Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2013/2014

First Test

8 April, 2014, 20 h. – room C01

Duration 2 hours

It is not allowed neither consultation of any kind nor programmable calculators

Quotação: P1 a)1 b)2 c)2 d)1 P2 a) 3 b) 1 c) 2 P3 a) 1 b) 1 c) 1 d) 1 P4-4.

P1. Figure P1-1 represents a side “cut” view of a water delivery canal in which there is a gate (“comporta”) that is moved by a motor.

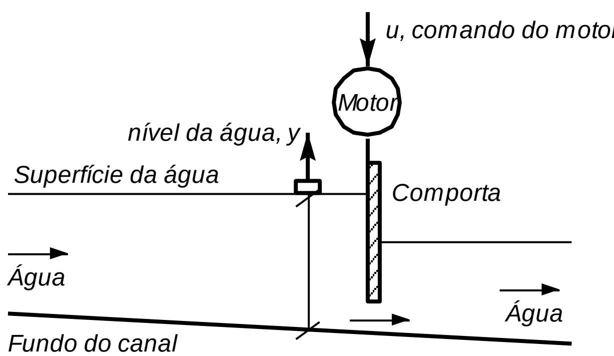


Figure P1-1. Problem 1.

The transfer function that relates the signal u (gate command)with the gate position, v, is

$$G_1(s) = \frac{1}{s^2 + 2s + 3}.$$

The transfer function that relates the gate position, v, with the water level y measured by a sensor located upstream the gate is

$$G_2(s) = \frac{s+4}{s^3 + 3s^2 + 2s + 4}.$$

Answer the following questions:

a) Write a state realization of G_1.

b) Write a state realization of G_2.

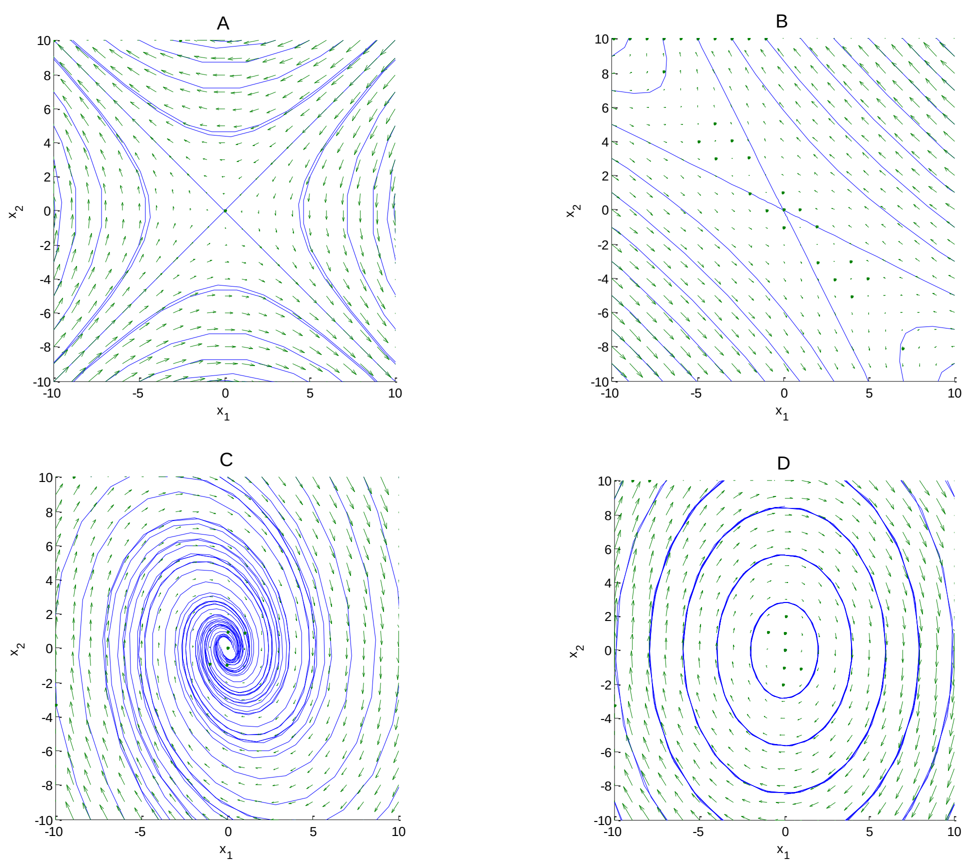
c) Write a state realization of the global system (given by the series of G_1.and G_2), and such that the state of the global system is the concatenation of the states of G_1.and G_2 that you have defined in a) and b).

d) Draw a block diagram of G_2, using only integrators, gains and algebraic sums, all **scalar**.

