Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2018/2019

First Test

April 10, 2019, 20 h – Duration 2 hours

Consultation of any kind is not allowed

Grades: P1a)3 b)2 P2 a)4 b)2 P3 a)2 b)1 c)1 d) 1 e) 1 P4 a)1 b)2

P1. Consider the system with input u and output y, with transfer function

$$G(s) = \frac{1}{s^2 + 4s + 3}$$

a) Write a state realization for this system using phase variables (the output and its derivatives).

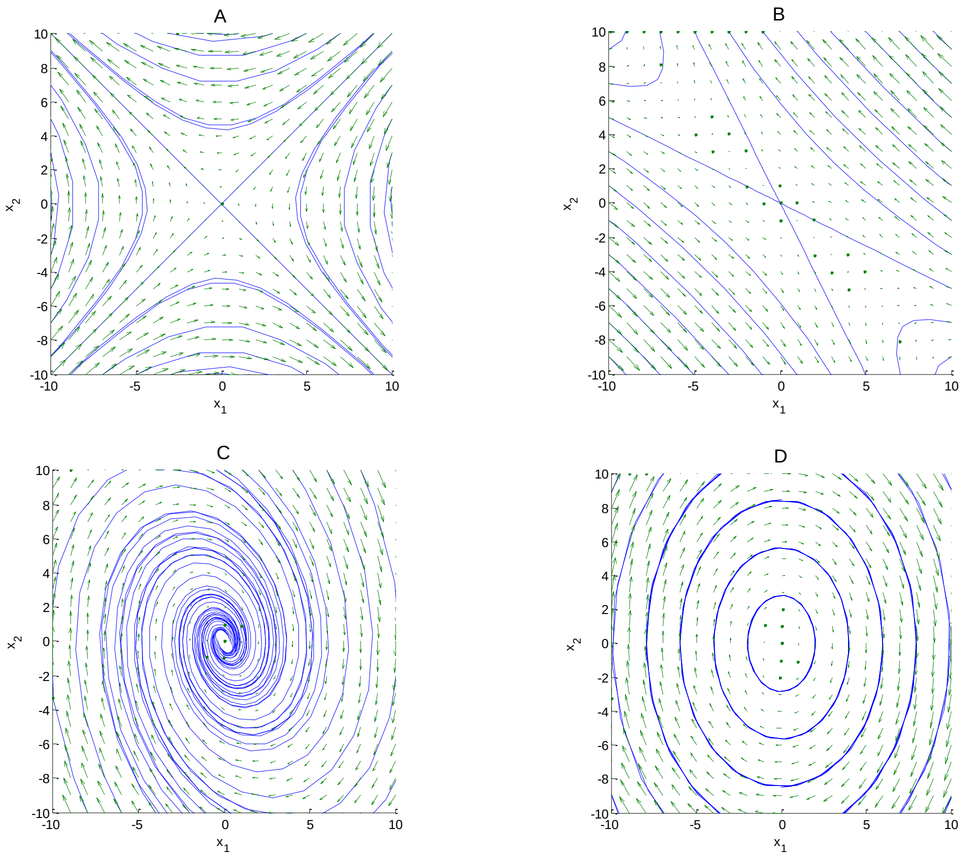
b) Consider now the system, whose transfer function is modified from the one of a) by the addition of a zero, and define conveniente state variables. Write the state equations in matrix form.

$$G(s) = \frac{s+1}{s^2 + 4s + 3}$$

P2. In relation to the state linear $\dot{x} = Ax$, consider the matrices, numbered from 1 to 4:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -0.6 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & -1.5 \\ -1.5 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} -5/3 & -4/3 \\ 4/3 & 5/3 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$$

Consider also the phase portraits shown in figure P2-1, that are identified with the letters A, B, C and D.



a) Say which matrix is associated to which phase poortrait. Justify.

b) For A_2 compute the expression that yieçlds tha state as a function of time, given the initial condition $x(0) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

P3. Consider the state model of order

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad\qquad y = [1 \quad \alpha]\, x(t).$$

Parameter α has a known value. Answer the following questions:

a) Design the vector of the gains of a linear state variable regulatror that places the closed-loop poles $-4 \pm j4$

b) Say whether i tis possible or not to compute the gains of an asynmptotic observer that places the error eigenvalues in whatever values are specified. Express your answer as a function of α.

c) Give an interpretation of the answer to the previous alínea b) in terms of the process transfer function.

d) Assume that $\alpha = 2$. Design na asymptotic observer that places the poles of the state estimation error at $-10 \pm j10$.

e) Draw a block diagramo f the controller, including the observer, using onçly basic **scalar** blocks (integrators, gains, sums).

P4. Consider the homogeneous state equation of order 2nd order

$$\dot{x} = Ax$$

In relation to this equation, we know two solutions for linearly independent initial conditions, that are given, for all positive t, by

$$x^1(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

and

$$x^2(t) = \begin{bmatrix} -1 + e^t \\ e^t \end{bmatrix}$$

a) Write the state transition matrix (matrix exponential), e^{At}.

b) From e^{At} compute the matrix A (the entries of A are constants).