Scalar Mixing in New Physics Models

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Resumo

A descoberta do bosão de Higgs em 2012 foi um feito importante na física de partículas. Esta partícula escalar é essencial no Modelo Padrão para explicar a massa das outras partículas. No entanto, não existe nada na teoria que restrinja o setor escalar do Modelo Padrão a ter apenas uma partícula física.

Nesta tese, consideramos um modelo geral com um número arbitrário de singletos escalares de $SU(2)$ com hipercargas $Y = 0, 1, 2$; um número arbitrário de dubletos escalares de $SU(2)$ com hipercarga $Y = 1/2$ e um número arbitrário de tripletos escalares de $SU(2)$ com hipercargas $Y = 0, 1$. Os escalares deste modelo podem misturar-se de forma arbitrária.

Para este modelo geral, começamos por calcular os parâmetros oblíquos S, T, U, V, W e X. Encontramos uma prescrição para que os parâmetros oblíquos S e U sejam finitos e mostramos que essa prescrição é válida para um modelo com qualquer conteúdo escalar.

Ainda neste modelo geral, calculamos as correções a um loop ao vértice $Zb\bar{b}$.

Aplicamos então os nossos resultados a um modelo concreto: o modelo de Georgi-Machacek, calculando para este modelo os parâmetros oblíquos e as correções a um loop ao vértice $Zb\bar{b}$.

Finalmente, comparamos os resultados das correções a um loop ao vértice $Zb\bar{b}$ para o modelo de Georgi-Machacek com resultados experimentais e descobrimos que não obtemos maior concordância do que no Modelo Padrão.

Palavras-Chave

Tripletos Escalares, Nova Física, Parâmetros Oblíquos, Vértice $Zb\bar{b}$, Modelo de Georgi-Machacek

Abstract

The discovery of the Higgs boson in 2012 was an important achievement in particle physics. This scalar particle is essential in the Standard Model to explain the mass of the other particles. However, there is nothing in the theory that restricts the scalar sector of the Standard Model to have only one physical particle.

In this thesis, we consider a general model with an arbitrary number of scalar $SU(2)$ singlets with hypercharges $Y = 0, 1, 2$; an arbitrary number of scalar $SU(2)$ doublets with hypercharge $Y = 1/2$ and an arbitrary number of scalar $SU(2)$ triplets with hypercharges $Y = 0, 1$. We let the scalars with the same electric charge mix arbitrarily.

For this general model, we start by computing the oblique parameters S, T, U, V, W and X. We find a prescription for the oblique parameters S and U to be finite and show that this prescription is valid in a model with any scalar content.

Still in this general model, we compute the one-loop corrections to the $Zb\bar{b}$ vertex.

We apply then our results to a concrete model: the Georgi-Machacek model, computing for this model both the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex.

We compare the results for the one-loop corrections to the $Zb\bar{b}$ vertex in the Georgi-Machacek model with experimental results and find that we do not get a better agreement than in the Standard Model.

Keywords

Scalar triplets, New Physics, Oblique parameters, $Zb\bar{b}$ vertex, Georgi-Machacek Model

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Acronyms

2HDM two-Higgs-doublet model

BFB Boundedness From Below

CKM Cabibbo-Kobayashi-Maskawa

GM Georgi-Machacek

H.c. Hermitian Conjugate

LHC Large Hadron Collider

MHDM multi-Higgs-doublet model

NP New Physics

QED Quantum Electrodynamics

QCD Quantum Chromodynamics

SM Standard Model

VEV vacuum expectation value

Chapter 1

Introduction

Richard Feynman once said [1] ”We do not know what the rules of the game are; all we are allowed to do is to watch the playing. Of course, if we watch long enough, we may eventually catch on to a few of the rules. The rules of the game are what we mean by fundamental physics.” In particle physics we are interested in these ”rules of the game” and we search them by studying the basic constituents of nature and their interactions.

The most successful theory in particle physics nowadays is the Standard Model (SM) [2–4]. The SM is a $SU(3) \times SU(2) \times U(1)$ gauge theory that describes every fundamental particle observed until now and the way they interact with each other.

The SM is one of the theories in science with the best predictive power. As an example, the anomalous magnetic moment of the electron was theoretically predicted – using the SM – to be [5] $a_e(\text{theory}) = 1159.652181643(763) \times 10^{-6}$, while it was measured to be [6] $a_e(\text{experiment}) = 1159.65218091(26) \times 10^{-6}$, which means that the theoretical value agrees with the experimental value to 9 significant figures which is a remarkable result. Some of the discoveries that supported the SM were the discovery of neutral currents in 1973 [7,8], the discovery of the charm quark in 1974 [9,10], the discovery of the bottom quark in 1977 [11], the discovery of the W [12,13] and the Z [14,15] bosons in 1983 and the discover of the top quark in 1995 [16,17] and the discovery of the Higgs boson at the CERN Large Hadron Collider (LHC) in 2012 [18,19] which had first been theoretically predicted in 1964 [20,21].

There are, however, some things that the SM cannot explain. Some examples of these are dark matter, the matter-antimatter asymmetry, why are there three generations of fermions or why do the masses of the elementary particles have such distinct values between them.

Having the SM such a big predictive power, the approach of theoretical particle physicists nowadays is not to discard the SM and formulate a new theory only because the SM fails to explain some phenomena. Theoretical particle physicists are trying to improve the SM by adding to it new features that could help explaining what the SM cannot without compromising its admiring results.

The scalar sector of the SM contains only one scalar $SU(2)$ doublet. There is not, however, anything in the theory that requires its scalar sector sector to have only one doublet[1]. As such, theoretical particle physicists have been proposing additions to the scalar sector of the SM. The most common addition to this part of the SM is to add another scalar $SU(2)$ doublet, such that we get a two-Higgs-doublet model (2HDM). A review of this kind of models can be found in [22]. We can also add to the Standard Model more than one doublet, such that we get multi-Higgs-doublet model (MHDM) or also add scalar multiplets with other dimensions. A formalism has been developed to work with models with an arbitrary number of scalar $SU(2)$ singlets and doublets [23–25].

In this thesis, we will extend this formalism such that it can include scalar triplets and will then, for a generic model with an arbitrary number of scalar singlets, doublets and triplets, try to find a prescription for computing the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex. We will then use our results to compute these quantities in the Georgi-Machacek model [27], which is a model which contains one $SU(2)$ doublet with hypercharge $Y = \frac{1}{2}$ and two $SU(2)$ triplets, one of them with hypercharge $Y = 0$ and the other one with hypercharge $Y = 1$.

This thesis is outlined as follows. In chapter 2, we describe custodial symmetry, a feature of the SM that will be important in the rest of the thesis. In chapter 3, we enlarge the formalism presented in [23–25] for it to include also scalar triplets. In chapter 4, we find a prescription for computing the oblique parameters for a general model with an arbitrary number of scalar singlets, doublets and triplets, show that the photon propagator is transverse at one-loop level and make some remarks on the divergent parts of the oblique parameters S and U in models with multiplets of any dimension. In chapter 5, we compute the one-loop corrections to the $Zb\bar{b}$ vertex for a general model with an arbitrary number of scalar singlets, doublets and triplets. In chapter 6, we apply the results of chapters 4 and 5 to compute the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex in the Georgi-Machacek (GM) model. In this chapter, we also make a fit of the results obtained for the one-loop corrections to the $Zb\bar{b}$ vertex to the experimental results. Finally, in chapter 7 we make a conclusion about the work done and the results obtained.

[1] In fact, the theory must have at least one scalar doublet such that the fermions acquire mass.

Chapter 2

Custodial Symmetry

The SM of particle physics describes the behaviour of elementary particles and how they interact with each other through strong, weak and electromagnetic interactions. The part of the SM that describes the strong interaction is called Quantum Chromodynamics (QCD). QCD predicts the existence of quarks and gluons. The quarks are fermions which carry color charge. The gluons are the bosons that mediate the strong interaction between particles that carry color charge. The gluons themselves also carry color charge. The remaining parts of the SM describe the electroweak interactions, which are a unification of the electromagnetic and weak interactions. The electroweak interactions will be the main focus of this thesis. In appendix A, we present a short review of the SM, focusing mainly on its electroweak part.

The scalar potential of the SM can be written as $V = \mu^2 \operatorname{Tr}\big(\Phi^\dagger\Phi\big) + \lambda \operatorname{Tr}\big(\Phi^\dagger\Phi\big)^2$, where

$$\Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^- & \varphi^0 \end{bmatrix}. \tag{2.1}$$

This potential is invariant under a global $SU(2)_L \times SU(2)_R$ symmetry with Φ transforming as $\Phi \to U_L \Phi U_R^\dagger$, where U_L and U_R are matrices of $SU(2)_L$ and $SU(2)_R$, respectively.

The group $SU(2) \times SU(2)$ is isomorphic to $SO(4)$. It can be seen that the scalar potential is invariant under $SO(4)$ as we can write it as

$$\begin{aligned} V =&\mu^2 \left((\operatorname{Re}\varphi^+)^2 + (\operatorname{Im}\varphi^+)^2 + (\operatorname{Re}\varphi^0)^2 + (\operatorname{Im}\varphi^0)^2\right) \\ &+ \lambda \left((\operatorname{Re}\varphi^+)^2 + (\operatorname{Im}\varphi^+)^2 + (\operatorname{Re}\varphi^0)^2 + (\operatorname{Im}\varphi^0)^2\right)^2, \end{aligned} \tag{2.2}$$

which means that it is a function of the square of the norm of the $SO(4)$ quadruplet

$$\begin{pmatrix} \operatorname{Re}\varphi^+ \\ \operatorname{Im}\varphi^+ \\ \operatorname{Re}\varphi^0 \\ \operatorname{Im}\varphi^0 \end{pmatrix}. \tag{2.3}$$

The vacuum expectation value (VEV) of Φ is given by

$$\langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} v & 0 \\ 0 & v \end{bmatrix}. \tag{2.4}$$

Thus, the vacuum is not invariant under the full group $SU(2)_L \times SU(2)_R$. However, as $\langle 0|\Phi|0\rangle$ is proportional to the identity matrix, it preserves a group $SU(2)_V$ corresponding to $U_L = U_R$. This symmetry preserved by the vacuum under the group $SU(2)_V$ is called custodial symmetry [30]. This custodial symmetry is the reason behind the relation $m_W = m_Z c_W$ between the masses of the gauge bosons.

However, this $SU(2)_L \times SU(2)_R$ is not a symmetry of the whole SM Lagrangian. This symmetry is violated by the Yukawa Lagrangian and by the terms involving the weak hypercharge coupling g'. For example, the Yukawa Lagrangian for the quarks breaks this symmetry if we have up- and down-type quarks with different masses and if we have quark mixing (*i.e.*, if $V_{CKM} \neq \mathbb{1}_{3\times 3}$). In fact, we can write $\mathcal{L}_{\text{Yukawa quarks}}$ as

$$\mathcal{L}_{\text{Yukawa quarks}} = -\frac{1}{v}\sum_{j=1}^{3}\sum_{k=1}^{3}(M_d)_{jk}\left((\overline{\mathcal{U}_L}V_{CKM})_j \quad \overline{\mathcal{D}_{Lj}}\right)\mathcal{D}_{Rk}\begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix} \tag{2.5a}$$

$$-\frac{1}{v}\sum_{j=1}^{3}\sum_{k=1}^{3}(M_u)_{jk}\left((\overline{\mathcal{U}_L})_j \quad (\overline{\mathcal{D}_L}V^\dagger_{CKM})_j\right)\mathcal{U}_{Rk}\begin{pmatrix}\varphi^{0*} \\ -\varphi^-\end{pmatrix} + \text{H.c.}, \tag{2.5b}$$

where H.c. stands for Hermitian conjugate. If we had $M_d = M_u \equiv M$ and $V_{CKM} = \mathbb{1}_{3\times 3}$, then we could write

$$\mathcal{L}_{\text{Yukawa quarks}} = -\frac{1}{v}\sum_{j=1}^{3}\sum_{k=1}^{3}M_{jk}\left(\overline{\mathcal{U}_{Lj}} \quad \overline{\mathcal{D}_{Lj}}\right)\begin{pmatrix}\varphi^{0*} & \varphi^+ \\ -\varphi^- & \varphi^0\end{pmatrix}\begin{pmatrix}\mathcal{U}_{Rk} \\ \mathcal{D}_{Rk}\end{pmatrix} + \text{H.c.}. \tag{2.6a}$$

Thus, by transforming the quark fields under $SU(2)_L \times SU(2)_R$ as

$$\begin{pmatrix}\mathcal{U}_{Lj} \\ \mathcal{D}_{Lj}\end{pmatrix} \to U_L\begin{pmatrix}\mathcal{U}_{Lj} \\ \mathcal{D}_{Lj}\end{pmatrix}, \qquad \begin{pmatrix}\mathcal{U}_{Rj} \\ \mathcal{D}_{Rj}\end{pmatrix} \to U_R\begin{pmatrix}\mathcal{U}_{Rj} \\ \mathcal{D}_{Rj}\end{pmatrix}, \tag{2.7}$$

we would get an invariant Yukawa Lagrangian for the quarks. As experience tells us we do not have $M_d = M_u$ and $V_{CKM} = \mathbb{1}_{3\times 3}$, then the Yukawa Lagrangian for the quarks breaks the $SU(2)_L \times SU(2)_R$ symmetry.

Therefore, custodial symmetry is only an approximate symmetry of the SM. That is why the relation $m_W = m_Z c_W$ is only valid at tree-level.

Chapter 3

Formalism

3.1 Field Content

We consider an $SU(2) \times U(1)$ electroweak model in which the scalar sector includes n_d $SU(2)$ doublets with hypercharge $Y = \frac{1}{2}$,

$$\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix}, \quad k = 1, ..., n_d, \tag{3.1}$$

n_{t_1} $SU(2)$ triplets with hypercharge $Y = 1$,

$$\Xi_p = \begin{pmatrix} \xi_p^{++} \\ \xi_p^+ \\ \xi_p^0 \end{pmatrix}, \quad p = 1, ..., n_{t_1}, \tag{3.2}$$

n_{t_0} $SU(2)$ real triplets with hypercharge $Y = 0$,

$$\Lambda_q = \begin{pmatrix} \lambda_q^+ \\ \lambda_q^0 \\ -\lambda_q^- \end{pmatrix}, \quad q = 1, ..., n_{t_0}, \tag{3.3}$$

where λ_0 is a real scalar field, n_{s_1} complex $SU(2)$ singlets with hypercharge $Y = 1$,

$$\chi_j^+, \quad j = 1, ..., n_{s_1}, \tag{3.4}$$

n_{s_0} real $SU(2)$ singlets with hypercharge $Y = 0$,

$$\chi_l^0, \quad l = 1, ..., n_{s_0}, \tag{3.5}$$

and n_{s_2} complex $SU(2)$ singlets with hypercharge $Y = 2$,

$$\chi_r^{++}, \quad r = 1, ..., n_{s_2}. \tag{3.6}$$

Alternatively, we could have considered doublets with hypercharge $Y = -\frac{1}{2}$ instead of doublets with hypercharge $Y = \frac{1}{2}$ and triplets with hypercharge $Y = -1$ instead of triplets with hypercharge $Y = 1$ as the complex conjugate of a representation of $SU(2)$ is equivalent to that representation.

We have then a total of $n_1 = n_d + n_{t_1} + n_{t_0} + n_{s_1}$ complex scalar fields with electric charge $+1$, $n_0 = 2n_d + 2n_{t_1} + n_{t_0} + n_{s_0}$ real scalar fields with electric charge 0 and $n_2 = n_{t_1} + n_{s_2}$ complex scalar fields with electric charge $+2$.

The neutral fields are allowed to have non-zero VEVs, such that

$$\langle 0|\varphi_k^0|0\rangle = \frac{v_k}{\sqrt{2}}, \qquad \langle 0|\xi_p^0|0\rangle = \frac{w_p}{\sqrt{2}}, \tag{3.7a}$$

$$\langle 0|\lambda_q^0|0\rangle = x_q, \qquad \langle 0|\chi_l^0|0\rangle = u_l, \tag{3.7b}$$

where the VEVs v_k and w_p are in general complex and the VEVs x_q and u_l are real. We can then expand the neutral fields around their VEVs as

$$\varphi_k^0 = \frac{1}{\sqrt{2}}(v_k + \varphi_k^{0\prime}), \qquad \xi_p^0 = \frac{1}{\sqrt{2}}(w_p + \xi_p^{0\prime}), \tag{3.8a}$$

$$\lambda_q^0 = x_q + \lambda_q^{0\prime}, \qquad \chi_l^0 = u_l + \chi_l^{0\prime}. \tag{3.8b}$$

If we have in our model a general complex multiplet of isospin T and hypercharge Y, with a VEV $\frac{v}{\sqrt{2}}$ in the component with $T_3 = -Y$ (which is the component with zero electric charge, such that the $U(1)_Q$ symmetry remains unbroken), then the contributions to the masses of the gauge bosons will be given by

$$m_Z^2 = \frac{g^2|v|^2}{c_W^2}Y^2, \qquad m_W^2 = \frac{g^2|v|^2}{2}(T(T+1) - Y^2). \tag{3.9}$$

If we have a real multiplet, as the product of its covariant derivative by the respective conjugate transpose appears in the Lagrangian multiplied by a factor of $\frac{1}{2}$, then its contributions to the masses of the gauge bosons will also be multiplied by a factor of $\frac{1}{2}$. Alternatively, if we define the VEV on the neutral component of the real multiplet to be v (without the factor of $\frac{1}{\sqrt{2}}$), as we did in equation 3.8, then we get contributions to the masses of the gauge bosons with the same form as in equation 3.9.

Thus, in our model, the masses of the $W^\pm$ and Z bosons are given in terms of the VEVs of the scalar fields as

$$m_Z^2 = \frac{g^2}{c_W^2}\Big(\frac{1}{4}v^2 + w^2\Big), \qquad m_W^2 = g^2\Big(\frac{1}{4}v^2 + \frac{1}{2}w^2 + x^2\Big), \tag{3.10}$$

where we defined $v = \sqrt{\sum_{k=1}^{n_d} |v_k|^2}$, $w = \sqrt{\sum_{p=1}^{n_{t_1}} |w_p|^2}$ and $x = \sqrt{\sum_{q=1}^{n_{t_0}} x_q^2}$. We note that the relation $m_W = m_Z \cos\theta_W$ is, in general, no longer verified due to the introduction of triplets in the model.

These scalar fields will mix according to their mass matrices. We call the fields that are eigenstates of the mass matrices with electric charges $+2$, $+1$ and 0, S_c^{++} ($c = 1, ..., n_2$), S_a^+ ($a = 1, ..., n_1$) and S_b^0 ($b = 1, ..., n_0$), respectively. The neutral fields S_b^0 are reals fields. We can then write

$$\varphi_k^+ = \sum_{a=1}^{n_1} (U_1)_{ka} S_a^+, \qquad \chi_j^+ = \sum_{a=1}^{n_1} (U_2)_{ja} S_a^+, \qquad \lambda_q^+ = \sum_{a=1}^{n_1} (U_3)_{qa} S_a^+, \tag{3.11a}$$

$$\xi_p^+ = \sum_{a=1}^{n_1} (U_4)_{pa} S_a^+, \qquad \varphi_k^{0\prime} = \sum_{b=1}^{n_0} (V_1)_{kb} S_b^0, \qquad \xi_p^{0\prime} = \sum_{b=1}^{n_0} (V_2)_{pb} S_b^0, \tag{3.11b}$$

$$\lambda_q^{0\prime} = \sum_{b=1}^{n_0} (R_1)_{qb} S_b^0, \qquad \chi_l^{0\prime} = \sum_{b=1}^{n_0} (R_2)_{lb} S_b^0, \qquad \xi_p^{++} = \sum_{c=1}^{n_2} (T_1)_{pc} S_c^{++}, \tag{3.11c}$$

$$\chi_r^{++} = \sum_{c=1}^{n_2} (T_2)_{rc} S_c^{++}, \tag{3.11d}$$

where the matrices U_1, U_2, U_3, U_4, V_1, V_2, R_1, R_2, T_1 and T_2 have dimensions $n_d \times n_1$, $n_{s_1} \times n_1$, $n_{t_0} \times n_1$, $n_{t_1} \times n_1$, $n_d \times n_0$, $n_{t_1} \times n_0$, $n_{t_0} \times n_0$, $n_n \times n_0$, $n_{t_1} \times n_2$ and $n_{s_2} \times n_2$, respectively. The matrices R_1 and R_2 are real, while the others are complex. The matrix

$$\tilde{U} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} \tag{3.12}$$

is $n_1 \times n_1$ unitary and it diagonalizes the mass matrices of the scalars with charge $+1$. The matrix

$$\tilde{V} = \begin{pmatrix} \mathrm{Re}\, V_1 \\ \mathrm{Im}\, V_1 \\ \mathrm{Re}\, V_2 \\ \mathrm{Im}\, V_2 \\ R_1 \\ R_2 \end{pmatrix} \tag{3.13}$$

is real and is $n_0 \times n_0$ orthogonal. It diagonalizes the mass matrix of the real components of the neutral scalar fields. The matrix

$$\tilde{T} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \tag{3.14}$$

is $n_2 \times n_2$ unitary and it diagonalizes the mass matrices of the scalars with charge $+2$.

Due to the unitarity of the matrix $\tilde{U}$ we can write the relations

$$\sum_{i=1}^{4} U_i^\dagger U_i = \mathbb{1}_{n_1\times n_1}, \qquad U_1 U_1^\dagger = \mathbb{1}_{n_d\times n_d}, \qquad U_2 U_2^\dagger = \mathbb{1}_{n_{s_1}\times n_{s_1}}, \tag{3.15a}$$

$$U_3 U_3^\dagger = \mathbb{1}_{n_{t_0}\times n_{t_0}}, \qquad U_4 U_4^\dagger = \mathbb{1}_{n_{t_1}\times n_{t_1}}, \qquad U_i U_j^\dagger = 0\,\forall\, i\neq j. \tag{3.15b}$$

Similarly, we can write due to the unitarity of the matrix $\tilde{T}$

$$\sum_{i=1}^{2} T_i^\dagger T_i = \mathbb{1}_{n_2\times n_2}, \qquad T_1 T_1^\dagger = \mathbb{1}_{n_{t_1}\times n_{t_1}}, \tag{3.16a}$$

$$T_2 T_2^\dagger = \mathbb{1}_{n_{s_2}\times n_{s_2}}, \qquad T_1 T_2^\dagger = 0. \tag{3.16b}$$

Due to the orthogonality of the matrix $\tilde{V}$ we can write

$$\sum_{i=1}^{2}(\mathrm{Re}\,V_i^T\,\mathrm{Re}\,V_i + \mathrm{Im}\,V_i^T\,\mathrm{Im}\,V_i) + \sum_{i=1}^{2} R_i^T R_i = \mathbb{1}_{n_0\times n_0}, \tag{3.17a}$$

$$\mathrm{Re}\,V_1\,\mathrm{Re}\,V_1^T = \mathrm{Im}\,V_1\,\mathrm{Im}\,V_1^T = \mathbb{1}_{n_d\times n_d}, \tag{3.17b}$$

$$\mathrm{Re}\,V_2\,\mathrm{Re}\,V_2^T = \mathrm{Im}\,V_2\,\mathrm{Im}\,V_2^T = \mathbb{1}_{n_{t_1}\times n_{t_1}}, \tag{3.17c}$$

$$R_1 R_1^T = \mathbb{1}_{n_{t_0}\times n_{t_0}}, \tag{3.17d}$$

$$R_2 R_2^T = \mathbb{1}_{n_n\times n_n}, \tag{3.17e}$$

$$R_1 R_2^T = 0, \tag{3.17f}$$

$$\mathrm{Re}\,V_i\,\mathrm{Im}\,V_j^T = 0\,\forall\, i,j, \tag{3.17g}$$

$$\mathrm{Re}\,V_i R_j^T = 0\,\forall\, i,j, \tag{3.17h}$$

$$\mathrm{Im}\,V_i R_j^T = 0\,\forall\, i,j, \tag{3.17i}$$

$$\mathrm{Re}\,V_1\,\mathrm{Re}\,V_2^T = 0, \tag{3.17j}$$

$$\mathrm{Im}\,V_1\,\mathrm{Im}\,V_2^T = 0. \tag{3.17k}$$

In this theory, where the gauge group $SU(2)\times U(1)$ is broken to $U(1)$, we will have three Goldstone bosons, $G^\pm$ and G^0. We will identify them as $S_1^\pm$ and S_1^0, respectively:

$$S_1^\pm \equiv G^\pm, \qquad S_1^0 \equiv G^0. \tag{3.18}$$

This means that only the $S_a^\pm$ with $a\geq 2$ and the S_b^0 with $b\geq 2$ will be physical particles, as well as the S_c^{++} or S_c^{--} for all values of c. We will denote the mass of the scalars $S_a^\pm$ by m_a, the mass of the scalars S_b^0 by μ_b and the mass of the scalars S_c^{++} by M_c.

3.2 Covariant Derivatives

We can write the covariant derivative for a gauge theory with $SU(2)\times U(1)$ as gauge group as [1]

$$D^\mu = \partial^\mu + ieQA^\mu - i\frac{g}{c_W}(T_3 - Qs_W^2)Z^\mu - ig(W^{\mu+}T_+ + W^{\mu-}T_-). \tag{3.19}$$

Applying it to the doublets we get

$$D_\mu\phi_k = \begin{pmatrix} \partial_\mu\varphi_k^+ + ieA_\mu\varphi_k^+ + ig\frac{(s_W^2-c_W^2)}{2c_W}Z_\mu\varphi_k^+ - i\frac{g}{\sqrt{2}}W_\mu^+\varphi_k^0 \\ \partial_\mu\varphi_k^0 + i\frac{g}{2c_W}Z_\mu\varphi_k^0 - i\frac{g}{\sqrt{2}}W_\mu^-\varphi_k^+ \end{pmatrix}. \tag{3.20}$$

Applying it to the triplets with hypercharge $Y=0$, we get

$$D_\mu\Lambda_q = \begin{pmatrix} \partial_\mu\lambda_q^+ + ieA_\mu\lambda_q^+ - igc_W Z_\mu\lambda_q^+ - igW_\mu^+\lambda_q^0 \\ \partial_\mu\lambda_q^0 + ig(W_\mu^+\lambda_q^- - W_\mu^-\lambda_q^+) \\ -\partial_\mu\lambda_q^- + ieA_\mu\lambda_q^- - igc_W Z_\mu\lambda_q^- - igW_\mu^-\lambda_q^0 \end{pmatrix}. \tag{3.21}$$

Applying it to the triplets with hypercharge $Y=1$, we get

$$D_\mu\Xi_p = \begin{pmatrix} \partial_\mu\xi_p^{++} + 2ieA_\mu\xi_p^{++} + ig\frac{s_W^2-c_W^2}{c_W}Z_\mu\xi_p^{++} - igW^{\mu+}\xi_p^+ \\ \partial_\mu\xi_p^+ + ieA_\mu\xi_p^+ + ig\frac{s_W^2}{c_W}Z^\mu\xi_p^+ - ig(W_\mu^+\xi_p^0 + W_\mu^-\xi_p^{++}) \\ \partial_\mu\xi_p^0 + i\frac{g}{c_W}Z_\mu\xi_p^0 - igW_\mu^-\xi_p^+ \end{pmatrix}. \tag{3.22}$$

Applying it to the singlets with charge $+1$, we get

$$D_\mu\chi_j^+ = \partial_\mu\chi_j^+ + ieA_\mu\chi_j^+ + ig\frac{s_W^2}{c_W}Z_\mu\chi_j^+. \tag{3.23}$$

Applying it to the singlets with charge $+2$, we get

$$D_\mu\chi_r^{++} = \partial_\mu\chi_r^{++} + 2ieA_\mu\chi_r^{++} + 2ig\frac{s_W^2}{c_W}Z_\mu\chi_r^{++}. \tag{3.24}$$

3.3 Goldstone Bosons

The gauge group of this model is $SU(2)\times U(1)$, which has 4 generators. These 4 generators can be written as Q, T_3, T_- and T_+. The VEVs of the neutral fields break this symmetry to a $U(1)$ symmetry generated by Q. This means that when we apply the operator Q to the vacuum it gives 0, such that an element of the group, which has the form $e^{i\theta Q}$, leaves the vacuum invariant. In the case of the other

[1] Here and in the rest of the thesis, we will use the sign conventions of [28] which correspond to setting $\eta_Z = 1$, $\eta = -1$ and $\eta_e = 1$ in [29].

three generators, which are the generators of the broken symmetry, their action on the vacuum will give the Goldstone bosons. In fact, if T is a generator of a group, then $e^{i\theta T}$ is an element of that group. If θ is an infinitesimal parameter we can write $e^{i\theta T} \simeq 1 + i\theta T$. Therefore, acting with this element of the group on the vacuum will give us the vacuum plus an additional term that will correspond to the Goldstone boson.

Applying $i\theta T_3$ to the vacuum gives

$$i\theta T_3 \begin{pmatrix} 0 \\ \frac{v_k}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -i\theta \frac{v_k}{2\sqrt{2}} \end{pmatrix} \Rightarrow (V_1)_{k1} = i\frac{A}{2\sqrt{2}} v_k, \tag{3.25a}$$

$$i\theta T_3 \begin{pmatrix} 0 \\ 0 \\ \frac{w_p}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -i\theta \frac{w_p}{\sqrt{2}} \end{pmatrix} \Rightarrow (V_2)_{p1} = i\frac{A}{\sqrt{2}} w_p, \tag{3.25b}$$

$$i\theta T_3 \begin{pmatrix} 0 \\ x_q \\ 0 \end{pmatrix} = 0 \Rightarrow (R_1)_{q1} = 0, \tag{3.25c}$$

$$i\theta T_3 u_l = 0 \Rightarrow (R_2)_{l1} = 0, \tag{3.25d}$$

where A is a normalization constant. Using equation 3.17a, we get that $A = \frac{2\sqrt{2}}{\sqrt{v^2+4w^2}}$, where we chose the phase of A to be real. Thus, we can write

$$(V_1)_{k1} = i\frac{v_k}{\sqrt{v^2+4w^2}}, \qquad (V_2)_{p1} = 2i\frac{w_p}{\sqrt{v^2+4w^2}}. \tag{3.26}$$

If now we apply $i\theta T_+$ to the vacuum, we get

$$i\theta T_+ \begin{pmatrix} 0 \\ \frac{v_k}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} i\theta \frac{v_k}{2} \\ 0 \end{pmatrix} \Rightarrow (U_1)_{k1} = i\frac{B}{2} v_k \tag{3.27a}$$

$$i\theta T_+ \begin{pmatrix} 0 \\ x_q \\ 0 \end{pmatrix} = \begin{pmatrix} i\theta x_q \\ 0 \\ 0 \end{pmatrix} \Rightarrow (U_3)_{q1} = iBx_q, \tag{3.27b}$$

$$i\theta T_+ \begin{pmatrix} 0 \\ 0 \\ \frac{w_p}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ i\theta \frac{w_p}{\sqrt{2}} \\ 0 \end{pmatrix} \Rightarrow (U_4)_{p1} = i\frac{B}{\sqrt{2}} w_p, \tag{3.27c}$$

where B is a normalization constant. As the fields χ_j^+ have no VEV, then we have $(U_2)_{j1} = 0$. Using the first equation from 3.15a, we get $B = -\frac{2i}{\sqrt{v^2+2w^2+4x^2}}$, where we chose the phase of the normalization constant to be $-\frac{\pi}{2}$. Thus, we can write

$$(U_1)_{k1} = \frac{v_k}{\sqrt{v^2+2w^2+4x^2}}, \tag{3.28a}$$

$$(U_3)_{q1} = 2\frac{x_q}{\sqrt{v^2+2w^2+4x^2}}, \tag{3.28b}$$

$$(U_4)_{p1} = \sqrt{2}\frac{w_p}{\sqrt{v^2+2w^2+4x^2}}. \tag{3.28c}$$

3.4 Lagrangian

Taking into consideration everything that was presented earlier in this chapter, the gauge-kinetic Lagrangian becomes

$$\begin{aligned}
&\sum_{k=1}^{n_d}(D^\mu\phi_k)^\dagger(D_\mu\phi_k) + \sum_{p=1}^{n_{t_1}}(D^\mu\Xi_p)^\dagger(D_\mu\Xi_p) + \frac{1}{2}\sum_{q=1}^{n_{t_0}}(D^\mu\Lambda_q)^\dagger(D_\mu\Lambda_q) + \sum_{j=1}^{n_{s_1}}(D^\mu\chi_j^+)^\dagger(D_\mu\chi_j^+)\\
&\quad + \frac{1}{2}\sum_{l=1}^{n_{s_0}}(\partial^\mu\chi_l^0)(\partial_\mu\chi_l^0) + \sum_{r=1}^{n_{s_2}}(D^\mu\chi_r^{++})^\dagger(D_\mu\chi_r^{++})\\
&= \sum_{a=1}^{n_1}(\partial^\mu S_a^-)(\partial_\mu S_a^+) + \frac{1}{2}\sum_{b=1}^{n_0}(\partial^\mu S_b^0)(\partial_\mu S_b^0) + \sum_{c=1}^{n_2}(\partial^\mu S_c^{--})(\partial_\mu S_c^{++}) && \text{(3.29a)}\\
&\quad + m_W^2 W^{\mu-}W_\mu^+ + \frac{m_Z^2}{2}Z_\mu Z^\mu && \text{(3.29b)}\\
&\quad + im_W(W_\mu^-\partial^\mu G^+ - W_\mu^+\partial^\mu G^-) && \text{(3.29c)}\\
&\quad + m_Z Z_\mu\partial^\mu G^0 && \text{(3.29d)}\\
&\quad - em_W A_\mu(W^{\mu-}G^+ + W^{\mu+}G^-) && \text{(3.29e)}\\
&\quad - g\frac{m_W}{c_W}Z_\mu\sum_{a=1}^{n_1}\Big((s_W^2\delta_{1a} + (U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a})W^{\mu-}S_a^+\\
&\qquad + (s_W^2\delta_{a1} + (U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1})W^{\mu+}S_a^-\Big) && \text{(3.29f)}\\
&\quad + ieA_\mu\sum_{a=1}^{n_1}(S_a^+\partial^\mu S_a^- - S_a^-\partial^\mu S_a^+) && \text{(3.29g)}\\
&\quad + i\frac{g}{c_W}Z_\mu\sum_{a,a'=1}^{n_1}\Big(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a}\Big)(S_a^+\partial^\mu S_{a'}^- - S_{a'}^-\partial^\mu S_a^+) && \text{(3.29h)}\\
&\quad + \frac{g}{2c_W}Z_\mu\sum_{b,b'=1}^{n_0}\Big(\frac{1}{2}\operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'}\Big)(S_b^0\partial^\mu S_{b'}^0 - S_{b'}^0\partial^\mu S_b^0) && \text{(3.29i)}\\
&\quad + ig\sum_{a=1}^{n_1}\sum_{b=1}^{n_0}\Bigg(\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big)W_\mu^-(S_b^0\partial^\mu S_a^+ - S_a^+\partial^\mu S_b^0)\\
&\qquad - \Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big)W_\mu^+(S_b^0\partial^\mu S_a^- - S_a^-\partial^\mu S_b^0)\Bigg) && \text{(3.29j)}\\
&\quad + 2gm_W W_\mu^+ W^{\mu-}\sum_{b=2}^{n_0}S_b^0\Big(\frac{1}{2}\operatorname{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\operatorname{Re}(U_4^\dagger V_2)_{1b} + \operatorname{Re}(U_3^\dagger R_1)_{1b}\Big) && \text{(3.29k)}
\end{aligned}$$

$$- g\frac{m_Z}{c_W}\frac{Z_\mu Z^\mu}{2}\sum_{b=2}^{n_0} S_b^0(\mathrm{Im}(V_1^\dagger V_1)_{1b} + 2\,\mathrm{Im}(V_2^\dagger V_2)_{1b}) \tag{3.29l}$$

$$+ 2ieA_\mu \sum_{c=1}^{n_2}(S_c^{++}\partial^\mu S_c^{--} - S_c^{--}\partial^\mu S_c^{++}) \tag{3.29m}$$

$$+ i\frac{g}{c_W}Z_\mu \sum_{c,c'=1}^{n_2}(2s_W^2\delta_{cc'} - (T_1^\dagger T_1)_{c'c})(S_c^{++}\partial^\mu S_{c'}^{--} - S_{c'}^{--}\partial^\mu S_c^{++}) \tag{3.29n}$$

$$+ ig\sum_{a=1}^{n_1}\sum_{c=1}^{n_2}((T_1^\dagger U_4)_{ca}W_\mu^+(S_c^{--}\partial^\mu S_a^+ - S_a^+\partial^\mu S_c^{--}) - (U_4^\dagger T_1)_{ac}W_\mu^-(S_c^{++}\partial^\mu S_a^- - S_a^-\partial^\mu S_c^{++})) \tag{3.29o}$$

$$+ gm_W\sum_{c=1}^{n_2}\Big((U_4^\dagger T_1)_{1c}W_\mu^- W^{\mu -}S_c^{++} + (T_1^\dagger U_4)_{c1}W_\mu^+ W^{\mu +}S_c^{--}\Big) \tag{3.29p}$$

$$\begin{aligned} - egA_\mu \sum_{a=1}^{n_1}\sum_{b=1}^{n_0} S_b^0 \Bigg(& W^{\mu +}S_a^-\left(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\right) \\ & + W^{\mu -}S_a^+\left(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\right)\Bigg) \end{aligned} \tag{3.29q}$$

$$\begin{aligned} - \frac{g^2}{c_W}Z^\mu \sum_{a=1}^{n_1}\sum_{b=1}^{n_0} S_b^0 \Bigg(& W_\mu^+ S_a^-\left(\frac{s_W^2}{2}(U_1^\dagger V_1)_{ab} + \frac{1+s_W^2}{\sqrt{2}}(U_4^\dagger V_2)_{ab} - c_W^2(U_3^\dagger R_1)_{ab}\right) \\ & + W_\mu^- S_a^+\left(\frac{s_W^2}{2}(V_1^\dagger U_1)_{ba} + \frac{1+s_W^2}{\sqrt{2}}(V_2^\dagger U_4)_{ba} - c_W^2(R_1^T U_3)_{ba}\right)\Bigg) \end{aligned} \tag{3.29r}$$

$$+ g^2 W_\mu^+ W^{\mu -}\sum_{b,b'=1}^{n_0} S_b^0 S_{b'}^0\left(\frac{1}{4}(V_1^\dagger V_1)_{b'b} + \frac{1}{2}(V_2^\dagger V_2)_{b'b} + (R_1^T R_1)_{b'b}\right) \tag{3.29s}$$

$$+ \frac{g^2}{2c_W^2}Z_\mu Z^\mu \sum_{b,b'=1}^{n_0} S_b^0 S_{b'}^0\left(\frac{1}{4}(V_1^\dagger V_1)_{b'b} + (V_2^\dagger V_2)_{b'b}\right) \tag{3.29t}$$

$$+ g^2 W_\mu^+ W^{\mu -}\sum_{a,a'=1}^{n_1} S_a^+ S_{a'}^-\left(\frac{1}{2}(U_1^\dagger U_1)_{a'a} + (U_3^\dagger U_3)_{a'a} + 2(U_4^\dagger U_4)_{a'a}\right) \tag{3.29u}$$

$$+ e^2 A_\mu A^\mu \sum_{a=1}^{n_1} S_a^- S_a^+ \tag{3.29v}$$

$$+ \frac{eg}{c_W}A^\mu Z_\mu \sum_{a,a'=1}^{n}\Big(2s_W^2\delta_{aa'} - (U_1^\dagger U_1)_{a'a} - 2(U_3^\dagger U_3)_{a'a}\Big)S_{a'}^- S_a^+ \tag{3.29w}$$

$$+ \frac{g^2}{c_W^2}Z_\mu Z^\mu \sum_{a,a'=1}^{n_1}\left(s_W^4\delta_{aa'} + \left(\frac{1}{4} - s_W^2\right)(U_1^\dagger U_1)_{a'a} + (c_W^2 - s_W^2)(U_3^\dagger U_3)_{a'a}\right)S_{a'}^- S_a^+ \tag{3.29x}$$

$$+ 4e^2 A^\mu A_\mu \sum_{c=1}^{n_2} S_c^{++}S_c^{--} \tag{3.29y}$$

$$+ 4\frac{eg}{c_W}Z_\mu A^\mu \sum_{c,c'=1}^{n_2}(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c})S_c^{++}S_{c'}^{--} \tag{3.29z}$$

$$+ \frac{g^2}{c_W^2}Z_\mu Z^\mu \sum_{c,c'=1}^{n_2}(4s_W^4\delta_{c'c} + (1 - 4s_W^2)(T_1^\dagger T_1)_{c'c})S_c^{++}S_{c'}^{--} \tag{3.29aa}$$

$$+ g^2 W_\mu^- W^{\mu+} \sum_{c,c'=1}^{n_2} (T_1^\dagger T_1)_{c'c} S_c^{++} S_{c'}^{--} \tag{3.29ab}$$

$$- \left(3egA^\mu + g^2 \frac{(2s_W^2 - c_W^2)}{c_W} Z^\mu\right) \sum_{a=1}^{n_1} \sum_{c=1}^{n_2} ((T_1^\dagger U_4)_{ca} W_\mu^+ S_c^{--} S_a^+ + (U_4^\dagger T_1)_{ac} W_\mu^- S_c^{++} S_a^-) \tag{3.29ac}$$

$$+ \frac{g^2}{\sqrt{2}} \sum_{b=1}^{n_0} \sum_{c=1}^{n_2} ((V_2^\dagger T_1)_{bc} W_\mu^- W^{\mu-} S_c^{++} S_b^0 + (T_1^\dagger V_2)_{cb} W_\mu^+ W^{\mu+} S_c^{--} S_b^0) \tag{3.29ad}$$

$$- \frac{g^2}{2} \sum_{a,a'=1}^{n_1} ((U_3^T U_3)_{a'a} W_\mu^- W^{\mu-} S_a^+ S_{a'}^+ + (U_3^\dagger U_3^*)_{a'a} W_\mu^+ W^{\mu+} S_a^- S_{a'}^-). \tag{3.29ae}$$

