Minimal Solutions to the CKM Unitarity Problem with a Vector-like Quark Isosinglet

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Abstract

Recent results and calculations provide strong evidence that the CKM matrix is not unitary, contradicting the Standard Model (SM) and suggesting New Physics (NP). In this thesis, we propose a minimal extension of the SM where an up-type vector-like quark isosinglet, denoted by T, is introduced, leading to a simple solution to the CKM unitarity problem (CKM-UP). We adopt the Botella-Chau parametrization for the 4×3 quark mixing-matrix, containing the usual three angles and phase of the 3×3 SM-mixing, plus three extra angles s_{14},s_{24},s_{34} and two phases δ_{14},δ_{24}. To achieve a minimal solution to the CKM-UP, we assume that the mixing of T with standard quarks is dominated by s_{14}. Interesting features and a novel pattern of T decays, predominantly to the first generation, are obtained. However, one has to make sure that the limits of Electroweak Precision Measurements (EWPM), are not exceeded. We have found that ε_K, sensitive to CP violation, plays a crucial role in constraining these type of models. Imposing a (recently derived) restrictive upper-bound on NP to ε_K, we find, in the limit of exact s_{14}-dominance where $s_{24}, s_{34} = 0$, that $\varepsilon_K^{\text{NP}}$ is too large. If, however, one relaxes this exact s_{14}-dominance limit, a significantly large parameter region is obtained, where $\varepsilon_K^{\text{NP}}$ is in agreement with experiment, maintaining previously encountered features. Other important EWPM associated quantities are also studied. To a good approximation, these results are independent of s_{34}, allowing for solutions solely using three NP parameters: s_{14},s_{24} and $\delta' = \delta_{24} - \delta_{14}$.

Keywords: New physics, CKM unitarity problem, vector-like quarks, flavour changing neutral currents, electroweak precision measurements.

1. Introduction

The Standard Model (SM) of particle physics is one of the most successful theories in all of physics, however, it has become increasingly evident that the SM is incomplete and, thus, cannot be a final theory. During the last decades, theoretical particle physicists have deeply invested their efforts on exploiting some of the unrestricted aspects of the SM in order to develop potential extensions to this theory that may be able to improve it, i.e. that might circumvent some of its incompatibilities with experiments and increase its overall predictive power.

Recently, new results and theoretical calculations seem to indicate that the 3×3 unitarity of the CKM matrix in the SM might be violated, and the normalisation of the first its row is such that $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1$ [1, 2, 3, 4, 5, 6, 7, 8]. If confirmed, this would be a major result, providing evidence for NP beyond the SM. This problem is typically referred to as the CKM unitarity problem (CKM-UP), or Cabibbo angle anomaly. Some of the most elegant and simpler solutions for this problem rely on extensions of the SM quark sector with the introduction of a new type of quarks, the vector-like quarks (VLQs), which necessarily leads to deviations to CKM unitarity. Nonetheless, models with VLQs also provide a natural mechanism for the suppression of these unitarity deviations, while leading to a rich phenomenology due to the large enhancement of the parameter space [9, 10].

It has also been pointed out that the simple addition of either one down-type [11] or one up-type [12, 13] VLQ isosinglet may account for this New Physics (NP). In both cases, the parameter space is very large, involving six mixing angles and three CP violating phases. This is the type of extension we will be interested in exploring here, in particular, extensions with one up-type VLQ, which, as we shall demonstrate, appear to present a more a natural solution to the CKM-UP. More concretely, the ultimate goal of this thesis is to look for a minimal solution to the CKM-UP i.e. a solution involving the least number of new fields and parameters added to the SM. To do this, we work in the framework of the SM extended with one up-type VLQ isosinglet, in a minimal mixing limit case, and which we refer to as the s_{14}-dominance limit. This minimal solution not only addresses the CKM anomaly and obeys the stringent constraints coming from experiment, and which arise from processes such as neutral meson mixings or kaon decays and others, but also, most importantly, maintains the main predictions of SM.

2. A brief review of the Standard Model

In the SM, after the spontaneous symmetry breaking of the electroweak gauge group $SU(2)_L \times U(1)_Y$ into the electromagnetic gauge group $U(1)_Q$, as described by the Higgs mechanism, one can write the EW Lagrangian for the quark sector as

$$\mathscr{L}^q_{\rm EW} = \mathscr{L}^q_{\rm kin} + \mathscr{L}^q_m + \mathscr{L}^q_h + \mathscr{L}^q_{\rm NC} + \mathscr{L}^q_{\rm CC}. \quad (1)$$

In general, the mass terms are given by

$$\mathscr{L}^q_m = -\overline{\boldsymbol{u}}'_{\boldsymbol{L}} m^u\ \boldsymbol{u}'_{\boldsymbol{R}} - \overline{\boldsymbol{d}}'_{\boldsymbol{L}} m^d\ \boldsymbol{d}'_{\boldsymbol{R}} + h.c., \quad (2)$$

where $m^{u,d}$, the quark mass matrices, are proportional to the EW scale v. Here, $\boldsymbol{u}'$ and $\boldsymbol{d}'$ are vectors containing all quarks flavours of the up and down sectors, respectively. In terms of the mass eigenstates $\boldsymbol{u}_{\boldsymbol{L},\boldsymbol{R}} = (u,c,t)^T_{L,R}$ and $\boldsymbol{d}_{\boldsymbol{L},\boldsymbol{R}} = (d,s,b)^T_{L,R}$, one has

$$\mathscr{L}^q_m = -\overline{\boldsymbol{u}}_{\boldsymbol{L}} d^u_0\ \boldsymbol{u}_{\boldsymbol{R}} - \overline{\boldsymbol{d}}_{\boldsymbol{L}} d^d_0\ \boldsymbol{d}_{\boldsymbol{R}} + h.c., \quad (3)$$

with $d^q_0 = V^{q\dagger}_L m^q V^q_R$ being the diagonal matrices that contain the definite masses of the respective sector's quark fields and $V^q_{L,R}$ are unitary matrices, that keep the kinetic terms invariant, i.e.

$$\mathscr{L}^q_{\rm kin} = i\overline{\boldsymbol{u}}'\not{\partial}\boldsymbol{u}' + i\overline{\boldsymbol{d}}'\not{\partial}\boldsymbol{d}' = i\overline{\boldsymbol{u}}\not{\partial}\boldsymbol{u} + i\overline{\boldsymbol{d}}\not{\partial}\boldsymbol{d}. \quad (4)$$

