Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2016/2017

Exam

23 June 2017, 11h30 – Room Ea2

Duration 3 hours

Consultation not allowed nor programmable calculators

Points: P1a)2b)1c)2d)1 P2 a)1b)1c)1d)1e)1 P3 a)1b)1c)1 P4-3 P5-3

P1. Consider the system with input u and output y, with transfer function

$$G(s) = \frac{s+5}{s^2+3s+2}$$

a) Write a state realization using phase variables.

b) Compute the corresponding eigenvalues and eigenvectors..

c) Using the modal decomposition, write the solution $x(t)$ of the state equation when the initial condition is $x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

d) Using only basic **scalar** blocks (integrators, adders and gains), draw a block diagram that represents the state model..

P2. Consider the system of figure P2-1.

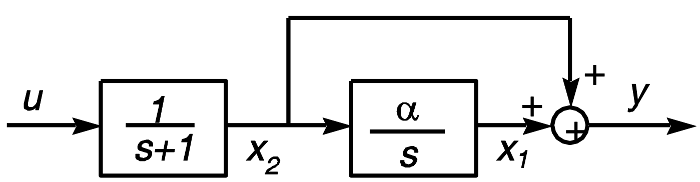


Fig. P2-1. Model of the system to control

a) Obtain a 2nd order state model. Take as state variables x_1 and x_2, as inbdicated in fig. P2-1.

b) For the above state model, state for which values of parameter α it is posiible to compute the gains of a linear state feedbak (LSF) such as to place arbitrarily the closed loop poles. Do not the closed loop characteristic polynomial.

c) For $\alpha = 2$, write the equations and compute the gains of a LSF to locate the closed loop poles at $-4 \pm j$. Assume in this question that you have access to x_1 and x_2.

d) Assume now that you don't have access to the direct measure of x_1 and x_2. State for which values of α is it possible to design a state observer such that the error converges to zero with an arbitrarily specified dynamics.

e) For $\alpha = 2$ write the equations and design the gains of an observer such that the error in the state estimate converges to zero with eigenvcalues of the error equation given by $-10 \pm j$.

P3. A pendulum with length 1m and mass 1kg is described by the nonlinear state model

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = -10\sin(x_1) - \beta x_2 \end{cases}$$

where x_1 is the angle of deviation with respect to the vertical, x_2 is the angular velocity and β is the friction coefficient..

a) Compute the matrix of the linearized system around the origin.

b) Write the algebraic equation satisfied by the eigenvalues of the linearized dynamics of the linearized system as a function of parameter β

c) Using the linearized model, what can you say about the stability of the origin of the nonlinear system in the following cases: $\beta = 0$ and $\beta = 8$? Justify..

P4. Consider a vehicle that moves along a single direction, along a graduated line, as shown in fig. P4-1.

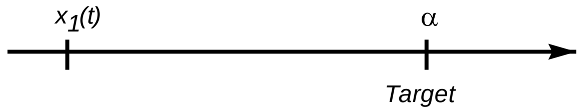


Figura P4-1. Vehicle that moves along a line.

At time t the vehicle is in the position $x_1(t)$, has a velocity $x_2(t)$.and is actuated by a force (input manipulated variable) $u(t)$. These variables are relayted by the state model

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u$$

We want to compute a control law that brings the vehicle to the point with coordinate α, and zero velocity. For that sake, we consider the following candidate Lyapunov function

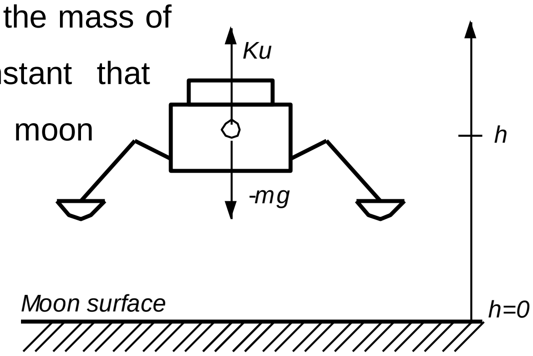
$$V(x_1, x_2) = \frac{1}{2}\left(K_1(x_1 - \alpha)^2 + x_2^2\right)$$

Based on this function, on Lyapunov's Direct Method, and on the Invariant set Theorem, find a feedback control law that ensures that $x_1(t) \to \alpha$ and $x_2(t) \to 0$. Justify.

P5. Under simplifaction assumptions, the movement of the LEM (Lunar Excursion Module) of the APOLLO 11 mission in its descent to the moon (see figure) is described by

$$m\frac{d^2}{dt^2}h = \ddot{K}u - mg,$$

where h is the altitude (function of time t), m is the mass of the vehicle (assumed constant), $K$is a constant that measures the effect of the jet, and g is the moon gravity at the surface (assumed constant).



a) Taking as state variables $x_1 = h$ and $x_2 = \dot{h}$, and using $\frac{K}{m} = 1$, write the vehicle state equations as a function of g.

b) Using Pontryagin’s Principle, find the function $u(t), 0 \leq t \leq T$, with $T = 10$ that minimizes

$$J(u) = \frac{1}{2}\int_0^T u^2(t)dt$$

Subject toi the constraints

$h(0) = \alpha, \dot{h}(0) = \beta$ (inicial posituion and velocity)

$h(T) = 0$, $\dot{h}(T) = 0$ (the vehicle reaches the moon surface with zero velocity)

Use the following values for the parameters

$\frac{K}{m} = 1$, $\alpha = 5000$, $\beta = gT = 16$.

Aid:

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-(\frac{d\lambda}{dt})' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda(T) = \Psi_x(x(T))$$

$$H(\lambda,x,u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$