# Dual Resource Constrained Flexible Job Shop Problem using Hybrid Genetic Algorithm 

Application to Quality Control Laboratory Scheduling

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## Abstract

Quality control laboratory scheduling has huge potential to reduce costs and increase production. However, this is not widely discussed or supported by academics. This thesis proposes a hybrid genetic algorithm with and without variable neighborhood search to minimize the total completion time.

The problem is formulated as an extension of a Dual Resource Constrained Flexible Job Shop Problem for the allocation of both machine and worker resources. This extension, divides each job in three phases allowing the worker to perform other jobs in between these mandatory presential stages. While maintaining the traditional structure of genetic algorithms, this work presents a way of creating the Initial Population, based on increment tables which groups the most compatible jobs together. Also, rules regarding the allocation of resources to sequential tasks were introduced. A new combination of crossover operations with weighted probabilities are proposed, proving that combining multiple types of crossovers achieves fitter results. Additionally, a novel combination of mutation operations is implemented and a variable neighborhood search with four different structures was introduced alongside a dynamic termination criteria that adapts the parameters of the algorithm. A tuning of the parameters was performed and the final results were compared to the benchmark. This study is competitive with the benchmark for small instances, achieving the optimal solution for seven of them. For medium sized instances the proposed genetic algorithm surpasses the literature. For large sized instances this study generated significant results surpassing the compared results by $57 \%$.

## Keywords

Quality Control Laboratory; Dual Resource Constrained Scheduling; Flexible Job Shop Problem; Hybrid Genetic Algorithm; Total Completion Time.

## Resumo

O escalonamento de um laboratório de controlo de qualidade tem um enorme potencial para reduzir custos e aumentar a produção. Todavia, este problema tem tido pouco suporte por parte da academia. Esta tese propõe um algoritmo genético híbrido com a possibilidade de realizar uma procura local para minimizar o tempo total de conclusão. Este problema é formulado como uma extensão do escalonamento flexível de duplo constrangimento de recursos para a alocação de tanto máquinas como trabalhadores. Esta extensão, divide cada trabalho em três fases, possibilitando ao analista ser alocado em mais do que uma tarefa ao mesmo tempo, em máquinas diferentes, durante os períodos em que este está livre.

Mantendo a estrutura típica de um algoritmo genético, o trabalho desenvolvido combina novas maneiras de criar a População Inicial, baseadas em tabelas incrementais que junta os trabalhos mais compatíveis. Ainda, regras para alocação sequencial de recursos foram introduzidas. Uma nova combinação de operações de crossover com probabilidades baseadas em pesos é proposta, provando que combinando diferentes tipos de crossover obtém-se melhores resultados. Adicionalmente, uma nova combinação de mutações é também aplicada e uma procura local com quatro estruturas distintas foi introduzida juntamente com um critério de paragem dinâmico que adapta os parâmetros do algoritmo. Um afinação dos parâmetros foi realizada e os resultados finais foram comparados com os da literatura. Este estudo é competitivo com a literatura para pequenas instâncias, alcançando soluções óptimas para sete destas. Para instâncias médias o algoritmo genético ultrapassa os resultados da literatura. Para grandes instâncias este estudo gerou resultados significantemente melhores ultrapassando os estudos

## Palavras Chave

Laboratório de Controlo de Qualidade; Escalonamento Flexível de Duplo Constrangimento de Recursos; Problema de Job shop Flexível, Algoritmo Genético Híbrido; Tempo Total de Conclusão

## Nomenclature

## Indices

$j \quad$ subscript for jobs
subscript for machines
subscript for workers
subscript for tasks

## Dimensions

| $n$ | number of jobs |
| :--- | :--- |
| $q_{j}$ | number of operations of job $j$ |
| $m$ | number of machines |
| $w$ | number of workers |
| $N_{i j}$ | number of tasks in operation $i$ of job $j$ |

## Sets

| $J$ | set of jobs |
| :--- | :--- |
| $K$ | set of machines |
| $K$ | set of workers |
| $O_{j}$ | set of operations of job $j$ |
| $K_{i j}$ | set of machines that can be used for operation $O_{i j}$ |
| $W_{i j}$ | set of workers that can be used for operation $O_{i j}$ |

## Parameters

| $p_{i j}$ | processing time of operation $i$ of job $j$ |
| :--- | :--- |
| $\rho_{i j s}^{s}$ | time after the start of processing of operation $i$ of job $j$ |
| $\rho_{i j s}^{d}$ | duration of intervention $s$ in operation $i$ of job $j$ |
| $M$ | very large number |

## Variables

| $t_{i j}$ | start time of operation $i$ of job $j$ |
| :--- | :--- |
| $x_{i j k}$ | binary assignment of operation $i$ of job $j$ to machine $k$ |
| $\alpha_{i j h}$ | binary assignment of operation $i$ of job $j$ to worker $h$ |
| $\beta_{i j i^{\prime} j^{\prime}}$ and $\gamma i j i^{\prime} j^{\prime}$ sequencing variables |  |
| $\mathcal{J}$ | total completion time |
| $c_{j}$ | completion time of job $j$ |

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## Acronyms

| DRC | Dual Resource Constrained |
| :--- | :--- |
| DRCFJSP | Dual Resource Constrained Flexible Job Shop Problem |
| FJSP | Flexible Job Shop Problem |
| GA | Genetic Algorithm |
| JSP | Job Shop Problem |
| SA | Simulated Annealing |
| VNS | Variable Neighborhood Search |
| VDO | Vibration Damping Optimization |

## Introduction

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### 1.1 Motivation

Rome wasn't built in a day - or is it the case that Rome wasn't built without scheduling? For over thousands of years, humans have been using scheduling as a tool to build and coordinate operations. Scheduling is central to how jobs are planned and completed and provides order to businesses. Without scheduling, workers simply cannot work. In its infancy, scheduling was grasped as a concept, for example comprehending activities and sequencing of operations. This transitioned to executing wide-scale operations such as building pyramids ( 2780 BCE ), to the great wall of China ( 7 BCE ) which required scheduling hundreds of thousands of workers. Mosteiro dos Jerónimos is a masterpiece of construction of the 16th century which took over a century to plan and build. Convento de Mafra, dates to the 18th century and has over 1200 rooms. None of these monuments could ever have been accomplished without some form of scheduling.

However, most empirical evidence and sources suggest that formal scheduling processes were only implemented during the 19th Century. In today's scheduling tools, a commonly used chart to show the allocation of resources in time is the Gantt Chart, propagated by Henry Gantt during the early 20th century. The purpose of this tool was to help organisations plan repetitive tasks, measure productivity levels, and demonstrate how employees' resources can be allocated more efficiently. The Gantt Chart revolutionised the way people work and is now deployed as a powerful tool to help project managers schedule and plan projects. Therefore, utilising scheduling processes are instrumental in aiding efficiency and productivity in the workplace. [3]

Also, one of the earliest scheduling tools is Karol Adamiecki's 'Harmonygraph' (figure 1.1). Karol sought to create 'work harmonization' and demonstrated the significance of creating practical scheduling and it has been argued that companies who implemented Karol's methods experienced an increase in productivity of up to $400 \%$ [3].

| Time | From | -* | - | - | A-1 | B-1 | B-1 | D-1 | A-2 | B-2 | C, D-2 | $\begin{gathered} \text { A-3 } \\ \text { E-1DUM } \\ \hline \end{gathered}$ | C, D-2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | To | A-2 | B-2, C | D-2 | A-3 | E-1 | $\begin{aligned} & \hline \mathrm{D}-3 \\ & \text { DUM } \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{D}-3 \\ & \mathrm{DUM} \\ & \hline \end{aligned}$ | E-2 | E-2 | E-2 | - | - |
|  | Activity | A-1(4) | B-1(4) | D-1(2) | A-2(4) | B-2(3) | C(3) | D-2(3) | A-3(1) | E-1(4) | DUM(0) | E-2(e) | D-3(8) |
| $1$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $4$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 1 i i VA VA VA VA : |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 1 1 : 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 i |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 l |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 I |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 l |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 l |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 l |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 14 l |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 5 |  | $1$ |  |  | 1 |  | 1 | I |  |  |  | d |

Figure 1.1: The 'Harmonygraph' by Karol Adamiecki's [1]

The Y axis shows the time scale, and X axis shows a list of tasks. The duration of the tasks is shown by a vertical sliding tab. This chart also depicts the activities the predecessors and successors participated in. Although this was not published until 1931, Adamiecki's harmonygraph inspired the CPM (Critical Path Method) and PERT (Project Evaluation and Review Technique) systems.

The evolution of the scheduling science allows systems to be more complex, with a higher number of activities and sequencing of operations. Scheduling is required for complex manufacturing systems. Manufacturing systems have evolved and become increasingly industrialized, more reliant on technology with state-of-the-art thinking and machinery. Since industrial manufacturing first started it has been gone through four pivotal developments. Each revolution is unique and serves as major turning points within society. Each revolution adopted new methods of manufacturing, which resulted in significant improvement to products and competitiveness of companies.

Dating back to the middle of the 18th century, the First Industrial Revolution relied on the transition to new manufacturing processes using water and steam. Industry 2.0, also known as "The Technological Revolution" introduced superior electrical technology. The Third Industrial Revolution (1970) began with the first computer era, where automated systems became a commonality, although still very dependent on human input and intervention. In the XX century companies targeted for large batches, low costs and standardization. [4]

Currently, society is experiencing the fourth wave of the Industrial revolution, Industry 4.0, and academia is already looking forward to a fifth. In the XXI century, standardization of the internet paved way for a fast-paced world and accelerated the expansion of globalisation. The internet enabled the inter-
connectedness of countries, goods, news, and people on an unprecedented scale. With easier access to news and trends, this led to rapidly changing consumer demands, and resulted in industries needing to become more adaptable to satisfy demand and allow for greater customization The era of mass production is gone, more and more industries focus on the concept of mass customization. Therefore, the integration of smart systems, machines embedded with sensors, software and other technologies with the purpose of connecting and exchanging data with other devices and systems over the internet is of major importance. With this technological advances, industries are capable of producing smaller batches with smaller inventories, different products at the same time and have a better control of the material flow in the supply chain while maintaining profitable margins. [5]

A particular industry that requires the adaptation and transition to the loT and Industry 4.0 is the Pharmaceutical Industry. Pharmaceutical companies operate in one of the most competitive and regulated markets, where compliance with Good Manufacturing Practices (GMP) and Good Laboratory Practices (GLP) is mandatory to commercialize any drug. Without such tight regulations, medicines could become serious dangerous to public health [6]. Therefore, companies that successfully integrate in their production systems the loT may obtain considerable better margins over the remaining market, despite the tight regulations and the abundance of competition.

On the other hand, the future of the pharmaceutical sector is predicted to be prosperous. For example, the race for the Covid-19 vaccine has stimulated innovation across the industry, but the sector has been building momentum for years. Prior to the current pandemic, between 2006 and 2020, the Industry's Research and Development (R\&D) was growing at an annual rate of $7.6 \%$ in the USA [7]. These growing costs have been mainly associated with the shifting paradigm forcing the industry to change from a one-drug-fits-all approach to more targeted drugs for small and specific patient and therapeutic groups (mass customization) [4].

The challenge related to the high R\&D spend is the rising expectations of investors for a high return of investment (ROI). Margins are shrinking which dissuades potential investors from investing in new medicines. R\&D has not been living up to the expectations as the total number of molecular entities commercialized in past years did not match with the extraordinary high R\&D costs [6].

On the other side, there is a growing expectation for pharmaceutical companies to have a higher proportion of developed products regulated and accepted by markets, and developed within faster time frames. This in turn leads to higher profit margin. With both arguments in mind, corporations begin to follow the principles of Industry 4.0 in order to apply them to the pharmaceutical industry which is known as Pharma 4.0. Pharma 4.0 has brought a series of challenges including the scheduling and rescheduling problem. The scheduling and rescheduling problem entails the search for the best possible set of decisions which allows for an increase in profits, by cutting costs in the production and quality control phases. This achieves higher throughput rates, lower energy consumption and more
effective and planned maintenance, whilst maintaining high quality levels. [4].
During its development life cycle, a drug must be constantly monitored with laboratory tests to assess its quality. In this scenario, quality control laboratories serves as a critical function in pharmaceutical production and control [8]. It is responsible for ensuring that goods comply with predefined standards by following guidelines and practices. More specifically, it is responsible for monitoring manufacturing processes through the assessment of samples taken at different stages of the manufacturing process. These can be taken at the early stages to raw materials, at mid stages and also at the final stages when the finished product is developed. The assessment of these samples is completed by comparing with the standard pre-defined quality metrics. Quality control makes it possible to meet high product quality standards [9] by promptly detecting deviations in the quality of a manufacturing stream.

The role of managing jobs within QC laboratories is a complicated and complex task. For example, there are thousands of different tasks, which each require the delegation of specific skills and specialised instruments in order to execute each test. Laboratory management involves resources (both personal and equipment) planning and scheduling, analysis prioritization results evaluation and documentation.

The management of jobs within contract manufacturing organisations (manufacturers that produces goods under the brand of its clients), causes greater complications. This is because these manufacturers deal with a large array of projects and handle a variety of materials. Given the high mix of products and tests it is important to develop effective strategies for laboratory management. Laboratory information management systems (LIMS) used in pharmaceutical industry often lack on essential features such as schedule planning and stock management [6].

The purpose of this thesis is to propose a solution to automatically schedule real-sized QC laboratories. In this chapter, a literature review on scheduling methods will be presented mainly focusing on flexible and dual resource constrained job shop problems as it represents the problem at hand. At the end of the chapter a guideline of the thesis is given.

### 1.2 Scheduling in Manufacturing System

### 1.2.1 Flexible Job Shop Problem

Scheduling is a challenging problem in manufacturing shop floors and is known to be strongly an NPhard problem. It can be defined as the allocation of finite resources over a period of time in order to optimize one or more objectives.

Shop-floors have limitations in the amount of different tasks a certain machine and worker can perform. Therefore, studies have included flexibility as an additional limitation to better simulate real manufacturing complex conditions. Brucker and Schlie (1991) [10] introduced a Job Shop Problem (JSP) with machines as the only flexibility parameter achieving reasonable results for instances smaller than 3 jobs.

Later, in 1997, Dauzère-Pérès and Paulli [11] formulated a Flexible Job Shop Problem (FJSP) in which an extended version of the disjunctive graph model was presented, allowing for no distinction between reassigning or resequencing an operation. This problem has been studied significantly by researchers utilising genetic algorithms [12], [13] .

### 1.2.2 Dual Resource Constrained Problem

The dual resource constraint formulation is a generalisation of the multi-resource constraint, and includes three sub-problems: (i) Assigning operations to resources of machines, (ii) Assigning operations to resources of workers and (iii) Sequencing the operations on the machines considering workers in order to optimize the performance measure [14]. This allows for simplifications and can be utilised for simpler computational algorithms and in turn decreases computational time for larger problems [2].

Most of the literature predominantly focuses on equipment as the sole limiting constraint within JSP. However, workers often appear as the bottleneck in many shop-floors, especially if both workers and machines reach capacity constraints. Kher [15] focused on evaluating the policies for deploying workers and the dispatching rules ("when" and "where") to offer a near-perfect delivery performance for vital customers with very rigid delivery guidelines in the Dual Resource Constrained (DRC) job shop environments.

### 1.2.3 Dual Resource Constrained Flexible Job Shop Problem

In 1997, Patel [16] proposed, in his master thesis, a combination of the two problems, a Scheduling of flexible Manufacturing Systems Under Dual-Resource Constraints using Genetic Algorithms. Six different dispatching rules were formulated and used with eight performance criteria. These were compared with single resource constrained JSP. The results demonstrated that depending on the number of resources being constrained, different dispatching rules proved better results. For the DRC problem, the shortest processing time performed the best. In 2000, ElMaraghy et al. [17] proposed an algorithm where machines were set in work-centers of a 2 by 2 formation. Also using six different dispatching rules, EIMaraghy et al., drew a similar conclusion to Patel's, except EIMaraghy et al demonstrated that the DRC did not prove better results for any dispatching rule in particular.

Labour flexibility is often expensive, as more training is required to be administered. In 2001 Felan and Fry [18] investigated the effect of having different levels of training across the workforce (Multi-level heterogeneous flexibility). Their results revealed that a combination of workers with very high flexibility and workers with no flexibility, performed higher than those with near equal flexibility.

Yue et al. [19] investigated cross-training policies in DRC parallel job shop. The workers were required to learn new skills, as additional parts were added to the system. Moreover, if workers stopped
performing a certain job for a significant period of time, a forgetting model would be implemented to simulate the time they would take to perform it once they were assigned to that job again. This curve reflected on the experience they have gathered and the time they had stopped doing the given work. In 2011, Xianzhou and Zhenhe [20], presented a new immune Genetic Algorithm which proved to have a higher level of convergence precision when solving a DRC flexible job shop problem, through merging immune and genetic algorithms.

Other meta-heuristic algorithms have proved to yield higher quality results in the Dual Resource Constrained Flexible Job Shop Problem (DRCFJSP) while minimizing the makespan. Lei and Guo (2014) [21] proposed a Variable Neighborhood Search (VNS) composed of two neighbourhood search procedures and a restarting mechanism. In the same year Yazdani et al. [22] presented a Simulated Annealing (SA) and a Vibration Damping Optimization (VDO) algorithms with the same minimization criteria.

