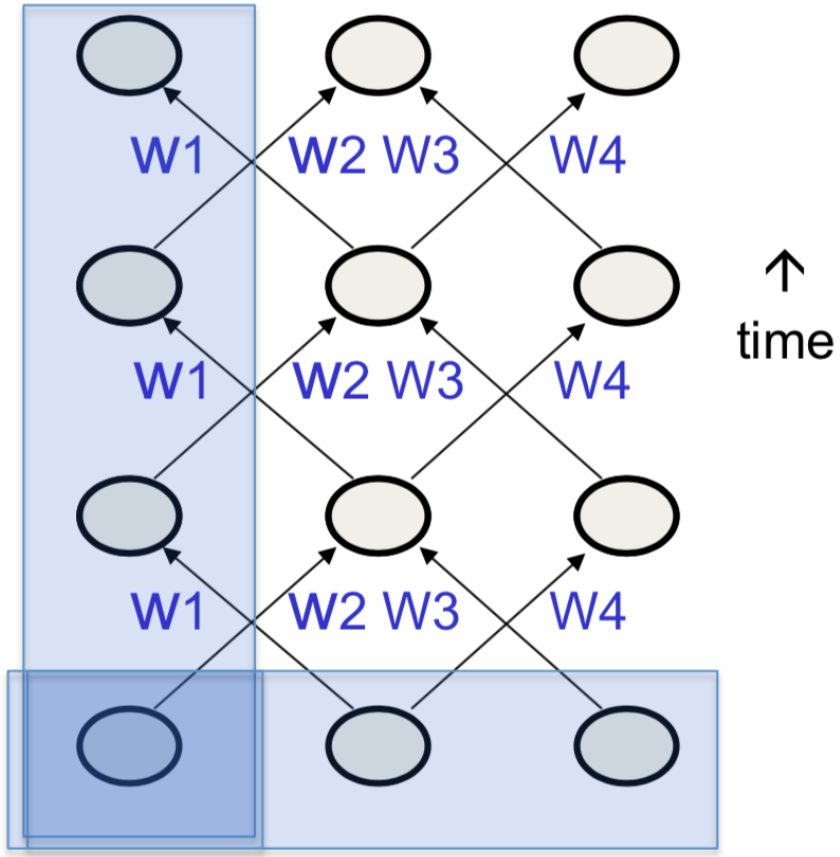
# The equivalence between feedforward nets and recurrent nets



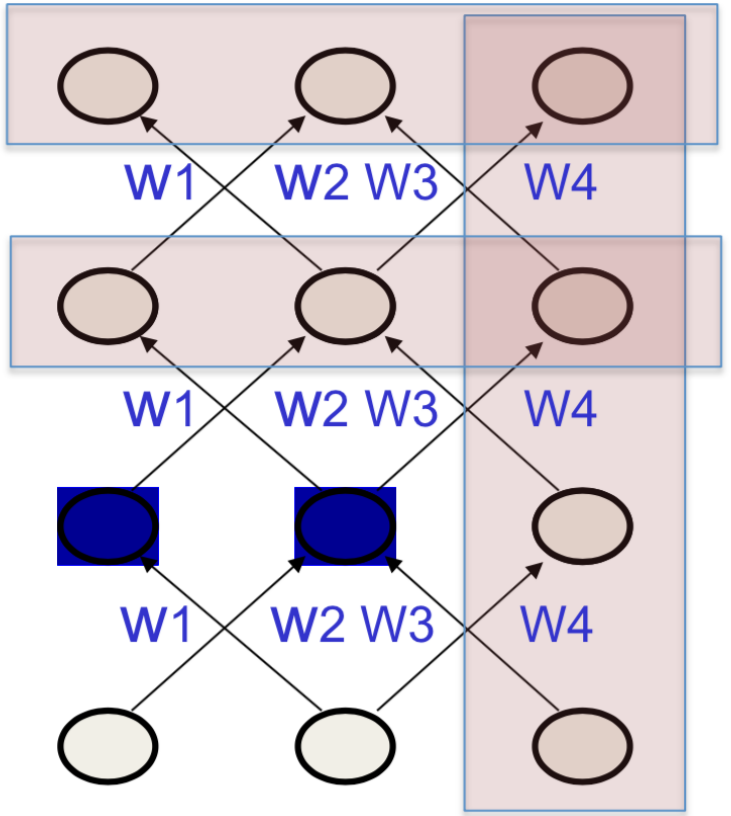
# Providing input to recurrent networks

- We can specify inputs in several ways:
  - Specify the initial states of all the units.
  - Specify the initial states of a subset of the units.
  - Specify the states of the same subset of the units at every time step.
- This is the natural way to model most sequential data.

