Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2018/2019

24/May/2019, 20h00 – 22h00

Second Test

Duration 2 hours

Not allowed consultation of any kind

Grades: P1a)2b)1,5c)1,5d)1 P2Aa)1b)1c)1d)1e)1P2B-2 P3a)3b)2 P4 a)3 b)1

P1. The leaf Spring, shown in the picture is frequently used in suspension systems for a wide range of vehicles, either motorize or with animal traction. Opposite to linear springs, that cause a force that is linerar with respect to displacement, these springs exert a force that is a nonlinear function of the amplitude of displacement. Taking as state variables the position (in a given referencial), x_1, and the velocity, x_2 , of the mass attached to the spring, consider the following nonlinear state model of a suspension of this type

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\beta x_2 - K_1 x_1 - K_2 x_1^3 \end{cases}$$

Take as parameter values $\beta = 0{,}1$, $K_1 = 1$, $K_2 = -0{,}25$.

a) This system has 3 equilibrium points. Find them.

b) Write the Jacobian matrix for each of the equilibrum points

c) Compute the eigenvalues of the system linearized around each of the equilibrum points. Show all your computations.

d) On the basis of the linearized system, what can you say about the stability of each of the equilibrioum points?

P2. *This problem has two alternatives, labeled A and B. The alternative A is more complicated, but has a higher gtrade (5 points). The alternative B is value only 2 points.* ***You must indicate in a clear way which is the altermative that you choose. If you answer both alternatives, only A will be consider and the other alternative is not considered.***

A) The deviation of the crown temperature (the temperature measured at the "roof") of a flass fornace with respect to the desired value, x (scalar), is related to the inflow of fuel to be burned, u, through the linear 1st order model described by the state equation

$$\dot{x} = ax + u$$

where a is a parameter that corresponds to the pole of the system, that is to say, it corresponds to the inverse of the time constant. The figure shows a photo of ione burner in action, inside a glass furnace.

a) Start by assuming that parameter a is known. Compute the gain K of a controller that feeds back x, that is to say, for which the control law is $u = -Kx$, such that the controlled system behaves as having a pole at the specified position a_m, that is to say, that it verifies $\dot{x} = a_m x$. I tis remarked that that x is the temperature deviation with respect to the desired value, which means that we want to drive x to zero and simplifies the problem.

b) Assume now that the value of a is unknown, and we resort to an adaptive rule. $\hat{a}$ be the estimate of a and $\tilde{a}$ is the estimation error. These quantities are related by $\tilde{a} = \hat{a} - a$. Writte a differential equation that relates x and $\tilde{a}$, when the control law obtained in the previous question a) is used with a replaced by its estimate $\hat{a}$.

c) Consider the candidate Lyapunov function for control and estimation given by

$$V(x, \tilde{a}) = \frac{1}{2}\left(x^2 + \frac{1}{\gamma}\tilde{a}^2\right).$$

Obtain a law to adjust the estimate $\tilde{a}$ that ensures that this is indeed a Lyapunov function. . Write this law in a form that envolves na integral and makes explicit the initial estimate, $\hat{a}(0)$.

d) Sketch a block diagram of the system with the control law and the adaptation rule.

e) What can you say on the value for which the temperature deviation x converges, on the basis iof the invariant set theorem?

B) Consider the system defined by the nonlinear state equations

$$\frac{dx_1}{dt} = -x_1 - x_1 x_2^2$$

$$\frac{dx_2}{dt} = -x_2 - x_1^2 x_2$$

Fpor this ystem, the origin ($x_1 = 0$, $x_2 = 0$) is na equilibrium point. In relation to this equilibrium, take as candidate Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2).$$

Ansqwer tghe question: What can you say about the stability of this eqyuilibrium point? Show all your compoutations.

P3. The reign of Napoleon III took place between 1852 and 1870 and was a singular period in the history of France. First because Napoleon III (a nephew of Napoleon Bonaparte) started by being elected as the first president of the French Republic, before he proclaimed himself emperor. Also because this is a time of deep contracictions, between na enormous socio-economic progress (for instance, in this period the lenght of the french railway is muitplied by 5; very advanced social legislation is published) and a hard cultural repression, under which many writters, today famous and well respoected, have been prosecuted in court because of the subjects of their work, either in prose or poetry. One of those writers was Guy de Maupassant, because of his work *Madame Bauvary*, considered a scandal (but that according to present day standards can be offered to innocent youngsters). This novel contains the description of a grave digger who had two forms of income: By the funerals that he performed and by planting potatoes ion the part of the cemetery that was not yet occupied by graves. For na engineer trained at IST that reads this story, the problem arises: Qhen the grave digger accepts a funeral, he receives a payment of this service, but the total area in which he can plant potatoes is reduced. What is then the optimal policy of funeral acceptance that maximizes the grade digger's profits in a given time perior? The presente problema answers this question.

Let x be the área occupied by the graves. This variable (scalar) satisfies the state model

$$\frac{dx}{dt} = u, \quad x(0) = 0,$$

where u is the rate of funeral acceptances (control variable) and i tis assumed that the dfuneral, initially, is completely unoccupied. Furthermore, i tis assumed that the área varies continuously. Since the grave digger has a limited capacity to work, the control o u satisfies

$$0 \leq u \leq u_{max}.$$

Let T be the period duriong which the grave digger manages the cemetery. His total profit is given by

$$J(u) = \int_0^T [A - x(t) + \rho u(t)]\, dt,$$

where A is the total área of the cemetery and ρ is the quotient of the prioces of funerals and potatoes (that is to say, assuming that the unitary price of potatoes is 1, ρ is the cost of each funeral). To simplify the problema, i tis assumed that the cemetery is not fully occupied by graves at time T. Answer the following questions:

a) Using Pontryagin's Maximum Principle, compute the optimal control, that maximizes $J(u)$. Show all your computations. Draw sketches of of the Hamiltonian as a function of the control, and of the co-state, the optimal control and the state, as a function of time (see the help at the end of the test).

b) Compute the Hamiltonian as a function of time.

P4. Consider the unstable system described by the scalar state model

$$\dot{x} = x + u, \quad x(0) = 0,$$

In which the state $x(t) \in \mathbb{R}$, at each time instant t, that is to say, x is acalar.

a) Using Pontryagin's Maximum Principle, find the control law that minimizes the cost

$$J(u) = \frac{1}{2}\int_0^{\infty} (x(t) - r)^2 + \rho(u(t) + r)^2 dt,$$

where r is a const reference to track and $\rho > 0$ is a parameter.

Sugestion: Assume that the co-state verifies

$$\lambda(t) = -px(t) + g$$

and find constants for p and g.

b) Compute the value of the equilibrium state, as a function of r, and show that, when $t \to \infty$, the state tends to this equilibrium value..

Help

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-(\frac{d\lambda}{dt})' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda'(T) = \Psi_x(x(T))$$

$$H(\lambda, x, u) = \lambda' f(x,u) + L(x,u)$$

In general for a 2nd order system:

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$