Modeling, simulation, and altitude control of a weather meteorological balloon

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Abstract: Weather balloons are important tools for the study and prediction of different meteorological characteristics such as humidity, wind, temperatures and atmospheric pressure. These balloons, usually made of latex, fly freely in the atmosphere through a buoyant force, resulting from the difference between the densities of air and the lifting gas of the balloon. Based on the SONDA project proposal, the idea of altitude control for a weather balloon was developed. This project suggests that the balloon's altitude should be controlled so it can exploit the wind currents present in the atmosphere using them as guidance to interest research areas. Balloon altitude control is a common research topic for high altitude balloons, however, for latex balloons, this study is not a very explored area. Thus, this thesis has two main parts. The first is to model a weather balloon. The modelling starts with the description of the main components of the balloon, then, the definition of an atmospheric model to represent the atmospheric conditions the balloon would face and the study of the nonlinear model representing the balloon's elasticity is done. In addition to these models, a thermal model, a wind model and a dynamics model were also developed. The second part of the project was the development of altitude control strategy for a latex balloon. Control was done through a cascade control strategy to perform velocity and altitude control. Finally, the altitude control and free dynamics of the modelled balloon were simulated.
Keywords: Weather Balloon, Balloon Modelling, Altitude Control, Velocity Control

I. Introduction

Weather balloons are launched from about 900 locations around the world two times every day to collect information on temperature, humidity, and winds at various levels in the atmosphere [12]. Remaining as an indispensable tool in atmospheric science, meteorology and other applications requiring stratospheric observations, they also became a low-cost alternative for meteorological research since maintenance and repair of satellites cannot be provided without extremely expensive equipment. These balloons, usually made of latex, fly freely in the atmosphere through a buoyant force, resulting from the difference between the densities of the atmospheric air and the lifting gas inside the balloon. An ongoing project named SONDA (Synchronous Oceanic and Atmospheric Data Acquisition) proposes the idea of altitude control for a weather balloon that acts as a carrier for a probe. This project suggests the combination of altitude control with the wind currents profile through the atmosphere to lead the weather balloon with the probe into interest research areas. Following that idea, in this work, a latex balloon filled with helium will be modelled and an altitude control strategy will be implemented. Altitude control is a field of interest to high altitude balloons since balloons are a cheaper option of atmospheric study than other means. There are different high altitude balloon types, such as super pressure balloons, zero pressure balloons and dual-balloons and they can be controlled with different mechanisms. To control the altitude of a weather balloon it is necessary to either change the buoyancy of the balloon or its mass. Buoyancy change is obtained by changing the volume of the balloon (which can be done with mass or temperature change) [13]. In the balloon altitude control field, works have been developed for zero pressure balloons such as [9], in which a PID control strategy with a gas compress–release mechanism was implemented. For a super pressure balloon, works such as [3], in which reinforcement learning was used to train a flight controller from simulations, was made. Some work on station keeping, considering that wind directions vary with altitude and exploring the natural wind field, such as in [8] and [5] have also been developed. However, for latex balloons, this field is yet to be explored. And this is the main focus of the present work.

II. Weather Balloon Modelling and Altitude Control

A. Weather balloon characteristics

For this work, it is intended to firstly model a latex balloon filled with helium. For a usual weather balloon that only has the balloon film mass, gas and a payload, the following approach can be followed using tabled values for a specific weather balloon. Knowing some specifications such as the balloon envelope/film mass (m_b), the payload mass (m_p), initial balloon radius at launch (R_i), its uninflated radius (R_0), latex density (ρ_b), neck size and maximum radius before it bursts (R_{max}), posterior calculations of its parameters at launch can be made. With all these balloon parameters as well as the lift gas density (ρ_{0Gas}), it's possible to calculate the values of the following variables at launching:

Thickness of the uninflated balloon:

$$t_0 = \frac{m_b}{4\pi \cdot R_0^2 \cdot \rho_b} \tag{1}$$

Initial volume of the balloon at release:

$$V_0 = \frac{4}{3}\pi \cdot R_i^3 \tag{2}$$

Initial mass of gas:

$$m_{0Gas} = \rho_{0Gas} \cdot V_0 \tag{3}$$

The weather balloon will also carry a gas cylinder of weight m_{cil} and a control valve (m_{valve}). And the correct way to find the previous parameters will be through lift. Lift is the difference between buoyancy and the weight of the balloon.

$$L = V_0 \cdot g \cdot (\rho_{0Atm} - \rho_{0Gas}) \tag{4}$$

Considering that the weather balloon will carry the balloon film mass, the payload, a gas cylinder and a control valve, the lift should compensate all these weights.

$$m \cdot g = V_0 \cdot g \cdot (\rho_a - \rho_g) \Longleftrightarrow V_0 = \frac{m}{\rho_a - \rho_g} \tag{5}$$

Here, m represents the sum of all components of the weather balloon that doesn't include the mass gas.

$$m = m_b + m_p + m_{cil} + m_{valve} \tag{6}$$

The parameter ρ_a, represents the density of the air surrounding the balloon. Introducing this volume into equation 3, the initial mass can be found and verified.

B. Hyperelastic Theory

Since rubber applies a restoring force inwards, the pressure inside any elastic balloon is always a little bit greater than the outside pressure. It could be useful to analyse whether this pressure difference affects the ascent of the balloon. This pressure variation is called the membrane pressure (ΔP) and it is the difference between the pressure inside the balloon (P_{in}) and the atmosphere pressure (P_{out}):

$$P_{in} = \Delta P + P_{out} \tag{7}$$

Rubber (latex) is a nonlinear elastic material, therefore, nonlinear models (Mooney-Rivlin and Gent models) should be used to represent its pressure variation. The Mooney-Rivlin model is useful for rubber materials at small stretches, but it is not adequate for large strain subjection. Gent can model the stiffening that the rubber undergoes as it approaches its breaking point. However, the parameters are more laborious to measure precisely without destructing the balloon. These theoretical models and parameters can be consulted in [11].

