TÉCNICO LISBOA

Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2017/2018

25/Maio/2018, 20h00 – 22h00, Ea1, Ea2, E1, E2, E3, E4

Second Test

Duration 2 hours

Not allowed consultation of any kind

Grades: P1 a)1 b)4 c)2 P2 A) a) 1 b)1 c)2 d)1 B)2 P3-5 P4-3

P1. Under certain conditions, a CO2 gas laser can be described by the nonlinear state

$$\dot{x}_1 = x_1 x_2,$$

$$\dot{x}_2 = 1 - \alpha x_2 - (1 + \beta x_2) x_1.$$

>In these equations, α and β are positive parameters that verify

$$\alpha + \beta < 2,$$

x_1 is the normalized intensity of the light beam and x_2 is proportional to the difference between excited and non-excited gas atoms. Answer the following questions:

a) Show that $(0, {}^{1}/_{\alpha})$ is an equilibrium point.

b) Compute the dynamics of the linearized system around the equilibrium point referred in a), as well as the corresponding eigenvalues. What can you say about the stability of the nonlinear point?

c) Compute the other equilibrium point. What can you say about the stability of this point based on the linearized model?

P2. In this problem you have two alternatives, denoted A and B. Alternative A is more complicated, but has a highjer grade (5 points). The grade of alternative B is just 2 points. **You must indicate clearly what is the alternative that you choose to answer. If you answer both only the answer to A is considered.**

A) Consider the system of figP2-1 that represents a flow controller with a nonlinear valve, where y is the measure of the flow across the valve (system output) and u is the command of the valve (manipulated variable).

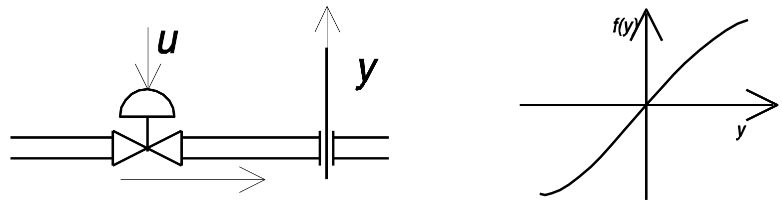

Fig. P2-1: Problem 2 A). Nonlinear flow control valve.

The valve is described by the 1st order nonlinear system

$$\frac{dy}{dt} = u - \theta\ f(y)$$

Where the nonlinear function $f(.)$ is known, being represented in fig. P2-1 (wright) and the constant parameter θ is unknown.

Answer the following questions:

a) Find a static output feedback such that, assuming a perfect knowledge of θ, the system (valve + feedback) behaves as a pure integrator.

b) Find a linear control law to apply to the resulting iontegrator such that, assuming perfect knowledge of θ, the tracking error $e(t) = r - y(t)$ converges to zero with a time constant of 1 second. Assume that the reference r is constant.

c) Using Klyapunov's Direct Method find an adjustment law of the estimate of θ such that the overall system is stable.

d) Using the invariant set theorem say if you can ensure that the tracking error $e(t)$ converges to zero when t converges to infinity.

B) Consider the system defined by the nonlinear state equations

$$\frac{dx_1}{dt} = -x_1 - x_1 x_2^2$$

$$\frac{dx_2}{dt} = -x_2 - x_1^2 x_2$$

For this system, the origin ($x_1 = 0$, $x_2 = 0$) is an equilibrium point. In relation to this point, consider the candidate Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2).$$

Question: What can you say about the stability of this equilibrioum point, using Lyapunov's Direct Method. Shjow all your computations and justify your answer.

P3. In this problem you have two alternatives, denoted A and B. Alternative A is more complicated, but has a highjer grade (5 points). The grade of alternative B is just 3 points. **You must indicate clearly what is the alternative that you choose to answer. If you answer both only the answer to A is considered.**

A) Consider the scalar system described by the state equation

$$\dot{x} = x + u,$$

where x is the state (scalar) and u is the manipulated variable (scalar). Applying Pontryagin's Maximum Principle compute the state feedback control law that minimizes

$$J = \int_0^\infty \left(x^2(t) + u^2(t)\right)dt$$

Assume that at each time t, the state $x(t)$ and the co-state $\lambda(t)$ are related by

$$\lambda(t) = -p\,x(t)$$

where p is a constant that you must compute.

B) Consider the scalar system described by the state equation

$$\dot{x} = x - 0{,}2u,$$

where x is the state (scalar) and u is the manipulated variable (scalar). By applying Pontryagin's Maximum Principle, obtain a control law that minimizes

$$J(u) = x(5) + \int_0^5 u(t)dt,$$

where, at each time t, the control variable verifies the constraint

$$0 \leq u(t) \leq 1.$$

P4. Consider a mobile robot that moves only along one direction and is modeled as a double integrator in which the state x_1 and x_2 are the position and the velocity in a coordinate system. The state equations are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

Find the optimal control that transfers the ste at time 0, given by

$$x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

to the state at time 2 given by

$$x(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

minimizing

$$J(u) = \frac{1}{2}\int_0^2 u^2(t)dt.$$

Useful information

$$\boxed{\begin{array}{l} \dot{x} = -ax + b \quad a, b \text{ constant} \\ \qquad x(t) = \dfrac{b}{a} + Ce^{-at} \end{array}}$$

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-(\frac{d\lambda}{dt})' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda'(T) = \Psi_x(x(T))$$

$$H(\lambda, x, u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \dfrac{\partial L}{\partial x_1} & \dfrac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \dfrac{\partial \Psi}{\partial x_1} & \dfrac{\partial \Psi}{\partial x_2} \end{bmatrix}$$

Laplace transforms

$$1 \;\rightarrow\; \frac{1}{s}, \qquad e^{-at} \;\rightarrow\; \frac{1}{s+a}, \qquad 1 - e^{-at} \;\rightarrow\; \frac{a}{s(s+a)}$$