Localization
Fundamentals

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Course Handouts
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Bibliography

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Localization

Where Am I?

Localization Problem:
Determine the robot
*posture* (position + orientation)
at each time instant
Localization

What do I know about this place?

I have a map !!!

Nothing ..... 

Structured Environments
- Industry
- Hospitals
- Laboratories

Unstructured Environments
- Open air
- Underwater
- Space
Localization

What do I know about this place?

I have a map !!!

Nothing ..... Do I need a map ?

SLAM
Simultaneous Localization and Mapping

I can build a map while moving and simultaneously self-localizing

Yes No
Localization Methodologies

Relative Localization (Localization with relative measurements)
- Odometry
  Mobile robot localization through wheel motion evaluation
- Inertial Navigation
  Mobile robot localization through its motion state evaluation (velocities and accelerations)

Absolute Localization
- Active beacons
  Computes absolute location by measuring the direction of incidence (or the distance to) 3 or more active beacons. Transmitter locations must be known in inertial frame
- Artificial and Natural Landmarks
  Landmarks are located in known environment places, or they are detected in the environment. Same methods used for active beacons apply.
- Model matching
  Information from robot sensors is compared to a map or world model. Matching sensor-based and world model maps, vehicle’s absolute pose is estimated
  This can be used to update the world map over time

Relative Localization + Absolute Localization
Localization Methodologies

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**Relative Localization + Absolute Localization**
Relative Localization: Odometry

- Uses encoders to measure the distance travelled by each wheel
  - From the robot kinematics, the translation and rotation of the robot frame is evaluated.
- Absolute position estimation results from the integration of relative translation and orientation between two encoder readings.
- Odometry performance is a function of the vehicle’s kinematics.

\[
v_r(t) = v_d(t) - \text{linear velocity of right wheel}
\]
\[
v_l(t) = v_e(t) - \text{linear velocity of left wheel}
\]

- Each drive wheel has an associated encoder.
- Encoder measures the travelled distance in a given time interval \(\Delta T\).
Relative Localization: Odometry

Kinematics model (for unicycle)

\[
\begin{align*}
\dot{x}(t) &= \frac{v_r(t) + v_l(t)}{2} \cos(\theta(t)) \\
\dot{y}(t) &= \frac{v_r(t) + v_l(t)}{2} \sin(\theta(t)) \\
\dot{\theta}(t) &= \frac{v_r(t) - v_l(t)}{L}
\end{align*}
\]

Assume that

- we know
  \[
  \begin{bmatrix}
  x(t = k\Delta T) \\
  y(t = k\Delta T) \\
  \theta(t = k\Delta T)
  \end{bmatrix} = \begin{bmatrix}
  x(k) \\
  y(k) \\
  \theta(k)
  \end{bmatrix}
  \]
- we measure
  \[\Delta D_l(k), \ \Delta D_r(k)\]

We want to estimate

\[
\begin{bmatrix}
x(k + 1) \\
y(k + 1) \\
\theta(k + 1)
\end{bmatrix} = ?
\]

Distance travelled by each wheel in the interval

\[\Delta T, (k + 1)\Delta T\]
Discretization of Kinematics Model

\begin{align*}
\dot{x}(t) &= \frac{v_r(t) + v_l(t)}{2} \cos(\theta(t)) \\
\dot{y}(t) &= \frac{v_r(t) + v_l(t)}{2} \sin(\theta(t)) \\
\dot{\theta}(t) &= \frac{v_r(t) - v_l(t)}{L}
\end{align*}

For \( t > t_k \)

\begin{align*}
x(t) &= x(t_k) + \int_{t_k}^{t} \frac{v_r(\tau) + v_l(\tau)}{2} \cos(\theta(\tau)) \, d\tau \\
y(t) &= y(t_k) + \int_{t_k}^{t} \frac{v_r(\tau) + v_l(\tau)}{2} \sin(\theta(\tau)) \, d\tau \\
\theta(t) &= \theta(t_k) + \int_{t_k}^{t} \frac{v_r(\tau) - v_l(\tau)}{L} \, d\tau
\end{align*}

For \( t = t_k + 1 \)

\begin{align*}
x(t_{k+1}) &= x(t_k) + \int_{t_k}^{t_{k+1}} \frac{v_r(\tau) + v_l(\tau)}{2} \cos(\theta(\tau)) \, d\tau \\
y(t_{k+1}) &= y(t_k) + \int_{t_k}^{t_{k+1}} \frac{v_r(\tau) + v_l(\tau)}{2} \sin(\theta(\tau)) \, d\tau \\
\theta(t_{k+1}) &= \theta(t_k) + \int_{t_k}^{t_{k+1}} \frac{v_r(\tau) - v_l(\tau)}{L} \, d\tau
\end{align*}
Discretization of Kinematics Model

What is the path in the interval $[(k\Delta T_k) \Delta T]$?

What is the profile of the velocity of each wheel in the interval $[(k\Delta T_k) \Delta T]$?