The interactions of quarks with the Higgs boson can now be written as

$$\mathscr{L}^q_h = -\overline{\boldsymbol{d}}_{\boldsymbol{L}}\frac{h}{v}d^d_0\boldsymbol{d}_{\boldsymbol{R}} - \overline{\boldsymbol{u}}_{\boldsymbol{L}}\frac{h}{v}d^u_0\boldsymbol{u}_{\boldsymbol{R}} + h.c, \quad (5)$$

while the EW neutral currents (NC) describing interactions of quarks with the photon A_μ and the Z boson, are given by

$$\begin{aligned}\mathscr{L}^q_{\rm NC} = &-eA_\mu J^\mu_{q,\rm em} - \frac{g}{2c_W}Z_\mu(\bar{\boldsymbol{u}}_{\boldsymbol{L}}\gamma^\mu\boldsymbol{u}_{\boldsymbol{L}} \\ &- \bar{\boldsymbol{d}}_{\boldsymbol{L}}\gamma^\mu\boldsymbol{d}_{\boldsymbol{L}} - 2s^2_W J^\mu_{q,\rm em}),\end{aligned} \quad (6)$$

where we introduced the electromagnetic current

$$J^\mu_{q,\rm em} = \frac{2}{3}\bar{\boldsymbol{u}}\gamma^\mu\boldsymbol{u} - \frac{1}{3}\bar{\boldsymbol{d}}\gamma^\mu\boldsymbol{d}. \quad (7)$$

The charged current Lagrangian, describing interactions of quarks with the $W^\pm$ boson can be written as

$$\mathscr{L}^q_{\rm CC} = -\frac{g}{\sqrt{2}}\left(\bar{\boldsymbol{u}}_{\boldsymbol{L}}\not{W}^+ V_{\rm CKM}\boldsymbol{d}_{\boldsymbol{L}} + h.c.\right), \quad (8)$$

where we find the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V_{\rm CKM} \equiv V^{u\dagger}_L V^d_L \equiv= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}, \quad (9)$$

i.e. the 3×3 (charged current) unitary matrix with elements corresponding to the mixings of different quark flavours in the SM and the W bosons.

From (6-9) it is clear that, in the SM, the NC in (6) and (5) are flavour conserving, while the CC are flavour changing in general.

In the SM, the unitary CKM matrix is written in the PDG parametrization in terms of three mixing angles θ_{12}, θ_{13} and θ_{23} and a single physical phase δ [14]. This parametrization of $V_{\rm CKM}$ is given by

$$\begin{aligned}V_{\rm PDG} = &\begin{pmatrix}1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix} \\ &\cdot\begin{pmatrix}c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{pmatrix}.\end{aligned} \quad (10)$$

A no less useful parametrization of the CKM matrix is the Wolfenstein parametrization [15] where the mixings are expanded in terms of $\lambda \simeq |V_{us}|$:

$$\begin{aligned}V_{\rm Wolf} = &\begin{pmatrix}1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1\end{pmatrix} \\ &+ O(\lambda^4)\end{aligned} \quad (11)$$

By comparing (10) with (11) one can easily check that $s_{12} \simeq \lambda$, $s_{13} \sim \lambda^3$, and $s_{23} \sim \lambda^2$.

3. Extensions with VLQ isosinglets and the CKM unitarity problem

The charged currents are controlled by the CKM matrix, which in the case of the SM is strictly unitary. Therefore, detection of deviations of unitarity should constitute compelling evidence of the existence of New Physics (NP) beyond the SM [8, 11, 13]. Considering that $|V_{ub}|^2 \simeq 1.6\times 10^{-5}$, to good approximation, the SM predicts for the first row of the CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 = 1. \quad (12)$$

Given the current level of experimental precision and control of theoretical uncertainties, which has allowed $|V_{ud}|$ and $|V_{us}|$ to be determined with considerable precision, this has become the most promising test of CKM unitarity.

Currently, $|V_{us}|$ is calculated from experimental data on kaon decays, whereas results regarding neutron decay are most relevant for $|V_{ud}|$. The ratio $|V_{us}/V_{ud}|$ can be independently determined by comparing radiative decay rates of certain pion and kaon decays. Recent calculations for these quantities [11] show deviations from the condition (12) by more than 4σ, disfavouring the CKM unitarity at a 99.998% CL. In fact, these results are much more compatible with

$$|V_{ud}|^2 + |V_{us}|^2 = 1 - \Delta^2, \tag{13}$$

where at a 95% confidence level, one has $\Delta = 0.04 \pm 0.01$. This suggests the need for an extra mixing, for instance a V_{14} with $|V_{14}| = \Delta$ which in turn would allow for a new unitarity condition

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{14}|^2 = 1. \tag{14}$$

This is very interesting as it is exactly what one obtains with the addition of VLQs to the SM, which are already common in some extensions of the SM and are not excluded by experiment. VLQs consist of quark fields where the LH and RH chiral components transform in the same manner under the SM gauge group.

Consider the general case were we introduction to the theory n_u up-type VLQ isosinglets $U'_i = U'_{iL} + U'_{iR}$ and n_d down-type VLQ isosinglets $D'_i = D'_{iL} + D'_{iR}$. In these types of extensions we will have two new types of mass terms. One coming from the Higgs mechanism

$$-\mathscr{L}_m^{\text{VLQ}} = \overline{\boldsymbol{d}}'_{\boldsymbol{L}} \omega^d \boldsymbol{D}'_{\boldsymbol{R}} + \overline{\boldsymbol{u}}'_{\boldsymbol{L}} \omega^u \boldsymbol{U}'_{\boldsymbol{R}} + h.c, \tag{15}$$

where $\omega^{u,d}$ are matrices of sizes $3 \times n_u$ and $3 \times n_d$, and an other, corresponding to the bare mass terms

$$\begin{aligned} -\mathscr{L}_b^{\text{VLQ}} = & \overline{\boldsymbol{D}}'_{\boldsymbol{L}} X^d \boldsymbol{d}'_{\boldsymbol{R}} + \overline{\boldsymbol{U}}'_{\boldsymbol{L}} X^u \boldsymbol{u}'_{\boldsymbol{R}} \\ & + \overline{\boldsymbol{D}}'_{\boldsymbol{L}} M^d \boldsymbol{D}'_{\boldsymbol{R}} + \overline{\boldsymbol{U}}'_{\boldsymbol{L}} M^u \boldsymbol{U}'_{\boldsymbol{R}} + h.c, \end{aligned} \tag{16}$$

which have no SM analogous.

