TÉCNICO LISBOA

Mestrado Integrado em

Engenharia Electrotécnica e de Computadores

Controlo Em Espaço de Estados

2015/2016

Second Test

25 May 2016, 18h30 – rooms EA5, EA2

Duration 2 hours

No consultation of any kind allowed nor programmable means

Grading: P1-a)1 b)3 c)2, P2-a)1 b)3 c)1, P3-5, P4-a)3 b)1.

P1. The pendulum is described by equations that are the same for other systems of practical interest, such as an electrical power generator connected to a power transmission line. Consider the model of the pendulum that is described by the following nonlinear state equations:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = 9\sin(x_1)$$

a) Show that the states $\bar{x}^1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ e $\bar{x}^2 = \begin{bmatrix} \pi \\ 0 \end{bmatrix}$ are two equilibrium points.

b) Get the equations of the linearized system around each of the equilibrium points.

c) Based on the results obtained on b) what can you say about the stability of each of the equilibrium points of the **nonlinear system** referred on a)?

P2. In this problem you have two alternatives, named A and B. Alternative A is more complicated, but has an higher grading (5 points), whereas alternative B is evaluated only 2 points. **You must indicate which alternative you choose. If you answer to both alternatives only the answer to A is considered.**

A) In this problem the stability of a satelite actuated by a nonlinear controller is considered. For the sake of simplicity, it is assumed that the satellite moves only in one plane. Let θ be the satellite angle with respect to a reference direction, ω the corresponding angular velocity, and u the torque (manipulated variable) that is applied to the satellite by jets. The state model is therefore

$$\frac{d\theta}{dt} = \omega$$

$$\frac{d\omega}{dt} = u$$

The control law computes u as a function of the measures of θ and ω, and is given by

$$u = -g(\omega) - h(\theta),$$

Where the functions g and h (there are several possibilities) satisfy

$$g(0) = 0 \text{ and } \omega g(\omega) > 0 \text{ for } \omega \neq 0$$

$$h(0) = 0 \text{ and } \theta h(\theta) > 0 \text{ for } \theta \neq 0$$

a) Show that, with this control law, the origin $\theta = 0$ and $\omega = 0$ (the satellite stoped and turned to the reference direction) is an equilibrium point.

b) What can you say about the stability of this equilibrium point using the Lyapunov function $(\theta, \omega) = \frac{1}{2}\omega^2 + \int_0^\theta h(\sigma)d\sigma$?

c) At each time instant t, the manipulated variable $u(t)$ must be within the boundary values: $-u_{max} \leq u(t) \leq u_{max}$. Give an example of functions g and h that ensure that the origin is asymptoticaly stable and that at the same time ensure this interval of values for $u(t)$.

B) Consider the system described by the nonlinear state equations

$$\frac{dx_1}{dt} = -x_1 - x_1 x_2^2$$

$$\frac{dx_2}{dt} = -x_2 - x_1^2 x_2$$

For this system, the origin ($x_1 = 0$, $x_2 = 0$) is an equilibroum point. With respect to this equilibrium point, take as candidate Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2).$$

What can you say about the stability of this equiloibrium point for the nonlinear system? Show all computations.

P3. Consider an electrical energy production plant that uses a thermal solar collector. The fluid to be heated by the sun circulates by the action of a pump and is used to produce electrical energy in a thermoelectric group. The model (very simplified) that relates the power produced x with the pump command u (in normalized units, with $u = 0$ corresponding to the pump stop and $u = 1$ corresponding to the maximum velocity) is given by the scalar differential equation

$$\frac{dx}{dt} = -0{,}2x + u$$

Starting from a situation in which no power is being produced (meaning that $x(0) = 0$), we want to find the function u in the time interval between $t = 0$ and $t = 5$ that maximizes the functional

$$J(u) = \int_0^5 [x(t) - u(t)]dt, \quad 0 \leq u \leq 1.$$

This functional represents a compromise between producing the maximum of energy at the end of the optimization interval, on one side, and consuming less energy on the pump. By using Pontryagin's Principle **obtain the optimal control**.

Sketch qualitatively, but indicating the important points, the plots of the Hamiltonian, of the co-state, of the optimal control, and of the power produced in the time interval between $t = 0$ e $t = 5$.

P4. Consider a vehicle that moves along a straight line, always in the same direction, and where the v is related with the rate of fuel consumption u by

$$\frac{dv}{dt} = -v + u.$$

Initially the velocity is zero, $v(0) = 0$. We want to control the vehicle in the time interval $[0, T]$, so that the final velocity has the value $v(T) = V_2$, and such as to minimize the fuel spent in the manoeuver

$$J(u) = \int_0^T u(t)dt.$$

The following constraint must be respected

$$0 \le u(t) \le u_{max}$$

Above, V_2, T and u_{max} are parameters with well defined and assumed known values. The answer must be expressed in terms of them.

a) Using Pontryagin's Principle, find the optimal control that solves this problem. Sketch qualitatively, but indicating the important points, the plots of the Hamiltonian, of the co-state, and of the optimal control.

b) Discuss the existence of solution for the optimal control problem as a function of the parameters V_2, T e u_{max}.

Useful help

$\dot{x} = -ax + b \quad a, b$ constants

$$x(t) = \frac{b}{a} + Ce^{-at}$$

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-(\frac{d\lambda}{dt})' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda(T) = \Psi_x(x(T))$$

$$H(\lambda, x, u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$