Problem 1

Consider the single isolated bored pile constructed in a sandy soil as illustrated in Figure 1. The concrete pile has a length, \( L = 15 \text{ m} \) and circular cross-section with the diameter, \( B = 800 \text{ mm} \), and the Young’s modulus for the concrete is \( E = 30 \text{ GPa} \). The sandy soil gives a bearing capacity factor, \( N_q = 60 \), lateral pressure coefficient, \( K_s = 0.45 \), pile-soil friction angle, \( \delta = 28^\circ \), saturated unit weight \( \gamma_{sat} = 20 \text{ kN/m}^3 \) and Young’s modulus, \( E_s = 30 \text{ MPa} \).

a) Determine the ultimate resistance of the pile in compression and tension.

b) Using Eurocode 7, determine the design resistance for the pile.

c) Given that the pile is subject to compressive vertical loading comprising a permanent action, \( F_G = 1500 \text{ kN} \) and a variable action, \( F_Q = 500 \text{ kN} \); verify the safety of the pile with respect to the bearing capacity ULS.

d) Evaluate the settlement of the pile for the actions indicated in (c).

Problem 2

Consider the single isolated driven pile constructed in a clayey soil as illustrated in Figure 2. The concrete pile has a length, \( L = 15 \text{ m} \) and square cross-section of width, \( B = 350 \text{ mm} \). The clayey soil has an undrained shear strength, \( c_u = 20 + 5z \text{ kPa} \) (\( z \) is the depth from surface), a bearing capacity factor, \( N_c = 9 \), adhesion factor, \( \alpha = 0.80 \), and saturated unit weight \( \gamma_{sat} = 20 \text{ kN/m}^3 \).

a) Determine the ultimate resistance of the pile in compression and tension.

b) Using Eurocode 7, determine the design resistance for the pile.

c) Given that the pile is subject to compressive vertical loading comprising a permanent action, \( F_G = 380 \text{ kN} \) and a variable action, \( F_Q = 150 \text{ kN} \); verify the safety of the pile with respect to the bearing capacity ULS.
Problem 3

Consider the hyperbolic functions in Table 1 & Figure 3 to represent the load-displacement response observed for a set of four static compression load tests on 600 mm diameter continuous flight auger (CFA) type piles. Using these results determine the pile design compression resistance using Eurocode 7.

<table>
<thead>
<tr>
<th>Ensaio de carga</th>
<th>Relação carga-assentamento (curvas de ajustamento)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Q(s) = 3900s/(s+20) )</td>
</tr>
<tr>
<td>2</td>
<td>( Q(s) = 3700s/(s+22) )</td>
</tr>
<tr>
<td>3</td>
<td>( Q(s) = 4200s/(s+24) )</td>
</tr>
<tr>
<td>4</td>
<td>( Q(s) = 3820s/(s+27) )</td>
</tr>
</tbody>
</table>

![Graph of load-displacement response](image-url)
Problem 4

Consider the single isolated bored pile constructed in a silty sand using bentonite mud to support the pile bore during excavation. The concrete pile has a length, \( L = 12 \) m and circular cross-section with the diameter, \( B = 600 \) mm, and the Young’s modulus for the concrete is \( E = 30 \) GPa. The silty sand has a saturated unit weight \( \gamma_{sat} = 20 \) kN/m\(^3\).

The soil profile has been characterized with a 16 m deep borehole with Standard Penetration Tests (SPT) at 2 m centres (Table 2). In addition to the borehole, a single Cone Penetration Test (CPT) was also completed and the following profile of CPT cone resistance was obtained:

\[
q_c = 3.1 + 0.54z \text{ MPa, } 0 < z < 10 \text{ m}
\]
\[
q_c = 16 \text{ MPa, } z > 10 \text{ m}
\]

Based on these in situ test results, calculate the compression resistance of the pile.

<table>
<thead>
<tr>
<th>( z \text{ (m)} )</th>
<th>( N_{SPT} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>40</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
</tr>
</tbody>
</table>
Formulas

