Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2016/2017

First Test – English version

19th April 2017, 20 horas – Rooms: Ea2, Ea1, E4, E3, E2

Duration 2 hours

Consulting docs. or the use of programmable calculators are not allowed

Rating: P1 a) 1 b) 2 c) 3 d) 1 e) 1 **P2** a) 1 b) 4 c) 1 d) 1 **P3** a) 1 b) 0,5 c) 0,5 **P4** a) 1,5 b)0,5 c)1. P3A a) 0,2 b) 0,3 c) 0,2 d) 0,3

P1. In the island of Tonga-Bonda there are rabbits and foxes and an unlimited supply of food for rabbits. Local scientists came to the conclusion that, being $x_1(t)$ the number of rabbits at time t and $x_2(t)$ the number of foxes, these variables (that form the state of the system) satisfy the linear state equations (without external input)

$$\begin{cases} \dfrac{dx_1}{dt} = x_1 - 2x_2 \\ \dfrac{dx_2}{dt} = -2x_1 + 4x_2 \end{cases}$$

a) Write the state equation in matrix form. Write the matrix A (the matrix of dynamics of the system). Remark that there is no exogenous input.

b) Compute the eigenvalues and the corresponding eigenvectors of A.

c) Write the modal decomposition of the system for arbitrary initial conditions. (that is to say, considering generic k_1 and k_2 for each of the modes).

d) Assume that the initial number of rabbits is 100 (that is to say, $x_1(0) = 100$). Find the initial number of foxes, $x_2(0)$, such that the number of foxes and the number of rabbits is kept constant in time, that is to say, such that $x_1(t) = x_1(0)$ and $x_2(t) = x_2(0)$ for all t.

e) Assume now that the initial number of rabbits is 100, and the initial number of foxes 45. Write the time functions that allow you to compute $x_1(t)$ and $x_2(t)$ as a function of time, and find the time t_f for which foxes become extinct.

P2. The transfer function of a permanent magnet direct current motor that drives a robot arm joint is

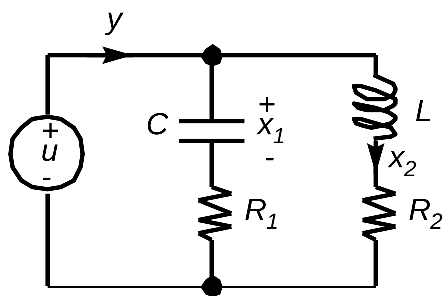
$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}$$

where U is the Laplace transform of the electric tension applied to the motor, that corresponds to the input signal u, and Y is the Laplace transform of the robot arm joint angle, that corresponds to the output signal y.

a) Take as state variables $x_1 = y$ and $x_2 = \dot{y}$ and write the corresponding state equations in matrix form.
b) Compute the gains K_1 and K_2 such that $u = -K_1 x_1 - K_2 x_2$ places the roots of the controlled system at the roots of $\alpha_c(s) = s^2 + 3s + 9$.
c) Design a state estimator such that the estimation errors have poles at the roots of the polynomial $\alpha_o(s) = s^2 + 15s + 225$.
d) Find the transfer function of the controller obtained by the combination of the two precedent questions. Indicate numerical values for the coefficients of the numerator and denominator.

P3. Consider the electrical circuit of figure P3-1.

a) Write the state equations for the state variables indicated. You don't have to write the output equation.
b) Give a condition on R_1, R_2, L and C so that the state realization that you obtained is not controllable.
c) Interpret the loss of controllability condition that you have obtained on b) in terms of the time constants of the circuit (inverse of the eigenvalues of the dynamical matrix). Indicate your computation.



Figur P3-1

As an alternative to problem P3 you may solve problem P3A. It is remarked that problem P3A is rated 1 less point. You must clearly indicate in your solution sheet which problem you solved (P3 or P3A). **If you present both solutions, only P3 will be considered**.

P3A. Consider the system whose block diagram is shown in figure P3-2, α is a parameter (a number assumed to be known).

a) Write the state equations (dynamic equations and output equation) for the state and output variables indicated.
b) Tell if there are values of α for which there is loss of controllability .
c) Tell if there are values of α for which there is loss of observability.
d) Give an interpretation of the loss of controllability and/or observability in terms of the transfer function.

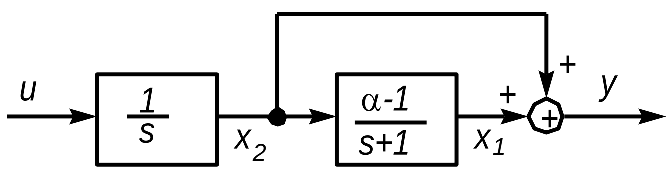


Figure P3-2.

P4. Model Based Predictive Control (MPC) is a very powerful technique to design controllers that has its origins in the petro-chemical industry in the years 60 of the XX century but that has generalized, since thee 80'ies to a variety of fields such as robotics, automotive, or even Marketing. This problem aims at designing a predictive control law in a simple case (linear model, no constraints, no forced integrator).

Consider the plant described by the linear state model in discrete time

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

In order to simplify, it is assumed that, in addition to the output y, we have also access to the state x at each instant of discrete time k ($k = 0, 1, 2, 3, \dots$).

Assume that you are at instant k, and that you know $x(k)$. We want to compute a sequence of manipulated variable values $u(k)$, $u(k+1)$, ..., $u(k+H-1)$, where H is an integer number selected by the designer (called the "control horizon"). The computation of these control samples is made by minimizing

$$J(u(k), \dots, u(k+H-1)) = \sum_{i=1}^{H} y^2(k+i) + Ru^2(k+i-1)$$

a) In order to express J as a function of only $x(k)$ and $u(k), \dots, u(k+H-1)$, write $y(k+i)$ as a function of $x(k)$ and $u(k), \dots, u(k+H-1)$. Suggestion: use the state model to express $x(k+i)$ as a function of $x(k)$ and $u(k), \dots, u(k+H-1)$. **Make $\boldsymbol{H} = \mathbf{4}$ in all these alíneas.**

b) Define $Y = [y(k+1) \quad \dots \quad y(k+H)]^T$, $U = [u(k) \quad \dots \quad u(k+H-1)]^T$. Using the result of a), show that there are matrices W and Π such that $Y = WU + \Pi x(k)$. Write the elements of these matrices for $H = 4$.

c) Find, as a function of $x(k)$, R, W, and Π the value of U that minimizes J.

Remark: In MPC, of all the sequence U only the first element, $u(k)$, is applied to the process, the whole process being repeated at discrete $k+1$. This procedure is the so called "receding horizon strategy".