

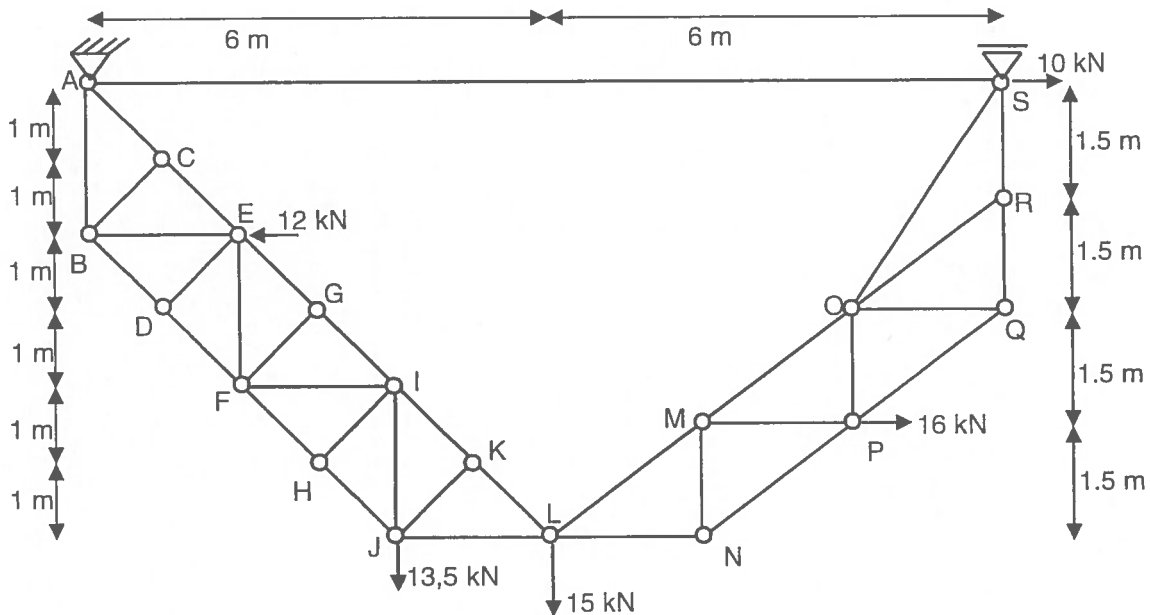


Desligue o telemóvel
Sem consulta, excepto do formulário fornecido
Identifique todas as folhas com o número e nome
Entregue cada problema em folhas separadas
Justifique adequadamente todas as respostas
Duração: 1h30m

Problema 1 (6,0)

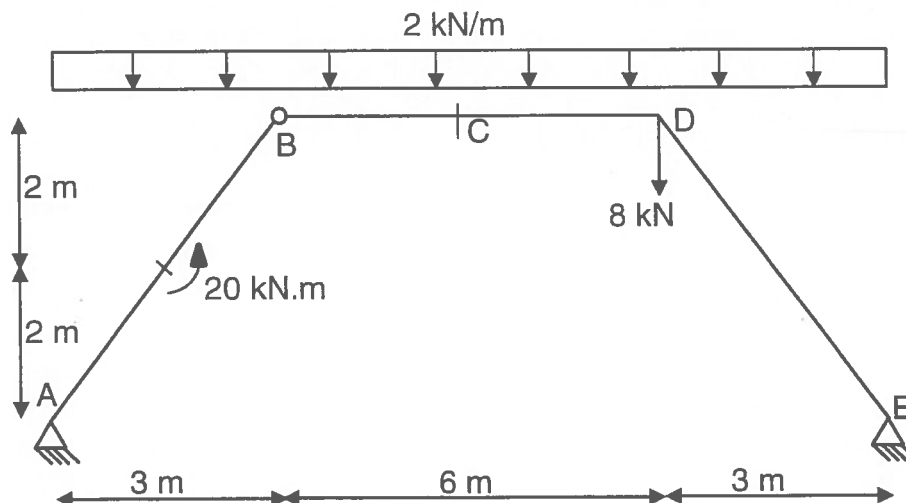
Considere a seguinte estrutura articulada

- a) Calcule as reacções de apoio. (1,0)
- b) Calcule a força na barra MO (3,0)
- c) Calcule a força na barra FI (2,0)



Problema 2 (6,0)