Bearing capacity of single pile in compression and tension

<table>
<thead>
<tr>
<th>Compression resistance:</th>
<th>Effective stress design:</th>
<th>Total stress design:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_c = R_b + R_s )</td>
<td>( R_b = A_b.q_b = A_b.N_q \sigma_{v,b} )</td>
<td>( R_c ) Total compression resistance, kN</td>
</tr>
<tr>
<td>Tension resistance:</td>
<td>( R_s = A_s.q_s = A_s.K_s \sigma_{v,avg} \tan \delta )</td>
<td>( R_t ) Tension resistance, kN</td>
</tr>
<tr>
<td>( R_t = R_s )</td>
<td></td>
<td>( R_b ) Base resistance, kN</td>
</tr>
<tr>
<td></td>
<td>Total stress design:</td>
<td>( R_s ) Shaft resistance, kN</td>
</tr>
<tr>
<td></td>
<td>( R_b = A_b.q_b = A_b.N_c \sigma_{u,b} )</td>
<td>( A_b ) Base area of pile, m^2</td>
</tr>
<tr>
<td></td>
<td>( R_s = A_s.q_s = A_s.\alpha \sigma_{u,avg} )</td>
<td>( A_s ) Surface area of pile shaft, m^2</td>
</tr>
<tr>
<td></td>
<td>( \sigma_{v,b} ) Vertical effective stress at pile tip, kPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma_{v,avg} ) Average vertical effective stress, kPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma_n ) Confining pressure on pile shaft, kPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( N_q ) Bearing capacity factor, -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( K_s ) Ratio ( \sigma_n / \sigma_{v,avg} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \delta ) Pile-soil friction angle, degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( c_{u,b} ) Undrained shear strength at pile tip, kPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( c_{u,avg} ) Average undrained shear strength, kPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( N_c ) Bearing capacity factor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \alpha ) Pile adhesion factor, ( q_s / c_{u,avg} )</td>
<td></td>
</tr>
</tbody>
</table>
Settlement of floating pile in elastic half-space

The settlement of a pile may be calculated using the following expression:

\[ S = \frac{Q \cdot I}{E_s d} \]
where,
\[ Q = \text{applied load} \]
\[ D = \text{pile diameter} \]
\[ E_s = \text{Soil Young's modulus of elasticity} \]
\[ I = \text{Influence factor for an incompressible pile in an elastic half-space with Poisson's ratio,} \ \nu = 0.5 \]
\[ R_k = \text{correction factor for pile compressibility} \]
\[ R_h = \text{correction factor for layer of finite thickness} \]
\[ R_v = \text{correction factor for Poisson's ratio} \]

\[ K = \frac{E}{E_s} R_A \text{ with } R_A = \frac{A}{\pi d^2 / 4} \]

\[ E = \text{Pile Young's modulus of elasticity} \]
\[ A = \text{Cross-sectional area of pile} \]
### Eurocode 7 partial factors for axially loaded piles

**Design Approach 1**
- Combination 1 (DA1-C1): “A1” + “M1” + “R1”
- Combination 2 (DA1-C2): “A2” + “M1” + “R4”

#### Table A.3 - Partial factors on actions ($\gamma_f$) or the effect of actions ($\gamma_E$):

<table>
<thead>
<tr>
<th>Action</th>
<th>Effect</th>
<th>Symbol</th>
<th>“A1”</th>
<th>“A2”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
<td>Unfavourable</td>
<td>$\gamma_{G,unf}$</td>
<td>1.35</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Favourable</td>
<td>$\gamma_{G,fav}$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Variable</td>
<td>Unfavourable</td>
<td>$\gamma_{Q,unf}$</td>
<td>1.50</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>Favourable</td>
<td>$\gamma_{Q,fav}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

#### Table A.4 - Partial factors for soil parameters ($\gamma_M$):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Set “M1”</th>
<th>“M2”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of shearing resistance</td>
<td>$\varphi'$</td>
<td>1.00</td>
<td>1.25</td>
</tr>
<tr>
<td>Effective cohesion</td>
<td>$c'$</td>
<td>1.00</td>
<td>1.25</td>
</tr>
<tr>
<td>Undrained shear strength</td>
<td>$c_u$</td>
<td>1.00</td>
<td>1.40</td>
</tr>
<tr>
<td>Unconfined shear strength</td>
<td>$c_q$</td>
<td>1.00</td>
<td>1.40</td>
</tr>
<tr>
<td>Weight density</td>
<td>$\gamma_T$</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td><strong>Note:</strong> applied to $\tan \varphi'$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Table A.6 - Partial resistance factors ($\gamma_R$) for driven piles:

<table>
<thead>
<tr>
<th>Resistance</th>
<th>Symbol</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>$\gamma_b$</td>
<td>1.00</td>
<td>1.10</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>Shaft</td>
<td>$\gamma_s$</td>
<td>1.00</td>
<td>1.10</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>Total (compression)</td>
<td>$\gamma_t$</td>
<td>1.00</td>
<td>1.10</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>Tension</td>
<td>$\gamma_{s;t}$</td>
<td>1.25</td>
<td>1.15</td>
<td>1.10</td>
<td>1.60</td>
</tr>
</tbody>
</table>