Note that, for term 3.29d to be real, the normalization constant A from equation 3.25 had to be real. Thus, we could only have chosen it to be positive or negative. We chose it to be positive and recovered the usual term mixing the Z boson and the neutral Goldstone boson. If we had chosen it to be negative it would have no consequences on the results of physically meaningful quantities as we can change the phase of the neutral Goldstone boson field arbitrarily. The normalization constant B from equation 3.27 could have also been the symmetric of the one we chose but we also chose it like this so that the term 3.29c would have its usual form. As before, this choice would not have add any consequence to the results of physically meaningful quantities.

The Feynman rules resulting from the Lagrangian in equation 3.29 can be found in Appendix B. The vertex in B.2a corresponds to 3.29e, the vertices in B.2b and B.2c correspond to 3.29f, the vertex in B.2d corresponds to 3.29g, the vertex in B.2e corresponds to 3.29h, the vertex in B.2f corresponds to 3.29i, the vertices in B.2g and B.2h correspond to 3.29j, the vertex in B.2i corresponds to 3.29k, the vertex in B.2j corresponds to 3.29l, the vertex in B.2k corresponds to 3.29m, the vertices in B.2m and B.2n correspond to 3.29o, the vertices in B.2o and B.2p correspond to 3.29p, the vertices in B.3a and B.3b correspond to 3.29q, the vertices in B.3c and B.3d correspond to 3.29r, the vertex in B.3e corresponds to 3.29s, the vertex in B.3f corresponds to 3.29t, the vertex in B.3g corresponds to 3.29u, the vertex in B.3h corresponds to 3.29v, the vertex in B.3i corresponds to 3.29w, the vertex in B.3j corresponds to 3.29x, the vertex in B.3k corresponds to 3.29y, the vertex in B.3l corresponds to 3.29z, the vertex in B.3m corresponds to 3.29aa, the vertex in B.3n corresponds to 3.29ab, the vertices in B.3o, B.3p, B.3q and B.3r correspond to 3.29ac, the vertices in B.3s and B.3t correspond to 3.29ad, the vertices in B.3u and B.3v correspond to 3.29ae.

Chapter 4

Oblique Parameters

4.1 Definition of the Oblique Parameters

When the following criteria are satisfied [31]

- The electroweak gauge group is $SU(2) \times U(1)$;
- The New Physics (NP) particles have suppressed couplings to the light fermions with which experiments are performed and couple mainly to the SM gauge boson;
- The relevant measurements are those made at energy scales $q^2 \approx 0$, $q^2 = m_Z^2$ and $q^2 = m_W^2$;

then, the NP effects can be parametrized by six quantities. These quantities are the oblique parameters. Three of them were defined by Peskin and Takeuchi [32,33], are called S, T and U and are given by [33]

$$\alpha T = \frac{1}{m_Z^2}\left(\frac{1}{c_W^2}\delta A_{WW}(0) - \delta A_{ZZ}(0)\right), \tag{4.1a}$$

$$\frac{\alpha}{4 s_W^2 c_W^2} S = \left.\frac{\partial\, \delta A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=0} - \left.\frac{\partial\, \delta A_{AA}(q^2)}{\partial q^2}\right|_{q^2=0} + \frac{c_W^2 - s_W^2}{c_W s_W}\left.\frac{\partial\, \delta A_{AZ}(q^2)}{\partial q^2}\right|_{q^2=0}, \tag{4.1b}$$

$$\begin{aligned}\frac{\alpha}{4 s_W^2} U = &\left.\frac{\partial\, \delta A_{WW}(q^2)}{\partial q^2}\right|_{q^2=0} - c_W^2 \left.\frac{\partial\, \delta A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=0} \\ &- s_W^2 \left.\frac{\partial\, \delta A_{AA}(q^2)}{\partial q^2}\right|_{q^2=0} + 2 c_W s_W \left.\frac{\partial\, \delta A_{AZ}(q^2)}{\partial q^2}\right|_{q^2=0},\end{aligned} \tag{4.1c}$$

where α is the fine-structure constant and $\delta A_{VV'}(q^2) = A_{VV'}(q^2)|_{NP} - A_{VV'}(q^2)|_{SM}$, where the $A_{VV'}(q^2)$ are the coefficients of $g^{\mu\nu}$ in the vacuum polarization tensors $\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}(q^2) + q^\mu q^\nu B_{VV'}(q^2)$, where VV' may be either AA, AZ, ZZ or WW and q is the four-momentum of the gauge boson.

Altarelli and Barbieri defined parameters ϵ_1, ϵ_2 and ϵ_3 [34, 35] which are related to S, T and U by

$$\epsilon_1 = \alpha T, \qquad \epsilon_2 = -\frac{\alpha}{4s_W^2}U, \qquad \epsilon_3 = \frac{\alpha}{4s_W^2}S. \tag{4.2}$$

The other three parameters were defined by Maksymyk, Burgess and London [36], are called V, W and X and are given by [36]

$$\alpha V = \frac{\partial\, \delta A_{ZZ}(q^2)}{\partial q^2}\bigg|_{q^2=m_Z^2} - \frac{\delta A_{ZZ}(m_Z^2) - \delta A_{ZZ}(0)}{m_Z^2}, \tag{4.3a}$$

$$\alpha W = \frac{\partial\, \delta A_{WW}(q^2)}{\partial q^2}\bigg|_{q^2=m_W^2} - \frac{\delta A_{WW}(m_W^2) - \delta A_{WW}(0)}{m_W^2}, \tag{4.3b}$$

$$\frac{\alpha}{s_W c_W} X = \frac{\partial\, \delta A_{AZ}(q^2)}{\partial q^2}\bigg|_{q^2=0} - \frac{\delta A_{AZ}(m_Z^2) - \delta A_{AZ}(0)}{m_Z^2}. \tag{4.3c}$$

4.2 Vacuum Polarization Tensors

The Feynman diagrams that contribute to the vacuum polarization tensors can be found in appendix C.

To compute the vacuum polarization tensors, we use dimensional regularization and use the integrals $I_{r,m}$ defined as

$$I_{r,m}(\Delta) \equiv \int \frac{d^d k}{(2\pi)^d} \frac{(k^2)^r}{(k^2 - \Delta + i\epsilon)^m}. \tag{4.4}$$

We can then write the contributions of each of the diagrams in C.1 to A_{ZZ} as

$$A_{ZZ\,1} = i\frac{g^2}{2c_W^2}\sum_{b=1}^{n_0}\Big(\frac{1}{2}(V_1^\dagger V_1)_{bb} + 2(V_2^\dagger V_2)_{bb}\Big)M^{4-d} I_{01}(\Delta = \mu_b^2), \tag{4.5a}$$

$$A_{ZZ\,2} = 2i\frac{g^2}{c_W^2}\sum_{a=1}^{n_1}\Big(s_W^4 + \Big(\frac{1}{4} - s_W^2\Big)(U_1^\dagger U_1)_{aa} + (c_W^2 - s_W^2)(U_3^\dagger U_3)_{aa}\Big)M^{4-d} I_{01}(\Delta = m_a^2), \tag{4.5b}$$

$$A_{ZZ\,3} = 2i\frac{g^2}{c_W^2}\sum_{c=1}^{n_2}\Big(4s_W^4 + (1 - 4s_W^2)(T_1^\dagger T_1)_{cc}\Big)M^{4-d} I_{01}(\Delta = M_c^2), \tag{4.5c}$$

$$A_{ZZ\,4} = -i\frac{g^2}{c_W^2}\sum_{b=1}^{n_0-1}\sum_{b'=b+1}^{n_0}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'} + \mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 M^{4-d}\int_0^1 dx \frac{4}{d} I_{12}(\Delta = D(q^2, \mu_b^2, \mu_{b'}^2)), \tag{4.5d}$$

$$\begin{aligned} A_{ZZ\,5} = -i\frac{g^2}{c_W^2}\sum_{b=2}^{n_0}(\mathrm{Im}(V_1^\dagger V_1)_{1b} + 2\,\mathrm{Im}(V_2^\dagger V_2)_{1b})^2 M^{4-d}\int_0^1 dx\Big(-\frac{1}{d} I_{12}(\Delta = D(q^2, \mu_b^2, \mu_1^2)) \\ - m_Z^2 I_{02}(\Delta = D(q^2, \mu_b^2, m_Z^2)) + \frac{1}{d} I_{12}(\Delta = D(q^2, \mu_b^2, m_Z^2))\Big), \end{aligned} \tag{4.5e}$$

$$A_{ZZ\,6} = -i\frac{g^2}{c_W^2}\sum_{a,a'=1}^{n_1}\Big(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'}\Big)\times$$
$$\times\Big(s_W^2\delta_{a'a} - \frac{1}{2}(U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a}\Big)M^{4-d}\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, m_a^2, m_{a'}^2)), \tag{4.5f}$$
$$A_{ZZ\,7} = -i\frac{g^2}{c_W^2}\sum_{a=1}^{n_1}(s_W^2\delta_{a1} + (U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1})(s_W^2\delta_{1a} + (U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a})\times$$
$$\times M^{4-d}\int_0^1 dx\Big(-\frac{1}{d}I_{12}(\Delta = D(q^2, m_a^2, m_1^2)) - m_W^2 I_{02}(\Delta = D(q^2, m_a^2, m_W^2))$$
$$+\frac{1}{d}I_{12}(\Delta = D(q^2, m_a^2, m_W^2))\Big), \tag{4.5g}$$
$$A_{ZZ\,8} = A_{ZZ\,7}, \tag{4.5h}$$
$$A_{ZZ\,9} = -i\frac{g^2}{c_W^2}\sum_{c,c'=1}^{n_2}(2s_W^2\delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c})M^{4-d}\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, M_c^2, M_{c'}^2)), \tag{4.5i}$$

where $D(q^2, A, B) \equiv q^2x^2 - q^2x + A(1-x) + Bx$ and M is an unphysical parameter with mass dimensions. The expression for $A_{ZZ\,1}$ was multiplied by a symmetry factor of $\frac{1}{2}$ because in that diagram we have a real internal particle.

We can write the contributions of each of the diagrams in C.2 to A_{WW} as

$$A_{WW\,1} = i\frac{g^2}{2}\sum_{b=1}^{n_0}\Big(\frac{1}{2}(V_1^\dagger V_1)_{bb} + (V_2^\dagger V_2)_{bb} + 2(R_1^T R_1)_{bb}\Big)M^{4-d}I_{01}(\Delta = \mu_b^2), \tag{4.6a}$$
$$A_{WW\,2} = ig^2\sum_{a=1}^{n_1}\Big(\frac{1}{2}(U_1^\dagger U_1)_{aa} + (U_3^\dagger U_3)_{aa} + 2(U_4^\dagger U_4)_{aa}\Big)M^{4-d}I_{01}(\Delta = m_a^2), \tag{4.6b}$$
$$A_{WW\,3} = ig^2\sum_{c=1}^{n_2}(T_1^\dagger T_1)_{cc}M^{4-d}I_{01}(\Delta = M_c^2), \tag{4.6c}$$
$$A_{WW\,4} = -ig^2\sum_{a=1}^{n_1}\sum_{b=1}^{n_0}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big)\times$$
$$\times\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big)M^{4-d}\times$$
$$\times\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, \mu_b^2, m_a^2)), \tag{4.6d}$$
$$A_{WW\,5} = -4ig^2\sum_{b=2}^{n_0}\Big(\frac{1}{2}\,\mathrm{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\,\mathrm{Re}(U_4^\dagger V_2)_{1b} + \mathrm{Re}(U_3^\dagger R_1)_{1b}\Big)^2\times$$
$$\times M^{4-d}\int_0^1 dx\Big(-\frac{1}{d}I_{12}(\Delta = D(q^2, \mu_b^2, m_1^2)) - m_W^2 I_{02}(\Delta = D(q^2, \mu_b^2, m_W^2))$$
$$+\frac{1}{d}I_{12}(\Delta = D(q^2, \mu_b^2, m_W^2))\Big), \tag{4.6e}$$
$$A_{WW\,6} = -ig^2\sum_{a=1}^{n_1}\sum_{c=1}^{n_2}(T_1^\dagger U_4)_{ca}(U_4^\dagger T_1)_{ac}M^{4-d}\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, m_a^2, M_c^2)), \tag{4.6f}$$

$$A_{WW\,7} = -i\frac{g^2 m_W^2}{c_W^2 m_Z^2}\sum_{a=1}^{n_1}(s_W^2\delta_{a1} + (U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1})(s_W^2\delta_{1a} + (U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a})M^{4-d}\times$$
$$\times\int_0^1 dx\Big(-\frac{1}{d}I_{12}(\Delta = D(q^2, m_a^2, \mu_1^2)) - m_Z^2 I_{02}(\Delta = D(q^2, m_a^2, m_Z^2))$$
$$+\frac{1}{d}I_{12}(\Delta = D(q^2, m_a^2, m_Z^2))\Big), \quad (4.6g)$$
$$A_{WW\,8} = -4ig^2\sum_{c=1}^{n_2}|(U_4^\dagger T_1)_{1c}|^2 M^{4-d}\int_0^1 dx\Big(-\frac{1}{d}I_{12}(\Delta = D(q^2, M_c^2, m_1^2))$$
$$- m_W^2 I_{02}(\Delta = D(q^2, M_c^2, m_W^2)) + \frac{1}{d}I_{12}(\Delta = D(q^2, M_c^2, m_W^2))\Big). \quad (4.6h)$$

The expression for $A_{WW\,1}$ was also multiplied by a symmetry factor of $\frac{1}{2}$ because in that diagram we have a real internal particle.

We can write the contributions of each of the diagrams in C.3 to A_{AA} as

$$A_{AA\,1} = 2ie^2 M^{4-d}\sum_{a=1}^{n_1} I_{01}(\Delta = m_a^2), \quad (4.7a)$$
$$A_{AA\,2} = 8ie^2 M^{4-d}\sum_{c=1}^{n_2} I_{01}(\Delta = M_c^2), \quad (4.7b)$$
$$A_{AA\,3} = -ie^2 M^{4-d}\sum_{a=1}^{n_1}\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, m_a^2, m_a^2)), \quad (4.7c)$$
$$A_{AA\,4} = -4ie^2 M^{4-d}\sum_{c=1}^{n_2}\int_0^1 dx\frac{4}{d}I_{12}(\Delta = D(q^2, M_c^2, M_c^2)). \quad (4.7d)$$

We can write the contributions of each of the diagrams in C.4 to A_{AZ} as

$$A_{AZ\,1} = i\frac{eg}{c_W}\sum_{a=1}^{n_1}(2s_W^2 - (U_1^\dagger U_1)_{aa} - 2(U_3^\dagger U_3)_{aa})M^{4-d}I_{01}(\Delta = m_a^2), \quad (4.8a)$$
$$A_{AZ\,2} = 4i\frac{eg}{c_W}\sum_{c=1}^{n_2}(2s_W^2 - (T_1^\dagger T_1)_{cc})M^{4-d}I_{01}(\Delta = M_c^2), \quad (4.8b)$$
$$A_{AZ\,3} = -i\frac{eg}{c_W}\sum_{a=1}^{n_1}\Big(s_W^2 - \frac{1}{2}(U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa}\Big)M^{4-d}\int_0^1 dx\frac{4}{d}g^{\mu\nu}I_{12}(\Delta = D(q^2, m_a^2, m_a^2)), \quad (4.8c)$$
$$A_{AZ\,4} = -2i\frac{eg}{c_W}\sum_{c=1}^{n_2}(2s_W^2 - (T_1^\dagger T_1)_{cc})M^{4-d}\int_0^1 dx\frac{4}{d}g^{\mu\nu}I_{12}(\Delta = D(q^2, M_c^2, M_c^2)). \quad (4.8d)$$

4.3 Parameter T

To compute the oblique parameter T we need the part proportional ot $g^{\mu\nu}$ of the vacuum polarization tensors $\Pi_{ZZ}^{\mu\nu}$ and $\Pi_{WW}^{\mu\nu}$ at $q^2 = 0$. For that, we use the results of the integrals I_{01}, I_{02} and I_{12} at $q^2 = 0$, expanded up to order ϵ^0, where $\epsilon = 4 - d$ given by [25]

$$M^{4-d} I_{01}(\Delta) = \frac{i}{(4\pi)^2}\Delta(\text{div} - \log\Delta), \tag{4.9a}$$

$$M^{4-d}\int_0^1 dx\, I_{02}(\Delta = D(0,B,A)) = \frac{i}{(4\pi)^2}\frac{1}{A}\Big(A(\text{div} - \log A) - \frac{A+B}{2} + F(A,B)\Big), \tag{4.9b}$$

$$\frac{4}{d}M^{4-d}\int_0^1 dx\, I_{12}(\Delta = D(0,B,A)) = \frac{i}{(4\pi)^2}\Big(A(\text{div} - \log A) + B(\text{div} - \log B) + F(A,B)\Big), \tag{4.9c}$$

where div $= \frac{2}{\epsilon} - \gamma + 1 + \log\big(4\pi M^2\big)$, being γ the Euler-Mascheroni constant and the function F is defined as

$$F(x,y) \equiv \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y}\log\left(\frac{x}{y}\right) & x \neq y, \\ 0 & x = y. \end{cases} \tag{4.10}$$

This function is symmetric under exchange of variables, making the integrals I_{02} and I_{12} also symmetric under the exchange $A \leftrightarrow B$.

Using the definition from 4.1a, equation 4.9, the relations from equations 3.15 – 3.17, we get for T [1]

$$\alpha T = \Big(\frac{g}{4\pi c_W m_Z}\Big)^2 \Bigg(\sum_{a=2}^{n_1}\sum_{b=2}^{n_0}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big)\times$$
$$\times\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big)F(m_a^2, \mu_b^2) \tag{4.11a}$$

$$+\sum_{b=2}^{n_0}\Big(\frac{1}{2}\operatorname{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\operatorname{Re}(U_4^\dagger V_2)_{1b} + \operatorname{Re}(U_3^\dagger R_1)_{1b}\Big)^2\times$$
$$\times\Big(2(m_W^2 + \mu_b^2) - 3F(m_W^2, \mu_b^2)\Big) \tag{4.11b}$$

$$+\sum_{a=2}^{n_1}\sum_{c=1}^{n_2}|(T_1^\dagger U_4)_{ca}|^2 F(M_c^2, m_a^2) \tag{4.11c}$$

$$+\frac{m_W^2}{4c_W^2 m_Z^2}\sum_{a=2}^{n_1}\Big((U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a}\Big)\Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1}\Big)\times$$
$$\times\Big(-3F(m_Z^2, m_a^2) + 2(m_Z^2 + m_a^2)\Big) \tag{4.11d}$$

$$+\sum_{c=1}^{n_2}|(U_4^\dagger T_1)_{1c}|^2\Big(-2(m_W^2 + M_c^2) - 3F(m_W^2, M_c^2)\Big) \tag{4.11e}$$

$$-\frac{1}{2}(m_h^2 + m_W^2) + \frac{3}{4}F(m_h^2, m_W^2) \tag{4.11f}$$

$$-\sum_{b=2}^{n_0-1}\sum_{b'=b+1}^{n_0}\Big(\frac{1}{2}\operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 F(\mu_b^2, \mu_{b'}^2) \tag{4.11g}$$

[1] To compute the vacuum polarization tensors in the SM, we used, for some vertices, Feynman rules that are different from the usual SM Feynman rules. The Feynman rules for those vertices can be found in appendix D. For the vertices that are not present in appendix D, we used the usual SM Feynman rules that can be found in [28] or [29].

$$- \frac{1}{4}\sum_{b=2}^{n_0}\Big(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2\operatorname{Im}(V_2^\dagger V_2)_{1b}\Big)^2 (2(m_Z^2+\mu_b^2) - 3F(m_Z^2,\mu_b^2)) \tag{4.11h}$$

$$- 2\sum_{a=2}^{n_1-1}\sum_{a'=a+1}^{n_1}\Big(\frac{1}{2}(U_1^\dagger U_1)_{aa'} + (U_3^\dagger U_3)_{aa'}\Big)\times$$

$$\times\Big(\frac{1}{2}(U_1^\dagger U_1)_{a'a} + (U_3^\dagger U_3)_{a'a}\Big)F(m_a^2,m_{a'}^2) \tag{4.11i}$$

$$- \frac{1}{2}\sum_{a=2}^{n_1}((U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a})((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1})\times$$

$$\times (2(m_W^2+m_a^2) - 3F(m_W^2,m_a^2)) \tag{4.11j}$$

$$- 2\sum_{c=1}^{n_2-1}\sum_{c'=c+1}^{n_2}|(T_1^\dagger T_1)_{cc'}|^2 F(M_c^2,M_{c'}^2) \tag{4.11k}$$

$$+ \frac{1}{2}(m_h^2+m_Z^2) - \frac{3}{4}F(m_h^2,m_Z^2) \tag{4.11l}$$

$$- m_1^2(\mathsf{div} - \log m_1^2)\Big(\frac{m_Z^2 c_W^2}{4m_W^2} - \frac{1}{4}$$

$$- \frac{3}{4}(U_3^\dagger U_3)_{11} + \frac{7}{4}(U_4^\dagger U_4)_{11} + 2s_W^2\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\Big) \tag{4.11m}$$

$$- \frac{3}{4}m_W^2(\mathsf{div} - \log m_W^2)\Big((U_3^\dagger U_3)_{11} - 3(U_4^\dagger U_4)_{11}$$

$$+ 2((U_3^\dagger U_3)_{11} - (U_4^\dagger U_4)_{11})^2\Big) \tag{4.11n}$$

$$+ M_1^2(\mathsf{div} - \log M_1^2)\frac{m_W^2}{m_Z^2 c_W^2}(U_4^\dagger U_4)_{11} \tag{4.11o}$$

$$- \frac{3}{4}m_Z^2(\mathsf{div} - \log m_Z^2)\Big(1 - \frac{m_W^2}{c_W^2 m_Z^2}\Big(1 - 2(U_3^\dagger U_3)_{11}$$

$$+ 6(U_4^\dagger U_4)_{11} + \Big(1 - \frac{m_Z^2 c_W^2}{m_W^2}\Big)^2\Big)\Big)\Bigg). \tag{4.11p}$$

Thus, in a model with triplets, the T parameter has a divergent result. This was expected because parameter T is divergent for models that violate custodial symmetry at one-loop level [26, 37], as is the case of the models with triplets whose neutral components have a non-zero VEV.

4.4 Parameter S

To compute the oblique parameter S (as well as the oblique parameter U) we need the derivatives with respect to q^2 of the part proportional to $g^{\mu\nu}$ of the vacuum polarization tensors $\Pi_{ZZ}^{\mu\nu}$ and $\Pi_{WW}^{\mu\nu}$ at $q^2 = 0$. For that we use the expansion of the derivatives with respect to q^2 of the integrals I_{02} and I_{12} at $q^2 = 0$, expanded up to order ϵ^0, where $\epsilon = 4 - d$ given by

$$M^{4-d}\int_0^1 dx \frac{4}{d}\frac{\partial}{\partial Q}\Big(I_{12}(\Delta = D(Q,I,J))\Big)_{Q=0} = \frac{i}{48\pi^2}(1 - \mathsf{div} + K(I,J)), \tag{4.12}$$

where we defined

$$K(I,J) = -\frac{5}{6} + \frac{2IJ}{(I-J)^2} - \frac{J^2(3I-J)}{(I-J)^3}\log\left(\frac{I}{J}\right) + \log I. \tag{4.13}$$

This function is symmetric under the exchange $I \leftrightarrow J$. When $J = I$, we have $K(I,I) = \log I$ and thus, equation 4.12 becomes [31]

$$M^{4-d}\int_0^1 dx \frac{4}{d}\frac{\partial}{\partial Q}\Big(I_{12}(\Delta = D(Q,I,I))\Big)_{Q=0} = \frac{i}{48\pi^2}(1 - \text{div} + \log I). \tag{4.14}$$

We will also need

$$M^{4-d}\int_0^1 dx \frac{\partial}{\partial Q}\Big(I_{02}(\Delta = D(Q,I,J))\Big)_{Q=0} = \frac{i}{2(4\pi)^2}\tilde{K}(I,J), \tag{4.15}$$

where

$$\tilde{K}(I,J) = \frac{I^2 - J^2 - 2IJ\log\left(\frac{I}{J}\right)}{(I-J)^3}. \tag{4.16}$$

Using the definition from 4.1b, equations 4.12 – 4.15, the relations from equations 3.15 – 3.17, we get for S

$$\frac{\alpha}{4s_W^2c_W^2}S =$$

$$= \frac{g^2}{192\pi^2c_W^2}\left(4\sum_{b=2}^{n_0-1}\sum_{b'=b+1}^{n_0}\left(\frac{1}{2}\operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'}\right)^2 K(\mu_b^2,\mu_{b'}^2)\right. \tag{4.17a}$$

$$+\sum_{b=2}^{n_0}\Big(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2\operatorname{Im}(V_2^\dagger V_2)_{1b}\Big)^2(K(\mu_b^2,m_Z^2) - 6m_Z^2\tilde{K}(\mu_b^2,m_Z^2)) \tag{4.17b}$$

$$+4\sum_{a,a'=2}^{n_1}\Big(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'}\Big)\times$$

$$\times\Big(s_W^2\delta_{a'a} - \frac{1}{2}(U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a}\Big)K(m_a^2,m_{a'}^2) \tag{4.17c}$$

$$+2\sum_{a=2}^{n_1}\Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1}\Big)\Big((U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a}\Big)\times$$

$$\times(K(m_a^2,m_W^2) - 6m_W^2\tilde{K}(m_a^2,m_W^2)) \tag{4.17d}$$

$$+4\sum_{c,c'=1}^{n_2}(2s_W^2\delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c})K(M_c^2,M_{c'}^2) \tag{4.17e}$$

$$-K(m_h^2,m_Z^2) + 6m_Z^2\tilde{K}(m_h^2,m_Z^2) \tag{4.17f}$$

$$-4s_W^2c_W^2\sum_{a=2}^{n_1}\log m_a^2 - 16s_W^2c_W^2\sum_{c=1}^{n_2}\log M_c^2 \tag{4.17g}$$

$$+ 4(c_W^2 - s_W^2) \sum_{a=2}^{n_1} \Big(s_W^2 - \frac{1}{2} (U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa} \Big) \log m_a^2 \tag{4.17h}$$

$$+ 8(c_W^2 - s_W^2) \sum_{c=1}^{n_2} (2s_W^2 - (T_1^\dagger T_1)_{cc}) \log M_c^2 \tag{4.17i}$$

$$- \left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)^2 (1 - \text{div}) \Bigg). \tag{4.17j}$$

where we have used $e = g\, s_W$.