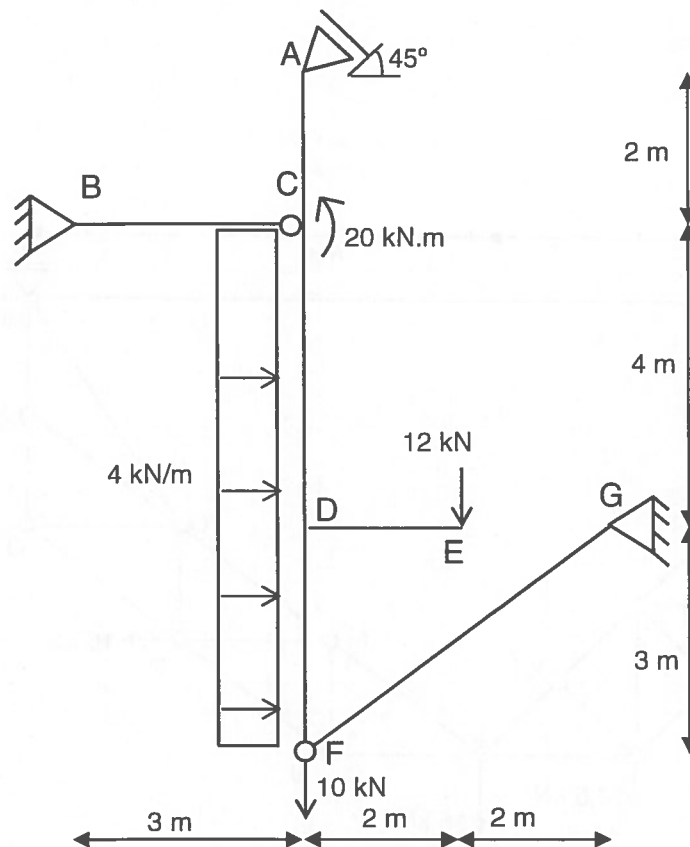
Utilizando o PTV calcule o momento e na secção C (a meio da barra BCD) e a reacção horizontal em E



Problema 3 (8,0)

Considere a estrutura da figura seguinte.

- a) Calcule as reacções de apoio (3,0)
 b) Trace os diagramas de corpo livre de todas as barras, apresentando as forças nas direcções paralela e perpendicular ao eixo de cada barra (5,0)



Formulário:

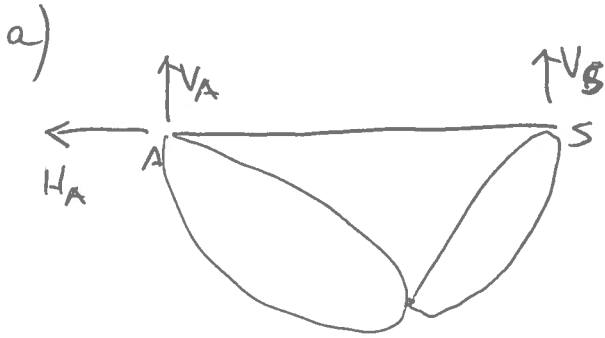
$$\vec{F}_{AB} = F_{AB} \vec{\lambda}_{AB} \quad \vec{\lambda}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} \quad \vec{M}_A = \vec{AP} \times \vec{F} \quad M_{AB} = \vec{\lambda}_{AB} \cdot \vec{M}_A$$

$$\vec{M}_B = \vec{M}_A + \vec{BA} \times \vec{R} \quad \vec{AQ} = \frac{\vec{R} \times \vec{M}_A}{R^2} + \lambda \vec{R}$$

$$F_a \leq \mu_e N \quad T_2 = T_1 e^{\mu_e \beta}$$

$$\delta \vec{r}_B = \delta \vec{r}_A + \delta \theta \times \vec{AB}$$

1º Problema



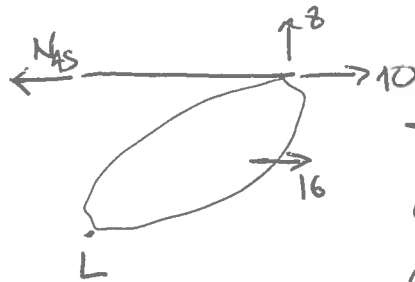
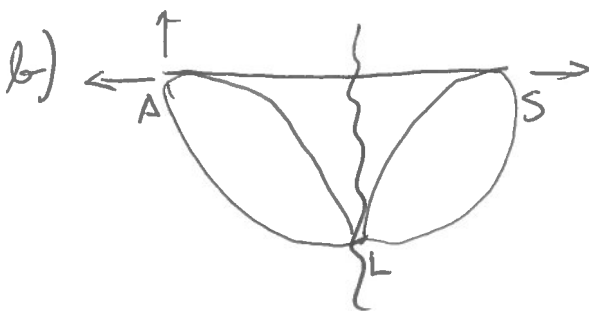
$$\sum F_x = 0 \quad -H_A - 12 + 16 + 10 = 0 \quad H_A = 14$$