Collecting all up-type and down-type quarks of the theory into the flavour vectors of dimension $n_u + 3$ and $n_d + 3$

$$\mathcal{U}' \equiv \begin{pmatrix} \boldsymbol{u}' \\ \boldsymbol{U}' \end{pmatrix}, \quad \mathcal{D}' \equiv \begin{pmatrix} \boldsymbol{d}' \\ \boldsymbol{D}' \end{pmatrix}, \tag{17}$$

one can write all mass terms as

$$-\mathscr{L}_M^q = \overline{\mathcal{U}}'_{\boldsymbol{L}} \mathcal{M}^u \mathcal{U}'_{\boldsymbol{R}} + \overline{\mathcal{D}}'_{\boldsymbol{L}} \mathcal{M}^d \mathcal{D}'_{\boldsymbol{R}} + h.c, \tag{18}$$

where the two new quark mass matrices

$$\mathcal{M}^{u,d} = \begin{pmatrix} m^{u,d} & \omega^{u,d} \\ X^{u,d} & M^{u,d} \end{pmatrix}, \tag{19}$$

represented here in block form, are matrices of size $(3+n_u)\times(3+n_u)$ and $(3+n_d)\times(3+n_d)$, respectively. The fields in (17) are related to the mass eigenstates through

$$\mathcal{U}'_{\boldsymbol{L,R}} = \mathcal{V}^u_{L,R} \mathcal{U}_{\boldsymbol{L,R}} \equiv \begin{pmatrix} A^u_{L,R} \\ B^u_{L,R} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_{\boldsymbol{L,R}} \\ \boldsymbol{U}_{\boldsymbol{L,R}} \end{pmatrix}, \tag{20}$$

and similarly for the down-type quarks, so that $\mathscr{D} = \mathcal{V}_L^\dagger \mathcal{M} \mathcal{V}_R$, are the diagonalised mass matrices of each sector. The matrices $\mathcal{V}^{u,d}_{L,R}$ are unitary to ensure the invariance of the kinetic terms, while their first 3 rows, $A^{u,d}_{L,R}$, and their last $n_{u,d}$ rows, $B^{u,d}_{L,R}$, are not.

Being isospin singlets both $\boldsymbol{U}'_{\boldsymbol{L,R}}$ and $\boldsymbol{D}'_{\boldsymbol{L,R}}$ will play a role similar to $\boldsymbol{u}'_{\boldsymbol{R}}$ and $\boldsymbol{d}'_{\boldsymbol{R}}$ in interactions with the gauge bosons. Therefore, one has

$$\begin{aligned} \mathscr{L}^q_{\text{CC}} & = -\frac{g}{\sqrt{2}} \left(\overline{\boldsymbol{u}}'_L \not{W}^+ \boldsymbol{d}'_{\boldsymbol{L}} + h.c. \right) \\ & = -\frac{g}{\sqrt{2}} \left(\overline{\mathcal{U}}_{\boldsymbol{L}} \not{W}^+ (A^{u\dagger}_L A^d_L) \mathcal{D}_{\boldsymbol{L}} + h.c. \right), \end{aligned} \tag{21}$$

so that we now have a larger and non-unitary $(3 + n_u) \times (3 + n_d)$ mixing matrix $\mathcal{V}_{\text{CKM}} \equiv A^{u\dagger}_L A^d_L$. This motivates the study of VLQs models as solutions to the CKM-UP.

However, this now leads to interactions of quarks with the Z and Higgs bosons no longer being flavour diagonal. In fact, now

$$\begin{aligned} \mathscr{L}^q_{\text{NC}} = & - e A_\mu J^\mu_{q,\text{em}} - \frac{g}{2c_W} (\overline{\mathcal{U}}_{\boldsymbol{L}} \not{Z} F^u \mathcal{U}_{\boldsymbol{L}} \\ & - \overline{\mathcal{D}}_{\boldsymbol{L}} \not{Z} F^d \mathcal{D}_{\boldsymbol{L}} - 2 s_W^2 J^\mu_{q,\text{em}} Z_\mu). \end{aligned} \tag{22}$$

with $J^\mu_{q,\text{em}} = \frac{2}{3} \overline{\mathcal{U}} \gamma^\mu \mathcal{U} - \frac{1}{3} \overline{\mathcal{D}} \gamma^\mu \mathcal{D}$, and

$$-\mathscr{L}^q_h = \overline{\mathcal{U}}_{\boldsymbol{L}} \frac{h}{v} F^u \mathscr{D}^u \mathcal{U}_{\boldsymbol{R}} + \overline{\mathcal{D}}_{\boldsymbol{L}} \frac{h}{v} F^d \mathscr{D}^d \mathcal{D}_{\boldsymbol{R}} + h.c, \tag{23}$$

where $F^u = \mathcal{V}_{\text{CKM}} \mathcal{V}^\dagger_{\text{CKM}}$ and $F^d = \mathcal{V}^\dagger_{\text{CKM}} \mathcal{V}_{\text{CKM}}$ are the matrices that control these tree-level flavour changing neutral currents (FCNCs), which were absent from the SM.

Currently, the lower-bound for the mass of a heavy-quark is of the order of the TeV. Assuming then a new mass scale $v' \gg v$ associated to the terms in (16) and the heavy-top mass, one can show, using $\mathscr{D}^2 = \mathcal{V}_L^\dagger \mathcal{M} \mathcal{M}^\dagger \mathcal{V}_L$ and the block form

$$\mathcal{V}_L = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \tag{24}$$

that (omitting the u and d sector indices)

$$F = \begin{pmatrix} \mathbb{1}_3 - S^\dagger S & K^\dagger R \\ R^\dagger K & R^\dagger R \end{pmatrix}, \tag{25}$$

with $S \sim v/v'$, $R \sim v/v'$ and

$$K^\dagger K = \mathbb{1}_3 - S^\dagger S, \qquad KK^\dagger = \mathbb{1}_3 - RR^\dagger, \tag{26}$$

leading to a $(v/v')^2 \ll 1$ suppression of FCNCs, so that modifications to the SM EW theory are naturally small in these extensions.

