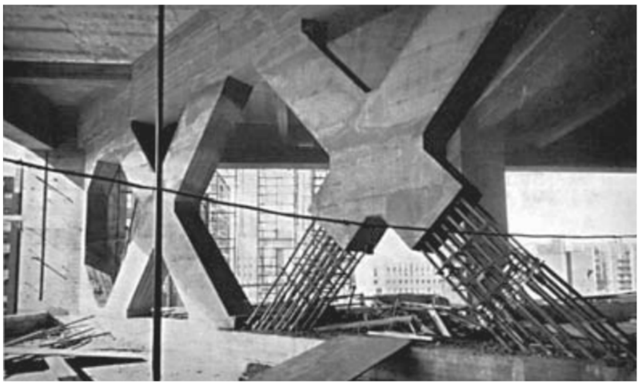

Tall Buildings — Lateral Load Resisting Systems

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Thesis to obtain the Master of Science Degree in

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Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Abstract

Lateral load resisting systems are extremely important in tall buildings, because the lateral loads represent a major concern in tall and slender structures. There are several different systems to resist the lateral loads in tall buildings, each of them with their specific characteristics. Some systems are very intrusive in the façade of the building, imposing an architectural expression, and some are more discreet but either interfere with space or have low efficiency.

An outrigger frame structural system is a lateral load resisting system that transforms the bending moment present in the core of the building into axial load in the perimeter columns. It is an interior system that synergizes with the external elements of the structure. In this way, outrigger systems can interfere with rentable space of floors but are also very efficiency, allowing for taller buildings and with more architectural freedom when comparing with the other systems.

This thesis intends to compare the efficiency of the different types of outrigger frame systems. For this purpose, a comparative study of different solutions were applied to the original outrigger system presented in the Montreal Stock Exchange Tower, in Montreal, Canada, and the most important factors are highlighted.

Keywords: outriggers; belts; structural stiffness; tall buildings; lateral loads

Resumo

Os sistemas resistentes às ações laterais são extremamente importantes em edifícios altos, porque as ações horizontais representam uma grande condicionante a estruturas altas e esbeltas. Existem diversos sistemas de resistência às ações laterais em edifícios altos, cada um com as suas características específicas. Alguns sistemas são muito intrusivos na fachada do edifício, impondo uma expressão arquitetónica, e alguns são mais discretos, mas interferem no espaço ou apresentam baixa eficiência.

Um sistema outrigger é um sistema de resistência às ações laterais que transforma o momento fletor presente no núcleo do edifício em forças axiais nos pilares periféricos. É um sistema interno em sinergia com os elementos externos da estrutura. Desta forma, os sistemas estabilizadores podem interferir no espaço útil do edifício, mas também são muito eficientes, permitindo edifícios mais altos e com mais liberdade arquitetónica em comparação com os outros sistemas.

Esta dissertação pretende comparar a eficiência dos diferentes tipos de sistemas outrigger. Para isso, um estudo comparativo de diferentes soluções foi feito ao sistema outrigger original presente no Montreal Stock Exchange Tower, em Montreal, Canadá, e os fatores mais importantes são destacados.

Palavras-chave: outriggers; belts; rigidez estrutural; edifícios altos; ações laterais

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1. Introduction

1.1. Context and objectives

As cities became more populated, the need for space has been a major priority of city planners and promoters. Since, the horizontal space is limited, the only way to increase space was to go higher and to make structures taller. Together with the increase in height of buildings, the forces increase as well, both the vertical and lateral, but specially the lateral forces. These lateral loads become a priority. If in small structures they are so small that sometimes can be disregarded, in tall buildings they are a major concern and present a few problems and challenges for structural engineers.

There are several different lateral force resisting systems, developed along the years, to be employed in tall buildings. Some systems are simple and cannot reach a considerate height and others can but present other challenges like onerous and arduous building processes and architectural restraints.

Outrigger systems provide strong and stiff structures without interfering much with the facade of the building, subjecting it to more architectural freedom. Some structures are very efficient in resisting the lateral loads but compromise, either by approximating the vertical elements and reducing the views or by interfering with the architecture and aesthetics of the facade, thus outrigger systems present a valuable solution comparing to the other systems. The are many examples of applications of different lateral load resisting systems but few examples and comparisons of different systems under the same conditions and with the same primary vertical system. This thesis intends to address the different types of outrigger systems and compare them with each other and with other types of lateral load resisting systems such as the tube systems.

