CHAPTER 7
SIGNAL ENCODING TECHNIQUES

These slides are made available to faculty in PowerPoint form. Slides can be freely added, modified, and deleted to suit student needs. They represent substantial work on the part of the authors; therefore, we request the following.

If these slides are used in a class setting or posted on an internal or external www site, please mention the source textbook and note our copyright of this material.

All material copyright 2016 Cory Beard and William Stallings, All Rights Reserved
7.1 ENCODING AND MODULATION TECHNIQUES

(a) Encoding onto a digital signal

(b) Modulation onto an analog signal
REASONS FOR CHOOSING ENCODING TECHNIQUES

• Digital data, digital signal
  – Equipment less complex and expensive than digital-to-analog modulation equipment

• Analog data, digital signal
  – Permits use of modern digital transmission and switching equipment
REASONS FOR CHOOSING ENCODING TECHNIQUES

• Digital data, analog signal
  – Some transmission media will only propagate analog signals
  – E.g., optical fiber and unguided media

• Analog data, analog signal
  – Analog data in electrical form can be transmitted easily and cheaply
  – Done with voice transmission over voice-grade lines
**SIGNAL ENCODING CRITERIA**

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio
  - Data rate
  - Bandwidth
- An increase in data rate increases bit error rate
- An increase in SNR decreases bit error rate
- An increase in bandwidth allows an increase in data rate
FACTORS USED TO COMPARE ENCODING SCHEMES

• Signal spectrum
  – With lack of high-frequency components, less bandwidth required
  – With no DC component, AC coupling via transformer is possible
  – Transfer function of a channel is worse near band edges

• Clocking
  – Ease of determining beginning and end of each bit position
FACTORS USED TO COMPARE ENCODING SCHEMES

• Signal interference and noise immunity
  – Performance in the presence of noise

• Cost and complexity
  – The higher the signal rate to achieve a given data rate, the greater the cost
BASIC ENCODING TECHNIQUES

- Digital data to analog signal
  - Amplitude-shift keying (ASK)
    - Amplitude difference of carrier frequency
  - Frequency-shift keying (FSK)
    - Frequency difference near carrier frequency
  - Phase-shift keying (PSK)
    - Phase of carrier signal shifted
7.2 MODULATION OF ANALOG SIGNALS FOR DIGITAL DATA

(a) ASK

(b) BFSK

(c) BPSK
AMPLITUDE-SHIFT KEYING

• One binary digit represented by presence of carrier, at constant amplitude
• Other binary digit represented by absence of carrier

\[ s(t) = \begin{cases} 
A \cos(2\pi f_c t) & \text{binary 1} \\
0 & \text{binary 0}
\end{cases} \]

• where the carrier signal is \( A \cos(2\pi f_c t) \)
AMPLITUDE-SHIFT KEYING

- Susceptible to sudden gain changes
- Inefficient modulation technique
- Used to transmit digital data over optical fiber
BINARY FREQUENCY-SHIFT KEYING (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

\[ s(t) = \begin{cases} 
A\cos\left(2\pi f_1 t\right) & \text{binary 1} \\
A\cos\left(2\pi f_2 t\right) & \text{binary 0}
\end{cases} \]

- where \( f_1 \) and \( f_2 \) are offset from carrier frequency \( f_c \) by equal but opposite amounts \( f_d \)
BINARY FREQUENCY-SHIFT KEYING (BFSK)

• Less susceptible to error than ASK
• Used for high-frequency (3 to 30 MHz) radio transmission
• Can be used at higher frequencies on LANs that use coaxial cable
7.3 FULL-DUPEX FSK TRANSMISSION ON A VOICE GRADE CHANNEL
MULTIPLE FREQUENCY-SHIFT KEYING (MFSK)

• More than two frequencies are used
• More bandwidth efficient but more susceptible to error

\[ s_i(t) = A \cos(2\pi f_i t) \quad 1 \leq i \leq M \]

- \( f_i = f_c + (2i - 1 - M)f_d \)
- \( f_c \) = the carrier frequency
- \( f_d \) = the difference frequency
- \( M \) = number of different signal elements = \( 2^L \)
- \( L \) = number of bits per signal element
MULTIPLE FREQUENCY-SHIFT KEYING (MFSK)

- To match data rate of input bit stream, each output signal element is held for:
  
  \[ T_s = LT \text{ seconds} \]

  - where \( T \) is the bit period (data rate = \( 1/T \))

- So, one signal element encodes \( L \) bits
MULTIPLE FREQUENCY-SHIFT KEYING (MFSK)

• Total bandwidth required
  \[ 2Mf_d \]

• Minimum frequency separation required
  \[ 2f_d = \frac{1}{T_s} \]

• Therefore, modulator requires a bandwidth of
  \[ W_d = 2^L/LT = M/T_s \]
PHASE-SHIFT KEYING (PSK)