Thus, we get a gauge invariant result for the oblique parameters S. However, this result is divergent for models with $m_W \neq m_Z c_W$ (which is the case of a general model with scalar $SU(2)$ triplets). This divergence can be cancelled if we multiply the Feynman rules for the SM vertices ZG^0H and ZZH [2] by $\sqrt{1 - \left(\frac{c_W^2 m_Z^2}{m_W^2} - 1\right)^2}$ (which is equal to 1 in models where $m_W = m_Z c_W$, as is the case of the SM). This is true for models with any scalar multiplets, as we show in section 4.10. After the multiplication of these SM Feynman rules by this factor we get

$$\frac{\alpha}{4 s_W^2 c_W^2} S =$$

$$= \frac{g^2}{192 \pi^2 c_W^2} \Bigg(4 \sum_{b=2}^{n_0-1} \sum_{b'=b+1}^{n_0} \Big(\frac{1}{2} \operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'} \Big)^2 K(\mu_b^2, \mu_{b'}^2) \tag{4.18a}$$

$$+ \sum_{b=2}^{n_0} \Big(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2 \operatorname{Im}(V_2^\dagger V_2)_{1b} \Big)^2 (K(\mu_b^2, m_Z^2) - 6 m_Z^2 \tilde{K}(\mu_b^2, m_Z^2)) \tag{4.18b}$$

$$+ 4 \sum_{a,a'=2}^{n_1} \Big(s_W^2 \delta_{aa'} - \frac{1}{2} (U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'} \Big) \times$$

$$\times \Big(s_W^2 \delta_{a'a} - \frac{1}{2} (U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a} \Big) K(m_a^2, m_{a'}^2) \tag{4.18c}$$

$$+ 2 \sum_{a=2}^{n_1} \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1} \Big) \Big((U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a} \Big) \times$$

$$\times (K(m_a^2, m_W^2) - 6 m_W^2 \tilde{K}(m_a^2, m_W^2)) \tag{4.18d}$$

$$+ 4 \sum_{c,c'=1}^{n_2} (2 s_W^2 \delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2 s_W^2 \delta_{c'c} - (T_1^\dagger T_1)_{c'c}) K(M_c^2, M_{c'}^2) \tag{4.18e}$$

$$- \left(1 - \left(\frac{c_W^2 m_Z^2}{m_W^2} - 1\right)^2\right) K(m_h^2, m_Z^2) + 6 \left(1 - \left(\frac{c_W^2 m_Z^2}{m_W^2} - 1\right)^2\right) m_Z^2 \tilde{K}(m_h^2, m_Z^2) \tag{4.18f}$$

$$- 4 s_W^2 c_W^2 \sum_{a=2}^{n_1} \log m_a^2 - 16 s_W^2 c_W^2 \sum_{c=1}^{n_2} \log M_c^2 \tag{4.18g}$$

$$+ 4(c_W^2 - s_W^2) \sum_{a=2}^{n_1} \Big(s_W^2 - \frac{1}{2} (U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa} \Big) \log m_a^2 \tag{4.18h}$$

[2] These two vertices are related by gauge invariance. If we multiply their Feynman rules by different factors, the result for S becomes gauge dependent.

$$+ 8(c_W^2 - s_W^2) \sum_{c=1}^{n_2} (2s_W^2 - (T_1^\dagger T_1)_{cc}) \log M_c^2 \Bigg). \tag{4.18i}$$

4.5 Parameter U

Using the definition from 4.1c, equations 4.12 – 4.15, the relations from equations 3.15 – 3.17, we get for U

$$\frac{\alpha}{4s_W^2} U = \frac{g^2}{192\pi^2} \Bigg(4 \sum_{a=2}^{n_1} \sum_{b=2}^{n_0} \Big(\frac{1}{2} (U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}} (U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab} \Big) \times$$

$$\times \Big(\frac{1}{2} (V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}} (V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba} \Big) K(\mu_b^2, m_a^2) \tag{4.19a}$$

$$+ 4 \sum_{b=2}^{n_0} \Big(\frac{1}{2} \operatorname{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}} \operatorname{Re}(U_4^\dagger V_2)_{1b} + \operatorname{Re}(U_3^\dagger R_1)_{1b} \Big)^2 \times$$

$$\times (K(\mu_b^2, m_W^2) - 6m_W^2 \tilde{K}(\mu_b^2, m_W^2)) \tag{4.19b}$$

$$+ 4 \sum_{a=2}^{n_1} \sum_{c=1}^{n_2} (T_1^\dagger U_4)_{ca} (U_4^\dagger T_1)_{ac} K(m_a^2, M_c^2) \tag{4.19c}$$

$$+ \frac{m_W^2}{c_W^2 m_Z^2} \sum_{a=2}^{n_1} \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1} \Big) \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{1a} \Big) \times$$

$$\times (K(m_a^2, m_Z^2) - 6m_Z^2 \tilde{K}(m_a^2, m_Z^2)) \tag{4.19d}$$

$$+ 4 \sum_{c=1}^{n_2} |(U_4^\dagger T_1)_{1c}|^2 (K(M_c^2, m_W^2) - 6m_W^2 \tilde{K}(M_c^2, m_W^2)) \tag{4.19e}$$

$$- \Big(K(m_h^2, m_W^2) - 6m_W^2 \tilde{K}(m_h^2, m_W^2) \Big) \tag{4.19f}$$

$$- 4 \sum_{b=2}^{n_0 - 1} \sum_{b'=b+1}^{n_0} \Big(\frac{1}{2} \operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'} \Big)^2 K(\mu_b^2, \mu_{b'}^2) \tag{4.19g}$$

$$- \sum_{b=2}^{n_0} \Big(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2 \operatorname{Im}(V_2^\dagger V_2)_{1b} \Big)^2 \times$$

$$\times (K(\mu_b^2, m_Z^2) - 6m_Z^2 \tilde{K}(\mu_b^2, m_Z^2)) \tag{4.19h}$$

$$- 4 \sum_{a,a'=2}^{n_1} \Big(s_W^2 \delta_{aa'} - \frac{1}{2} (U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'} \Big) \times$$

$$\times \Big(s_W^2 \delta_{a'a} - \frac{1}{2} (U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{a'a} \Big) K(m_a^2, m_{a'}^2) \tag{4.19i}$$

$$- 2 \sum_{a=2}^{n_1} \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1} \Big) \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{1a} \Big) \times$$

$$\times (K(m_a^2, m_W^2) - 6m_W^2 \tilde{K}(m_a^2, m_W^2)) \tag{4.19j}$$

$$- 4 \sum_{c,c'=1}^{n_2} (2s_W^2 \delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2s_W^2 \delta_{cc'} - (T_1^\dagger T_1)_{cc'}) K(M_c^2, M_{c'}^2) \tag{4.19k}$$

$$+ \Big(K(m_h^2, m_Z^2) - 6m_Z^2 \tilde{K}(m_h^2, m_Z^2) \Big) \tag{4.19l}$$

$$- 4s_W^4 \sum_{a=2}^{n_1} \log m_a^2 - 16 s_W^4 \sum_{c=1}^{n_2} \log M_c^2 \tag{4.19m}$$

$$+ 8 s_W^2 \sum_{a=2}^{n_1} \Big(s_W^2 - \frac{1}{2}(U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa} \Big) \log m_a^2 \tag{4.19n}$$

$$+ 16 s_W^2 \sum_{c=1}^{n_2} (2 s_W^2 - (T_1^\dagger T_1)_{cc}) \log M_c^2 \tag{4.19o}$$

$$+ \left(\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)^2 + 3\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right) \right) (1 - \text{div}) \Bigg). \tag{4.19p}$$

Similarly to what happened to the oblique parameter S, we get a result for the oblique parameter U which is gauge invariant but divergent for models with $m_W \neq m_Z c_W$ (which is the case of a general model with scalar $SU(2)$ triplets). This divergence can also be cancelled if, besides multiplying the Feynman rules for the SM vertices ZG^0H and ZZH by $\sqrt{1 - \left(\frac{c_W^2 m_Z^2}{m_W^2} - 1\right)^2}$ (as was done to obtain a finite result for S), we also multiply the SM vertices $W^\pm G^\mp H$ and $W^\pm W^\mp H$ (which are also related by gauge invariance) by $\sqrt{4 - 3\frac{c_W^2 m_Z^2}{m_W^2}}$ (which is also equal to 1 in models where $m_W = m_Z c_W$). This is also true for models with any scalar multiplets and we will also show it in section 4.10. After the multiplication of these SM Feynman rules by this factor we get

$$\frac{\alpha}{4 s_W^2} U = \frac{g^2}{192\pi^2} \Bigg(4 \sum_{a=2}^{n_1} \sum_{b=2}^{n_0} \Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab} \Big) \times$$

$$\times \Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba} \Big) K(\mu_b^2, m_a^2) \tag{4.20a}$$

$$+ 4 \sum_{b=2}^{n_0} \Big(\frac{1}{2} \operatorname{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}} \operatorname{Re}(U_4^\dagger V_2)_{1b} + \operatorname{Re}(U_3^\dagger R_1)_{1b} \Big)^2 \times$$

$$\times (K(\mu_b^2, m_W^2) - 6 m_W^2 \tilde{K}(\mu_b^2, m_W^2)) \tag{4.20b}$$

$$+ 4 \sum_{a=2}^{n_1} \sum_{c=1}^{n_2} (T_1^\dagger U_4)_{ca} (U_4^\dagger T_1)_{ac} K(m_a^2, M_c^2) \tag{4.20c}$$

$$+ \frac{m_W^2}{c_W^2 m_Z^2} \sum_{a=2}^{n_1} \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1} \Big) \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{1a} \Big) \times$$

$$\times (K(m_a^2, m_Z^2) - 6 m_Z^2 \tilde{K}(m_a^2, m_Z^2)) \tag{4.20d}$$

$$+ 4 \sum_{c=1}^{n_2} |(U_4^\dagger T_1)_{1c}|^2 (K(M_c^2, m_W^2) - 6 m_W^2 \tilde{K}(M_c^2, m_W^2)) \tag{4.20e}$$

$$- \left(4 - 3 \frac{m_Z^2 c_W^2}{m_W^2} \right) \Big(K(m_h^2, m_W^2) - 6 m_W^2 \tilde{K}(m_h^2, m_W^2) \Big) \tag{4.20f}$$

$$- 4 \sum_{b=2}^{n_0 - 1} \sum_{b'=b+1}^{n_0} \Big(\frac{1}{2} \operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'} \Big)^2 K(\mu_b^2, \mu_{b'}^2) \tag{4.20g}$$

$$- \sum_{b=2}^{n_0} \Big(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2 \operatorname{Im}(V_2^\dagger V_2)_{1b} \Big)^2 (K(\mu_b^2, m_Z^2) - 6 m_Z^2 \tilde{K}(\mu_b^2, m_Z^2)) \tag{4.20h}$$

$$
\begin{aligned}
&- 4 \sum_{a,a'=2}^{n_1} \Big(s_W^2 \delta_{aa'} - \frac{1}{2} (U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'} \Big) \times \\
&\quad \times \Big(s_W^2 \delta_{a'a} - \frac{1}{2} (U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{a'a} \Big) K(m_a^2, m_{a'}^2) && \text{(4.20i)} \\
&- 2 \sum_{a=2}^{n_1} \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1} \Big) \Big((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{1a} \Big) \times \\
&\quad \times \left(K(m_a^2, m_W^2) - 6 m_W^2 \tilde{K}(m_a^2, m_W^2) \right) && \text{(4.20j)} \\
&- 4 \sum_{c,c'=1}^{n_2} (2 s_W^2 \delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2 s_W^2 \delta_{cc'} - (T_1^\dagger T_1)_{cc'}) K(M_c^2, M_{c'}^2) && \text{(4.20k)} \\
&+ \left(1 - \left(\frac{c_W^2 m_Z^2}{m_W^2} - 1 \right)^2 \right) \Big(K(m_h^2, m_Z^2) - 6 m_Z^2 \tilde{K}(m_h^2, m_Z^2) \Big) && \text{(4.20l)} \\
&- 4 s_W^4 \sum_{a=2}^{n_1} \log m_a^2 - 16 s_W^4 \sum_{c=1}^{n_2} \log M_c^2 && \text{(4.20m)} \\
&+ 8 s_W^2 \sum_{a=2}^{n_1} \Big(s_W^2 - \frac{1}{2} (U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa} \Big) \log m_a^2 && \text{(4.20n)} \\
&+ 16 s_W^2 \sum_{c=1}^{n_2} (2 s_W^2 - (T_1^\dagger T_1)_{cc}) \log M_c^2 \Bigg). && \text{(4.20o)}
\end{aligned}
$$

4.6 Parameter V

To compute parameters V and W we will need [31]

$$
\begin{aligned}
&\frac{\partial}{\partial Q} \left(M^{4-d} \frac{4}{d} \int_0^1 dx \, I_{12}(\Delta = D(Q, I, J)) \right) \\
&- \frac{M^{4-d} \frac{4}{d} \int_0^1 dx \, I_{12}(\Delta = D(Q, I, J)) - M^{4-d} \frac{4}{d} \int_0^1 dx \, I_{12}(\Delta = D(0, I, J))}{Q} && \text{(4.21)} \\
= &\frac{i}{96 \pi^2} H(I, J, Q),
\end{aligned}
$$

where

$$
\begin{aligned}
H(I, J, Q) \equiv &2 - 9 \frac{I+J}{Q} + 6 \frac{(I-J)^2}{Q^2} \\
&+ \frac{3}{Q} \left[- \frac{I^2 + J^2}{I - J} + 2 \frac{I^2 - J^2}{Q} - \frac{(I-J)^3}{Q^2} \right] \log \frac{I}{J} \\
&+ \left(I + J - \frac{(I-J)^2}{Q} \right) \frac{3 f(t, r)}{Q^2}. && \text{(4.22a)}
\end{aligned}
$$

In the definition of $H(I, J, Q)$, we used a function $f(t, r)$, being $t \equiv I + J - Q$, $r \equiv Q^2 - 2Q(I + J) + (I - J)^2$ and

$$f(t,r) = \begin{cases} \sqrt{r}\log\left|\frac{t-\sqrt{r}}{t+\sqrt{r}}\right|, & r>0, \\ 0, & r=0, \\ 2\sqrt{-r}\arctan\frac{\sqrt{-r}}{t}, & r<0. \end{cases} \tag{4.23}$$

We will also need [31]

$$\begin{aligned} &\frac{\partial}{\partial Q}\left(M^{4-d}\int_0^1 dx\, I_{02}(\Delta = D(Q,I,J))\right) \\ &- \frac{M^{4-d}\int_0^1 dx\, I_{02}(\Delta = D(Q,I,J)) - M^{4-d}\int_0^1 dx\, I_{02}(\Delta = D(0,I,J))}{Q} \\ =& -\frac{i}{32\pi^2}\frac{1}{Q}\tilde{H}(I,J,Q), \end{aligned} \tag{4.24}$$

where

$$\tilde{H}(I,J,Q) \equiv 4 + \left(\frac{I+J}{I-J} - 2\frac{I-J}{Q}\right)\log\frac{I}{J} + \frac{-Q^2+3Q(I+J)-2(I-J)^2}{rQ}f(t,r). \tag{4.25}$$

Using the definition from 4.3a, equations 4.21 and 4.24, the relations from equations 3.15 – 3.17, we get for V

$$\alpha V = \frac{g^2}{384\pi^2 c_W^2}\left(4\sum_{b=2}^{n_0-1}\sum_{b'=b+1}^{n_0}\left(\frac{1}{2}\operatorname{Im}(V_1^\dagger V_1)_{bb'} + \operatorname{Im}(V_2^\dagger V_2)_{bb'}\right)^2 H(\mu_b^2,\mu_{b'}^2,m_Z^2)\right. \tag{4.26a}$$

$$+\sum_{b=2}^{n_0}(\operatorname{Im}(V_1^\dagger V_1)_{1b} + 2\operatorname{Im}(V_2^\dagger V_2)_{1b})^2\big(12\tilde{H}(\mu_b^2,m_Z^2,m_Z^2) + H(\mu_b^2,m_Z^2,m_Z^2)\big) \tag{4.26b}$$

$$\begin{aligned} &+4\sum_{a,a'=2}^{n_1}\left(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'}\right)\times \\ &\times\left(s_W^2\delta_{a'a} - \frac{1}{2}(U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a}\right)H(m_a^2,m_{a'}^2,m_Z^2) \end{aligned} \tag{4.26c}$$

$$\begin{aligned} &+2\sum_{a=2}^{n_1}((U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1})((U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a})\times \\ &\times\left(12\tilde{H}(m_a^2,m_W^2,m_Z^2) + H(m_a^2,m_W^2,m_Z^2)\right) \end{aligned} \tag{4.26d}$$

$$+4\sum_{c,c'=1}^{n_2}(2s_W^2\delta_{cc'} - (T_1^\dagger T_1)_{cc'})(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c})H(M_c^2,M_{c'}^2,m_Z^2) \tag{4.26e}$$

$$\left. -12\tilde{H}(m_h^2,m_Z^2,m_Z^2) - H(m_h^2,m_Z^2,m_Z^2)\right). \tag{4.26f}$$

This result is both gauge indenpendent and finite.

4.7 Parameter W

Using the definition from 4.3b, equations 4.21 and 4.24, the relations from equations 3.15 – 3.17, we get for W

$$\begin{aligned}
\alpha W =&\frac{g^2}{384\pi^2}\Bigg(4\sum_{a=2}^{n_1}\sum_{b=2}^{n_0}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab}+\frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab}+(U_3^\dagger R_1)_{ab}\Big)\times \\
&\times\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba}+\frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba}+(R_1^T U_3)_{ba}\Big)H(\mu_b^2,m_a^2,m_W^2) \qquad (4.27a)\\
&+4\sum_{b=2}^{n_0}\Big(\frac{1}{2}\operatorname{Re}(U_1^\dagger V_1)_{1b}+\frac{1}{\sqrt{2}}\operatorname{Re}(U_4^\dagger V_2)_{1b}+\operatorname{Re}(U_3^\dagger R_1)_{1b}\Big)^2\times \\
&\times\Big(12\tilde{H}(\mu_b^2,m_W^2,m_W^2)+H(\mu_b^2,m_W^2,m_W^2)\Big) \qquad (4.27b)\\
&+4\sum_{a=2}^{n_1}\sum_{c=1}^{n_2}(T_1^\dagger U_4)_{ca}(U_4^\dagger T_1)_{ac}H(m_a^2,M_c^2,m_W^2) \qquad (4.27c)\\
&+\frac{m_W^2}{m_Z^2 c_W^2}\sum_{a=2}^{n_1}((U_4^\dagger U_4)_{a1}-(U_3^\dagger U_3)_{a1})((U_4^\dagger U_4)_{1a}-(U_3^\dagger U_3)_{1a})\times \\
&\times\Big(12\tilde{H}(m_a^2,m_Z^2,m_W^2)+H(m_a^2,m_Z^2,m_W^2)\Big) \qquad (4.27d)\\
&+4\sum_{c=1}^{n_2}|(U_4^\dagger T_1)_{1c}|^2\Big(12\tilde{H}(M_c^2,m_W^2,m_W^2)+H(M_c^2,m_W^2,m_W^2)\Big) \qquad (4.27e)\\
&-12\tilde{H}(m_h^2,m_W^2,m_W^2)-H(m_h^2,m_W^2,m_W^2)\Bigg). \qquad (4.27f)
\end{aligned}$$

4.8 Parameter X

To compute the parameter X we will need equation 4.14, which is a specific case of 4.12, when $J = I$. We will also need [31]

$$\begin{aligned}
&\frac{M^{4-d}\,\frac{4}{d}\int_0^1 dx\, I_{12}(\Delta=D(Q,I,J))-M^{4-d}\,\frac{4}{d}\int_0^1 dx\, I_{12}(\Delta=D(0,I,J))}{Q}\\
=&\frac{i}{96\pi^2}\left(2-2\text{div}+\log I+\log J+G(I,J,Q)\right),
\end{aligned} \qquad (4.28)$$

and

$$\frac{M^{4-d}\,\frac{4}{d}\int_0^1 dx\, I_{02}(\Delta=D(Q,I,J))-M^{4-d}\,\frac{4}{d}\int_0^1 dx\, I_{02}(\Delta=D(0,I,J))}{Q}=-\frac{i}{32\pi^2}\frac{1}{Q}\tilde{G}(I,J,Q), \qquad (4.29)$$

where

$$G(I,J,Q) \equiv -\frac{16}{3} + 5\frac{I+J}{Q} - 2\frac{(I-J)^2}{Q^2}$$
$$+\frac{3}{Q}\left(\frac{I^2+J^2}{I-J} - \frac{I^2-J^2}{Q} + \frac{(I-J)^3}{3Q^2}\right)\log\frac{I}{J} + \frac{r}{Q^3}f(t,r), \tag{4.30a}$$

and

$$\tilde{G}(I,J,Q) \equiv -2 + \left(\frac{I-J}{Q} - \frac{I+J}{I-J}\right)\log\frac{I}{J} + \frac{f(t,r)}{Q}. \tag{4.31a}$$

Using the definition from 4.3c, equations 4.28 and 4.29, the relations from equations 3.15 – 3.17, we get for X

$$\frac{\alpha}{s_W c_W}X = -\frac{eg}{96\pi^2 c_W}\left(\sum_{a=2}^{n_1}\left(s_W^2 - \frac{1}{2}(U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa}\right)G(m_a^2, m_a^2, m_Z^2)\right. \tag{4.32a}$$
$$\left. + 2\sum_{c=1}^{n_2}\left(2s_W^2 - (T_1^\dagger T_1)_{cc}\right)G(M_c^2, M_c^2, m_Z^2)\right). \tag{4.32b}$$

4.9 Notes on A_{AA}

Due to the Ward-Takahashi identities of Quantum Electrodynamics (QED) [38,39], the photon propagator must be transverse to all orders. This means that we can write

$$\Pi_{AA}^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)A_{AA}(q^2), \tag{4.33}$$

being $\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$ the transverse projector. The fact that we are able to write the photon propagator as in 4.33 has the consequence that we must have $A_{AA}(q^2=0)=0$. We will compute here $A_{AA}(q^2=0)$ to check that it is in fact equal to 0.

At $q^2=0$ we get then

$$A_{AA\,1}(q^2=0) = -\frac{e^2}{8\pi^2}\sum_{a=1}^{n_1} m_a^2(\text{div} - \log m_a^2), \tag{4.34a}$$
$$A_{AA\,2}(q^2=0) = -\frac{e^2}{2\pi^2}\sum_{c=1}^{n_2} M_c^2(\text{div} - \log M_c^2), \tag{4.34b}$$
$$A_{AA\,3}(q^2=0) = \frac{e^2}{8\pi^2}\sum_{a=1}^{n_1} m_a^2(\text{div} - \log m_a^2), \tag{4.34c}$$

$$A_{AA\,4}(q^2=0) = \frac{e^2}{2\pi^2}\sum_{c=1}^{n_2} M_c^2(\text{div} - \log M_c^2), \tag{4.34d}$$

$$A_{AA\,5}(q^2=0) = \frac{e^2}{64\pi^2}\left(-m_1^2(\text{div} - \log m_1^2) - 3m_W^2(\text{div} - \log m_W^2) + 2(m_W^2 + m_1^2) - 3F(m_1^2, m_W^2)\right), \tag{4.34e}$$

$$A_{AA\,6}(q^2=0) = A_{AA\,5}(q^2=0), \tag{4.34f}$$

$$\begin{aligned}A_{AA\,7}(q^2=0) = -\frac{e^2}{16\pi^2}\Bigg(&\left(\frac{m_1^2}{m_W^2}-1\right)\Big(\frac{3}{4}(m_1^2+2m_W^2)(1-\text{div})\\
&+\frac{-5m_1^6+9m_1^2m_W^4-4m_W^6-6(3m_1^2m_W^4-2m_W^6)\log m_W^2+6m_1^6\log m_1^2}{8(m_1^2-m_W^2)^2}\Big)\\
&+\frac{1}{m_W^2}\Big(\frac{3}{4}\Big(\frac{1}{2}-\text{div}\Big)(m_1^4+m_1^2m_W^2+m_W^4)\\
&+\frac{m_W^6-m_1^6+3m_1^6\log m_1^2-3m_W^6\log m_W^2}{4(m_1^2-m_W^2)}\Big)\\
&+m_W^2\Big(\frac{39}{8}-\frac{27}{4}\text{div}+\frac{27}{4}\log m_W^2\Big)\Bigg),\end{aligned} \tag{4.34g}$$

$$A_{AA\,8}(q^2=0) = \frac{e^2}{16\pi^2}\left(\frac{3}{2}\frac{m1^4}{m_W^2}\Big(\frac{1}{6}-\text{div}+\log m_1^2\Big)+m_W^2\Big(\frac{15}{4}-\frac{9}{2}\text{div}+\frac{9}{2}\log m_W^2\Big)\right) \tag{4.34h}$$

$$A_{AA\,9}(q^2=0) = \frac{e^2}{16\pi^2}\frac{m_1^2}{2}\Big(\text{div}-\log m_1^2\Big), \tag{4.34i}$$

$$A_{AA\,10}(q^2=0) = A_{AA\,9}(q^2=0). \tag{4.34j}$$

We can see that $A_{AA\,1}(q^2=0) + A_{AA\,3}(q^2=0) = 0$ and $A_{AA\,2}(q^2=0) + A_{AA\,4}(q^2=0) = 0$. The other diagrams cancel each other such that

$$\begin{aligned}&A_{AA\,5}(q^2=0) + A_{AA\,6}(q^2=0) + A_{AA\,7}(q^2=0)\\
&+ A_{AA\,8}(q^2=0) - A_{AA\,9}(q^2=0) - A_{AA\,10}(q^2=0) = 0.\end{aligned} \tag{4.35}$$

Therefore, we get $A_{AA}(q^2=0)=0$ as expected.

4.10 Notes on the divergent parts of S and U

In this section we will show that if we multiply the usual SM Feynman rules (which can be found, for example, in [28] or in [29]) for the vertices ZG^0H and ZZH by $\sqrt{1-(K-1)^2}$ and for the vertices $W^\pm G^\mp H$ and $W^\pm W^\mp H$ by $\sqrt{4-3K}$, where $K \equiv \frac{m_Z^2 c_W^2}{m_W^2}$ then we get a finite result for the oblique parameters S and U for a model with any scalar content.

We use the $SU(2)$ representation with weak isospin J:

$$(T_3)_{rc} = \delta_{rc}\,(J+1-r)\,, \tag{4.36a}$$

$$(T_+)_{rc} = \delta_{r+1,c} \sqrt{\frac{r\,(2J+1-r)}{2}}, \tag{4.36b}$$

$$(T_-)_{rc} = \delta_{r-1,c} \sqrt{\frac{(r-1)\,(2J+2-r)}{2}}, \tag{4.36c}$$

where r stands for the row of the matrix and c stands for the column of the matrix, with $0 \leq r, c \leq 2J+1$.

Consider an $SU(2) \times U(1)$ electroweak model, in which the scalar sector includes $SU(2)$ multiplets M_{JY} labeled by their weak isospin J and their weak hypercharge Y, such that $J + Y \in \mathbb{N}_0$. Each multiplet M_{JY} has VEV v_{JY} in its component with electric charge zero, *i.e.*, in the component with third component of weak isospin $T_3 = -Y$. Writing the multiplets as column vectors, we will denote by M^0_{JY} the component in row $J+1+Y$, which has electric charge 0, by M^{-Q}_{JY} $(Q > 0)$ the component in row $J+Y+1+Q$ which has electric charge $-Q$ and by M^{+Q}_{JY} $(Q > 0)$ the component in row $J+Y+1-Q$ which has electric charge $+Q$. We will consider only complex multiplets, such that $M^{+Q}_{JY} \neq \left(M^{-Q}_{JY}\right)^*$. [3]

We can then write

$$\begin{aligned}
D_\mu M^{-Q}_{JY} &= \partial_\mu M^{-Q}_{JY} - ieQA_\mu M^{-Q}_{JY} + i\,\frac{g}{c_w}\,Z_\mu M^{-Q}_{JY}\left(Y + Qc_w^2\right) && (4.37a)\\
&\quad -ig\,W^+_\mu M^{-Q-1}_{JY}\sqrt{\frac{(J+Y+Q+1)\,(J-Y-Q)}{2}} && (4.37b)\\
&\quad -ig\,W^-_\mu M^{-Q+1}_{JY}\sqrt{\frac{(J+Y+Q)\,(J-Y-Q+1)}{2}}, && (4.37c)\\
D_\mu M^0_{JY} &= \partial_\mu M^0_{JY} - i\,\frac{g}{c_w}\,Z_\mu M^0_{JY}\,(-Y) && (4.37d)\\
&\quad -ig\,W^+_\mu M^-_{JY}\sqrt{\frac{(J+Y+1)\,(J-Y)}{2}} && (4.37e)\\
&\quad -ig\,W^-_\mu M^+_{JY}\sqrt{\frac{(J+Y)\,(J-Y+1)}{2}}, && (4.37f)\\
D_\mu M^{+Q}_{JY} &= \partial_\mu M^{+Q}_{JY} + ieQA_\mu M^{+Q}_{JY} - i\,\frac{g}{c_w}\,Z_\mu M^{+Q}_{JY}\left(-Y + Qc_w^2\right) && (4.37g)\\
&\quad -ig\,W^+_\mu M^{+Q-1}_{JY}\sqrt{\frac{(J+Y-Q+1)\,(J-Y+Q)}{2}} && (4.37h)\\
&\quad -ig\,W^-_\mu M^{+Q+1}_{JY}\sqrt{\frac{(J+Y-Q)\,(J-Y+Q+1)}{2}}. && (4.37i)
\end{aligned}$$

A scalar multiplet with weak isospin J and weak hypercharge Y has a component with zero electric charge if and only if $-J \leq Y \leq J$. Let Q be a non-negative number. A scalar multiplet with weak isospin J and weak hypercharge Y has a component with electric charge $+Q$ if and only if $Q - J \leq Y \leq Q + J$. A scalar multiplet with weak isospin J and weak hypercharge Y has a component with electric charge $-Q$ if and only if $-Q - J \leq Y \leq J - Q$.

[3] If some of the multiplets are real, the same conclusions are still valid but there are some modifications in the intermediate steps.

The masses of the gauge boson are given in terms of the VEVs of the scalar fields by

$$m_Z^2 = \frac{g^2}{c_W^2} \sum_{-J \leq Y \leq J} |v_{JY}|^2 \left(2Y^2\right), \tag{4.38a}$$

$$\begin{aligned} m_W^2 = & g^2 \left[\sum_{-J \leq Y \leq J-1} |v_{JY}|^2 \frac{(J+Y+1)(J-Y)}{2} \right. \\ & \left. + \sum_{-J+1 \leq Y \leq J} |v_{JY}|^2 \frac{(J+Y)(J-Y+1)}{2} \right] \end{aligned} \tag{4.38b}$$

$$\begin{aligned} = & g^2 \left[\sum_{-J \leq Y \leq J} |v_{JY}|^2 \frac{(J+Y+1)(J-Y)}{2} \right. \\ & \left. + \sum_{-J \leq Y \leq J} |v_{JY}|^2 \frac{(J+Y)(J-Y+1)}{2} \right] \end{aligned} \tag{4.38c}$$

$$= g^2 \sum_{-J \leq Y \leq J} |v_{JY}|^2 \left(J^2 - Y^2 + J\right). \tag{4.38d}$$

We can then write

$$M_{JY}^{+Q} = R_{JY}^{Q} \begin{pmatrix} S_1^{+Q} \\ S_2^{+Q} \\ \vdots \\ S_{n_Q}^{+Q} \end{pmatrix}, \qquad \left(M_{JY}^{-Q}\right)^* = S_{JY}^{Q} \begin{pmatrix} S_1^{+Q} \\ S_2^{+Q} \\ \vdots \\ S_{n_Q}^{+Q} \end{pmatrix}, \tag{4.39}$$

where n_Q is the total number of charge-Q scalars, R_{JY}^Q and S_{JY}^Q are $1 \times n_Q$ mixing matrices, S_a^{+Q} ($a = 1, \ldots, n_Q$) are the eigenstates of the mass matrix of the scalars with charge Q and $S_1^+ \equiv G^+$ is the charged Goldstone boson. We will denote by m_a^Q the mass of the S_a^Q scalar. We form the $n_Q \times n_Q$ matrices U^Q by stacking all the rows R_{JY}^Q and S_{JY}^Q for a fixed Q on top of each other; those matrices are unitary.

The unitarity of U^Q implies

$$\sum_{Q-J \leq Y \leq Q+J} (R_{JY}^Q)_{1a} (R_{JY}^Q)^*_{1a'} + \sum_{-Q-J \leq Y \leq J-Q} (S_{JY}^Q)_{1a} (S_{JY}^Q)^*_{1a'} = \delta_{aa'} \tag{4.40a}$$

$$\sum_{a=1}^{n_Q} (R_{JY}^Q)_{1a} (R_{J'Y'}^Q)^*_{1a} = \delta_{JJ'} \delta_{YY'} \tag{4.40b}$$

$$\sum_{a=1}^{n_Q} (S_{JY}^Q)_{1a} (S_{J'Y'}^Q)^*_{1a} = \delta_{JJ'} \delta_{YY'} \tag{4.40c}$$

$$\sum_{a=1}^{n_Q} (R_{JY}^Q)_{1a} (S_{J'Y'}^Q)^*_{1a} = 0 \tag{4.40d}$$

We can also write

$$M_{JY}^0 = v_{JY} + \frac{A_{JY} + iB_{JY}}{\sqrt{2}} \begin{pmatrix} G^0 \\ S_2^0 \\ \vdots \\ S_{n_0}^0 \end{pmatrix}, \tag{4.41}$$

where n_0 is the total number of neutral scalars and A_{JY} and B_{JY} are real $1 \times n_0$ matrices. We will denote by μ_b the mass of the S_b^0 scalar. We form the $n_0 \times n_0$ matrix V by stacking all the rows A_{JY} and B_{JY} on top of each other. The matrix V is real and orthogonal.

The orthogonality of V implies

$$\sum_{-J \le Y \le J} ((A_{JY})_{1b}(A_{JY})_{1b'} + (B_{JY})_{1b}(B_{JY})_{1b'}) = \delta_{bb'}, \tag{4.42a}$$

$$\sum_{b=1}^{n_0} (A_{JY})_{1b}(A_{J'Y'})_{1b} = \delta_{JJ'}\delta_{YY'}, \tag{4.42b}$$

$$\sum_{b=1}^{n_0} (B_{JY})_{1b}(B_{J'Y'})_{1b} = \delta_{JJ'}\delta_{YY'}, \tag{4.42c}$$

$$\sum_{b=1}^{n_0} (A_{JY})_{1b}(B_{J'Y'})_{1b} = 0. \tag{4.42d}$$

We get for the mixing of the W boson with the charged scalars

$$\mathcal{L}_{W^\pm G^\mp} = \sum_{-J \le Y \le J-1} \left[-ig\, W_\mu^- v_{JY} \sqrt{\frac{(J+Y+1)(J-Y)}{2}}\, \partial^\mu \left(M_{JY}^-\right)^* \right] \tag{4.43a}$$

$$+ \sum_{-J+1 \le Y \le J} \left[ig\, W_\mu^- v_{JY}^* \sqrt{\frac{(J+Y)(J-Y+1)}{2}}\, \partial^\mu M_{JY}^+ \right] + \text{H.c.} \tag{4.43b}$$

$$\equiv i m_W \left(W_\mu^- \partial^\mu G^+ - W_\mu^+ \partial^\mu G^-\right). \tag{4.43c}$$

Therefore, the charged Goldstone boson is given by

$$G^+ = \frac{g}{m_W} \left[- \sum_{-J \le Y \le J-1} v_{JY} \sqrt{\frac{(J+Y+1)(J-Y)}{2}} (M_{JY}^-)^* \right. \tag{4.44a}$$

$$\left. + \sum_{-J+1 \le Y \le J} v_{JY}^* \sqrt{\frac{(J+Y)(J-Y+1)}{2}} M_{JY}^+ \right], \tag{4.44b}$$

such that

$$(R_{JY})^*_{11} = \frac{g}{m_W}\sqrt{\frac{J^2-Y^2+J+Y}{2}}\, v^*_{JY}, \qquad (S_{JY})^*_{11} = -\frac{g}{m_W}\sqrt{\frac{J^2-Y^2+J-Y}{2}}\, v_{JY}. \tag{4.45}$$

We get for the mixing of the Z boson with the neutral scalars

$$\mathcal{L}_{ZG^0} = i\,\frac{g}{c_W}\, Z_\mu \sum_{-J\le Y\le J} Y v_{JY}\, \partial^\mu \left(M^0_{JY}\right)^* + \text{H.c.} \equiv m_Z Z_\mu \partial^\mu G^0. \tag{4.46}$$

Therefore, the neutral Goldstone boson is given by

$$G^0 = i\,\frac{g}{c_w m_Z} \sum_{-J\le Y\le J} Y\left[v_{JY}\,\partial^\mu\left(M^0_{JY}\right)^* - v^*_{JY}\,\partial^\mu M^0_{JY}\right], \tag{4.47}$$

such that

$$(A_{JY})_{11} = \frac{-\sqrt{2}g}{c_W m_Z}\, Y\,\text{Im}\, v_{JY}, \qquad (B_{JY})_{11} = \frac{\sqrt{2}g}{c_W m_Z}\, Y\,\text{Re}\, v_{JY}. \tag{4.48}$$

As we saw for the model with triplets, the diagrams for which $\left.\frac{\partial A_{VV'}(q^2)}{\partial q^2}\right|_{q^2=0}$ (where VV' may be either AA, AZ, ZZ or WW) is divergent are those for which the internal particles are two scalar particles. The ones which have as internal particles one gauge boson and one scalar are finite. The tadpole diagrams do not contribute for $\left.\frac{\partial A_{VV'}(q^2)}{\partial q^2}\right|_{q^2=0}$ as they do not depend on the momentum q of the external gauge boson. Thus, we need the Feynman rules for the vertices with one gauge boson and two scalars.