1.2. Scope of the thesis

This thesis will focus on the lateral resisting systems in general and their behavior, with an emphasis on outrigger frame systems and their different types condensed in direct or conventional outriggers and indirect or virtual outriggers.

The scope of the document will exclude the vertical loads and vertical load paths other than when originated from horizontal loads or when needed as secondary minor verifications.

Another topic that the document will not focus is the definition and detailing of the member elements and their reinforcement. Instead, the analysis is more focused on the placement of the elements, the efficiency of the structure as a whole and the conception of the structural system.

1.3. Document outline

The document is organized into six chapters. The first two chapters introduce the topic and present the problem of the lateral loads and their resisting systems. In this introductory chapter — chapter 1, an overall context is presented as is the scope of the thesis, and the objectives are outlined. The chapter 2 presents the different types of structural systems in an order from which they were developed, their behavior, some details of certain systems and some examples of applications. This intends to introduce the logic and main concerns of the lateral load resisting systems.

Chapters 3 and 4 present in more detail the outrigger frame systems and their behavior. While Chapter 3 explores the different types of outrigger structures, how they work their benefits and their disadvantages, Chapter 4 concerns more about the design and construction considerations that must apply when dimensioning outrigger systems, like shortening effects, design of connections or seismic design.

In chapter 5, a comparison is made between several alternatives to the original outrigger structure present in the Montreal Stock Exchange Tower in Montreal, Canada. The main objective of this is to evaluate the previously studied solutions and assess the efficiency of each solution, comparing them. Lastly, Chapter 6 present the results and conclusions of the thesis and suggestions of future developments are given.

2. Types of Lateral Resisting Systems

There are several types of structural systems that are designed to resist horizontal loads. There are also several ways to classify these systems and divide them. For example, according to Mir M. Ali and Kyoung Sun Moon [1], they can be divided into two broad categories, the interior structures and the exterior structures, and each of the categories can be subdivided into smaller groups as well. This distinction refers to the location and distribution of the components of the primary lateral load-resisting system over the building [1]. A system can be categorized as an interior structure if the majority of the lateral load-resisting system is located within the interior of the building. Likewise, if the majority of the lateral load-resisting system is located on the external perimeter of the building, the system is categorized as an exterior structure. Still, it can be found in any interior structure components located in the perimeter of the building and in an exterior structure there can be internal components also stiffening the structure.

The structural systems of tall buildings were developed over time, starting with rigid frame systems as in the 42m high Home Insurance Building (completed in 1885 in Chicago) considered by many as the world's first skyscraper (Figure 2.1). Shear walls were also an initial system being combined later in a shear-frame interaction system and then working their way until the outrigger frames and mega cores we see today.

At first, the main concern was to support the vertical loads of the building, but with the advance of technology, the increase of strength of the materials used, the reach for higher buildings and the decrease of the buildings weight per volume, horizontal loads such as wind and seismic action became also one of the main concerns. The axial stress of a building was still a focus point of the design along with the bending moment of it. In this way, a tall or super-tall building can be compared to a vertical cantilever and as the height of the cantilever increases, the axial stress increases linearly with its increased weight, but the bending moment increases more rapidly.

The classification of the structural systems and especially their efficient height is only a guideline. It differs with the buildings aspect ratio, shape, load conditions, site constraints, buildings stability, etc., but since the rigidity of each system is different and some are stronger and stiffer than others, as the height of the building increases, the choice of the structural system decreases. For a high-rise building, the choice of a structural system strong enough is limited and it is frequently combined of a few systems whereas for a low-rise building, there can be many choices available. For a high-rise building, since the alternatives are limited, the structural design and the architectural design should go hand-in-hand and be considered together. Also, the structural system of a building tends to be connected to the form and function of it, and this becomes more important in taller building in order that sometimes the structural design has to define the architecture.