• Two-level PSK (BPSK)
  - Uses two phases to represent binary digits

\[ s(t) = \begin{cases} 
A\cos(2\pi f_c t) & \text{binary 1} \\
A\cos(2\pi f_c t + \pi) & \text{binary 0}
\end{cases} \]

\[ = \begin{cases} 
A\cos(2\pi f_c t) & \text{binary 1} \\
-A\cos(2\pi f_c t) & \text{binary 0}
\end{cases} \]
PHASE-SHIFT KEYING (PSK)

• Differential PSK (DPSK)
  – Phase shift with reference to previous bit
    • Binary 0 – signal burst of same phase as previous signal burst
    • Binary 1 – signal burst of opposite phase to previous signal burst
7.5 DIFFERENTIAL PHASE-SHIFT KEYING
QUADRATURE PHASE-SHIFT KEYING (PSK)

- Four-level PSK (QPSK)
  - Each element represents more than one bit

\[ s(t) = \begin{cases} 
A \cos \left( 2\pi f_c t + \frac{\pi}{4} \right) & 11 \\
A \cos \left( 2\pi f_c t + \frac{3\pi}{4} \right) & 01 \\
A \cos \left( 2\pi f_c t - \frac{3\pi}{4} \right) & 00 \\
A \cos \left( 2\pi f_c t - \frac{\pi}{4} \right) & 10 
\end{cases} \]
7.6 QPSK CONSTELLATION DIAGRAM
7.7 QPSK AND OQPSK MODULATORS

Signal Encoding Techniques 7-23
PHASE-SHIFT KEYING (PSK)

- Multilevel PSK
  - Using multiple phase angles with each angle having more than one amplitude, multiple signal elements can be achieved

\[ D = \frac{R}{L} = \frac{R}{\log_2 M} \]

- \( D \) = modulation rate, baud or symbols/sec
- \( R \) = data rate, bps
- \( M \) = number of different signal elements = \( 2^L \)
- \( L \) = number of bits per signal element
PERFORMANCE

• Bandwidth of modulated signal ($B_T$)
  – ASK, PSK  $B_T = (1+r)R$
  – FSK  $B_T = 2\Delta f + (1+r)R$

• $R =$ bit rate
• $0 < r < 1;$ related to how signal is filtered
• $\Delta f = f_2 - f_c = f_c - f_1$
PERFORMANCE

• Bandwidth of modulated signal ($B_T$)

  – MPSK

  \[
  B_T = \left( \frac{1+r}{L} \right) R = \left( \frac{1+r}{\log_2 M} \right) R
  \]

  – MFSK

  \[
  B_T = \left( \frac{(1+r)M}{\log_2 M} \right) R
  \]

• $L =$ number of bits encoded per signal element

• $M =$ number of different signal elements
BIT ERROR RATE (BER)

• Performance must be assessed in the presence of noise
• “Bit error probability” is probably a clearer term
  – BER is not a rate in bits/sec, but rather a probability
  – Commonly plotted on a log scale in the y-axis and $E_b/N_0$ in dB on the x-axis
  – As $E_b/N_0$ increases, BER drops
• Curves to the lower left have better performance
  – Lower BER at the same $E_b/N_0$
  – Lower $E_b/N_0$ for the same BER
• BPSK outperforms other schemes in Figure 7.9
BER CALCULATION EXAMPLES

- Bit Error Rate (BER) calculation based on the modulation scheme:

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>$Q\left(\sqrt{\frac{2 \cdot E_b}{N_0}}\right)$</td>
</tr>
<tr>
<td>DPSK</td>
<td>$0.5 \cdot e^{-\frac{E_b}{N_0}}$</td>
</tr>
<tr>
<td>QPSK</td>
<td>$Q\left(\sqrt{\frac{2 \cdot E_b}{N_0}}\right)$</td>
</tr>
<tr>
<td>MFSK (coherently detected)</td>
<td>$\leq (M - 1) \cdot Q\left(\sqrt{\frac{\log_2(M) \cdot E_b}{N_0}}\right)$</td>
</tr>
</tbody>
</table>

- Q-function: the area under the Gaussian tail
  
  $Q(k) = P(X > \mu + k\sigma) = \frac{1}{\sqrt{2\pi}} \int_k^{+\infty} e^{-\lambda^2/2} d\lambda$
Q-FUNCTION TABLE