Mooney-Rivlin Model:

$$\begin{aligned} 0 =& p_a(z) - \frac{nR_gT_g(z)}{\frac{4}{3}\pi R^3} + \\ & 2s_+\Delta\left(1 - \frac{s_-}{s_+}\left(\frac{r}{r_0}\right)^2\right) \end{aligned} \tag{8}$$

Gent Model:

$$\begin{aligned} 0 =& p_a(z) - \frac{nR_gT_g(z)}{\frac{4}{3}\pi R^3} + \\ & 2\eta\Delta\left(\frac{J_m}{J_m - 2\left(\frac{R_0}{R}\right)^{-2} - \left(\frac{R_0}{R}\right)^4 - 3}\right) \end{aligned} \tag{9}$$

where Δ represents the following:

$$\Delta = \frac{t_0}{r_0}\left(\left(\frac{R_0}{R}\right) - \left(\frac{R_0}{R}\right)^7\right)$$

C. Atmospheric Model

The dynamics of the balloon are heavily influenced by the density, pressure and temperature of the air surrounding it. The typical weather balloon will reach altitudes of approximately 30-35 km before it bursts [7]. To study the influence of atmosphere in the balloon the International Standard Atmosphere model [1] will be used. This model expresses the temperature, pressure and density variation with altitude. The equations for the troposphere (0 to 11000 meters), lower stratosphere (11000 to 25000 meters) and upper stratosphere (above 25000 meters) are the following:

$$T_a(z) = \begin{cases} 288.15 - 0.0065 \cdot z, & \text{if } A \\ 216.65, & \text{if } B \\ 141.94 + 0.00299 \cdot z, & \text{if } C \end{cases} \tag{10}$$

$$p_a(z) = \begin{cases} 101290 \cdot (\frac{288.15-0.0065 \cdot z}{288.15})^{5.256}, & \text{if } A \\ 22650 \cdot \mathrm{e}^{(1.73-0.000157 \cdot z)}, & \text{if } B \\ 2488 \cdot (\frac{141.94+0.00299 \cdot z}{216.65})^{-11.388}, & \text{if } C \end{cases} \tag{11}$$

$$A = z \in [0, 11000[$$

$$B = z \in [11000, 25000[$$

$$C = 25000 \leq z$$

D. Thermal Model

The temperature inside the balloon changes during the different phases of the balloon flight. As a consequence of, mostly, direct solar radiation and infrared radiation from the Earth, the envelope heats the gas inside the balloon through convection and self glow and as a result, the gas inside the balloon expands, the balloon radius increases which leads the buoyancy to also increase. The used model, illustrated by Figure 1, is based on the one presented in [6]. The temperature variables are T_a, for atmospheric temperature, T_f, for balloon film temperature and T_g for the gas temperature.

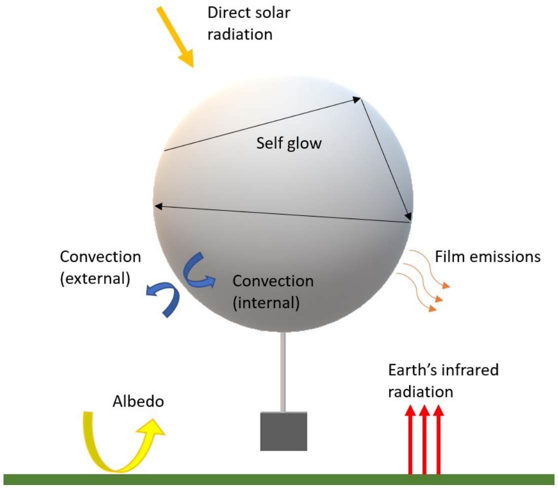


Fig. 1. Thermal model of the weather balloon. The model is composed by radiation (solar or infrared), albedo, convection (internal and external), film emissions and self glow.

*1) **Direct solar radiation**:* The solar radiation depends only on the solar constant (which is here assumed to be E_{Sun} =1367.5 W/m^2) and on the solar elevation angle, ψ:

1) Mean Anomaly (MA)

$$MA = 2\pi \frac{Day_{number}}{Days_{PerYear}} \tag{12}$$

The mean anomaly depends on the variable Day_{number} (which is how many days have past since the beginning of the year until the balloon date of launch) and the parameter $Days_{PerYear}$ (which is the total number of days in that year).

2) True Anomaly (TA)

$$TA \approx MA + 2e \sin(MA) + \frac{5}{4}e^2 \sin(2MA) \tag{13}$$

3) Solar irradiance flux at the top of the atmosphere

$$I_{Sun} = \frac{E_{Sun}}{R_{AU}^2} \cdot \left(\frac{1 + e\cos(TA)}{1 - e^2}\right)^2 \tag{14}$$

The parameters $e = 0.016708$ and $R_{AU} = 1$ are related to the planet (in this case, the values presented are for the Earth).

4) Direct solar irradiance at altitude Z

$$I_{SunZ} = I_{Sun}\tau_{atm} \tag{15}$$

The direct solar irradiance at a certain height depends on the transmissivity of the atmosphere, τ_{atm} (explained in section II-D8), at such height.

5) Direct solar flux acting on the balloon

$$q_{Sun} = I_{SunZ} \tag{16}$$

The total heat from the direct solar influence is expressed by:

$$Q_{Sun} = \alpha \cdot A_p \cdot q_{Sun}\beta \tag{17}$$

where, A_p is the projected area, $\beta = 1 + \frac{\tau}{1-r}$, α is the solar absorptivity of the external surface, τ is the atmosphere transmissivity and the reflection coefficient is given by $r = 1 - \alpha - \tau$.