$$\sum F_y = 0 \quad V_A + V_S - 13.5 - 15 = 0$$

$$\sum M_A = 0 \quad 12V_S - 2 \times 12 + 4.5 \times 16 - 4 \times 13.5 - 6 \times 15 = 0$$

$$12V_S = 96 \quad V_S = 8$$

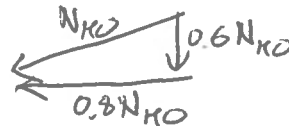
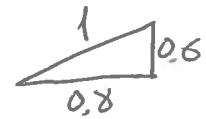
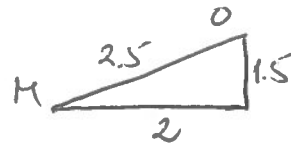
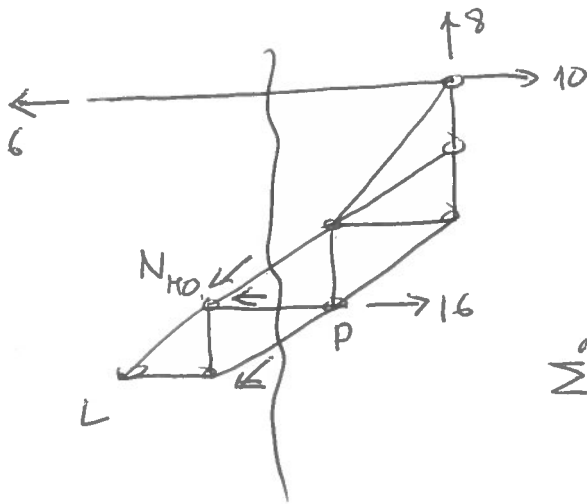
$$V_A + 8 - 13.5 - 15 = 0 \quad V_A = 20.5$$



$$\sum M_L^{dir} = 0$$

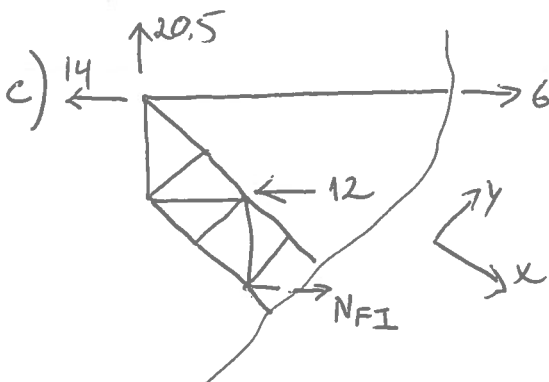
$$6N_{LS} - 16 \times 1.5 - 10 \times 6 + 8 \times 6 = 0$$

$$6N_{LS} = 36 \quad N = 6$$



$$\sum M_P^{dir} = 0 \quad 0.8N_{HO} \times 1.5 - 10 \times 4.5 + 8 \times 2 + 6 \times 4.5 = 0$$

$$1.2N_{HO} = 45 - 16 - 27 = 2 \quad N_{HO} = 1.67 \quad (\leftarrow \text{compressão})$$

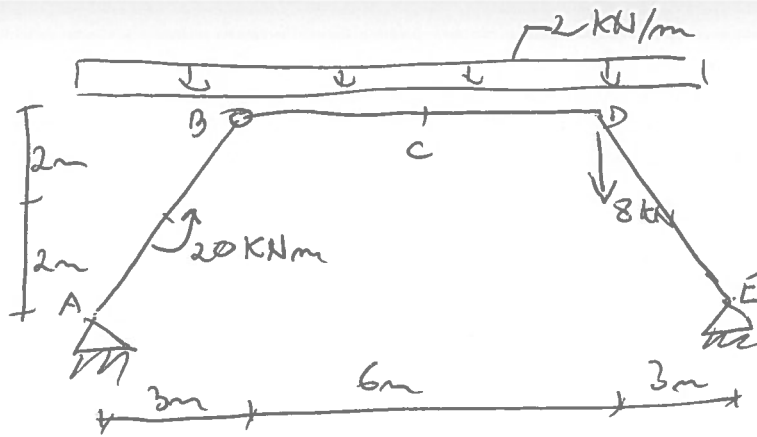


$$\sum F_x = 0 \quad \frac{N_{FI}}{\sqrt{2}} - \frac{14}{\sqrt{2}} + \frac{6}{\sqrt{2}} - \frac{12}{\sqrt{2}} + \frac{20.5}{\sqrt{2}} = 0$$