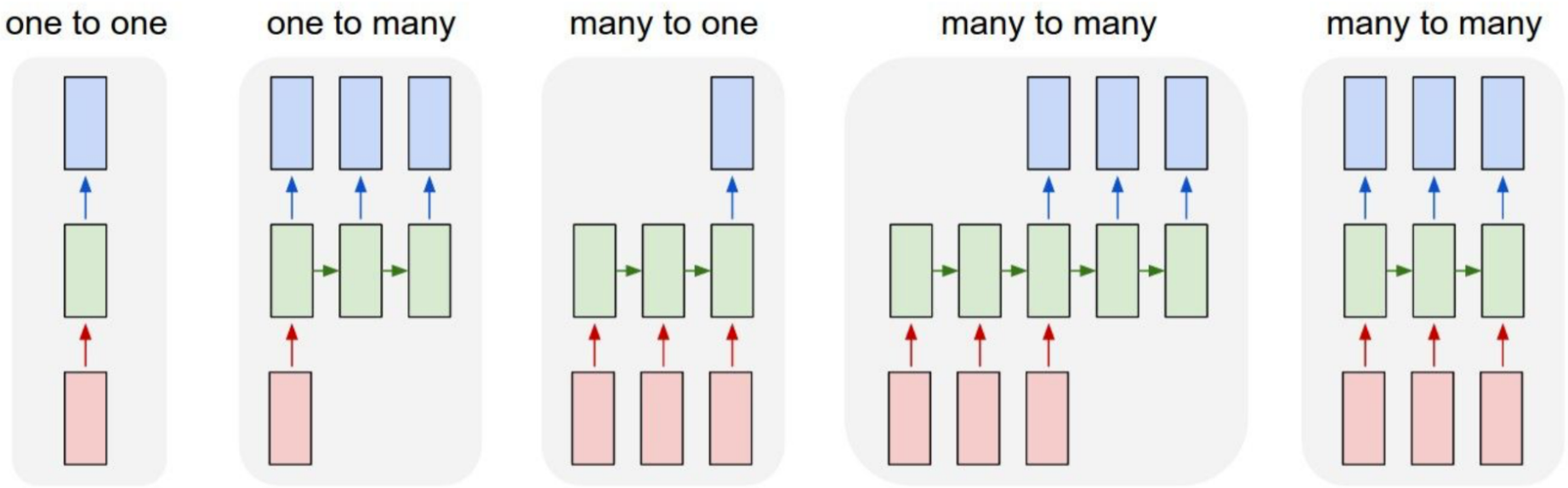


# Teaching signals for recurrent networks

- We can specify targets in several ways:
  - Specify desired final activities of all the units
  - Specify desired activities of all units for the last few steps
    - Good for learning attractors
    - It is easy to add in extra error derivatives as we backpropagate.
  - Specify the desired activity of a subset of the units.
- The other units are input or hidden units.