Figure 2.1: Home Insurance Building, Chicago

For buildings of 40 stories or less, many systems can be adopted and the most commonly used are rigid frame systems, core systems and shear wall systems. On the other hand, for higher buildings these systems do not provide the strength and stability necessary to satisfy structural safety and serviceability requirements. To solve this, a combination of the effect of a rigid frame and a shear wall was implemented and gave the possibility to go higher. Nowadays, the most commonly used systems that can be seen in super-tall buildings are shear-frame systems, mega-core systems, outrigger frame systems and tube systems. These systems were invented to satisfy the structural safety and the occupancy comfort, in terms of lateral sway, in an effective and economic way [1].

2.1. Rigid frames

Also known as moment resisting frames, rigid frames are one of the two basic structural systems to resist lateral loads [1]. It consists of vertical elements — the columns, connected to each other by thick beams — the girders, in each floor creating a planar grid frame, offering a certain rigidity to the structure. This planar grid frame can also be the external frame of a building and if all the external frames are rigid, this creates a framed-tube.

Moment resisting frames rely on the premise that the nodes are rigid. For this reason, reinforced concrete is the preferable material for this type of structures because of its naturally monolithic behavior, whereas for steel structures rigid framing is achieved by the strengthening of beam-column connections. These frames resist the lateral loads through the combination of the flexure resistance of its elements [7] and it is as rigid as the elements composing it. Thus, it depends directly on the cross section of the elements and inversely on the length and spacing of them. The cross-section of the columns is controlled by both the gravity loads and the lateral loads since they also contribute to the stiffness of the building. The gravity loads are greater at the base of the building thus, sometimes the cross-section of columns tends to be enlarged from the top to the bottom. At this time, the columns were placed where there was the least restriction to architectural planning but at the same time, they needed to be closely spaced to assure the lateral stiffness of the structure. The cross-section of the beams is controlled by the stiffness of the building in order to offer sufficient resistance to lateral loads and ensure acceptable lateral sway. The cross-section of the columns is thereafter slightly increased as well to assure sufficient stiffness.

The lateral sway is probably the biggest disadvantage of the rigid frame system in tall buildings. This occurs because of two reasons: the cantilever moment of the building and the bending of the building's elements. Both these effects can be perceived in Figure 2.2. The cantilever moment of the building is the building behaving as a vertical cantilever drifting at the top because of the moment caused by the lateral loads which is represented in Figure 2.2a. On the other hand, the bending of the building's elements is caused by the shear effect and is manifested in the bending of the columns and beams represented in Figure 2.2b. Due to the nature of the rigid frames, the deformation due to the bending of the building's elements is much greater than the deformation due to the cantilever bending.

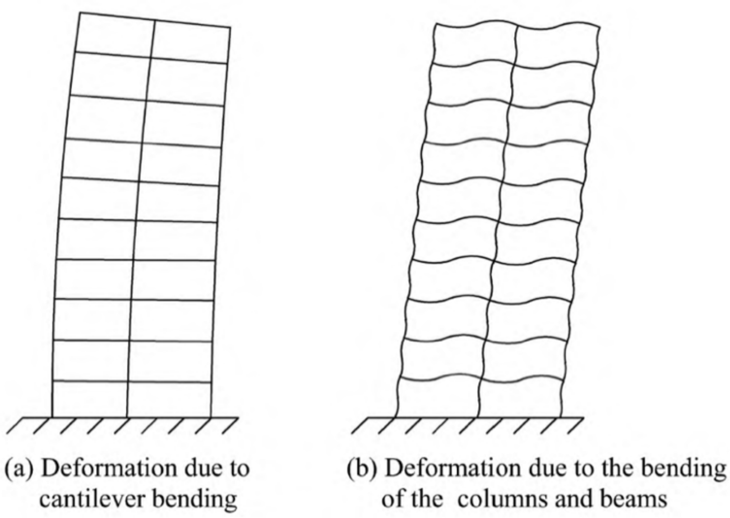


Figure 2.2: Types of deformation of rigid frames

Rigid frame systems efficiently and economically provide sufficient stiffness and rigidity to buildings until 25 stories high. This type of structures started to be applied in 1885 in the construction of the Home Insurance Building in Chicago (Figure 2.1). They continued to be applied in buildings with steel structures until the 64m high Ingalls Building, completed in 1903 in Cincinnati, which is the first tall building with a reinforced concrete structural system (Figure 2.3).