Apart from the makespan minimization, Paksi and Ma'ruf (2015) [23] developed a GA algorithm utilising indirect chromosome representation with two layers in order to reduce delays. In 2017 Zhang et al. [24] proposed a Particle Swarm Optimization with a three layered encoding scheme in order to reduce the production period and cost. Zhong et al. (2018) [25] presented an algorithm that could lower the makespan and total processing cost. They stated that the local search algorithms, tabu search, classical metaheuristic methods, or binary particle swarm optimization could not achieve the same results. A branch population genetic algorithm based on compressed time-window scheduling strategy was implemented, which did not hamper the quality of the initial population. An improvement in the makespan was achieved by $7 \%$.

### 1.2.4 Human Factors and Uncertainty

Researchers have extensively explored whether human factors should be considered in optimization algorithms, i.e. methodologies which include factors such as fatigue and productivity. Despite the reasonable considerations in favour of the human factors, Helander in 2000 [26] presented seven reasons for not considering human factor in the production system development process which, among others, included the unpredictability of human behaviour. This hardly quantifiable factor leads to uncertainty in the production times which often complicates planning.

Scheduling with uncertain production times is also know as fuzzy scheduling. Lang and Li [27] explore the uncertain operation time constraints using grey simulation technology and Non-dominated Sorting Genetic Algorithm II considering delivery satisfaction, cost, energy consumption and noise pollution as the optimized objects. In 2016 Gao et al. [28] presented an artificial bee colony algorithm considering fuzzy processing time and new job insertion which demanded new rescheduling operations. This operation of performing new assignments based on newly arrived information is often called
dynamic scheduling and it is another branch of scheduling problems.

### 1.2.5 A wider overview into Scheduling problems

This dissertation cannot possibly cover all different types of manufacturing scheduling. Nevertheless, there are some that, due to its similarities or relevant considerations to the problem at hand, inspired the development of this thesis and therefore, are worth to be mentioned.

Unpredictable events such as failure of a machine, sickness of a worker or new urgent batches arriving at the shop floor, happen on any industry. This forces operations to be rescheduled, also known as dynamic scheduling. In 2010, Araz and Salum [29] proposed a real-time (dynamic) scheduling approach to select a predetermined scheduling rule in DRC manufacturing systems. The model requires a combination of Artificial Neural Networks, fuzzy inference system and simulation to provide the knowledge base. Cunha (2017) [4], proposed a dynamic scheduling master thesis in which a flexible dynamic environment existed. To cope with the dynamic environment two ant inspired multi-agent algorithms were proposed. Recently in 2020, Andrade-Pineda et al. [30] proposed a solution for an automobile collision repair shop where the re-scheduling needs, like due-date changes delay in arrival, changes in job processing time and rush jobs were common. Reasonable schedules could be achieved under five seconds.

In order to minimize the makespan, Zheng and Wang (2016) [31] developed a knowledge-guided fruit fly optimization algorithm (KGFOA) which is an improvement from the simpler fruit fly optimization (FOA) which enabled guiding the search process. In 2018, Guo et al. [32] proposed a hybrid genetic algorithm which hybridizes genetic algorithm (GA) with variable neighborhood search (VNS) to overcome GA's slow convergence speed due to it's unguided mutation.

Some studies also included loading and unloading times of fixtures, considering the influence of resource requirement similarity among different operations, as well as the time to shift workers between assignments. Wu et al. [33], used similarity-based scheduling algorithm for setup-time reduction (SSA4STR) and later introduced an improved non-dominated sorting genetic algorithm II (NSGA-II) to optimize the DRFJSP when loading and unloading fixtures, also called, DRFJSP-LU. A goal in this research was to minimize the total time fixtures were mounted and dismounted on/off the machines, i.e. tasks using the same fixtures were encouraged to be completed sequentially, as it could save time loading and unloading the fixture.

When more than a single objective is desirable to be optimized at the same time, often the problem is called a multi-objective optimization. In 2017, Gong et al. [34] proposed an extension to the traditional genetic algorithm, a memetic algorithm with the objective to minimize the maximum completion time, the maximum workload of machines and the total workload of all machines. Approaching human factor indicators, green production and processing time simultaneously, Gong et al. (2018) [35] proposed a
hybrid genetic algorithm to solve the problem. In 2019 Yazdani and Zandieh [36] proposed two types of multi-objective evolutionary algorithm including fast elitist non-dominated sorting genetic algorithm and non-dominated ranking genetic algorithm. The objective was to solve Multi-Objective DRCFJSP by minimizing the makespan and the critical machine workload simultaneously.

Some researchers have even drawn their attention to other constrained problems rather than exclusively focusing on single and dual resource constraints. For example Gao and Pan (2016) [37] proposed a multi-resource constrained flexible job shop scheduling problem by using a shuffled multi-swarm micromigrating birds optimizer. The constraints were labor, maintenance equipment, tooling and machinery. This study was the first reported application of micro-evolutionary algorithms to solve flexible job shop problems. The experimental results and statistical analyses demonstrated that the presented SM-MBO algorithm clearly surpassed all of the other compared algorithms by a substantial margin. The authors also pointed that their simple solution representation (only machine assignment and operation sequence presented in the encoding), which was capable of decreasing the search space, was also a factor to be taken into consideration in the algorithm's success.

### 1.2.6 Laboratory Scheduling - Quality control specific applications

The scholars previously mentioned, haven't included the context of the Quality control laboratory scheduling. Whereby, workers can work on several jobs simultaneously, as it is only necessary to be present in 3 different time instances on the assigned machine for a given job, representing the setup, intermediate and disassembly/data processing tasks. Each and every study previously mentioned, did not allow workers to leave a certain machine unattended while it was performing a job.

In 2019 Cunha et al. [38] formulated a Mixed integer linear programming (MILP) algorithm to solve the DRCFJSP in the Quality control laboratory scheduling (QCLS) environment in order to minimize the makespan. The obtained solution could only reach the optimum for small instances where the number of jobs considered was equal or less than three.

Recently, in 2020 Akbar and Irohara [39] proposed seven metaheuristic algorithms with six different decoding schemes to solve what they named a multi-task simultaneous supervision dual resourceconstrained scheduling problem. The multi-tasking refers to the ability of a machine to work independently of the worker and allows for parallel scheduling after an initial set up and a final supervision, much like this thesis problem, (without the sampling operation in middle). Although, it does not allowing for different skill levels in workers and machines (flexibility). The metaheuristics that proved the best results were the modified Permutation-based Genetic Algorithm and the Modified Bees Algorithm. The same authors, Akbar and Irohara [40], two year prior, presented a study explaining that this type of multitasking ability of workers could yield workload imbalances between operators. Social sustainability was therefore acquainted in a multiple objective model to minimize makespan and workload unbalances.

In 2021, Martins et al. [2], sought to investigate the QC scheduling problem, formulated a three-level dynamic heuristic with both a branch and cut and a tabu-search algorithm implemented for comparison. The results showed that the heuristic outperforms the other algorithms for large-sized instances. The present thesis results will be compared with the ones from Martins et al. [2] research.

### 1.2.7 Literature Review Scheme

In this section, a table review (1.1 and 1.2) of the different scheduling methods approached in the literature is presented in chronological order based on the criteria proposed by Dhiflaoui et al. (2018) [14].

The structure of the approach is composed of four criteria:
(1) Are the Machines Flexible? (Can machines perform more than one task but are limited in the amount of tasks they can perform? "yes" or "no").
(2) Are Workers Flexible? (Can Workers perform more than one task but are limited in the amount of tasks they can perform? "yes" or "no").
(3) The optimization criteria (Makespan, Cost, Lateness, among others).
(4) The implemented approaches (Mixed integer programming, Genetic algorithm, Tabu search, among others).

### 1.3 Contributions

The added-value proposal can be summarized in the following topics:

## Evaluation and development of previous literature work

The proposed framework is the first study conducted with a meta-heuristic algorithm in scheduling problems for the Quality Control laboratory environment. Therefore, the first hybrid Genetic Algorithm with Variable Neighborhood Search to solve such a problem is proposed and compared to bench mark results.

## Adaptation to the Quality Control Scheduling Problem

Apart from operating outside the common literature field, the present work includes six novel implementations: (1) - A probabilistic choice in the Initial Population based on weights which correspond to different paths when selecting tasks. It proved to be best when kept on a random selection (section 5.3.1).
(2) - A Prohibition condition which controls the resources allocation which proved to improve results by $4 \%$ (section 5.3.2).

Table 1.1: Literature Review - part 1

| Date | Author(s) | Machines Flexible? | Workers Flexible? | Optimization Criteria | Approach |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1991 | Brucker and Schlie | yes | no | Makespan | 2D - Network |
| 1997 | Patel | yes | yes | Makespan | Genetic |
| 1997 | Dauzère-Pérès and Paulli | yes | no | Makespan | Tabu Search |
| 2000 | Eimaraghy et al. | yes | yes | Makespan | Genetic |
| 2000 | Kher | no | no | Percentage of tardy jobs and Root mean squared tardiness | ANOVA |
| 2001 | Felan and Fry | no | yes | Labour cost vs Labour perform given the flexibility | ANOVA |
| 2007 | Pezzellaa et al. | yes | no | Makespan | Genetic |
| 2008 | Yue et al. | no | yes | Mean flow time of jobs | ANOVA |
| 2010 | Araz and Salum | no | no | Multiple Criteria | Artificial neural networks and Fuzzy inference system |
| 2011 | Xianzhou and Zhenhe | yes | yes | Malespan | Genetic + Immune algorithm |
| 2011 | Lang and Li | yes | yes | Delivery satisfaction, Cost, <br> Energy consumpiton and Noise pollution | Grey simulation technology and Non-dominated Sorting <br> Genetic Algorithm- II (NSGA)-II |
| 2014 | Driss et al. | yes | no | Makespan | New Genetic Algorithm (NGA) |
| 2014 | Lei and Guo | yes | yes | Makespan | Variable Neighbourhood Search (VNS) |
| 2015 | Yazdani et al. | yes | yes | Makespan | Simulated annealing and Vibration Damping Optimization |
| 2016 | $\begin{gathered} \text { Paksi } \\ \text { and Ma'ruf } \end{gathered}$ | yes | yes | Total Tardiness | Genetic |
| 2016 | Zheng and Whang | yes | yes | Makespan | Knowledge guided Fuit Fly Optimization |
| 2016 | Gao and Pan | yes | yes | Makespan | Shuffled Multi-swarm Micro-migrating Birds Optimization (SM 2 - MBO) |

Table 1.2: Literature Review - Part 2

| Date | Author(s) | Machines <br> Flexible? | Workers <br> Flexible? | Optimization <br> Criteria | Approach |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2016 | Gao et al. | yes | yes | Minimize maximum <br> fuzzy completion time | Artificial Bee Colony (ABC) |
| 2017 | Cunha | yes | yes | Optimizing unforseen events | Genetic and <br> Variable Neighbourhood <br> Search (VNS) |
| 2017 | Gong et al. | yes | yes | Maximum <br> completion time, <br> Maximum workload <br> and Total workload | Memetic Algorithm |
| 2017 | Zhang et al. | yes | yes | Makespan and Cost | Hybrid discrete particle <br> swarm optimization |
| 2018 | Gong et al. | yes | yes | Maximum Completion Time, <br> maximum total worker cost, <br> green-production related indicator | Newly Hybrid <br> Genetic Algorithm (NHGA) |
| 2018 | Akbar <br> and Irohara | no | no | Workloal smoothnability, <br> Makespan index, | NSGA-II |
| 2018 | Zhong et al. | no al. | yes | yes | yes |
| 2019 | Cunha et al. | yes | yes | Makespan and Cost | Makespan |

(3) - An Increment Initialization that combines compatible tests together which improves results by 5.3\% (section 5.3.3).
(4) - A probabilistic choice in the Crossover operation (with the same logic as the Initial Solution) which improves the results when compared to performing Crossover operations with only one type of crossover. Improvements from a single point crossover are $5.8 \%$, from a double point crossover are $4.8 \%$ and from an MPX crossover are $1.1 \%$ (section 5.4).
(5) - A Variable Neighborhood Search is implemented to improve GA's slow convergence speed which combines four different structures. This implementation proved fitter results for small instances ( $0.1 \%$ better) when compared to a GA without VNS (section 5.5).
(6) - A Dynamic Termination Criteria is proposed to allow the algorithm to adapt based on the number of non-improving iterations (section 5.6).

### 1.4 Organization of the Document

The remainder of this document is structured as follows:

## Chapter 2 - The Dual Resource Constrained Flexible Job Shop Problem

This chapter presents the description of the problem, it's assumptions and it's mathematical formulation.

## Chapter 3 - Genetic Algorithm

In this chapter the genetic algorithm is introduced and it's principles explained covering the five main phases of a GA: Initial Population (3.1), Fitness evaluation (3.2), Selection (3.3), Crossover (3.4), and Mutation (3.5). Also different encoding schemes presented in literature are described (3.1.1).

## Chapter 4 - Proposed Hybrid Genetic Algorithm

Chapter 4 details the proposed Hybrid Genetic Algorithm explaining the logic behind each specific implementation (4.1 to 4.8). The variable neighborhood search is introduced and explained (4.9) as well as the Dynamic Termination Criteria (4.10).

## Chapter 5-Results

This chapter presents the description of the experiments, explaining the reason behind the different test instances utilized (5.1). Also, the studies conducted to tune the algorithm are presented (5.2 to 5.6) and a comparison between the final results this Hybrid GA produced and bench mark results is shown (5.7).

## Chapter 6 - Conclusion

At last, in chapter 6 the conclusions for the present dissertation are drawn and possible future work is discussed.

## The Dual Resource Constrained Flexible Job Shop Problem

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### 2.1 Application to the Quality Control Laboratory Scheduling

As mentioned in section 1, the classical flexible job shop scheduling problem (FJSSP) solely treats machines as the only resource constraint, failing to consider the importance of the dominant role of qualified labour in production and manufacturing. Quality control laboratories in pharmaceutical industry requires both resources to be scheduled and often have a flexible workforce and machinery capable of performing different tasks.

The quality control laboratories scheduling problem can be summarised as follows: assign each operation (test) of each job (sample) to a machine (analytical instrument) and worker (analyst/chemist), to minimise an objective function. Each operation can only be processed by predetermined subsets of machines and workers. The machine is required for the full processing time of the test while the worker is only required at a number of predetermined intervals (tasks) during the processing time of the operation. These tasks are the machine setup, the preparation of sample and the materials and data processing at the end. All of these tasks must be performed by the same worker. In contrast to the literature reviewed, in this Quality Control Laboratory, workers can switch between tests, if they are present when they are required by the operations they are allocated to. Additionally, all tasks in an operation must be carried out by the same worker.

Some assumption are put forward:

- Each Machine can only process one operation at a time on any job.
- Each Worker may operate in more than one machine at a time, given that the worker respects the times he needs to be present at the machines he is assigned to.
- Each operation can be performed only once on one machine and its sequence is respected for every job.
- The operations of different jobs do not have precedence constraints, only the ones in the same job.
- A temporary interruption of an operation is not allowed after it has started.
- An operation of any job cannot be processed until its preceding operations are completed.
- The processing time corresponding to each operation are given in advance and are the same regardless the machine or worker that performs it.


### 2.2 Formulation of the Scheduling Problem

The following mathematical formulation was performed by [2]: an instance of the QC labs scheduling problem consists of a set of jobs, $J=\left\{J_{1}, \ldots, J_{n}\right\}$ with $n$ being the total number of jobs. The set of machines is represented by: $K=\left\{K_{1}, \ldots, K_{m}\right\}$, the set of workers by $W=\left\{W_{1}, \ldots, W_{w}\right\}$ and the set of
operations by $O_{j}=\left\{O_{1 j}, \ldots, O_{q j}\right\}$ for each job $j$. The number of operations in job $j$ is $q_{j}$. $K_{i j}$ and $W_{i j}$ are, respectively, the subsets of machines and workers that can process operation $i$ of job $j$. Additionally, each operation can be divided by $N_{i j}$ individual tasks, $s$ that requires a worker present, either partially or fully. Each operation is characterised by its processing time $p_{i j}$, the start time, $\rho_{i j s}^{s}$, and the duration time, $\rho_{i j s}^{d}$. The machine processing time is always greater than the sum of the duration of the worker tasks and is the same as the processing time $p_{i j}$.

The main decision variables are the start time of operation $i$ of job $j, t_{i j}$, the binary assignment of operations to machines, $x_{i j h} \in\{0,1\}$ and the binary assignment of operations to workers, $\alpha_{i j h} \in\{0,1\}$. Additionally, the following sequencing variables are used: $\beta_{i j i^{\prime} j^{\prime}} \in\{0,1\}$ is equal to 1 if $O_{i j}$ is scheduled before $O_{i^{\prime} j^{\prime}}$, it is 0 otherwise; and $\gamma_{i j s i^{\prime} j^{\prime}} \in\{0,1\}$ is equal to 1 if the task (worker intervention) $s$ of $O_{i j}$ is scheduled before the task $s^{\prime}$, it is 0 otherwise.