*2) **Infrared radiation**:* The longwave radiation flux from the ground at the balloon altitude Z follows the Stefan-Boltzman law (with σ =5.67×10^{-8} Wm^{-2}K^{-4}) and is written as:

$$\begin{aligned} q_{IREarth} &= q_{IRgroundZ} = q_{IRground} \cdot \tau_{atmIR} \\ &= \varepsilon_{ground} \cdot \sigma \cdot T_{ground}^4 \cdot \tau_{atmIR} \end{aligned} \tag{18}$$

Ground emissivity is usually low in the hottest places while it is usually high in the coldest ones. However, it is possible to consider an average value ε_{ground}=0.95. The total heat resulting from the infrared radiation is given by:

$$Q_{IREarth} = \alpha_{IR} \cdot A_s \cdot q_{IREarth} \cdot VF\beta_{IR} \tag{19}$$

where A_s is the balloon surface area, α_{IR} is the absorptivity of the external surface in the infrared spectrum, $\beta_{IR} = 1 + \frac{\tau_{IR}}{1-r_{IR}}$, $r_{IR} = 1 - \alpha_{IR} - \tau_{IR}$. τ_{IR} is the transmissivity of the atmosphere in the infrared spectrum (explained in section II-D8). The VF component represents the "View Factor", which is the effective balloon area exposed to the planet surface. This component depends on the Earth radius R_{Earth} and the height z. Its expression is given by:

$$HalfCone = \sin^{-1}\left[\frac{R_{Earth}}{R_{Earth+z}}\right] \tag{20}$$

$$VF = \frac{1 - \cos(HalfCone)}{2} \tag{21}$$

*3) **Albedo**:* Albedo represents the fraction of solar radiation that is reflected by the planet surface and atmosphere.

$$q_{Albedo} = Albedo \cdot I_{Sun} \cdot \sin(\psi) \tag{22}$$

$$Q_{Albedo} = \alpha A_s q_{Albedo} VF\beta \tag{23}$$

*4) **Film emissions**:* The emitted energy from both the interior and exterior of the balloon skin depends on the film temperature itself, T_f, and the emissivity of the film ε.

$$Q_{IRout} = \sigma\varepsilon A_s T_f^4 \beta_{IR} \tag{24}$$

*5) **External Convection**:* The Nusselt number for a sphere in free convection is:

$$Nu_{free} = 2 + 0.45 \cdot (Gr_a \cdot Pr_a)^{0.25} \tag{25}$$

The Nusselt number for forced convection is:

$$Nu_{forced} = 2 + 0.41 \cdot Re^{0.55} \tag{26}$$

The Grashof number is given by:

$$Gr_a = \frac{\rho_a^2 \cdot g \cdot |T_f - T_a| \cdot D^3}{T_a \cdot \mu_a^2} \tag{27}$$

Reynolds number is given by:

$$Re = \frac{v_z \cdot D \cdot \rho_a}{\mu_a} \tag{28}$$

where, D is the balloon's diameter.

- Free convection

$$H_{efree} = \frac{Nu_{free} \cdot k_a}{D} \tag{29}$$

- Forced convection

$$H_{eforced} = \frac{Nu_{forced} \cdot k_a}{D} \tag{30}$$

The heat transfer coefficient used for eternal convection should be the maximum value of the free and forced convection heat transfer coefficients:

$$H_e = max(H_{efree}, H_{eforced}) \tag{31}$$

And the heat through external convection is then:

$$Q_{ConvExt} = H_e \cdot A_s(T_a - T_f) \tag{32}$$

*6) **Self-Glow**:* Absorbed infrared self-glow from the interior.

$$Q_{IRfilm} = \sigma\varepsilon\alpha_{IR}A_sT_f^4\frac{1}{1-r_{IR}} \tag{33}$$

*7) **Internal Convection**:* The internal convection is only result of free convection. The Nussest number is obtained by the expression:

$$Nu_{int} = 2 + 0.45 \cdot (Gr_g \cdot Pr_g)^{0.25} \tag{34}$$

The Grashof number is given by:

$$Gr_g = \frac{\rho_g^2 \cdot g \cdot |T_f - T_g| \cdot D^3}{T_g \cdot \mu_g^2} \tag{35}$$

Once again, the heat transfer coefficient can be written as:

$$H_i = \frac{Nu_{int} \cdot k_g}{D} \tag{36}$$

Finally, the heat produced by internal and external convection are defined as following:

$$Q_{ConvInt} = H_i \cdot A_s(T_f - T_g) \tag{37}$$

*8) **Atmosphere Transmissivity**:* Irradiance of both the solar radiation and the long-wave radiation is influenced by the presence of the atmosphere. The atmosphere transmissivity can be calculated for both the direct solar radiation and the ground infrared radiation as following:

- Transmissivity of a solar beam through the atmosphere

$$\tau_{atm} = 0.5\left[e^{-0.65 \cdot AirMass} + e^{-0.095 \cdot AirMass}\right] \tag{38}$$

- Infrared transmissivity of the ground

$$\tau_{atmIR} = 1.716 - 0.5\left[e^{-0.65\frac{p_a}{p_0}} + e^{-0.095\frac{p_a}{p_0}}\right] \tag{39}$$

- Air Mass Factor:

$$AirMass = \left(\frac{p_a}{p_0}\right) \cdot (\sqrt{1229 + (614\sin(\psi))^2} - 614\sin(\psi)) \tag{40}$$

Note:
- p_a is ambient atmosphere pressure;
- p_0 is the Earth surface atmosphere pressure;

*9) **Heat Transfer Coefficient Parameters**:* The procedure to compute the internal and external convection requires the heat transfer coefficients. This coefficient depends on different air and gas properties that are represented by the following expressions:

- Dynamic viscosity ($\frac{N \cdot s}{m^2}$)

$$\mu_a = \frac{1.458 \cdot 10^{-6}\ T_a^{1.5}}{T_a + 110.4} \tag{41}$$

$$\mu_g = 1.895 \cdot 10^{-5} \cdot \left(\frac{T_g}{273.15}\right)^{0.647} \tag{42}$$

- Conductivity ($\frac{W}{m \cdot K}$)

$$k_a = 0.0241 \cdot \left(\frac{T_a}{273.15}\right)^{0.9} \tag{43}$$

$$k_g = 0.144 \cdot \left(\frac{T_g}{273.15}\right)^{0.7} \tag{44}$$

- Prandtl number

$$Pr_a = 0.804 - 3.25 \cdot 10^{-4} \cdot T_a \tag{45}$$

$$Pr_g = 0.729 - 1.6 \cdot 10^{-4} \cdot T_g \tag{46}$$

*10) **Thermal Balance**:* The thermal balance on the balloon film is represented by the following equation:

$$\dot{T}_f = \frac{Q_f}{c_f \cdot m_b} \tag{47}$$

where, c_f can be obtained by a correlation presented in [14]. The balloon envelope/film is affected by the following heats:

$$\begin{aligned} Q_f = {} & Q_{Sun} + Q_{IREarth} + Q_{Albedo} + Q_{IRfilm} \\ & + Q_{ConvExt} - Q_{IRout} - Q_{ConvInt} \end{aligned} \tag{48}$$

The thermal balance on the gas is represented by:

$$\dot{T}_g = \frac{Q_{ConvInt}}{\gamma \cdot c_v \cdot m_g} + (\gamma - 1) \cdot T_g \cdot \left(\frac{dm_g}{dt} \cdot \frac{1}{m_g} - \frac{dV}{dt} \cdot \frac{1}{V}\right) \tag{49}$$

For all previous calculations $A_p = \pi R^2$ and $A_s = 4A_p$.