Figure 2.3: Ingalls Building, Cincinnati

2.2. Shear Trusses and Shear Walls

Shear trusses and shear walls are the second basic interior structural system to resist lateral loads. It consists of a vertical truss/wall capable of resisting the vertical and horizontal loads, sometimes eliminating the need for columns, and approximating the behavior of the building to a vertical cantilever rigidly fixed at the base of the building. Due to the cantilever behavior, the inter-story drift between adjacent floors is greater in the upper floors than the others. This is why, in very tall buildings, it is difficult to control the lateral sway of the top of the building. This system can be used in steel, reinforced concrete or composite structures.

For steel structures, shear trusses are used. It consists of braced frames creating a vertical truss that resists the horizontal loads through axial deformation of the diagonals on the braces.

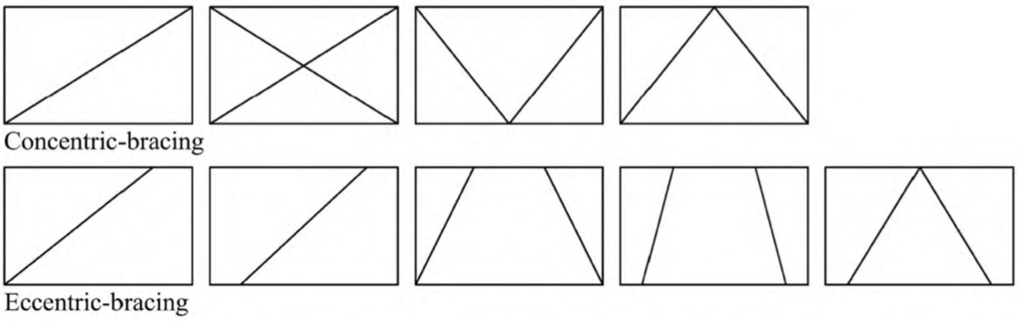


Figure 2.4: Types of Concentric Braced Frames and Eccentric Braced Frames

These braces can have several formats and according to its structural behavior they can be categorized as concentric braced frames or eccentric braced frames (Figure 2.4). Concentric braced frames are braced frames that the braces meet the beams and columns at a single point — the nodes, leaving the structure members with primarily axial forces. On the other hand, eccentric braced frames are frames where the braces end point is eccentric to the beam-column node. These braces cause bending moments on the members due to the eccentricity from the nodes. Concentric braced frames are more rigid and they contribute to the lateral stiffness of the structure within elastic limits but, in general, in seismic regions eccentric braced frames are preferred due to their energy dissipation capacity and ductility [2]. The external shear forces are dissipated by the system's ductility through the bending of the truss members. Eccentric braced frames are also used to accommodate door because of their wider span but, in general, because the diagonals of a braced frame are an obstacle to external sights, they are usually encased within walls and located at the core of the building. Eccentric braced frames are preferred in seismic regions but they have been used in other non-seismic regions as well due to their natural easiness to accommodate doors and other openings [3], [1] which can be seen in Figure 2.5 that represents a door opening placement in an eccentric braced frame and in a concentric braced frame.

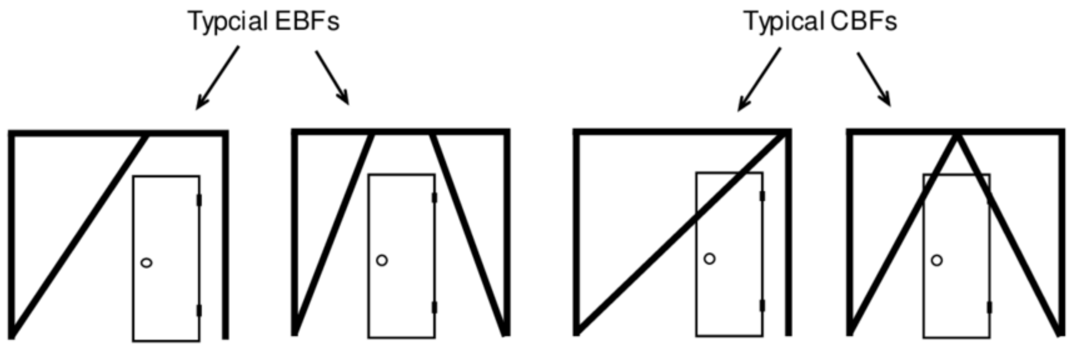


Figure 2.5: Schematic representation of architectural insertion of openings in braced frames