### 2.2.1 Objective Functions

The Objective function of the problem is to minimise the total completion time $\mathrm{J}(2.1)$ where $c_{j}$ is the completion time of job $j$, i.e., the time required to complete the operations of that job.

$$
\begin{equation*}
\min \mathcal{J}=\sum_{j \in J} c_{j}, \quad \forall j \in J \tag{2.1}
\end{equation*}
$$

### 2.2.2 Constraints

The completion time of each job, $c_{j}$ must be greater or equal than the completion time of the last operation of that job as expressed below in:

$$
\begin{equation*}
c_{j} \geq t_{c_{j} j}+p_{c_{j} j} \tag{2.2}
\end{equation*}
$$

Each test can only be assigned to one suitable machine (2.3) and worker (2.4):

$$
\begin{align*}
& \sum_{k \in K} x_{i j k}=1, \quad \forall j \in J, i \in O_{j}, k \in K_{i j}  \tag{2.3}\\
& \sum_{h \in W} \alpha_{i j k}=1, \quad \forall j \in J, i \in O_{j}, h \in W_{i j} \tag{2.4}
\end{align*}
$$

Constraint (2.5) guarantees the precedence is respected between operations of the same job.

$$
\begin{equation*}
t_{i j} \geq t_{(i-1) j}+p_{(i-1) j} \tag{2.5}
\end{equation*}
$$

For the sequencing of operations in machines, constraint (2.6) ensures that the start time of any $O_{i j}$ is greater or equal than the finish time of any $O_{i^{\prime} j^{\prime}}$ that is scheduled before $\left(\beta_{i j i^{\prime} j^{\prime}}=0\right)$ in the same
machine $\left(x_{i j k}=x_{i^{\prime} j^{\prime} k}\right)$.

$$
\begin{equation*}
t_{i j} \geq t_{i^{\prime} j^{\prime}}+p_{i^{\prime} j^{\prime}}-\left(2-x_{i j k}-x_{i^{\prime} j^{\prime} k^{\prime}}+\beta_{i j i^{\prime} j^{\prime}}\right) M, \quad \forall j \in J, i \in O_{j}, O_{i j} \neq O_{i^{\prime} j^{\prime}}, k \in K_{i j} \cap K_{i^{\prime} j^{\prime}} \tag{2.6}
\end{equation*}
$$

As the prior constraint is disjunctive, the big M formulation is adopted, Similarly, constraint (2.7) guarantees that any $O_{i j}$ scheduled before any other $O_{i^{\prime} j^{\prime}}$ in the same machine ( $\beta_{i j i^{\prime} j^{\prime}}=1$ and $x_{i j k}=$ $\left.x_{i^{\prime} j^{\prime} k}\right)$ is finished before the later starts.

$$
\begin{equation*}
t_{i^{\prime} j^{\prime}} \geq t_{i j}+p_{i j}-\left(3-x_{i j k}-x_{i^{\prime} j^{\prime} k^{\prime}}-\beta_{i j i^{\prime} j^{\prime}}\right) M, \quad \forall j \in J, i \in O_{j}, O_{i j} \neq O_{i^{\prime} j^{\prime}}, k \in K_{i j} \cap K_{i^{\prime} j^{\prime}} \tag{2.7}
\end{equation*}
$$

Two constraints are required to bound for $\beta_{i j i^{\prime} j^{\prime}}=0$ and $\beta_{i j i^{\prime} j^{\prime}}=1$. In the same way, constraints (2.8) and (2.9) ensure that a worker cannot perform overlapping tasks.

$$
\begin{equation*}
t_{i^{\prime} j^{\prime}}+\rho_{i j s}^{s} \geq t_{i^{\prime} j^{\prime}}+\rho_{i^{\prime} j^{\prime} s^{\prime}}^{s}+\rho_{i^{\prime} j^{\prime} s^{\prime}}^{d}-\left(2-\alpha_{i j h}-\alpha_{i^{\prime} j^{\prime} h}+\gamma i j s i^{\prime} j^{\prime} s^{\prime}\right) M, \quad \forall j \in J, i \in O_{j}, s \in N_{i j}, O_{i j} \neq O_{i^{\prime} j^{\prime}} \tag{2.8}
\end{equation*}
$$

$$
\begin{equation*}
t_{i^{\prime} j^{\prime}}+\rho_{i^{\prime} j^{\prime} s^{\prime}}^{s} \geq t_{i j}+\rho_{i j s}^{s}+\rho_{i j s}^{d}-\left(3-\alpha_{i j h}-\alpha_{i^{\prime} j^{\prime} h}-\gamma i j s i^{\prime} j^{\prime} s^{\prime}\right) M, \quad \forall j \in J, i \in O_{j}, s \in N_{i j}, O_{i j} \neq O_{i^{\prime} j^{\prime}} \tag{2.9}
\end{equation*}
$$

## Genetic Algorithm

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A Genetic Algorithm (GA) is a meta-heuristic search algorithm inspired by Charles Darwin's theory of natural evolution. This algorithm emulates the process of natural selection where the fittest individuals (with favourable genetics) have higher chances to propagate their genes to the next generation. The fittest individuals have a set of adaptive traits in nature, these advantageous differences can appear in size, shape/form and intelligence. These adaptive traits increase the creature's chance of survival in a certain environment, thus, making the individual superior and a more sought-after candidate for reproduction purposes. In contrast, the genetically inferior individuals are likely to disappear from the gene pool altogether.

For each offspring that is born, besides the inherited genes from the parents, offspring also generate random mutations in their genes which can lead to better adaptation and consequentially higher probabilities of gene propagation. With this natural mechanism species are able to keep the strongest genes in the gene pool, therefore, increasing the species chances to prosper whilst adapting to an ever changing environment.

It is worth mentioning that the artificial GA, while replicating the main concepts of natural evolution has some relevant differences. First and foremost, an initial solution has to be created. Therefore, a spontaneous and often random generation of the individuals is performed. Secondly, the individuals do not age in the artificial algorithm. They are simply discarded or kept in the population depending on their fitness value. This means that the fittest individuals may exist in numerous generations until the termination criteria is met and the algorithm stops.

Bearing these similarities and differences in mind, in an artificial Genetic Algorithm there are five main phases to be considered:

1)     - Initial Population creation
2)     - Fitness Evaluation of the population
3)     - Selection
4)     - Crossover
5)     - Mutation

The following sections from 3.1 to 3.5 explain the main principles behind each phase:

### 3.1 Initial Population

This process begins with a set of generated individuals which together form the Population. Each individual is a solution to the problem and is generally randomly generated in accordance to the problem restrictions. Individuals are characterized by a set of parameters also known as Genes. A set of predefined number of genes constitutes a chromosome (solution) which in turn is the same as an individual. This correct ordering of genes forms a chromosome which is also known as the process of encoding.

The following figure 3.1 illustrates this concepts. In blue, the population, which consists of four individuals/chromosomes/solutions represented in green. Each chromosome has six genes, represented in red.


Figure 3.1: Population, Chromosomes and Genes definition

To form the Initial Population each solution needs to be correctly encoded. This can be done in various ways and depending on the problem, one path may be better adapted to it. The following section 3.1.1 presents the four main types of encoding schemes found in the literature for scheduling problems.

### 3.1.1 Encoding schemes

In scheduling problems, each solution is often represented by a code that must contain an enough amount of information to enable it to be converted into a scheduling solution when the decoding process is later applied. If this code has less information than it should, the feasible solution space narrows. On the other hand, if the code contains unnecessary information, the solution becomes more complex and may delay or complicate the entire process.

Mainly, the Encoding scheme in the literature is divided by 4 major representations which does not necessarily translate into 4 different decoding mechanisms since these, can vary from problem to problem in order to better accommodate to it's peculiarities.
(1) In this encoding method, each gene is represented by a quadruple string (i,j,k,l), one for each operation, where $\boldsymbol{i}$ signifies the job which an operation belongs to; $\boldsymbol{j}$ characterizes the progressive number of that operation within job $\boldsymbol{i} ; \boldsymbol{k}$ indicates the machine assigned to that operation and I indicates the worker assigned [21] [36]. An example of this representation follows:

Table 3.1: Decoding Methods - quadruple string

| $(1,1,4,2)$ | $(2,1,2,2)$ | $(4,1,1,1)$ | $(4,2,5,1)$ | $(2,2,1,2)$ | $(3,1,2,2)$ | $(1,2,3,1)$ | $(1,3,1,1)$ | $(3,2,1,2)$ | $(3,3,2,2)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(2) A single chromosome is used by Akbar and Irohara [39] to encode one scheduling solution. The code contains information about the job sequence when evaluated in the decoding scheme where a
repetition of the same number in the encoding table represents the next operation of that job. In the particular case of table 3.2, the $8^{\text {th }}$ gene represents job 1 operation 2. This representation is also known as the operation-based encoding method. [33]

Table 3.2: Single Chromosome encoding

| 1 | 2 | 4 | 4 | 2 | 3 | 1 | 1 | 3 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

To perform the decoding, in this case, the attributed worker and machine may change every time a change in the operations order is done. For this representation [39] proposed 6 different decoding schemes and studied their effectiveness. The differences of this processes were dependent on the procedure used to select the machine and or operator i.e., whether the first available machine should be selected or the first available operator, or even the pair of machine and operation combined that could start the earliest or finish first.
(3) On a third type of encoding, Zheng and Wang (2016) [31] proposed two vectors, an operation sequence vector (OSV) and a resource assignment vector (RAV). OSV is used to denote a sequence of all the operations of all the jobs. RAV is used to denote an assignment of machines and workers. An example of this representation is seen below 3.3:

Table 3.3: Two Vector encoding
OSV:
RAV:

| $O_{1,1}$ | $O_{2,1}$ | $O_{4,1}$ | $O_{4,2}$ | $O_{2,2}$ | $O_{3,1}$ | $O_{1,2}$ | $O_{1,3}$ | $O_{3,2}$ | $O_{3,3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(M_{4}, W_{2}\right)$ | $\left(M_{2}, W_{2}\right)$ | $\left(M_{1}, W_{1}\right)$ | $\left(M_{5}, W_{1}\right)$ | $\left(M_{1}, W_{2}\right)$ | $\left(M_{2}, W_{2}\right)$ | $\left(M_{3}, W_{1}\right)$ | $\left(M_{1}, W_{1}\right)$ | $\left(M_{1}, W_{2}\right)$ | $\left(M_{2}, W_{2}\right)$ |

(4) Finally, the fourth type of encoding mechanism is a three layered chromosome (table 3.4) adding from the encoding type three a decomposition of the RAV vector into a Machine Assignment vector (MAS) and a Worker assignment vector (WAS). [34]

Table 3.4: Three Layered Chromosome
OSV:

| 1 | 2 | 4 | 4 | 2 | 3 | 1 | 1 | 3 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 2 | 1 | 5 | 1 | 2 | 3 | 1 | 1 | 2 |
| 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 |

### 3.2 Fitness Evaluation

The Fitness Evaluation quantifies all individuals in terms of the chosen objective function. A fitness score is assigned to each solution, according to the chosen objective function, which will correlate with the probability that the individual has of being selected for reproduction and thus remaining or not in the
gene pool. In order to allow the evaluation of a solution an individual needs to be correctly encoded with the genes (encoding process). After the encoding is completed it is then possible to decompose the encoding scheme (decoding), in accordance with the problem constraints, to allow for the fitness evaluation. The following scheme 3.2 illustrates this concepts:




Figure 3.2: Fitness Evaluation Process

In figure 3.2 a certain solution is encoded (on a three layer encoding scheme) which then follows a process of decoding, which respects the equations defined in chapter 2, resulting in the Finish Times of each operations. With this information, based on a certain objective function, the fitness score of this individual is calculated and compared to the rest individuals in the same population. The represented individual is the second best in the population with a total fitness score of 54 time instances.

### 3.3 Selection

The selection phase is about choosing which population will be able to produce offspring. As it happens in the natural world, the fittest individuals have higher chances of passing on their genes. Often in artificial algorithms, the selection is performed by tournament or roulette wheel selection. In 2005, Zhang et al. [41] compared both methods and concluded that tournament selection is more efficient due to its higher convergence speed, which is important for large problems (which is the case). Therefore, in the present work, the former was chosen. Also, due to the tournament selection the best solutions might be discarded (as they might not be chosen for a tournament) and therefore an elitism operation is implemented to ensure the continuation of the best genes of the population.

### 3.4 Crossover

Crossover operations are about selecting which genes from which parents are passed to the off-springs (2 parents will produce 2 offspring). Often Crossover techniques rely on selecting random points in the parents chromosomes and swapping the genes they correspond to in order to build the off-springs encoding. The most common in literature are the single-point and the double-point crossover which will be covered in the next chapter.

### 3.5 Mutation

Mutation in an artificial genetic algorithm is the process of randomly selecting one or more genes from an offspring and then changing these genes while maintaining the solution feasibility.

After all of these steps the standard genetic algorithm repeats until a termination criteria is reached.

## Proposed Hybrid Genetic Algorithm

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The novel contribution of this thesis arises from using a Hybrid genetic algorithm to solve the DRC flexible job shop problem in a Quality Control Laboratory Environment which, to the author's best knowledge, has never been done before. The implemented Hybrid Genetic Algorithm contains the traditional phases in ( 3.1 to 3.5 ) with additional problem specific implementations. A flow chart of the general implementation of the hybrid GA is presented in figure 4.1.


Figure 4.1: Implemented GA - flowchart

At first sight the implemented hybrid GA resembles the traditional one. Although, the differences to
a standard GA do not lie in the sequencing of the steps, but in the steps itself. In this chapter these different phases will be explained in the following sections 4.1 to 4.10 . It is worth mentioning that the Variable Neighbourhood Search is either chosen at the beginning of the algorithm or not, it does not depend on any probabilities.

### 4.1 Reading the Instance

In this phase, apart from the instance loading, two binary tables, the compatibility tables are produced. This idea was based on Xiao et al. (2021) [33] which applied this concept to the loading and unloading of fixtures in a scheduling problem and is now applied to the Quality Control Laboratory Scheduling environment. The tables have zeros (meaning can't perform) or ones (meaning can perform) which inform which jobs can be performed by which machines and workers. It is worth remarking that the Jobs appear with decimal numbers which mean the task of the current job to be performed. This was the notation chosen for the encoding table as it will be explained in section 4.3 Considering an example with 4 Jobs, 2 Workers and 7 machines the compatibility tables could be:

Table 4.1: Machines Compatibility Table

| Job | $M_{1}$ | $M_{2}$ | $M_{3}$ | $M_{4}$ | $M_{5}$ | $M_{6}$ | $M_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 . 1}$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 2.1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 3.1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 3.2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 4.1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |

Table 4.2: Analysts Compatibility Table

| Job | Worker 1 | Worker 2 |
| :---: | :---: | :---: |
| $\mathbf{1 . 1}$ | 1 | 0 |
| 2.1 | 1 | 0 |
| 3.1 | 0 | 1 |
| 3.2 | 0 | 1 |
| 4.1 | 1 | 1 |

This implementation is particularly useful to easily access information to ascertain whether a re-
source is able to perform a certain task or not. With tables 4.1 and 4.2 one can notice, for example, that job 1.1 is able to be performed by machine 1 , machine 5 and worker 1 .

### 4.2 Initial Population

In the Initial Population phase two distinct implementations which rely on different logic were created: the Initial Population Job Randomization (IPJR) and the Incremental Initial Population Worker Randomization (IIPWR). In both cases, a novel limitation in allocating resources was also implemented, namely a Prohibition Condition (PC) which restricted both workers and machines to work more than $X$ and $Y$ times in a row (respectively). The rationale behind the PC concept is that while assigning workers and machines to a certain job, both analysts and appliances may repeat themselves infinitely, as long as they can perform the chosen job, leading to inferior solutions solutions.

### 4.2.1 Initial Population with Job Selection

The logic behind this implementation is to produce an Initial Population in the most intuitive way possible. This is accomplished by firstly choosing an available job, and then allocating a random machine and worker that can perform the selected task.

In order to first select each job, a novel Job Selection Procedure is utilized, which combined three different methods: a Random Initialization, a Longest Processing Time selection and a Most Number Of Tests selection. All three of these methods can only choose from the pool of available tests previously defined. Random Initialization is as it's name suggests, each job is selected randomly. The Longest Processing Time selects the one with the longest processing time. The Most Number Of Tests chooses a job with the most amount of tests needing to be performed. In the latter two selections, if there are multiple jobs with the same conditions one of those is then chosen by randomization. In order to choose which implementation is used, weighs $\left(w_{1}, w_{2}, w_{3}\right)$ are assigned to each alternative corresponding to the probability that each selection method has to be chosen.

In the figure 4.2, the Initial Population Job Randomization implementation can be seen:


Figure 4.2: Initial Population with Job Selection flowchart

### 4.2.2 Incremental Initial Population with Worker Selection

The logic behind this implementation is based on the concept of puzzle pieces where, ideally, there is a sequence of jobs that can be perfectly grouped together, much like a puzzle. In the context of this problem, once a worker is assigned to a job and a machine, the worker is automatically assigned to three time instances on that machine that he must respect. The time in between these time instances is called Idle Gap and the problem is best solved when this interval of time is minimized. This means that if one could find a series of jobs able to be performed by the same worker on different machines where each job could perfectly fit in the Idle Gap of the others this problem would be optimally solved.

In order to understand which jobs best fit together, an Increment Table was calculated with it's results being the time that the worker has to wait before he can work on the next assigned job.

In the following figure 4.3 this concept is illustrated. Two jobs are presented and one worker is assigned to perform both. Since the Idle Gap of the first operation matches perfectly with the work time
of the second, the total idle time of the worker in this operation is zero (i.e. the increment is zero).


Figure 4.3: Puzzle Piece concept

In this particular example, considering these are the only two available jobs, the increment table can be seen in table 4.3. As demonstrated, test 1 of job 1 has a zero increment to perform the test 1 of job 2. However, the reverse would mean an increment of 19 time instances. This illustrates the importance of allocating compatible jobs together.

