TÉCNICO LISBOA

Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2013/2014

Second Test

29 May 2018, 20 horas – rooms F2,F3,FA1

Duration 2 hours

You may not use neither programmable calculators nor consult anything

Grades: P1-6 P2-5 P3-5 P4-4

P1. Under certain conditions, a CO2 laser of gas phase de fase can be described by the following nonlinear model

$$\dot{x}_1 = x_1 x_2$$
$$\dot{x}_2 = 1 - \alpha x_2 - (1 + \beta x_2) x_1$$

where α and β positive parameters, x_1 is the normalised intensity of the light beam and x_2 is proportional to the difference between the numbers of excited and non-excited gas atoms. Answer the following questions:

a) Compute all the equilibrium states of the system.
b) Obtain the equations of the linearized systems around each of the equilibrium states you have found in a).
c) Based on the linearization models that you obtained in b), state whether each of the equilibrium points are asymptotically stable, unstable, or that you cannot say anything about stability. Assume that $\beta + \alpha < 2$.

P2. Consider the block diagram of a feedback servomechanism shown on figure P2-1. In this position control system, the input signal u of a ditect current motor is generated by an actuator whose characteristics is described by a nonlinear function f whose argument is the tracking error.

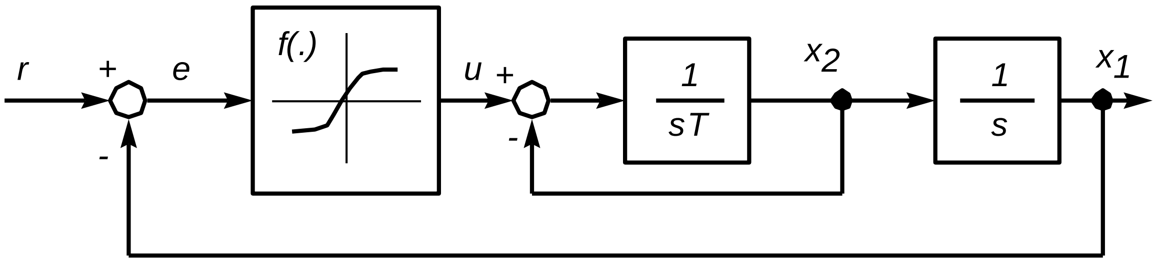

Fig. P2-1 A feedback servomechanism with a nonlinear actuator.

It is known that this function is such that

$$f(e) > 0 \text{ for } e > 0; \quad f(e) = 0 \text{ for } e = 0; \quad f(e) < 0 \text{ for } e < 0$$

This means that:

$$\int_0^e f(\sigma)d\sigma > 0 \quad \text{for} \quad e \neq 0$$

The reference r is constant in time.

The variables x_1 and x_2 are, respectively, the angular position and the angular velocity of the motor shaft. The parameter $T > 0$ is the motor dominant time constant.

Answer the following questions:

a) Consider the state defined by the tracking error e and the angular velocity x_2. Write the corresponding nonlinear state equations.

b) Show that

$$V(e, x_2) = \frac{T}{2}x_2^2 + \int_0^e f(\sigma)d\sigma$$

Is a Lyapunov function for the origin of the system described in a). What can you conclude about the stability of this equilibrium point using the Lyapunov Theorem?

c) What conclusions can you draw for the same problem by applying the invariant set theorem?.

P3. One class of control applications that attracts major interest is related to Medicine. In this problem we aim at optimizing the therapy that allows to counteract the growth of a tumor. It is assumed that the growth of the tumor may be modelled through the following scalar differential

$$\dot{x} = x - bu,$$

where x is the tumour mass, u is the therapy used, assumed to be the rate of administration of a drug (manipulated variable) and b is a positive parameter, assumed known, that reflects the sensitivity of the tumour to the therapy.

It is assumed that the therapy is applied during a period of duration T. The objective is a compromise between minimizing the tumour mass at the end of the treatment, $x(T)$, and the total quantity of drug administered to the patient (the drug has toxic secondary effects). This objective is translated is the following cost functional, that is to be minimized

$$J(u) = x(T) + \int_0^T \rho\, u(t)dt,$$

where ρ is a positive parameter that measures the relative importance of minimizing the total qyantity of drug administered to the patient, versus the objective of reducing the tumour.

The therapy must verify the condition

$$0 \leq u \leq u_{max},$$

where u_{max} is the maximum allowed drug administration rate. It is assumed that $\rho > b$. Find the optimal control function that minimizes J. Use Pontryagin's Principle.

Help:

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-(\frac{d\lambda}{dt})' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda(T) = \Psi_x(x(T))$$

$$H(\lambda, x, u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$

P4. A submarine moves along a straight line, with a longitudinal velocity V in an aquatic ambient where there are no currents, moved by a propulsion force F_p. In this problem we want to design a controller that keeps the incremental velocity of the submarine with respect to the equilibrium velocity close to zero (this means that the total velocity should be close to V, even in the presence of disturbances). The design is made assuming that the following model holds

$$\dot{v} = -0.5v + u \,,$$

where u is the increment of the propulsion force with respect to the equilibrium F_p and v is the corresponding velocity increment with respect to the velocity equilibrium V. Consider the quadratic cost defined over an infinite horizon

$$J = \frac{1}{2}\int_0^\infty \left(v^2(t) + u^2(t)\right) dt \,.$$

By applying Pontryagin's principle, obtain a feedback control law that minimizes J.

Sugestion: Assume that, at each time instant t, the velocity $v(t)$ and the co-state $\lambda(t)$ are related by

$$\lambda(t) = -p\,v(t) \,,$$

where p is a positive constant that you should compute, and write an algebraic equation verified by p.