In principle, the simplest solutions to the CKM-UP are obtained with the addition of either one down-type (heavy-bottom B) or up-type (heavy-top T) VLQ isosinglet. In the former case, we obtain a 3×4 mixing matrix

$$\mathcal{V}_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{uB} \\ V_{cd} & V_{cs} & V_{cb} & V_{cB} \\ V_{td} & V_{ts} & V_{tb} & V_{tB} \end{pmatrix}, \tag{27}$$

and the solution to this problem is achieved by having $|V_{uB}| = \Delta$. However, in a suitable weak basis where the 3×3 up-sector mass matrix is diagonal, one can have

$$\mathcal{M}^d = \begin{pmatrix} m_d V_{ud} & m_s V_{us} & m_b V_{ub} & m_B V_{uB} \\ m_d V_{cd} & m_s V_{cs} & m_b V_{cb} & m_B V_{cB} \\ m_d V_{td} & m_s V_{ts} & m_b V_{tb} & m_B V_{tB} \\ m_d V_{41} & m_s V_{42} & m_b V_{43} & m_B V_{44} \end{pmatrix}. \tag{28}$$

Thus, we find with $m_B \sim 1$ TeV and $|V_{uB}| \approx 0.04$ that $|\mathcal{M}^d_{14}|$ is about 10 times larger than any other mass term originating from the Higgs mechanism. This seems to us to be an inconsistency and thus very unnatural. This result, coupled with recent results from [13] demonstrating that models with an heavy-bottom are close to being excluded, leads us to exclusively focus on the study of models with an heavy-top.

In that case, the mixing matrix is a 4×3 matrix of the form

$$\mathcal{V}_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{pmatrix}. \tag{29}$$

Here, the first row of CKM no longer respects the unitary condition (14), but it can still be verified for the first row of

$$\mathcal{V}^{u\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{14} \\ V_{cd} & V_{cs} & V_{cb} & V_{24} \\ V_{td} & V_{ts} & V_{tb} & V_{34} \\ V_{Td} & V_{Ts} & V_{Tb} & V_{44} \end{pmatrix}, \tag{30}$$

the unitarity matrix which diagonalises $\mathcal{H}^u = \mathcal{M}^u \mathcal{M}^{u\dagger}$ in the WB where the 3×3 down sector mass matrix is diagonal. The CKM unitarity problem may then be solved by having $|V_{14}| = \Delta$.

4. Phenomenological effects of mixing with a heavy-top

In a model with just one heavy-top, the existing FCNCs involve only up-type quarks. Also, $\mathcal{V}_{\rm CKM}$ will be a non-unitary 4×3 matrix and the matrices controlling the FCNCs are

$$\begin{aligned} F^u &= \mathcal{V}_{\rm CKM} \mathcal{V}^\dagger_{\rm CKM} = \mathcal{V}^{u\dagger}_L K_0 K_0^T \mathcal{V}^u_L, \\ F^d &= \mathcal{V}^\dagger_{\rm CKM} \mathcal{V}_{\rm CKM} = V^{d\dagger}_L V^d_L = \mathbb{1}_3, \end{aligned} \tag{31}$$

where

$$K_0^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{32}$$

and V^d_L and $\mathcal{V}^u_L$ are, respectively, the 3×3 and 4×4 unitary matrices that diagonalize $H_d = m_d m_d^\dagger$ and $\mathcal{H}_u = \mathcal{M}_u \mathcal{M}_u^\dagger$.

Throughout this chapter we chose to work with parametrizations of $\mathcal{V}_{\rm CKM}$ where the quantity

$$\lambda^K_u \equiv V^*_{us} V_{ud}, \tag{33}$$

is real. This is because with this choice, the expression for several of the most important quantities become easier to calculate than in the general case.

Following [16, 17, 18], one can show that the short distance (SD) expressions for the NP contributions to the mass differences Δm_N associated to neutral meson mixings $N^0 - \overline{N}^0$, with $N = K, B_{d,s}$, can be written as

$$\Delta m^{\rm NP}_N \approx \frac{G_F^2 M_W^2 m_N f_N^2 B_N \left|\mathcal{S}^{\rm NP}_N\right|}{6\pi^2}, \tag{34}$$

where G_F is the Fermi constant and M_W is the mass of the W-boson. Moreover, m_N, B_N and f_N are the average mass, bag parameter and decay constant of the meson N, respectively, and

$$\mathcal{S}^{\rm NP}_N = \sum_{i=c,t,T} \eta^N_{iT} \lambda^N_i \lambda^N_T S_0(x_i, x_T), \tag{35}$$

where $S_0(x_i, x_j)$ are the gauge-invariant Inami-Lim (IL) box functions with $x_i \equiv (m_i/M_W)^2$ [19] and η^N_{ij} are $O(1)$ QCD corrections.

Similarly, for the CP violation parameter ε_K, dominated by SD contributions, one can write

$$|\varepsilon^{\rm NP}_K| = \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\varepsilon}{12\sqrt{2}\pi^2 \Delta m_K} |{\rm Im}\left(\mathcal{S}^{\rm NP}_K\right)|, \tag{36}$$

where $\kappa_\varepsilon = 0.92 \pm 0.02$ [20].

These expressions can be used to constraint our model. To consider the model safe with regard to the neutral kaon system we establish the following criteria:

$$\Delta m_K^{\rm NP} \lesssim \Delta m_K^{\rm exp} \simeq (3.484 \pm 0.006) \times 10^{-12}\ {\rm MeV}, \quad (37)$$

$$|\varepsilon_K^{\rm NP}| \lesssim \delta\varepsilon_K = 2.48 \times 10^{-4}. \quad (38)$$

For $\Delta m_K^{\rm NP}$ we require simply that it is lower than the experimental value, given the theoretical uncertainty still associated with the SM prediction and its long-distance (LD) piece. For ε_K we impose a much stringent condition than the usual requirement $|\varepsilon_K^{\rm NP}| \lesssim |\varepsilon_K^{\rm exp}|$. This choice is motivated by recent theoretical calculations which lead to the SM prediction $|\varepsilon_K^{\rm SM}| = (2.16 \pm 0.18) \times 10^{-3}$ [21] being very close to the experimental value $|\varepsilon_K^{\rm NP}| = (2.228 \pm 0.011) \times 10^{-3}$ [14]. Thus, although significantly more restrictive, we consider that this new criterion is much more appropriate.

For the B_q meson systems ($q = d, s$), in the SM, the mass difference is dominated by SD contributions and one has to very good approximation

$$\Delta m_{B_q}^{\rm SM} \approx \frac{G_F^2 M_W^2 m_{B_q} f_{B_q}^2 B_{B_q} \eta_{tt}^{B_q} \left(\lambda_t^{B_q}\right)^2 S_0(x_t)}{6\pi^2}, \quad (39)$$

and it can be shown that $\Delta m_{B_q}^{\rm SM} \approx \Delta m_{B_q}^{\rm exp}$, leaving little room for NP. Hence, we simply require that

$$\delta m_{B_q} \equiv \frac{\Delta m_{B_q}^{\rm NP}}{\Delta m_{B_q}^{\rm SM}} \lesssim 0.1, \quad (40)$$

i.e. the size of NP contributions should be around one order of magnitude smaller than those of the SM pieces.