Table 4.3: Increment Table

|  | 1.1 | 2.1 |
| :---: | :---: | :---: |
| 1.1 | - | 0 |
| 2.1 | 19 | - |

In order to select and combine the jobs with the least amount of increment time together the reverse of the previous method (IPJS) must be performed. Firstly, the algorithm has to select a worker (respecting the prohibition conditions as well) and then it chooses a machine where this worker can perform at. Afterwards, it obtains the pool of available jobs that both the machine and worker can execute. After this process is completed, depending on the number of times that workers has been chosen before, several different ramifications exist which ultimately leads to the assignment of the chosen job. This is stored in a variable called "Worker Number"

The "Work Number" variable shows the number of times a certain worker was already chosen in the assignment cycle. This variable increases to a maximum of two, therefore, when the "Work Number" surpasses this value it returns to the number one. This means that the increment cycle of the initial solution is capped to assign two workers in a row. These parameters were implemented, as the initial solution is assumed to be improved by the genetic algorithm later on. Increasing the complexity of the algorithm on the Initial Solution would lead to increases in computational time and would risk falling
into local optimums. Flowcharts, 4.4 and 4.5 illustrate the Incremental Initial Population with Worker Selection (IIPWS) procedure.

The machine and worker routine is a set of condition cycles intended to qualify or disqualify the reason behind not having any available machines to be chosen. The deeper the layer of questioning, the upper the cycle returns to in the IIPWS. Figure 4.6 illustrates this routine.


Figure 4.6: Machine and Worker Routine

### 4.3 Encoding

A three-layered encoding mechanism is proposed based on encoding type 1 and 4 (section 3.1.1). This encoding mechanism was implemented, as it indicates which test is being performed in each job, whilst having the same three-layered chromosome as encoding type 4. The implemented encoding


Figure 4.4: Incremental Initial Population with Worker Selection flowchart - part one


Figure 4.5: Incremental Initial Population with Worker Selection flowchart - part two
scheme has therefore, three chromosomes: the operation sequence chromosome (OS), representing the process sequencing of operations; the machine selection chromosome (MS), which represents the allocation of machines and the Analysts selection chromosome (AS), representing the worker assignment layer. The number of genes in the chromosome equals the total number of operations in all jobs. This representation forces the algorithm to keep the same assignment of resources when a test changes it's position respecting the allocation of machines and workers.

Considering an example with 4 Jobs, 2 Analysts and 7 machines, the following table Table 4.4 represents a possible configuration for the encoding mechanism.

Table 4.4: Three layered Encoding Chromosome

| Operation Sequence (OS) | 1.1 | 3.1 | 2.1 | 3.2 | 4.1 |
| ---: | :---: | :---: | :---: | :---: | :--- |
| Machine Selection (MS) | 2 | 3 | 5 | 4 | 6 |
| Worker Selection (AS) | 1 | 1 | 1 | 2 | 1 |

In Table 4.4 in the Operation Sequence (OS) chromosome, the unit number represents the job and the decimal number represents the test of that job. More specifically, the forth gene in the OS chromosome is 3.2. This means the test 2 of job 3 is being allocated to this particular position.

The encoding process is performed with the initial population creation and following it is the decodification process.

### 4.4 Decoding

The Decoding process can be divided into four steps: firstly, define the temporal vectors of the machines and workers (Worker and Machine available time). The vectors were considered to have a resolution of 0.1 representing the workers' and machine times necessary in each operation. This was an approximation done to the problem in order to allow the decodification of the tasks. Secondly, select the operations from the OS vector individually from left to right and obtain the corresponding machine and analyst. Afterwards, define a third temporal vector of the selected job, the operation work times vector with the same 0.1 time resolution. Subsequently, compare the vectors and place the operation as early as possible. Update the temporal vectors and repeat until all jobs are assigned. The decoding process with the temporal vectors is exemplified in figure 4.7 for two distinct consecutive operations where the machine is not repeated.

| Operation 1 | Start | End |
| :---: | :---: | :---: |
| Task 1 | 0 | 0.1 |
| Task 2 | 0.3 | 0.4 |
| Task 3 | 0.9 | 1 |


| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Operation 2 | Start | End |
| :---: | :---: | :---: |
| Task 1 | 0 | 0.1 |
| Task 2 | 0.4 | 0.5 |
| Task 3 | 0.8 | 1 |



Job allocation procedure:

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Worker Available Times

## Worker Available

 Times Times

Figure 4.7: Job allocation example with two consecutive operations

### 4.5 Fitness Evaluation

Following the decoding process, the time at which each job is finished is calculated, so that, the fitness value of the individual can be calculated. In figureTh 4.8 this procedure is illustrated:


Figure 4.8: Fitness Evaluation - Flowchart

### 4.6 Selection

In the existing algorithm the Selection phase is achieved through an Elitism operation (which starts once the number of iterations is superior $1 / 8$ of the total population) followed by a Tournament Selection. The Elitism procedure ensures that the best (alpha) solutions are present in the parent population and are not discarded by the Tournament Selection. The delay added to the elitism is to guarantee that the solution has been improved prior to defining any elite population. This aids in eliminating local optimums at the beginning of the algorithm where the solution presents worse fitness values. The Tournament Selection chooses five random solutions from the main population and selects the best one to be added to crossover population. This procedure is repeated until the final population has a certain predefined amount of tournament winning individuals. The bigger the population selected for the tournaments, the less likely it is for weaker individuals to be selected. This is because, if a weak individual is selected to be in a tournament, there is a higher probability that a stronger individual is also in that tournament. Consequentially this implementation mainly chooses the best population to be featured in the Crossover while also giving a chance to the weaker individuals to be present. The lower the number of individuals in a tournament the higher variety of solutions available for crossover, thus reducing the convergence speed of the algorithm.

Figure 4.9 illustrates this two methods used in the selection phase.


Figure 4.9: Techniques used in the Selection phase - Flowchart

### 4.7 Crossover

The novel implemented approach explores three different crossover operations: Single Point Crossover, Double Point Crossover and MPX crossover. As previously utilised in the Population Initialization phase (4.2) weights were implemented to account for different probabilities of choosing a certain path. In the next chapter this weight selection is compared and studied. The following figure 4.10 illustrates this phase:


Figure 4.10: Crossover - Flowchart

### 4.7.1 Single Point Crossover

The Single Point Crossover is performed by firstly, choosing a random point in both parents genes. This divides the parents chromosomes in two sections identical sections. Afterwards, the crossover is completed by swapping parents chromosomes right sides. The following figure 4.11 illustrates this Crossover operation:


Figure 4.11: Single Point Crossover example

### 4.7.2 Double Point Crossover

In the Double Point Crossover, two different random numbers (in the chromosome length) are chosen which correspond to a certain interval gap in the parents genes. This middle section is swapped between the two to produce both off-springs. The following figure 4.12 illustrates this Crossover operation:


Figure 4.12: Double Point Crossover example

### 4.7.3 MPX Crossover

The MPX Crossover was based on [36] and is responsible for the change in the allocation of workers and machines. It revolves around the creation of a random binary vector with the same length of the chromosomes, which, depending on the value it assumes on a certain gene, uses the machines and workers from one of the parents to assemble the off-springs. The procedure is the following: assign to the offspring with the same number of the parent the same order of jobs. Going from left to right until the end of the chromosome, retrieve the number of the binary value on that position. If the value of the binary vector is equal to one, parent 1 will assign offspring 2 with the machine and worker he has been assigned to the job the offspring 2 has in that position. The opposite happens for parent 2 and offspring 1. If the value on the random vector is equal to zero, parent 1 assigns the machine and worker he has for that job to the offspring of the same number. Likewise for the second parent and offspring.

The following figure 4.13 illustrates this Crossover operation:

| Job Machine Worker |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Offspring 1 | $(2.1,2,1)$ | $(4.1,5,1)$ | $(2.2,2,1)$ | $(1.1,1,1)$ | $(3.1,1,1)$ | $(4.2,2,1)$ | $(1.2,4,1)$ | $(3.2,1,2)$ | $(1.3,2,1)$ |
| Parent 1 | $(2.1,1,1)$ | $(4.1,5,1)$ | $(2.2,3,2)$ | $(1.1,4,3)$ | $(3.1,1,1)$ | $(4.2,6,3)$ | $(1.2,4,1)$ | $(3.2,3,2)$ | $(1.3,2,1)$ |


| Random <br> Binary Vector | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Parent 2 | $(1.1,1,1)$ | $(3.1,4,1)$ | $(2.1,2,1)$ | $(3.2,1,2)$ | $(4.1,5,2)$ | $(2.2,2,1)$ | $(1.2,3,2)$ | $(1.3,3,2)$ | $(4.2,2,1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Offspring 2 | $(1.1,4,3)$ | $(3.1,4,1)$ | $(2.1,1,1)$ | $(3.2,3,2)$ | $(4.1,5,2)$ | $(2.2,3,2)$ | $(1.2,3,2)$ | $(1.3,2,1)$ | $(4.2,2,1)$ |

Figure 4.13: MPX Crossover example

### 4.8 Mutation

Offspring population have a certain chance to mutate. The mutation phase starts with a Shift Mutation (section 4.8.1) based on [4] followed by an "Intelligent Mutation" [21] (section 4.8.2). Depending on a certain predetermined probability, a machine and worker mutation is performed (section 4.8.3). Afterwards, the non-mutated off-springs are combined with the mutated ones to form the final off-springs population, as it can be seen in the following figure 4.14


Figure 4.14: Mutation - Flowchart

### 4.8.1 Shift mutation

The Shift mutation resembles the Double Point Crossover 4.7.2 where two different random numbers with the length of the chromosome are firstly obtained. Each number will correspond to a gene (with a job, worker and machine) and will swap positions with the other number's position. Afterwards, feasibility and precedence checking is required.

### 4.8.2 Intelligent mutation

The concept behind the Intelligent mutation is to select one of the operations from the machine with maximum workload and assign it to the least used machine. If the machine is not compatible no mutation occurs.

### 4.8.3 Worker and Machine mutation

This mutation operation uses the same shifted genes from the Shift Mutation 4.8.1. Here both machines and workers are changed to new compatible workforce to perform that job. If no compatibility exists no change is performed.

### 4.9 Variable Neighbourhood Search

The logic behind Variable Neighbourhood Search (VNS) is to explore neighborhoods of a current incumbent solution. If the new solution proves fitter than the previous one, then the algorithm selects it as the incumbent solution and explores further neighbor solutions until a termination criteria is achieved. This "exploration" is usually performed by operations similar to the ones implemented in the mutation phase in section 4.8. VNS was implemented in an effort to improve GA's slow convergence speed due to unguided mutations.

In the implemented VNS, 4 neighborhood structures were implemented: Exchange, Replace, Change and Intelligent structure. Different search algorithms were implemented as it allows for different neighborhood searches which, can potentially improve the convergence speed of the algorithm thus reducing the makespan.

The Exchange structure is analogous to the same exact principle as the shift mutation. The Change structure is equivalent to the Worker and Machine Mutation and the Intelligent structure follows the same logic as the one with the same name in the Mutation phase. The Replace mutation consists of selecting two different random numbers within the chromosome length which consists of two different genes. Subsequently, the gene position that corresponds to the smallest number is placed in the position on the encoding table of the biggest number. The gene in the newly occupied position moves one position backwards as well as the rest of the genes until no more overlapping occurs.

The following implemented algorithm is the following:

## Algorithm 4.1: Variable Neighbourhood Search (VNS)

Data: Parent and Offspring Population
Result: VNS population
1: Sort the population;
2: Select the best percentage of individuals to perform VNS (VNS population);
3: Obtain individual $x$;
4: while Not all individuals in the VNS population had undergone the VNS do
5: Set: $a=0 ; b=0 ; c=0 ; d=0 ; t=0$;
6: for $t=1 \xrightarrow{t}$ iteration $\mathbf{d o}$
if $\mathrm{a=0} \mathrm{or} \mathrm{(a=b=c}=d=1)$ then
Produce new individual y by Swap operation. Evaluate the individual y;
if y is better than x then
$x=y ; a=0 ; t=0 ;$
else
$a=1-a ; t=t+1 ;$
if $b=c=d=1$ then
$L b=c=d=0$
if $b=0$ then
Generate new solution y with Insert. Evaluate the individual y;
if y is better then x then
let $x=y, b=0, t=0$;
else
$b=1-b ; t=t+1 ;$
if $\mathrm{c}=0$ then
Produce new individual y with Assign. Evaluate the individual y;
if y is better then x then
let $x=y, c=0, t=0$;
else
$\mathrm{c}=1-\mathrm{c} ; \mathrm{t}=\mathrm{t}+1 ;$
if $d=0$ then
Generate new solution y with Intelligent. Evaluate the individual y;
if y is better then x then
let $x=y, d=0, t=0 ;$
else
$d=1-d ; t=t+1 ;$
if $\mathrm{t}>$ iteration/4 then
Terminate
7: Return individual x ;

### 4.10 Dynamic Termination Criteria

The termination criteria was implemented in the event the algorithm did not yield any improvements after $N$ iterations. Since this lack of improvement may be related to a local optima, a procedure was implemented to avoid the algorithm being trapped in one. In this case, a counter of the successive non
improving iterations was implemented, that when it surpasses $\frac{1}{4}$ of the termination criteria variable, the percentage of offspring and the mutation percentages increases to $80 \%$ and $50 \%$ respectively. Also if the VNS is active it increases to $30 \%$ of the best population. Afterwards, an intelligent mutation is performed to all the population. If the total completion time of the instance is reduced, this new individual replaces the previous one. If the algorithm improves these parameters return to the normal values and the counter is returns to zero. The algorithm may end after a predefined number of nonimproving consecutive iterations is reached (termination criteria). The algorithm is also limited to 3 h of total computational time. The following algorithm (2) illustrates this logic:

```
Algorithm 4.2: Termination Criteria
    Data: Best Solution
    Result: Continuation or Termination of the algorithm
    while Computational time \(l<3 \mathrm{~h}\) do
        if algorithm did not improve from last iteration then
            number of non-improving iterations \(+=1\) (counter);
            if counter \(\geqslant \frac{1}{4}\) termination criteria then
                Mutation coefficients \(=50 \%\);
                Percentage of best population in VNS \(=30 \%\);
            Percentage of Offspring \(=80 \%\);
            if counter \(\geqslant\) termination criteria then
                    terminate algorithm;
        else
            counter \(=0\);
            All values return to the original state;
```



## Results

## Contents

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### 5.1 Description of the Experiments

The experiments developed encompass different configurations of the QC labs scheduling problem. To allow for realistic data, a study by Martins et al. [2] was conducted on real quality control laboratory and an instance generator was developed. The instances used in this work are the same ones used by [2]. The instance generator aims to mimic the conditions faced on a regular working day in this Quality Control Laboratory environment. The instance generation approach followed makes use of the sample type (job type) concept, which has direct relation to what might happen in real life scenario in QC labs. Each combination of product and source (e.g. raw material, final product) usually results on a predetermined set of tests (operations) that need to be completed. This way, jobs come from a set of predetermined possible job types, each characterised by a number of operations and their processing times.

Regarding the aforementioned job types, $J$, three are considered in all experiments. For each job type, the number of operations is determined using a discrete uniform distribution (DUD), ranging from 1 to 3 . Each operation has a processing time $p_{i j}$ ranging from 1 to 5 and the worker tasks start from three time points: the start of the operation, at $30 \%$ of operation processing and at $90 \%$ of operation processing. The duration of the worker tasks are: $5 \%, 10 \%$ and $10 \%$ of the total processing time, representing the setup, intermediate and disassembly/data processing tasks.

The parameters that characterise experiments are the number of jobs $n$, number of machines $m$, number of workers $w$ and the flexibility. Experiments have been developed for 5,10 and 70 jobs. The smaller instances have been used to facilitate the implementation of the algorithm and are not representative of the QC lab dimension. The medium 12 instances of 10 jobs represent the daily workload of a QC laboratory. The larger 70 job instances mimic the realistic QC labs weekly scheduling problem. The number of machines is set at 7 , representing different types of equipment present at laboratories. For this thesis, the implementation developed is flexible. i.e., it can be used for any number of machines.

The number of workers can be 2, 3 and 7 , representing cases that are respectively, worker restricted, balanced and machine restricted. The flexibility parameter is used to compute the machines and workers that are able to carry out a certain operation. When generating the instance, each one of the workers and machines has a probability equal to their flexibility of being eligible to carry out the operation. Experiments are done for flexibilities of $30 \%$ and $60 \%$, encompassing cases where machines and workers have less or more general competences. If more than one machine and worker can perform a certain job, the time they take is the same, i.e., there is no heterogeneity between workers and machines that can perform the same operation.

The experiments developed are summarised in Table 5.1. Three replications of each experiment have been generated to improve the reliability of results. With $n$ taking three possible values, $w$ taking three possible values and the flexibility with two possible values, the total number of instances with different
experimental parameters is eighteen. Considering that each of these instances have three distinct unique parameter variations, this is three replications, there are a total of fifty-four distinct experiments. [2]

All experiments were performed on a computer running Windows 10 with an Intel Core i5-8265U processor ( 1.60 GHz base frequency) and 8 GB of RAM.