For reinforced concrete or composite buildings, shear walls are generally used. They consist of reinforced concrete walls uninterrupted from bottom to top that can be perforated or solid. It is one of the most used forms for tall buildings to create lateral stiffness. There can be combined in the same plane two or more shear walls connected to each other by beams. In this case the total stiffness exceeds the sum of each individual wall's stiffness because the beams make the walls act as one single unit by restricting their individual behavior. This is called a coupled shear wall.

Usually the shear walls and coupled shear walls are located in the core of the building. Since these shear walls have a great influence on the lateral stiffness, its position affects greatly the behavior of the structure and there are innumerous ways to arrange the shear walls and the cores according to the number, position, shape and direction.

When the shear wall or shear trusses are solely located in the core of the building this system can be called a core system. In core systems, floor slabs are either cantilevered from the core wall or can be arranged in modules of floors if there are exterior discontinuous columns. Both these cases can be represented in Figure 2.6 and in this second case (Figure 2.6b), the slabs are fixed to the core wall and supported by the discontinuous columns and these columns are then supported by strengthened cantilever slabs at the bottom of each module. Core systems can have open cores, closed cores or partially closed cores. Closed cores have a better performance due to the torsion stiffness but partially closed cores are preferred due to architectural reasons. Partially closed cores are open cores that is converted into partially closed core by the means of floor beams. The flexural rigidity of the core in a core system is limited by the flexural depth of the core wall and therefore, in supertall buildings, core systems are not enough to resist lateral loads and so mega cores are used instead.

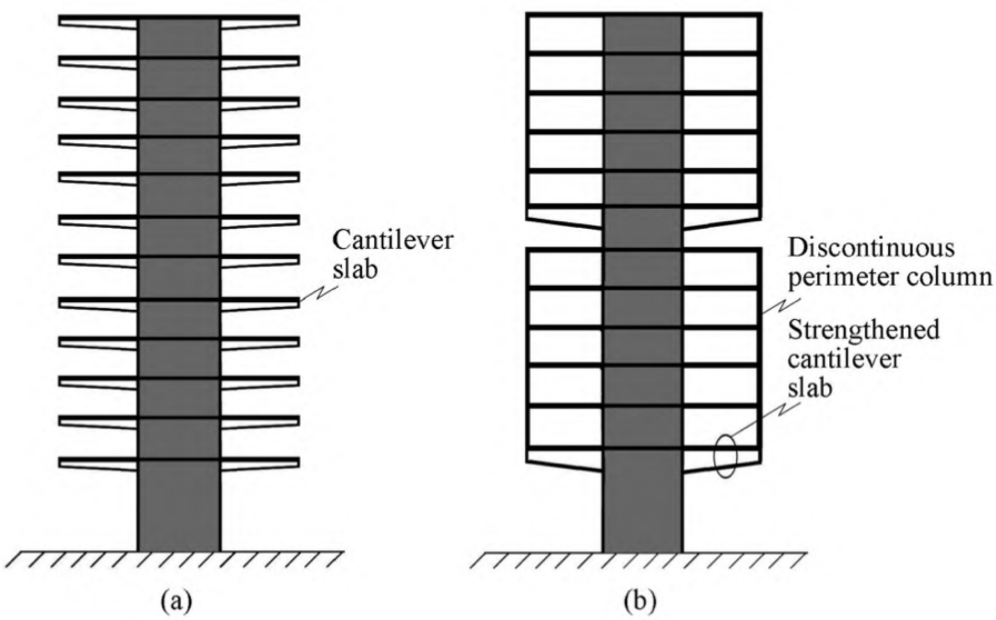


Figure 2.6: Core systems: (a) with cantilevered slabs, (b) with external discontinuous columns

Shear walls and shear trusses efficiently and economically provide sufficient stiffness to resist lateral loads on buildings until 35 stories. Meanwhile, core systems only provide sufficient stiffness to buildings up to 20 stories [2].

