Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo em Espaço de Estados

2015/2016

First Test

8 April 2015, 20:00 – Rooms: V1.23, V1.24, V1.25

Duration 2 hours

The use of programmable calculators is not allowed

Gradings: P1a)**3**b)**1** c)**1 d)1** P2a) **2** b)**1** c)**1 d)1 e)1 f)1** P3a)**2** b)**1** c)**1 d)1** P4 a)**2**

Alternative to P2 (P2A): a) 2, b) 2. You may solve either P2 or P2A, but only one is used for the grading.

P1. Consider the linear time invariant system shown figure P1-1, where α is a constant parameters, assumed to be known.

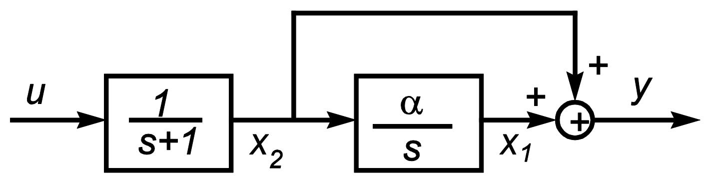


Fig. P1-1. Problem P1. Block diagram.

a) Write, in matrix form, the state model of the system, using as state the variables indicated.

b) **Without computing** neither the controllability matrix, nor the observability matrix, show that there are values α that lead to a loss of observability or observability for the model you obtained.

c) Compute the observability matrix and state whether or not the state model is observable.

d) Compute the controllability matrix and state whether or not the state model is controllable.

P2. Figure P2-1 shows a circuit used to make the tuning of an antenna, in which the coil resistance is neglected.

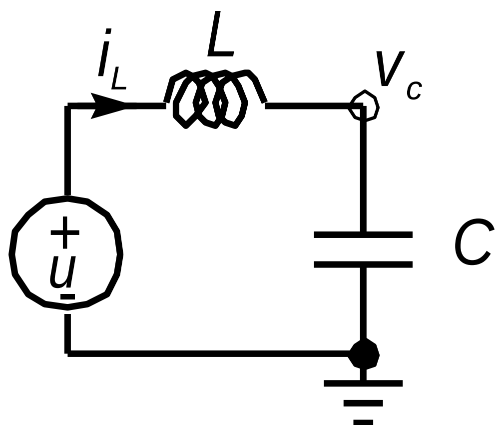


Fig. P2-1. Resonance circuit of an antenna.

The circuit consists of an electrical tension source of u that may vary in time, and that corresponds to the electrical tension induced by the electromagnetic wave that illuminates the antenna, in series with a coil of inductance L and a capacitor of capacitance C.

a) Taking as state variables $x_1 = v_c$ (the tension across the terminals of the capacitor) and $x_2 = i_L$ (the current through the coil), and as input u *and outputy* $= v_c$, write the state model equations for this system.
b) Compute the eigenvalues, and the corresponding eigenvectors of the matrix of dynamics of the system. Assume generic values of L and C.
c) From the state model, compute, compute the transfer function and indicate what are their poles.
d) Assume now that, in a system with normalized units, $L = C = 1$. Assume also that $u(t) = 0$ and that initially there is no current through the coil and that the tension across the capacitor is $1V$. Using the expression that you have studied for the modal decomposition, write the solution of the homogeneous state with the initial condition as indicated. Provide your answer as real functions of time for $x_1(t)$ and $x_2(t)$.

e) Sketch the time response, in the conditions of d), for t between 0 and π. Draw separate graphics for $x_1(t)$ and $x_2(t)$. Sketch also the corresponding trajectory in the state space, corresponding to this initial condition and for this period of time.

f) Compute the field vectors associated to the homogeneous equation in the two points indicated next:

  a. The initial condition;

  b. At the point where the trajectory that you found in d) and e) crosses the axis x_2.

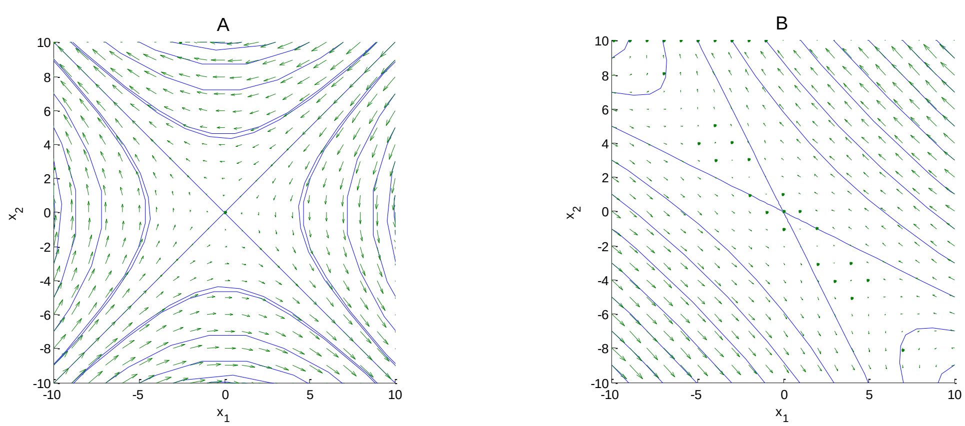
  Plot the vectors on the trajectory sketch that you did in e).