Table 5.1: Parameters for the experiments designed [2]

| Parameters |  | Values |
| :---: | :---: | :---: |
| Job generation parameters |  |  |
| Job Types | $N$ | 3 |
| Operations in job | $q_{j}$ | $D U D(1,3)$ |
| Operation processing time | $p_{i j}$ | $D U D(1,5)$ |
| Starting point of worker tasks | 1 | $\left[0,0.3 p_{i j}, 0.9 p_{i j}\right]$ |
| Duration of worker tasks | $p_{i j s}^{d}$ | $\left[0.05 p_{i j}, 0.1 p_{i j}, 0.1 p_{i j}\right]$ |
| Experiment Parameters |  |  |
| Number of jobs |  |  |
| Number of machines | $n$ | $[5,7,10]$ |
| Number of workers |  |  |
| Flexibility | $w$ | 7 |
|  |  |  |

### 5.2 General GA Parameters

The general genetic algorithm parameters are considered to be the essential parameters, present in every GA. The correct tuning of this constants is not a trivial task, since these variables are often codependent. This section will cover how the tuning of these parameters could potentially improve the algorithm's results and present the values considered in this dissertation. These tests ran without VNS and a sample instance was considered ( 70 jobs, 7 machines, 3 workers, 0.3 flexibility and number 0 ) since to understand the degree of convergence of the algorithm the tests need to run for long periods of time, therefore, computing all 18 instances of 70 jobs proved to be unfeasible. The results are the average of the the best result in 5 runs.

### 5.2.1 Population Size

Regarding the population size tuning, a total of 50 iterations were considered and kept constant as well as an offspring percentage of $70 \%$ and a mutation chance of $50 \%$. The average standard deviation of the results was 57.2 hours. The results are presented in figure 5.1 below:


Figure 5.1: Study of the Total Completion Time with different population sizes

From 5.1 the most suitable population numbers are 150 with an average of 811 seconds of computational time, 400 with 2032 seconds and 700 with 3219 seconds.

Since the final results are meant to run the instances for 3 h ( 10800 seconds), with a population of 700 individuals, the total number of iterations could only be about 3.35 times the number of iterations used in this tests (meaning roughly 160 iterations) which is a low number to allow a proper convergence of the algorithm. As for the 400 individuals it allows for roughly 5 times the 50 iterations, i.e., 250 total iterations. For the 150 individuals it allows up to 13 times the 50 iterations (i.e., around 650 iterations). Since the objective is to obtain the best possible result in 3 h the 400 off population was chosen since it provides a good compromise between the results and the computational time.

### 5.2.2 Offspring Percentage

Regarding the offspring percentage tuning, apart from the other parameters, a total of 100 iterations with a total population of 400 was considered. The percentage of population to be selected for the offspring
operation was changed from 10 to $100 \%$. The results can be seen in the following figure 5.2 , in which the average standard deviation is 62.4 hours.

## Tuning of the Percentage of Offspring



Figure 5.2: Study of the Total Completion Time with different percentage of Offspring

From figure 5.2 one can remark that the values presenting the best results for the offspring percentage are the $50 \%$ and $70 \%$. Since the objective is to obtain the best possible results in 3 h , the latter value was considered.

### 5.2.3 Mutation Percentage

For the percentage of mutation tuning, a total of 50 iterations were considered while the other parameters were the ones obtained from the previous tuning ( 400 of population and $70 \%$ of offspring). The following figure 5.3 shows the results in which the average standard deviation is 67.4 hours.


Figure 5.3: Study of the Total Completion Time with different percentage of mutation

From figure 5.3 one can conclude that from the study the mutation is not very relevant for this instance (results differ by $1 \%$ ). Nevertheless the best percentage of mutation is $30 \%$ as it induces the least total completion time in the model instance.

### 5.3 Initial Population

In order to obtain the best possible results, every weight must be tuned. Therefore the adjustments start in the initial population. The goal is to obtain the best solutions without falling into a local optima.

### 5.3.1 Job Selection Procedure

Tests were performed to the Job Selection Procedure (section 4.2.1) in order to verify which combination of weights ( $w_{1}, w_{2}, w_{3}$ ) proved better results. The following tables 5.2 and 5.3 contain the average values followed by the respective standard deviation returned by the algorithm for the 70 job instances. These were obtained from a population of 500 individuals. In green is the best value and in yellow the second best for the same instance.

Table 5.2: Weight Tuning part 1

| $\begin{aligned} & \text { № } \\ & \text { Jobs } \end{aligned}$ | Workers | Flex | Rep | $\begin{gathered} 100 \mid 0 \\ \mid 0 \end{gathered}$ | $\begin{gathered} 0 \mid 100 \\ \mid 0 \end{gathered}$ | $\begin{gathered} 0 \mid 0 \\ \hline 100 \end{gathered}$ | $\begin{gathered} 40 \mid 30 \\ \mid 30 \end{gathered}$ | $\begin{gathered} \hline 60 \mid 20 \\ \mid 20 \end{gathered}$ | $\begin{gathered} \hline 60 \mid 30 \\ \mid 10 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | $5277+$ +-211 | 5558 +/-225 | $6428+/-306$ | 5957 +/-261 | 5786 +/-254 | 5696 +/-255 |
|  |  |  | 1 | $7369+/-283$ | $7492+/-317$ | $8250+$ +-392 | 8095 +/-319 | 7860 +/-324 | $7750+/-277$ |
|  |  |  | 2 | $4541+/-137$ | 4568 +/-152 | 4938 +/-176 | 4629 +/-145 | 4624 +/-150 | 4583 +/-142 |
|  |  | 0.6 | 0 | $5194+/-206$ | $5303+/-195$ | $6243+$ +-239 | 5336 +/-202 | $5298+/-197$ | $5212+/-188$ |
|  |  |  | 1 | 5651 +/-256 | 5923 +/-310 | $6627+1-248$ | 5773 +/-260 | $5696+/-247$ | 5649 +/-255 |
|  |  |  | 2 | $5681+/-276$ | $6240+/-305$ | $6690+$ +-290 | 6278 +/-307 | $6104+/-305$ | $6000+/-287$ |
|  | 3 | 0.3 | 0 | 4615 +/-237 | $4208+/-235$ | $5765+/-252$ | 5314 +/-310 | 4983 +/-272 | 4930 +/-285 |
|  |  |  | 1 | $13050+/-825$ | $13645+/-873$ | $14596+/-843$ | 13920 +/-924 | $13695+/-879$ | $13610+/-906$ |
|  |  |  | 2 | $6208+/ 256$ | $7211+/-341$ | $7195+/-239$ | $6909+/-297$ | $6710+/-288$ | $6750+/-313$ |
|  |  | 0.6 | 0 | $3695+/-176$ | $4104+/-224$ | $4151+/-165$ | 4024 +/-182 | 3961 +/-180 | 3961 +/-182 |
|  |  |  | 1 | $3863+/-187$ | $4300+/-247$ | $4325+$ +-188 | 4108 +/-230 | $4033+/-192$ | 4004 +/-194 |
|  |  |  | 2 | 2718 +/-163 | $2937+/-205$ | $3064+1-203$ | 2955 +/-199 | $2892+/-193$ | $2862+/-192$ |
|  | 7 | 0.3 | 0 | $4362+/-273$ | $4877+1-346$ | 4880 +/-304 | 4708 +/-335 | $4606+/-296$ | 4596 +/-304 |
|  |  |  | 1 | $6612+/-450$ | $6835+$ +/482 | $6957+/-456$ | $6909+/-509$ | 6834 +/-456 | $6794+/-458$ |
|  |  |  | 2 | 3627 +/-177 | $3739+/-212$ | $3805+$ +-171 | 3655 +/-204 | $3608+/-180$ | $3600+/-191$ |
|  |  | 0.6 | 0 | $5680+/-281$ | 5427 +/-269 | $6376+/-303$ | 6082 +/-304 | $5937+/-312$ | $5878+/-286$ |
|  |  |  | 1 | $2831+/-174$ | $3324+/-234$ | $3480+/-213$ | 3388 +/-221 | $3220+/-219$ | $3156+/-217$ |
|  |  |  | 2 | $4564+/-286$ | $5180+/-279$ | $5281+/-281$ | $5106+/-317$ | 4960 +/-290 | 4944 +/-301 |
| Total |  |  |  | 95536 | 100871 | 109048 | 103146 | 100805 | 99976 |

Table 5.3: Weight Tuning part 2

| $\begin{array}{\|c} \hline \text { № } \\ \text { Jobs } \end{array}$ | Workers | Flex | Rep | $\begin{gathered} 60 \mid 10 \\ \mid 30 \end{gathered}$ | $\begin{gathered} \hline 80 \mid 10 \\ \mid 10 \end{gathered}$ | $\begin{gathered} 90 \mid 5 \\ \mid 5 \end{gathered}$ | $\begin{gathered} 95 \mid 5 \\ \mid 0 \end{gathered}$ | $\begin{gathered} 95 \mid 0 \\ \mid 5 \end{gathered}$ | $\begin{gathered} 98 \mid 2 \\ \mid 0 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | $5815+/-256$ | $5549+$ /-248 | 5413 +/-233 | $5341+$ +-244 | 5334 +/-215 | $\begin{aligned} & 5289+/-228 \\ & 7384+/-287 \end{aligned}$ |
|  |  |  | 1 | 7938 +/-344 | $7603+/-312$ | $7504+/-291$ | $7441+/-288$ | 7466 +/-314 |  |
|  |  |  | 2 | 4683 +/-156 | 4582 +/-155 | 4565 +/-150 | $4553+/-143$ | 4547 +/-144 | 4548 +/-145 |
|  |  | 0.6 | 0 | $5407+/-211$ | 5254 +/-197 | 5219 +/-191 | $5180+/-189$ | 5217 +/-212 | $5194+/-206$ |
|  |  |  | 1 | $5821+$ +-264 | $5632+/-255$ | 5626 +/-245 | $5599+$ +-258 | 5716 +/-248 | $5641+/-256$ |
|  |  |  | 2 | $6204+$ +-295 | 5925 +/-278 | 5797 +/-275 | $5703+/-264$ | 5764 +/-285 | 5676 +/- 275 |
|  | 3 | 0.3 | 0 | $5038+/-262$ | $4762+/-255$ | 4656 +/-232 | $4604+/-236$ | 4634 +/-239 | 4602+/-218 |
|  |  |  | 1 | $13895+$ +/860 | $13462+/-897$ | $13252+/-871$ | $13129+/-827$ | $13206+/-850$ | $13101+/-838$ |
|  |  |  | 2 | $6751+/-260$ | $6516+/-296$ | $6355+/-284$ | 6275 +/- 251 | $6289+/-261$ | $6235+/-272$ |
|  |  | 0.6 | 0 | 3987 +/-177 | 3871 +/-172 | $3792+/-168$ | $3755+/-181$ | 3752 +/-195 | $3706+/-177$$3886+/-206$ |
|  |  |  | 1 | $4100+$ +-191 | 3986 +/-190 | 3931 +/-189 | $3897+$ +-193 | 3931 +/-179 |  |
|  |  |  | 2 | 2896 +/-186 | 2818 +/-180 | 2786 +/-173 | $2752+$ +-180 | 2737 +/-167 | $2731+/-167$ |
|  | 7 | 0.3 | 0 | $4613+/-309$ | 4500 +/-282 | 4439 +/-272 | $\begin{aligned} & 4411+/-289 \\ & 6619+/-485 \end{aligned}$ | 4417 +/-274 | $4411+/-268$ |
|  |  |  | 1 | $6805+/-468$ | $6727+/-460$ | 6689 +/-465 |  | $6651+/-455$ | $6633+/-1437$ |
|  |  |  | 2 | 3657 +/-171 | 3620 +/-178 | 3626 +/-172 | 3586 +/-155 | 3631 +/-170 | 3609 +/-166 |
|  |  | 0.6 | 0 | $5979+$ +-300 | $5826+/-306$ | 5749 +/-287 | $5667+/-276$ | $5707+/-300$ | $5683+/-285$ |
|  |  |  | 1 | 3238 +/-217 | $3047+$ +-205 | 2951 +/-202 | 2869 +/-196 | 2910 +/-190 | $\begin{aligned} & 2853+/-185 \\ & 4579+/-271 \end{aligned}$ |
|  |  |  | 2 | 4974 +/-299 | $4812+$ +-288 | 4682 +/-299 | 4627 +/-196 | 4590 +/-256 |  |
| Total |  |  |  | 101799 | 98488 | 97031 | 96009 | 70093 | 95759 |

The reasoning behind the different experimented weights was: firstly, assume the maximum probability to each path and rank them from best to worst. This likely means that the more the best paths are followed by the algorithm the better the results are. Therefore the following tests were conducted bearing in mind that $w_{1}$ and $w_{2}$ proved the best and second best results (respectively).

It can be noticed that when the weight one, $w_{1}$, has $100 \%$ of probability of being chosen, which corresponds to the random initialization, the results proved to be the best for nearly all the instances.

It is worth mentioning that the 70 Job instance is used throughout all tuning tests in this work as it has the most job allocation and consequentially, more reliability, and is the target instance of study for this work.

### 5.3.2 Prohibition Condition Study

The prohibition Condition as explained in section 4.2 assures that there is a limit for allocating a certain $X$ number of workers and $Y$ number of machines in a row. In this section, a tuning of these parameters is performed the same way as it was done in the section 5.3.1 above.

## Isolated Example with and without Prohibition Conditions

Firstly, to prove that the prohibition condition improves the results, let's consider a small instance with 5 jobs, 3 workers and flexibility 0.6 . A possible first configuration for the encoding table without any Prohibition Conditions applied could be:

Table 5.4: Example with no Prohibition Condition applied

| 1.1 | 5.1 | 1.2 | 3.1 | 5.2 | 4.1 | 2.1 | 2.2 | 3.2 | 3.3 | 4.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 1 | 7 | 7 | 7 | 5 | 5 | 1 | 7 | 2 | 7 |
| 1 | 3 | 3 | 2 | 3 | 1 | 3 | 1 | 1 | 2 | 3 |

The present configuration 5.4 has a total completion time of 58 time units. Now, if the reasoning provided is correct, if one applies the prohibition condition to encoding table, with both workers and machine not able to be chosen more than 2 times in a row, the fitness value should improve or in the very least stay the same. In the present example, on the job 5.2 , machine 7 needs to be replaced by another suitable machine. The suitable machines for job 5.2. are: Machines 1, 2, 4 or 5 . Applying the decoding and using the same objective function, the total completion time for machine 1,2 and 4 are, respectively: 44.4, 44.4 and 44.4 time units. For machine 5 it must be taken into account that this creates another prohibition condition in job 2.1 as it can be seen in the table below 5.5 :

Table 5.5: Prohibition applied in job 2.1

| 1.1 | 5.1 | 1.2 | 3.1 | 5.2 | 4.1 | 2.1 | 2.2 | 3.2 | 3.3 | 4.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 1 | 7 | 7 | 5 | 5 | 5 | 1 | 7 | 2 | 7 |
| 1 | 3 | 3 | 2 | 3 | 1 | 3 | 1 | 1 | 2 | 3 |

The present configuration in table 5.5 has a total completion time of 49.4 time units. Following the same reasoning as before, the available machines for job 2.1 are: 1, 3, 4 or 7 . Since changing to any of these values does not induce any other prohibition conditions, the total completion time for these machines are, respectively: $41.4,40.5,40.5$ and 40.5 , proving that introducing a prohibition condition on the machines improves the initial solution.

Now let's consider another possible first configuration for the encoding table without any Prohibition Conditions where a worker is chosen 3 times in a row. A possible configuration for the encoding table could be:

Table 5.6: Example with no Prohibition Condition applied - worker

| 1.1 | 5.1 | 1.2 | 3.1 | 5.2 | 4.1 | 2.1 | 2.2 | 3.2 | 3.3 | 4.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 1 | 7 | 7 | 5 | 5 | 5 | 1 | 7 | 2 | 7 |
| 1 | 1 | 1 | 2 | 3 | 1 | 3 | 1 | 1 | 2 | 3 |

The present configuration in table 5.6 has a total completion time of 49.4 time units. With the same reasoning, another possible worker to perform job 1.2 is worker 3 which makes the total completion time equal to 40.7 time units, thus improving the solution.

## Weight tuning of the Prohibition Conditions

The reasoning for the following experimented weights was: first check if there is some correlation between the number of times the prohibition conditions are fired and the mean results obtained. Secondly, while maintaining one of the conditions fixed (either the worker or the machines) check if by increasing or decreasing the other, it changes the results for better or worse. The results on table 5.7 were obtained by performing the average of five runs on a population size of 500 for each instance. The average standard deviation across all instances was +/- 259.