# Recurrent Neural Networks: Process Sequences




Vanilla Neural Networks

e.g. Image Captioning image -> sequence of words

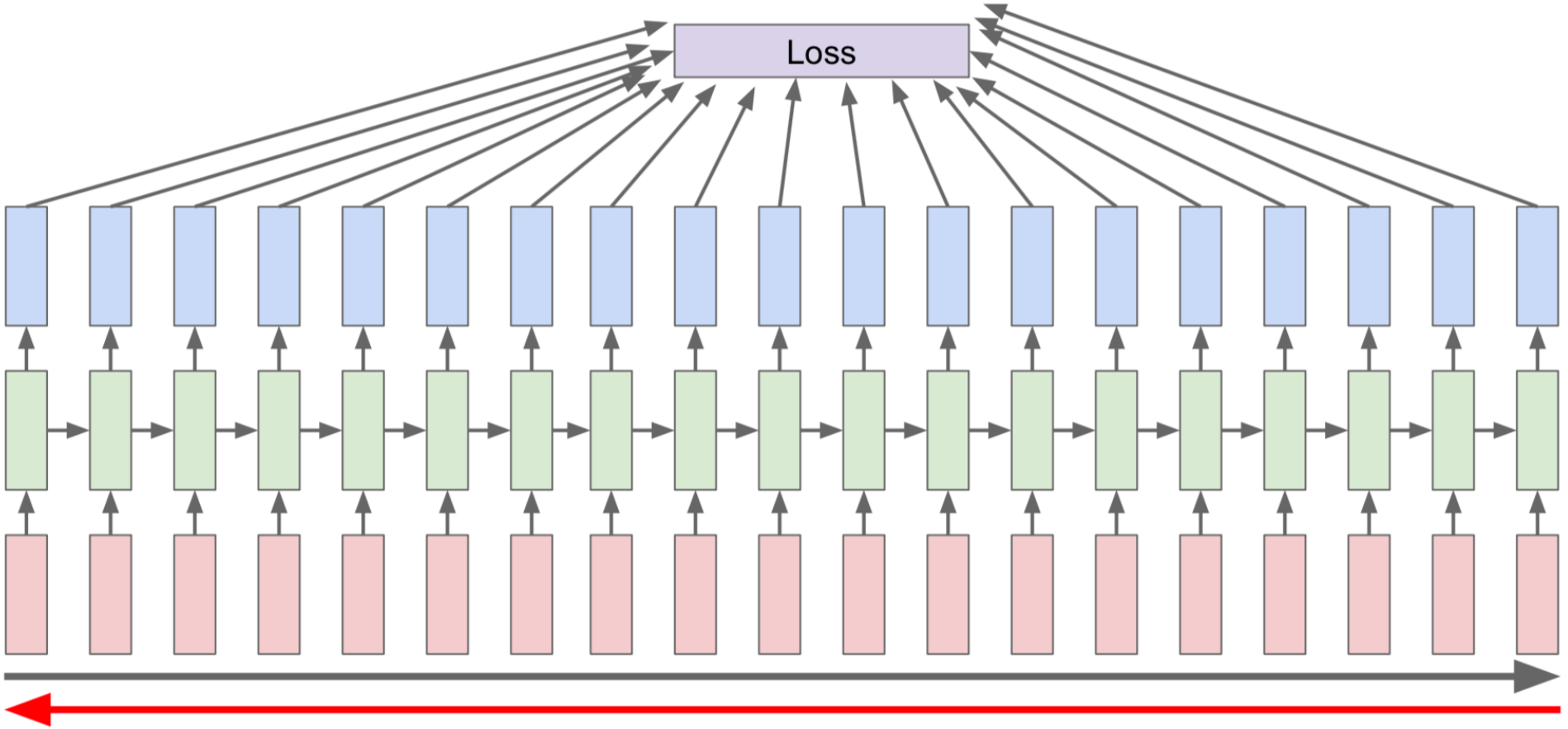
e.g. Sentiment Classification sequence of words -> sentiment