P2. In relation to the linear state model $\dot{x} = Ax$, consider the matrices numbered from 1 to 4:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -0.6 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & -1.5 \\ -1.5 & 0 \end{bmatrix} \qquad A_3 = \begin{bmatrix} -5/3 & -4/3 \\ 4/3 & 5/3 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$$

Consider also the phase portraints shown in figure P2-1 and identified with the letters A, B, C and D.



a) State which matrix matches which plot. Justify..

b) In relation to A_2 compute an expression that yields the state as an explicit function of time, knowing that the initial condition is $x(0) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

c) **What really happened at the Waterloo battle**. The Waterloo battle took place in 18 June 1815 (close to the village with the same name, in Belgium). In this battle the allied army, commanded by the Duke of Wellington (most soldiers in this army were british, but 17 different languages were spoken!) beated the french army commanded by Napoleon. The victory only felt to the side of Wellington after the arrival of general Bucher (a prussian that exclaimed when he visited London: "What a beatifull city to be saked") and his troops, that joinned the allied forces. Matrix A_2 above represents the state model for the evolution of the number of french and allied troops during the battle. In this model, x_1 represents the number of french soldiers (divided by 10000, meaning that $x_1 = 1$ means 10000 french soldiers) and x_2 the number of allied soldiers (also divided by 10000). Inicially, the french had 30000 soldiers and the allies 25000. General Bucher brough 10000 soldiers that joinned the allies. Show that, if Bucher did not arrived, Napoleon would win the Waterloo battle (meanin that, when time passed, the troops would reach a situation in which $x_1 > 0$ and $x_2 = 0$), but, with the reinforcement of Bucher soldiers, is Wellington that wins (meaning that, when time passes, one reaches a situation in which $x_1 = 0$ and $x_2 > 0$). In order to simplify things, assume that Bucher and his soldiers arrived at the beginning of the battle. Use a sketch of the phase portrait to justify your answer.

P3. A heat exchanger allows to transfer heat from a fluid (heat source) to another that we want to heat. It consists of two separate circuits. In one of the circuits circulates the fluid to heat and in the other circulates the fluid that is the heat source. By adjusting the flow of the hot fluid one can vary the quantity of energy transferred to the fluid to heat, by unit of time, thereby changing its temperature. Figure P3-1 shows a schematic view of a heat exchanger.

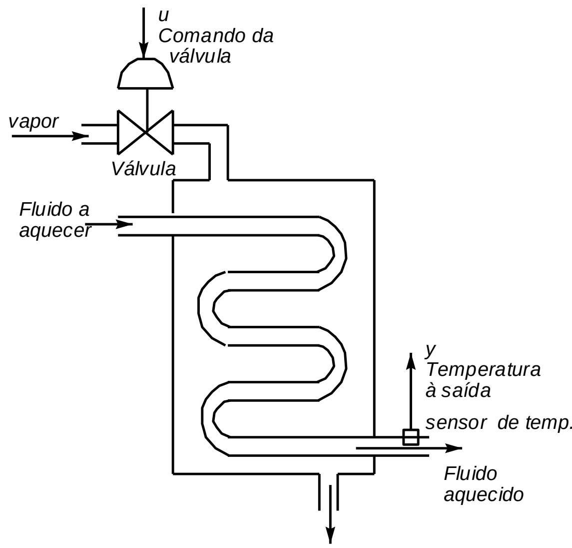


Fig. P3-1 Schematic view of a heat exchanger.

In this problem, we want to design a linear state feedback controller to regulate the temperature of the outlet fluid, by acting on a steam valve. For that purpose, we know the following state model that relates increments around a working point in the steam admission valve command u with the increments y of the fluid temperature at the outlet:

$$\dot{x}(t) = \begin{bmatrix} -0.017 & 0.017 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Answer the following questions:

a) Design a state feedback controller that places the closed-loop poles at $-0.05 \pm j0.087$

b) Tell if the system is or is not controllable. Justify your answer.

c) Design a state observer with the poles of the estimation error dynamics at $-0.15 \pm j0.26$

d) Tell if the system is or is not observable. Justify your answer.

P4. Consider the system described by the linear state equation

$$\frac{dx}{dt} = Ax + bu \text{ , with initial condition } x(0) \text{ ,}$$

where $x \in R^n$ (column vector) is the state, $u \in R$ (scalar), is the input, $t \in R$ denotes time, and $A \in R^{n \times n}$ and $b \in R^n$ are matrices of parameters. Using the change of variables

$$x(t) = e^{At} z(t) \text{ ,}$$

where $z \in R^n$ is a new state variable, get an expression for the solution of the state equation as a function of t, of the initial condition, the input and of the matrices that define the system.

Aids: $\frac{d}{dt} e^{At} = Ae^{At}$, $\frac{d}{dt}\left(M(t)N(t)\right) = \dot{M}N + M\dot{N}$

Glossary

Comporta – Gate

Água – Water

Vapor – Steam

Nível da água – Water level

Comando do motor – Motor command

Fluído a aquecer – Fluid do heat

Fluído aquecido – Heated fluid

Válvula – Valve

Temperatura à saída – Output temperature