Assumption

Translation followed by a rotation

Approximation

$$
\begin{align*}
\Delta D_l(k) &= T(k) - \Delta \theta(k) \frac{L}{2} \\
\Delta D_r(k) &= T(k) + \Delta \theta(k) \frac{L}{2}
\end{align*}
$$

$$
\begin{align*}
\Delta D(k) &= \frac{\Delta D_r(k) + \Delta D_l(k)}{2} = T(k) \\
\Delta \theta(k) &= \frac{\Delta D_r(k) - \Delta D_l(k)}{L} = \Delta \theta(k)
\end{align*}
$$
Given the encoder measurements \( \Delta D_l(k), \Delta D_r(k) \):

\[
\Delta D(k) = \frac{\Delta D_r(k) + \Delta D_l(k)}{2} \quad \Delta \theta(k) = \frac{\Delta D_r(k) - \Delta D_l(k)}{L}
\]

Relative to the mid-point between rear wheels

State Noise accounts for uncertainty in the model:
- wheel deformation and slippage
- vibration
- encoder readings
Odometry | Pose estimation

- **Given**
  - Model of the system
  - Localization estimate at time instant $k$
  - Associated uncertainty
  - Odometry readings
  - Error characterization

- **Question:**
  - Which is the estimate at time instant $k+1$? $\hat{X}(k+1)$
  - Which is the associated uncertainty? $\Sigma_X(k+1)$

\[
X(k+1) = f(X(k), \Delta D(k), \Delta \theta(k)) + v(k)
\]

Assumed Gaussian
Odometry | Pose estimation

- Location estimate

\[
\hat{X}(k + 1) = f(\hat{X}(k), \Delta D(k), \Delta \theta(k))
\]

- Uncertainty – covariance matrix (with approximations)

- Expansion of \( f(X(k), \Delta D, \Delta \theta) \) in Taylor series around \( \hat{X}(k), \Delta D(k), \Delta \theta(k) \)
- Neglecting higher order terms
- \( X(k), \Delta D e \Delta \theta \) are uncorrelated

\[
\Sigma_X(k + 1) \approx F \Sigma_X(k) F^T + Q(k)
\]

\[
F = \frac{\partial f}{\partial X} \bigg|_{\hat{X}(k), \Delta D(k), \Delta \theta(k)}
\]

This is the prediction stage of an EKF
Odometry Errors (1)

• Because the sensor measurements are integrated, the position error by odometry accumulates over time

• Sources of errors
  ➢ Systematic Errors
    o Limited resolution during integration
    o Misalignment of the wheels
    o Uncertainty on the wheel diameter
    o Variation in the contact point of the wheel
    o Unequal floor contact
  ➢ Non Systematic Errors
    o Wheel slippage due to soil conditions
    o Fast rotation motion
    o Large vehicle accelerations
• Motion command → equal velocity in both wheels
• Differential control

Odometry Errors (2)

Non systematic error

Real trajectory

Trajectory estimated by odometry
Odometry Experiments

Odometry is not accurate during large time intervals

To decrease the error, it is necessary to anchor to any feature in the environment.

When to do that?
• Periodically
• When uncertainty is too large

Elipsis - 90% probability of the robot being inside

The uncertainty increases along the motion
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**Relative Localization + Absolute Localization**
Active Beacon and Landmark-Based Absolute Localization Systems

- **Active Beacons**
  - Most common navigation aids on ships and airplanes
  - Provide very accurate positioning information with minimal processing
  - High cost in installation and maintenance (require power)
- **Landmarks**
  - **Natural**: no installation cost, recognition may be difficult
  - **Artificial**: passive, easier recognition, lower installation costs

- Two different types of absolute localization methods:
  - **Trilateration**
  - **Triangulation**
Trilateration (1)

**TRILATERATION**

Determination of vehicle’s position based on distance measurements to known beacon (active or passive) sources

**Usual configurations**

- **active beacons**: 3 or more transmitters mounted at known locations in the environment and one receiver on board the robot
- **landmarks**: one transmitter on-board and receivers/transponders/retroreflectors mounted on the environment; on board camera to acquire landmark images and recognize them

**Examples**

- GPS
- LIDAR
Trilateration (2)

How can TRILATERATION presented as a ranging method, support the evaluation of the ABSOLUTE POSITION of a land mobile robot moving in 2D?

• PROBLEM
  • I know the position of points P1, P2, P3
  • I am at a distance
    • d1 from P1
    • d2 from P2
    • d3 from P3
  • Where am I (x,y)?

\[
\begin{align*}
(x - x_1)^2 + (y - y_1)^2 &= d_1^2 \\
(x - x_2)^2 + (y - y_2)^2 &= d_2^2 \\
(x - x_3)^2 + (y - y_3)^2 &= d_3^2 \\
\end{align*}
\]

Removes ambiguity

With only 2, there is (might be) ambiguity

(\text{I am here})
Trilateration with Passive Artificial Landmarks

LGV - Laser Guided Vehicle, Soporcel – Fábrica de Papel
2D Trilateration: Particular Case I

- Robot colinear with 2 beacons

In this case, the distance to the 2 beacons yields with no ambiguity the robot position.