$$N_{FI} - 14 + 6 - 12 + 20.5 = 0$$

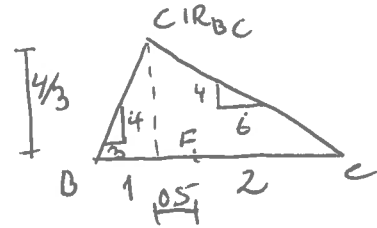
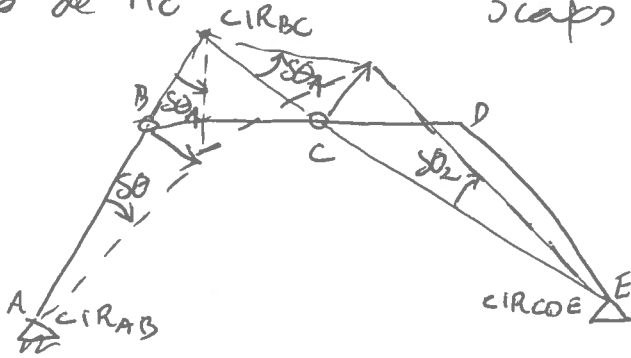
$$N_{FI} = -0.5 \quad (\leftarrow \text{compressão})$$

2º Problema



Cálculo de M_C

3 caps - AB, BC e CDE



$$\sum V_B = 3\delta\theta_1 = 1.5\delta\theta_1 \quad \delta\theta_1 = 3\delta\theta_2$$

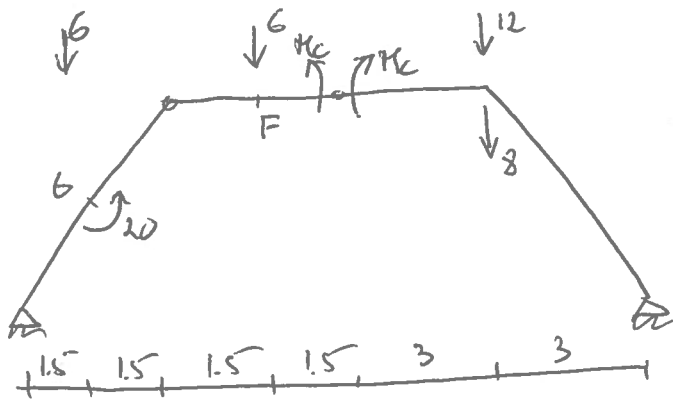
$$\sum V_C = 2\delta\theta_1 = 6\delta\theta_2$$

$$2 \times 3\delta\theta_1 = 6\delta\theta_2 \quad \delta\theta_1 = \delta\theta_2$$

$$\sum V_B = 1.5\delta\theta \downarrow$$

$$\sum V_F = 0.5 \cdot \delta\theta_1 = 0.5 \times 3\delta\theta = 1.5\delta\theta \uparrow$$