Table 5.7: Prohibition Condition values tuning

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | GA no Prohibition | GA <br> W = 3 <br> M = 2 | GA <br> W = 3 <br> $\mathbf{M}=3$ | GA $\begin{aligned} & \mathrm{W}=3 \\ & \mathrm{M}=4 \\ & \hline \end{aligned}$ | GA <br> $\mathrm{W}=3$ <br> $M=6$ | GA <br> W = 4 <br> M = $\mathbf{3}$ | GA <br> W = 4 <br> M = 4 | GA <br> W = 4 <br> M = 5 | GA $w=5$ $\mathbf{M}=5$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | 5413.141 | 4985 | 5001 | 5007 | 5005 | 5067 | 5056 | 5057 | 5112 |
|  |  |  | 1 | 7500.56 | 7316 | 7356 | 7384 | 7378 | 7378 | 7381 | 7384 | 7392 |
|  |  |  | 2 | 4558.991 | 4554 | 4544 | 4541 | 4543 | 4537 | 4546 | 4548 | 4547 |
|  |  | 0.6 | 0 | 5211.68 | 5138 | 5134 | 5134 | 5149 | 5141 | 5149 | 5143 | 5163 |
|  |  |  | 1 | 5639.7 | 5441 | 5455 | 5449 | 5450 | 5533 | 5529 | 5529 | 5552 |
|  |  |  | 2 | 5803.67 | 5642 | 5648 | 5650 | 5648 | 5637 | 5665 | 5643 | 5661 |
|  | 3 | 0.3 | 0 | 4675.081 | 4521 | 4548 | 4593 | 4585 | 4546 | 4568 | 4570 | 4594 |
|  |  |  | 1 | 13412.95 | 12096 | 12497 | 12783 | 12980 | 12552 | 12720 | 12931 | 12905 |
|  |  |  | 2 | 6349.8 | 6134 | 6172 | 6198 | 6204 | 6172 | 6216 | 6206 | 6201 |
|  |  | 0.6 | 0 | 3792.893 | 3647 | 3679 | 3673 | 3674 | 3676 | 3683 | 3680 | 3680 |
|  |  |  | 1 | 3923.9 | 3850 | 3854 | 3876 | 3870 | 3871 | 3863 | 3862 | 3875 |
|  |  |  | 2 | 2763.56 | 2686 | 2701 | 2704 | 2729 | 2703 | 2717 | 2733 | 2710 |
|  | 7 | 0.3 | 0 | 4403.69 | 4310 | 4343 | 4369 | 4374 | 4358 | 4387 | 4359 | 4364 |
|  |  |  | 1 | 6714.08 | 6502 | 6606 | 6609 | 6598 | 6597 | 6579 | 6610 | 6594 |
|  |  |  | 2 | 3623.75 | 3565 | 3585 | 3615 | 3610 | 3596 | 3599 | 3627 | 3625 |
|  |  | 0.6 | 0 | 5738.35 | 5558 | 5628 | 5644 | 5667 | 5629 | 5663 | 5660 | 5653 |
|  |  |  | 1 | 2957.89 | 2816 | 2836 | 2839 | 2831 | 2837 | 2847 | 2855 | 2846 |
|  |  |  | 2 | 4682.125 | 4512 | 4538 | 4529 | 4553 | 4532 | 4520 | 4552 | 4543 |
| Average |  |  |  | 5398 | 5182 | 5229 | 5255 | 5269 | 5242 | 5260 | 5275 | 5279 |

From table 5.7 it can be noticed that the prohibition condition achieves the best results for $\mathrm{W}=3$ and $M=2$, improving results by $4 \%$ when comparing with a GA with no prohibition condition.

It is worth mentioning that values below $W=3$ and $M=2$ were not tested since this could potentially risk falling into local optimum in the very beginning of the algorithm. The values of the prohibitions means it is the value when the prohibition starts. Therefore if $W=3$, it means that the algorithm will not allow 4 equal workers to be sequentially assigned .

As it can be seen in Appendix B, in tables B. 1 and B. 2 there is no correlation between the amount of times a certain prohibition is fired and the results on that particular instance.

### 5.3.3 IP with Job Selection vs Incremental IP with worker selection

The final addition to the Initial Population is the Incremental Initial Population with worker selection as it is explained in section 4.2.2. The following table 5.8 contains a comparison between the average values of the Normal initialization (IPJS) with the average values of this new approach. These are obtained from a population of 500 individuals and the following values are the respective standard deviation. As it happened previously, the best values for each instance are marked in green.

Table 5.8: IP with Job Selection (normal initialization) vs Incremental IP with worker selection (increment initialization)

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | Normal Initialization | Increment <br> Initialization |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | 5006.7 +/-184 | $\begin{aligned} & 4958.5+/-192 \\ & 7245.2+/-248 \end{aligned}$ |
|  |  |  | 1 | $7324.3+/-287$ |  |
|  |  |  | 2 | $4540.2+/-143$ | 4579.7 +/-171 |
|  |  | 0.6 | 0 | $5141.4+/-182$ | $5184.9+/-214$ |
|  |  |  | 1 | $5435.3+/-216$ | 5193.9 +/-183 |
|  |  |  | 2 | $5600.7+/-245$ | 5531.8 +/-196 |
|  | 3 | 0.3 | 0 | $4519.9+/-203$ | $4900.3+/-211$ |
|  |  |  | 1 | 12024.1 +/-675 | 10027.1 +/-566 |
|  |  |  | 2 | $6108.2+/-243$ | 6057.6 +/-321 |
|  |  | 0.6 | 0 | $3655.2+/-182$ | 3098.7 +/-120 |
|  |  |  | 1 | $3850.4+/-177$ | 3677.4 +/-158 |
|  |  |  | 2 | $2689.9+/-164$ | 2524.3 +/-141 |
|  | 7 | 0.3 | 0 | $4331.1+/-251$ | 4088.6 +/-247 |
|  |  |  | 1 | $6503+/-415$ | 5934.4 +/-372 |
|  |  |  | 2 | 3562.1 +/-153 | 3572.3 +/-184 |
|  |  | 0.6 | 0 | $5603.2+/-275$ | $5141.9+/-233$ |
|  |  |  | 1 | 2822.8 +/-177 | 2414.6 +/-112 |
|  |  |  | 2 | $4490.2+/-250$ | 4142.5 +/-172 |
| Total: |  |  |  | 93208.7 | 88273.7 |

From table 5.8 it can be noticed that for the majority of the instances the Increment Initialization (IPJS) obtains fitter results thus proving the accuracy of this implementation. When comparing the total completion time of all instances, the Increment Initialization improves the results by a margin of 5.3\% when comparing to the Normal Initialization.

## Convergence Speed

The results from table 5.8 prove that the Increment Initialization reaches fitter solutions in the Initial Population. This section studies whether this solutions are capable of having a faster convergence speed in the final solution of the Genetic Algorithm. In this case, a study was conducted for the first instance where the best initial and final objective functions for both the normal and incremental initialization were registered. The algorithm ran with a population of 10 individuals for 500 iterations with no VNS. The results can be seen in the following table (5.9) and in the figures 5.4 and 5.5 :

Table 5.9: Convergence Study

|  | Best Initial Population Solution | Best Final Solution | Difference (\%) |
| :---: | :---: | :---: | :---: |
| Normal Initialization | 4774.9 | 4327.0 | 9.38 |
| Increment Initialization | 4678.7 | 4131.9 | 11.69 |



Figure 5.4: Normal Initialization convergence


Figure 5.5: Increment Initialization convergence

As it can be seen, the increment Initialization has a faster convergence speed while also achieving fitter solutions thus proving the strength of this implementation.

### 5.4 Crossover Tuning

Tests were performed to the Crossover methods (section 4.7) in order to identify which combination of weights ( $c_{1}$ - Single Point Crossover, $c_{2}$ - , Double Point Crossover and $c_{3}$ - MPX crossover) proved fitter results. The following tables 5.10 and 5.11 present the best objective function obtained from a population size of 75 with 35 total iterations. In green is the best value and in yellow the second best for the same instance.

Table 5.10: Crossover Tuning part 1

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | $\begin{gathered} 100 \mid 0 \\ \mid 0 \end{gathered}$ | $\begin{gathered} 0 \mid 100 \\ \mid 0 \\ \hline \end{gathered}$ | $\begin{gathered} 0 \mid 0 \\ \mid 100 \end{gathered}$ | $\begin{gathered} 30 \mid 30 \\ \mid 40 \end{gathered}$ | $\begin{gathered} 20 \mid 20 \\ \mid 60 \end{gathered}$ | $\begin{gathered} 10 \mid 10 \\ \mid 80 \end{gathered}$ | $\begin{aligned} & 5 \mid 5 \\ & \mid 90 \end{aligned}$ | $\begin{gathered} 0 \mid 50 \\ \mid 50 \end{gathered}$ | $\begin{gathered} 0 \mid 25 \\ \mid 75 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | 4168 | 4277 | 3844 | 3991 | 4301 | 3986 | 3793 | 3976 | 3904 |
|  |  |  | 1 | 6141 | 5940 | 5963 | 5978 | 5875 | 6184 | 5976 | 6014 | 6062 |
|  |  |  | 2 | 3738 | 3724 | 3802 | 3649 | 3668 | 3657 | 3745 | 3599 | 3792 |
|  |  | 0.6 | 0 | 4448 | 4162 | 4303 | 4159 | 4303 | 4263 | 4161 | 4192 | 4226 |
|  |  |  | 1 | 4756 | 4660 | 4298 | 4498 | 4250 | 4353 | 4193 | 4302 | 4195 |
|  |  |  | 2 | 4558 | 4502 | 4485 | 4587 | 4496 | 4456 | 4406 | 4449 | 4474 |
|  | 3 | 0.3 | 0 | 3638 | 3746 | 3675 | 3441 | 3698 | 3634 | 3607 | 3712 | 3645 |
|  |  |  | 1 | 8343 | 7873 | 7180 | 7881 | 7801 | 7802 | 7446 | 7670 | 7298 |
|  |  |  | 2 | 4942 | 5042 | 4827 | 4868 | 4896 | 4790 | 4788 | 4880 | 4810 |
|  |  | 0.6 | 0 | 2976 | 2914 | 2772 | 2817 | 2841 | 2855 | 2950 | 2772 | 2916 |
|  |  |  | 1 | 3133 | 3097 | 3041 | 3181 | 3147 | 3061 | 3025 | 3099 | 3132 |
|  |  |  | 2 | 2041 | 2060 | 1954 | 2024 | 1925 | 1960 | 1940 | 1969 | 1983 |
|  | 7 | 0.3 | 0 | 3337 | 3331 | 3279 | 3291 | 3307 | 3293 | 3337 | 3250 | 3339 |
|  |  |  | 1 | 4757 | 4637 | 4496 | 4669 | 4618 | 4426 | 4496 | 4598 | 4642 |
|  |  |  | 2 | 2902 | 2811 | 2888 | 2790 | 2920 | 2847 | 2726 | 2865 | 2821 |
|  |  | 0.6 | 0 | 4669 | 4679 | 4268 | 4268 | 4118 | 4157 | 4192 | 4265 | 4447 |
|  |  |  | 1 | 2041 | 2189 | 2024 | 2153 | 2025 | 2127 | 2078 | 2079 | 2088 |
|  |  |  | 2 | 3346 | 3548 | 3342 | 3604 | 3309 | 3375 | 3212 | 3456 | 3271 |
| Total |  |  |  | 73931 | 73189 | 70439 | 71849 | 71497 | 71222 | 70071 | 71146 | 71045 |

Table 5.11: Crossover Tuning part 2

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | $\begin{gathered} 0 \mid 15 \\ \mid 85 \end{gathered}$ | $\begin{gathered} 0 \mid 10 \\ \mid 90 \end{gathered}$ | $\begin{gathered} 10 \mid 0 \\ \mid 90 \end{gathered}$ | $\begin{gathered} 10 \mid 5 \\ \mid 85 \end{gathered}$ | $\begin{gathered} 15 \mid 5 \\ \mid 80 \end{gathered}$ | $\begin{gathered} 20 \mid 5 \\ \mid 75 \end{gathered}$ | $\begin{gathered} 50 \mid 0 \\ \mid 50 \end{gathered}$ | $\begin{gathered} 70 \mid 0 \\ \mid 30 \end{gathered}$ | $\begin{gathered} 70 \mid 5 \\ \mid 25 \end{gathered}$ | $\begin{gathered} \hline 33.3 \mid 33.3 \\ \mid 33.4 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | 4029 | 3962 | 3843 | 3911 | 3825 | 3966 | 3980 | 4047 | 4009 | 3841 |
|  |  |  | 1 | 5975 | 6141 | 6037 | 6095 | 6077 | 6037 | 6133 | 6182 | 6138 | 6026 |
|  |  |  | 2 | 3601 | 3711 | 3724 | 3641 | 3600 | 3656 | 3730 | 3748 | 3640 | 3732 |
|  |  | 0.6 | 0 | 4107 | 4279 | 4023 | 4041 | 4260 | 4047 | 4280 | 4285 | 4364 | 4257 |
|  |  |  | 1 | 4254 | 4312 | 4383 | 4372 | 4194 | 4492 | 4648 | 4252 | 4659 | 4452 |
|  |  |  | 2 | 4346 | 4369 | 4435 | 4370 | 4368 | 4261 | 4392 | 4485 | 4611 | 4512 |
|  | 3 | 0.3 | 0 | 3692 | 3651 | 3702 | 3616 | 3584 | 3595 | 3606 | 3609 | 3619 | 3572 |
|  |  |  | 1 | 7514 | 7660 | 7291 | 7024 | 7717 | 7391 | 7463 | 8099 | 7670 | 8171 |
|  |  |  | 2 | 4651 | 4763 | 4660 | 4616 | 4776 | 4521 | 4788 | 4700 | 4631 | 4746 |
|  |  | 0.6 | 0 | 2872 | 2978 | 2823 | 2738 | 2903 | 2856 | 2813 | 2953 | 3005 | 3052 |
|  |  |  | 1 | 3015 | 2945 | 3033 | 3032 | 3049 | 3061 | 3090 | 3222 | 3055 | 3018 |
|  |  |  | 2 | 1949 | 1908 | 1937 | 1957 | 1957 | 1929 | 1990 | 2106 | 2002 | 2051 |
|  | 7 | 0.3 | 0 | 3285 | 3241 | 3276 | 3322 | 3302 | 3306 | 3269 | 3260 | 3337 | 3274 |
|  |  |  | 1 | 4649 | 4644 | 4498 | 4582 | 4556 | 4574 | 4568 | 4656 | 4662 | 4569 |
|  |  |  | 2 | 2806 | 2804 | 2804 | 2841 | 2810 | 2818 | 2763 | 2908 | 2828 | 2861 |
|  |  | 0.6 | 0 | 4138 | 4144 | 4131 | 4133 | 4217 | 4247 | 4228 | 4369 | 4401 | 4209 |
|  |  |  | 1 | 2178 | 2099 | 2054 | 2052 | 2091 | 2116 | 2110 | 2118 | 2080 | 2056 |
|  |  |  | 2 | 3356 | 3325 | 3438 | 3304 | 3276 | 3384 | 3357 | 3270 | 3504 | 3312 |
| Total |  |  |  | 70415 | 70935 | 70093 | 69645 | 70562 | 70256 | 71208 | 72269 | 72214 | 71711 |

The reasoning behind the different experimented weights was similar to (5.3.1): firstly, assume the maximum weight to each path and rank them from best to worst. This likely means that the more the best path is followed by the algorithm the better the results are. Therefore, the following tests were conducted bearing in mind that $c_{3}$ proved the best results. The next four tests with an incremental increase towards the 3rd weight demonstrate this reasoning and obtain the second best result for the weights: $c_{1}=5$, $c_{2}=5, c_{3}=90$. Afterwards, a disruptive weight selection $\left(c_{1}=0, c_{2}=50, c_{3}=50\right)$ was done to
check whether results would improve. This was followed by a progressive increase in the 3rd weight while marginally changing the other two. The best results were obtained for $c_{1}=10, c_{2}=5, c_{3}=85$. Afterwards, the remaining tests were attempts to give extra weight to the first path proving the logic that the worthiest weight to increase is the 3rd, corresponding to the MPX crossover.

### 5.5 Variable Neighborhood Search

### 5.5.1 Tuning

The parameters to tune in the VNS are the following:

- Percentage of best population to undergo VNS.
- Number of iterations in the VNS.

These must be tuned so that the increase in computational time provided by the VNS is compensated by an increase in the convergence speed. In order to tune these parameters the algorithm ran with a total population of 20 individuals for 10 iterations. All the values were calculated from an average of the 5 best individuals on each instance. Also, to allow comparison in the same plot, all the values are relative to the maximum value in each instance. The instance plots can be read in the blue left vertical axis (corresponding to the total completion time), whereas the computational time plot is read in the right orange vertical axis.

## Percentage of best population to perform VNS

Chart 5.6 contains a study (first three instances) for the percentage of the best population to undergo the VNS, while maintaining the number of iterations in the VNS constant and equal to six. The remaining plots for instances 4 to 18 can be consulted in the appendix A.2.1.

After analysing each instance, the percentage of population to undergo VNS that presents the fitter results can be seen in the following table 5.12. Also the number of occurrences on each percentage is presented in table 5.13.


Figure 5.6: Study of the Total Completion Time with different percentage of population accepted for VNS (Instances 1-3)

Table 5.12: Best Percentage of VNS in each instance

| Instance | Best VNS Percentage (\%) |
| :---: | :---: |
| 1 | 20 or 40 |
| 2 | 30 |
| 3 | 30 |
| 4 | 30 |
| 5 | 10,40 or 60 |
| 6 | 40 |
| 7 | 30 |
| 8 | 10 or 30 |
| 9 | 10 |
| 10 | 20 or 50 |
| 11 | 30 |
| 12 | 30 |
| 13 | 30 or 40 |
| 14 | 20 |
| 15 | 20 or 30 |
| 16 | 20 or 60 |
| 17 | 5 |
| 18 | 20 |

Table 5.13: Number of Occurrences on each percentage

| Best VNS Percentage (\%) | $\mathbf{5}$ | $\mathbf{1 0}$ | $\mathbf{2 0}$ | $\mathbf{3 0}$ | $\mathbf{4 0}$ | $\mathbf{5 0}$ | $\mathbf{6 0}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Occurrences | 2 | 3 | 7 | 7 | 4 | 1 | 2 |

The criteria to select the best percentage of VNS was to select the lowest value with the lowest possible amount of computational time. Therefore, for the case in figure 5.6, in instance 1 the $20 \%$ mark and the $40 \%$ mark were both selected. Table 5.13 indicates that the best percentage of VNS is found between 20 and 30 percent. Therefore, a percentage of $25 \%$ for the VNS was chosen.

## Number of iterations in the VNS

In order to tune the number of iterations, the same procedure was implemented resulting in the following figure 5.7 for the first 3 instances:

Tuning of the number of VNS Iterations


Figure 5.7: Study of the Total Completion Time with different VNS iterations (Instances 1-3)

After analysing each instance, the number of iterations that presents the fitter results can be seen in the following table 5.14. Also the number of occurrences on each percentage is presented in table 5.15.