The $AS^{+Q}S^{-Q}$ interaction terms in the Lagrangian are

$$\begin{aligned}
\mathcal{L}_{AS^{+Q}S^{-Q}} =& ieQA^\mu \sum_{-J-Q\le Y\le J-Q} \Big((M^{-Q}_{JY})^*\partial_\mu M^{-Q}_{JY} - M^{-Q}_{JY}\partial_\mu (M^{-Q}_{JY})^*\Big) \\
&+ ieQA^\mu \sum_{Q-J\le Y\le J+Q} \Big((M^{+Q}_{JY})\partial_\mu (M^{+Q}_{JY})^* - (M^{+Q}_{JY})^*\partial_\mu M^{+Q}_{JY}\Big) \qquad (4.49a)\\
=& ieQA^\mu \sum_{-J-Q\le Y\le J-Q}\sum_{a,a'=1}^{n_Q} (S^Q_{JY})^*_{1a'}(S^Q_{JY})_{1a}\left(S^{+Q}_a\partial_\mu S^{-Q}_{a'} - S^{-Q}_{a'}\partial_\mu S^{+Q}_a\right) \\
&+ ieQA^\mu \sum_{Q-J\le Y\le J+Q}\sum_{a,a'=1}^{n_Q} (R^Q_{JY})^*_{1a'}(R^Q_{JY})_{1a}\left(S^{+Q}_a\partial_\mu S^{-Q}_{a'} - S^{-Q}_{a'}\partial_\mu S^{+Q}_a\right) \qquad (4.49b)\\
=& ieQA^\mu \sum_{a=1}^{n_Q}\left(S^{+Q}_a\partial_\mu S^{-Q}_a - S^{-Q}_a\partial_\mu S^{+Q}_a\right). \qquad (4.49c)
\end{aligned}$$

The $ZS^{+Q}S^{-Q}$ interaction terms in the Lagrangian are

$$
\begin{aligned}
\mathcal{L}_{ZS^{+Q}S^{-Q}} =& - i\frac{g}{c_W}Z^\mu \sum_{-J-Q\le Y\le J-Q} \left(Y + Qc_W^2\right) \left((M_{JY}^{-Q})^*\partial_\mu M_{JY}^{-Q} - M_{JY}^{-Q}\partial_\mu (M_{JY}^{-Q})^*\right) \\
& - i\frac{g}{c_W}Z^\mu \sum_{Q-J\le Y\le J+Q} \left(-Y + Qc_W^2\right) \left((M_{JY}^{+Q})\partial_\mu (M_{JY}^{+Q})^* - (M_{JY}^{+Q})^*\partial_\mu M_{JY}^{+Q}\right) \qquad (4.50a) \\
=& - igc_W Q Z^\mu \sum_{a=1}^{n_Q} \left(S_a^{+Q}\partial_\mu S_a^{-Q} - S_a^{-Q}\partial_\mu S_a^{+Q}\right) \\
& - i\frac{g}{c_W}Z^\mu \sum_{Q-J\le Y\le J-Q} \sum_{a,a'=1}^{n_Q} Y \left((S_{JY}^Q)_{1a}(S_{JY}^Q)^*_{1a'} - (R_{JY}^Q)_{1a}(R_{JY}^Q)^*_{1a'}\right) \times \\
& \quad \times \left(S_a^{+Q}\partial_\mu S_{a'}^{-Q} - S_{a'}^{-Q}\partial_\mu S_a^{+Q}\right) \qquad (4.50b) \\
& - i\frac{g}{c_W}Z^\mu \sum_{Y\ge -Q-J\wedge Y\le J-Q\wedge Y<Q-J} \sum_{a,a'=1}^{n_Q} Y (S_{JY}^Q)_{1a}(S_{JY}^Q)^*_{1a'} \left(S_a^{+Q}\partial_\mu S_{a'}^{-Q} - S_{a'}^{-Q}\partial_\mu S_a^{+Q}\right) \qquad (4.50c) \\
& + i\frac{g}{c_W}Z^\mu \sum_{Y\ge Q-J\wedge Y>J-Q\wedge Y\le Q+J} \sum_{a,a'=1}^{n_Q} Y (R_{JY}^Q)_{1a}(R_{JY}^Q)^*_{1a'} \left(S_a^{+Q}\partial_\mu S_{a'}^{-Q} - S_{a'}^{-Q}\partial_\mu S_a^{+Q}\right). \qquad (4.50d)
\end{aligned}
$$

The ZS^0S^0 interaction terms in the Lagrangian are

$$
\begin{aligned}
\mathcal{L}_{ZS^0S^0} &= i\frac{g}{c_W}Z^\mu \sum_{-J\le Y\le J} Y \left((M_{JY}^0)\partial_\mu (M_{JY}^0)^* - (M_{JY}^0)^*\partial_\mu M_{JY}^0\right) \qquad (4.51a) \\
&= i\frac{g}{2c_W}Z^\mu \sum_{-J\le Y\le J} \sum_{b,b'=1}^{n_0} Y \left((A_{JY})_{1b} + i(B_{JY})_{1b}\right)\left((A_{JY})_{1b'} - i(B_{JY})_{1b'}\right) \times \\
& \qquad \times \left(S_b^0\partial_\mu S_{b'}^0 - S_{b'}^0\partial_\mu S_b^0\right) \qquad (4.51b) \\
&= -\frac{g}{2c_W}Z^\mu \sum_{-J\le Y\le J} \sum_{b,b'=1}^{n_0} Y \left((A_{JY})_{1b'}(B_{JY})_{1b} - (A_{JY})_{1b}(B_{JY})_{1b'}\right) \times \\
& \qquad \times \left(S_b^0\partial_\mu S_{b'}^0 - S_{b'}^0\partial_\mu S_b^0\right). \qquad (4.51c)
\end{aligned}
$$

The $W^\pm S^\mp S^0$ interaction terms in the Lagrangian are

$$
\begin{aligned}
\mathcal{L}_{W^\pm S^\mp S^0} =& ig \sum_{-J\le Y\le J-1} \sqrt{\frac{(J+Y+1)(J-Y)}{2}} \Bigg(W^{\mu +}(M_{JY}^0)^*\partial_\mu M_{JY}^- \\
& - W^{\mu -}M_{JY}^0\partial_\mu (M_{JY}^-)^* + W^{\mu -}(M_{JY}^-)^*\partial_\mu M_{JY}^0 - W^{\mu +}M_{JY}^-\partial_\mu (M_{JY}^0)^* \Bigg) \qquad (4.52a) \\
& + ig \sum_{-J+1\le Y\le J} \sqrt{\frac{(J+Y)(J-Y+1)}{2}} \Bigg(W^{\mu +}(M_{JY}^+)^*\partial_\mu M_{JY}^0
\end{aligned}
$$

$$- W^{\mu-} M^{+}_{JY} \partial_\mu (M^0_{JY})^* + W^{\mu-} (M^0_{JY})^* \partial_\mu M^+_{JY} - W^{\mu+} M^0_{JY} \partial_\mu (M^+_{JY})^* \Big) \tag{4.52b}$$

$$= i\frac{g}{2} \sum_{-J\le Y\le J-1} \sum_{a=1}^{n_1} \sum_{b=1}^{n_0} \sqrt{(J+Y+1)(J-Y)} \Big(W^{\mu+} (A_{JY} - iB_{JY})_{1b} (S^1_{JY})^*_{1a} \times$$

$$\times (S^0_b \partial_\mu S^-_a - S^-_a \partial_\mu S^0_b) + W^{\mu-} (A_{JY} + iB_{JY})_{1b} (S^1_{JY})_{1a} (S^+_a \partial_\mu S^0_b - S^0_b \partial_\mu S^+_a) \Big) \tag{4.52c}$$

$$+ i\frac{g}{2} \sum_{-J+1\le Y\le J} \sum_{a=1}^{n_1} \sum_{b=1}^{n_0} \sqrt{(J+Y)(J-Y+1)} \Big(W^{\mu+} (A_{JY} + iB_{JY})_{1b} (R^1_{JY})^*_{1a} \times$$

$$\times (S^-_a \partial_\mu S^0_b - S^0_b \partial_\mu S^-_a) + W^{\mu-} (A_{JY} - iB_{JY})_{1b} (R^1_{JY})_{1a} (S^0_b \partial_\mu S^+_a - S^+_a \partial_\mu S^0_b) \Big). \tag{4.52d}$$

The $W^{\pm} S^{\pm Q} S^{\mp Q \mp 1}$ interaction terms in the Lagrangian are

$$\mathcal{L}_{W^{\pm} S^{\pm Q} S^{\mp Q \mp 1}} = i\frac{g}{\sqrt{2}} \sum_{-Q-J\le Y\le J-Q-1} \sum_{a=1}^{n_Q} \sum_{a'=1}^{n_{Q+1}} \sqrt{(J+Y+Q+1)(J-Y-Q)} \times$$

$$\times \Big(W^{\mu-} (S^Q_{JY})^*_{1a} (S^{Q+1}_{JY})_{1a'} (S^{Q+1}_{a'} \partial_\mu S^{-Q}_a - S^{-Q}_a \partial_\mu S^{Q+1}_{a'})$$

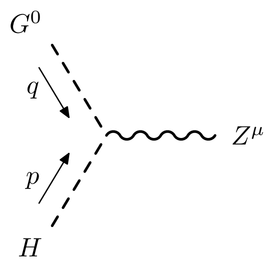
$$+ W^{\mu+} (S^{Q+1}_{JY})^*_{1a'} (S^Q_{JY})_{1a} (S^Q_a \partial_\mu S^{-Q-1}_{a'} - S^{-Q-1}_{a'} \partial_\mu S^Q_a) \Big) \tag{4.53a}$$

$$+ i\frac{g}{\sqrt{2}} \sum_{Q-J+1\le Y\le J+Q} \sum_{a=1}^{n_Q} \sum_{a'=1}^{n_{Q+1}} \sqrt{(J+Y-Q)(J-Y+Q+1)} \times$$

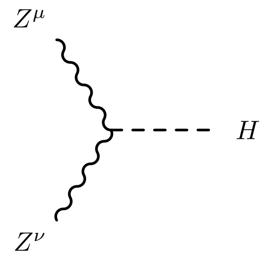
$$\times \Big(W^{\mu+} (R^{Q+1}_{JY})^*_{1a'} (R^Q_{JY})_{1a} (S^{-Q-1}_{a'} \partial_\mu S^Q_a - S^Q_a \partial_\mu S^{-Q-1}_{a'})$$

$$+ W^{\mu-} (R^{Q+1}_{JY})_{1a'} (R^Q_{JY})^*_{1a} (S^{-Q}_a \partial_\mu S^{Q+1}_{a'} - S^{Q+1}_{a'} \partial_\mu S^{-Q}_a) \Big). \tag{4.53b}$$

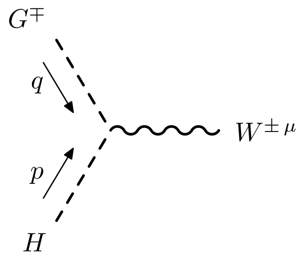
Besides using the Feynman rules required by gauge invariance for the triple vertices with gauge and Goldstone bosons, we will use the following Feynman rules to compute the SM amplitudes:



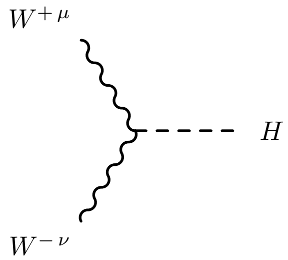
$$= \frac{g}{2c_W} \sqrt{X} (q-p)^\mu, \tag{4.54a}$$



$$= i\frac{g}{c_W} m_Z \sqrt{X}\, g^{\mu\nu}, \tag{4.54b}$$



$$= \mp i\frac{g}{2}\sqrt{Z}(q-p)^\mu, \tag{4.54c}$$



$$= igm_W \sqrt{Z}\, g^{\mu\nu}. \tag{4.54d}$$

Gauge invariance requires that the vertices ZG^0H and ZZH are multiplied by the same factor $\sqrt{X}$. The same happens with the vertices $W^\pm G^\mp H$ and $W^\pm W^\mp H$ that are both multiplied by $\sqrt{Z}$. In the SM we have $X = Z = 1$. Here we are assuming that X and Z can be different from 1 in a model where $m_W \neq m_Z c_W$.

Therefore, we get

$$\begin{aligned}
\left.\frac{\partial \delta A_{ZZ}}{\partial q^2}\right|_{q^2=0} =& \frac{g^2}{192\pi^2 c_W^2}\left(2\sum_{b,b'=1}^{n_0}\left(\sum_{-J\leq Y\leq J} Y\left((A_{JY})_{1b'}(B_{JY})_{1b} - (A_{JY})_{1b}(B_{JY})_{1b'}\right)\right)^2\right. \\
&+ 4\sum_{Q}\sum_{a,a'=1}^{n_Q}\Big(Qc_W^2\delta_{aa'} + \sum_{Q-J\leq Y\leq J-Q} Y((S_{JY}^Q)_{1a}(S_{JY}^Q)^*_{1a'} - (R_{JY}^Q)_{1a}(R_{JY}^Q)^*_{1a'}) \\
&+ \sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J} Y(S_{JY}^Q)_{1a}(S_{JY}^Q)^*_{1a'} \\
&- \sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y(R_{JY}^Q)_{1a}(R_{JY}^Q)^*_{1a'}\Big)\times \\
&\times\Big(Qc_W^2\delta_{a'a} + \sum_{Q-J\leq Y\leq J-Q} Y((S_{JY}^Q)_{1a'}(S_{JY}^Q)^*_{1a} - (R_{JY}^Q)_{1a'}(R_{JY}^Q)^*_{1a}) \\
&+ \sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J} Y(S_{JY}^Q)_{1a'}(S_{JY}^Q)^*_{1a}
\end{aligned} \tag{4.55a}$$

$$- \sum_{Y \geq Q-J \wedge Y > J-Q \wedge Y \leq Q+J} Y(R^Q_{JY})_{1a'}(R^Q_{JY})^*_{1a}\Big) \tag{4.55b}$$

$$- X - 4\left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)^2\Bigg)(1-\text{div}) + \text{finite terms} \tag{4.55c}$$

$$= \frac{g^2}{192\pi^2 c_W^2}\Bigg(4\sum_{-J\leq Y\leq J} Y^2 + 4\sum_Q \Big(Q^2 c_W^4 n_Q + 2\sum_{Q-J\leq Y\leq J-Q} Y^2$$

$$+ 2Qc_W^2 \sum_{Y\geq -Q-J \wedge Y\leq J-Q \wedge Y<Q-J} Y - 2Qc_W^2 \sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y$$

$$+ \sum_{Y\geq -Q-J \wedge Y\leq J-Q \wedge Y<Q-J} Y^2 + \sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y^2\Big)$$

$$- X - 4\left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)^2\Bigg)(1-\text{div}) + \text{finite terms}, \tag{4.55d}$$

$$\left.\frac{\partial \delta A_{AZ}}{\partial q^2}\right|_{q^2=0} = -\frac{eg}{48\pi^2 c_W}\Bigg(\sum_Q \sum_{a=1}^{n_Q} Q\Big(Qc_W^2 + \sum_{Q-J\leq Y\leq J-Q} Y\left((S^Q_{JY})_{1a}(S^Q_{JY})^*_{1a} - (R^Q_{JY})_{1a}(R^Q_{JY})^*_{1a}\right)$$

$$+ \sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J} Y(S^Q_{JY})_{1a}(S^Q_{JY})^*_{1a}$$

$$- \sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y(R^Q_{JY})_{1a}(R^Q_{JY})^*_{1a}\Big) \tag{4.56a}$$

$$+ \left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)\Bigg)(1-\text{div}) + \text{finite terms} \tag{4.56b}$$

$$= -\frac{eg}{48\pi^2 c_W}\Bigg(\sum_Q \Big(Q^2 n_Q c_W^2 + Q\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J} Y$$

$$- Q\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y\Big) \tag{4.56c}$$

$$+ \left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)\Bigg)(1-\text{div}) + \text{finite terms}, \tag{4.56d}$$

$$\left.\frac{\partial \delta A_{AA}}{\partial q^2}\right|_{q^2=0} = \frac{e^2}{48\pi^2}\left(\sum_Q Q^2 n_Q - 1\right)(1-\text{div}) + \text{finite terms}, \tag{4.57a}$$

$$\left.\frac{\partial \delta A_{WW}}{\partial q^2}\right|_{q^2=0} = \frac{g^2}{192\pi^2}\Bigg(\sum_{a=1}^{n_1}\sum_{b=1}^{n_0}\Big(\sum_{-J\leq Y\leq J-1}\sqrt{(J+Y+1)(J-Y)}((A_{JY})_{1b} - i(B_{JY})_{1b})(S^1_{JY})^*_{1a} -$$

$$- \sum_{-J+1\leq Y\leq J}\sqrt{(J+Y)(J-Y+1)}((A_{JY})_{1b} + i(B_{JY})_{1b})(R^1_{JY})^*_{1a}\Big)\times$$

$$\times \Big(\sum_{-J\le Y\le J-1} \sqrt{(J+Y+1)(J-Y)}((A_{JY})_{1b}+i(B_{JY})_{1b})(S^1_{JY})_{1a}-$$

$$- \sum_{-J+1\le Y\le J} \sqrt{(J+Y)(J-Y+1)}((A_{JY})_{1b}-i(B_{JY})_{1b})(R^1_{JY})_{1a}\Big) \quad (4.58a)$$

$$+2\sum_Q \sum_{a=1}^{n_Q}\sum_{a'=1}^{n_{Q+1}} \Big(\sum_{-J-Q\le Y\le J-Q-1} \sqrt{(J+Y+Q+1)(J-Y-Q)}(S^{Q+1}_{JY})^*_{1a'}(S^Q_{JY})_{1a}-$$

$$- \sum_{Q-J+1\le Y\le Q+J} \sqrt{(J+Y-Q)(J-Y+Q+1)}(R^{Q+1}_{JY})^*_{1a'}(R^Q_{JY})_{1a}\Big)\times$$

$$\times \Big(\sum_{-J-Q\le Y\le J-Q-1} \sqrt{(J+Y+Q+1)(J-Y-Q)}(S^{Q+1}_{JY})_{1a'}(S^Q_{JY})^*_{1a}-$$

$$- \sum_{Q-J+1\le Y\le Q+J} \sqrt{(J+Y-Q)(J-Y+Q+1)}(R^{Q+1}_{JY})_{1a'}(R^Q_{JY})^*_{1a}\Big) \quad (4.58b)$$

$$-Z-\frac{m_Z^2c_W^2}{m_W^2}\Bigg)(1-\text{div})+\text{finite terms} \quad (4.58c)$$

$$=\frac{g^2}{192\pi^2}\Bigg(2\sum_{-J\le Y\le J-1}(J+Y+1)(J-Y)+2\sum_{-J+1\le Y\le J}(J+Y)(J-Y+1)$$

$$+2\sum_Q\Big(\sum_{-Q-J\le Y\le J-Q-1}(J+Y+Q+1)(J-Y-Q)$$

$$+\sum_{Q-J+1\le Y\le J+Q}(J+Y-Q)(J-Y+Q+1)\Big)-Z-\frac{m_Z^2c_W^2}{m_W^2}\Bigg)\times \quad (4.58d)$$

$$\times(1-\text{div})+\text{finite terms.} \quad (4.58e)$$

Using equations 4.55, 4.56 and 4.57, we can write

$$\frac{\alpha}{4s_W^2c_W^2}S=\frac{g^2}{192\pi^2c_W^2}\Bigg(4\sum_{-J\le Y\le J}Y^2+4\sum_Q\Big(Q\Big(\sum_{Y\ge -Q-J\wedge Y\le J-Q\wedge Y<Q-J}Y-\sum_{Y\ge Q-J\wedge Y>J-Q\wedge Y\le Q+J}Y\Big)$$

$$+2\sum_{Q-J\le Y\le J-Q}Y^2+\sum_{Y\ge -Q-J\wedge Y\le J-Q\wedge Y<Q-J}Y^2+\sum_{Y\ge Q-J\wedge Y>J-Q\wedge Y\le Q+J}Y^2\Big) \quad (4.59a)$$

$$-X-4\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)^2+4s_W^2c_W^2$$

$$-4(c_W^2-s_W^2)\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)\Bigg)(1-\text{div})+\text{finite terms.} \quad (4.59b)$$

Let us define

$$\gamma \equiv \sum_{-J \leq Y \leq J} Y^2 + \sum_Q \Big(Q \Big(\sum_{Y \geq -Q-J \wedge Y \leq J-Q \wedge Y < Q-J} Y - \sum_{Y \geq Q-J \wedge Y > J-Q \wedge Y \leq Q+J} Y \Big) + 2 \sum_{Q-J \leq Y \leq J-Q} Y^2 + \sum_{Y \geq -Q-J \wedge Y \leq J-Q \wedge Y < Q-J} Y^2 + \sum_{Y \geq Q-J \wedge Y > J-Q \wedge Y \leq Q+J} Y^2 \Big). \tag{4.60}$$

We will now prove that γ is equal to 0.

Given a multiplet (J, Y), the maximum value for Q is $J+|Y|$ and the minium value for Q is $\max(1, |Y|-J)$.

Consider the case where we have a multiplet with $J = Y$. In this case we have

$$\gamma = Y^2 - (1 + 2 + \ldots + 2Y)\, Y + (2Y)\, Y^2 \tag{4.61a}$$
$$= (2Y+1)\, Y^2 - \frac{(2Y)\,(2Y+1)}{2} Y = 0. \tag{4.61b}$$

Consider the case where we have a multiplet with $J = -Y$. In this case we have

$$\gamma = Y^2 + (1 + 2 + \ldots - 2Y)\, Y + (-2Y)\, Y^2 \tag{4.62a}$$
$$= (-2Y+1)\, Y^2 + \frac{(-2Y)\,(-2Y+1)}{2} Y = 0. \tag{4.62b}$$

Consider the case where we have a multiplet with $Y > 0$ and $J > Y$. In this case we have

$$\gamma = Y^2 - ((J-Y+1) + (J-Y+2) + \ldots + (J+Y))\, Y + 2\,(J-Y)\, Y^2 + ((J+Y) - (J-Y))\, Y^2 \tag{4.63a}$$
$$= (2J+1)\, Y^2 - \frac{(2Y)\,(J-Y+1+J+Y)}{2} Y = 0. \tag{4.63b}$$

Consider the case where we have a multiplet with $Y < 0$ and $J > -Y$. In this case we have

$$\gamma = Y^2 - ((J+Y+1) + (J+Y+2) + \ldots + (J-Y))\, Y + 2\,(J+Y)\, Y^2 + ((J-Y) - (J+Y))\, Y^2 \tag{4.64a}$$
$$= (2J+1)\, Y^2 + \frac{(-2Y)\,(J+Y+1+J-Y)}{2} Y = 0. \tag{4.64b}$$

Consider the case where we have a multiplet with $Y > 0$ and $J < Y$. In this case we have

$$\gamma = -((Y-J) + (Y-J+1) + \ldots + (J+Y))\, Y + ((J+Y) - (Y-J) + 1)\, Y^2 \tag{4.65a}$$
$$= (2J+1)\, Y^2 + \frac{(2J+1)\,(2Y)}{2} Y = 0. \tag{4.65b}$$

Consider the case where we have a multiplet with $Y > 0$ and $J < Y$. In this case we have

$$\begin{aligned}
\gamma &= -\left((Y-J)+(Y-J+1)+\ldots+(J+Y)\right)Y \\
&\quad +\left((J+Y)-(Y-J)+1\right)Y^2 \qquad (4.66a)\\
&= (2J+1)\,Y^2+\frac{(2J+1)\,(2Y)}{2}\,Y=0. \qquad (4.66b)
\end{aligned}$$

Consider the case where we have a multiplet with $Y < 0$ and $J < -Y$. In this case we have

$$\begin{aligned}
\gamma &= -\left((-Y-J)+(-Y-J+1)+\ldots+(-Y+J)\right)Y \\
&\quad +\left((-J-Y)-(J-Y)+1\right)Y^2 \qquad (4.67a)\\
&= (-2J+1)\,Y^2+\frac{(-2J+1)\,(-2Y)}{2}\,Y=0. \qquad (4.67b)
\end{aligned}$$

We conclude that $\gamma = 0$ for all multiplets. This means that we get

$$\begin{aligned}
\frac{\alpha}{4s_W^2c_W^2}S =& \frac{g^2}{192\pi^2c_W^2}\Bigg(-X-4\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)^2+4s_W^2c_W^2 \\
&-4(c_W^2-s_W^2)\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)\Bigg)\times \qquad (4.68a)\\
&\times(1-\text{div})+\text{finite terms}. \qquad (4.68b)
\end{aligned}$$

Thus, we get a finite result for S in a model with any scalar multiplets if $X = 1-\left(\frac{m_Z^2c_W^2}{m_W^2}-1\right)^2$. Using 4.55, 4.56, 4.57 and 4.58 we get for parameter U

$$\begin{aligned}
\frac{\alpha}{4s_W^2}U =& \frac{g^2}{192\pi^2}\Bigg(2\sum_{-J\le Y\le J-1}(J+Y+1)(J-Y)+2\sum_{-J+1\le Y\le J}(J+Y)(J-Y+1) \qquad (4.69a)\\
&+2\sum_Q\Big(\sum_{-Q-J\le Y\le J-Q-1}(J+Y+Q+1)(J-Y-Q)\\
&+\sum_{Q-J+1\le Y\le J+Q}(J+Y-Q)(J-Y+Q+1)\Big)-Z-\frac{m_Z^2c_W^2}{m_W^2} \qquad (4.69b)\\
&-4\sum_{-J\le Y\le J}Y^2-4\sum_Q\Big(Q^2c_W^4n_Q+2\sum_{Q-J\le Y\le J-Q}Y^2\\
&+2Qc_W^2\sum_{Y\ge -Q-J\wedge Y\le J-Q\wedge Y<Q-J}Y-2Qc_W^2\sum_{Y\ge Q-J\wedge Y>J-Q\wedge Y\le Q+J}Y\\
&+\sum_{Y\ge -Q-J\wedge Y\le J-Q\wedge Y<Q-J}Y^2+\sum_{Y\ge Q-J\wedge Y>J-Q\wedge Y\le Q+J}Y^2\Big)\\
&+X+4\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)^2 \qquad (4.69c)
\end{aligned}$$

$$
\begin{aligned}
&-4s_W^4\left(\sum_Q Q^2 n_Q - 1\right) - 8s_W^2\sum_Q\Big(Q^2 n_Q c_W^2 + \\
&\quad + Q\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J} Y - Q\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J} Y\Big) && \text{(4.69d)}\\
&-8s_W^2\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)\Bigg)(1-\text{div})+\text{finite terms} && \text{(4.69e)}\\
=&\frac{g^2}{192\pi^2}\Bigg(2\sum_{-J\leq Y\leq J-1}(J+Y+1)(J-Y)+2\sum_{-J+1\leq Y\leq J}(J+Y)(J-Y+1) && \text{(4.69f)}\\
&+2\sum_Q\Big(\sum_{-Q-J\leq Y\leq J-Q-1}(J+Y+Q+1)(J-Y-Q)\\
&\quad+\sum_{Q-J+1\leq Y\leq J+Q}(J+Y-Q)(J-Y+Q+1)\Big) && \text{(4.69g)}\\
&-4\sum_{-J\leq Y\leq J}Y^2-4\sum_Q\Big(Q^2n_Q+2\sum_{Q-J\leq Y\leq J-Q}Y^2\\
&\quad+2Qc_W^2\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J}Y-2Qc_W^2\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J}Y\\
&\quad+\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J}Y^2+\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J}Y^2\Big) && \text{(4.69h)}\\
&-8s_W^2\sum_Q Q\Big(\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J}Y-\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J}Y\Big) && \text{(4.69i)}\\
&-Z-\frac{m_Z^2c_W^2}{m_W^2}+X+4\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)^2 && \text{(4.69j)}\\
&-8s_W^2\left(s_W^2-\frac{1}{2}-\frac{1}{2}\left(1-\frac{m_Z^2c_W^2}{m_W^2}\right)\right)+4s_W^4\Bigg)(1-\text{div})+\text{finite terms.} && \text{(4.69k)}
\end{aligned}
$$

Let us define

$$
\begin{aligned}
\theta\equiv&2\sum_{-J\leq Y\leq J-1}(J+Y+1)(J-Y)+2\sum_{-J+1\leq Y\leq J}(J+Y)(J-Y+1) && \text{(4.70a)}\\
&+2\sum_Q\Big(\sum_{-Q-J\leq Y\leq J-Q-1}(J+Y+Q+1)(J-Y-Q)\\
&\quad+\sum_{Q-J+1\leq Y\leq J+Q}(J+Y-Q)(J-Y+Q+1)\Big)-4\sum_{-J\leq Y\leq J}Y^2 && \text{(4.70b)}\\
&-4\sum_Q\Big(Q^2n_Q+2\sum_{Q-J\leq Y\leq J-Q}Y^2+2Qc_W^2\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J}Y\\
&\quad-2Qc_W^2\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J}Y+\sum_{Y\geq -Q-J\wedge Y\leq J-Q\wedge Y<Q-J}Y^2\\
&\quad+\sum_{Y\geq Q-J\wedge Y>J-Q\wedge Y\leq Q+J}Y^2\Big) && \text{(4.70c)}
\end{aligned}
$$

$$- 8s_W^2 \sum_Q Q\Big(\sum_{Y\geq -Q-J \wedge Y \leq J-Q \wedge Y<Q-J} Y - \sum_{Y \geq Q-J \wedge Y > J-Q \wedge Y \leq Q+J} Y\Big). \tag{4.70d}$$

We will now prove that θ is equal to 0.

Consider the case where we have a multiplet with $J = Y$. In this case we have

$$\begin{aligned} \theta &= 4Y + 2\left(\sum_{Q=1}^{2Y-1} (2Y-Q)(Q+1)\right) - 4Y^2 - 4\sum_{Q=1}^{2Y} Q^2 \\ &\quad +8c_W^2 Y \sum_{Q=1}^{2Y} Q - 8Y^3 + 8s_W^2 Y \sum_{Q=1}^{2Y} Q \qquad (4.71a) \\ &= 4Y + \frac{4}{3}\left(-2Y + 3Y^2 + 2Y^3\right) - 4Y^2 - \frac{4}{3}Y(1+2Y)(1+4Y) \\ &\quad +8c_W^2 Y^2(1+2Y) - 8Y^3 + 8s_W^2 Y^2 (1+2Y) = 0. \qquad (4.71b) \end{aligned}$$

Consider the case where where we have a multiplet with $J = -Y$. In this case we have

$$\begin{aligned} \theta &= -4Y + 2\left(\sum_{Q=1}^{2Y-1} (Q+1)(-2Y-Q)\right) - 4Y^2 - 4\sum_{Q=1}^{-2Y} Q^2 \\ &\quad -8c_W^2 Y \sum_{Q=1}^{-2Y} Q + 8Y^3 - 8s_W^2 Y \sum_{Q=1}^{-2Y} Q \qquad (4.72a) \\ &= -4Y - \frac{4}{3}\left(-2Y - 3Y^2 + 2Y^3\right) - 4Y^2 + \frac{8}{3}Y(-1+2Y)(-1+4Y) \\ &\quad -8c_W^2 Y^2(-1+2Y) + 8Y^3 - 8s_W^2 Y^2(-1+2Y) = 0. \qquad (4.72b) \end{aligned}$$

Consider the case where we have a multiplet with $Y > 0$ and $J > Y$. In this case we have

$$\begin{aligned} \theta &= 2(J+Y+1)(J-Y) + 2(J-Y+1)(J+Y) \\ &\quad +2\sum_{Q=1}^{J-Y-1}(J+Y+Q+1)(J-Y-Q) + 2\sum_{Q=1}^{J+Y-1}(J+Y-Q)(J-Y+Q+1) \\ &\quad -4Y^2 - 8\sum_{Q=1}^{J-Y} Q^2 - 4\sum_{Q=J-Y+1}^{J+Y} Q^2 - 8(J-Y)Y^2 + 8c_W^2 Y \sum_{Q=J-Y+1}^{J+Y} Q \\ &\quad -8Y^3 + 8s_W^2 Y \sum_{Q=J-Y+1}^{J+Y} Q \qquad (4.73a) \\ &= 2(J+Y+1)(J-Y) + 2(J-Y+1)(J+Y) \\ &\quad +\frac{2}{3}(J-Y-1)(J-Y)(Y+2+2J) + \frac{2}{3}(J+Y-1)(J+Y)(-Y+2+2J) \\ &\quad -4Y^2 - \frac{4}{3}(1+2J-2Y)(J-Y)(1+J-Y) - \frac{4}{3}Y(1+6J+6J^2+2Y^2) \\ &\quad -8(J-Y)Y^2 + 8c_W^2 Y^2(2J+1) - 8Y^3 + 8s_W^2 Y^2(2J+1) = 0. \qquad (4.73b) \end{aligned}$$

Consider the case where we have a multiplet with $Y < 0$ and $J > -Y$. In this case we have

$$\begin{aligned}
\theta &= 2(J+Y+1)(J-Y) + 2(J-Y+1)(J+Y) \\
&\quad +2\sum_{Q=1}^{J-Y-1}(J+Y+Q+1)(J-Y-Q) + 2\sum_{Q=1}^{J+Y-1}(J+Y-Q)(J-Y+Q+1) \\
&\quad -4Y^2 - 8\sum_{Q=1}^{J+Y}Q^2 - 4\sum_{Q=J-Y+1}^{J-Y}Q^2 - 8(J+Y)Y^2 - 8c_W^2 Y\sum_{Q=J+Y+1}^{J-Y}Q \\
&\quad +8Y^3 - 8s_W^2 Y\sum_{Q=J+Y+1}^{J-Y}Q \qquad \text{(4.74a)} \\
&= 2(J+Y+1)(J-Y) + 2(J-Y+1)(J+Y) \\
&\quad +\frac{2}{3}(J-Y-1)(J-Y)(Y+2+2J) + \frac{2}{3}(J+Y-1)(J+Y)(-Y+2+2J) \\
&\quad -4Y^2 - \frac{4}{3}(1+2J+2Y)(J+Y)(1+J+Y) + \frac{4}{3}Y(1+6J+6J^2+2Y^2) \\
&\quad -8(J+Y)Y^2 + 8c_W^2 Y^2(2J+1) + 8Y^3 + 8s_W^2 Y^2(2J+1) = 0. \qquad \text{(4.74b)}
\end{aligned}$$

Consider the case where we have a multiplet with $Y > 0$ and $J < Y$. In this case we have

$$\begin{aligned}
\theta &= 2\sum_{Q=Y-J}^{J+Y-1}(J+Y-Q)(J-Y+Q+1) - 4\sum_{Q=Y-J}^{J+Y}Q^2 \\
&\quad +8c_W^2 Y\sum_{Q=Y-J}^{J+Y}Q - 4(2J+1)Y^2 + 8s_W^2 Y\sum_{Q=Y-J}^{J+Y}Q \qquad \text{(4.75a)} \\
&= \frac{4}{3}J(1+J)(1+2J) - \frac{4}{3}(1+2J)(J+J^2+3Y^2) \\
&\quad +8c_W^2 Y^2(1+2J) - 4(2J+1)Y^2 + 8s_W^2 Y^2(1+2J) = 0. \qquad \text{(4.75b)}
\end{aligned}$$

Consider the case where we have a multiplet with $Y < 0$ and $J < -Y$. In this case we have

$$\begin{aligned}
\theta &= 2\sum_{Q=-Y-J}^{J-Y-1}(J+Y+Q+1)(J-Y-Q) - 4\sum_{Q=-Y-J}^{J-Y}Q^2 \\
&\quad -8c_W^2 Y\sum_{Q=-Y-J}^{J-Y}Q - 4(2J+1)Y^2 - 8s_W^2 Y\sum_{Q=-Y-J}^{J-Y}Q \qquad \text{(4.76a)} \\
&= \frac{4}{3}J(1+J)(1+2J) - \frac{4}{3}(1+2J)(J+J^2+3Y^2) \\
&\quad +8c_W^2 Y^2(1+2J) - 4(2J+1)Y^2 + 8s_W^2 Y^2(1+2J) = 0. \qquad \text{(4.76b)}
\end{aligned}$$

Therefore, we can conclude that $\theta = 0$ for all multiplets. This means that we get

$$\frac{\alpha}{4s_W^2}U = \frac{g^2}{192\pi^2}\Bigg(- Z - \frac{m_Z^2 c_W^2}{m_W^2} + X + 4\left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)^2 \tag{4.77a}$$

$$- 8s_W^2\left(s_W^2 - \frac{1}{2} - \frac{1}{2}\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right) + 4s_W^4\Bigg)(1 - \text{div}) + \text{finite terms} \tag{4.77b}$$

$$= \frac{g^2}{192\pi^2}\left(X - Z + \left(\frac{m_Z^2 c_W^2}{m_W^2} - 1\right)^2 + 3\left(1 - \frac{m_Z^2 c_W^2}{m_W^2}\right)\right)(1 - \text{div}) + \text{finite terms.} \tag{4.77c}$$

Thus, if we put $X = 1 - \left(\frac{m_Z^2 c_W^2}{m_W^2} - 1\right)^2$ (the value that makes S finite), then we get a finite result for U in a model with any scalar multiplets by putting $Z = 4 - \frac{3m_Z^2 c_W^2}{m_W^2}$.

4.11 Comparison with results from the literature

In reference [40], a model is considered with one complex $SU(2)$ scalar doublet with hypercharge $Y = \frac{1}{2}$

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \tag{4.78}$$

and one real $SU(2)$ scalar triplet with hypercharge $Y = 0$

$$\Lambda = \begin{pmatrix} \lambda^+ \\ \lambda^0 \\ -\lambda^- \end{pmatrix}. \tag{4.79}$$

In this model, the field φ^0 acquires a VEV $v/\sqrt{2}$ (where $v \in \mathbb{R}$), such that we can write $\varphi^0 = \frac{1}{\sqrt{2}}(v + \operatorname{Re}\varphi^{0\prime} + i\operatorname{Im}\varphi^{0\prime})$, and the field λ^0 acquires a VEV equal to $\frac{1}{2}v\tan\beta$, such that we can write $\lambda^0 = \frac{1}{2}v\tan\beta + \lambda^{0\prime}$. The masses of the W and Z bosons can then be written as

$$m_W = \frac{gv}{2\cos\beta}, \qquad m_Z = \frac{gv}{2c_W}. \tag{4.80}$$

The neutral fields are assumed to have no mixing, such that $S_1^0 = \operatorname{Im}\varphi^{0\prime}$ is the neutral Goldstone boson, $S_2^0 = \operatorname{Re}\varphi^{0\prime}$ and $S_3^0 = \lambda^{0\prime}$. The field $S_2^0 = \operatorname{Re}\varphi^{0\prime}$ is the SM Higgs boson field. The charged fields are assumed to mix by an angle β (which is the same that appears on the quotient between the VEVs of φ^0 and λ^0), such that

$$\begin{pmatrix} S_1^\pm \\ S_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \varphi^\pm \\ \lambda^\pm \end{pmatrix}. \tag{4.81}$$

We can then write the mixing matrices as

$$U_1 = \begin{pmatrix}\cos\beta & -\sin\beta\end{pmatrix}, \qquad U_3 = \begin{pmatrix}\sin\beta & \cos\beta\end{pmatrix}, \tag{4.82a}$$

$$V_1 = \begin{pmatrix}i & 1 & 0\end{pmatrix}, \qquad R_1 = \begin{pmatrix}0 & 0 & 1\end{pmatrix}, \tag{4.82b}$$

while the matrices U_2, U_4, V_2, R_2, T_1 and T_2 are not present in this model.