#### Table A.7 - Partial resistance factors ($\gamma_R$) bored piles:

<table>
<thead>
<tr>
<th>Resistance</th>
<th>Symbol</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>$\gamma_b$</td>
<td>1.25</td>
<td>1.10</td>
<td>1.00</td>
<td>1.60</td>
</tr>
<tr>
<td>Shaft</td>
<td>$\gamma_s$</td>
<td>1.00</td>
<td>1.10</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>Total (compression)</td>
<td>$\gamma_t$</td>
<td>1.15</td>
<td>1.10</td>
<td>1.00</td>
<td>1.50</td>
</tr>
<tr>
<td>Tension</td>
<td>$\gamma_{s;t}$</td>
<td>1.25</td>
<td>1.15</td>
<td>1.10</td>
<td>1.60</td>
</tr>
</tbody>
</table>

#### Table A.8 - Partial resistance factors ($\gamma_R$) continuous flight auger (CFA) piles:

<table>
<thead>
<tr>
<th>Resistance</th>
<th>Symbol</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>$\gamma_b$</td>
<td>1.10</td>
<td>1.00</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>Shaft</td>
<td>$\gamma_s$</td>
<td>1.10</td>
<td>1.00</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>Total (compression)</td>
<td>$\gamma_t$</td>
<td>1.10</td>
<td>1.00</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>Tension</td>
<td>$\gamma_{s;t}$</td>
<td>1.15</td>
<td>1.10</td>
<td>1.60</td>
<td></td>
</tr>
</tbody>
</table>
Table A.9 - Correlation factors $\xi$ to derive characteristic values from static pile load tests  
(n - number of tested piles)

<table>
<thead>
<tr>
<th>$\xi$ for n =</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$\geq$5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>1.40</td>
<td>1.30</td>
<td>1.20</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>1.40</td>
<td>1.20</td>
<td>1.05</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table A.10 - Correlation factors $\xi$ to derive characteristic values from ground test results  
(n - number of tested piles)

<table>
<thead>
<tr>
<th>$\xi$ for n =</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>$\geq$10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>1.40</td>
<td>1.35</td>
<td>1.33</td>
<td>1.31</td>
<td>1.29</td>
<td>1.27</td>
<td>1.25</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>1.40</td>
<td>1.27</td>
<td>1.23</td>
<td>1.20</td>
<td>1.15</td>
<td>1.12</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Table A.11 - Correlation factors $\xi$ to derive characteristic values from dynamic impact tests  
(n - number of tested piles)

<table>
<thead>
<tr>
<th>$\xi$ for n =</th>
<th>$\geq$2</th>
<th>$\geq$5</th>
<th>$\geq$10</th>
<th>$\geq$15</th>
<th>$\geq$20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>1.60</td>
<td>1.50</td>
<td>1.45</td>
<td>1.42</td>
<td>1.40</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>1.50</td>
<td>1.35</td>
<td>1.30</td>
<td>1.25</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Problem 5

Consider a circular pile, 0.8 m diameter with a length of 20 m embedded in a homogeneous soil with a reaction modulus, k of 20000 kPa. The pile is formed from reinforced concrete with a Young’s modulus, E of 29 GPa, the pile head is free and subject to a horizontal load, V₀ of 100 kN, Figure 5.

a) Calculate the horizontal displacement at the pile head (y₀) using the general expressions (semi-flexible behaviour).
b) Repeat the calculation in a) but assuming that the pile is flexible.
c) Calculate the maximum moment, M_max in the pile.
d) Now assume that k = 10000 kPa and recalculate y₀ and M_max. Comment on the results.
e) Calculate the critical length, i.e. the length beyond which the pile is considered to have flexible behaviour (for k = 20000 kPa). Comment of the value obtained.

Problem 6

Consider the pile in Problem 5, but now consider that the head of the pile is fixed, i.e. is prevented from rotating, Figure 6.

a) Deduce the general function for displacements along the shaft of the pile and calculate the horizontal displacement at the pile head.
b) Calculate the maximum bending moment and compare with the value obtained in Problem 5.

Problem 7

Consider once again the pile in Problem 5, but now consider that the pile is subjected to loads V₀ = 100 kN and M₀ = 50 kNm, Figure 7.

a) Calculate the horizontal displacement at the pile head.
b) Calculate the maximum bending moment.