e.g. Machine Translation seq of words -> seq of words

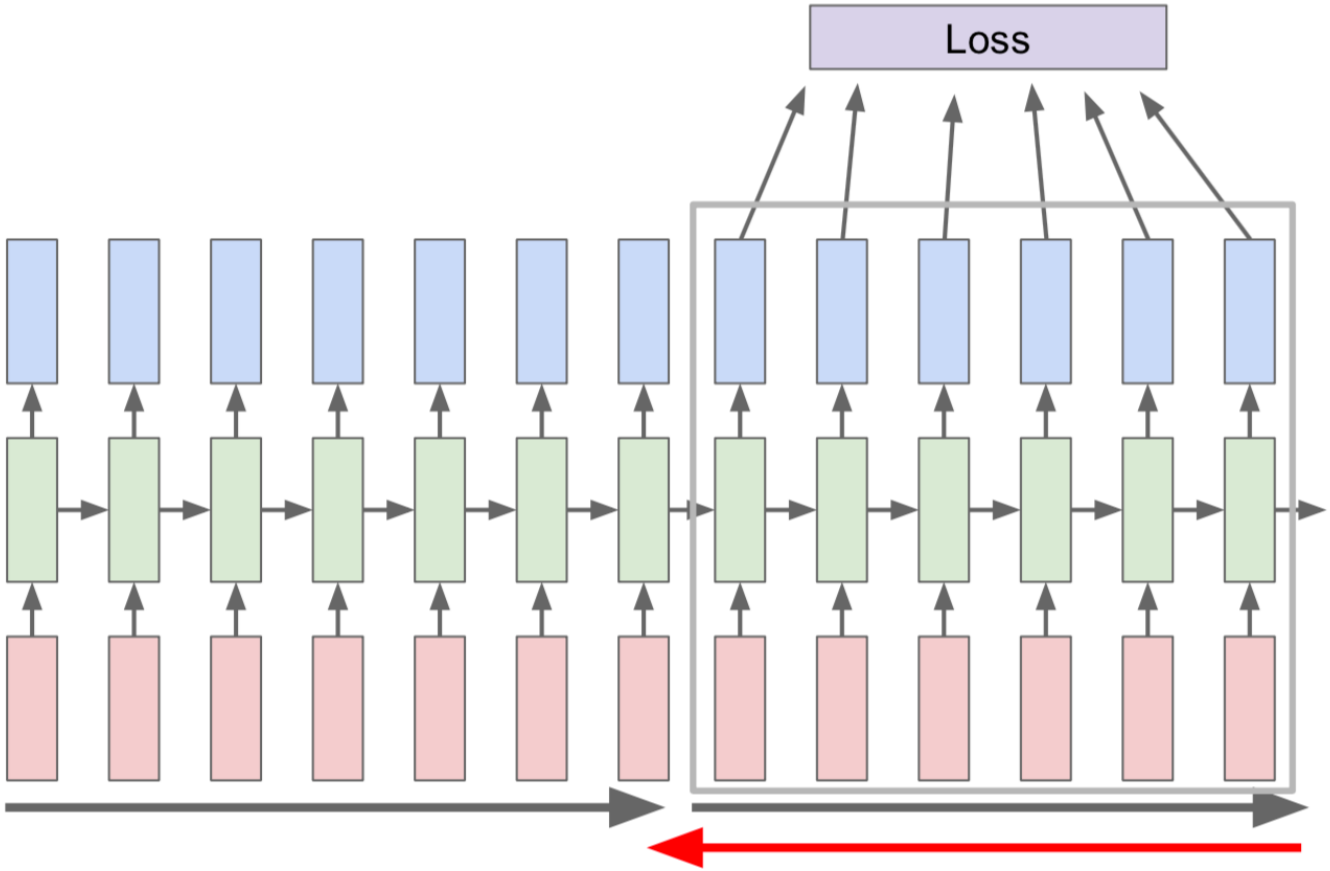
e.g. Video classification on frame level

# Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

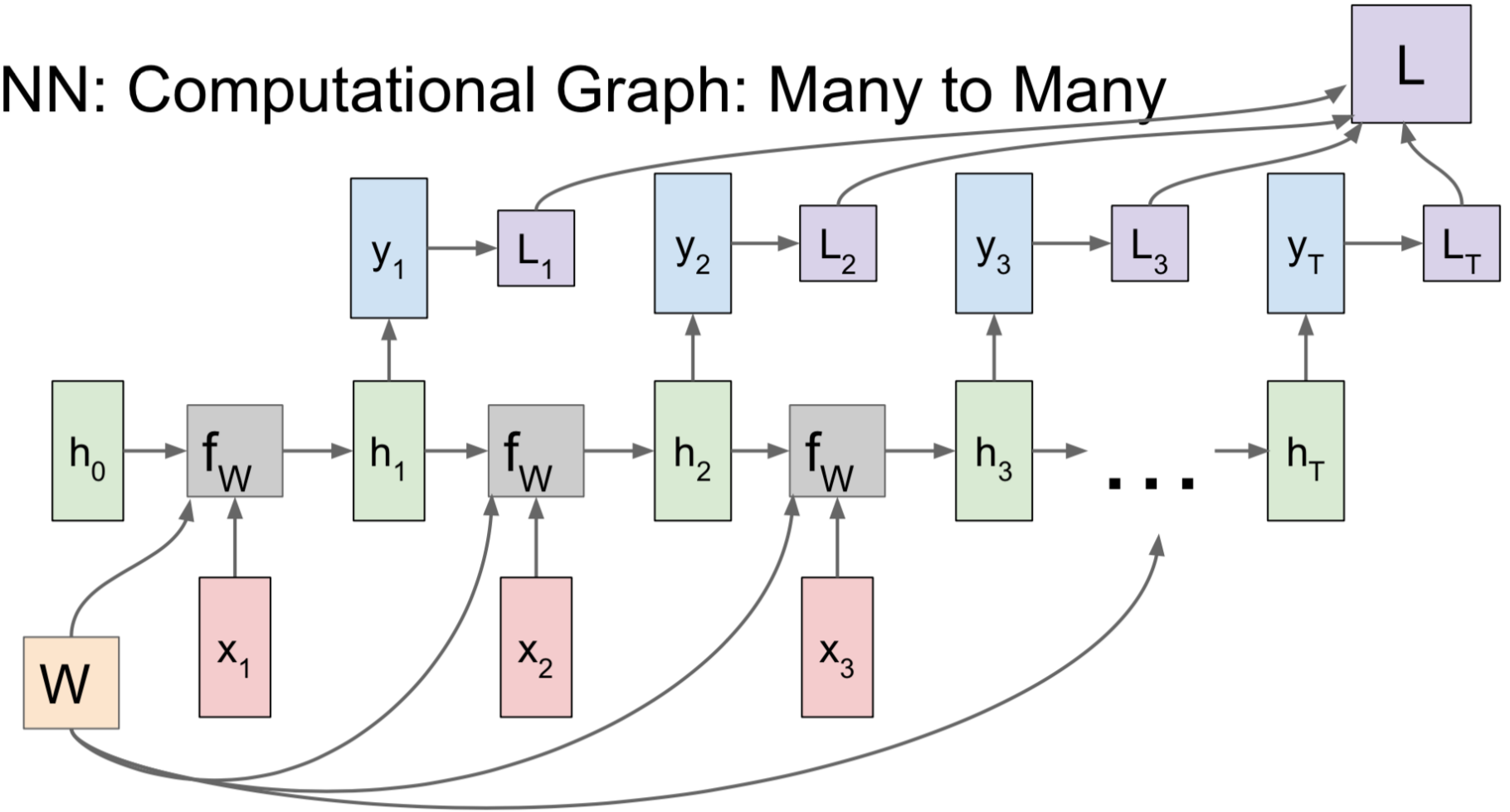


# **Truncated** Backpropagation through time




Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