2.3. Shear-Frame Systems

Shear-frame is the name given to the interaction of both rigid frames and shear trusses or shear walls like the structures seen in Figure 2.7. This combination results in a significant increase of the lateral stiffness of the structure which can lead to higher structures. This simple interaction can double the efficient height of the building. Moment resisting frames are only efficient for buildings up to 25 stories high and shear walls are only efficient for buildings up to 30 or 35 stories high depending on the width and thickness of the wall [2]. For both these systems, higher structures than the ones before mentioned will result in unacceptable lateral displacements, creating discomfort for the users.

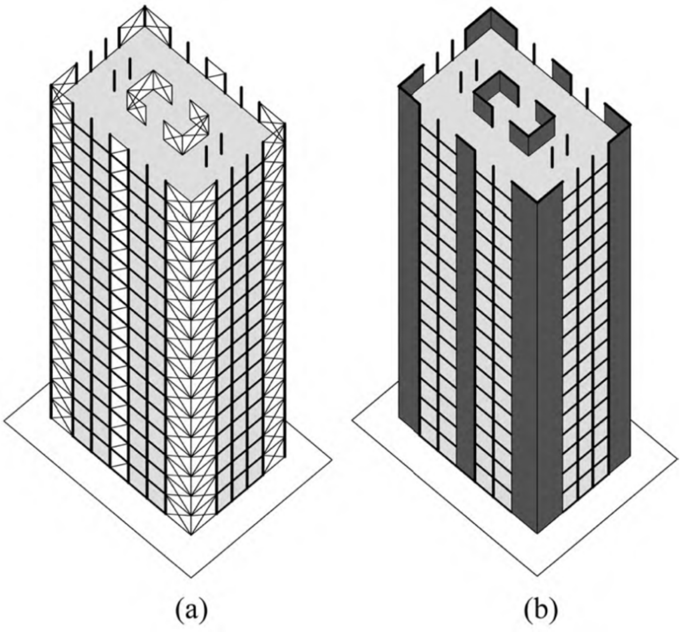


Figure 2.7: Shear frame systems: (a) Shear trussed-frame (braced-frame) system; (b) Shear walled-frame system (source: [2])

The interaction of a frame system and a shear wall or shear truss was the innovative system that made it possible to build higher buildings and created the perspective of interaction between different systems to improve structural strength and stiffness. A paper by Khan and Sbarounis (1964) presented the mechanics of a shear-frame interaction system that led to the development of innovative structural systems that are cost-effective [4]. A rigid frame resists lateral loads through the ductility of the beams and columns and the inter-story drift is higher at the base of the structure because it is where the shear force is higher. On the other hand, shear wall structures are less ductile, but they resist lateral loads within elastic limits because they have a greater area subjected to the shear force, thus they have a great stiffness. As they behave as a vertical cantilever, naturally the higher inter-story drift is at the top of the building while at the bottom is where the structure is more rigid.

It can be seen that the weakness of each system is compensated by the other and that both make a stronger and stiffer structure. While at the bottom rigid frames tend to have bigger displacements, shear walls are more rigid, thus restraining the frame, and at the top the displacements induced by the behavior of the shear walls are compensated by the stiffness of the nodes, beams and columns of the frame, that in its turn restrain the shear wall. The functioning of these two systems acting together is represented in Figure 2.8. In this way, a structure with a shear-frame interaction has greater stiffness than a structure with a shear truss, a shear wall or a rigid frame acting alone.