Problem 8

Consider a circular pile, 0.8 m diameter with a length of 20 m embedded in a sandy soil where the reaction modulus increases linearly with depth with, \( \eta = 5000 \) kPa/m. The pile is formed from reinforced concrete with a Young’s modulus, E of 29 GPa, the pile head is free and subject to a horizontal load, V₀ of 100 kN.

a) Calculate the horizontal displacement at the pile head and the maximum bending moment.
b) Compare these with the results obtained in Problem 5.
Formulas

Single pile in Winkler medium subject to transverse, \( V_0 \) load and/or moment, \( M_0 \) at the pile head

Relative stiffness of pile-soil system

a) Constant modulus of reaction, \( k \) (typically assumed for clayey soils):

\[
\lambda = 4 \sqrt[4]{\frac{k}{4E_p I_p}}
\]

b) Modulus of reaction increasing linearly with depth (\( k = n_x \cdot x \); typically for granular soils):

\[
\eta = \sqrt[5]{\frac{n_x}{E_p I_p}}
\]

where

- \( E_p \) Young’s modulus for the pile
- \( I_p \) Moment of inertia of pile
- \( x \) depth
- \( y \) horizontal displacement
- \( L \) pile length
- \( x' \) (L-x)
- \( \theta \) rotation
- \( V \) Transverse load
- \( M \) Applied moment
Free headed pile in uniform soil (reaction modulus, k constant), subject to transverse load, \( V_0 \) at the pile head

a) Flexible piles (\( \lambda L > 3.0 \))

\[
y = \frac{2V_0 \lambda}{k} \left( e^{-\lambda x} \cos \lambda x \right)
\]

\[
\theta = \frac{-2V_0 \lambda^2}{k} e^{-\lambda x} (\cos \lambda x + \sin \lambda x)
\]

\[
M = \frac{V_0}{k} (e^{-\lambda x} \sin \lambda x) \quad \text{&} \quad M_{max}(x = 0.79/\lambda) = 0.32 \frac{V_0}{k}
\]

\[
V = V_0 e^{-\lambda x} (\cos \lambda x - \sin \lambda x)
\]

b) Semi-flexible piles (\( 1.0 < \lambda L < 3.0 \))

\[
y = \frac{2V_0 \lambda}{k} K_{yV}
\]

\[
K_{yV} = \frac{\sinh \lambda L \cos \lambda x \cosh \lambda x' - \sin \lambda L \cosh \lambda x \cos \lambda x'}{\sinh^2 \lambda L - \sin^2 \lambda L}
\]

\[
\theta = \frac{-2V_0 \lambda^2}{k} K_{\theta V}
\]

\[
K_{\theta V} = \frac{\sinh \lambda L (\sin \lambda x \cosh \lambda x' + \cos \lambda x \sinh \lambda x') + \sin \lambda L (\sin \lambda x \cos \lambda x' + \cosh \lambda x \sin \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}
\]

\[
M = \frac{V_0}{k} K_{MV}
\]

\[
K_{MV} = \frac{\sinh \lambda L \sin \lambda x \sinh \lambda x' - \sin \lambda L \sin \lambda x \cos \lambda x'}{\sinh^2 \lambda L - \sin^2 \lambda L}
\]

\[
V = V_0 K_{VV}
\]

\[
K_{VV} = \frac{\sinh \lambda L (\cos \lambda x \sinh \lambda x' - \sin \lambda x \cosh \lambda x') - \sin \lambda L (\cosh \lambda x \sin \lambda x' - \sin \lambda x \cos \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}
\]

c) Rigid piles (\( \lambda L < 1.0 \))

\[
y = \frac{2V_0}{Lk} \left( 2 - 3 \frac{x}{L} \right)
\]

\[
\theta = \frac{-6V_0}{L^2 k}
\]

\[
M = V_0 L \left( \frac{x}{L} - 2 \left( \frac{x}{L} \right)^2 + \left( \frac{x}{L} \right)^3 \right) \quad \text{&} \quad M_{max}(x = L/3) = 4 \frac{4}{27} V_0 L
\]

\[
V = V_0 \left( 1 - 4 \left( \frac{x}{L} \right) + 3 \left( \frac{x}{L} \right)^2 \right)
\]
Free headed pile in uniform soil (reaction modulus, k constant), subject to moment, $M_o$ applied at the pile head

**a) Flexible piles ($\lambda L > 3.0$)**

$$y = \frac{2M_o\lambda^2}{k} (e^{-\lambda x} \cos \lambda x - \sin \lambda x)$$

$$\theta = \frac{-4M_o\lambda^3}{k} (e^{-\lambda x} \cos \lambda x)$$

$$M = M_o e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$$

$$V = -2M_o \lambda (e^{-\lambda x} \sin \lambda x)$$

**b) Semi-flexible piles (1.0 < $\lambda L < 3.0$)**

$$y = \frac{2M_o\lambda^2}{k} K_{yV}$$

$$K_{yM} = \frac{\sinh \lambda L (\sin \lambda x \cosh \lambda x' - \cos \lambda x \sinh \lambda x') + \sin \lambda L (\sin \lambda x \cos \lambda x' + \cosh \lambda x \sin \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}$$