In the SM, the mixing $D^0 - \overline{D}^0$ is associated to non-negligible LD contributions, which are difficult to compute. Therefore, we establish the criterion

$$x_D^{\rm NP} \leq x_D^{\rm exp} = 0.39^{+0.11}_{-0.12}\%, \quad (41)$$

for the NP contribution to the mixing parameter [22, 23]

$$x_D^{\rm NP} \equiv \frac{\Delta m_D}{\Gamma_D} \simeq \frac{\sqrt{2} G_F}{3\Gamma_D} r(m_c, M_Z) B_D f_D^2 m_D \left|F_{12}^u\right|^2, \quad (42)$$

where $r(m_c, M_Z)$ is a factor which accounts for RG effects.

For the rare kaon decays $K_L \to \pi^0 \nu\overline{\nu}$ and $K^+ \to \pi^+ \nu\overline{\nu}$, which are dominated by SD contributions, one can write

$$\frac{k^0}{k^0_{\rm SM}} = \left|1 + \frac{{\rm Im}\lambda_T^K X_0(x_T) + {\rm Im}A_{ds}}{{\rm Im}\lambda_t^K X_0(x_t)}\right|^2 \quad (43)$$

$$\frac{k^+}{k^+_{\rm SM}} = \left|1 + \frac{\lambda_T^K X_0(x_T) + A_{ds}}{\lambda_c^K X^{\rm NNL}(x_c) + \lambda_t^K X_0(x_t)}\right|^2, \quad (44)$$

where, using $\xi = \{0, +\}$, one has defined

$$\frac{k^\xi}{k^\xi_{\rm SM}} = \frac{{\rm Br}(K^\xi \to \pi^\xi \overline{\nu}\nu)}{{\rm Br}(K^\xi \to \pi^\xi \overline{\nu}\nu)_{\rm SM}}. \quad (45)$$

In these expressions, $X_0(x_i)$ is another kind of IL function and for $K^+ \to \pi^+ \nu\overline{\nu}$ it is relevant to introduce the next-to-next-to-leading (NNL) order charm-quark contribution, $X^{\rm NNL}(x_c)$. In the case of the decay of K_L, given that the experimental upper-bound to its branching ratio is about 100 times larger than the SM prediction, we just look for a possible enhancement to ${\rm Br}(K_L \to \pi^0 \overline{\nu}\nu)$. However, for K^+ we require that

$$0.24 \lesssim \left(\frac{k^+}{k^+_{\rm SM}}\right)_{2\sigma} \lesssim 2.28, \quad (46)$$

which is a rough 2σ range we establish for this ratio, resulting from the experimental value ${\rm Br}(K^+ \to \pi^+ \overline{\nu}\nu)_{\rm exp} = \left(10.6^{+4.0}_{-3.4} \pm 0.9\right) \times 10^{-11}$, and the SM prediction ${\rm Br}(K^+ \to \pi^+ \overline{\nu}\nu)_{\rm SM} = (8.4 \pm 1.0) \times 10^{-11}$ [24].

It is important to mention the role of the factor A_{ds} introduced in (43) and (44) and defined as

$$A_{ds} = \sum_{ij} V_{id} \left(F^u - \mathbb{1}\right)_{ij} V_{js}^* N(x_i, x_j), \quad (47)$$

where

$$N(x_i, x_j) = \frac{x_i x_j}{8} \left(\frac{\log x_i - \log x_j}{x_i - x_j}\right), \quad (48)$$

$$N(x_i, x_i) \equiv \lim_{x_j \to x_i} N(x_i, x_j) = \frac{x_i}{8}. \quad (49)$$

This usually overlooked factor, accounts for the decoupling behaviour associated to the mixing of the heavy-top with other quarks and the FCNCs that slightly modify the EW penguin loop diagrams describing these type of decays.

The parameter ε'/ε quantifies direct CP violation in $K_L \to \pi\pi$ decays, has in this type of models NP contribution given by

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm NP} \simeq F(x_T){\rm Im}\lambda_T^K + (P_X + P_Y + P_Z) A_{ds}, \quad (50)$$

where $F(x_t)$ corresponds to the following linear combinations of Inami-Lim functions

$$F(x_i) = P_0 + P_X X_0(x_i) + P_Y Y_0(x_i) + P_Z Z_0(x_i) + P_E E_0(x_i), \quad (51)$$

where P_0, P_X, P_Y, P_Z and P_E are constants.

Here, we require that the NP contribution is contained in the rough 1σ range [25]

$$-4 \times 10^{-4} \lesssim \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm NP} \lesssim 10 \times 10^{-4}. \quad (52)$$

Finally, we look at the possible decay channels of the heavy-top in this type of model. In the limit of $m_T^2 \gg M_Z^2, M_W^2, m_{q_i}^2$, one has

$$\Gamma(T \to u_i Z) \simeq \Gamma(T \to u_i h) \simeq \frac{m_T^2}{32\pi v^2} \left|F^u_{4i}\right|, \quad (53)$$

$$\Gamma(T \to d_i W) \simeq \frac{m_T^2}{16\pi v^2} \left|V_{Td_i}\right|^2. \quad (54)$$

Typically, searches for heavy quarks are involve assumptions made to these decay widths. Most commonly, it is assumed that decays involving the third generation $u_i = t$ and $d_i = b$ are the dominant decay channels, leading to lower-bounds for the heavy-top mass of about $m_T \gtrsim 1$ TeV [26, 27]. This assumption, however, is not entirely justified.

5. The s14-dominance hypothesis: a minimal solution to the CKM-UP with an heavy-top

Consider again the SM with the minimal addition of one VLQ sisosinglet, the heavy-top quark T. With respect to its mass, we are interested in searching for regions that may be accessible in upcoming generations of accelerators and therefore we shall restrict ourselves to the study of regions where $m_T \leq 2.5$ TeV, which is compatible with the rough upper-bound presented in [13] for models with an heavy-top where $|V_{14}| \simeq 0.04$. In this framework, and in the WB where the 3×3 down sector mass matrix is diagonal, i.e. $M_d = \mathrm{diag}(m_d, m_s, m_b)$, the mixing matrix is given by $\mathcal{V}_{\rm CKM} = \mathcal{V}^\dagger K_0$, where, with the Botella-Chau (BC) parameterization [28], $\mathcal{V}^\dagger$ can be written as

$$\mathcal{V}^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \begin{pmatrix} & V_{\rm PDG} & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (55)$$

whith $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $\theta_{ij} \in [0, \pi/2]$, $\delta \in [0, 2\pi]$. The normalisation of the first row leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - s_{14}^2 \quad (56)$$

making it evident that a solution for the observed CKM-UP implies that $s_{14} = \Delta \in [0.03, 0.05]$. We adopt this parametrization which, as stated, also yields a real CKM factor $\lambda_u^K = s_{12}c_{12}c_{13}^2c_{14}^2$, allowing us to use the results from last chapter.