$$\sum V_D = 3\delta\theta_2 = 3\delta\theta$$



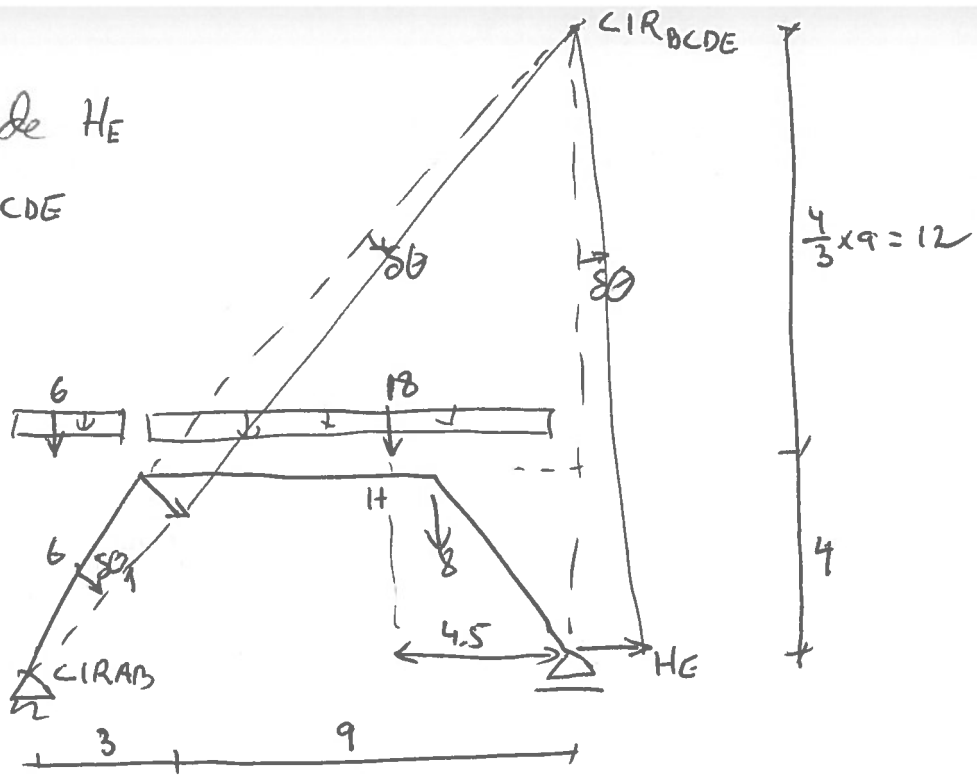
$$W = -20\delta\theta + 6 \times 1.5\delta\theta + 6 \times (-1.5\delta\theta) + M_C \cdot 3\delta\theta + M_C \delta\theta + (12+8)(-3\delta\theta) = 0$$

$$= -20 + 4M_C - 60 = 0 \quad M_C = 20 \text{ kNm}$$



Cálculo de H_E

2 casos - AB e BCDE



$$\delta_{VB} = 3\delta\theta_1 = 9\delta\theta \quad \delta\theta_1 = 3\delta\theta$$

$$\delta_{V6} = 1.5 \times 3\delta\theta = 4.5\delta\theta \downarrow$$

$$\delta_{VH} = 4.5\delta\theta \downarrow$$

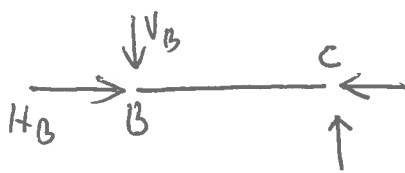
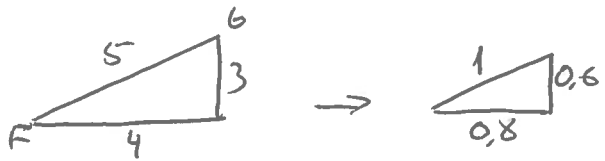
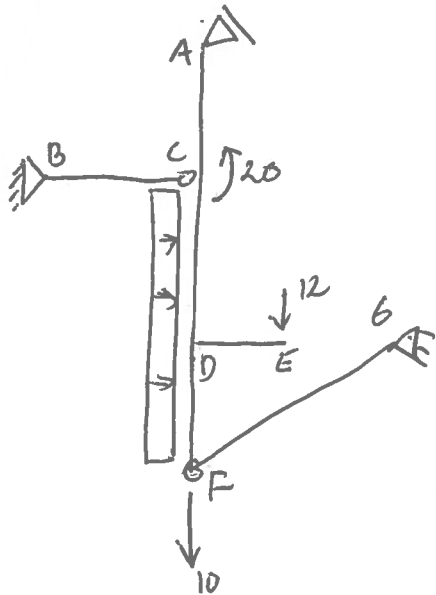
$$\delta_{VD} = 3\delta\theta \downarrow$$

$$\delta_{HE} = 16\delta\theta (\rightarrow)$$

$$W = 6 \times 4.5\delta\theta + 18 \times 4.5\delta\theta + 8 \times 3\delta\theta - 20 \times 3\delta\theta + H_E \times 16\delta\theta = 0$$

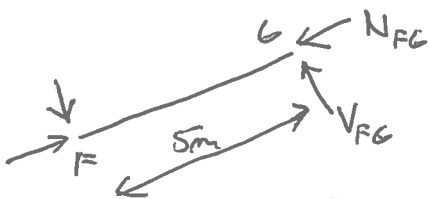
$$27 + 81 + 24 - 60 + 16H_E = 0 \quad H_E = \frac{-72}{16} = -4.5 \text{ KN} (\leftarrow)$$