$$\theta = \frac{-4M_o\lambda^3}{k} K_{\theta M}$$

$$K_{\theta M} = \frac{\sinh \lambda L (\cos \lambda x \cosh \lambda x') + \sin \lambda L (\cosh \lambda x \cos \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}$$

$$M = M_o K_{MM}$$

$$K_{MM} = \frac{\sinh \lambda L (\cos \lambda x \sinh \lambda x' + \sin \lambda x \cosh \lambda x') - \sin \lambda L (\cosh \lambda x \sin \lambda x' + \sin \lambda x \cos \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}$$

$$V = -2M_o \lambda K_{VM}$$

$$K_{yM} = \frac{\sinh \lambda L (\sin \lambda x \sinh \lambda x') - \sin \lambda L (\sin \lambda x \sin \lambda x')}{\sinh^2 \lambda L - \sin^2 \lambda L}$$

**c) Rigid piles ($\lambda L < 1.0$)**

$$y = \frac{6M_o}{L^2 k} \left(1 - 2 \frac{x}{L}\right)$$

$$\theta = \frac{-12M_o}{L^3 k}$$

$$M = M_o \left(1 - 3 \left(\frac{x}{L}\right)^2 + 2 \left(\frac{x}{L}\right)^3\right)$$

$$V = -\frac{6M_o}{L} \left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right)$$
Free head pile subject to transverse load, $V_0$ at the pile head with reaction modulus, $k = n_h x$

### a) Flexible piles ($\eta L > 4.0$) & semi-flexible piles ($1.5 < \eta L < 4.0$)

$$y = \frac{V_0 \eta^2}{n_h} A_y V \quad A_y V(x = 0) = 2.44 \quad A_y V = 2.44 S_1 - 1.62 S_2 + S_4$$

$$S_1 = 1 - \frac{5}{5!} (\eta x)^5 + \frac{6}{10!} (\eta x)^{10} - \frac{6 \cdot 11}{15!} (\eta x)^{15} + \ldots$$

$$S_2 = \left(\frac{\eta x}{6!}\right)^6 + \frac{2}{11!} (\eta x)^{11} - \frac{2 \cdot 7 \cdot 12}{16!} (\eta x)^{16} + \ldots$$

$$S_4 = \left(\frac{\eta x}{8!}\right)^3 - \frac{4 (\eta x)^8}{13!} + \frac{4 \cdot 9 (\eta x)^{13}}{18!} - \frac{4 \cdot 9 \cdot 14 (\eta x)^{18}}{18!} + \ldots$$

$$\theta = \frac{V_0 \eta^3}{n_h} A_{\theta V} \quad A_{\theta V}(x = 0) = -1.62 \quad A_{\theta V} = \frac{dA_y V}{dx}$$

$$M = \frac{V_0 \eta}{\eta} A_{MV} \quad M_{\text{max}}(x = 1.3/\eta) = \frac{0.77 V_0}{\eta} \quad A_{MV} = \frac{d^2 A_y V}{d^2 x}$$

$$V = V_0 A_{V V} \quad A_{V V} = \frac{d^3 A_y V}{d^3 x}$$

### b) Rigid piles ($\eta L < 1.5$)

$$y = \frac{V_0}{L^2 n_h} \left(18 - 24 \frac{x}{L}\right)$$

$$\theta = \frac{-24 V_0}{L^3 n_h}$$

$$M = V_0 L \left(\frac{x}{L} - 3 \left(\frac{x}{L}\right)^3 + 2 \left(\frac{x}{L}\right)^4\right) \quad \& M_{\text{max}}(x = 0.42L) = 0.26 V_0 L$$

$$V = V_0 \left(1 - 9 \left(\frac{x}{L}\right)^2 + 8 \left(\frac{x}{L}\right)^3\right)$$
Free head pile subject to moment, \( M_0 \) applied at the pile head with reaction modulus, \( k = n_h x \),

\[ y = \frac{M_0 \eta^3}{n_h} A_{yM} \quad A_{yM}(x = 0) = 1.62 \quad A_{yM} = 1.62 S_1 - 1.75 S_2 + S_3 \]

\[ S_1 = 1 - \frac{(\eta x)^5}{5!} + \frac{6(\eta x)^{10}}{10!} - \frac{6 \cdot 11(\eta x)^{15}}{15!} + \ldots \]

\[ S_2 = \eta x - \frac{2(\eta x)^6}{6!} + \frac{2 \cdot 7(\eta x)^{11}}{11!} - \frac{2 \cdot 7 \cdot 12(\eta x)^{16}}{16!} + \ldots \]

\[ S_3 = \frac{(\eta x)^2}{2!} \frac{3(\eta x)^7}{7!} + \frac{3 \cdot 8(\eta x)^{12}}{12!} - \frac{3 \cdot 8 \cdot 13(\eta x)^{17}}{17!} + \ldots \]

\[ \theta = \frac{M_0 \eta^4}{n_h} A_{\theta M} \quad A_{\theta M}(x = 0) = -1.75 \quad A_{\theta M} = \frac{dA_{yM}}{dx} \]

\[ M = M_0 A_{MM} \quad A_{MM} = \frac{d^2 A_{yM}}{d^2 x} \]

\[ V = M_0 A_{VM} \quad A_{VM} = \frac{d^3 A_{yM}}{d^3 x} \]