- Which is the uncertainty due to measurement uncertainty?

Geometric reasoning

How can we handle this uncertainty in a probabilistic framework?
2D Trilateration: particular case II

- 3 colinear beacons

With 3 colinear beacons, the ambiguity is not solved.

I am here or here.
The **Global Positioning System** is a worldwide radio-navigation system formed from a constellation of 24 satellites and their ground stations.

- 24 satellites
- 1 orbit = 12 hours
- Altitude = 11 000 nautical miles above the earth, app. 20 200 Km)

Each satellite transmits data via high frequency radio waves back to Earth that can be received by anyone with a GPS receiver.

**Ground Station (control segments)**

- These stations monitor the GPS satellites, checking both their operational health and their exact position in space.
- There is a Master Station and a set of monitor stations.
GPS | Working Principle

- **POSITION** of GPS receiver (user) determined by *trilateration* based on TOF

- **POSITION**
  - Latitude - unknown
  - Longitude - unknown
  - Altitude – unknown

- **3 unknowns. Are 3 satellites enough?**
  - Distance to 3 satellites is measured → Trilateration

- Distance to satellite
  - Obtained from travel time of signals emitted by the satellites
  - GPS needs very accurate timing

- **PROBLEM**
  - Receiver clock and satellites clocks have different offsets w.r.t. standard time

- **SOLUTION**
  - 4 satellites are used
  - And $\Delta T = (t_{\text{clock receiver}} - t_{\text{clock satellite}})$ is included in the unknowns
  - 4 equations, 4 unknowns
GPS | Technical Challenges

1. Time synchronization between individual satellites and GPS receivers
2. Precise real-time location of satellite position
3. Accurate measurement of signal propagation time
4. Sufficient signal-to-noise ratio for reliable operation in the presence of interference and possible jamming

Examples of sources of errors

Atmospheric refraction and multipath reflections contribute to pseudorange measurements errors, specially at low elevation angles

Worst case geometric errors occur when the receiver and satellite approach a collinear configuration

Taken from Everett
GPS may have large errors – low accuracy (e.g., 100m)

**DGPS**

- A second GPS receiver is installed fairly close (several tens of kilometers) to the first (user).
- This second GPS will experience basically the same error effects when viewing identical reference satellites.
  - Satellite constellation is far away
- If the second GPS is fixed at a precise surveyed location, its calculated solution can be compared to the known position to generate a composite error vector
- This differential correction can then be passed to the first receiver to null out the unwanted effects
  - Reduces positioning errors
  - Commercial DGPS have errors well under 10 meters.
Determination of **vehicle’s pose** \((x, y, \theta)\) based on the evaluation of the **angles**, \(\alpha_1\), \(\alpha_2\), \(\alpha_3\) between the robot longitudinal axis and the direction with which three beacons installed on the environment at known positions are detected.
TRIANGULATION with three colinear beacons

First Step: Locate the vehicle in the beacons frame

A priori knowledge
Coordinates of $P_1$, $P_2$ and $P_3$

Unkowns
$x_L, y_L, \theta_L$

Acquired data
Angles: $\alpha_1, \alpha_2, \alpha_3$

Vehicle localization in the frame L
TRIANGULATION with three colinear beacons

\[
\begin{align*}
\beta_1 &= \alpha_1 - \alpha_2 \\
\beta_2 &= \alpha_2 - \alpha_3
\end{align*}
\]

From the law of Sines:

From triangle \( PP_2 P_3 \):

\[
\frac{d_2}{\sin \beta_2} = \frac{b}{\sin [180^\circ - (\theta_2 + \beta_2)]}
\] (1)

From triangle \( PP_2 P_1 \):

\[
\frac{d_1}{\sin \beta_1} = \frac{b}{\sin \theta_1}
\] (2)

Also,

\[
\theta_1 + \beta_1 + (180^\circ - \theta_2) = 180^\circ
\]

\[
\theta_1 = \theta_2 - \beta_1
\]

Replacing (3) in (1) and (2):

\[
\tan \theta_2 = \frac{(d_1 + d_2) \tan \beta_2 \cdot \tan \beta_1}{d_1 \tan \beta_2 - d_2 \tan \beta_1}
\]
TRIANGULATION with three colinear beacons

\[ x_L = b \cos \theta_2 \]
\[ y_L = x_L \tan \theta_2 \]

\[ x_L = \frac{d_1}{\tan \beta_1} \cdot \frac{\tan \theta_2 - \tan \beta_1}{1 + \tan^2 \theta_2} \]

\[ y_L = \frac{d_1}{\tan \beta_1} \cdot \frac{\tan \theta_2 - \tan \beta_1}{1 + \tan^2 \theta_2} \cdot \tan \theta_2 \]

\[ \theta_L = 2\pi - \alpha_1 - \theta_2 + \beta_1 \]