Using these mixing matrices, we can compute the oblique parameters S and U for this model using our results from equations 4.18 and 4.20. For parameter S we get

$$\frac{\alpha}{4s_W^2c_W^2}S =$$

$$=\frac{g^2}{192\pi^2c_W^2}\Bigg(4\Big(s_W^2-\frac{1}{2}s_\beta^2-c_\beta^2\Big)^2\log m_2^2+2c_\beta^2s_\beta^2(K(m_2^2,m_W^2)-6m_W^2\tilde{K}(m_2^2,m_W^2)) \tag{4.83a}$$

$$+s_\beta^4K(\mu_2^2,m_Z^2)-6s_\beta^4m_Z^2\tilde{K}(\mu_2^2,m_Z^2)-4s_W^2c_W^2\log m_2^2 \tag{4.83b}$$

$$+4(c_W^2-s_W^2)\Big(s_W^2-\frac{1}{2}s_\beta^2-c_\beta^2\Big)\log m_2^2\Bigg) \tag{4.83c}$$

$$=\frac{g^2}{192\pi^2c_W^2}\beta^2\Bigg(-2\log m_2^2+2(K(m_2^2,m_W^2)-6m_W^2\tilde{K}(m_2^2,m_W^2))\Bigg)+\mathcal{O}(\beta^3). \tag{4.83d}$$

where s_β and c_β stand for $\sin\beta$ and $\cos\beta$, respectively. For parameter U we get

$$\frac{\alpha}{4s_W^2}U = \frac{g^2}{192\pi^2}\Bigg(4\left(\frac{1}{4}s_\beta^2K(\mu_2^2,m_2^2)+c_\beta^2K(\mu_3^2,m_2^2)\right) \tag{4.84a}$$

$$+4\Big(\frac{1}{4}c_\beta^2(K(\mu_2^2,m_W^2)-6m_W^2\tilde{K}(\mu_2^2,m_W^2))$$

$$+s_\beta^2(K(\mu_3^2,m_W^2)-6m_W^2\tilde{K}(\mu_3^2,m_W^2))\Big) \tag{4.84b}$$

$$+\frac{m_W^2}{c_W^2m_Z^2}c_\beta^2s_\beta^2(K(m_2^2,m_Z^2)-6m_Z^2\tilde{K}(m_2^2,m_Z^2)) \tag{4.84c}$$

$$-(1+3s_\beta^2)(K(\mu_2^2,m_W^2)-6m_W^2\tilde{K}(\mu_2^2,m_W^2)) \tag{4.84d}$$

$$-4\Big(s_W^2-\frac{1}{2}s_\beta^2-c_\beta^2\Big)^2\log m_2^2 \tag{4.84e}$$

$$-2c_\beta^2s_\beta^2(K(m_2^2,m_W^2)-6m_W^2\tilde{K}(m_2^2,m_W^2)) \tag{4.84f}$$

$$-s_\beta^4(K(\mu_2^2,m_Z^2)-6m_W^2\tilde{K}(\mu_2^2,m_Z^2))-4s_W^4\log m_2^2 \tag{4.84g}$$

$$+8s_W^2\left(s_W^2-\frac{1}{2}s_\beta^2-c_\beta^2\right)\log m_2^2\Bigg) \tag{4.84h}$$

$$=\frac{g^2}{48\pi^2}\left(K(\mu_3^2,m_2^2)-\log m_2^2\right) \tag{4.84i}$$

$$+\frac{g^2}{192\pi^2}\beta^2\Bigg(K(\mu_2^2, m_2^2) - 4K(\mu_3^2, m_2^2) \tag{4.84j}$$

$$+4\Big(-\frac{1}{4}(K(\mu_2^2, m_W^2) - 6m_W^2\tilde{K}(\mu_2^2, m_W^2))$$

$$+(K(\mu_3^2, m_W^2) - 6m_W^2\tilde{K}(\mu_3^2, m_W^2))\Big) \tag{4.84k}$$

$$+(K(m_2^2, m_Z^2) - 6m_Z^2\tilde{K}(m_2^2, m_Z^2)) \tag{4.84l}$$

$$-3(K(\mu_2^2, m_W^2) - 6m_W^2\tilde{K}(\mu_2^2, m_W^2)) + 4\log m_2^2 \tag{4.84m}$$

$$-2(K(m_2^2, m_W^2) - 6m_W^2\tilde{K}(m_2^2, m_W^2))\Bigg) + \mathcal{O}(\beta^3). \tag{4.84n}$$

For both parameter S and parameter U, we expanded the result up to second order in β. In equation 26 of [40] they have

$$S = 0, \tag{4.85a}$$

$$U = -\frac{1}{3\pi}\left(m_k^2\log\left(\frac{m_k^2}{m_c^2}\right)\frac{(3m_c^2 - m_k^2)}{(m_k^2 - m_c^2)^3} + \frac{5(m_k^4 + m_c^4) - 22m_k^2m_c^2}{6(m_k^2 - m_c^2)^2}\right) + \mathcal{O}\left(\frac{m_Z}{m_c}\right), \tag{4.85b}$$

where we have $m_c \equiv m_2$ and $m_k \equiv \mu_3$. These results are only in zeroth order in β.

Comparing our results with the ones from [40], we can see that the result for S agrees up to order β^0. The result for U agrees up to order β^0 and $(m_Z/m_c)^0$. The fact that the results agree only to zeroth order in m_Z/m_c was to be expected as in [40] a different definition is used for the oblique parameters. Where, in the definition for the oblique parameters in equations 4.1, a derivative with respect to q^2 is used, in the definition used in [40], they use

$$\frac{A(m_V^2) - A(0)}{m_V^2}, \tag{4.86}$$

where m_V is either the mass of the Z or of the W bosons. When m_Z approaches 0 (and, consequently, m_W approaches 0 too as they are related by $m_W = \frac{m_Z c_W}{c_\beta}$) the expression in 4.86 becomes equal to $\frac{\partial A}{\partial q^2}\Big|_{q^2=0}$ and, therefore, the two definitions coincide. Thus, we should expect that the two results coincide to zeroth order in m_Z/m_c, as they do.

For parameter T, reference [40] presents a finite result in zeroth order in β. This result coincides with the result from this thesis in the same conditions. However, as for higher order in β we get a divergent result for this parameter, then this result has no physical meaning.

Chapter 5

One-loop corrections to the $Zb\bar{b}$ vertex

5.1 Introduction

Another way to indirectly detect heavy scalars can be through radiative corrections to the $Zb\bar{b}$ vertex. The coupling of the Z boson with the b quark and its anti-particle can be written as

$$\mathcal{L}_{Z\,b\,b} = \frac{g}{c_W} Z_\lambda \bar{b}\gamma^\lambda (g_{Lb} P_L + g_{Rb} P_R) b, \tag{5.1}$$

where $P_{L,R} = (1 \pm \gamma_5)/\,2$ are the chirality projectors and, at tree level, $g^0_{Lb} = \frac{s_W^2}{3} - \frac{1}{2}$ and $g^0_{Rb} = \frac{s_W^2}{3}$.

By considering the one-loop corrections to the $Zb\bar{b}$ vertex in our model with scalar particles in singlets, doublets and triplets of $SU(2)_L$, then the corrected couplings can be written as $g_{\aleph b} = g^{SM}_{\aleph b} + \delta g_{\aleph b} \quad (\aleph = L, R)$, where $g^{SM}_{\aleph b}$ is the coupling computed in the Standard Model and $\delta g_{\aleph b}$ are the New Physics contributions to the coupling. The two observables which are influenced by these correction due to New Physics are the hadronic branching ratio of Z to b quarks:

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}, \tag{5.2}$$

and the b quark asymmetry (measured in the process $e^- e^+ \to b\bar{b}$),

$$A_b = \frac{\sigma(e^-_L \to b_F) - \sigma(e^-_L \to b_B) + \sigma(e^-_R \to b_B) - \sigma(e^-_R \to b_F)}{\sigma(e^-_L \to b_F) + \sigma(e^-_L \to b_B) + \sigma(e^-_R \to b_B) + \sigma(e^-_R \to b_F)}, \tag{5.3}$$

where $e^-_{L,R}$ are left and right handed initial-state electrons and $b_{F,B}$ are final-state b-quarks moving in the forward and backward directions with respect to the direction of the initial-state electrons [41].

5.2 Couplings

We will use the approximation where the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{tb} = 1$, which means that we will only have to consider quarks bottom and top. We will also neglect the mass of the bottom quark m_b.

We will use

$$\mathcal{L}_{Z\,t\,t} = \frac{g}{c_W} Z_\lambda \bar{t}\gamma^\lambda (g_{Lt}P_L + g_{Rt}P_R)t, \tag{5.4}$$

$$\mathcal{L}_{W\,t\,b} = \frac{g}{\sqrt{2}} (\bar{t}\gamma^\lambda P_L b W^+_\lambda + \bar{b}\gamma^\lambda P_L t W^-_\lambda), \tag{5.5}$$

where, at tree level, $g^0_{Lt} = \frac{1}{2} - \frac{2s^2_W}{3}$ and $g^0_{Rt} = -\frac{2s^2_W}{3}$.

The terms in the Lagrangian for the interaction of the scalars with the quarks can be written as

$$\mathcal{L}_{S^\pm\,t\,b} = \sum_{a=1}^{n_1} (S^+_a \bar{t}(c^*_a P_L - d_a P_R)b + S^-_a \bar{b}(c_a P_R - d^*_a P_L)t), \tag{5.6}$$

$$\mathcal{L}_{S^0\,b\,b} = \sum_{l=1}^{n_0} S^0_l \bar{b}(r_l P_R + r^*_l P_L)b. \tag{5.7}$$

The terms in the Lagrangian for the interaction of the scalars with the Z boson can be written as

$$\mathcal{L}_{Z\,S^+\,S^-} = -\frac{g}{c_W} Z_\lambda \sum_{a,a'=1}^{n_1} X_{aa'} (S^+_a i\partial^\lambda S^-_{a'} - S^-_{a'} i\partial^\lambda S^+_a), \tag{5.8}$$

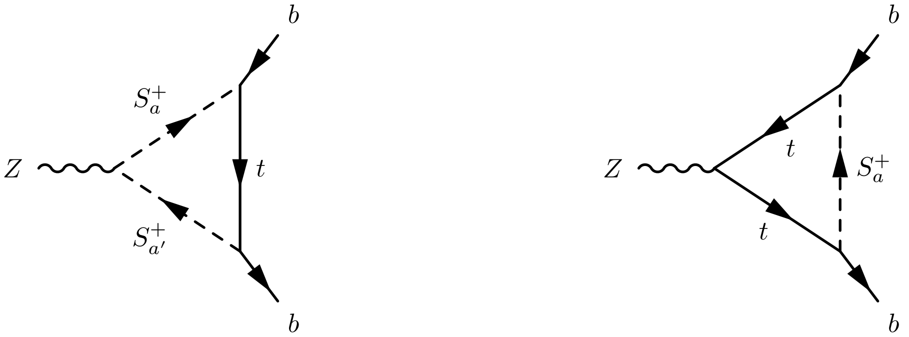
$$\mathcal{L}_{Z\,S^0\,S^0} = -\frac{ig}{c_W} Z_\lambda \sum_{l,l'=1}^{n_0} Y_{ll'} (S^0_l i\partial^\lambda S^0_{l'} - S^0_{l'} i\partial^\lambda S^0_l), \tag{5.9}$$

$$\mathcal{L}_{Z\,W\,S^\pm} = -\frac{g m_W}{c_W} Z_\lambda \sum_{a=1}^{n_1} \left(s_a W^{\lambda -} S^+_a + s^*_a W^{\lambda +} S^-_a \right), \tag{5.10}$$

where c_a, d_a, r_l and s_a are coefficients that are, in general, complex. The matrix X is $n_1 \times n_1$ Hermitian and the matrix Y is $n_0 \times n_0$ real and antisymmetric.

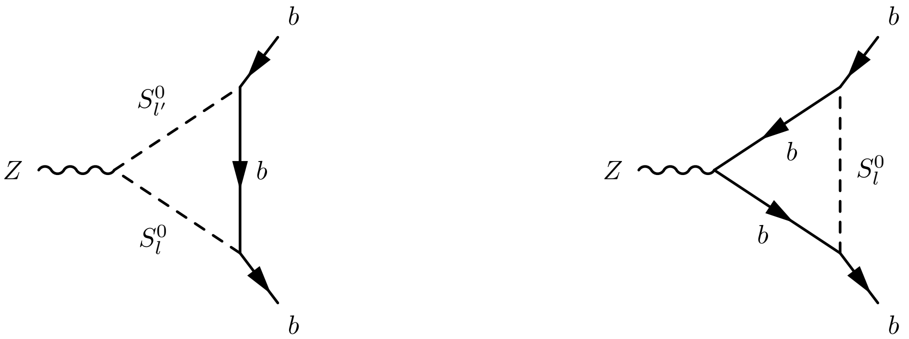
5.3 Feynman Diagrams

In figures 5.1 and 5.2, there are the diagrams that contribute at one-loop level to the $Zb\bar{b}$ vertex that contain charged and neutral scalars, respectively.



(a) **(b)**

Figure 5.1: Diagrams with charged scalars contributing to the $Zb\bar{b}$ vertex at one loop



(a) **(b)**

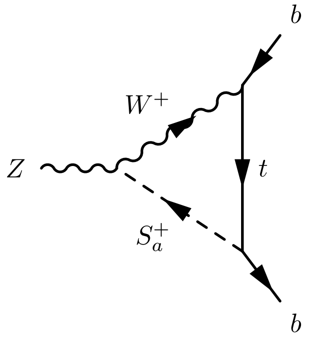
Figure 5.2: Diagrams with neutral scalars contributing to the $Zb\bar{b}$ vertex at one loop

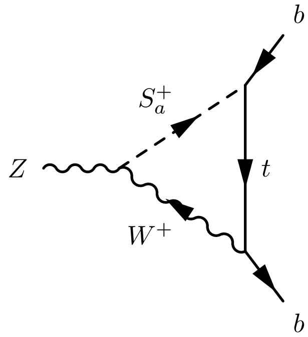
As argued in [42], the diagrams in 5.3 which contain neutral scalars are proportional to m_b, because the coupling of the Z boson to the b quarks preserves chirality, while the coupling of the neutral scalar to the b quarks changes their chirality. Thus, in these diagrams there must be a mass insertion in the b quark propagator in order to change the chirality of the b quark once again. As we are neglecting m_b, then we will not consider these diagrams. The diagrams in 5.3 which contain charged scalars do not give contributions beyond the Standard Model in models with only scalar singlets and doublets, as in these models the only $ZW^{\pm}S^{\mp}$ couplings are the $ZW^{\pm}S^{\mp}$ couplings present in the Standard Model. However, in our model, which also contains scalar triplets, these diagrams will give a New Physics contribution.

We follow the on-shell renormalization scheme from Hollik [43, 44]. We are looking for terms that change the tree-level couplings, which, after renormalization may be written as

$$i\Gamma_\mu^{Zbb} = i\gamma_\mu \frac{g}{c_W}\left((g_{Lb}^0 + \Delta g_L)P_L + (g_{Rb}^0 + \Delta g_R)P_R\right), \tag{5.11}$$

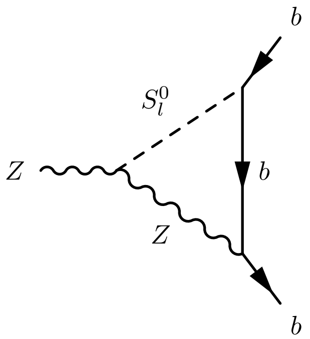
where $\Delta g_\aleph$ ($\aleph = L, R$) are the one-loop corrections after renormalization, including the ones that are

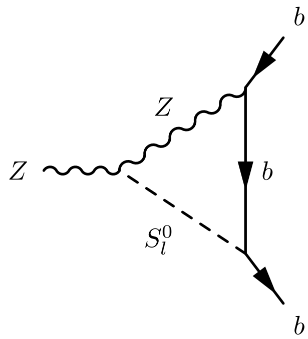




(a)

(b)





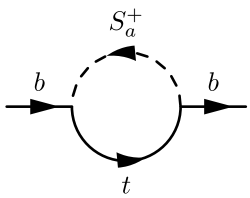
(c)

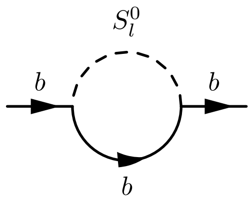
(d)

Figure 5.3: Diagrams with virtual gauge bosons

present in the Standard Model. Thus, we are not interested in terms proportional to p_i^μ, being p_i, with $i = 1, 2, 3$, the momenta of each of the external particles in the vertex. To perform renormalization, we need to evaluate the contributions of both the charged and the neutral scalars to the self-energy of the b quark. These diagrams are in figure 5.4. We will be interested on the part of the self-energy $i\Sigma(p)$ proportional to $\not{p}$, which we may write as $\Sigma(p) = \not{p}\left(\Omega_L(p^2)P_L + \Omega_R(p^2)P_R\right)$.

According to Hollik's renormalization scheme [43,44], the self-energy produces contributions to Δg_{Lb} and Δg_{Rb} given by $\Delta g_{Lb} = -g_{Lb}^0 \Omega_L(p^2 = m_b^2)$ and $\Delta g_{Rb} = -g_{Rb}^0 \Omega_R(p^2 = m_b^2)$.





(a)

(b)

Figure 5.4: Diagrams containing scalars that contribute to the self energy of the b quark

5.4 Results for the one-loop diagrams

To compute the correction for the couplings at one-loop we will use the Passarino-Veltman functions [45] defined by

$$
\begin{aligned}
&M^{\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m_0^2}\frac{1}{(k+r)^2-m_1^2}k^{\lambda}=\frac{i}{16\pi^2}r^{\lambda}B_1(r^2,m_0^2,m_1^2), &\text{(5.12a)}\\
&M^{\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m_0^2}\frac{1}{(k+r_1)^2-m_1^2}\frac{1}{(k+r_2)^2-m_2^2}=\\
=&\frac{i}{16\pi^2}C_0(r_1^2,(r_1-r_2)^2,r_2^2,m_0^2,m_1^2,m_2^2), &\text{(5.12b)}\\
&M^{\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m_0^2}\frac{1}{(k+r_1)^2-m_1^2}\frac{1}{(k+r_2)^2-m_2^2}k^{\lambda}k^{\nu}\\
=&\frac{i}{16\pi^2}(g^{\lambda\nu}C_{00}+r_1^{\lambda}r_1^{\nu}C_{11}+r_2^{\lambda}r_2^{\nu}C_{22}\\
&+(r_1^{\lambda}r_2^{\nu}+r_2^{\lambda}r_1^{\nu})C_{12})(r_1^2,(r_1-r_2)^2,r_2^2,m_0^2,m_1^2,m_2^2). &\text{(5.12c)}
\end{aligned}
$$

5.4.1 Diagrams with charged scalars

In the following results, the terms involving $\not{p}_1$ and $\not{p}_2$, being p_1 and p_2 the momenta of the b quarks were not considered because applying them to the b quark spinors would give, according to the Dirac equation, a term proportional to m_b which we are neglecting.

The diagrams in 5.1a lead to [42] [1]

$$
\begin{aligned}
&\Delta g_{Lb}(1a)=-\frac{1}{8\pi^2}\sum_{a,a'=1}^{n_1}c_aX_{aa'}c^*_{a'}C_{00}(m_Z^2,0,0,m_{a'}^2,m_a^2,m_t^2), &\text{(5.13a)}\\
&\Delta g_{Rb}(1a)=\Delta g_{Lb}(1a)(c_a\to d_a^*). &\text{(5.13b)}
\end{aligned}
$$

The diagrams in 5.1b lead to [42]

$$
\begin{aligned}
\Delta g_{Lb}(1b)=&\frac{1}{16\pi^2}\sum_{a=1}^{n_1}|c_a|^2\Bigg(-m_t^2g_{Lt}^0C_0(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)\\
&+g_{Rt}^0\Big(2C_{00}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)\\
&-\frac{1}{2}-m_Z^2C_{12}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)\Big)\Bigg), &\text{(5.14a)}\\
\Delta g_{Rb}(1b)=&\Delta g_{Lb}(1b)(c_a\to d_a^*,g_{Lt}^0\leftrightarrow g_{Rt}^0). &\text{(5.14b)}
\end{aligned}
$$

[1] There is a sign difference in our result compared to the result from [42] as we use a different convention. The physically meaningful quantities must, however, give the same result.

The diagrams in 5.4a lead to [42]

$$\Delta g_{Lb}(4a) = \frac{g^0_{Lb}}{16\pi^2}\sum_{a=1}^{n_1}|c_a|^2 B_1(0,m_t^2,m_a^2), \tag{5.15a}$$
$$\Delta g_{Rb}(4a) = \Delta g_{Lb}(4a)(c_a \to d_a, g^0_{Lb} \to g^0_{Rb}). \tag{5.15b}$$

The diagrams in 5.3a and 5.3b in an arbitrary gauge lead to

$$\Delta g_{Lb}(3a,b) = -\frac{g m_W m_t}{8\sqrt{2}\pi^2}\sum_{a=1}^{n_1}\mathrm{Re}(s_a c_a)\Big(C_0(m_Z^2,0,0,m_W^2,m_a^2,m_t^2)$$
$$-\frac{1}{m_W^2}\left(C_{00}(m_Z^2,0,0,m_W^2,m_a^2,m_t^2) - C_{00}(m_Z^2,0,0,m_1^2,m_a^2,m_t^2)\right)\Big), \tag{5.16a}$$
$$\Delta g_{Rb}(3a,b) = 0. \tag{5.16b}$$

Among the Passarino-Veltman functions used here, only B_1 and C_{00} are divergent. We have for those functions

$$B_1(r^2,m_0^2,m_1^2) = -\frac{\text{div}}{2} + \text{finite terms}, \tag{5.17a}$$
$$C_{00}(r_1^2,(r_1-r_2)^2,r_2^2,m_0^2,m_1^2,m_2^2) = \frac{\text{div}}{4} + \text{finite terms}. \tag{5.17b}$$

Therefore, the divergent terms in 5.13, 5.14, 5.15 and 5.16 are

$$\Delta g_{Lb}(1a) + \Delta g_{Lb}(1b) + \Delta g_{Lb}(4a) + \Delta g_{Lb}(3a,b) =$$
$$= \frac{\text{div}}{32\pi^2}\left(-\sum_{a,a'=1}^{n_1} c_a X_{aa'} c^*_{a'} + (g^0_{Rt} - g^0_{Lb})\sum_{a=1}^{n_1}|c_a|^2\right) + \text{finite terms}, \tag{5.18a}$$
$$\Delta g_{Rb}(1a) + \Delta g_{Rb}(1b) + \Delta g_{Rb}(4a) + \Delta g_{Rb}(3a,b) =$$
$$= \frac{\text{div}}{32\pi^2}\left(-\sum_{a,a'=1}^{n_1} d^*_a X_{aa'} d_{a'} + (g^0_{Lt} - g^0_{Rb})\sum_{a=1}^{n_1}|d_a|^2\right) + \text{finite terms}. \tag{5.18b}$$

Thus, as $g^0_{Rt} - g^0_{Lb} = \frac{c_W^2 - s_W^2}{2}$ and $g^0_{Lt} - g^0_{Rb} = \frac{c_W^2 - s_W^2}{2}$, in a consistent theory we must have

$$\sum_{a,a'=1}^{n_1} c_a X_{aa'} c^*_{a'} = \frac{c_W^2 - s_W^2}{2}\sum_{a=1}^{n_1}|c_a|^2, \qquad \sum_{a,a'=1}^{n_1} d^*_a X_{aa'} d_{a'} = \frac{c_W^2 - s_W^2}{2}\sum_{a=1}^{n_1}|d_a|^2. \tag{5.19a}$$

5.4.2 Diagrams with neutral scalars

The diagrams in 5.2a lead to [42]

$$\Delta g_{Lb}(2a) = -\frac{i}{4\pi^2}\sum_{l,l'=1}^{n_0} r_l Y_{ll'} r_{l'}^* C_{00}(0, m_Z^2, 0, 0, \mu_{l'}^2, \mu_l^2), \tag{5.20a}$$

$$\Delta g_{Rb}(2a) = \Delta g_{Lb}(2a)(r_l \to r_l^*). \tag{5.20b}$$

The diagrams in 5.2b lead to [42]

$$\begin{aligned}\Delta g_{Lb}(2b) =& \frac{g_{Rb}^0}{16\pi^2}\sum_{l=1}^{n_0} |r_l|^2 \Big(2C_{00}(0, m_Z^2, 0, \mu_l^2, 0, 0) - \frac{1}{2} \\ &- m_Z^2 C_{12}(0, m_Z^2, 0, \mu_l^2, 0, 0)\Big),\end{aligned} \tag{5.21a}$$

$$\Delta g_{Rb}(2b) = \Delta g_{Lb}(2b)(g_{Rb}^0 \to g_{Lb}^0). \tag{5.21b}$$

The diagrams in 5.4b lead to [42]

$$\Delta g_{Lb}(4b) = \frac{g_{Lb}^0}{16\pi^2}\sum_{l=1}^{n_0} |r_l|^2 B_1(0, 0, \mu_l^2), \tag{5.22a}$$

$$\Delta g_{Rb}(4b) = \Delta g_{Lb}(4b)(g_{Lb}^0 \to g_{Rb}^0). \tag{5.22b}$$

Using again equations 5.17, we have

$$\begin{aligned}&\Delta g_{Lb}(2a) + \Delta g_{Lb}(2b) + \Delta g_{Lb}(4b) = \\ =&\frac{\text{div}}{32\pi^2}\left(-2i\sum_{l,l'=1}^{n_0} r_l Y_{ll'} r_{l'}^* + (g_{Rb}^0 - g_{Lb}^0)\sum_{l=1}^{n_0}|r_l|^2\right) + \text{finite terms},\end{aligned} \tag{5.23a}$$

$$\begin{aligned}&\Delta g_{Rb}(2a) + \Delta g_{Rb}(2b) + \Delta g_{Rb}(4b) = \\ =&\frac{\text{div}}{32\pi^2}\left(-2i\sum_{l,l'=1}^{n_0} r_l^* Y_{ll'} r_{l'} + (g_{Lb}^0 - g_{Rb}^0)\sum_{l=1}^{n_0}|r_l|^2\right) + \text{finite terms}.\end{aligned} \tag{5.23b}$$

Thus, as $g_{Rb}^0 - g_{Lb}^0 = \frac{1}{2}$, in a consistent theory we must have

$$\sum_{l,l'=1}^{n_0} r_l Y_{ll'} r_{l'}^* = -\frac{i}{4}\sum_{l=1}^{n_0}|r_l|^2. \tag{5.24}$$

5.5 Results for the model with triplets

Similarly to what happens for the singlets, the scalar triplets have no Yukawa couplings. Thus, the Yukawa Lagrangian can be written as

$$\mathcal{L}_{\text{Yukawa}} = -\begin{pmatrix}\overline{t_L} & \overline{b_L}\end{pmatrix}\sum_{k=1}^{n_d}\left(f_k\begin{pmatrix}\varphi_k^+\\ \varphi_k^0\end{pmatrix}b_R + e_k\begin{pmatrix}\varphi_k^{0*}\\ -\varphi_k^-\end{pmatrix}t_R\right) + \text{H.c.}, \tag{5.25}$$

where e_k and f_k ($k = 1, ..., n_d$) are the Yukawa coupling constants. Developing this expression and comparing it with equations 5.6 and 5.7, we get the following relations

$$d_a = \sum_{k=1}^{n_d} f_k (U_1)_{ka}, \qquad c_a = \sum_{k=1}^{n_d} e_k (U_1)^*_{ka}, \qquad r_l = -\frac{1}{\sqrt{2}}\sum_{k=1}^{n_d} f_k (V_1)_{kl}. \tag{5.26}$$

Using the relations above and equations 3.15 and 3.17 we get $\sum_{a=1}^{n_1}|c_a|^2 = \sum_{k=1}^{n_d}|e_k|^2$, $\sum_{a=1}^{n_1}|d_a|^2 = \sum_{k=1}^{n_d}|f_k|^2$ and $\sum_{l=1}^{n_0}|r_l|^2 = \sum_{k=1}^{n_d}|f_k|^2$.

Due to an arbitrariness on the phase of the fields b_R and t_R, we can choose c_1 and d_1 to be real such that

$$c_1 = \frac{g\, m_t}{\sqrt{2} m_W}, \qquad d_1 = \frac{g\, m_b}{\sqrt{2} m_W} \approx 0, \qquad r_1 = \frac{-igm_b}{2c_W m_Z} \approx 0. \tag{5.27}$$

Comparing equation 5.8 with equation 3.29h, equation 5.9 with equation 3.29i and equation 5.10 with equation 3.29f, we get that for this class of models

$$Y_{ll'} = \frac{1}{4}\,\text{Im}(V_1^\dagger V_1)_{ll'} + \frac{1}{2}\,\text{Im}(V_2^\dagger V_2)_{ll'}, \tag{5.28a}$$

$$\begin{aligned} X_{aa'} &= -s_W^2\delta_{aa'} + \frac{1}{2}(U_1^\dagger U_1)_{a'a} + (U_3^\dagger U_3)_{a'a} = \\ &= \left(\frac{1}{2} - s_W^2\right)\delta_{aa'} - \frac{1}{2}\left((U_2^\dagger U_2)_{a'a} - (U_3^\dagger U_3)_{a'a} + (U_4^\dagger U_4)_{a'a}\right), \end{aligned} \tag{5.28b}$$

$$s_a = s_W^2\delta_{1a} + (U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a}. \tag{5.28c}$$

Using again equations 3.15 and 3.17 together with equations 5.26 and 5.28, we verify equations 5.19 and 5.24. Therefore, in this model with triplets, the divergences cancel, as they should.

Using equation 5.28 we get the following results for the contribution from the diagrams with charged scalars to Δg_{Lb} and Δg_{Rb}:

$$\Delta g_{Lb}^c = \Delta g_{Lb}(1a) + \Delta g_{Lb}(1b) + \Delta g_{Lb}(4a) + \Delta g_{Lb}(3a, b) = \tag{5.29a}$$

$$=\frac{1}{16\pi^2}\left(\sum_{a=1}^{n_1}|c_a|^2 f_L^c(m_a^2)\right. \tag{5.29b}$$

$$+\sum_{a,a'=1}^{n_1} c_{a'}^*\left((U_2^\dagger U_2)_{a'a}-(U_3^\dagger U_3)_{a'a}+(U_4^\dagger U_4)_{a'a}\right)c_a C_{00}(m_Z^2,0,0,m_{a'}^2,m_a^2,m_t^2) \tag{5.29c}$$

$$-g^2 s_W^2 m_t^2\Big(C_0(m_Z^2,0,0,m_W^2,m_1^2,m_t^2)$$

$$-\frac{1}{m_W^2}\left(C_{00}(m_Z^2,0,0,m_W^2,m_1^2,m_t^2)-C_{00}(m_Z^2,0,0,m_1^2,m_1^2,m_t^2)\right)\Big) \tag{5.29d}$$

$$-\sqrt{2}gm_W m_t\sum_{a=1}^{n_1}\mathrm{Re}\left(((U_4^\dagger U_4)_{1a}-(U_3^\dagger U_3)_{1a})c_a\right)\Big(C_0(m_Z^2,0,0,m_W^2,m_a^2,m_t^2)$$

$$\left.-\frac{1}{m_W^2}\left(C_{00}(m_Z^2,0,0,m_W^2,m_a^2,m_t^2)-C_{00}(m_Z^2,0,0,m_1^2,m_a^2,m_t^2)\right)\Big)\right), \tag{5.29e}$$

$$\Delta g_{Rb}^c=\Delta g_{Rb}(1a)+\Delta g_{Rb}(1b)+\Delta g_{Rb}(4a)+\Delta g_{Rb}(3a,b)= \tag{5.30a}$$

$$=\frac{1}{16\pi^2}\left(\sum_{a=2}^{n_1}|d_a|^2 f_R^c(m_a^2)\right. \tag{5.30b}$$

$$\left.+\sum_{a,a'=2}^{n_1} d_{a'}\left((U_2^\dagger U_2)_{a'a}-(U_3^\dagger U_3)_{a'a}+(U_4^\dagger U_4)_{a'a}\right)d_a^* C_{00}(m_Z^2,0,0,m_{a'}^2,m_a^2,m_t^2)\right). \tag{5.30c}$$

where we defined

$$f_L^c(m_a^2)=-g_{Lt}^0 m_t^2 C_0(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)+g_{Rt}^0\Big(2C_{00}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)$$
$$-\frac{1}{2}-m_Z^2 C_{12}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)\Big)+g_{Lb}^0 B_1(0,m_t^2,m_a^2)$$
$$+(2s_W^2-1)C_{00}(m_Z^2,0,0,m_a^2,m_a^2,m_t^2), \tag{5.31a}$$

$$f_R^c(m_a^2)=-g_{Rt}^0 m_t^2 C_0(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)+g_{Lt}^0\Big(2C_{00}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)$$
$$-\frac{1}{2}-m_Z^2 C_{12}(0,m_Z^2,0,m_a^2,m_t^2,m_t^2)\Big)+g_{Rb}^0 B_1(0,m_t^2,m_a^2)$$
$$+(2s_W^2-1)C_{00}(m_Z^2,0,0,m_a^2,m_a^2,m_t^2). \tag{5.31b}$$

For the contribution from the diagrams with neutral scalars to Δg_{Lb} and Δg_{Rb} we get

$$\Delta g_{Lb}^n=\Delta g_{Lb}(2a)+\Delta g_{Lb}(2b)+\Delta g_{Lb}(4b)= \tag{5.32a}$$

$$=\frac{1}{16\pi^2}\left(\sum_{l=2}^{n_0}|r_l|^2 f_L^n(\mu_l^2)\right. \tag{5.32b}$$

$$- i \sum_{l,l'=2}^{n_0} r_l \left(\mathrm{Im}(V_1^\dagger V_1)_{ll'} + 2\,\mathrm{Im}(V_2^\dagger V_2)_{ll'} \right) r_{l'}^* C_{00}(0, m_Z^2, 0, 0, \mu_{l'}^2, \mu_l^2) \Bigg), \tag{5.32c}$$

$$\Delta g_{Rb}^n = \Delta g_{Rb}(2a) + \Delta g_{Rb}(2b) + \Delta g_{Rb}(4b) = \Delta g_{Lb}^n (f_L^n \to f_R^n, r_l \to r_l^*). \tag{5.33}$$

where we defined

$$f_L^n(\mu_l^2) = g_{Rb}^0 \left(2C_{00}(0, m_Z^2, 0, \mu_l^2, 0, 0) - \frac{1}{2} - m_Z^2 C_{12}(0, m_Z^2, 0, \mu_l^2, 0, 0) \right) + g_{Lb}^0 B_1(0, 0, \mu_l^2), \tag{5.34a}$$

$$f_R^n(\mu_l^2) = g_{Lb}^0 \left(2C_{00}(0, m_Z^2, 0, \mu_l^2, 0, 0) - \frac{1}{2} - m_Z^2 C_{12}(0, m_Z^2, 0, \mu_l^2, 0, 0) \right) + g_{Rb}^0 B_1(0, 0, \mu_l^2). \tag{5.34b}$$

Until now we have been working with $g_{\aleph b}$ ($\aleph = L, R$) parametrized as $g_{\aleph b} = g_{\aleph b}^0 + \Delta g_{\aleph b}$, being $g_{\aleph b}^0$ the tree-level coupling and $\Delta g_{\aleph b}$ the one-loop contribution. To get a gauge independent result it is now convenient to switch to a parametrization that splits the SM and the New Physics parts. We will write it as $g_{\aleph b} = g_{\aleph b}^{SM} + \delta g_{\aleph b}$, where $g_{\aleph b}^{SM}$ is the SM part and $\delta g_{\aleph b}$ is the NP part. To obtain $\delta g_{\aleph b}$, we subtract from equations 5.29, 5.30, 5.32 and 5.33 the one-loop contribution to $g_{\aleph b}$ in the SM. In the limit of $m_b \to 0$, we get $\delta g_{Rb}^c = \Delta g_{Rb}^c$, $\delta g_{Lb}^n = \Delta g_{Lb}^n$ and $\delta g_{Rb}^n = \Delta g_{Rb}^n$ because the SM results for these one-loop contributions to the couplings are proportional to m_b^2. Thus, the results for δg_{Rb}^c, δg_{Lb}^n and δg_{Rb}^n are finite (as we have shown before) and, from equations 5.30, 5.32 and 5.33, are also gauge independent. For δg_{Lb}^c we get [2]

$$\delta g_{Lb}^c = \sum_{a=2}^{n1} \frac{|c_a|^2}{16\pi^2} \Bigg(g_{Lb}^0 B_1(0, m_t^2, m_a^2) - g_{Rt}^0 m_Z^2 C_{12}(0, m_Z^2, 0, m_a^2, m_t^2, m_t^2)$$

$$- g_{Lt}^0 m_t^2 C_0(0, m_Z^2, 0, m_a^2, m_t^2, m_t^2) + 2 g_{Rt}^0 C_{00}(0, m_Z^2, 0, m_a^2, m_t^2, m_t^2) - \frac{1}{2} g_{Rt}^0 \Bigg) \tag{5.35a}$$

$$- \sum_{a,a'=2}^{n_1} \frac{c_a X_{aa'} c_{a'}^*}{8\pi^2} C_{00}(m_Z^2, 0, 0, m_{a'}^2, m_a^2, m_t^2) \tag{5.35b}$$

$$- \frac{g m_W m_t}{8\sqrt{2}\pi^2} \sum_{a=2}^{n_1} \mathrm{Re}(s_a c_a) \left(C_0(m_Z^2, 0, 0, m_W^2, m_a^2, m_t^2) - \frac{1}{m_W^2} C_{00}(m_Z^2, 0, 0, m_W^2, m_a^2, m_t^2) \right) \tag{5.35c}$$

$$= \frac{\text{div}}{128\pi^2} \frac{g^2 m_t^2}{m_W^2} \left(1 - \frac{c_W^2 m_Z^2}{m_W^2} \right) + \text{finite terms.} \tag{5.35d}$$

Thus, we get a divergent result for δg_{Lb}^c for models with $m_W \neq m_Z c_W$.

[2] To compute the SM contribution to the coupling we used the Feynman rules from appendix D.

Chapter 6

The Georgi-Machacek Model

In this chapter we will apply the results from the previous chapters to a specific model containing scalar triplets: the Georgi-Machacek model to which we will impose an additional $\mathbb{Z}_2$ symmetry which will eliminate the cubic terms, making the model simpler without changing significantly the physics [26]. Namely, we will compute for this model the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex and we will make a fit of δg_{Lb} and δg_{Rb} to the experimental data.