E. Dynamics Model

As is shown in figure 2, the balloon is affected by the following forces:
Weight: $F_G = m_t \cdot g$
Buoyancy: $F_B = \rho_a \cdot V_b \cdot g$
Drag: $F_{Dz} = \frac{1}{2} \cdot C_D \cdot A_b \cdot \rho_a v_z^2$

$$\sum F = m \cdot a \Longleftrightarrow m_t \cdot \ddot{z} = F_G + F_{Dz} - F_B$$

The differential equation governing the vertical motion is then:

$$\ddot{z} = \frac{F_B - F_{Dz} - F_G}{m_t + m_v} \tag{50}$$

where, $m_v = C_m \cdot \rho_a \cdot V$ and represents the virtual mass the balloon carries during the flight. The coefficient C_m has a typical value of 0.5.

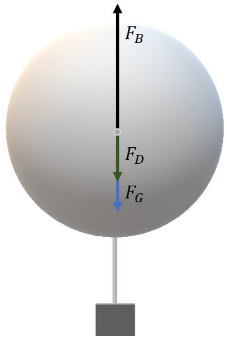


Fig. 2. Acting forces on the weather balloon.

Correlation: Drag coefficient and Reynolds number
The drag coefficient will not be constant. For this purpose, a correlation developed in [10], will be used with some adaptation to the weather balloon. Adapting this correlation developed for a sphere to a weather balloon, it was chosen to lower the curve to have drag coefficients near to 0.3 values. This adapted correlation will then be:

$$C_D = \frac{24}{Re} + \frac{2.6\left(\frac{Re}{5}\right)}{1+\left(\frac{Re}{5}\right)^{1.52}} + \frac{0.411\left(\frac{Re}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{Re}{2.63\times10^5}\right)^{-8}} + \frac{0.25\left(\frac{Re}{10^6}\right)}{1+\left(\frac{Re}{10^6}\right)} - 0.04 \tag{51}$$

F. Wind Model

It is assumed that there is no horizontal slip between the balloon and the surrounding air mass, so the horizontal velocity of the balloon is equal to that of the incident wind itself. In this case, it is only necessary to convert the wind components v_x and v_y to the LLA coordinate system. Defining the mean Earth radius as $r_{Earth} = 6371009$ m, and with z being the current altitude of the balloon, the total distance to the centre of the Earth at each instant is given by $R_{total} = r_{Earth} + z$. The variations in latitude ($\frac{\delta\phi}{\delta t}$) and longitude ($\frac{\delta\lambda}{\delta t}$) in each instant can be obtained by:

$$\frac{\partial\phi}{\partial t} = \frac{180}{\pi}\frac{v_{wy}}{R_{total}} \tag{52a}$$

$$\frac{\partial\lambda}{\partial t} = \frac{180}{\pi}\frac{v_{wx}}{R_{total}\cdot cos(\phi\frac{\pi}{180})} \tag{52b}$$

where ϕ is the current latitude in degrees (°).

G. Altitude Control

The control strategy proposed in section III-E, will be performed in the first two states of the system, the altitude and the velocity, having the final objective of positioning the weather balloon in reference altitude. A cascade control strategy was chosen with controller gains for velocity and altitude developed with the Linear Quadratic Regulator algorithm (section III-E2) implemented through Gain Scheduling a technique that allows the nonlinear system to be controlled in a simple and effective way.

III. Implementation

A. Initial Parameters

The tabled parameters specified in [2] of the model TA450 are made for a simple balloon with no additional structure. Only the payload, film, and initial gas mass. These tabled parameters are the following:

TABLE I
TA450 TABLED PARAMETERS FROM [2]

Parameter	Symbol	Value	Units
Balloon mass	m_b	0.450	kg
Payload mass	m_p	0.250	kg
Uninflated radius	R_0	0.43	m
Radius at release	R_i	0.65	m
Burst radius	R_b	2.36	m
Latex (rubber) density	ρ_b	1100	kg/m^3

An additional weight is added to the weather balloon, which is the structure responsible for controlling the mass of gas (a gas cylinder and a control valve). The cylinder will add a weight of m_{cil} =0.8 kg and the valve will add m_{valve} =0.15 kg. Therefore, the initial necessary mass of gas and volume of the weather balloon will be the ones presented in Table II.

TABLE II
BALLOON LAUNCH PARAMETERS

Parameter	Symbol	Value	Units
Initial Volume	V_0	1.5767	m^3
Initial Mass of Gas	m_{g0}	0.2814	kg

The atmosphere at launch will be considered the one at the altitude of 0 m. And the parameters are presented in table III.

TABLE III
ATMOSPHERE PARAMETERS AT LAUNCH

Parameter	Symbol	Value	Units
Temperature	T_0	288.15	K
Air pressure	P_{a0}	101325	Pa
Air density	ρ_{a0}	1.225	kg/m^3

As for helium, which is the gas considered in this simulation, the used parameters were:

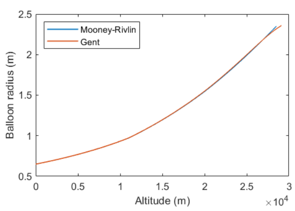
TABLE IV
GAS PARAMETERS AT LAUNCH

Parameter	Symbol	Value	Units
Molecular mass	M_{He}	$4.003\cdot10^{-3}$	kg/mol
Perfect gas constant	R_g	8.315684	JK^{-1} mol^{-1}
Temperature	T_0	288.15	K
Gas pressure	P_{g0}	106913	Pa
Gas density	ρ_{g0}	0.1785	g/m^3

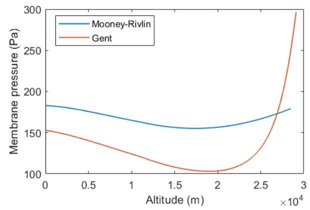
Note: Helium density is obtained using the perfect gas law.