3. Problema



$$\sum M_C^{\text{eq}} = 0 \quad V_B \times 3 = 0 \quad V_B = 0$$

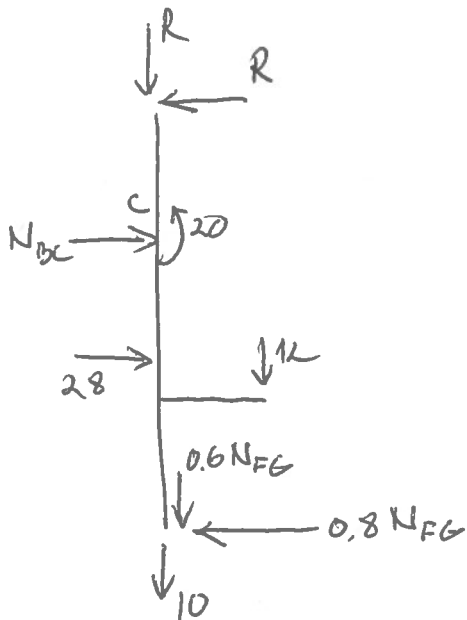
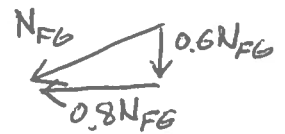
$$\sum F_x = 0 \quad N_{BC} = H_B$$



$$\sum M_F^{\text{dir}} = 0 \quad V_{FG} \times 5 = 0 \quad V_{FG} = 0$$

$$\sum F_x = 0 \quad N_{FG} = 0$$

$$\sum F_y = 0 \quad 0.6N_{FG} = 0$$



$$(1) \sum F_x = 0 \quad N_{BC} + 28 - R - 0.8N_{FG} = 0$$

$$(2) \sum F_y = 0 \quad -R - 0.6N_{FG} - 12 - 10 = 0$$

$$(3) \sum M_C = 0 \quad 20 - 2R - 7 \times 0.8N_{FG} + 28 \times 3.5 - 12 \times 2 = 0$$

$$(2) \quad -R - 0.6N_{FG} - 22 = 0$$

$$(3) \quad 2R - 5.6N_{FG} + 94 = 0$$

$$2 \times (2) + (3) \quad 2 \times (-0.6N_{FG}) + 2 \times (-22) - 5.6N_{FG} + 94 = 0$$

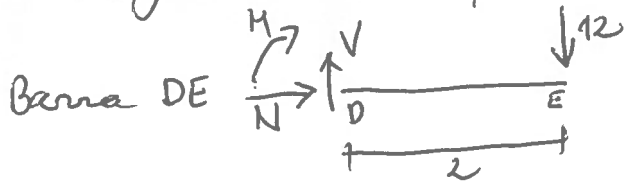
$$-6.8N_{FG} + 50 = 0 \quad N_{FG} = \frac{50}{6.8} = 7.353 \text{ kN}$$

$$(2) \quad -R - 0.6 \times 7.353 - 22 = 0 \quad R = -26.412 \text{ kN}$$

$$(1) \quad N_{BC} + 28 + 26.412 - 0.8 \times 7.353 = 0$$

$$N_{BC} = -48.53 \text{ kN}$$

Diagramas de corpo livre



$$\sum F_x = 0$$

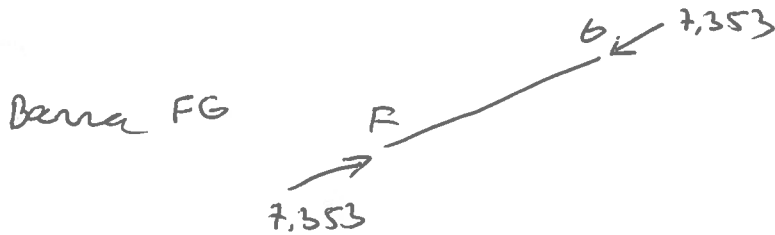
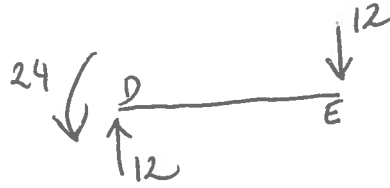
$$N = 0$$

$$\sum F_y = 0$$

$$V - 12 = 0 \quad V = 12$$

$$\sum M_D = 0$$

$$-H - 12 \times 2 = 0 \quad H = -24$$



Barras ACDF