\( a) \) Flexible piles (\( \eta L > 4.0 \)) & semi-flexible piles (\( 1.5 < \eta L < 4.0 \))

\( b) \) Rigid piles (\( \eta L < 1.5 \))

\[ y = \frac{M_0}{L^3 n_h} \left( 24 - 36 \frac{x}{L} \right) \]

\[ \theta = \frac{-36 M_0}{L^4 n_h} \]

\[ M = M_0 \left( 1 - 4 \left( \frac{x}{L} \right)^3 + 3 \left( \frac{x}{L} \right)^4 \right) \]

\[ V = \frac{M_0}{L} \left( -12 \left( \frac{x}{L} \right)^2 + 12 \left( \frac{x}{L} \right)^3 \right) \]
Problem 9

In order to support the loads from a bridge pier, a solution using a group of piles as shown in Figure 8 has been proposed. The piles are formed from reinforced concrete and will be constructed integrally with the rigid pile cap (full moment connection).

a) Calculate the distribution of load in the piles ignoring the flexibility of the piles (i.e. considering only static equilibrium).

b) Calculate considering the flexibility of the piles:
   (i) The coordinates and the forces acting at the elastic centre.
   (ii) The displacements and rotation at the elastic centre.
   (iii) The loads acting at the heads of the piles.

c) Compare and comment on the results.
Formulas

1 - STATIC EQUILIBRIUM METHOD

For the case of a group of “m” number piles can demonstrate that:

\[
N_i = \frac{Y}{m} \pm \frac{M \cdot x_i}{\sum_{i=1}^{m} x_i^2}
\]

\[
T_i = \frac{X}{m}
\]

\[
M_i = 0
\]

where

- \( X, Y \& M \) are the transverse and vertical forces, and moment acting at the base of the pile cap.
- \( T_i, N_i \) and \( M_i \) are the transverse and vertical forces and moment acting at the head of the \( i \)-th pile respectively.
- \( e_i \) is the distance from the \( i \)-th pile to the centre of rotation

2 - VESIC METHOD

Sign convention:

![VESIC Diagram](image)

where

- \( \alpha_i \) is the angle that the pile make to the vertical
- \( x_{oi}, y_{oi} \) are the coordinates of the pile heads relative to the base of the pile cap
- \( x_i, y_i \) are the coordinates of the pile heads relative to the “elastic centre”
- \( K_{ni} \) the axial stiffness of each pile
- \( K_{ti} \) the transverse stiffness of each pile
- \( t_i \) the elastic length (ratio between the moment at pile head to the transverse load in pure translation)
- \( s_i \) the ratio between the moment at pile head to the transverse load in pure rotation
- \( m \) is the number of piles
For the case of a group of "m" number vertical piles can demonstrate:

(i) Coordinates of the "elastic centre"

\[ x_c = \frac{\sum_{i=1}^{m} x_{oi} \cdot K_{ni}}{\sum_{i=1}^{m} K_{ni}} \]

\[ y_c = \frac{\sum_{i=1}^{m} t_i \cdot K_{ti}}{\sum_{i=1}^{m} K_{ti}} \]

(ii) Displacements and rotation of the "elastic centre"

\[
\begin{bmatrix}
\delta_{cx} \\
\delta_{cy} \\
\theta_c
\end{bmatrix}
= \begin{bmatrix}
1/\sum_{i}^{m} K_{ti} & 0 & 0 \\
0 & 1/\sum_{i}^{m} K_{ni} & 0 \\
0 & 0 & 1/M'''
\end{bmatrix}
\begin{bmatrix}
X_c \\
Y_c \\
M_c
\end{bmatrix}
\]

\[ M''' = \sum_{i=1}^{m} \left[ K_{ni}x_i^2 + K_{ti}(y_i + t_i)^2 + K_{ti}\left(\frac{s_i}{t_i} - 1\right) t_i^2 \right] \]

(iii) Forces in head of each pile

\[
\begin{bmatrix}
N_i \\
T_i \\
M_i
\end{bmatrix}
= \begin{bmatrix}
0 & K_{ni}/\sum K_{ni} & -K_{ni}x_i/M'''
\frac{K_{ti}}{\sum K_{ti}} & 0 & K_{ti}y_i + t_i/M'''
\frac{K_{ti}}{\sum K_{ti}} & 0 & t_iK_{ti}y_i + s_i/M'''
\end{bmatrix}
\begin{bmatrix}
X_c \\
Y_c \\
M_c
\end{bmatrix}
\]
LOADS on STRUTS SUPPORTING FLEXIBLE WALLS