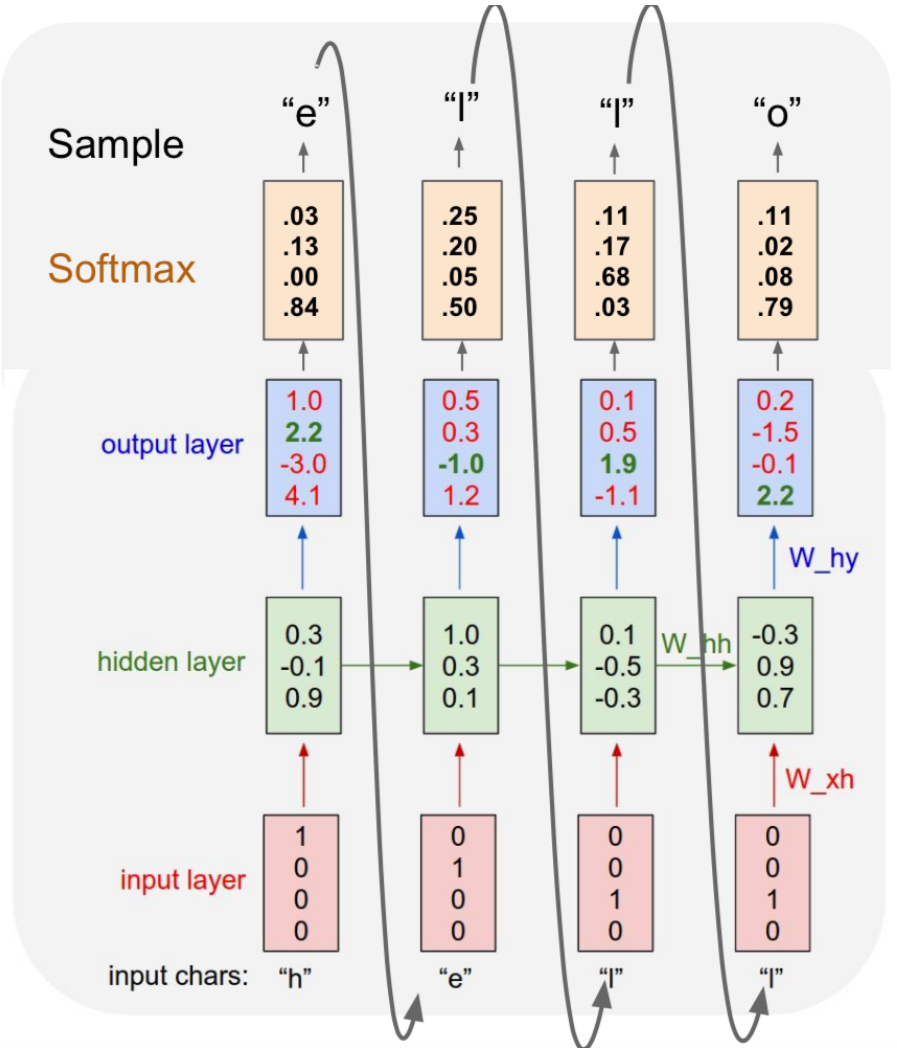
# RNN: Computational Graph: Many to Many



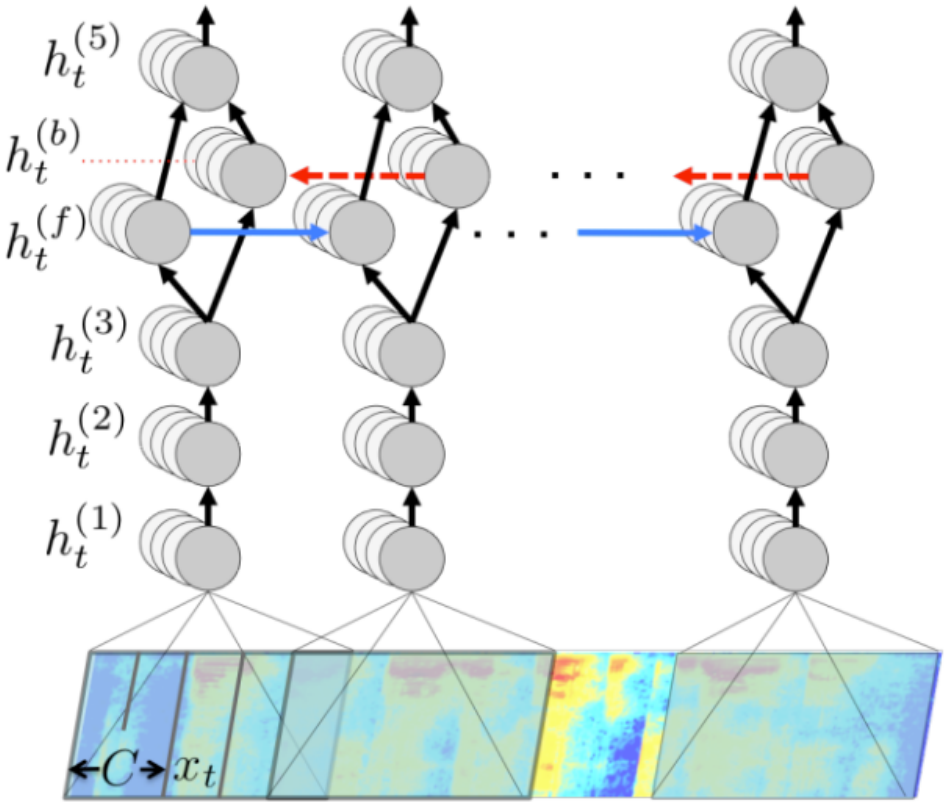
# Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

