**Mestrado Integrado em**

**Engenharia Electrotécnica e de Computadores**

# Controlo Em Espaço de Estados

**2017/2018**

## First Test

18 April 2018, 20h00 – Duration 2 hours

**Grading: P1**a)3 b)2 P2 a)4 b)2 c)1 P3 a)2 b)2 c)1 P4 a)1 b)2

**P1.** Consider the system with input $u$ and output $y$, and transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

a) Writte a state realization using phase variables (the output and its derivatives).

b) Using only **scalar** blocks (integrators, algebraic sums and gains) sketch a block diagram that allows to simulate the state model.

**P2.** In relation to the linear model $\dot{x} = Ax$, consider the matrices, numbered 1 to 4:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & 1.5 \\ 1.5 & 0 \end{bmatrix} \quad A_4 = \begin{bmatrix} -5/3 & -4/3 \\ 4/3 & 5/3 \end{bmatrix}$$

Consider also the phase portraits shown in figure P2-1, and identified with the letters A, B, C and D.

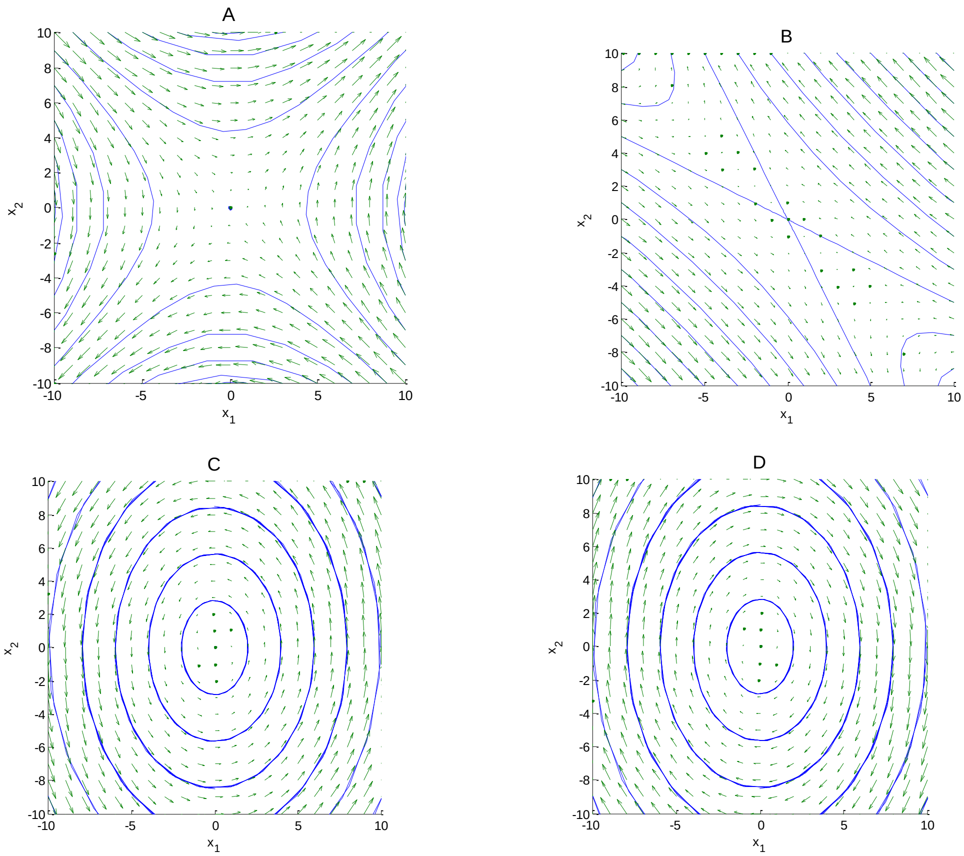


Fig. P2-1. Problem P2. Phase portraits.

a) State which matrix is associated to each phase-portrait. Justify

b) In relation to $A_3$ compute the expression that gives the state as a function of time for the initial condition $x(0) = \begin{bmatrix}1\\0\end{bmatrix}$. Repete withy the initial condition $x(0) = \begin{bmatrix}0\\1\end{bmatrix}$.

c) Fort the same matrix coinsidered in b), compute $e^{A_3 t}$ by the method that you prever. Observe that there is a method for which the answer is immeadiate.

**P3.** Figure P3-1 depicts a pendulum whose oscillation is influenced by a force (manipulated variable) $u$. In this problem we want to design a state feedback controller to regulate the position of the pendulum.

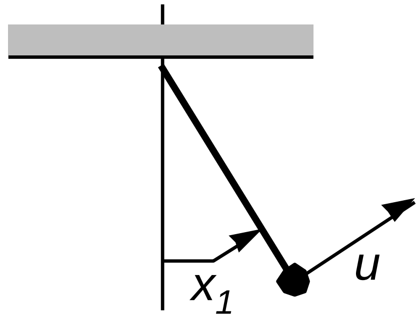


Fig. P3-1 Problem 3. The pendulum actuated by a manipulated force.

For this purpose, we know the following model that relates the force $u$ with the output $y$ given by the pendulum angular velocity

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).$$

Answer the following questions:

a) Design a state feedback regulator that places the closed-loop poles at $-4 \pm j4$

b) Design an asymptotic observer that places the state estimation error dynamics at $-10 \pm j10$

c) Sketch a block diagram of the controller, including the observer, that uses only basic **scalar** blocks (integrators, gains, sums).

**P4.** The mechanical system shown in figure P4-1 is a simplification of the sustension of an autobomile vehicle. The vehicle is represented by the mass $m$, assumed to be a point mass, and is linked to road profile through a spring of coefficient $k$ and a damper placed in parallel with a friction coefficient $\beta$. Point *A* is connected to the road surface, that is not flat. This assumption corresponds to move the point *A* vertically by a distance $u$, that may vary in time. We consider only movements in the vertical. Due to the change of the road $u$, the vehicle moves vertically by a $y$.

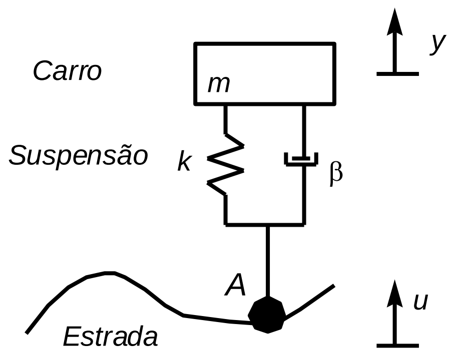


Figura P4-1. Problem P4.

When the spring is stretched or compressed with a change of length $\Delta$, the spring exerts a force that opposes the movement with strengt $k\Delta$. The damper is such that, when the difference of velocity between its ends is $v$, it exerts a force that opposes movement with a strength $\beta v$.

a) Using Newton's law, write a differential equation that relates $u$ (input signal) with $y$ (output signal). Consider a situation in which, from the start, the spring equilibrates the vehicle weight, so that the weigh does not enter the equation.

b) Define a set of convenient state variables, with the minimum number of variables, and write the state equation in matrix form. Present all intermediate calculations that are relevant.