6.1 The model

In 1985, Georgi and Machacek proposed a model [27] which contains one complex doublet with hypercharge $Y = \frac{1}{2}$,

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tag{6.1}$$

one real triplet with hypercharge $Y = 0$,

$$\Lambda = \begin{pmatrix} \lambda^+ \\ \lambda^0 \\ -\lambda^- \end{pmatrix}, \tag{6.2}$$

and one complex triplet with hypercharge $Y = 1$,

$$\Xi = \begin{pmatrix} \xi^{++} \\ \xi^+ \\ \xi^0 \end{pmatrix}. \tag{6.3}$$

These fields can be written in the matrix form

$$\Phi = \begin{pmatrix} \varphi^{0*} & \varphi^{+} \\ -\varphi^{-} & \varphi^{0} \end{pmatrix}, \qquad \Psi = \begin{pmatrix} \xi^{0*} & \lambda^{+} & \xi^{++} \\ -\xi^{-} & \lambda^{0} & \xi^{+} \\ \xi^{--} & -\lambda^{-} & \xi^{0} \end{pmatrix}. \tag{6.4}$$

These matrices transform under a global $SU(2)_L \times SU(2)_R$ symmetry as $\Phi \to U_{L2}\Phi U_{R2}^\dagger$ and $\Psi \to U_{L3}\Psi U_{R3}^\dagger$, where $U_{R,L\,2} = \exp\left(it^a\theta^a_{L,R}\right)$ and $U_{R,L\,3} = \exp\left(iT^a\theta^a_{L,R}\right)$, being t^a and T^a the two and three dimensional generators of $SU(2)$, respectively.

The most general potential which is invariant under the global $SU(2)_L \times SU(2)_R$ symmetry and also under a $\mathbb{Z}_2$ symmetry $\Psi \to -\Psi$ is [46]

$$\begin{aligned} V =& \frac{\alpha_2^2}{2}\operatorname{Tr}\left(\Phi^\dagger\Phi\right) + \frac{\alpha_3^2}{2}\operatorname{Tr}\left(\Psi^\dagger\Psi\right) + \beta_1(\operatorname{Tr}(\Phi^\dagger\Phi))^2 + \beta_2\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\operatorname{Tr}\left(\Psi^\dagger\Psi\right) \\ &+ \beta_3\operatorname{Tr}\left(\Psi^\dagger\Psi\Psi^\dagger\Psi\right) + \beta_4(\operatorname{Tr}\left(\Psi^\dagger\Psi\right))^2 - \beta_5\operatorname{Tr}\left(\Phi^\dagger t^a\Phi t^b\right)\operatorname{Tr}\left(\Psi^\dagger T^a\Psi T^b\right), \end{aligned} \tag{6.5}$$

where α_i for $i \in \{2,3\}$ and β_j for $j \in \{1, ..., 5\}$ are all real parameters because each trace term is also real.

This potential admits a vacuum structure such that $\langle 0|\varphi^0|0\rangle = a/\sqrt{2}$, $\langle 0|\lambda^0|0\rangle = b$ and $\langle 0|\xi^0|0\rangle = b$, where $a, b \in \mathbb{R}$ and are related to the parameters couplings of the model by

$$\frac{\alpha_2^2}{2} + 2a^2\beta_1 + 3b^2\beta_2 - \frac{3}{2}b^2\beta_5 = 0, \qquad \alpha_3^2 + 2a^2\beta_2 + 4b^2\beta_3 + 12b^2\beta_4 - a^2\beta_5 = 0. \tag{6.6}$$

This means that, in our notation, we have $n_d = 1$, $n_{t_1} = 1$, $n_{t_0} = 1$, $n_{s_1} = 0$, $n_{s_0} = 0$, $n_{s_2} = 0$, $n_0 = 5$, $n_1 = 3$, $n_2 = 1$. We also have $v = v_1 = a$, $x = x_1 = b$, $w = w_1 = \sqrt{2}b$. We can then write the masses of the W and Z bosons as $m_W^2 = \frac{g^2}{4}(a^2 + 8b^2)$ and $m_Z^2 = \frac{g^2}{4c_W^2}(a^2 + 8b^2)$, such that we have $m_W = m_Z c_W$. The fact that we have $m_W = m_Z c_W$ at tree-level in the GM model is due to a custodial $SU(2)$ symmetry that remains unbroken by the VEVs in equation 6.6.

Using equations 3.26 and 3.28, we can write for the GM model

$$(U_1)_{11} = \frac{a}{\sqrt{a^2 + 8b^2}}, \qquad (U_3)_{11} = \frac{2b}{\sqrt{a^2 + 8b^2}}, \qquad (U_4)_{11} = \frac{2b}{\sqrt{a^2 + 8b^2}}, \tag{6.7a}$$

$$(V_1)_{11} = i\frac{a}{\sqrt{a^2 + 8b^2}}, \qquad (V_2)_{11} = i\frac{2\sqrt{2}b}{\sqrt{a^2 + 8b^2}}. \tag{6.7b}$$

We also have that the matrix T_1, defined in 3.11c, is a 1×1 matrix that obeys $|T_1|^2 = 1$. Thus, T_1 is given by $e^{i\theta}$, where θ is an arbitrary real parameter which we choose to be 0, such that we get $T_1 = 1$. Changing the value of θ would only change the phase of the field S_1^{++} but this phase is arbitrary as it does not have consequences in the physical predictions of the model.

We can write the mass terms for the double charged scalars in the potential as

$$V_{M_{\pm\pm}} = \left(8\beta_3 b^2 + \frac{3}{2}\beta_5 a^2\right)\xi^{++}\xi^{--}. \tag{6.8}$$

Thus we have $M_1^2 = 8\beta_3 b^2 + \frac{3}{2}\beta_5 a^2$.

We can write the mass terms for the single charged scalars in the potential as

$$V_{M_\pm} = \begin{pmatrix}\varphi^- & \lambda^- & \xi^-\end{pmatrix} M_\pm^2 \begin{pmatrix}\varphi^+ \\ \lambda^+ \\ \xi^+\end{pmatrix}, \tag{6.9}$$

where

$$M_\pm^2 = \begin{pmatrix} 4b^2\beta_5 & -ab\beta_5 & -ab\beta_5 \\ -ab\beta_5 & 4b^2\beta_3 + a^2\beta_5 & -4\beta_3 b^2 - \frac{1}{2}\beta_5 a^2 \\ -ab\beta_5 & -4\beta_3 b^2 - \frac{1}{2}\beta_5 a^2 & 4b^2\beta_3 + a^2\beta_5 \end{pmatrix} \tag{6.10}$$

This matrix can be written as $M_\pm^2 = X^\dagger D_\pm X$, where

$$X = \begin{pmatrix} \frac{a}{\sqrt{a^2+8b^2}} & \frac{2b}{\sqrt{a^2+8b^2}} & \frac{2b}{\sqrt{a^2+8b^2}} \\ -\frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{2}\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{2}\sqrt{a^2+8b^2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{6.11}$$

and

$$D_\pm = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}\beta_5(a^2+8b^2) & 0 \\ 0 & 0 & 8b^2\beta_3 + \frac{3}{2}a^2\beta_5 \end{pmatrix}. \tag{6.12}$$

We can then write the mass terms for the single charged scalars in the potential as

$$V_{M_\pm} = \begin{pmatrix}\varphi^- & \lambda^- & \xi^-\end{pmatrix} X^\dagger D_\pm X \begin{pmatrix}\varphi^+ \\ \lambda^+ \\ \xi^+\end{pmatrix} = \begin{pmatrix}S_1^- & S_2^- & S_3^-\end{pmatrix} D_\pm \begin{pmatrix}S_1^+ \\ S_2^+ \\ S_3^+\end{pmatrix}. \tag{6.13}$$

We thus have $m_2^2 = \frac{1}{2}\beta_5(a^2+8b^2)$ and $m_3^2 = 8b^2\beta_3 + \frac{3}{2}a^2\beta_5$. We can also identify matrix $X^\dagger$ with matrix $\tilde{U}$ defined in equation 3.12, such that

$$U_1 = \begin{pmatrix} \frac{a}{\sqrt{a^2+8b^2}} & -\frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & 0 \end{pmatrix}, \qquad U_3 = \begin{pmatrix} \frac{2b}{\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{2}\sqrt{a^2+8b^2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{6.14a}$$

$$U_4 = \begin{pmatrix} \frac{2b}{\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{2}\sqrt{a^2+8b^2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{6.14b}$$

This confirms equation 6.7a.

We can write the mass terms for the neutral scalars in the potential as

$$V_{M_0} = \begin{pmatrix} \mathrm{Re}\,\varphi^{0\prime} & \mathrm{Im}\,\varphi^{0\prime} & \mathrm{Re}\,\xi^{0\prime} & \mathrm{Im}\,\xi^{0\prime} & \lambda^{0\prime} \end{pmatrix} M_0^2 \begin{pmatrix} \mathrm{Re}\,\varphi^{0\prime} \\ \mathrm{Im}\,\varphi^{0\prime} \\ \mathrm{Re}\,\xi^{0\prime} \\ \mathrm{Im}\,\xi^{0\prime} \\ \lambda^{0\prime} \end{pmatrix}, \tag{6.15}$$

where

$$M_0^2 = \begin{pmatrix} 8a^2\beta_1 & 0 & 4\sqrt{2}ab\beta_2 - 2\sqrt{2}ab\beta_5 & 0 & 4ab\beta_2 - 2ab\beta_5 \\ 0 & 4b^2\beta_5 & 0 & -\sqrt{2}ab\beta_5 & 0 \\ 4\sqrt{2}ab\beta_2 - 2\sqrt{2}ab\beta_5 & 0 & 8b^2\beta_3 + 16b^2\beta_4 + \frac{1}{2}a^2\beta_5 & 0 & 8\sqrt{2}b^2\beta_4 - \frac{1}{\sqrt{2}}a^2\beta_5 \\ 0 & -\sqrt{2}ab\beta_5 & 0 & \frac{1}{2}a^2\beta_5 & 0 \\ 4ab\beta_2 - 2ab\beta_5 & 0 & 8\sqrt{2}b^2\beta_4 - \frac{1}{\sqrt{2}}a^2\beta_5 & 0 & 8b^2\beta_3 + 8b^2\beta_4 + a^2\beta_5 \end{pmatrix}. \tag{6.16}$$

This matrix can be written as $M_0^2 = Y^T D_0 Y$, where

$$Y = \begin{pmatrix} 0 & \frac{a}{\sqrt{a^2+8b^2}} & 0 & \frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & 0 \\ 0 & -\frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & 0 & \frac{a}{\sqrt{a^2+8b^2}} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{3} \\ \frac{k-\sqrt{k^2+j^2}}{\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} & 0 & \frac{\sqrt{6}j}{3\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} & 0 & \frac{\sqrt{3}j}{3\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} \\ \frac{k+\sqrt{k^2+j^2}}{\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}} & 0 & \frac{\sqrt{6}j}{3\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}} & 0 & \frac{\sqrt{3}j}{3\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}} \end{pmatrix}, \tag{6.17}$$

where $k = 2a^2\beta_1 - 2b^2(\beta_3 + 3\beta_4)$ and $j = \sqrt{3}ab(2\beta_2 - \beta_5)$ and

$$D_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_2^2 & 0 & 0 & 0 \\ 0 & 0 & \mu_3^2 & 0 & 0 \\ 0 & 0 & 0 & \mu_4^2 & 0 \\ 0 & 0 & 0 & 0 & \mu_5^2 \end{pmatrix}, \tag{6.18}$$

with

$$\mu_2^2 = \frac{1}{2}\beta_5(a^2 + 8b^2), \tag{6.19a}$$

$$\mu_3^2 = 8b^2\beta_3 + \frac{3}{2}a^2\beta_5, \tag{6.19b}$$

$$\mu_4^2 = 4a^2\beta_1 + 4b^2(\beta_3 + 3\beta_4) - 2\sqrt{(2a^2\beta_1 - 2b^2(\beta_3 + 3\beta_4))^2 + 3a^2b^2(\beta_5 - 2\beta_2)^2}, \tag{6.19c}$$

$$\mu_5^2 = 4a^2\beta_1 + 4b^2(\beta_3 + 3\beta_4) + 2\sqrt{(2a^2\beta_1 - 2b^2(\beta_3 + 3\beta_4))^2 + 3a^2b^2(\beta_5 - 2\beta_2)^2}. \tag{6.19d}$$

We can then write the mass terms for the single charged scalars in the potential as

$$V_{M_0} = \begin{pmatrix}\mathrm{Re}\,\varphi^{0\prime} & \mathrm{Im}\,\varphi^{0\prime} & \mathrm{Re}\,\xi^{0\prime} & \mathrm{Im}\,\xi^{0\prime} & \lambda^{0\prime}\end{pmatrix} Y^T D_0 Y \begin{pmatrix}\mathrm{Re}\,\varphi^{0\prime}\\ \mathrm{Im}\,\varphi^{0\prime}\\ \mathrm{Re}\,\xi^{0\prime}\\ \mathrm{Im}\,\xi^{0\prime}\\ \lambda^{0\prime}\end{pmatrix} = \tag{6.20a}$$

$$= \begin{pmatrix}S_1^0 & S_2^0 & S_3^0 & S_4^0 & S_5^0\end{pmatrix} D_0 \begin{pmatrix}S_1^0\\ S_2^0\\ S_3^0\\ S_4^0\\ S_5^0\end{pmatrix}. \tag{6.20b}$$

We can also identify matrix Y^T with matrix $\tilde{V}$ from equation 3.13, such that

$$\mathrm{Re}\,V_1 = \begin{pmatrix}0 & 0 & 0 & \frac{k-\sqrt{k^2+j^2}}{\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} & \frac{k+\sqrt{k^2+j^2}}{\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}}\end{pmatrix}, \tag{6.21a}$$

$$\mathrm{Im}\,V_1 = \begin{pmatrix}\frac{a}{\sqrt{a^2+8b^2}} & -\frac{\sqrt{8}b}{\sqrt{a^2+8b^2}} & 0 & 0 & 0\end{pmatrix}, \tag{6.21b}$$

$$\mathrm{Re}\,V_2 = \begin{pmatrix}0 & 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{6}j}{3\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} & \frac{\sqrt{6}j}{3\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}}\end{pmatrix}, \tag{6.21c}$$

$$\mathrm{Im}\,V_2 = \begin{pmatrix}\frac{\sqrt{8}b}{\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{a^2+8b^2}} & 0 & 0 & 0\end{pmatrix}, \tag{6.21d}$$

$$R_1 = \begin{pmatrix}0 & 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}j}{3\sqrt{j^2+(k-\sqrt{k^2+j^2})^2}} & \frac{\sqrt{3}j}{3\sqrt{j^2+(k+\sqrt{k^2+j^2})^2}}\end{pmatrix}. \tag{6.21e}$$

This confirms equation 6.7b.

The Boundedness From Below (BFB) conditions for the GM model are [47, 48]

$$\beta_1 > 0, \tag{6.22a}$$

$$\beta_4 > \begin{cases}-\frac{1}{3}\beta_3, & \beta_3 \geq 0,\\ -\beta_3, & \beta_3 < 0,\end{cases} \tag{6.22b}$$

$$\beta_2 > \begin{cases}\frac{1}{2}\beta_5 - 2\sqrt{\beta_1(\frac{1}{3}\beta_3 + \beta_4)}, & \beta_5 \geq 0 \text{ and } \beta_3 \geq 0,\\ \omega_+(\zeta)\beta_5 - 2\sqrt{\beta_1(\zeta\beta_3 + \beta_4)}, & \beta_5 \geq 0 \text{ and } \beta_3 < 0,\\ \omega_-(\zeta)\beta_5 - 2\sqrt{\beta_1(\zeta\beta_3 + \beta_4)}, & \beta_5 < 0,\end{cases} \tag{6.22c}$$

where

$$\omega_\pm(\zeta) = \frac{1}{6}(1-B) \pm \frac{\sqrt{2}}{3}\left((1-B)\left(\frac{1}{2}+B\right)\right)^{1/2}, \tag{6.23}$$

and $B \equiv \sqrt{\frac{3}{2}\left(\zeta - \frac{1}{3}\right)}$. These conditions must be satisfied for all values $\zeta \in \left[\frac{1}{3}, 1\right]$.

Defining the quantities [49]

$$x_1^{\pm} = 12\beta_1 + 22\beta_4 + 14\beta_3 \pm \sqrt{(12\beta_1 - 22\beta_4 - 14\beta_3)^2 + 144\beta_2^2}, \tag{6.24a}$$

$$x_2^{\pm} = 4\beta_1 + 4\beta_4 - 2\beta_3 \pm \sqrt{(4\beta_1 - 4\beta_4 + 2\beta_3)^2 + 4\beta_5^2}, \tag{6.24b}$$

$$x_3^{\pm} = 4\beta_1 + 4\beta_4 \pm \sqrt{(4\beta_4 - 4\beta_1)^2 + 4\beta_5^2}, \tag{6.24c}$$

$$x_4^{\pm} = 8\beta_1 + 4\beta_4 - 2\beta_3 \pm \sqrt{(8\beta_1 - 4\beta_4 + 2\beta_3)^2 + 8\beta_5^2}, \tag{6.24d}$$

$$x_5^{\pm} = 12\beta_4 + 14\beta_3 \pm 2\sqrt{4\beta_4^2 + 4\beta_4\beta_3 + 17\beta_3^2}, \tag{6.24e}$$

$$y_1 = 8\beta_4 + 16\beta_3, \tag{6.24f}$$

$$y_2 = 8\beta_4 + 4\beta_3, \tag{6.24g}$$

$$y_3 = 4\beta_2 - \beta_5, \tag{6.24h}$$

$$y_4 = 4\beta_2 + 2\beta_5, \tag{6.24i}$$

$$y_5 = 4(\beta_2 - \beta_5), \tag{6.24j}$$

$$y_6 = 8\beta_4 + 4(2 + \sqrt{2})\beta_3, \tag{6.24k}$$

$$y_7 = 8\beta_4 + 4(2 - \sqrt{2})\beta_3, \tag{6.24l}$$

the unitarity conditions for the GM model are [49]

$$|x_1^{\pm}|,\ |x_2^{\pm}|,\ |x_3^{\pm}|,\ |x_4^{\pm}|,\ |x_5^{\pm}|,\ |y_1|,\ |y_2|,\ |y_3|,\ |y_4|,\ |y_5|,\ |y_6|,\ |y_7| < 8\pi. \tag{6.25}$$

6.2 Oblique parameters

Using the results presented in the previous section we can compute the oblique parameters in this model.

The oblique parameter T in the GM model becomes

$$\begin{aligned}
\alpha T = \left(\frac{g}{4\pi c_W m_Z}\right)^2 \Bigg(& \sum_{a=2}^{3}\sum_{b=2}^{5} \left(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\right) \times \\
& \times \left(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\right) F(m_a^2, \mu_b^2) && (6.26a) \\
& + \sum_{b=2}^{5} \left(\frac{1}{2}\,\mathrm{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\,\mathrm{Re}(U_4^\dagger V_2)_{1b} + \mathrm{Re}(U_3^\dagger R_1)_{1b}\right)^2 \times \\
& \times \Big(2(m_W^2 + \mu_b^2) - 3F(m_W^2, \mu_b^2)\Big) && (6.26b) \\
& + \sum_{a=2}^{3} |(U_4)_{1a}|^2 F(M_1^2, m_a^2) && (6.26c)
\end{aligned}$$

$$+ 2\frac{b^2}{a^2+8b^2}\left(-3F(m_Z^2,m_3^2)+2(m_Z^2+m_3^2)\right) \tag{6.26d}$$

$$+ \frac{4b^2}{a^2+8b^2}\Big(-2(m_W^2+M_1^2)-3F(m_W^2,M_1^2)\Big) \tag{6.26e}$$

$$- \frac{1}{2}(m_h^2+m_W^2)+\frac{3}{4}F(m_h^2,m_W^2) \tag{6.26f}$$

$$- \sum_{b=2}^{4}\sum_{b'=b+1}^{5}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'}+\mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 F(\mu_b^2,\mu_{b'}^2) \tag{6.26g}$$

$$- \frac{1}{4}\sum_{b=2}^{5}\Big(\mathrm{Im}(V_1^\dagger V_1)_{1b}+2\,\mathrm{Im}(V_2^\dagger V_2)_{1b}\Big)^2(2(m_Z^2+\mu_b^2)-3F(m_Z^2,\mu_b^2)) \tag{6.26h}$$

$$- \frac{a^2}{2(a^2+8b^2)}F(m_2^2,m_3^2) \tag{6.26i}$$

$$- 4\frac{b^2}{a^2+8b^2}\left(-3F(m_W^2,m_3^2)+2(m_W^2+m_3^2)\right)+\frac{1}{2}(m_h^2+m_Z^2) \tag{6.26j}$$

$$- \frac{3}{4}F(m_h^2,m_Z^2)-4\frac{b^2}{a^2+8b^2}m_1^2(\mathsf{div}-\log m_1^2) \tag{6.26k}$$

$$+ 6\frac{b^2}{a^2+8b^2}m_W^2(\mathsf{div}-\log m_W^2)+4\frac{b^2}{a^2+8b^2}\mu_1^2(\mathsf{div}-\log\mu_1^2) \tag{6.26l}$$

$$+ 3\frac{b^2}{a^2+8b^2}m_Z^2(\mathsf{div}-\log m_Z^2)\Bigg). \tag{6.26m}$$

The oblique parameter S in the GM model becomes

$$\frac{\alpha}{4s_W^2c_W^2}S =$$

$$=\frac{g^2}{192\pi^2c_W^2}\Bigg(4\sum_{b=2}^{4}\sum_{b'=b+1}^{5}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'}+\mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 K(\mu_b^2,\mu_{b'}^2) \tag{6.27a}$$

$$+\sum_{b=2}^{5}\Big(\mathrm{Im}(V_1^\dagger V_1)_{1b}+2\,\mathrm{Im}(V_2^\dagger V_2)_{1b}\Big)^2(K(\mu_b^2,m_Z^2)-6m_Z^2\tilde{K}(\mu_b^2,m_Z^2)) \tag{6.27b}$$

$$+4\sum_{a,a'=2}^{3}\Big(s_W^2\delta_{aa'}-\frac{1}{2}(U_1^\dagger U_1)_{aa'}-(U_3^\dagger U_3)_{aa'}\Big)\times$$

$$\times\Big(s_W^2\delta_{a'a}-\frac{1}{2}(U_1^\dagger U_1)_{aa'}-(U_3^\dagger U_3)_{a'a}\Big)K(m_a^2,m_{a'}^2) \tag{6.27c}$$

$$+\frac{16b^2}{a^2+8b^2}(K(m_3^2,m_W^2)-6m_W^2\tilde{K}(m_3^2,m_W^2)) \tag{6.27d}$$

$$-K(m_h^2,m_Z^2)+6m_Z^2\tilde{K}(m_h^2,m_Z^2)-4s_W^2c_W^2\sum_{a=2}^{3}\log m_a^2 \tag{6.27e}$$

$$+4(c_W^2-s_W^2)\sum_{a=2}^{3}\Big(s_W^2-\frac{1}{2}(U_1^\dagger U_1)_{aa}-(U_3^\dagger U_3)_{aa}\Big)\log m_a^2-4\log M_1^2\Bigg). \tag{6.27f}$$

Thus, we get (as expected) a gauge independent and finite result for oblique parameter S in the GM model.

The oblique parameter U in the GM model becomes

$$\frac{\alpha}{4s_W^2}U = \frac{g^2}{192\pi^2}\Bigg(4\sum_{a=2}^{3}\sum_{b=2}^{5}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big)\times$$

$$\times\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big)K(\mu_b^2, m_a^2) \tag{6.28a}$$

$$+ 4\sum_{b=2}^{5}\Big(\frac{1}{2}\,\mathrm{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\,\mathrm{Re}(U_4^\dagger V_2)_{1b} + \mathrm{Re}(U_3^\dagger R_1)_{1b}\Big)^2\times$$

$$\times\left(K(\mu_b^2, m_W^2) - 6m_W^2\tilde{K}(\mu_b^2, m_W^2)\right) \tag{6.28b}$$

$$+ 4\sum_{a=2}^{3}|(U_4)_{1a}|^2 K(m_a^2, M_1^2) \tag{6.28c}$$

$$+ \frac{8b^2}{a^2+8b^2}(K(m_3^2, m_Z^2) - 6m_Z^2\tilde{K}(m_3^2, m_Z^2)) \tag{6.28d}$$

$$+ \frac{16b^2}{a^2+8b^2}(K(M_1^2, m_W^2) - 6m_W^2\tilde{K}(M_1^2, m_W^2)) \tag{6.28e}$$

$$- K(m_h^2, m_W^2) + 6m_W^2\tilde{K}(m_h^2, m_W^2) \tag{6.28f}$$

$$- 4\sum_{b=2}^{4}\sum_{b'=b+1}^{5}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'} + \mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 K(\mu_b^2, \mu_{b'}^2) \tag{6.28g}$$

$$- \sum_{b=2}^{5}\Big(\mathrm{Im}(V_1^\dagger V_1)_{1b} + 2\,\mathrm{Im}(V_2^\dagger V_2)_{1b}\Big)^2(K(\mu_b^2, m_Z^2) - 6m_Z^2\tilde{K}(\mu_b^2, m_Z^2)) \tag{6.28h}$$

$$- 4\sum_{a,a'=2}^{3}\Big(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{aa'}\Big)\times$$

$$\times\Big(s_W^2\delta_{a'a} - \frac{1}{2}(U_1^\dagger U_1)_{aa'} - (U_3^\dagger U_3)_{a'a}\Big)K(m_a^2, m_{a'}^2) \tag{6.28i}$$

$$- \frac{16b^2}{a^2+8b^2}(K(m_3^2, m_W^2) - 6m_W^2\tilde{K}(m_3^2, m_W^2)) \tag{6.28j}$$

$$+ K(m_h^2, m_Z^2) - 6m_Z^2\tilde{K}(m_h^2, m_Z^2) - 4s_W^4\sum_{a=2}^{3}\log m_a^2 \tag{6.28k}$$

$$+ 8s_W^2\sum_{a=2}^{3}\Big(s_W^2 - \frac{1}{2}(U_1^\dagger U_1)_{aa} - (U_3^\dagger U_3)_{aa}\Big)\log m_a^2 - 4\log M_1^2\Bigg). \tag{6.28l}$$

We also obtain, in this model, a gauge independent and finite result for the U parameter.

The oblique parameter V in the GM model becomes

$$\alpha V = \frac{g^2}{384\pi^2 c_W^2}\Bigg(4\sum_{b=2}^{4}\sum_{b'=b+1}^{5}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'} + \mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)^2 H(\mu_b^2, \mu_{b'}^2, m_Z^2) \tag{6.29a}$$

$$+\sum_{b=2}^{5}(\mathrm{Im}(V_1^\dagger V_1)_{1b}+2\,\mathrm{Im}(V_2^\dagger V_2)_{1b})^2\big(12\tilde{H}(\mu_b^2,m_Z^2,m_Z^2)+H(\mu_b^2,m_Z^2,m_Z^2)\big) \tag{6.29b}$$

$$+4\sum_{a,a'=2}^{3}\Big(s_W^2\delta_{aa'}-\frac{1}{2}(U_1^\dagger U_1)_{aa'}-(U_3^\dagger U_3)_{aa'}\Big)\times$$

$$\times\Big(s_W^2\delta_{a'a}-\frac{1}{2}(U_1^\dagger U_1)_{a'a}-(U_3^\dagger U_3)_{a'a}\Big)H(m_a^2,m_{a'}^2,m_Z^2) \tag{6.29c}$$

$$+\frac{16b^2}{a^2+8b^2}\Big(12\tilde{H}(m_3^2,m_W^2,m_Z^2)+H(m_3^2,m_W^2,m_Z^2)\Big) \tag{6.29d}$$

$$+4(2s_W^2-1)^2H(M_1^2,M_1^2,m_Z^2) \tag{6.29e}$$

$$-12\tilde{H}(m_h^2,m_Z^2,m_Z^2)-H(m_h^2,m_Z^2,m_Z^2)\Bigg). \tag{6.29f}$$

The oblique parameter W in the GM model becomes

$$\alpha W=\frac{g^2}{384\pi^2}\Bigg(4\sum_{a=2}^{3}\sum_{b=2}^{5}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab}+\frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab}+(U_3^\dagger R_1)_{ab}\Big)\times$$

$$\times\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba}+\frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba}+(R_1^T U_3)_{ba}\Big)H(\mu_b^2,m_a^2,m_W^2) \tag{6.30a}$$

$$+4\sum_{b=2}^{5}\Big(\frac{1}{2}\,\mathrm{Re}(U_1^\dagger V_1)_{1b}+\frac{1}{\sqrt{2}}\,\mathrm{Re}(U_4^\dagger V_2)_{1b}+\mathrm{Re}(U_3^\dagger R_1)_{1b}\Big)^2\times$$

$$\times\Big(12\tilde{H}(\mu_b^2,m_W^2,m_W^2)+H(\mu_b^2,m_W^2,m_W^2)\Big) \tag{6.30b}$$

$$+4\sum_{a=2}^{3}|(U_4)_{1a}|^2H(m_a^2,M_1^2,m_W^2) \tag{6.30c}$$

$$+\frac{8b^2}{a^2+8b^2}\Big(12\tilde{H}(m_3^2,m_Z^2,m_W^2)+H(m_3^2,m_Z^2,m_W^2)\Big) \tag{6.30d}$$

$$+\frac{16b^2}{a^2+8b^2}\Big(12\tilde{H}(M_1^2,m_W^2,m_W^2)+H(M_1^2,m_W^2,m_W^2)\Big) \tag{6.30e}$$

$$-12\tilde{H}(m_h^2,m_W^2,m_W^2)-H(m_h^2,m_W^2,m_W^2)\Bigg). \tag{6.30f}$$

The oblique parameter X in the GM model becomes

$$\frac{\alpha}{s_W c_W}X=-\frac{eg}{96\pi^2 c_W}\left(s_W^2-\frac{1}{2}\right)\left(G(m_2^2,m_2^2,m_Z^2)+G(m_3^2,m_3^2,m_Z^2)+4G(M_1^2,M_1^2,m_Z^2)\right) \tag{6.31a}$$

In fact, the potential 6.5 is not the most general one invariant under $SU(2)_L\times U(1)$ that we might write. The $SU(2)_R$ symmetry is imposed by hand and will thus be violated at one-loop level. Therefore, despite having $m_W=m_Z c_W$ at the tree-level, when performing one-loop calculations we get divergences. That is why we get a divergent result for the oblique parameter T. The most general scalar potential invariant under $SU(2)_L \times U(1)$ with one scalar doublet with hypercharge $Y = \frac{1}{2}$, one scalar triplet with hypercharge $Y = 0$ and scalar triplet with hypercharge $Y = 1$ (the scalar content of the GM model) that we can write is

$$V = \mu_1^2\, \phi^\dagger \phi + \mu_2^2\, \Xi^\dagger \Xi + \mu_3^2\, \Lambda^\dagger \Lambda + \lambda_1\, (\phi^\dagger \phi)^2 + \lambda_2\, (\Xi^\dagger \Xi)^2 + \lambda_3\, (\Lambda^\dagger \Lambda)^2 \tag{6.32a}$$

$$+ \lambda_4\, (\phi^\dagger \phi)(\Xi^\dagger \Xi) + \lambda_5\, (\phi^\dagger \phi)(\Lambda^\dagger \Lambda) + \lambda_6\, (\Xi^\dagger \Xi)(\Lambda^\dagger \Lambda) + \lambda_7\, (\Xi^\dagger \Lambda)(\Lambda^\dagger \Xi) \tag{6.32b}$$

$$+ \lambda_8\, \theta^\dagger \Xi + \lambda_8^*\, \Xi^\dagger \theta + \lambda_9\, \gamma^\dagger \Lambda + \lambda_9\, \Lambda^\dagger \gamma + \lambda_{10}\, \rho^\dagger \sigma + \lambda_{10}^*\, \sigma^\dagger \rho + \lambda_{11}\, \sigma^\dagger \sigma \tag{6.32c}$$

$$+ \lambda_{12}\, \zeta^\dagger \zeta + \lambda_{13}\, \omega^\dagger \Lambda + \lambda_{13}^*\, \Lambda^\dagger \omega + \lambda_{14}\, \omega^\dagger \omega + \lambda_{15}\, \gamma^\dagger \omega \tag{6.32d}$$

$$+ \lambda_{16}\, \eta^\dagger \tau + \lambda_{16}^*\, \tau^\dagger \eta + \lambda_{17}\, \tau^\dagger \tau, \tag{6.32e}$$

where

$$\theta \equiv (\phi \otimes \phi)_3 = \begin{bmatrix} \varphi^+ \varphi^+ \\ \sqrt{2}\varphi^+ \varphi^0 \\ \varphi^0 \varphi^0 \end{bmatrix} \tag{6.33}$$

is a $SU(2)$ triplet with hypercharge $Y = 1$,

$$\gamma \equiv (\phi \otimes \tilde{\phi})_3 = \begin{bmatrix} \varphi^+ \varphi^{0*} \\ \frac{1}{\sqrt{2}}(\varphi^{0*}\varphi^0 - \varphi^+ \varphi^-) \\ -\varphi^0 \varphi^- \end{bmatrix} \tag{6.34}$$

is a $SU(2)$ triplet with hypercharge $Y = 0$,

$$\rho \equiv (\phi \otimes \Lambda)_2 = \begin{bmatrix} \sqrt{\frac{2}{3}}\lambda^+ \varphi^0 - \sqrt{\frac{1}{3}}\lambda^0 \varphi^+ \\ \sqrt{\frac{1}{3}}\lambda^0 \varphi^0 + \sqrt{\frac{2}{3}}\lambda^- \varphi^+ \end{bmatrix} \tag{6.35}$$

is a $SU(2)$ doublet with hypercharge $Y = \frac{1}{2}$,

$$\sigma \equiv (\phi \otimes \Xi)_2 = \begin{bmatrix} -\sqrt{\frac{2}{3}}\varphi^- \xi^{++} - \sqrt{\frac{1}{3}}\xi^+ \varphi^{0*} \\ -\sqrt{\frac{1}{3}}\varphi^- \xi^+ - \sqrt{\frac{2}{3}}\varphi^{0*} \xi^0 \end{bmatrix} \tag{6.36}$$

is a $SU(2)$ doublet with hypercharge $Y = \frac{1}{2}$,

$$\zeta \equiv (\Xi \otimes \Xi)_1 = \frac{1}{\sqrt{3}}(2\xi^{++}\xi^0 - \xi^+ \xi^+) \tag{6.37}$$

is a $SU(2)$ singlet with hypercharge $Y = 2$,

$$\omega \equiv (\Xi \otimes \tilde{\Xi})_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} -\xi^{++}\xi^- - \xi^+ \xi^{0*} \\ \xi^{++}\xi^{--} - \xi^0 \xi^{0*} \\ \xi^+ \xi^{--} + \xi^0 \xi^- \end{bmatrix} \tag{6.38}$$

is a $SU(2)$ triplet with hypercharge $Y = 0$,

$$\tau \equiv (\Xi \otimes \tilde{\Xi})_5 = \begin{bmatrix} \xi^{++}\xi^{0*} \\ \frac{1}{\sqrt{2}}(\xi^{0*}\xi^{+} - \xi^{++}\xi^{-}) \\ \frac{1}{\sqrt{6}}(\xi^{++}\xi^{--} + \xi^{0}\xi^{0*} - 2\xi^{+}\xi^{-}) \\ \frac{1}{\sqrt{2}}(\xi^{--}\xi^{+} - \xi^{0}\xi^{-}) \\ \xi^{--}\xi^{0} \end{bmatrix} \tag{6.39}$$

is a $SU(2)$ quintuplet with hypercharge $Y = 0$ and

$$\eta \equiv (\Lambda \otimes \Lambda)_5 = \begin{bmatrix} \lambda^{+}\lambda^{+} \\ \sqrt{2}\lambda^{+}\lambda^{0} \\ \sqrt{\frac{2}{3}}(\lambda^{0}\lambda^{0} - \lambda^{+}\lambda^{-}) \\ -\sqrt{2}\lambda^{-}\lambda^{0} \\ \lambda^{-}\lambda^{-} \end{bmatrix} \tag{6.40}$$

is a $SU(2)$ quintuplet with hypercharge $Y = 0$.