Problem 1

Consider the retaining wall represented in Figure 1. The soil (soil A) is sandy and has a volumetric unit weight, $\gamma = 18 \text{ kN/m}^3$, At-rest earth pressure coefficient, $K_0 = 0.55$ and an angle of shearing resistance, $\phi' = 32^\circ$.

a) Determine the design actions in the struts; assume the strut spacing is 4 m.
b) Consider that there is a uniform surcharge, $q$ of 20 kPa applied to the soil behind the wall.

Problem 2

Consider now, the situation represented in Figure 2. Soil A has the same properties as in Problem 1. The fill (aterro) has a volumetric unit weight, $\gamma = 16 \text{ kN/m}^3$ and an angle of shearing resistance, $\phi' = 25^\circ$. Determine the design actions in the struts; assume the strut spacing is 5 m.

Problem 3

Consider the situation represented in Figure 3. Soil A is a heavily overconsolidated clayey soil with an undrained shearing resistance, $c_u$ of 160 kPa and volumetric unit weight, $\gamma = 20 \text{ kN/m}^3$. The fill (aterro) has a volumetric unit weight, $\gamma = 18 \text{ kN/m}^3$ and an angle of shearing resistance, $\phi' = 30^\circ$. Determine the design actions in the struts; assume the strut spacing is 5 m.

Problem 4

Considering the situation in Figure 4, propose a distribution of struts that you consider adequate and the design forces for dimensioning them.

a) Assume that Soil A & B are sandy with $\gamma_A = 20 \text{ kN/m}^3$ and $\phi'_A = 35^\circ$, and $\gamma_B = 17 \text{ kN/m}^3$ and $\phi'_B = 32^\circ$.
b) Soil B has the same properties as in (a) but Soil A is now a clay with $c_u = 35 \text{ kPa}$ and $\gamma_A = 17 \text{ kN/m}^3$.
Two standard compaction tests have been undertaken using LNEC E-197, the results of which are detailed in the tables that follow. The results for Test 1 are also presented as the compaction curve in Figure 1.

a) Add the compaction curve for Test 2 to Figure 1, and discuss the relative positions of the two tests.

b) Assuming $G_s = 2.75$, add the saturation curves for $S_r = 100\%$ and $90\%$ to Figure 1.

c) Estimate the values for optimum water content and maximum dry density from each test.
### TEST 1

**Type:** Light  
**Weight of hammer:** 2.49 kg  
**Drop height:** 30.5 cm  
**Nº layers:** 3  
**Nº blows/layer:** 55

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#### Sample Tin

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<td>152.78</td>
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<td>8842</td>
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<tr>
<td>Weight of soil ($g$)</td>
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<td>4428</td>
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<td>Weight of hammer ($g$)</td>
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<td>18.7</td>
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### TEST 2

**Type:** Heavy  
**Weight of hammer:** 4.54 kg  
**Drop height:** 45.7 cm  
**Nº layers:** 5  
**Nº blows/layer:** 55

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#### Sample Tin

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<td>Dry density ($g/cm^3$)</td>
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Problem 1.2

For the construction of an embankment fill 5 km long, 15 m wide at the top and with side slopes of 2H:1V (Figure 2), soil from a borrow pit close to the work site was used. A laboratory compaction test was undertaken on this soil and the resulting compaction curve is presented in Figure 3.

The specification for compaction required two criteria to be satisfied: Degree of field compaction, GC ≥ 97% and field water content, \((w_{\text{opt}} - 2\%) < w < (w_{\text{opt}})\).

The soil in the borrow pit has a natural water content of 5% and a humid unit weight of 17.2 kN/m\(^3\).

a) Determine the volume of soil that needs to be excavated from the borrow pit to construct the embankment fill.

b) Determine the volume of water that needs to be added to the soil taken from the borrow pit in order for the embankment fill to meet the specified compaction criteria.

c) At the beginning of the embankment construction, a trial fill was constructed using the compaction equipment to be used in the works by the contractor. Various combinations of layer thickness (0.25 m and 0.40 m) and numbers of passes (2, 4 and 8) of the compaction plant were used and the results are indicted in Figure 4. Indicate which mode of operation should be chosen to ensure that the compaction specification is satisfied.
Problem 1.3

Consider a compacted fill with a unit weight of 21.5 kN/m$^3$ and a water content of 12%.
Assuming $G_s = 2.70$:

a) Calculate the dry unit weight, voids ratio and degree of saturation of the compacted soil.
b) Would it be possible to compact the same soil at a water content of 13.5% and achieve a
dry density of 20 kN/m$^3$? Justify.