Next, in an attempt to fully achieve a minimal solution of the CKM-UP, we impose the s_{14}-dominance limit for the mixing, defined as

$$s_{14} = \Delta \sim \lambda^2, \quad s_{24} = s_{34} = 0. \quad (57)$$

In this limit, (55) takes the very manageable form

$$\mathcal{V}_{\rm CKM} = \begin{pmatrix} V^0_{ud}c_{14} & V^0_{us}c_{14} & V^0_{ub}c_{14} \\ V^0_{cd} & V^0_{cs} & V^0_{cb} \\ V^0_{td} & V^0_{ts} & V^0_{tb} \\ -V^0_{ud}s_{14} & -V^0_{us}s_{14} & -V^0_{ub}s_{14} \end{pmatrix}, \quad (58)$$

where V^0_{ij} now refers to the SM mixings in (9). Note that the two new CP violating phases, δ_{14} and δ_{24}, can be eliminated through rephasings of the quark fields, so that in this limit we are effectively only adding a parameter to the SM mixing, the angle θ_{14}. Also, the first row of $\mathcal{V}_{\rm CKM}$ is only slightly modified by a factor of $c_{14} \simeq 1$, whereas the second and third rows remain intact. The new forth row of $\mathcal{V}_{\rm CKM}$ is suppressed by a factor of s_{14}.

The matrix controlling the FCNCs now reduces to

$$F^u = \begin{pmatrix} c_{14}^2 & 0 & 0 & -c_{14}s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -c_{14}s_{14} & 0 & 0 & s_{14}^2 \end{pmatrix}, \quad (59)$$

and in this case, at tree level only two new decays involving FCNCs arise: $T \to uZ$ and $T \to uh$. Thus, using (53) and (54) one can identify some of the most salient features of this framework as being the dominant coupling of T with up and down quarks, the significantly smaller coupling of T with the bottom quark and the vanishing of the coupling of T with the charm and top quarks. These large couplings of the very massive heavy-top to the lighter first generation, instead of the much heavier third generation is opposed to the usual "wisdom", making this a particularly intriguing and exciting limit to study. In fact, the assumption of dominant decays to the first generation (instead of to the third) allows one to have a lower-bound for m_T as low as $m_T = 0.685$ TeV [29], which might be accessible to the next generation of accelerators.

Another, and no less important feature of our model is the vanishing of NP contributions to ε'/ε

and to the branching ratio of the $K_L \to \pi^0\overline{\nu}\nu$ decay[1], which can be deduced from the fact that the CKM factor $\lambda_T^K = \lambda_u^K t_{14}^2$ is real. In addition, the fact that $F_{12}^u = 0$ means there is no NP contribution to $D^0 - \overline{D}^0$ mixing at tree level.

For $B_q^0 - \overline{B}_q^0$ mixing one has also very suppressed NP contributions. For instance, using

$$\begin{aligned} \theta_{12} &\simeq 0.2264, \quad \theta_{13} \simeq 0.0037, \\ \theta_{23} &\simeq 0.0405, \quad \quad \delta \simeq 1.215, \end{aligned} \tag{60}$$

for the SM mixing parameters, one obtains $\delta m_{B_d} \lesssim 0.72\%$ and $\delta m_{B_s} \lesssim 0.04\%$ for $m_T \leq 2.5$ TeV and $s_{14} \lesssim 0.05$. In the case of $K^0 - \overline{K}^0$ mixing one can achieve (37) in most of the (s_{14}, m_T) parameter space, but regions with $s_{14} \gtrsim 0.45$ and $m_T \gtrsim 2$ TeV seem to be disfavoured.

The decay $K^+ \to \pi^+\overline{\nu}\nu$ also receives significant NP contributions in this limit, so that the requirement in (46) ends up imposing somewhat stringent restrictions on the parameter space, but one can still have $s_{14} \gtrsim 0.04$. Finally, for ε_K one has

$$|\varepsilon_K^{\rm NP}| = \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\varepsilon}{12\sqrt{2}\pi^2 \Delta m_K} \, |\mathcal{F}| \,, \tag{61}$$

where

$$\begin{aligned} \mathcal{F} =& (\eta_{tT}^K S_{tT} - \eta_{cT}^K S_{cT}) \cdot \\ & \cdot c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} s_{14}^2 \sin\delta, \end{aligned} \tag{62}$$

which is completely incompatible with the new very restrictive condition (38), excluding all regions of parameter space within $s_{14} \in [0.03, 0.05]$ and $m_T \in [0.685, 2.5]$ TeV. For instance, for $m_T = 1.392$ TeV, $s_{14} = 0.042$ and (60) one has $|\varepsilon_K^{\rm NP}| = 4.959 \times 10^{-3} \simeq 20\delta\varepsilon_K$. Therefore, this stringent s_{14}-dominance limit with $s_{24}, s_{34} = 0$, is not a viable minimal solution to the CKM-UP.

6. The limit of realistic s14-dominance: solving the ε_K problem

Even if one would overlook the intrinsic problem with ε_K, the limit of strict s_{14}-dominance seems to be somewhat unnatural. In a more realistic scenario one expect that $s_{24}, s_{34} \neq 0$ and could reasonably expect $|\varepsilon_K|^{\rm NP} < \delta\varepsilon_K$ to be achievable. However, in order to keep the simplicity and other interesting features of s_{14}-dominance, one requires these angles to be small, i.e. $s_{24}, s_{34} \ll s_{14}$, which amounts to replacing the strict s_{14}-dominance limit by a more realistic version. Nonetheless, a priori, it is not obvious that a small deviation from $s_{24}, s_{34} = 0$ would lead to $|\varepsilon_K^{\rm NP}| < \delta\varepsilon_K$ and thus the framework of s_{14}-dominance could be entirely incompatible with the resolution of the ε_K problem.

[1]This is true because in this s_{14}-dominance limit, one has $A_{ds} \simeq -\lambda_T^K x_T/8$.

In this chapter we show that this is not the case. More concretely, we show that it is possible to achieve $|\varepsilon_K^{\rm NP}| < \delta\varepsilon_K$ in the region where $m_T \in [0.685, 2.5]$ TeV, while preserving the most important features of strict s_{14}-dominance. In a new relaxed s_{14}-dominance scenario, we maintain $s_{14} = 0.04 \pm 0.01 \sim \lambda^2$ but now have $s_{24}, s_{34} \lesssim \lambda^5$.