To obtain the GM potential (equation 6.5) from equation 6.32, we must set the coefficients λ_8, λ_9, λ_{13}, λ_{14}, λ_{15}, λ_{16} and λ_{17} to 0 and relate the other coefficients according to

$$\mu_2^2 = 2\mu_3^2, \qquad \lambda_2 = \lambda_6 - \frac{2\sqrt{2}}{3}\lambda_{10}, \qquad \lambda_3 = \frac{1}{4}\lambda_6 - \frac{\sqrt{2}}{3}\lambda_{10}, \tag{6.41a}$$

$$\lambda_4 = 2\lambda_5 + \frac{\sqrt{2}}{3}\lambda_{10}, \qquad \lambda_{11} = -\sqrt{2}\lambda_{10}, \qquad \lambda_{12} = \frac{3}{2}\lambda_6, \tag{6.41b}$$

such that we can write the GM potential as

$$V = \mu_1^2\, \phi^\dagger\phi + 2\mu_3^2\, \Xi^\dagger\Xi + \mu_3^2\, \Lambda^\dagger\Lambda + \lambda_1\, (\phi^\dagger\phi)^2 + \left(\lambda_6 - \frac{2\sqrt{2}}{3}\lambda_{10}\right)(\Xi^\dagger\Xi)^2 \tag{6.42a}$$

$$+ \left(\frac{1}{4}\lambda_6 - \frac{\sqrt{2}}{3}\lambda_{10}\right)(\Lambda^\dagger\Lambda)^2 + \left(2\lambda_5 + \frac{\sqrt{2}}{3}\lambda_{10}\right)(\phi^\dagger\phi)(\Xi^\dagger\Xi) \tag{6.42b}$$

$$+ \lambda_5\, (\phi^\dagger\phi)(\Lambda^\dagger\Lambda) + \lambda_6\, (\Xi^\dagger\Xi)(\Lambda^\dagger\Lambda) + \lambda_7\, (\Xi^\dagger\Lambda)(\Lambda^\dagger\Xi) + \lambda_{10}\, \rho^\dagger\sigma + \lambda_{10}^*\, \sigma^\dagger\rho \tag{6.42c}$$

$$- \sqrt{2}\lambda_{10}\, \sigma^\dagger\sigma + \frac{3}{2}\lambda_6\, \zeta^\dagger\zeta. \tag{6.42d}$$

6.3 One-loop corrections to the $Zb\bar{b}$ vertex

We will now present the results for the one-loop corrections to the $Zb\bar{b}$ vertex in the GM model.

In this model we do only have one scalar doublet. This implies that we will only have one coupling constant f and one coupling constant e, where f and e are defined in equation 5.25. We can choose these constants to be real due to a freedom on the phase of the fields b_R and t_R. These constants are related to the VEVs of the neutral fields and to the masses of the b and t quarks by

$$f = \frac{\sqrt{2}m_b}{a} \approx 0, \qquad e = \frac{\sqrt{2}m_t}{a}. \tag{6.43}$$

Thus, we can write the constants c_a, d_a and r_l as

$$c_a = e(U_1)^*_{1a} = \frac{\sqrt{2}m_t}{a}(U_1)^*_{1a}, \tag{6.44a}$$

$$d_a = f(U_1)_{1a} = \frac{\sqrt{2}m_b}{a}(U_1)_{1a} \approx 0, \tag{6.44b}$$

$$r_l = -\frac{f}{\sqrt{2}}(V_1)_{1l} = -\frac{m_b}{a}(V_1)_{1l} \approx 0. \tag{6.44c}$$

6.3.1 Charged scalar contribution

The contributions from the diagrams with charged scalars to the one-loop corrections to the $Zb\bar{b}$ vertex δg^c_{Lb} and δg^c_{Rb} can be written as

$$\delta g^c_{Lb} = \frac{m_t^2}{16\pi^2}\left(\frac{2}{a^2}\sum_{a=2}^{3}|(U_1)_{1a}|^2 f^c_L(m_a^2)\right) = \frac{m_t^2}{\pi^2}\frac{b^2}{a^2}\frac{1}{a^2+8b^2}f^c_L(m_2^2), \tag{6.45}$$

$$\delta g^c_{Rb} = 0. \tag{6.46}$$

6.3.2 Neutral scalar contribution

As in this model we have $r_l = 0$, then the contributions from the diagrams with neutral scalars to the one-loop corrections to the $Zb\bar{b}$ vertex δg^n_{Lb} and δg^n_{Rb} are equal to 0 in the approximation $m_b = 0$.

6.4 Numerical fit to the experimental results

The Standard Model predictions for the couplings $g_{L,R\,b}$ are $g_L^{SM} = -0.420875$ and $g_R^{SM} = 0.077362$ [50].

We can relate the observable A_b with the couplings $g_{L,R\,b}$ by [51]

$$A_b = \frac{2r_b\sqrt{1-4\mu_b}}{1-4\mu_b+(1+2\mu_b)r_b^2}, \tag{6.47}$$

where $r_b = \frac{g_{Lb}+g_{Rb}}{g_{Lb}-g_{Rb}}$ and $\mu_b = \frac{\left(m_b(m_Z^2)\right)^2}{m_Z^2}$. We use the numerical values $m_b(m_Z^2) = 3$ GeV and $m_Z = 91.1876$ GeV [6]. Inverting equation 6.47, we get [52]

$$\frac{g_{Lb}}{g_{Rb}} \equiv \varrho = \frac{\sqrt{1-4\mu_b}\left(1 \pm \sqrt{1-(1+2\mu_b)A_b^2}\right) + (1+2\mu_b)A_b}{\sqrt{1-4\mu_b}\left(1 \pm \sqrt{1-(1+2\mu_b)A_b^2}\right) - (1+2\mu_b)A_b}, \tag{6.48}$$

such that we have two solutions for ϱ. We can also relate the observable R_b with the couplings $g_{L,Rb}$ by [41, 52]

$$R_b = \frac{s_b c^{\text{QCD}} c^{\text{QED}}}{s_b c^{\text{QCD}} c^{\text{QED}} + s_c + s_u + s_s + s_d}, \tag{6.49}$$

where $c^{\text{QCD}} = 0.9953$ and $c^{\text{QED}} = 0.99975$ are QCD and QED corrections, respectively, $s_c + s_u + s_s + s_d = 1.3184$ and [41, 52]

$$s_b = (1-6\mu_b)(g_{Lb} - g_{Rb})^2 + (g_{Lb} + g_{Rb})^2 = g_{Rb}^2\left((2-6\mu_b)(1+\varrho^2) + 12\mu_b\varrho\right). \tag{6.50}$$

Using equations 6.49 and 6.50, we get

$$g_{Rb}^2 = \frac{s_c + s_u + s_s + s_d}{c^{\text{QCD}} c^{\text{QED}}\left((2-6\mu_b)(1+\varrho^2) + 12\mu_b\varrho\right)} \frac{R_b}{1-R_b}. \tag{6.51}$$

This equation allows for two signs for g_{Rb}.

Using the SM predictions for the couplings we get $A_b^{SM} = 0.9347$ and $R_b^{SM} = 0.21581$.

An overall fit of various electroweak observables gives [6]

$$R_b^{\text{fit}} = 0.21629 \pm 0.00066, \qquad A_b^{\text{fit}} = 0.923 \pm 0.020. \tag{6.52}$$

We have then that R_b^{fit} deviates from its SM value by 0.7σ and A_b^{fit} deviates from its SM value by 0.6σ. However, making the average of two direct measurements of A_b done at LEP1 and SLAC in two different ways, we get [53]

$$A_b^{\text{average}} = 0.901 \pm 0.013. \tag{6.53a}$$

We get then, a deviation of 2.6σ of A_b^{average} from the SM prediction.

Using the central values of equations 6.52 and 6.53 and equations 6.48 and 6.51, we get the values displayed in table 6.1, where we also present $\delta g_L = g_L + 0.420875$ and $\delta g_R = g_R - 0.077362$.

We can see that in solutions 3 and 4 the value of δg_{Lb} is too large, which indicates that solutions 1 and 2 might be preferred over solutions 3 and 4. Reference [54] claims that there are already a couple LHC points that favour solution 1 over solution 2 and that the high-luminosity-LHC can be decisive to understand which of the solutions is the correct one. On the other hand, reference [55] claims that the PETRA (35 GeV) data favours solution 2 over solution 1.

solution	g_{Lb}	g_{Rb}	δg_{Lb}	δg_{Rb}
1^{fit}	-0.420206	0.084172	0.000669	0.006810
2^{fit}	-0.419934	-0.082806	0.000941	-0.160168
3^{fit}	0.420206	-0.084172	0.841081	-0.161534
4^{fit}	0.419934	0.082806	0.840809	0.005444
1^{average}	-0.417814	0.095496	0.003061	0.018134
2^{average}	-0.417504	-0.094139	0.003371	-0.171501
3^{average}	0.417518	-0.095496	0.838688	-0.172858
4^{average}	0.417504	0.094139	0.838379	0.016777

Table 6.1: Results for g_{Lb} and g_{Rb} computed from the experimental values for A_b and R_b. Solutions labelled by ”fit” were computed using A_b^{fit}, while solutions labelled by ”average” were computed using A_b^{average}.

To make the numerical fit to the experimental data, we will make a further simplification: we will put $\beta_5 = 2\beta_2$ on the scalar potential. In this case, matrix M_0^2 defined in 6.16 becomes

$$M_0^2 = \begin{pmatrix} 8a^2\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 8b^2\beta_2 & 0 & -2\sqrt{2}ab\beta_2 & 0 \\ 0 & 0 & 8b^2\beta_3 + 16b^2\beta_4 + a^2\beta_2 & 0 & 8\sqrt{2}b^2\beta_4 - \sqrt{2}a^2\beta_2 \\ 0 & -2\sqrt{2}ab\beta_2 & 0 & a^2\beta_2 & 0 \\ 0 & 0 & 8\sqrt{2}b^2\beta_4 - \sqrt{2}a^2\beta_2 & 0 & 8b^2\beta_3 + 8b^2\beta_4 + 2a^2\beta_2 \end{pmatrix}. \tag{6.54}$$

The off-diagonal elements of the first line are equal to 0, such that we get alignment. We will consider $\mathrm{Re}(\varphi^0)$ as the Higgs boson present in the SM, such that we have $8a^2\beta_1 \equiv m_h^2 = (125.09\ \text{GeV})^2$.

In this aligned version of the GM model, the matrix Y defined in 6.17 becomes

$$Y = \begin{pmatrix} 0 & \frac{a}{\sqrt{a^2+8b^2}} & 0 & \frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & 0 \\ 0 & -\frac{2\sqrt{2}b}{\sqrt{a^2+8b^2}} & 0 & \frac{a}{\sqrt{a^2+8b^2}} & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{3} & 0 & \frac{\sqrt{6}}{3} \\ 0 & 0 & \frac{\sqrt{6}}{3} & 0 & \frac{\sqrt{3}}{3} \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{6.55}$$

The masses of the neutral scalars become $\mu_2^2 = \beta_2(a^2+8b^2)$, $\mu_3^2 = 8b^2\beta_3 + 3a^2\beta_2$, $\mu_4^2 = 8b^2(\beta_3+3\beta_4)$, $\mu_5^2 = 8a^2\beta_1$, and the matrices V_1, V_2 and R_1 become

$$\mathrm{Re}\,V_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathrm{Im}\,V_1 = \begin{pmatrix} \frac{a}{\sqrt{a^2+8b^2}} & -\frac{\sqrt{8}b}{\sqrt{a^2+8b^2}} & 0 & 0 & 0 \end{pmatrix}, \tag{6.56a}$$

$$\mathrm{Re}\,V_2 = \begin{pmatrix} 0 & 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{3} & 0 \end{pmatrix}, \qquad \mathrm{Im}\,V_2 = \begin{pmatrix} \frac{\sqrt{8}b}{\sqrt{a^2+8b^2}} & \frac{a}{\sqrt{a^2+8b^2}} & 0 & 0 & 0 \end{pmatrix}, \tag{6.56b}$$

$$R_1 = \begin{pmatrix} 0 & 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \end{pmatrix}. \tag{6.56c}$$

The strategy used to fit the experimental data was to scan the allowed regions for the potential parameters by the BFB conditions and the unitarity conditions and select the ones for which the deviation of the oblique parameters S and U from their experimental values were less than 1σ. For each of those points, we computed δg_L (δg_R is equal to 0). The result, as well as the experimental points and the SM prediction, can be seen in figure 6.1. We fitted only solution number 1 from table 6.1 as in the GM model we have $\delta g_{Rb} = 0$, which means that we will not be able to get a good fit to the other solutions. We have used `LoopTools` [56, 57] to perform the numerical integration of the Passarino-Veltman functions.

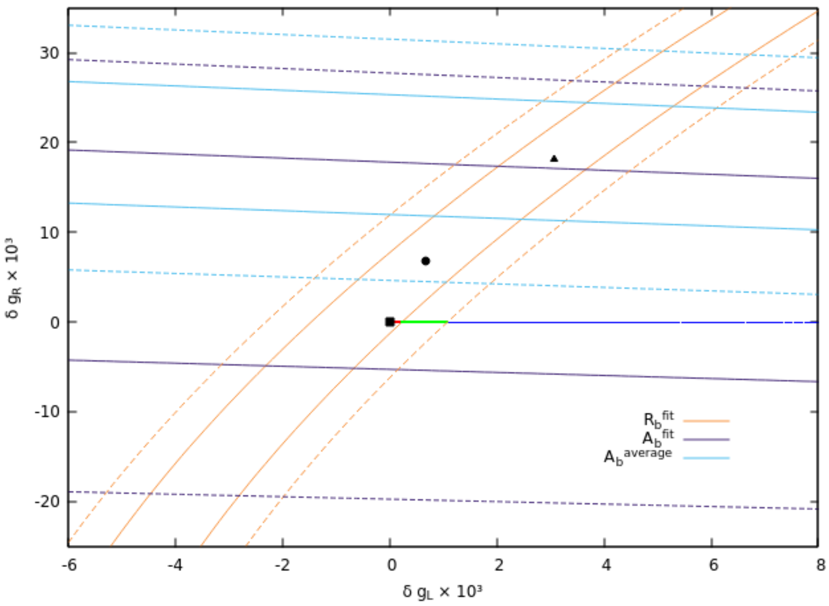


Figure 6.1: Scatter plot of values of δg_L and δg_R. The square marks the SM prediction, the circle marks the best-fit point of solution 1^{fit} and the triangle marks the best-fit point of solution 1^{average}. The orange lines mark the 1σ (full lines) and 2σ (dashed lines) boundaries of the region determined by the experimental value of R_b^{fit}, the light blue lines mark the 1σ (full lines) and 2σ (dashed lines) boundaries of the region determined by the experimental value of A_b^{average} and the purple lines mark the 1σ (full lines) and 2σ (dashed lines) boundaries of the region determined by the experimental value of A_b^{fit}. The red points are inside the 1σ region determined by the experimental value of R_b^{fit}, the green points are outside that 1σ region but inside the 2σ one and the dark blue points are more than 2σ away from the experimental value of R_b^{fit}.

From the figure above we see that we do not get a better agreement with solution 1 than in the SM. In fact, we cannot even reach the 2σ interval of A_b^{average}. This happens because in this model, as in any model with only one scalar doublet (and possibly other additional $SU(2)$ multiplets of higher dimension), in the limit $m_b \to 0$ the Yukawa coupling f vanishes, making $\delta g_R = 0$. This will make it difficult to find a better fit to the experimental data than in the SM.

Using the points from the previous plot that are less than 2σ away from the experimental value of R_b^{fit} (red and green points from figure 6.1), we made the three plots on figures 6.2, 6.3 and 6.4 showing the masses of the scalars used to obtain those points.

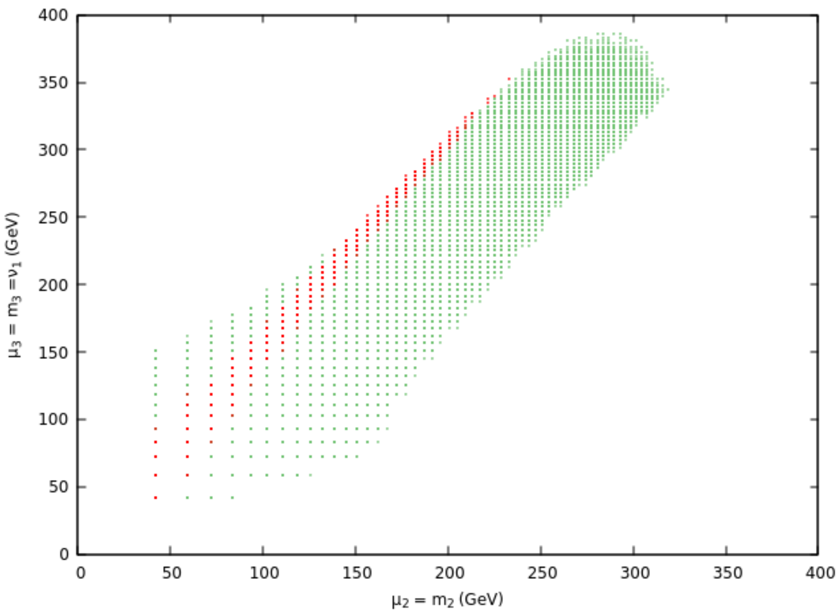


Figure 6.2: Plot of $\mu_3 = m_3 = M_1$ as a function of $\mu_2 = m_2$ for the points less than 2σ away from the experimental value of R_b^{fit} from the plot of figure 6.1. The red points are the ones for which R_b is less than 1σ away from its experimental value.

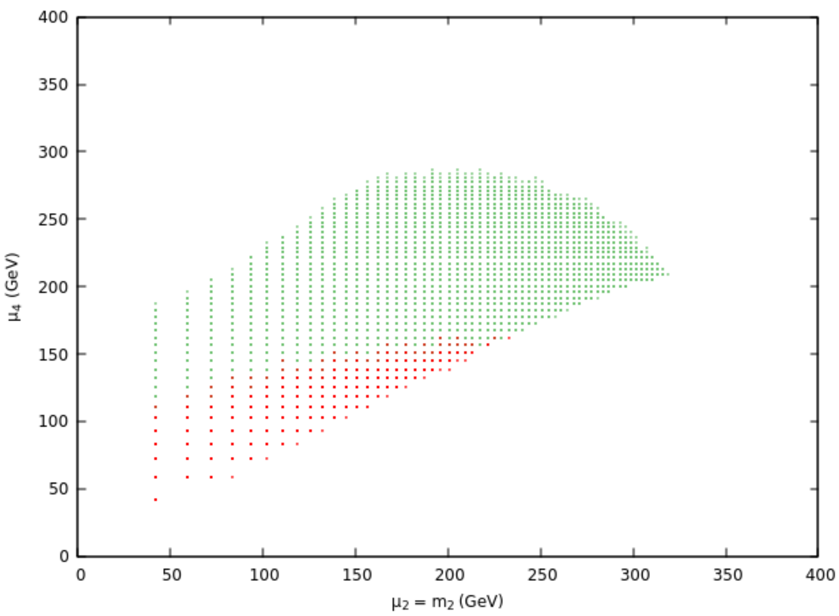


Figure 6.3: Plot of μ_4 as a function of $\mu_2 = m_2$ for the points less than 2σ away from the experimental value of R_b^{fit} from the plot of figure 6.1. The red points are the ones for which R_b is less than 1σ away from its experimental value.

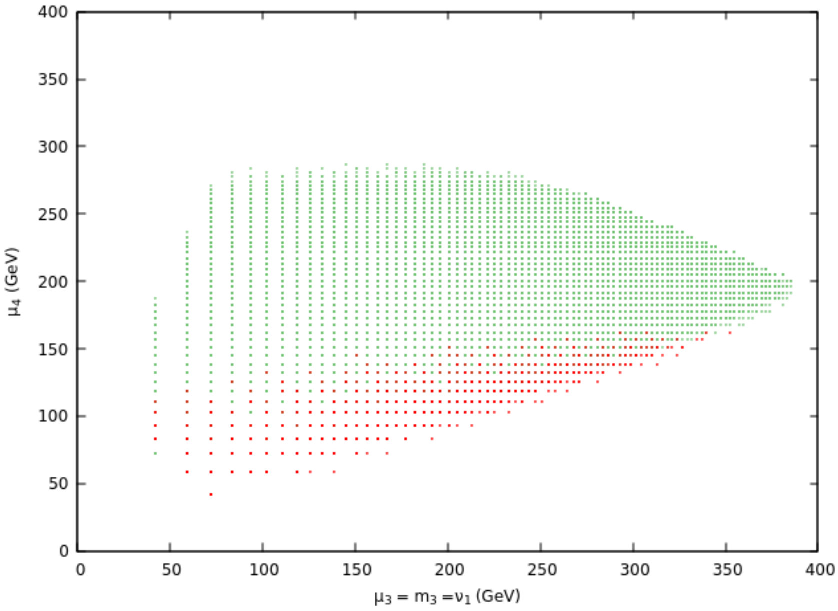


Figure 6.4: Plot of μ_4 as a function of $\mu_3 = m_3 = M_1$ for the points less than 2σ away from the experimental value of R_b^{fit} from the plot of figure 6.1. The red points are the ones for which R_b is less than 1σ away from its experimental value.

The plots from figures 6.2, 6.3 and 6.4 show us the range of masses for each of the new scalars that give a better fit to the experimental data.

Chapter 7

Conclusion

In this thesis, we presented a formalism to work with models with an arbitrary number of $SU(2)$ singlets with hypercharges $Y = 0, 1, 2$; $SU(2)$ doublets with hypercharge $Y = 1/2$ and $SU(2)$ triplets with hypercharge $Y = 0, 1$. We then applied this formalism to compute some observables in general models with this scalar content and computed then these observables in the concrete case of the GM model. The main problem with our formalism was that the relation $m_W = m_Z c_W$ is only valid for models whose multiplets with non-zero VEV obey the relation $T(T+1) = 3Y^2$, where T is the isospin of the multiplet and Y is its hypercharge. Thus, in a general model with triplets we have $m_W \neq m_Z c_W$. The quantities we computed required a subtraction of the result for that quantity in the SM from the result for that quantity in our NP model. As in the SM the masses of the gauge bosons obey the relation $m_W = m_Z c_W$ and in a general model with triplets that relation is not verified, then this subtraction was not trivial.

In chapter 2, we described a feature of the SM (and of other models), related to its scalar sector, which is custodial symmetry. Custodial symmetry is responsible for the relation $m_W = m_Z c_W$ between the masses of the Z and W gauge bosons.

In chapter 3, we presented the aforementioned formalism, defining its scalar content and the matrices that describe the mixing of the scalars. We also wrote the gauge-kinetic Lagrangian for a model with that scalar content and identified the relation between the fields of the Goldstone bosons and the fields of the scalars that appear in the multiplets.

In chapter 4, we used the formalism from chapter 3 to find a prescription to compute the oblique parameters in a model with scalar $SU(2)$ singlets, doublets and triplets. We started by identifying the relevant Feynman diagrams and computed then the vacuum polarization tensors and, when needed, their derivatives with respect to the square of the momentum of the external gauge bosons. We obtained a divergent and gauge dependent result for parameter T and finite and gauge independent results for parameters S, U, V, W and X. In computing the oblique parameters, due to the problem mentioned in the first paragraph of this chapter, we used some Feynman rules for the SM that do not look the same compared to the usual SM Feynman rules but that become the usual ones when we use the relation $m_W = m_Z c_W$. In this way, we were able to obtain a finite result for the oblique parameters S and U that we would not obtain otherwise. We proved then that, using these Feynman rules for the SM, we get a finite result for the oblique parameters S and U in a model with any scalar content. Still in chapter 4, we showed that the part proportional to the metric tensor of the vacuum polarization tensor of the photon at $q^2 = 0$ (being q^μ the four-momentum of the external gauge boson) is equal to 0, which is a consequence of the vacuum polarization tensor of the photon being transverse as required by the Ward-Takahashi identities. We also compared our result for the oblique parameters S and U with those from [40] for a model with one doublet with hypercharge $Y = 1/2$ and one triplet with hypercharge $Y = 0$. Our results agreed with the ones from [40].

In chapter 5, we computed the one-loop corrections to the $Zb\bar{b}$ vertex in a model with scalar singlets, doublets and triplets. We started by identifying the two observables that are influenced by these corrections. We identified the relevant Feynman diagrams and computed the contribution of each of the diagrams to the couplings of the Z boson with the b quark and its anti-particle using the formalism from chapter 3. The result that we obtained is divergent for models with $m_W \neq m_Z c_W$. This may also be related to the problems in the subtraction of SM quantities from the same quantities in our NP model mentioned in the first paragraph of this chapter. In the case of the one-loop corrections to the $Zb\bar{b}$ vertex we could not fix this problem.

Finally, in chapter 6, we applied the results from the previous chapters to compute the oblique parameters and the one-loop corrections to the $Zb\bar{b}$ vertex to the concrete case of the Georgi-Machacek model. We started with a short description of the model. Having identified the mixing matrices between scalars for this model, we proceeded to compute the oblique parameters for the GM model. We computed then the one-loop corrections to the $Zb\bar{b}$ vertex also for the GM model. Then, relating the corrections to the couplings with the relevant observables that had been identified in the previous chapter, and assuming alignment in the GM model, we compared our results with the experimental ones for different values of the masses of the NP scalars. We did not obtain a better fit to the experimental results than the one obtained by the SM. This happens because the GM model only contains one $SU(2)$ scalar doublet. In the approximation $m_b \to 0$, this will imply that the coupling g_{Rb} in the GM model will be equal to this coupling in the SM, such that only g_{Lb} will be changed by the additional scalar content of the GM model. However, the result for g_{Lb} in the GM model is always bigger than the SM one, such that the GM fit is always worse than the SM one.

A possible way to continue the work of this thesis would be to find a way to get a finite and gauge independent result for the one-loop corrections to the $Zb\bar{b}$ vertex. Using the work of this thesis, we will also try to publish a paper on an international journal with the results obtained for the oblique parameters.

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Appendix A

Short Review of the Standard Model

The Standard Model Lagrangian can be written as a sum of several terms as $\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}$. As mentioned before, the SM is a gauge theory with gauge group $SU(3) \times SU(2) \times SU(1)$. $SU(3)$ will be the group of color charge. It is a Lie group with a Lie algebra of dimension 8. This means that $SU(3)$ has 8 generators that will, according to the Goldstone theorem [58, 59], give rise to 8 real gluon fields G_a^μ ($a = 1, ..., 8$). The mathematical description of the behaviour of the gluons and the way they interact with the quarks is given by $\mathcal{L}_{QCD}$. This part of the SM Lagrangian will not be examined here as we will focus on the electroweak part of the Lagrangian.

$SU(2)$ and $U(1)$ are Lie groups with Lie algebras of dimensions 3 and 1, respectively. This means that, according to the Goldstone theorem, we will have 3 real gauge bosons W_a^μ ($a = 1, ..., 3$) due to $SU(2)$ and 1 real gauge boson B^μ due to $U(1)$. The gauge group $SU(2)$ has coupling constant $-g$ and the gauge group $U(1)$ has coupling constant $-g'$.

However, the gauge bosons W_a^μ and B^μ will not be the physical gauge bosons. Let us define the fields A^μ and Z^μ as

$$\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} B^\mu \\ W_3^\mu \end{pmatrix}, \tag{A.1}$$

where c_W and s_W are, respectively, the cosine and the sine of the Weinberg angle θ_W. This transformation is unitary, real and orthogonal. c_W and s_W are given in terms of the couplings g and g' by

$$c_W = \frac{g}{\sqrt{g^2 + g'^2}}, \qquad s_W = -\frac{g'}{\sqrt{g^2 + g'^2}}. \tag{A.2}$$

We define also the fields $W^{\mu+}$ and $W^{\mu-}$ as

$$W^{\mu\pm} = \frac{W_1^\mu \mp i W_2^\mu}{\sqrt{2}}. \tag{A.3}$$

The fields $W^{\mu\pm}$ are complex fields and are complex-conjugate of each other.

The gauge part of the SM Lagrangian is then given by $\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\left(F_{Y\mu\nu}F_Y^{\mu\nu} + \sum_{i=1}^3 F_{j\mu\nu}F_j^{\mu\nu}\right)$, where $F_{Y\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $F_{j\mu\nu} = \partial_\mu W_{j\nu} - \partial_\nu W_{j\mu} + g\sum_{k,l=1}^3 \epsilon_{jkl}W_{k\mu}W_{l\nu}$. Using the inverse relations of the ones from equations A.1 and A.3 we obtain the Lagrangian terms that describe the interaction of the physical electroweak gauge bosons.

For the kinetic term for a fermion field, we would like to have a real Lorentz-invariant term. For that, we can start by trying the usual Dirac term $\mathcal{L}_{\text{Dirac}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu\right)\psi$, where ψ is any fermion field, γ^μ are the Dirac matrices and $\bar{\psi} = \psi^\dagger A$, with A being a 4×4 matrix defined by $A\gamma_\mu = \gamma_\mu^\dagger A$, $A^\dagger = A$. In the Dirac, Weyl and Majorana representations of the Dirac matrices (the most common representations) $A = \gamma_0$.

However, the Dirac Lagrangian presented above would not be gauge invariant. To fix that, let us define the covariant derivative D_μ as $D^\mu = \partial^\mu - ig(W_1^\mu T_1 + W_2^\mu T_2 + W_3^\mu T_3) - ig'B^\mu Y$, where $T_a(a = 1,...,3)$ are the generators of the gauge group $SU(2)$, which obey $[T_j, T_k] = i\sum_{l=1}^3 \epsilon_{jkl}T_l$ and Y is the generator of the gauge group $U(1)$. The operators $T_a(a = 1,...,3)$ are called isospin operators, while the operator Y is hypercharge operator. Defining

$$e \equiv gs_W = -g'c_W = -\frac{gg'}{\sqrt{g^2 + g'^2}}, \tag{A.4}$$

and the operator Q, T_+ and T_- as

$$Q = T_3 + Y, \qquad T_\pm = \frac{T_1 \pm iT_2}{\sqrt{2}}, \tag{A.5}$$

and using equations A.1 and A.3 we can write the covariant derivative in terms of the physical gauge boson fields as

$$D^\mu = \partial^\mu + ieQA^\mu - i\frac{g}{c_W}(T_3 - Qs_W^2)Z^\mu - ig(W^{\mu+}T_+ + W^{\mu-}T_-). \tag{A.6}$$

Let us also define the left and right chirality projection operators $P_{L,R}$ as $P_{L,R} = \frac{1\mp\gamma_5}{2}$ where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The eigenvalue of γ_5 is the chirality. The operators $P_{L,R}$ are called projectors as they obey the usual relations of the projectors $P_R + P_L = \mathbb{1}$, $(P_{R,L})^2 = P_{R,L}$, $P_RP_L = P_LP_R = 0$.

The operator P_R projects the fermion field ψ into its component $\psi_R \equiv P_R\psi$ with chirality $+1$, such that $\gamma_5\psi_R = \psi_R$ and the operator P_L projects the fermion field ψ into its component $\psi_L \equiv P_L\psi$ with chirality -1, such that $\gamma_5\psi_L = -\psi_L$.

The weak interaction is chiral, which means that right-handed and left-handed fermions of the same type undergo different interactions. However, electromagnetism is not chiral, as right- and left-handed fermions undergo the same electromagnetic interaction. Therefore, in the SM, right- and left-handed fermions have different T_3 and different Y but they have the same $Q = T_3 + Y$. That can be achieved by putting the left-handed fermions in $SU(2)$ doublets while the right-handed fermions are put in $SU(2)$ singlets.

The fermion field content of the SM is then divided into quarks and leptons. The SM leptons are $\begin{pmatrix}\nu_{eL} & e_L\end{pmatrix}^T$, $\begin{pmatrix}\nu_{\mu L} & \mu_L\end{pmatrix}^T$, $\begin{pmatrix}\nu_{\tau L} & \tau_L\end{pmatrix}^T$, e_R, μ_R and τ_R. The left-handed leptons are placed in $SU(2)$ doublets of hypercharge $Y = -1/2$, while the right-handed leptons are placed in $SU(2)$ singlets of hypercharge $Y = -1$. The SM quarks are $\begin{pmatrix}u_L & d_L\end{pmatrix}^T$, $\begin{pmatrix}c_L & s_L\end{pmatrix}^T$, $\begin{pmatrix}t_L & b_L\end{pmatrix}^T$, u_R, c_R, t_R, d_R, s_R and b_R. The left-handed quarks are placed in $SU(2)$ doublets of hypercharge $Y = 1/6$, while the right-handed up-type quarks (u, c and t) are placed in $SU(2)$ singlets of hypercharge $Y = 2/3$ and the right-handed up-type quarks (d, s and b) are placed in $SU(2)$ singlets of hypercharge $Y = -1/3$.

We can now fix our attempt to write the Dirac Lagrangian by writing it as $\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L(i\gamma^\mu D_\mu)\psi_L + \bar{\psi}_R(i\gamma^\mu D_\mu)\psi_R$, where ψ can be any fermions (either lepton or quark). This Lagrangian is now invariant under $SU(2) \times U(1)$ and it contains the terms describing the interaction of the gauge bosons with the fermions.

Until now, we do not have mass terms for the gauge bosons and we know that the Z and the $W^\pm$ bosons are massive. Furthermore, in nature, we do not observe the $SU(2)\times U(1)_Y$ gauge symmetry. We just observe the gauge symmetry $U(1)_Q$, where Q is the electric charge. To solve both of these issues, we introduce a scalar field which we allow to have a non-zero VEV. This scalar field cannot be a $SU(2)$ singlet as this would mean that $SU(2)$ would remain unbroken. Furthermore, it must have 0 electric charge, so that $U(1)_Q$ remains unbroken. The simplest possible choice for breaking the $SU(2) \times U(1)_Y$ gauge symmetry meeting the previous conditions is to have one, and only one, doublet of $SU(2)$. This is the choice used in the SM and it turns out to give good predictions of physical observables.

Let us call ϕ to the SM scalar doublet. For it to have a component with 0 electric charge, its hypercharge must either be $+1/2$ or $-1/2$, as the components of a doublet have $T_3 = \pm 1/2$ and $Q = T_3 + Y$. As the representations of $SU(2)$ are equivalent to their complex conjugate representations, then it is arbitrary to choose between hypercharge $+1/2$ or $-1/2$. We will then choose $Y = +1/2$. Thus, we can write ϕ as $\phi = \begin{pmatrix}\varphi^+ & \varphi^0\end{pmatrix}^T$, where φ^+ and φ^0 are complex scalar fields.

To write a gauge invariant scalar Lagrangian we use again the covariant derivative, such that we get $\mathcal{L}_{\text{scalar}} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$, where $V(\phi)$ is the scalar potential of the SM. The scalar potential must be invariant under $SU(2) \times U(1)$ and must also be at most quartic in the scalar fields due to renormalizability. Thus, the most general scalar potential that we can write with only one scalar doublet is $V = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$, where μ^2 has dimension M^2 and λ is dimensionless. For the potential to be bounded from below (which means that it does not become infinitely negative when we increase the fields) we must have $\lambda > 0$. For spontaneous symmetry breaking to happen we must have $\mu^2 < 0$, so that the potential has a minimum for $\phi^\dagger\phi \neq 0$. When these conditions are met, V has a minimum at $\phi^\dagger\phi = v^2$ (where v can be chosen to be real without loss of generality), with $v = \sqrt{-\frac{\mu^2}{2\lambda}}$. As we want the non-zero VEV to be in the neutral component of the doublet, we can write

$$\varphi^0 = v + \frac{H + i\chi}{\sqrt{2}}, \tag{A.7}$$

where H and χ are real scalar fields with zero VEV. If we develop the SM potential using A.7 we find that the fields χ and $\varphi^\pm$ are massless and the field H has mass $m_h^2 = -2\mu^2$. Thus, the fields χ and $\varphi^\pm$ are called Goldstone bosons and appear because we started with a gauge symmetry with gauge group $SU(2) \times U(1)_Y$ whose algebra has 4 generators and this symmetry was broken into a symmetry with gauge group $U(1)_Q$ whose algebra has 1 generator, which means that we will have 3 Goldstone bosons that will get absorbed by the gauge bosons that acquire mass and will become their longitudinal components. The field H is the Higgs boson, which is a physical particle.

Thus, we can write $\mathcal{L}_{\text{scalar}} = (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$. By developing this Lagrangian, we get the terms describing the interaction of the scalars with the gauge bosons and of the scalars with each other and we find that the Z and $W^\pm$ gauge bosons acquire a mass. The mass of the Z boson is given by $m_Z = \frac{gv}{\sqrt{2}c_W}$ and the mass of the W boson is given by $m_W = \frac{gv}{\sqrt{2}}$. Thus, we obtain the relation $m_W = m_Z c_W$.

We still do not have mass terms for the fermions. Similarly to what happens to the gauge bosons, spontaneous symmetry breaking will be responsible for the mass of the fermions.

Let us start by the leptons. The $SU(2)$ left-handed lepton doublets have hypercharge $Y = -1/2$, the $SU(2)$ scalar doublet ϕ has hypercharge $Y = +1/2$ and the $SU(2)$ right-handed lepton singlets have hypercharge $Y = -1$. Thus, we can write

$$\mathcal{L}_{\text{Yukawa leptons}} = -y_l \begin{pmatrix} \overline{\nu_{lL}} & \overline{l_L} \end{pmatrix} \phi \, l_R - y_l^* \overline{l_R} \, \phi^\dagger \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \tag{A.8}$$

where l stands for any of the leptons and y_l is a dimensionless complex constant. It is, however, possible to rephase the field e_R such that the constant y_l becomes real and in this case, we find that the mass of the lepton l is given by $m_l = y_l v$. Using A.8, we get the terms describing the interaction of the leptons with the gauge bosons.