Table 5.14: Best Number of Iterations in VNS in each instance

| Instance | Best Number of Iterations (\%) |
| :---: | :---: |
| 1 | 2 |
| 2 | 5 or 8 |
| 3 | 3 or 9 |
| 4 | 4 or 6 |
| 5 | 4 or 8 |
| 6 | 5 |
| 7 | 2 or 8 |
| 8 | 5 |
| 9 | 4 |
| 10 | 4 |
| 11 | 6 |
| 12 | 4 |
| 13 | 2 |
| 14 | 4 |
| 15 | 5 or 7 |
| 16 | 6 |
| 17 | 4 |
| 18 | 5 |

Table 5.15: Number of Occurrences on each iteration

| Best Number of VNS Iterations | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Occurrences | 3 | 1 | 7 | 5 | 3 | 1 | 3 |

The criteria to select the most suitable number of iterations was the same as the one used for the percentage of best population for VNS. Therefore, for the case in figure 5.6, in instance 1, two iterations is the most suitable number. Table 5.15 indicates that most suitable number of iterations is four (with 7 occurrences).

The remaining plots for instances 4 to 18 can be consulted in appendix A.2.2.

### 5.6 Dynamic Termination Criteria

To understand whether the Dynamic Termination Criteria presented fitter results, a study was conducted in the medium instances (10 jobs) since, in these, for a computational time of 15 minutes one can obtain converged results that went through several iterations before reaching the final values. For the smaller instances the algorithm converges too fast not allowing for a proper comparison. For the instances of 70 jobs it takes much more time. The results were preformed with a total population of 400 individuals. The results are presented in 5.16.

Table 5.16: Termination Criteria Comparison

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | Termination Criteria | No Termination Criteria |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 2 | 0.3 | 0 | $\begin{gathered} 91.5 \\ 191.4 \\ 53.9 \\ 121.6 \end{gathered}$ | 93.7 |
|  |  |  | 1 |  | 193.2 |
|  |  |  | 2 |  | 56 |
|  |  | 0.6 | 0 |  | 122.8 |
|  |  |  | 1 | 107.9 | 107.2 |
|  |  |  | 2 | 130.6 | 131.7 |
|  | 3 | 0.3 | 0 | 108.4 | 108.1 |
|  |  |  | 1 | 123.1 | 123.1 |
|  |  |  | 2 | 96.3 | 96.4 |
|  |  | 0.6 | 0 | 60.7 | 60.8 |
|  |  |  | 1 | 95.4 | 96.7 |
|  |  |  | 2 | 72.9 | 72.9 |
|  | 7 | 0.3 | 0 | 103.0 | 103.4 |
|  |  |  | 1 | 120.6 | 120.1 |
|  |  |  | 2 | 93.1 | 93.0 |
|  |  | 0.6 | 0 | 105.1 | 104.9 |
|  |  |  | 1 | 99.9 | 102 |
|  |  |  | 2 | 89.6 | 89.6 |
| Total |  |  |  | 1865 | 1876 |

From table 5.16, as expected, the Termination Criteria proves fitter results for the total completion time and will therefore be used in the final results.

The following figure 5.8 depicts a case where, due to the Dynamic termination Criteria, the algorithm was able to be improved in the last iterations of the study. Also in figure 5.9, the convergence plots of the algorithm while not using the Dynamic Termination Criteria is shown:


Figure 5.8: Convergence plot of Instance 2 with Termination Criteria


Figure 5.9: Convergence plot of Instance 2 with no Termination Criteria

### 5.7 Final Results Comparison

The following table 5.18 contains the total completion time (in hours) obtained by the implemented GA with and without VNS, alongside with benchmark results from the work completed by Martins et al. [2]. Highlighted in green and yellow are the best and second best results for each instance, respectively. The parameters used for these results were the ones obtained from the tuning operations ( 5.2 to 5.5 ) and can be found in table 5.17.

Table 5.17: Parameters used in the final results

| Population Size | 400 |
| :---: | :---: |
| Initial Population Weights | $\mathrm{W} 1=100\|\mathrm{~W} 2=0\| \mathrm{W} 3=0$ |
| Incremental Initialization? | Yes |
| Elitism (\%) | 5 |
| Elite Start | $\frac{1}{4}$ Population size |
| Percentage of offspring (\%) | 70 |
| Crossover Weights | $\mathrm{C} 1=10\|\mathrm{C} 2=5\| \mathrm{C} 3=85$ |
| Mutation Percentage (\%) | 30 |
| Termination Criteria | 250 Iterations or 3h (big instances) |

From table 5.18 it is proven that both Genetic Algorithms surpass the Branch and Cut algorithm for large instances with the simple GA surpassing with a margin of $57 \%$. Between both GA's, the one without VNS proves fitter results for medium and large sized instances by a margin of $0.6 \%$ and $0.4 \%$ respectively. For small instances, the GA with VNS obtains fitter results by $0.1 \%$ when compared to the

Table 5.18: Final Results comparison

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | GA | GA + VNS | B\&C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 2 | 0.3 | 0 | 41.3 | 41.3 | 41.3 |
|  |  |  | 1 | 54.9 | 55.1 | 54.0 |
|  |  |  | 2 | 23.6 | 23.6 | 23.6 |
|  |  | 0.6 | 0 | 42.3 | 41.8 | 41.7 |
|  |  |  | 1 | 45.8 | 45.5 | 44.3 |
|  |  |  | 2 | 23.6 | 23.6 | 23.3 |
|  | 3 | 0.3 | 0 | 70.2 | 70.2 | 67.7 |
|  |  |  | 1 | 34.3 | 34.3 | 33.3 |
|  |  |  | 2 | 43.3 | 43.3 | 40.6 |
|  |  | 0.6 | 0 | 27.4 | 27.4 | 27.4 |
|  |  |  | 1 | 26.8 | 26.8 | 26.7 |
|  |  |  | 2 | 33.0 | 33.0 | 32.3 |
|  | 7 | 0.3 | 0 | 29.7 | 29.7 | 26.7 |
|  |  |  | 1 | 46.4 | 46.4 | 46.4 |
|  |  |  | 2 | 24.1 | 24.1 | 21.4 |
|  |  | 0.6 | 0 | 26.1 | 26.0 | 26.0 |
|  |  |  | 1 | 42.1 | 42.1 | 42.1 |
|  |  |  | 2 | 27.0 | 27.0 | 27.0 |
| 10 | 2 | 0.3 | 0 | 92.1 | 91.2 | 89.8 |
|  |  |  | 1 | 188.8 | 187.8 | 189.5 |
|  |  |  | 2 | 54.7 | 55.4 | 54.4 |
|  |  | 0.6 | 0 | 120.7 | 120.1 | 120.3 |
|  |  |  | 1 | 108.4 | 113.3 | 111.3 |
|  |  |  | 2 | 130.2 | 130.1 | 127.0 |
|  | 3 | 0.3 | 0 | 108.2 | 108.2 | 106.3 |
|  |  |  | 1 | 123.4 | 122.5 | 114.3 |
|  |  |  | 2 | 96.4 | 96 | 94.8 |
|  |  | 0.6 | 0 | 61.3 | 60.7 | 60.2 |
|  |  |  | 1 | 90.4 | 92.9 | 105.2 |
|  |  |  | 2 | 73.9 | 75.2 | 76.0 |
|  | 7 | 0.3 | 0 | 103 | 105.1 | 113.4 |
|  |  |  | 1 | 119.6 | 120 | 119.9 |
|  |  |  | 2 | 93.1 | 93.8 | 87.1 |
|  |  | 0.6 | 0 | 102.6 | 106.5 | 129.9 |
|  |  |  | 1 | 101.6 | 98.7 | 141.2 |
|  |  |  | 2 | 84.4 | 85.6 | 99.9 |
| 70 | 2 | 0.3 | 0 | 3526.9 | 3656.6 | 8149.5 |
|  |  |  | 1 | 5522.8 | 5544.4 | 9910.8 |
|  |  |  | 2 | 3539.5 | 3509.2 | - |
|  |  | 0.6 | 0 | 3744 | 3560.8 | 10172.7 |
|  |  |  | 1 | 3888.7 | 3887.9 | 11286.7 |
|  |  |  | 2 | 3906.4 | 4366.4 | 9030.0 |
|  | 3 | 0.3 | 0 | 3304.5 | 3366.4 | 8541.5 |
|  |  |  | 1 | 6401.2 | 6431.8 | - |
|  |  |  | 2 | 4077.1 | 4102.9 | 8423.6 |
|  |  | 0.6 | 0 | 2502.6 | 2466.8 | 5829.2 |
|  |  |  | 1 | 2828.1 | 2776.8 | 7738.7 |
|  |  |  | 2 | 1795.3 | 1827 | 3573.9 |
|  | 7 | 0.3 | 0 | 3111.2 | 3108.9 | 8255.7 |
|  |  |  | 1 | 4249.6 | 4178.9 | 9559.3 |
|  |  |  | 2 | 2551.2 | 2563.4 | 6130.3 |
|  |  | 0.6 | 0 | 3772.1 | 3724 | - |
|  |  |  | 1 | 1776.6 | 1811.2 | - |
|  |  |  | 2 | 3069.7 | 2926.5 | - |
| Total |  |  |  | 66082.2 | 66334.2 | - |

Table 5.20: Encoding table of the final solution for instance 1

| Job | 2.1 | 2.2 | 2.3 | 3.1 | 5.1 | 5.2 | 4.1 | 1.1 | 1.2 | 1.3 | 4.2 | 5.3 | 3.2 | 4.3 | 3.3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine | 1 | 6 | 7 | 4 | 4 | 6 | 3 | 7 | 3 | 1 | 5 | 5 | 4 | 7 | 7 |
| Worker | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 2 | 1 |

simple GA. For small instances the Branch and Cut algorithm surpasses both GA's (by a margin of 2.3\% to the $G A+V N S)$. For medium instances the BC presents better results when the number of workers is two, but as the instance size increases towards seven workers it looses it's advantage. Therefore, for medium instances the GA performs better than the BC by a margin of $0.6 \%$ and the GA + VNS obtains the same overall results as the BC. The following table 5.19 outlines these comparisons:

Table 5.19: Comparison between algorithms for the 3 types of Instances

| Instance Type | Best Algorithm | Second Best Algorithm | Worst Algorithm |
| :--- | :---: | :---: | :---: |
| Small (5 Jobs) | B\&C | GA + VNS | GA |
| Medium (10 Jobs) | GA | GA + VNS \& B\&C | - |
| Large (70 Jobs) | GA | GA + VNS | B\&C |

## Representation of the Solution

In order to represent the solution a Gantt Chart for both workers and machines is computed. For the solution of the first instance in table 5.18 with a total completion time of 41.3 hours (and a makespan of 12.6 hours), it's Gantt Charts are represented in figures 5.10 and 5.11 , to which corresponds the encoding table 5.20.


Figure 5.10: Worker Gantt Chart of first instance of the small instances


Figure 5.11: Machine Gantt Chart of first instance of the small instances

## 6

## Conclusions

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### 6.1 Conclusions

The objective of this thesis was to develop tools capable of creating a real life-size schedule for a Quality Control laboratory environment using a meta-heuristic algorithm. To achieve this objective, a hybrid Genetic algorithm was implemented to minimize the total completion time. This algorithm was designed specifically for the problem at hand and introduced novel approaches to former studies in literature, even when considering to the more general manufacturing scheduling. Compatibility tables were successfully used in the context of laboratory scheduling to inform upon the availability of resources (machines and workers).

The Initial Population had a system of weights which corresponded to probabilities of performing a certain path in the assignment of the genes. The random Initialization $\left(w_{1}\right)$ proved the best results overall. Additionally, the prohibition conditions proved fitter results regardless the value used in each of them. The fitter results were obtained while restricting the number of workers in a row to 4 and machines to 3. The Incremental Initial Population with worker selection also proved to improve the algorithm's convergence and overall results. Regarding the Crossover, unlike the initial population, the fitter results in this phase were found with a combination of the three weights: $c_{1}=10, c_{2}=5$ and $c_{3}=85$. The general GA parameters were studied and a tuning was performed for the total population size, the total percentage of offspring and percentage of mutation. The Variable Neighborhood Search proved to be best when the percentage of the best percentage of population chosen to perform VNS was $25 \%$ and when a total of four iterations were performed.

The GA with VNS proved fitter results when compared to the simple GA only for the small instances with five jobs but could not match with the optimal values from the Branch and Cut algorithm. The final results and comparison with bench mark tests was performed and the implemented algorithm proved fitter results for the GA without Variable Neighborhood search for large and medium instances. Since the objective of this dissertation was to create a real life-size schedule for a Quality Control laboratory, the 70 job instances are the ones mimicking this dimension. The Genetic Algorithm (GA) in this study improved the results from previous studies by a total of $57 \%$. Therefore, the objectives of this dissertation were met and a novel implementation was successfully introduced.

### 6.2 Future Work

The scheduling of resources extends further than the dual resource constrained flexible job shop problem. As introduced in the literature review in chapter 1.2.5, there are other factors that can be considered. For example, workers do not all perform with the same level of productivity, resource constraints are often not limited to workers and machines. Furthermore, in some cases it may be useful to consider other objective functions and optimization criteria, even simultaneously. Therefore, as for future work,
this thesis could be extended to a multi-resource or multi-objective scheduling problem. It is worth mentioning that the algorithm is also prepared to minimize the makespan which can be particularly useful if one decides to migrate to multi-objective scheduling. Heterogeneous workers may also be considered in future work and learning curves implemented in the study.

Due to unpredictable events, there may be shortage of supply or an urgent order in need to be attended. Therefore, dynamic scheduling is often of high importance in scheduling problems which could be a valuable extension to this thesis.

Furthermore, the optimal tuning of parameters in a genetic algorithm is often complicated to achieve since parameters are not mutually exclusive and often depend on each other. A further study could be performed in order to understand how these parameters could be better tuned and access which parameters have higher correlation between each other. Perhaps a cross-tuning of the parameters could be studied in future work, or a tuning using Bayesian optimization.

The study of the quality control laboratory scheduling is still very recent. This is known to be the first study in this environment to make use of a meta-heuristic algorithm. Further meta-heuristics could be developed and compared to the one in this work and possibly find which meta-heuristic provides fitter results to the problem at hand.

Also, it could be of interest to study how the current Genetic Algorithm could perform faster and more efficient. As an example, parallel computing could be explored in the algorithm in order to reduce the fitness evaluation phase computational time.

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## A

## Tuning

## A. 1 Initial Results - No Tuning

The first results for the Genetic Algorithm A. 1 were obtained with the simpler approaches where only the one point crossover was implemented, no initial population nor crossover calibration and no VNS. A stopping criteria was implemented that fires if after 30 iterations the algorithm has no improvements. Also, the algorithm is limited to run for a maximum of 15 minutes regardless of the number of iterations it performs.