Expanding the mixings in this more realistic limit in terms of λ and using $c_{13}, c_{23}, c_{24}, c_{34} \simeq 1$, one has up to $\mathcal{O}(\lambda^8)$

$$\begin{aligned} V_{cd} &= V_{21} - c_{12} s_{14} s_{24} e^{-i\delta'}, \\ V_{cs} &= V_{22} - s_{12} s_{14} s_{24} e^{-i\delta'}, \\ V_{td} &= V_{31} - c_{12} s_{14} s_{34} e^{i\delta_{14}}, \\ V_{Td} &= V_{41} + s_{12} s_{24} e^{i\delta'}, \\ V_{Ts} &= V_{42} - c_{12} s_{24} e^{i\delta'} - c_{12} s_{23} s_{34} e^{-i\delta_{14}}, \\ V_{Tb} &= V_{43} - s_{23} s_{24} e^{i\delta'} - s_{34} e^{-i\delta_{14}}, \end{aligned} \tag{63}$$

with the other mixings remaining unchanged up to this order[2]. Here we refer to V_{ij} with $i, j = 1, 2, 3, 4$ as the entries of (58) and introduced the phase difference $\delta' \equiv \delta_{24} - \delta_{14}$. One can check that there is still no meaningful change to the SM mixings in this limit and the considerations for the decay patterns of the heavy-top still apply.

Now, in this limit one has also

$$|\varepsilon_K^{\rm NP}| \simeq \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\varepsilon}{6\sqrt{2}\pi^2 \Delta m_K} \left|\mathcal{F} - \mathcal{F}'\right|, \tag{64}$$

where at leading order one has

$$\begin{aligned} \mathcal{F} - \mathcal{F}' \simeq & \, s_{12} c_{12} s_{14}^2 \cdot \\ & \cdot \left(\tilde{S}_{tT} s_{13} s_{23} \sin\delta - \tilde{S}_{TT} s_{14} s_{24} \sin\delta' \right), \end{aligned} \tag{65}$$

with $\tilde{S}_{iT} \equiv \eta_{iT} S_{iT}$ and where the first term is the leading order of (62). The new term proportional to s_{24} and δ', and (at this order) independent of s_{34}, can partially cancel the original term. In fact, we find that there exists now a significant range of s_{14}, s_{24}, δ' and m_T where (38) can be achieved (see figures 1-3). As an example, taking for $s_{14} = 0.042$, $s_{24} = 5 \times 10^{-4}$, $s_{34} = 1.5 \times 10^{-6}$, $\delta_{14} = 0$, $\delta_{24} = 0.60$ and (60), one achieves $|\varepsilon_K^{\rm NP}| \simeq 2.324 \times 10^{-4}$.

[2]Here, we even relax the $s_{24} \lesssim \lambda^5$ condition to $s_{24} \lesssim \lambda^4$.

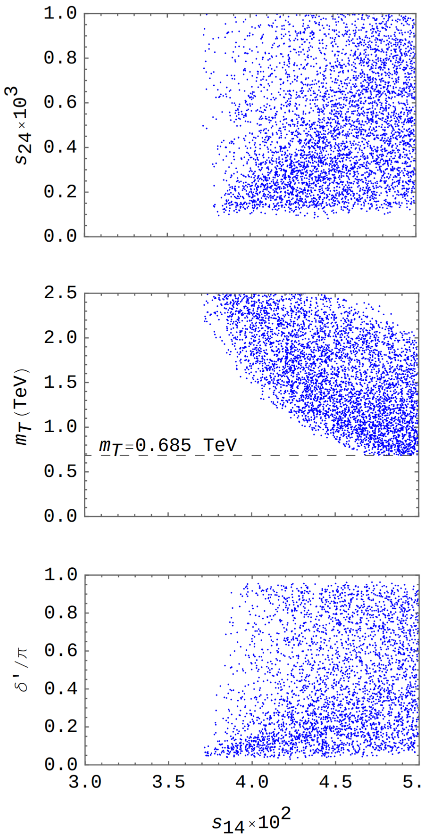

Figure 1: Allowed ranges for s_{24}, m_T and δ' against the allowed ranges for s_{14}.

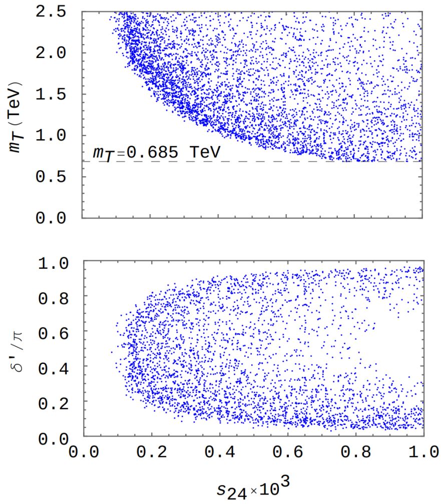

Figure 2: Allowed ranges for m_T and δ' against the allowed ranges for s_{24}.

For Δm_N and $k^+/k^+_{\rm SM}$, since the modifications in (63) do not change $|\lambda_i^N|$ meaningfully, we essentially recover the results of the strict $s_{24}, s_{34} = 0$ limit in all points of the parameter region of interest, i.e. $s_{14} \in [0.03, 0.05]$, $m_T \in [0.685, 2.5]$ TeV and $s_{24}, s_{34} \lesssim \lambda^5$. However, now (63) leads to

$$
\begin{aligned}
|F^u_{12}|^2 &= c_{14}^2 s_{14}^2 s_{24}^2, \\
{\rm Im}\lambda_T^K &\simeq -c_{12}^2 s_{14} s_{24} \sin\delta',
\end{aligned}
\tag{66}
$$

which in general are non-zero. This means that $D^0 - \overline{D}^0$, $K_L \to \pi^0 \overline{\nu}\nu$ and ε'/ε now receive non-vanishing NP contributions. For instance, we have $x_D^{\rm NP} \lesssim 0.066\%$ for $s_{24} \lesssim 1 \times 10^{-3}$ and $-4 \times 10^{-4} \lesssim \varepsilon'/\varepsilon < 0$ for $s_{24} \lesssim 7.8 \times 10^{-4}$. For $k^0/k^0_{\rm SM}$ we even obtain a significant reduction when compared to the result in the strict limit ($k^0/k^0_{\rm SM} = 1$) as one now has $0.25 \lesssim k^0/k^0_{\rm SM} \leq 1$.