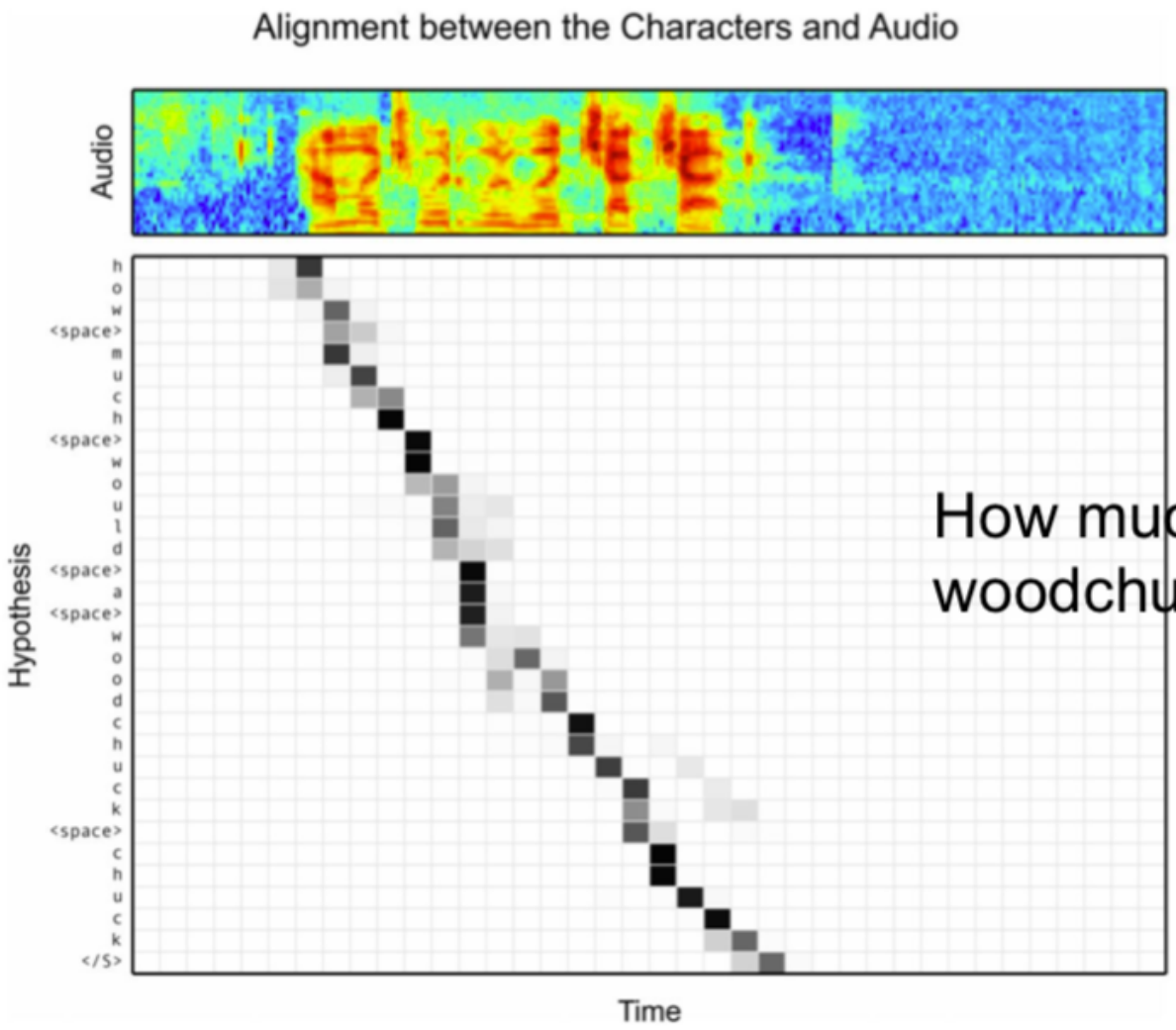
At test-time sample characters one at a time, feed back to model