B. Hyperelastic Model Implementation

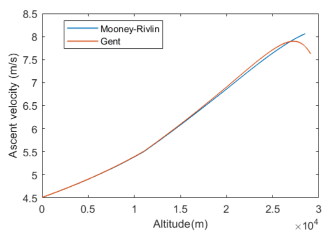
Implementing both hyperelastic models presented in section II-B, it can be seen that their behaviour is very similar in terms of radius evolution and ascent velocity as is shown in 3. The differences that can be pointed are relative to the ending phase of ascent velocity where the gent model represents the elastic behaviour of the balloon right before bursting differently than the Mooney-Rivlin model, and the small difference in terms of membrane pressure of one and the other. As these differences are not that relevant to influence the decision of which model to use, it was chosen to use the Mooney-Rivlin model since the Gent model parameters can be laborious to measure precisely and are very dependent on destructive testing (as was explained in section II-B).



(a) Radius variation with height for both models.



(b) Membrane Pressure variation with height for both models.



(c) Ascent Velocity variation with height for both models.

Fig. 3. Mooney-Rivlin and Gent models comparison of radius, membrane pressure and ascent velocity evolution.

C. Thermal Model Implementation

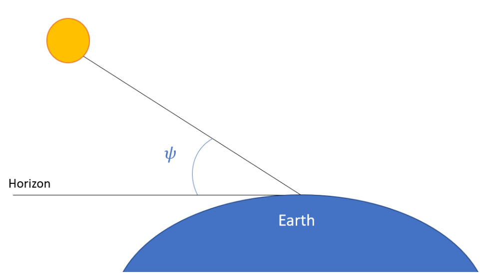


Fig. 4. Elevation angle of the sun (ψ) and the horizon representation.

Parameters:

An average value was chosen to implement the thermal contribution of albedo. For this purpose, a value of $Albedo = 0.12$ is the chosen mean value to represent this contribution. Another important note is that, since the heat the balloon receives from albedo depends on the elevation angle (shown in Figure 4), when is nighttime and the elevation angle is negative, the albedo is 0.

Algorithm 1 Elevation Angle of The Sun

function ELEVATION($lat, long, day, month, year, hour, min, sec$) a specific time of year

$g = g_1 + g_2 D$

$q = q_1 + q_2 D$

$L = q + L_1 \sin(g) + L_2 \sin(2g)$

$b = 0$

$e = e_1 - e_2 D$

$RA = \arctan\left(\frac{\cos(e)\sin(L)}{\cos(L)}\right)$

$\delta = \arcsin(\sin(e)\sin(L))$

$T = \frac{D}{t};$

$\theta_0 = a_1 + a_2 D + a_3 T^2 - \frac{T^3}{a_4}$

$\theta = \theta_0 + long$

$LHA = \theta - RA$

$\psi = \arcsin(\sin(lat)\sin(\delta) + \cos(lat)\cos(\delta)\cos(LHA))$

return ψ

end function

Fig. 5. Elevation angle of the sun (ψ) algorithm. The coefficients values and formulas description can be consulted in [4].

TABLE V
OPTICAL PROPERTIES OF THE FILM FOR DIRECT SOLAR RADIATION

Transmissivity	τ	0.61
Absorvity	α	0.35
Reflectivity	$r = 1 - \alpha - \tau$	0.04

TABLE VI
OPTICAL PROPERTIES OF THE FILM FOR INFRARED RADIATION

Transmissivity	τ_{IR}	0.86
Absorvity	α_{IR}	0.1
Reflectivity	$r_{IR} = 1 - \alpha_{IR} - \tau_{IR}$	0.04

TABLE VII
HEAT TRANSFER PARAMETERS

Parameter	Symbol	Value	Units
Adiabatic expansion coefficient	γ	1.667	-
Specific heat (helium)	c_v	3115.89	$\mathrm{Jg^{-1}K^{-1}}$

D. Wind Model Implementation

The wind information required to represent the horizontal movement of the balloon is obtained from NOAA (National Oceanic and Atmospheric Association) servers. NOAA provides weather forecasts, ocean and coast data and the wind database can be found in NOMADS (NOAA Operational Model Archive and Distribution System), where global and regional models of statistical and forecast wind data are stored. An example of the wind data acquired (with 0.25 degree precision) can be seen in figure 6. In the presented image, v_{wx} (representing the longitudinal velocity) and v_{wy} (representing the latitudinal velocity) for a fixed latitude, longitude and time, is shown. The data obtained for different pressures is represented graphically as a function of height (until $25Km$).

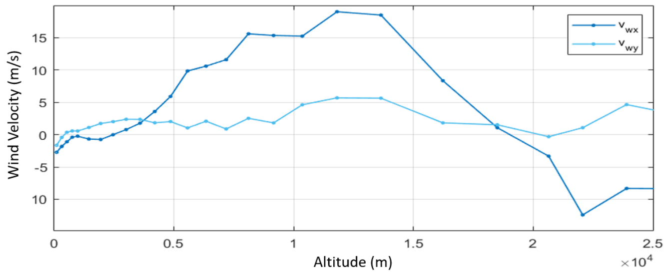


Fig. 6. Wind data from the NOMADS server for a fixed latitude (53º) and fixed longitude (350º).

After implementing the model presented in section II-F (named "Case 1") and another model that considered lateral drag (named "Case 2"), the difference between "Case 1" and "Case 2" was analysed. Since this error was not big enough

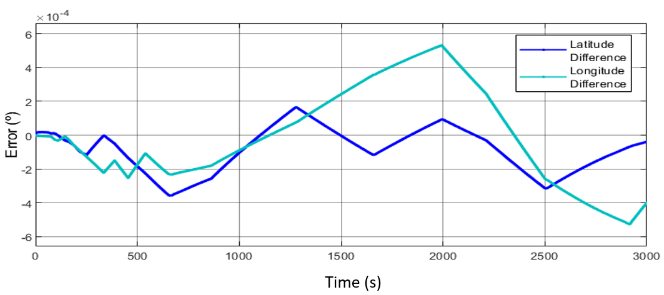


Fig. 7. Evolution of the difference between latitude and longitude (in degrees) obtained in "Case 1" and "Case 2" with time (in seconds).

to be considered in the model choice, the horizontal motion can be simplified to the simpler case (the one presented in section II-F). Therefore, after the shown results were obtained and analysed, it was decided to choose the first and simpler case as the one to continue in the implementation and to be included in the balloon model.