In the case of the quarks, the Yukawa interactions are a little different. Let us call p_j ($j = 1, 2, 3$) to three generic up-type quarks and n_j ($j = 1, 2, 3$) to three generic down-type quarks. These will not be the physical quarks. The $SU(2)$ doublets $\begin{pmatrix} p_{jL} & n_{jL} \end{pmatrix}^T$ ($j = 1, 2, 3$) have hypercharge $Y = -1/6$, the $SU(2)$ scalar doublet $\begin{pmatrix} \varphi^+ & \varphi^0 \end{pmatrix}^T$ has hypercharge $Y = +1/2$, the $SU(2)$ singlets p_{jR} have hypercharge $Y = +2/3$ and the $SU(2)$ singlets n_{jR} have hypercharge $Y = -1/3$. We also have the $SU(2)$ scalar doublet $\begin{pmatrix} \varphi^{0*} & -\varphi^- \end{pmatrix}^T$, which has hypercharge $Y = -1/2$. We can then write

$$\mathcal{L}_{\text{Yukawa quarks}} = -\sum_{j=1}^{3}\sum_{k=1}^{3}\Gamma_{jk}\begin{pmatrix}\overline{p_{jL}} & \overline{n_{jL}}\end{pmatrix} n_{kR}\begin{pmatrix}\varphi^{+}\\ \varphi^{0}\end{pmatrix} - \sum_{j=1}^{3}\sum_{k=1}^{3}\Delta_{jk}\begin{pmatrix}\overline{p_{jL}} & \overline{n_{jL}}\end{pmatrix} p_{kR}\begin{pmatrix}\varphi^{0*}\\ -\varphi^{-}\end{pmatrix} \tag{A.9a}$$

$$-\sum_{j=1}^{3}\sum_{k=1}^{3}\Gamma^{*}_{jk}\begin{pmatrix}\varphi^{-} & \varphi^{0*}\end{pmatrix}\overline{n_{kR}}\begin{pmatrix}p_{jL}\\ n_{jL}\end{pmatrix} - \sum_{j=1}^{3}\sum_{k=1}^{3}\Delta^{*}_{jk}\begin{pmatrix}\varphi^{0} & -\varphi^{+}\end{pmatrix}\overline{p_{kR}}\begin{pmatrix}p_{jL}\\ n_{jL}\end{pmatrix}, \tag{A.9b}$$

where Γ and Δ are 3×3 matrices of Yukawa coupling constants. They are in general complex and are not constrained by any symmetry. The mass matrices of the quarks will then be $(M_p)_{jk} = v\Delta_{jk}$ and $(M_n)_{jk} = v\Gamma_{jk}$. These matrices are not, however, diagonal. Hence, we will use a theorem that says that for any square matrix M there are two unitary matrices U and U' such that $D = U^{\dagger}MU'$ is a diagonal matrix with real and non-negative matrix elements. Using this theorem, we have the unitary matrices U^p_L, U^p_R, U^n_L, U^n_R, such that $U^{p\dagger}_L M_p U^p_R = M_u \equiv \mathrm{diag}(m_u, m_c, m_t)$, $U^{n\dagger}_L M_n U^n_R = M_d \equiv \mathrm{diag}(m_d, m_s, m_b)$.

Defining the physical fields in terms of the fields p_{jL}, p_{jR}, n_{jL} and n_{jR} as

$$\mathcal{U}_L \equiv \begin{pmatrix}u_L\\ c_L\\ t_L\end{pmatrix} = U^{p\dagger}_L p_L, \qquad \mathcal{U}_R \equiv \begin{pmatrix}u_R\\ c_R\\ t_R\end{pmatrix} = U^{p\dagger}_R p_R, \tag{A.10a}$$

$$\mathcal{D}_L \equiv \begin{pmatrix}d_L\\ s_L\\ b_L\end{pmatrix} = U^{n\dagger}_L n_L, \qquad \mathcal{D}_R \equiv \begin{pmatrix}d_R\\ s_R\\ b_R\end{pmatrix} = U^{n\dagger}_R n_R, \tag{A.10b}$$

we can write

$$\mathcal{L}_{\text{Yukawa quarks}} = -\frac{1}{v}\Big[\overline{\mathcal{U}_L}V_{CKM}M_d\mathcal{D}_R\varphi^{+} + \overline{\mathcal{D}_L}M_d\mathcal{D}_R\varphi^{0} \tag{A.11a}$$

$$+\overline{\mathcal{U}_L}M_u\mathcal{U}_R\varphi^{0*} - \overline{\mathcal{D}_L}V^{\dagger}_{CKM}M_u\mathcal{U}_R\varphi^{-} \tag{A.11b}$$

$$+\overline{\mathcal{D}_R}M_dV^{\dagger}_{CKM}\mathcal{U}_L\varphi^{-} + \overline{\mathcal{D}_R}M_d\mathcal{D}_L\varphi^{0*} \tag{A.11c}$$

$$+\overline{\mathcal{U}_R}M_u\mathcal{U}_L\varphi^{0} - \overline{\mathcal{U}_R}M_uV_{CKM}\mathcal{D}_L\varphi^{+}\Big], \tag{A.11d}$$

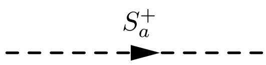
where $V_{CKM} \equiv U^{p\dagger}_L U^n_L$ is the CKM matrix [60, 61]. Using A.11, we get the terms describing the interaction of the quarks with the gauge bosons.

To obtain the full Yukawa Lagrangian, we just need to sum the Yukawa Lagrangian for the leptons with the Yukawa Lagrangian for the quarks, such that $\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Yukawa leptons}} + \mathcal{L}_{\text{Yukawa quarks}}$.

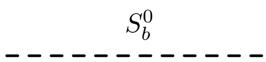
Appendix B

Feynman Rules for the General Formalism

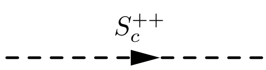
Here we present the Feynman rules for the general formalism computed from the gauge-kinetic Lagrangian in equation 3.29. The Feynman rules of the scalar propagators are:



$$= \frac{i}{p^2 - m_a^2 + i\epsilon} \tag{B.1a}$$

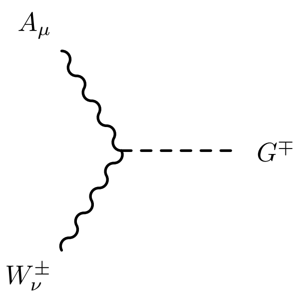


$$= \frac{i}{p^2 - \mu_b^2 + i\epsilon} \tag{B.1b}$$

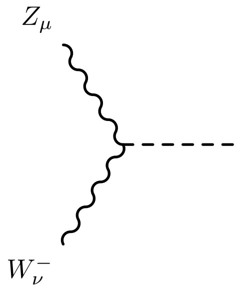


$$= \frac{i}{p^2 - M_c^2 + i\epsilon} \tag{B.1c}$$

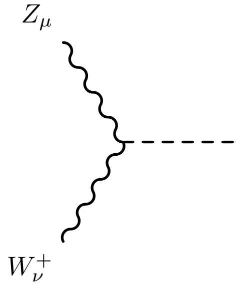
The Feynman rules for the three-particle vertices, where the particles indicated are entering the vertices, are:



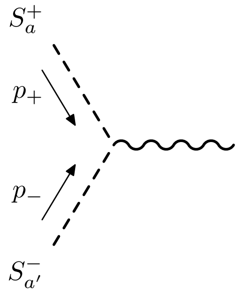
$$= -iem_W g_{\mu\nu} \tag{B.2a}$$



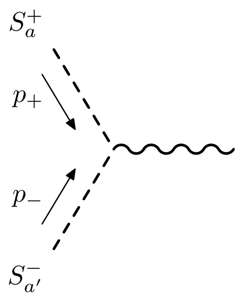
$$S^+_a \quad = -ig\frac{m_W}{c_W}g_{\mu\nu}\Big(s_W^2\delta_{1a} + (U_4^\dagger U_4)_{1a} - (U_3^\dagger U_3)_{1a}\Big) \qquad \text{(B.2b)}$$



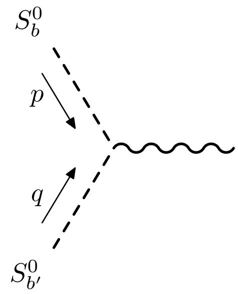
$$S^-_a \quad = -ig\frac{m_W}{c_W}g_{\mu\nu}\Big(s_W^2\delta_{a1} + (U_4^\dagger U_4)_{a1} - (U_3^\dagger U_3)_{a1}\Big) \qquad \text{(B.2c)}$$



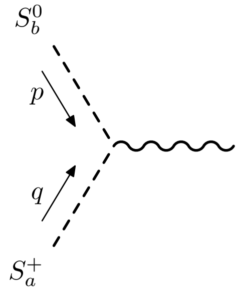
$$A_\mu \quad = ie\delta_{aa'}(p_- - p_+)_\mu \qquad \text{(B.2d)}$$



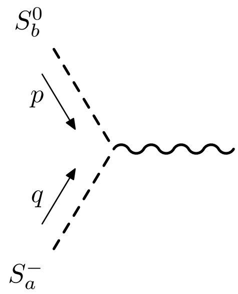
$$Z_\mu \quad = i\frac{g}{c_W}\Big(s_W^2\delta_{aa'} - \frac{1}{2}(U_1^\dagger U_1)_{a'a} - (U_3^\dagger U_3)_{a'a}\Big)(p_- - p_+)_\mu \qquad \text{(B.2e)}$$



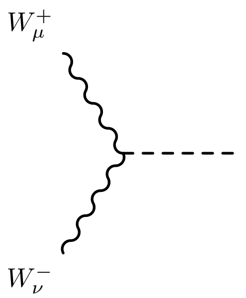
$$Z_\mu \quad = \frac{g}{c_W}\Big(\frac{1}{2}\,\mathrm{Im}(V_1^\dagger V_1)_{bb'} + \mathrm{Im}(V_2^\dagger V_2)_{bb'}\Big)(q-p)_\mu \qquad \text{(B.2f)}$$



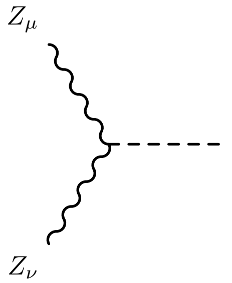
$$W_\mu^- \quad = ig\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big)(q-p)_\mu \tag{B.2g}$$



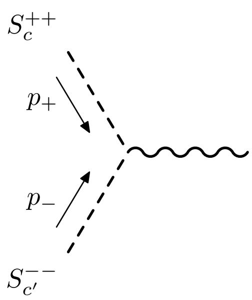
$$W_\mu^+ \quad = -ig\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big)(q-p)_\mu \tag{B.2h}$$



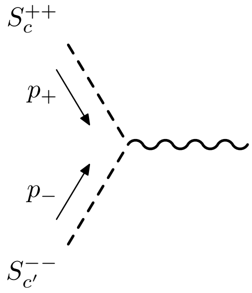
$$S_b^0 \quad = 2igm_W g_{\mu\nu}\Big(\frac{1}{2}\,\mathrm{Re}(U_1^\dagger V_1)_{1b} + \frac{1}{\sqrt{2}}\,\mathrm{Re}(U_4^\dagger V_2)_{1b} + \mathrm{Re}(U_3^\dagger R_1)_{1b}\Big), \quad \text{for } b \neq 1 \tag{B.2i}$$



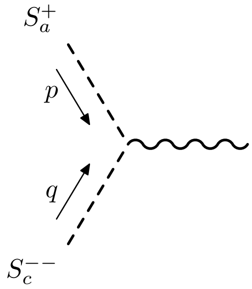
$$S_b^0 \quad = -ig\frac{m_Z}{c_W}g_{\mu\nu}\Big(\mathrm{Im}(V_1^\dagger V_1)_{1b} + 2\,\mathrm{Im}(V_2^\dagger V_2)_{1b}\Big), \text{ for } b \neq 1 \tag{B.2j}$$



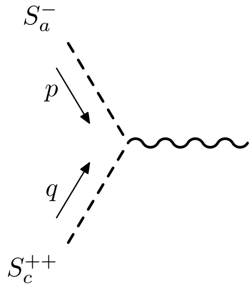
$$A_\mu \quad = 2ie\delta_{c'c}(p_- - p_+)_\mu \tag{B.2k}$$



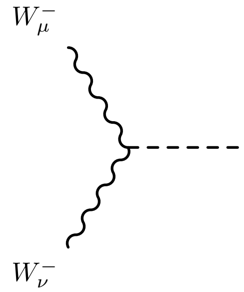
$$Z_\mu \quad = i\frac{g}{c_W}\Big(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c}\Big)(p_- - p_+)_\mu \tag{B.2l}$$



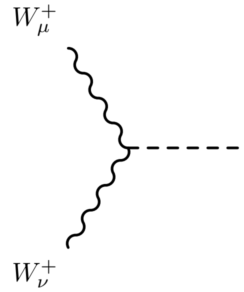
$$W_\mu^+ \;= -ig(T_1^\dagger U_4)_{ca}(q-p)_\mu \tag{B.2m}$$



$$W_\mu^- \;= ig(U_4^\dagger T_1)_{ac}(q-p)_\mu \tag{B.2n}$$

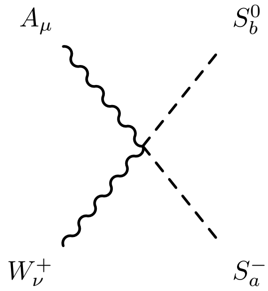


$$S_c^{++} = 2i\,g\,m_W\,g_{\mu\nu}(U_4^\dagger T_1)_{1c} \tag{B.2o}$$

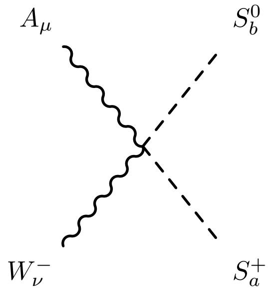


$$S_c^{--} = 2i\,g\,m_W\,g_{\mu\nu}(T_1^\dagger U_4)_{c1} \tag{B.2p}$$

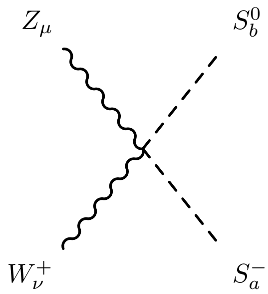
The Feynman rules for the four-particle vertices, where the particles indicated are entering the vertices, are:



$$= -iegg_{\mu\nu}\Big(\frac{1}{2}(U_1^\dagger V_1)_{ab} + \frac{1}{\sqrt{2}}(U_4^\dagger V_2)_{ab} + (U_3^\dagger R_1)_{ab}\Big) \tag{B.3a}$$

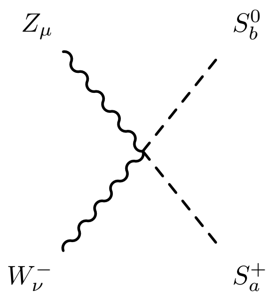


$$= -iegg_{\mu\nu}\Big(\frac{1}{2}(V_1^\dagger U_1)_{ba} + \frac{1}{\sqrt{2}}(V_2^\dagger U_4)_{ba} + (R_1^T U_3)_{ba}\Big) \tag{B.3b}$$



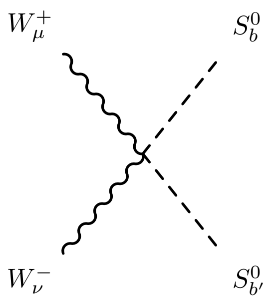
$$= -i\frac{g^2}{c_W}g_{\mu\nu}\Big(\frac{s_W^2}{2}(U_1^\dagger V_1)_{ab}$$

$$+ \frac{1+s_W^2}{\sqrt{2}}(U_4^\dagger V_2)_{ab} - c_W^2(U_3^\dagger R_1)_{ab}\Big) \tag{B.3c}$$



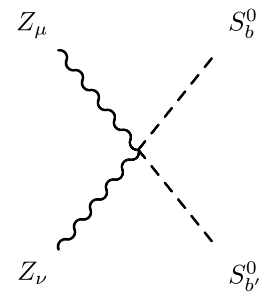
$$= -i\frac{g^2}{c_W}g_{\mu\nu}\Big(\frac{s_W^2}{2}(V_1^\dagger U_1)_{ba}$$

$$+ \frac{1+s_W^2}{\sqrt{2}}(V_2^\dagger U_4)_{ba} - c_W^2(R_1^T U_3)_{ba}\Big) \tag{B.3d}$$



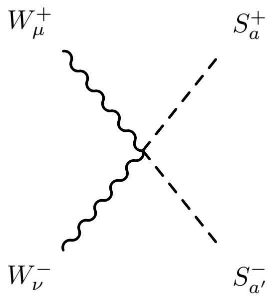
$$= ig^2 g_{\mu\nu}\Big(\frac{1}{4}((V_1^\dagger V_1)_{b'b} + (V_1^\dagger V_1)_{bb'})$$

$$+\frac{1}{2}((V_2^\dagger V_2)_{bb} + (V_2^\dagger V_2)_{bb'}) + 2(R_1^T R_1)_{b'b}\Big) \tag{B.3e}$$

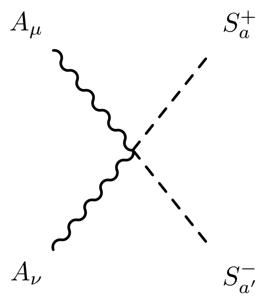


$$= i\frac{g^2}{c_W^2} g_{\mu\nu}\Big(\frac{1}{4}((V_1^\dagger V_1)_{b'b} + (V_1^\dagger V_1)_{bb'})$$

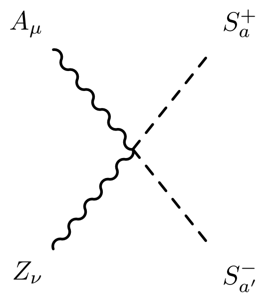
$$+ ((V_2^\dagger V_2)_{b'b} + (V_2^\dagger V_2)_{bb'})\Big) \tag{B.3f}$$



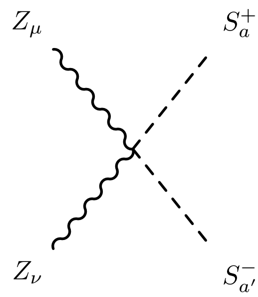
$$= ig^2 g_{\mu\nu}\Big(\frac{1}{2}(U_1^\dagger U_1)_{a'a} + (U_3^\dagger U_3)_{a'a} + 2(U_4^\dagger U_4)_{a'a}\Big) \tag{B.3g}$$



$$= 2ie^2 g_{\mu\nu}\delta_{a'a} \tag{B.3h}$$

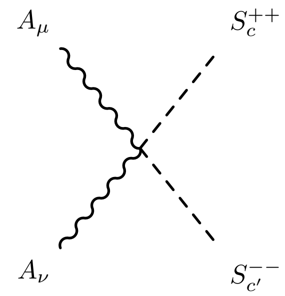


$$= i\frac{eg}{c_W} g_{\mu\nu}\Big(2s_W^2\delta_{a'a} - (U_1^\dagger U_1)_{a'a} - 2(U_3^\dagger U_3)_{a'a}\Big) \tag{B.3i}$$

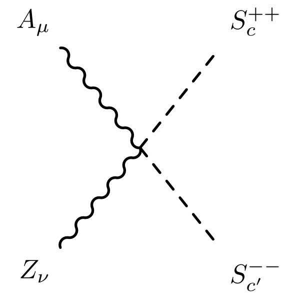


$$= 2i\frac{g^2}{c_W^2} g_{\mu\nu}\Big(s_W^4\delta_{a'a} + \Big(\frac{1}{4} - s_W^2\Big)(U_1^\dagger U_1)_{a'a}$$

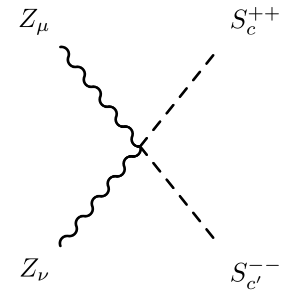
$$+\,(c_W^2 - s_W^2)(U_3^\dagger U_3)_{a'a}\Big) \tag{B.3j}$$



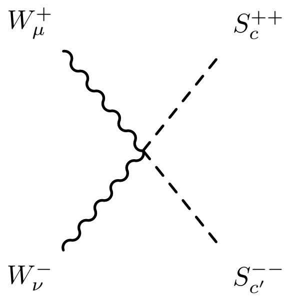
$$= 8ie^2 g_{\mu\nu}\delta_{c'c} \tag{B.3k}$$



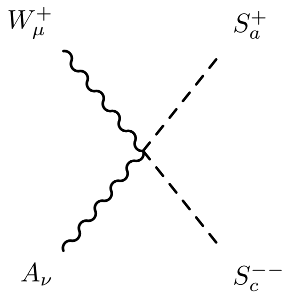
$$= 4i\frac{eg}{c_W} g_{\mu\nu}\Big(2s_W^2\delta_{c'c} - (T_1^\dagger T_1)_{c'c}\Big) \tag{B.3l}$$



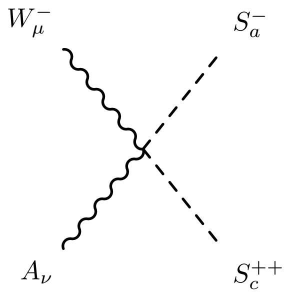
$$= 2i\frac{g^2}{c_W^2} g_{\mu\nu}\Big(4s_W^4\delta_{c'c} + (1-4s_W^2)(T_1^\dagger T_1)_{c'c}\Big) \tag{B.3m}$$



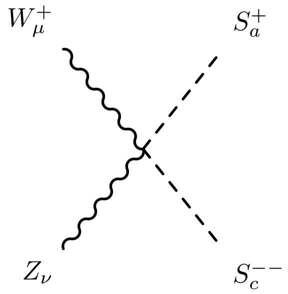
$$= ig^2 g_{\mu\nu}(T_1^\dagger T_1)_{c'c} \tag{B.3n}$$



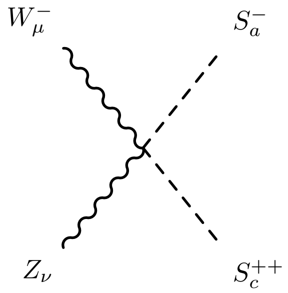
$$= -3iegg_{\mu\nu}(T_1^\dagger U_4)_{ca} \tag{B.3o}$$



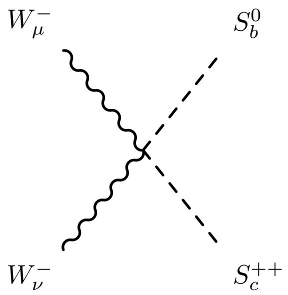
$$= -3iegg_{\mu\nu}(U_4^\dagger T_1)_{ac} \tag{B.3p}$$



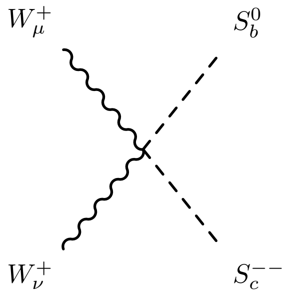
$$= -ig^2\frac{(2s_W^2 - c_W^2)}{c_W}g_{\mu\nu}(T_1^\dagger U_4)_{ca} \tag{B.3q}$$



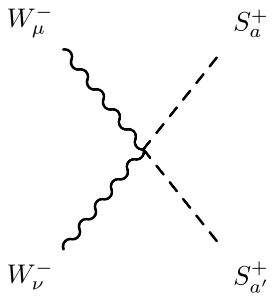
$$= -ig^2\frac{(2s_W^2 - c_W^2)}{c_W}g_{\mu\nu}(U_4^\dagger T_1)_{ac} \tag{B.3r}$$



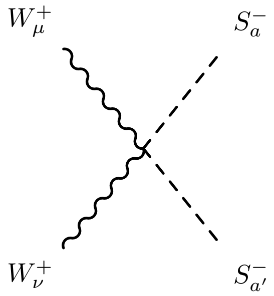
$$= ig^2\sqrt{2}g_{\mu\nu}(V_2^\dagger T_1)_{bc} \tag{B.3s}$$



$$= ig^2\sqrt{2}g_{\mu\nu}(T_1^\dagger V_2)_{cb} \tag{B.3t}$$



$$= -2ig^2\sqrt{2}g_{\mu\nu}(U_3^T U_3)_{a'a} \tag{B.3u}$$

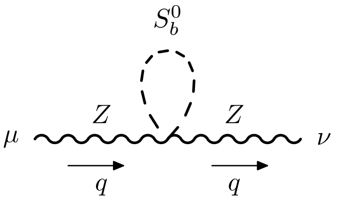


$$= -2ig^2\sqrt{2}g_{\mu\nu}(U_3^\dagger U_3^*)_{a'a} \tag{B.3v}$$

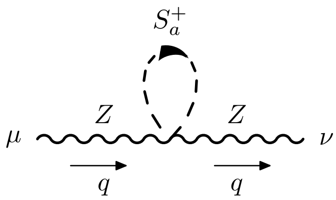
Appendix C

Feynman Diagrams for the One-loop Gauge Boson Propagators

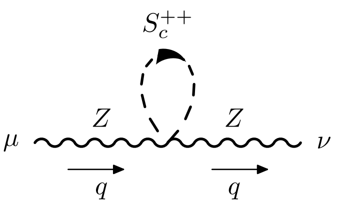
The diagrams that contribute to $\Pi^{\mu\nu}_{ZZ}$ at one-loop level are [1]



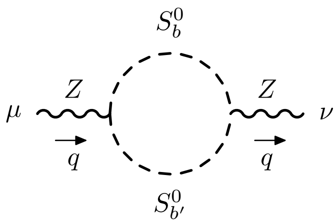
①



② (C.1a)

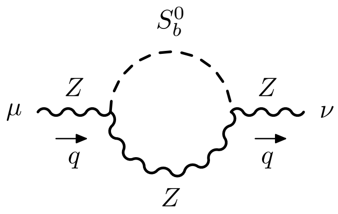


③

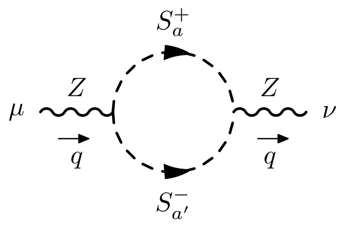


④ (C.1b)

[1]We do not include here the diagrams that have equal amplitudes to the same diagrams in the SM because we are interested only in the New Physics part of $A_{VV'}$ (with VV' being either AA, AZ, ZZ or WW), which means that these diagrams do not contribute to our results.

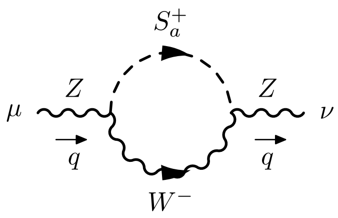


5

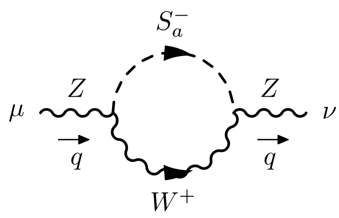


6

(C.1c)

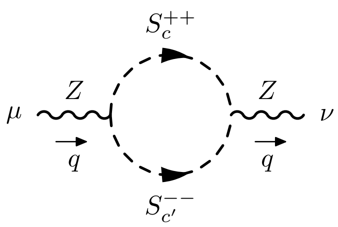


7



8

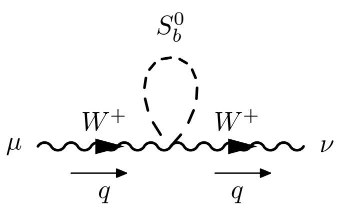
(C.1d)



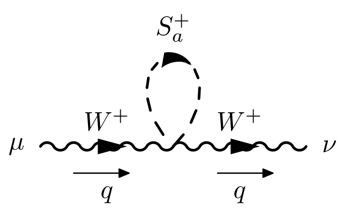
9

(C.1e)

The diagrams that contribute to $\Pi_{WW}^{\mu\nu}$ at one-loop level are

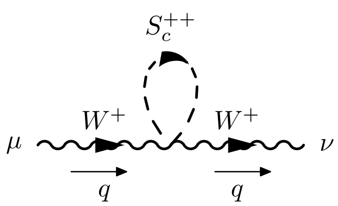


1

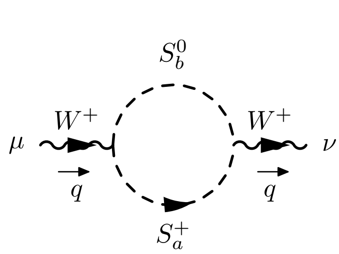


2

(C.2a)

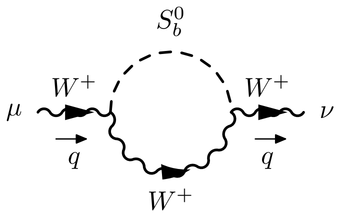


3

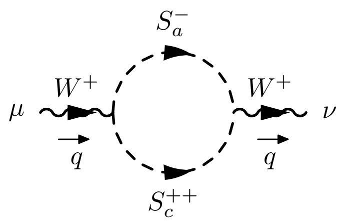


4

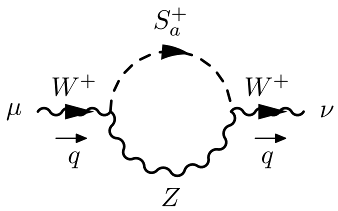
(C.2b)



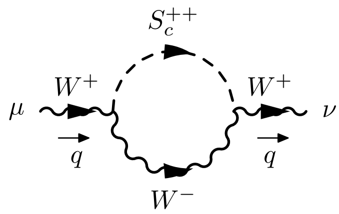
(5)



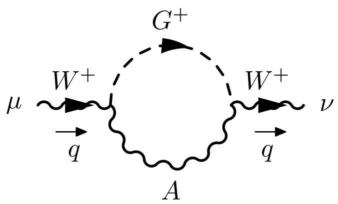
(6) (C.2c)



(7)

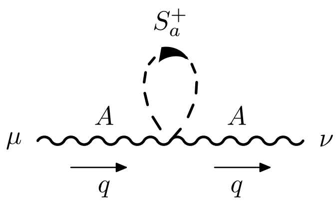


(8) (C.2d)

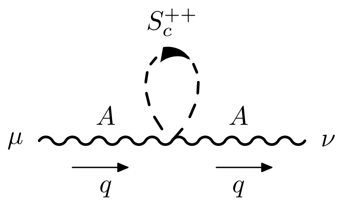


(9) (C.2e)

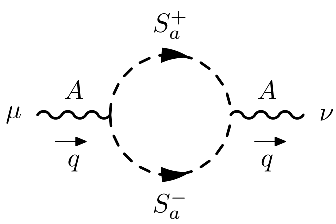
The diagrams that contribute to $\Pi^{\mu\nu}_{AA}$ at one-loop level are [2]



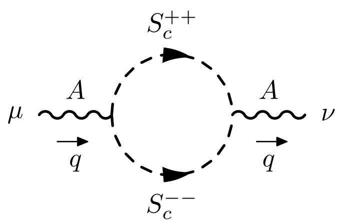
(1)



(2) (C.3a)

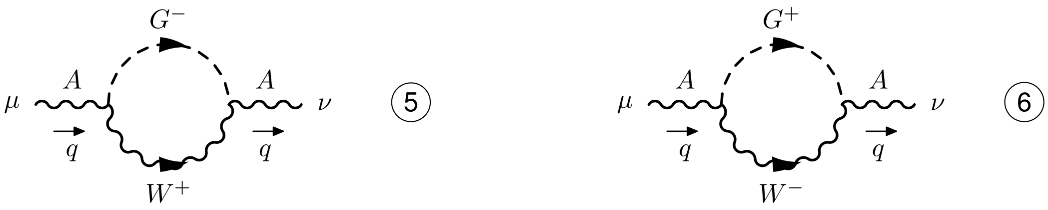


(3)

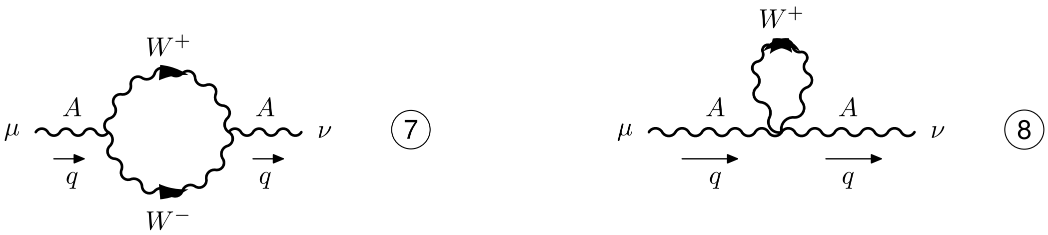


(4) (C.3b)

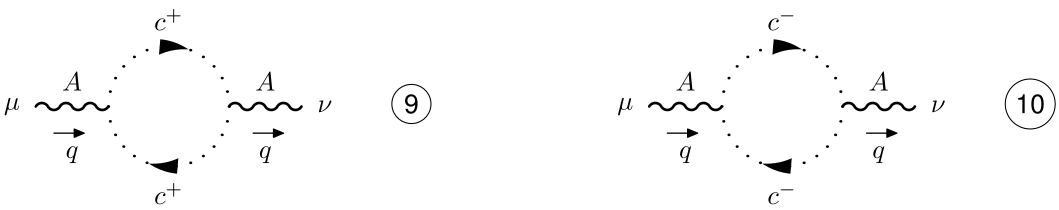
[2] Here we include all the diagrams with internal bosons as they are used in section 4.9.



(C.3c)



(C.3d)

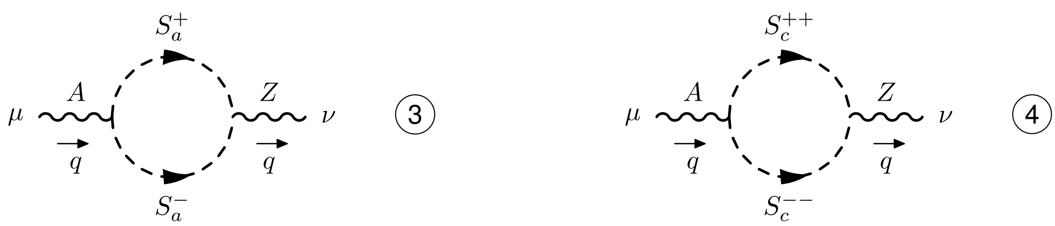


(C.3e)

The diagrams 5 to 10 are equal to the same diagrams in the SM. For that reason, we discard them in the calculation of the oblique parameters.

The diagrams that contribute to $\Pi^{\mu\nu}_{AZ}$ at one-loop level are

(C.4a)

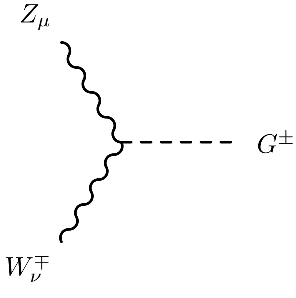


(C.4b)

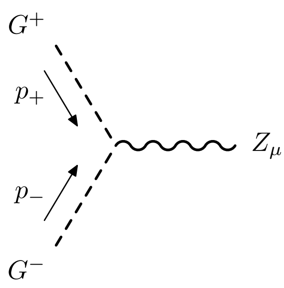
Appendix D

Standard Model Feynman Rules

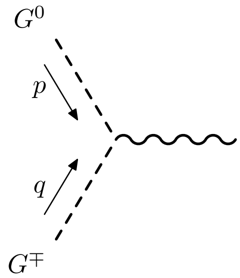
In this work we compute several quantities which are subtractions of the Standard Model values from those quantities from the values of those quantities in a New Physics model. For that purpose we need the Feynman rules for the Standard Model. Some of the Feynman rules we use for the SM vertices are not the same as the usual SM Feynman rules (which can be found, for example, in [28] or in [29]) as we subtract the SM quantities from the same quantities in models for which $m_W \neq m_Z c_W$ and some of the SM Feynman rules are simplifications for $m_W = m_Z c_W$. For that purpose, we present here the Feynman rules used in this work for the SM vertices that are different from those from [28] or [29].



$$= -igm_W c_W g_{\mu\nu} \left(\frac{m_Z^2}{m_W^2} - 1 \right) \tag{D.1a}$$



$$= -i\, g\, c_W \left(1 - \frac{m_Z^2}{2m_W^2} \right) (p_- - p_+)_\mu \tag{D.1b}$$



$$W^{\pm}_{\mu} = \frac{g}{2}\frac{m_Z c_W}{m_W}(q-p)_{\mu} \tag{D.1c}$$

The Feynman rule in D.1a can be obtained by requiring gauge invariance in the process $e^- \to \nu_e\ Z\ W^-$. Knowing this Feynman rule, we can obtain the Feynman rule in D.1b by requiring gauge invariance in the process $Z \to e^-\ \bar{\nu}_e\ \mu^+\ \nu_\mu$ and the Feynman rule in D.1c by requiring gauge invariance in the process $W^- \to e^-\ \bar{\nu}_e\ \nu_\mu\ \bar{\nu_\mu}$.