Table A.1: Initial Results - Not Tuned

| $\begin{gathered} \text { № } \\ \text { Jobs } \end{gathered}$ | Workers | Flex | Rep | GA | Time (s) | Pop | Num It | $\begin{gathered} \text { № of last } \\ \text { non } \\ \text { improving it: } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 2 | 0.3 | 0 | 41.7 | 12.3 | 150 | 40 | 30 |
|  |  |  | 1 | $\begin{aligned} & 55.1 \\ & 23.6 \end{aligned}$ | 12.7 | 150 | 42 | 30 |
|  |  |  | 2 |  | 6.8 | 150 | 35 | 30 |
|  |  | 0.6 | 0 | 42.6 | 13.6 | 150 | 43 | 30 |
|  |  |  | 1 | $\begin{aligned} & 45.9 \\ & 23.6 \end{aligned}$ | 12.7 | 150 | 42 | 30 |
|  |  |  | 2 |  | 5.5 | 150 | 31 | 30 |
|  | 3 | 0.3 | 0 | 70.234.3 | 12.7 | 150 | 35 | 30 |
|  |  |  | 1 |  | 10.1 | 150 | 39 | 30 |
|  |  |  | 2 | 43.3 | 9.4 | 150 | 33 | 30 |
|  |  | 0.6 | 0 | 27.426.8 | 7.7 | 150 | 38 | 30 |
|  |  |  | 1 |  | 8.7 | 150 | 40 | 30 |
|  |  |  | 2 | 33 | 10.2 | 150 | 39 | 30 |
|  | 7 | 0.3 | 0 | 29.7 | 6.6 | 150 | 32 | 30 |
|  |  |  | 1 |  | 9.6 | 150 | 32 | 30 |
|  |  |  | 2 | 24.1 | 8.2 | 150 | 39 | 30 |
|  |  | 0.6 | 0 | $\begin{gathered} 26.1 \\ 42.1 \\ 27 \end{gathered}$ | 8.2 | 150 | 40 | 30 |
|  |  |  | 1 |  | 10.6 | 150 | 39 | 30 |
|  |  |  | 2 |  | 6.8 | 150 | 30 | 30 |
| 10 | 2 | 0.3 | 0 | 91.6 | 32.2 | 150 | 64 | 30 |
|  |  |  | 1 | $\begin{gathered} 192.4 \\ 56.7 \end{gathered}$ | 70.1 | 150 | 83 | 30 |
|  |  |  | 2 |  | 27.2 | 150 | 68 | 30 |
|  |  | 0.6 | 0 | $\begin{gathered} 121.3 \\ 108 \\ 130.9 \end{gathered}$ | 56.7 | 150 | 64 | 30 |
|  |  |  | 1 |  | 56.8 | 150 | 67 | 30 |
|  |  |  | 2 |  | 50.3 | 150 | 57 | 30 |
|  | 3 | 0.3 | 0 | $\begin{aligned} & 108.4 \\ & 123.8 \end{aligned}$ | 28.1 | 150 | 42 | 30 |
|  |  |  | 1 |  | 47.1 | 150 | 57 | 30 |
|  |  |  | 2 | $\begin{gathered} 96.5 \\ 63 \end{gathered}$ | 25.4 | 150 | 43 | 30 |
|  |  | 0.6 | 0 |  | 23.8 | 150 | 59 | 30 |
|  |  |  | 1 | $\begin{aligned} & 92.9 \\ & 76.8 \end{aligned}$ | 25.6 | 150 | 42 | 30 |
|  |  |  | 2 |  | 32.5 | 150 | 68 | 30 |
|  | 7 | 0.3 | 0 | 103.9124.3 | 23.3 | 150 | 37 | 30 |
|  |  |  | 1 |  | 22.8 | 150 | 39 | 30 |
|  |  |  | 2 | $\begin{gathered} 95.3 \\ 111.2 \end{gathered}$ | 20.2 | 150 | 36 | 30 |
|  |  | 0.6 | 0 |  | 45.8 | 150 | 71 | 30 |
|  |  |  | 1 | 101.1 | 42.6 | 150 | 67 | 30 |
|  |  |  | 2 | 88.6 | 35.3 | 150 | 69 | 30 |
| 70 | 2 | 0.3 | 0 |  | 907.4 | 150 | 72 | 2 |
|  |  |  | 1 | 5957.2 | 903.3 | 150 | 38 | 0 |
|  |  |  | 2 | 3671.7 | 905.9 | 150 | 101 | 0 |
|  |  |  | 0 | 4152.7 | 904.5 | 150 | 52 | 1 |
|  |  | 0.6 | 1 | 4462.6 | 903.2 | 150 | 53 | 2 |
|  |  |  | 2 | 4501.6 | 911.7 | 150 | 53 | 0 |
|  | 3 | 0.3 | 0 | 3446.3 | 907.2 | 150 | 68 | 3 |
|  |  |  | 1 | 7094.6 | 907.6 | 150 | 44 | 0 |
|  |  |  | 2 | 4414.3 | 911.6 | 150 | 79 | 1 |
|  |  | 0.6 | 0 | 2766 | 906.8 | 150 | 92 | 1 |
|  |  |  | 1 | 3022.9 | 900.8 | 150 | 88 | 11 |
|  |  |  | 2 | 1893.5 | 900.6 | 150 | 138 | 1 |
|  | 7 | 0.3 | 0 | 3205 | 905.9 | 150 | 77 | 2 |
|  |  |  | 1 | 4507.3 | 921.3 | 150 | 54 | 1 |
|  |  |  | 2 | 2861.9 | 903.7 | 150 | 96 | 6 |
|  |  | 0.6 | 0 | 4301.5 | 907.7 | 150 | 55 | 1 |
|  |  |  | 1 | 2039.1 | 901.2 | 150 | 66 | 2 |
|  |  |  | 2 | 3325.4 | 914.4 | 150 | 41 | 4 |
| Total |  |  |  | 72085.3 |  |  |  |  |

It can be noticed that for the smaller instances of 5 and 10 jobs the algorithm reaches some kind of optimum. For the largest instance of 70 jobs the algorithm is far from converging into one.

## A. 2 VNS Tuning

## A.2.1 Percentage of best Population



Figure A.1: VNS Tuning Instances 4-6


Figure A.2: VNS Tuning Instances 7-9


Figure A.3: VNS Tuning Instances 10-12


Figure A.4: VNS Tuning Instances 13-15


Figure A.5: VNS Tuning Instances 16-18

## A.2.2 Number of Iterations

Tuning of the number of VNS Iterations


Figure A.6: VNS Iteration Tuning Instances 4-6


Figure A.7: VNS Tuning Instances 7-9

Tuning of the number of VNS Iterations


Figure A.8: VNS Iteration Tuning Instances 10-12

## Tuning of the number of VNS Iterations



Figure A.9: VNS Iteration Tuning Instances 13-15

Tuning of the number of VNS Iterations


Figure A.10: VNS Iteration Tuning Instances 16-18

## Prohibition Condition

Table B.1: Prohibition Condition values with counting part 1

| (No | Workers | Flex | Rep | $\begin{aligned} & \mathrm{GA} \\ & \mathrm{~W}=3 \\ & \mathrm{M}=3 \end{aligned}$ | Worker Count | Machine Count | Total Count | $\begin{gathered} \begin{array}{c} \text { AA } \\ W=3 \\ M=2 \end{array} \end{gathered}$ | Worker Count | Machine Count | Total Count | $\begin{aligned} & \mathrm{GA} \\ & \mathrm{~W}=3 \\ & \mathrm{M}=4 \end{aligned}$ | Worker Count | Machine Count | Total Count | $\begin{aligned} & \mathrm{GA} \\ & \mathrm{~W}=3 \\ & \mathrm{M}=6 \end{aligned}$ | Worker Count | Machine Count | Total Count |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | 0.3 | 0 | 5001 | 5118 | 1140 | 6258 | 4985 | 5123 | 4049 | 9172 | 5007 | 5113 | 446 | 5559 | 5005 | 5140 | 52 | 5192 |
|  |  |  | 1 | 7356 | 0 | ${ }_{8}^{1271}$ | ${ }_{8}^{1271}$ | 7316 | 0 | 5666 3419 | $\frac{5666}{3419}$ | ${ }^{7384}$ | 0 | ${ }^{367}$ | 367 | 7378 | 0 | ${ }^{27}$ | $\stackrel{27}{16}$ |
|  |  |  | ${ }_{0}^{2}$ | 4544 5134 | $\stackrel{0}{5537}$ | 816 1029 | 816 <br> 666 | 4554 5138 | $\stackrel{0}{5567}$ | 3419 6676 | ${ }^{3419} 12243$ | 4541 5134 | 56 | ${ }_{177}^{221}$ | ${ }_{5}^{221}$ | 4543 5149 | ${ }_{5} 5$ | 16 | $\stackrel{16}{5454}$ |
|  |  | 0.6 | 1 | 5455 | 9681 | 1496 | 11177 | 5441 | 9609 | ${ }^{8936}$ | 18545 | 5449 | 9684 | 286 | 9970 | 5450 | 9645 | 8 | ${ }_{9653}$ |
|  |  |  | 2 | 5648 | 2341 | 1507 | 3848 | 5642 | 2513 | 9645 | 12158 | 5650 | 2525 | 279 | 2804 | 5648 | 2399 | 11 | 2410 |
|  | 3 | 0.3 | 0 | 4548 | 0 | 1858 | 1858 | $\begin{aligned} & 4521 \\ & 12096 \\ & 6134 \end{aligned}$ | 0 | 6476 | 6476 | 4593 | 0 | 613 | 613 | 4585 | 0 | 111 | 171 |
|  |  |  | 1 | 12497 | 5087 | 4804 | 989 |  | 5026 | 10535 | 15561 | ${ }^{12783}$ | 5125 | 2485 | 7610 | 12980 | 4997 | 688 | 5685 |
|  |  |  | 2 | 6172 | 1252 | 1196 | 2448 |  | 1253 | 4843 | 6096 | 6198 | 1266 | 345 | 1611 | 6204 | 1136 | 28 | 1164 |
|  |  | 0.6 | 0 | 3679 | 4206 | 1452 | 5658 | 3647 | 4250 | 8783 | ${ }^{13033}$ | 3673 | 4183 | 295 | 4478 | 3674 | 4093 | 8 | 4101 |
|  |  |  | 1 | 3854 | 2204 | 1158 | 3362 | 3850 | 2118 | 7216 | 9334 | 3876 | 2133 | 208 | 2341 | 3870 | 2106 | 6 | 2112 |
|  |  |  | 2 | 2701 | 2678 | 1398 | 4076 | 2686 | 2705 | 7268 | 9973 | 2704 | 2668 | 266 | 2934 | 2729 | 2642 | 14 | 2656 |
|  | 7 | 0.3 | 0 | 4343 | 888 | 810 | 1698 | 4310 | 873 | 3805 | 4678 | 4369 | 895 | 200 | 1095 | 4374 | 842 | ${ }^{23}$ | 865 |
|  |  |  | 1 | 6606 | 2386 | 1389 | 3775 | 6502 | 2404 | 6594 | 8998 | 6609 | 2532 | 330 | 2862 | 6598 | 2487 | 30 | 2517 |
|  |  |  | ${ }^{2}$ | ${ }_{5}^{3585}$ | +1219 | ${ }_{203} 80$ | $\frac{2022}{3442}$ | 3565 5588 | 1300 <br> 1518 | 3533 <br> 10870 | ${ }^{4833}$ | ${ }_{5645}^{3615}$ | 1217 1411 | ${ }_{433}^{226}$ | $\stackrel{1443}{1844}$ | 3610 5667 | 1233 <br> 1387 | ${ }_{18}^{18}$ | $\begin{array}{r}1251 \\ \hline 1403 \\ \hline\end{array}$ |
|  |  | 0.6 | 0 | ${ }_{2628}^{5836}$ | ${ }^{1435}$ | ${ }_{1241}$ | 3442 <br> 3563 | 5558 2816 | 1518 2190 | 10870 8727 | ${ }^{12388} 10917$ | ${ }_{2649} 283$ | ${ }_{2181}^{141}$ | ${ }_{233} 23$ | 1844 2415 | ${ }_{2831}^{5667}$ | $\stackrel{1387}{2128}$ | ${ }_{10}^{16}$ | $\stackrel{1403}{2138}$ |
|  |  |  |  | 4538 | 1688 | 1683 | ${ }_{3371}$ | 4512 | 1620 | 9628 | 11248 | 4529 | 1600 | 317 | 1917 | 4553 | 1622 | 15 | ${ }^{1637}$ |
|  |  |  |  | 5229 |  |  |  | 5182 |  |  |  | 5255 |  |  |  | 5269 |  |  |  |

Table B.2: Prohibition Condition values tuning part 2

| (No | rikers | Flex | Rep | $\begin{aligned} & \text { GA } \\ & \text { W=4 } \\ & =3 \end{aligned}$ | Worke Count | $\begin{gathered} \text { Machine } \\ \text { Count } \end{gathered}$ | ${ }_{\text {count }}^{\substack{\text { Total } \\ \text { count }}}$ | $\begin{aligned} & \text { GA }=4 \\ & W=4 \\ & M=4 \end{aligned}$ | Worker Count | $\begin{gathered} \text { Machine } \\ \text { Count } \end{gathered}$ | Total Count | $\begin{aligned} & \text { GA } \\ & =4 \\ & M=5 \end{aligned}$ | Worker Count | $\begin{array}{\|c} \hline \text { Machine } \\ \text { Count } \end{array}$ | Total Count | $\begin{gathered} \text { GA } \\ W=5 \\ M=5 \end{gathered}$ | Worke Count | Machine | ${ }_{\text {count }}^{\text {Total }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 2 | ${ }^{0.3}$ | $\stackrel{1}{1}$ | - | $\frac{4012}{0}$ | ${ }_{\substack{1159 \\ 1343}}^{14}$ | ${ }_{\text {5171 }}^{1343}$ | $\underset{\substack{5056 \\ 7381}}{\text { cier }}$ | ${ }_{4052}^{4}$ | ${ }_{\substack{400 \\ 332}}$ | ${ }_{3452}^{4352}$ | $\underset{7384}{\substack{5057 \\ \hline 134}}$ | $\frac{3918}{0}$ | ${ }^{146}$ | $\stackrel{4064}{98}$ | $\underset{\substack{5112 \\ 7392}}{ }$ | ${ }_{3226}^{326}$ | ${ }^{170}$ | ${ }_{3}^{3396}$ |
|  |  |  |  | ${ }_{4537}$ |  | ${ }^{159}$ |  | ${ }_{4546}$ |  |  |  | 4548 |  |  |  |  |  | ${ }_{54}$ |  |
|  |  |  | 0 | ${ }_{5}^{5141}$ | ${ }^{3134}$ | ${ }_{1054}^{1044}$ | ${ }_{6}^{4188}$ | 5449 5599 | 3062 | ${ }^{194}$ | ${ }^{3256}$ | 5143 <br> 559 | ${ }_{\substack{3131 \\ 5322}}$ | ${ }_{4}^{34}$ | ${ }^{3165}$ | ${ }_{5163}^{555}$ | ${ }_{\substack{1846 \\ 3022}}^{1}$ | 33 <br> 39 <br> 8 | ${ }_{\text {1879 }}^{188}$ |
|  |  | 0.6 | $\frac{1}{2}$ | ${ }_{5637}^{5533}$ | 506 <br> 1299 <br> 129 | ${ }_{1}^{14478}$ | ${ }_{2033}^{6393}$ | ${ }_{5659}^{565}$ |  | ${ }_{263}$ | ${ }^{5150}$ | ${ }_{5629}$ | - | ${ }_{5}^{4}$ | ${ }^{54036} 1$ |  | ${ }_{\text {\% }}^{652}$ | ${ }_{53}^{39}$ | \% 3241 |
|  |  |  | ${ }^{\circ}$ | 4546 |  | ${ }_{1884}$ | ${ }_{1884} 88$ | ${ }_{4}^{4588}$ |  |  | 669 | 4570 |  | ${ }_{230}$ | ${ }^{230}$ | ${ }^{4594}$ |  |  | ${ }^{282}$ |
|  | 3 | 0.3 | 1 | ${ }_{1}^{12552}$ | ${ }^{2447}$ | ${ }_{4831}$ | ${ }^{7278}$ | ${ }^{12720}$ | ${ }^{2456}$ | +2435 | 4891 | ${ }^{12931}$ | ${ }^{2399}$ | ${ }^{\text {343 }}$ | ${ }^{3742}$ | ${ }^{12905}$ | ${ }^{1293}$ | ${ }^{1368}$ | ${ }^{2661}$ |
|  |  |  | ${ }_{2}$ | 6172 | ${ }^{543}$ | ${ }^{1244}$ | ${ }_{1787}^{1887}$ |  | ${ }_{492}^{403}$ |  | ${ }^{827}$ | - 6206 |  | ${ }_{96}^{98}$ | ${ }^{594}$ | ${ }^{6201}$ |  |  |  |
|  |  | ${ }^{0.6}$ | 0 | ${ }_{\substack{3676 \\ 3871}}$ | ${ }_{\text {1431 }}^{183}$ | 149 | ${ }_{2027}^{2250}$ | ${ }_{3683}^{3683}$ | - ${ }^{1403}$ | - | ${ }_{1091}^{1090}$ | ${ }_{3}^{3680}$ | +1432 | ${ }_{\substack{56 \\ 32}}$ | ${ }_{\text {1488 }}^{198}$ | ${ }^{3680}$ | 年541 <br> 380 | - ${ }^{53}$ | cos |
|  |  |  | 2 | ${ }_{2773}$ | ${ }^{937}$ | ${ }_{1249}$ | ${ }_{2}^{2386}$ | ${ }^{2777}$ | ${ }_{954}^{954}$ | ${ }_{302}$ | ${ }^{1256}$ | ${ }_{2733}$ | ${ }^{991}$ | ${ }_{60}$ | ${ }_{1031}$ | ${ }_{2710}$ | ${ }_{334}^{336}$ |  | ${ }_{420}^{440}$ |
|  | 7 | 0.3 | 0 | ${ }^{4358}$ | ${ }^{251}$ | 815 | ${ }^{1066}$ | ${ }^{4387}$ | ${ }^{255}$ | ${ }^{235}$ | 490 | 4359 | ${ }^{286}$ | ${ }^{80}$ | ${ }^{366}$ | 4364 | 79 | ${ }^{67}$ | ${ }^{146}$ |
|  |  |  | $\frac{1}{2}$ | ${ }^{6597}$ | ${ }^{\text {7388 }}$ | -1422 | ${ }^{2160}$ | 6579 | 668 <br> 68 <br> 29 | 317 <br> 318 <br> 188 | -985 | -6610 |  | 92 <br> 8 | - ${ }_{\text {856 }}^{344}$ | - 6594 | 186 <br> 18 | \% ${ }^{83}$ | ${ }^{269}$ |
|  |  |  | ${ }^{\circ}$ | ${ }_{5629}$ | ${ }^{262}$ | ${ }_{1037}$ | ${ }_{2199}$ | ${ }_{5663}$ | ${ }_{260}^{260}$ | ${ }_{379}$ | ${ }_{6} 639$ | 5660 | ${ }_{262}$ | ${ }_{80}$ | ${ }_{342}^{342}$ | ${ }_{5653}$ | ${ }_{47}$ | ${ }_{70}$ | ${ }_{1}^{128}$ |
|  |  | 0.6 | $\stackrel{1}{2}$ | ${ }^{2837}$ | ${ }_{5}^{534}$ | ${ }^{1429}$ | ${ }_{-1983}^{1988}$ | ${ }^{2847}$ |  | ${ }_{2}^{295}$ |  | ${ }^{2855}$ | ${ }^{548}$ | ${ }^{38}$ | ${ }^{586}$ | ${ }^{2846}$ | 150 <br> 15 | ${ }_{6}^{46}$ | ${ }^{196}$ |
|  | Average |  |  | 4532 |  |  |  | ${ }_{52200}$ | ${ }^{316}$ |  |  | ${ }_{5252}$ | ${ }^{304}$ |  | ${ }^{366}$ | - 5249 |  |  | 19 |