E. Altitude Control Design and Implementation

To execute altitude control of the weather balloon, it was decided to use a cascade control strategy and gain-scheduling with controller gains obtained by the LQR (Linear Quadratic Regulator) algorithm. LQR's implementation is relatively easy and its algorithm performs optimisations of the controller gains. LQR designs are only applicable for linear systems. Therefore, in the first step, the linearisation of the weather balloon nonlinear model is necessary. Noting a very big influence of the velocity (which presents a faster dynamics) in the altitude (a slower dynamics) of the weather balloon, the cascade control strategy is adequate. In this cascade control design, the altitude control is made on the primary (or outer) loop, while the vertical velocity control composes the secondary (or inner) loop.

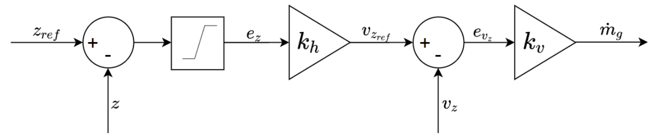


Fig. 8. Cascade controller applied to the weather balloon.

$$\dot{m}_g = k_v(k_h(z_{ref} - z) - v_z) \tag{53}$$

$$z_{ref} - z \in [-500, 500] \tag{54}$$

1) Linearisation: The state vector of the weather balloon system is given by $\boldsymbol{x} = \begin{bmatrix} v_z & z & \phi & \lambda & T_f & T_g & m_g \end{bmatrix}$. The input is $u = m_g$ and there will also a perturbation caused by the wind $\boldsymbol{w} = \begin{bmatrix} v_{wy} & v_{wx} \end{bmatrix}$. The output is defined by $\boldsymbol{y} = \begin{bmatrix} \phi & \lambda & z \end{bmatrix}$. Using the "Linear analysis tool" from MATLAB and introducing some restrictions to the velocity and altitude that were expected for the balloon a set of values that composes different operation points were obtained. It was decided to linearise the weather balloon model for a set of 9 operation points corresponding to different altitudes. These altitudes had a gap of 1 km between them and the points were from 15 km to 23 km. The LQR procedure will be demonstrated for one trim point (with an altitude of 15000 m). For this point,

$$u_o = 0$$

and

$$x_{o1} = \begin{bmatrix} 0, 15000, 307, 307, 0.181 \end{bmatrix}^T$$

.

This value of u_o was used for all cases of different trim states.

2) Linear Quadratic Regulator: After obtaining matrices A and B from the linearisation process, to calculate the gains with LQR algorithm 3 mais steps are needed:

1) Definition of the weights Q (relative importance of the energy associated with the state) and R (relative importance of the energy associated with the control action).

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 0.007 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{55}$$

$$R = 100$$

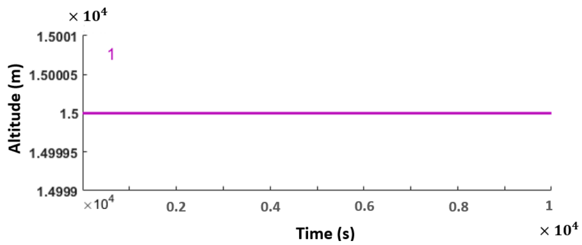
2) Resolution of the ARE (Riccati algebraic equation) given by $A^T P + PA - PBR^{-1}B^T P + Q = 0$
3) Finally, having the P matrix and knowing that $K = R^{-1}B^T P$, the gains K are found. The gains obtained with the shown data was:

$$K = \begin{bmatrix} 0.22692 & 0.00836 & 0.00173 & 0 & 2.97841 \end{bmatrix}$$

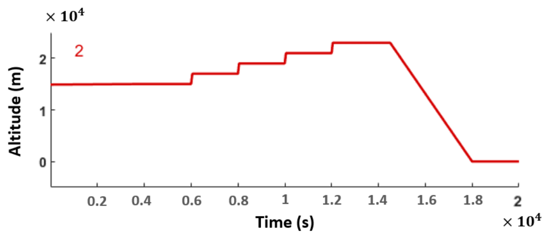
in which the vertical velocity gain is $k_v = 0.22692$ Kg/m and the altitude gain is $k_h = 0.00836\ s^{-1}$.

IV. Results

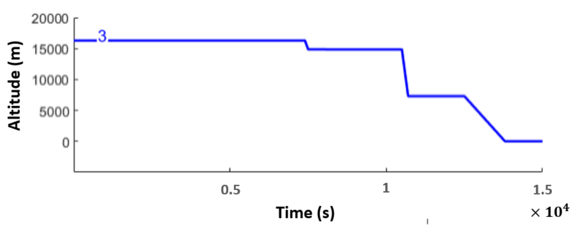
Three altitude references were used in this work to simulate the weather balloon flight, being that the last reference was manually chosen to simulate a trajectory from Lisbon to Seville to show how the altitude control could be combined with the study of the wind currents behaviour



(a) ”Constant” reference.



(b) ”To the sky” reference.



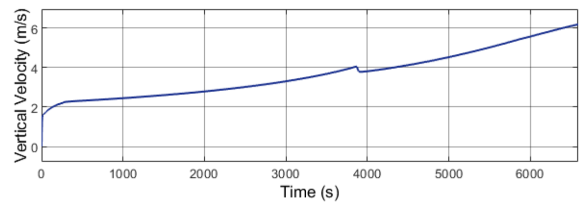
(c) ”Lisbon to Seville” reference.

Fig. 9. Used references for simulation.

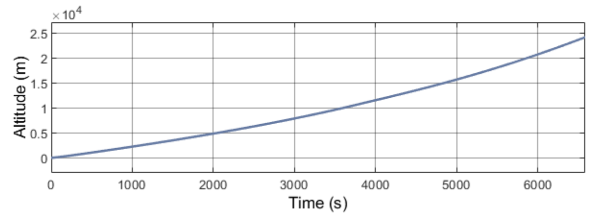
A. Weather balloon dynamics simulation

After implementing the models developed in section II, it was possible to obtain the evolution of temperature, LLA components, velocity and mass variation in the balloon through the flight. The simulation of the TA450 weather balloon free flight had a duration of approximately 1 hour and 48 minutes (6585s) and was done with the parameters presented in section III.