<table>
<thead>
<tr>
<th>Q</th>
<th>Q Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.000 000e-01</td>
</tr>
<tr>
<td>0.1</td>
<td>4.601 722e-01</td>
</tr>
<tr>
<td>0.2</td>
<td>4.207 403e-01</td>
</tr>
<tr>
<td>0.3</td>
<td>3.820 886e-01</td>
</tr>
<tr>
<td>0.4</td>
<td>3.445 783e-01</td>
</tr>
<tr>
<td>0.5</td>
<td>3.085 375e-01</td>
</tr>
<tr>
<td>0.6</td>
<td>2.742 531e-01</td>
</tr>
<tr>
<td>0.7</td>
<td>2.419 637e-01</td>
</tr>
<tr>
<td>0.8</td>
<td>2.118 554e-01</td>
</tr>
<tr>
<td>0.9</td>
<td>1.840 601e-01</td>
</tr>
<tr>
<td>1.0</td>
<td>1.586 553e-01</td>
</tr>
<tr>
<td>1.1</td>
<td>1.356 661e-01</td>
</tr>
<tr>
<td>1.2</td>
<td>1.150 697e-01</td>
</tr>
<tr>
<td>1.3</td>
<td>9.680 049e-02</td>
</tr>
<tr>
<td>1.4</td>
<td>8.075 666e-02</td>
</tr>
<tr>
<td>1.5</td>
<td>6.680 720e-02</td>
</tr>
<tr>
<td>1.6</td>
<td>5.479 929e-02</td>
</tr>
<tr>
<td>1.7</td>
<td>4.456 546e-02</td>
</tr>
<tr>
<td>1.8</td>
<td>3.593 032e-02</td>
</tr>
<tr>
<td>1.9</td>
<td>2.871 656e-02</td>
</tr>
<tr>
<td>2.0</td>
<td>2.275 013e-02</td>
</tr>
<tr>
<td>2.1</td>
<td>1.786 442e-02</td>
</tr>
<tr>
<td>2.2</td>
<td>1.390 345e-02</td>
</tr>
<tr>
<td>2.3</td>
<td>1.072 411e-02</td>
</tr>
</tbody>
</table>
7.9 THEORETICAL BIT ERROR RATE FOR VARIOUS ENCODING SCHEMES

![Graph showing the theoretical bit error rate for various encoding schemes. The graph plots probability of bit error (BER) against \((E_b/N_0)\) (dB). Curves are labeled for ASK, BFSK, DPSK, and BPSK.]
7.10 THEORETICAL BIT ERROR RATE FOR MULTILEVEL
FSK, PSK, AND QAM

(a) Multilevel FSK (MFSK)

(b) Multilevel PSK (MPSK) and 16 QAM
FRAME ERROR MODEL

- Frame Error Rate (FER) calculation based on the BER and frame size:
  - Consider a packet of length L bits.
  - Assume a Binary Symmetric Channel: binary channel with flipping probability equal to BER, constant along the frame.
  - Assume that the Frame Check Sequence (accounted for in the L) assures error detection with at least 1 flipped bit.

- The FER is calculated as:
  - $FER = 1 - (1 - BER)^L$

- Similar calculations apply to packets and blocks of data in general.
- Home Exercise: how about frames comprising blocks with different BER?

- ATTN: Sometimes FER is equivalently designated Packet Error Rate (PER). PER is generic to all layers. PHY and MAC PDUs in particular are called frames.
RECEIVER SENSITIVITY

• The minimum RF signal power level required at the input of a receiver for certain performance (e.g. BER or FER)
• E.g., IEEE 802.11 standard considers a maximum acceptable FER < 1% for 20-octet Physical layer Service Data Units (PSDUs), without interference.
• In order to simplify system analysis, it is considered that reception power below the receiver sensitivity does not allow communication.
• Spectral Efficiency for different modulations:

- In the figure, $W \equiv \text{Signal Bandwidth } (B_T), P_M \equiv \text{Symbol Error Rate (SER)}$
QUADRATURE AMPLITUDE MODULATION

- QAM is a combination of ASK and PSK
  - Two different signals sent simultaneously on the same carrier frequency

\[ s(t) = I(t) \cos(2\pi f_c t) + Q(t) \sin(2\pi f_c t) \]
7.11 16QAM CONSTELLATION DIAGRAM
7.12 QAM MODULATOR

Binary input $d(t)$ R bps

2-bit serial-to-parallel converter

$d_1(t)$ R/2 bps

Carrier oscillator

$\cos 2\pi f_c t$

Phase shift $-\pi/2$

$d_2(t)$ R/2 bps

$\sin 2\pi f_c t$

+ +

$\sum$

QAM signal out $s(t)$
HIERARCHICAL MODULATION

- DVB-T modulates two separate data streams onto a single DVB-T stream
- High Priority (HP) embedded within a Low Priority (LP) stream
- Multi carrier system, about 2000 or 8000 carriers
- QPSK, 16 QAM, 64QAM
- Example: 64QAM
  - good reception: resolve the entire 64QAM constellation
  - poor reception, mobile reception: resolve only QPSK portion
  - 6 bit per QAM symbol, 2 most significant determine QPSK
  - HP service coded in QPSK (2 bit), LP uses remaining 4 bit
REASONS FOR ANALOG MODULATION

• Modulation of digital signals
  – When only analog transmission facilities are available, digital to analog conversion required

• Modulation of analog signals
  – A higher frequency may be needed for effective transmission
  – Modulation permits frequency division multiplexing
BASIC ENCODING TECHNIQUES

• Analog data to analog signal
  – Amplitude modulation (AM)
  – Angle modulation
    • Frequency modulation (FM)
    • Phase modulation (PM)