# Deep Speech: Accurate Speech Recognition

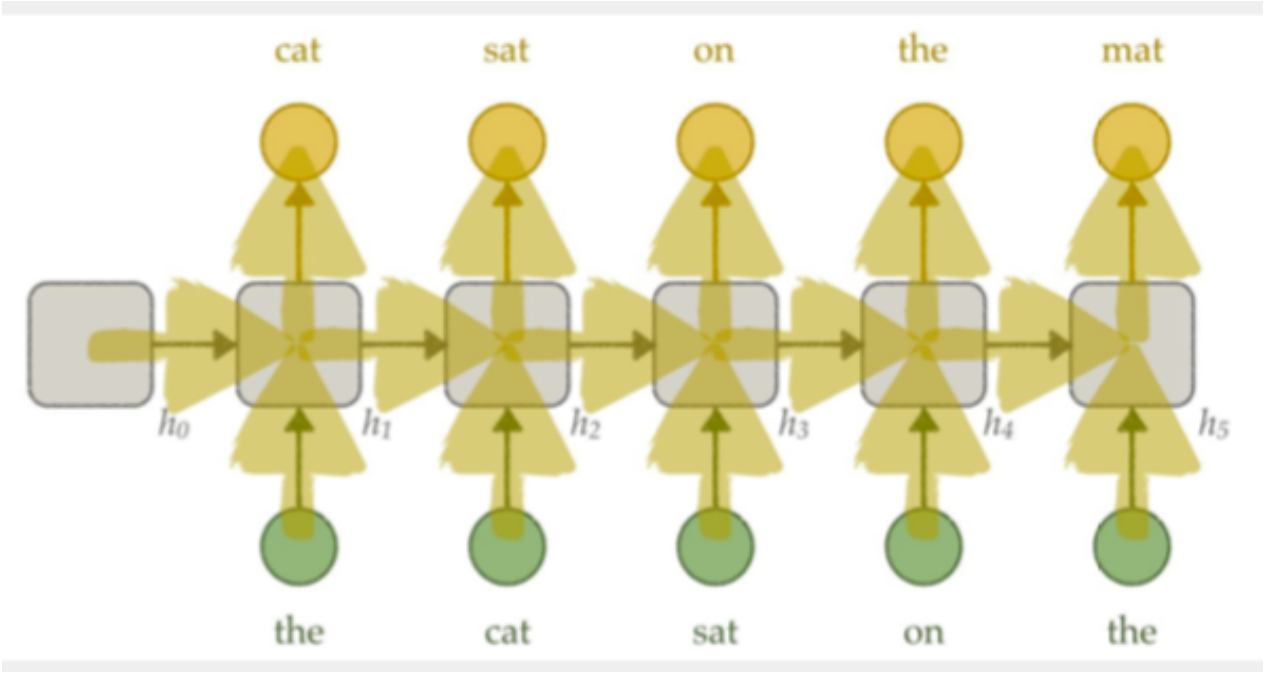



Bryan Catanzaro , 2015




How much would a woodchuck chuck

# Neural embedding



# Sequence to Sequence

- For learning to generate an output sequence $(y^{(1)},...,y^{(ny)})$ given an input sequence $(x^{(1)},x^{(2)},...,x^{(nx)})$
- It is composed of an encoder RNN that reads the input sequence decoder RNN that generates the output sequence
- The final hidden state of the encoder RNN is used to compute a generally fixed-size context variable ***C*** which represents a semantic summary of the input sequence and is given as input to the decoder RNN.