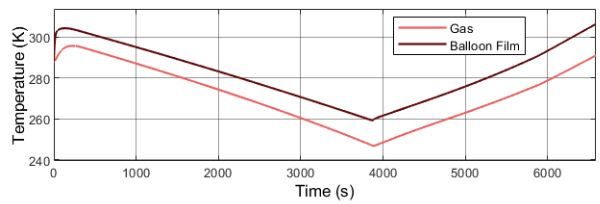
The free model dynamics present an increasing velocity for the through time with a point in time (when the balloon passes through a transition phase) that the velocity decreases slightly (as a consequence of a varying drag coefficient). The maximum value of velocity is $6.182m/s$. As for the altitude evolution, the curve evolves until the maximum altitude of $24140m$. The temperature decreases until altitudes slightly superior to 10Km as result of a decreasing atmospheric temperature. The latitude decreases and longitude increases for this flight.



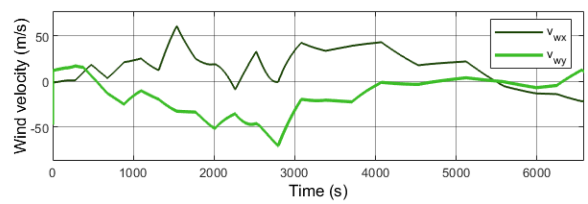
(a) Vertical velocity (m/s) for a free flight.



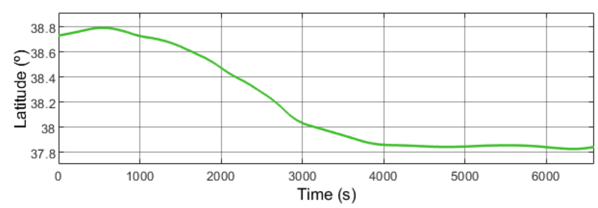
(b) Altitude (m) for a free flight.



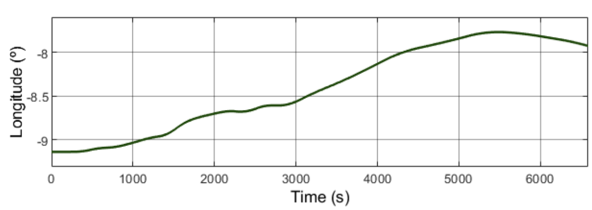
(c) Temperature (K) for a free flight.



(d) Wind Velocity (m/s) for a free flight.



(e) Latitude $(°)$ for a free flight.



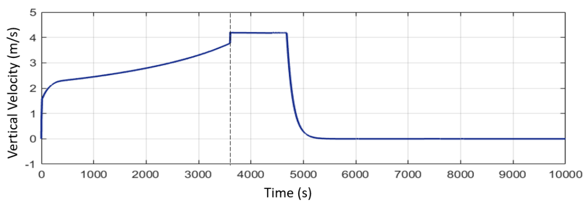
(f) Longitude $(°)$ for a free flight.

Fig. 10. Weather balloon free dynamics.

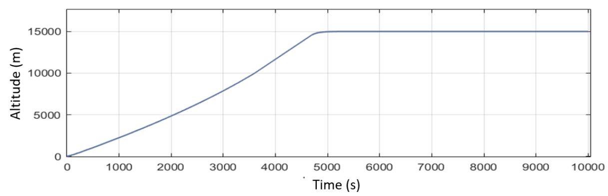
B. Altitude control results simulation

For all simulation with altitude control, the altitude is only controlled after the first hour of flight.

1) "Constant" Reference: Figure IV-B1 shows that at $5500s$ the balloon is $0.5m$ distant from the reference. As for the velocity profile, it can be seen the moment when the velocity transitions from the first hour of uncontrolled flight in which its velocity was near $3.8m/s$ to the control phase where this velocity is regulated to $4.2m/s$. This velocity is maintained as the maximum velocity that leads the weather balloon to the wanted position.



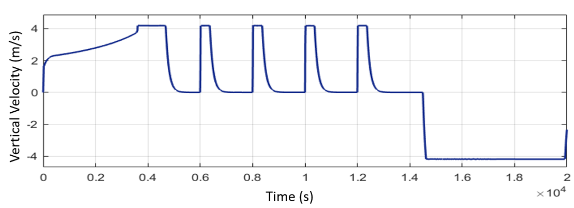
(a) Vertical velocity (m/s) for "Constant" reference.



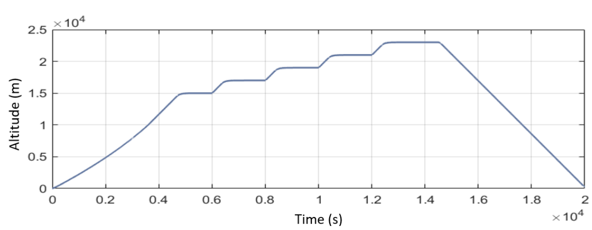
(b) Altitude (m) for "Constant" reference.

Fig. 11. Weather balloon velocity and altitude evolution with time for "Constant" reference.

2) "To the sky" Reference: Figure IV-B2, shows that the velocity that leads the weather balloon to the wanted altitude is the same in every situation since this maximum value is controlled to a maximum value close to $4.19m/s$. This is the velocity value for each increasing reference then decreases as the reference is reached and the velocity converges to $0m/s$.



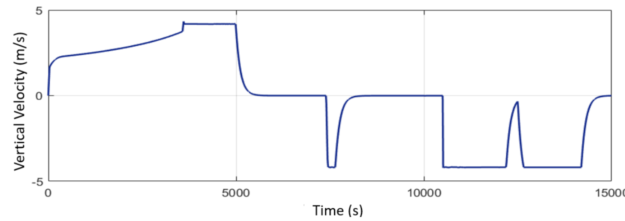
(a) Vertical velocity (m/s) for "To the sky" reference.