(Solution: a) $\gamma_d = 19.2$ kN/m$^3$, $e = 0.406$, $S_r = 80\%$)

Problem 1.4

A compaction test was undertaken in a mould with a volume equal to 1000 cm$^3$ and the
following results were obtained (assume $G_s = 2.67$):

<table>
<thead>
<tr>
<th>Weight (g)</th>
<th>2010</th>
<th>2092</th>
<th>2114</th>
<th>2100</th>
<th>2055</th>
</tr>
</thead>
<tbody>
<tr>
<td>w (%)</td>
<td>12.8</td>
<td>14.5</td>
<td>15.6</td>
<td>16.8</td>
<td>19.2</td>
</tr>
</tbody>
</table>

a) Draw the compaction curve and the saturation curve.
b) Determine the optimum water content and the maximum dry density.

(Solution: $\gamma_d = 18.3$ kN/m$^3$, $w_{opt} = 15\%$)

Problem 1.5

A layer of soil 0.30 m thick with a unit weight of 17.1 kN/m$^3$, a water content of 15% and soil
particle unit weight of 26.5 kN/m$^3$ was compacted using a cylindrical roller. After compaction,
the unit weight of the soil increased to 20.5 kN/m$^3$ with no change in the water content.

a) Calculate the change in degree of saturation and the change in layer thickness due to
compaction.

(Solution: before - $S_r = 51\%$ & $h = 0.30$ m, after - $S_r = 82\%$ & $h = 0.25$ m)
Problem 1.6

To construct an embankment with a total volume of 175000 m³, 182000 m³ of fill was obtained from a borrow pit. Laboratory compaction testing yielded values for $\gamma_{d,max} = 16.2 \text{kN/m}^3$ and $w_{opt} = 12\%$. Assuming that the soil was compacted to the optimum state, the natural water content of the borrow pit material is 6% and $G_s = 2.63$:

a) Determine the natural unit weight of the borrow pit material.
   (Solution: $\gamma_h = 16.5 \text{kN/m}^3$)

Problem 1.7

A laboratory compaction test has been undertaken for a motorway embankment construction for which the results are presented in Figure 5. The embankment has a total volume of 30000 m³.

a) Draw the compaction and saturation curves (consider $G_s = 2.70$) and determine the maximum dry density and optimum water content.
   (Solution: $\gamma_{d,max} = 18.8 \text{kN/m}^3$, $w_{opt} = 14.4\%$)

b) Knowing that the degree of field compaction of the fill is 97% and the borrow pit material used for the fill has the following characteristics - $\gamma_t = 17.2 \text{kN/m}^3$, $w_n = 12\%$, determine the minimum volume of soil to be supplied from the borrow pit.
   (Solution: $V_{exc} \geq 35624 \text{ m}^3$)

Figure 4 - Laboratory compaction curve
Formulas

V – Total volume
V_s – Volume of solid particles
V_v – Volume of voids
V_w – Volume of water
V_a – Volume of air

W – Total weight
W_s – Weight of solid particles
W_w – Weight of water
W_a – Weight of air

a) Volumetric relationships

\[ V = V_s + V_v = V_s + V_w + V_a \]

Porosity, \( n = \frac{V_v}{V} \times 100\% \)

Compactness, \( m = \frac{V_s}{V} \times 100\% \)

Voids ratio, \( e = \frac{V_v}{V_s} \)

Degree of saturation, \( S_r = \frac{V_w}{V_v} \times 100\% \)

b) Mass relationships

\[ W = W_s + W_w \]

Water content, \( w = \frac{W_w}{W_s} \times 100\% \)

c) Density/unit weight

Unit weight water, \( \gamma_w \)

Total/humid unit weight, \( \gamma = \frac{W}{V} \)

Saturated unit weight, \( \gamma_{sat} = \frac{W_{(S_r=100\%)}{V} = \frac{W_s}{V} \}

Dry unit weight, \( \gamma_d = \frac{W_s}{V} \)

Submerged unit weight, \( \gamma' = \gamma - \gamma_w \)

Unit weight solid particles, \( \gamma_s = \frac{W_s}{V_s} \)

d) Solid particle density

Solid particle density, \( G_s = \frac{\gamma_s}{\gamma_w} \)

e) Other relationships

\[ V = V_s(1 + e) \]

\[ n = \frac{e}{1 + e} \]

\[ \gamma = \gamma_d(1 + w) \]

\[ \gamma_d = \frac{\gamma_w G_s}{(1 + e)} \]

\[ S_r e = w G \]