At this point, it is instructive to present a global analysis of the parameter space of our realistic s_{14}-dominance case, subject to all the phenomenological constraints we just discussed. In particular, we look for the allowed ranges for s_{14}, s_{24}, δ' and m_T. In figures 1-3 we present the result of a simulation of 10^6 points, with $s_{14}, s_{24}, s_{34}, \delta_{14}, \delta_{24}$ and m_T as the free parameters. These span values of s_{14} that allow for the CKM unitarity problem to be solved and values of s_{24}, s_{34} compatible with the realistic case of s_{14}-dominance. More concretely the ranges used for the free parameters are

$$
\begin{aligned}
&s_{14} \in [0.03, 0.05], \quad s_{24}, s_{34} \in [0, 0.001], \\
&\delta_{14}, \delta_{24} \in [0, 2\pi], \quad m_T \in [0.685, 2.5]{\rm TeV}
\end{aligned}
\tag{67}
$$

where the range for s_{24} and s_{34} are compatible with our assumption that $s_{24}, s_{34} \lesssim \lambda^5$. The points displayed in these figures verify the conditions

$$
\begin{aligned}
&|\varepsilon_K^{\rm NP}| < \delta\varepsilon_K, \quad \left(\tfrac{\varepsilon'}{\varepsilon}\right)_{\rm NP} \in [-4, 10] \times 10^{-4}, \\
&\Delta m_K^{\rm NP} < \Delta m_K^{\rm exp}, \quad \frac{k^+}{k^+_{\rm SM}} \in [0.24, 2.28],
\end{aligned}
\tag{68}
$$

and we do not impose any constraint associated to other observables, because their NP contributions are extremely suppressed in both limits of s_{14}-dominance. We also omit plots involving s_{34}, as this parameter, within the chosen range $s_{34} \in [0, 0.001]$, has no noticeable influence of importance on the outcome of the allowed parameter region. This, coupled with the fact that in leading order, the expressions for the quantities that change significantly when going from $s_{24}, s_{34} = 0$ to $s_{24}, s_{34} \lesssim \lambda^5$, i.e. $|\varepsilon_K^{\rm NP}|$, $x_D^{\rm NP}$, $(\varepsilon'/\varepsilon)_{\rm NP}$ and $k^0/k^0_{\rm SM}$, do not depend on s_{34}, and depend only on one phase, δ', leads us to conclude that a minimal solution to the CKM-UP corresponds to the "two angle limit" where the two mixing angles s_{14} and s_{24} and the phase δ' are the only mixing parameters added to the SM.

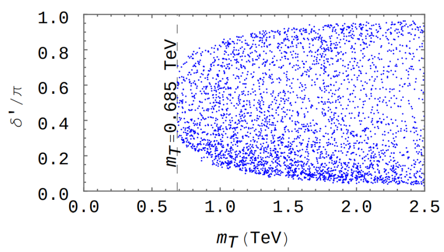

Figure 3: Allowed ranges for δ' against the allowed ranges for m_T.

It is also interesting to note that, although there still exists a considerable allowed region in parameter space, the regions $s_{14} \lesssim 0.37$ and $s_{24} \lesssim 1 \times 10^{-4}$ appear to be excluded, but the lower-bound for s_{24} can be lowered with a choice of a larger upper-bound for m_T (see the top panel in figure 2).

7. Conclusions

In this thesis we explored the possibility of having a minimal extension of the SM involving the sole introduction of an heavy-top quark T. We adopted the Botella-Chau parametrization for the 4×3 quark mixing-matrix, containing the usual three angles and phase of the 3×3 SM-mixing, plus three extra angles $\theta_{14}, \theta_{24}, \theta_{34}$ and two new phases δ_{14}, δ_{24}. We consider the limit of s14-dominance, where the introduction of $s_{14} \sim \lambda^2$ alone is sufficient to solve the CKM-UP, allowing the remaining mixing angles s_{24} and s_{34} to either be zero or much smaller.

In a first attempt, we explored the limit of strict s_{14}-dominance, where $s_{24} = s_{34} = 0$ and the NP phases δ_{14} and δ_{24} of the BC parametrization are unphysical. Within this exact limit, some extremely interesting features were encountered such as the dominant heavy-top decays to light quarks while decays to the third generation are very suppressed, which is a result that defies the usual assumption. In fact, this salient feature means that heavy-top masses as low as $m_T = 0.685$ TeV cannot be excluded, which is a value potentially accessible to the next generation of accelerators. Moreover, in this limit, not only does the 3×3 block of $V_{\rm CKM}$ containing the SM mixings remain essentially unchanged, but also the NP contributions to processes such as $D^0 - \overline{D}^0$, $K_L \to \pi^0 \overline{\nu} \nu$ and to the parameter ε'/ε are automatically zero, while for $B^0_{d,s} - \overline{B}^0_{d,s}$, although non-zero, are still exceedingly small. These results demonstrated that this limit can, to a very significant extent, recover many of the SM predictions. Even processes that may receive significant NP contributions like $K^0 - \overline{K}^0$ and $K^+ \to \pi^+ \overline{\nu} \nu$ do not strictly restrict the relatively small parameter space formed by the only two free parameters of the model: s_{14} and m_T. However, in the end, this limit fails to accommodate a new and more stringent constraint we set for the NP contribution of ε_K.

This result encouraged us to explore a slightly modified version of this limit, the limit of realistic s_{14}-dominance. The parameter region for s_{14} was maintained as a means to solve the CKM-UP, but the assumption of vanishing s_{24} and s_{34} was relaxed and instead $s_{24}, s_{34} \sim \lambda^5$ was used, so that now δ_{14} and δ_{24} have to taken into account. It is then shown that, in this limit, the problem previously encountered for ε_K could be satisfyingly solved leading to a reasonably large allowed region of parameters. Moreover, the results for the SM mixings, the heavy-top decays and the NP contributions to $K^0 - \overline{K}^0$, $B^0_{ds} - \overline{B}^0_{ds}$ and $K^+ \to \pi^+ \overline{\nu} \nu$, that where encountered in the strict limit where essentially recovered. The remaining processes which previously received no contribution, now do, but are still very small and do not comprise the safety of the model. Interestingly, in this analysis the leading order expressions for all the NP contributions of the studied processes/parameters depend solely on three of the five new mixing parameters, the mixing angles s_{14} and s_{24}, as well as the phase difference $\delta' = \delta_{24} - \delta_{14}$. This means that the addition of s_{34} might be superfluous and the "two angle limit" defined by having $s_{14} \sim \lambda^2$, $s_{24} \sim \lambda^5$ and $s_{34} = 0$ and consequently δ' as the only relevant NP phase, should constitute a true minimal solution to the CKM-UP for models with one heavy-top.

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