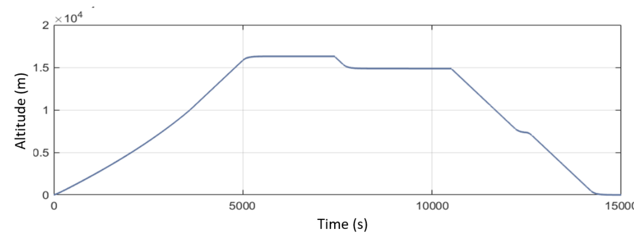
(b) Altitude (m) for "To the sky" reference.

Fig. 12. Weather balloon vertical velocity and altitude evolution with time for "To the sky" reference.

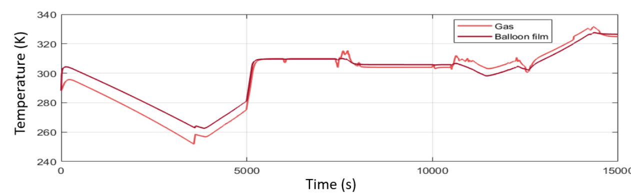
3) "Lisbon to Seville" Reference: Figure IV-B3 shows, in dashed line, the intended longitudinal trajectory for the weather balloon. The initial position of the weather balloon is in Lisbon, with geodetic coordinates $(\phi, \lambda) = (38.73°, -9.14°)$ (the negative value of longitude can also be read as a positive value into the west direction as the image shows). As for the final intended position, assuming that the balloon landing should be in "Plaza de España" with geodetic coordinates $(\phi, \lambda) = (37.377°, -5.987°)$.



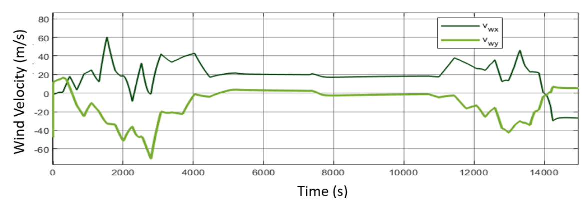
(a) Vertical velocity (m/s) for "Lisbon to Seville" reference.



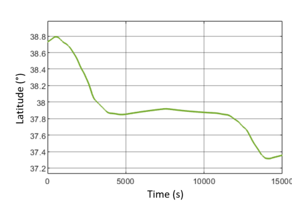
(b) Altitude (m) for "Lisbon to Seville" reference.



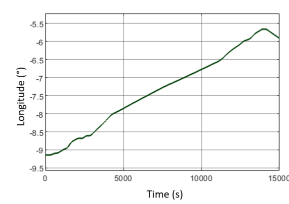
(c) Temperature (K) for "Lisbon to Seville" reference.



(d) Wind Velocity (m/s) for "Lisbon to Seville" reference.



(e) Latitude (°) for "Lisbon to Seville" reference.



(f) Longitude (°) for "Lisbon to Seville" reference.

Fig. 13. Weather balloon states response to altitude control for "Lisbon to Seville" reference.

The trajectory followed by the weather balloon (represented by the blue line in figure IV-B3) resulted in a landing at the geodetic coordinates of $(\phi, \lambda) = (37.320°, -5.875°)$. There

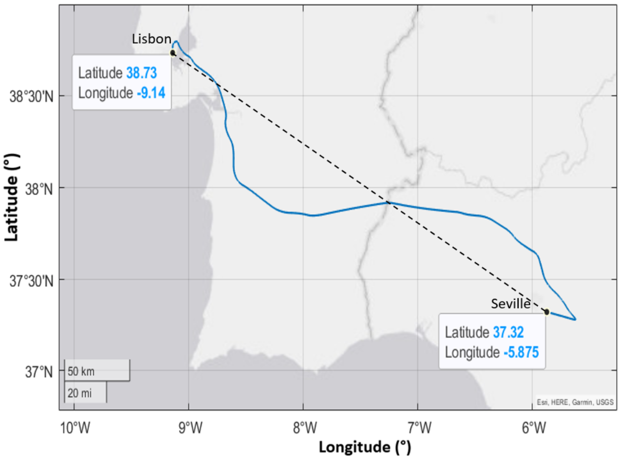


Fig. 14. Result of simulation for "Lisbon to Seville" reference.

is an error of 0.057 degrees in latitude and a 0.112 degrees error in longitude. This is equivalent to a error in distance of $11.76Km$. Considering the conditions in which this simulation was created, the wind uncertainty and the method in which the reference was chosen, this value can be considered good for a simulation and the following of wind currents through altitude control was accomplished.

V. Conclusions

A. Achievments

A complete model of a latex weather balloon resulted from this work. The TA450 model was assumed to carry, for control purposes, a gas cylinder and a control valve beyond the typical payload that weather balloons carry. For radius evolution modelling, since latex (natural rubber) is a hyperelastic material, a hyperelastic model was used. The atmospheric model representing the atmosphere behaviour for different ranges of altitude was also implemented in the modelling process. A complete thermal model accounting for direct solar radiation, infrared radiation and internal and external convection was also implemented. A dynamics model was obtained and a correlation between Reynolds number and the drag coefficient of the balloon was used. The wind currents were obtained from predictions made by GFS through the NOMADS servers and this data was implemented. The nonlinear balloon model was linearised and altitude control was implemented. The chosen control strategy was cascade control so not only the altitude but also the velocity could be controlled. This control strategy was implemented with gain scheduling in which the gains were found with the LQR algorithm and tuned after so the simulated curves could represent a normal behaviour for a weather balloon (trying to avoid drastic velocity or temperature changes). This control strategy was well implemented and all references were properly followed by the balloon as the simulated results show.

B. Future work

This work opens doors for many other developments in the latex balloon field. Some of which are listed below.

- Laboratory model validation and testing of the weather balloon simulated dynamics in a real word environment.
- Laboratory testing to obtain accurate latex thermodynamic and hyperelastic parameters.
- Valve modelling. The inclusion of a valve model could be an important evolution for a more accurate representation of the balloon model.
- Wind algorithm implementation. In this work the wind currents were chosen manually, however, there are methods to identify wind currents intensity and direction while the weather balloon flies. Although some methods already exist, new and less complex solutions can be created and implemented.
- Other control strategies could be implemented and then be compared to look for the better possible solution for the weather balloon.
- Development of other control mechanisms. Since the weather balloon is a very simple and air dependent flying aircraft, it could be hard to think about effective mechanisms for its control. However, developing alternative ways of control could open doors to extraordinary future results.

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