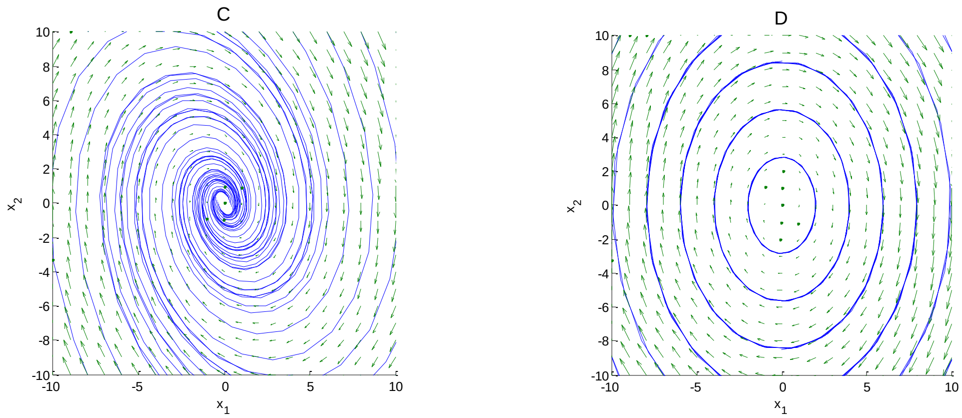
Remark: Problem P2A is an alternative to problem P2, but with a much lower grade. You may can only deliver only either P2 or P2A. If you deliver both, only P2 is considered.

P2A. In relation to $\dot{x} = Ax$, consider the matrices 1 until 4:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -0.6 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & -1.5 \\ -1.5 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} -5/3 & -4/3 \\ 4/3 & 5/3 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$$

Furthermore, consider the phase portraits shown below and identified with the A, B, C and D.





a) Associate the matrices with the phase portraits. Justify.

b) In relation to A_2 compute an expression that yields the state as a function of time, when the initial condition is $x(0) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

P3. Figure P3-1 shows a control system for the superheated steam temperature in a medium size boiler for electrical energy production.

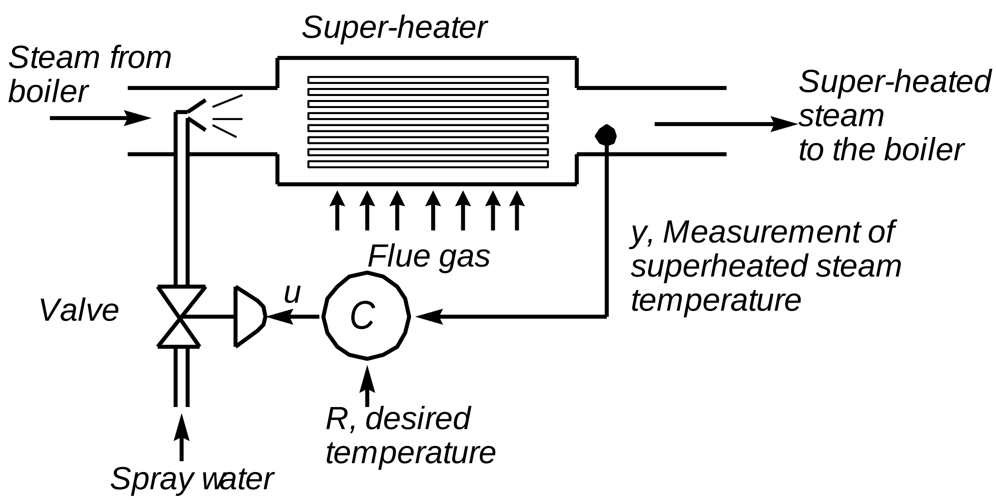


Fig. P3-1 Super-heated steam temperature control in a medium size boiler.

A spray of water that passes through the valve is injected in the steam that comes from the boiler. When the spray water is vaporized, it absorbs energy from the steam and reduces its temperature. The steam passes then through the super-heater that is made of a series of parallel pipes in the outside of which there are flue gases passing, increasing thereby the steam temperature before it passes to the turbine. Therefore, the injection of spray water allows to manipulate the steam temperature through the valve command signal. The valve command u is computed (in a scale between 0 and 100%, with 0 corresponding to the valve completely closed and 100%to the valve fully open) by the controller C based on R (reference signal that gives the desired temperature) and on the measurement y of steam temperature, performed through a thermocouple placed in the steam pipe, at the output of the super-heater.

In this problem, we want to design the temperature controller C using state variables feedback and an observer. We shall consider only the case in which the temperature reference is zero, that corresponds to keep an equilibrium temperature. For such a purpose, consider the following state model that relates the valve position u with the output corresponding to the steam temperature:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t) \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Answer the following questions:

a) Design a state feedback regulator that places the closed-loop poles at $-2 \pm j2$

b) Design an observer that places the estimator poles at $-10 \pm j10$

c) State whether it is reasonable to place the controller poles at $-50 \pm j50$, using the observer designed in b). Justify.

d) Draw a block diagram of the controller, including the observer using only basic blocs ((integrators, gains and sums) that are **scalar**.

P4. Consider the state model in discrete time, with zero initial conditions and where the $u(k)$ is zero for k negative

$$x(k+1) = Ax(k) + bu(k) \qquad \text{(P4-1)}$$

Write the input function as a sum of functions that are different from zero in only one point each:

$$u(k) = u(0)\delta(k) + u(1)\delta(k-1) + u(2)\delta(k-2) + \cdots + u(N)\delta(k-N)$$

Using the superposition principle, and the fact that the solution of the homogeneous equation

$$x(k+1) = Ax(k),\ x(k_0) \text{ given}$$

is

$$x(k) = A^{k-k_0}x(k_0),$$

To obtain an expression of the solution for the forced (non-homogeneous) equation (P4-1).