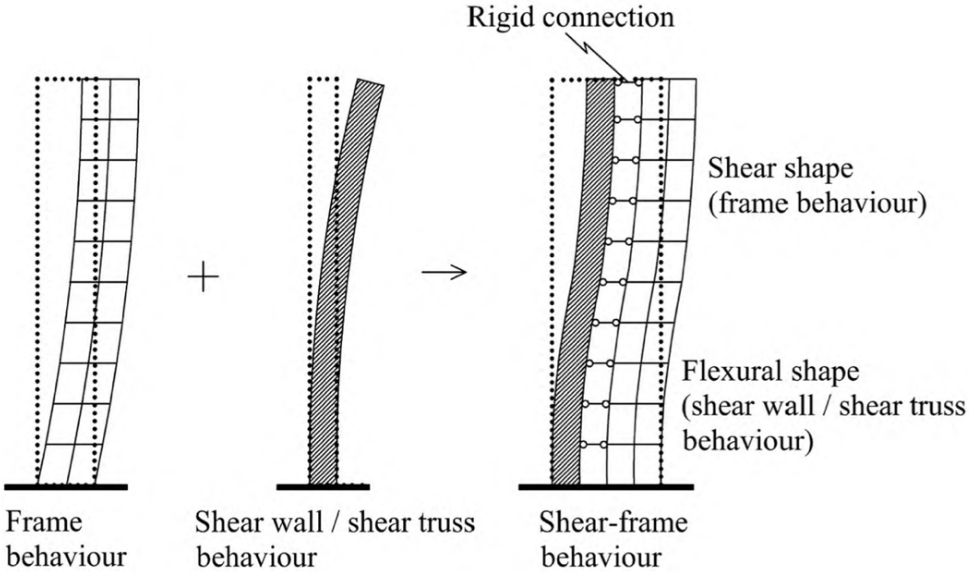


Figure 2.8: Behavior of the rigid frames and shear walls/trusses acting alone and together (source: [2])

Shear trusses or shear walls can usually be found in the core of the building. In the case of a building with a shear-frame structural system where the wall or the truss is located at the core, the system can be called a core-frame system, and if it is composed of a shear wall then is a core walled-frame system, or a core trussed-frame system in the case of a shear truss. Cores are usually partially closed, which means that they are not completely closed since they usually surround elevator shafts and stairwells but at the same time they have beams and slabs at each floor to increase the lateral and torsional stiffness.

The arrangement of the floor is of relevant importance to the behavior of the structure. If the shear walls and core are positioned in a way that the resultant lateral force acts close to the center of rigidity of the building, the system is not subjected to significant torsion, otherwise there is an eccentricity that can create high torsional forces which may be relevant in the design process.

When a shear-frame system consists of a rigid frame and a vertical shear truss the system is called a shear trussed-frame or a braced-frame system. Due to the natural cyclic and long-term periods of the lateral loads, the diagonals of the truss are under axial stress. Most of the braced-frame systems are in steel and some are in composite but rarely in concrete due to the tension stresses created in the diagonals and the poor behavior of concrete to tension forces. The diagonals of the braces can be single or double. If the diagonals are single, the buckling effect has to be taken into consideration when designing the structure but when the diagonals are doubled, the design of the structure is made in a way that only the diagonals that are tensioned are taken into consideration. The same process has to take place when designing a structure with a braced-frame system of reinforced concrete but with the variation that in this case only the diagonals in compression are taken into account and the diagonals in tension are ignored.

The first building ever to use a braced frame system was the Masonic Temple building in Chicago, built in 1892, but some of the most iconic buildings to have that system are the 319m high Chrysler Building, built in 1930, and the 381m high Empire State Building, built in 1931 (Figures 2.9 and 2.10, respectively), both in New York, completed in 1930 and 1931 respectively, and both held the title of the world's tallest building in their time [2].

Figure 2.9: Chrysler Building, New York City

Figure 2.10: Empire State Building, New York City

When a shear-frame system consists of a rigid frame and a shear wall, the structural system is called a shear walled-frame system. This shear wall is usually made of a solid or perforated reinforced concrete shear wall that is located inside the building and it is continued from the bottom to the top. This wall can be a coupled shear wall and can be also around the core of the building (core wall). In these cases, the structural shear wall is usually made of reinforced concrete, but it can also be a composite shear wall when made of steel beams encased in a concrete wall, or a steel shear wall when made of steel plates. The columns and beams of the rigid frame can be made of reinforced concrete, steel or composite. One example of a shear walled-frame system is the 127m high Pirelli Building, completed in 1958 in Milan, that has a structural system entirely made of reinforced concrete (Figure 2.11).

Figure 2.11: Pirelli Building, Milan

2.4. Mega Column, Mega Frame, Space Truss, Mega Core

These systems are made of columns and shear walls with cross sections much larger than the usual (Figure 2.12). They are reinforced concrete or composite buildings that can resist to vertical and lateral loads solely by their big columns and walls which can alone ensure the lateral stiffness of the building.

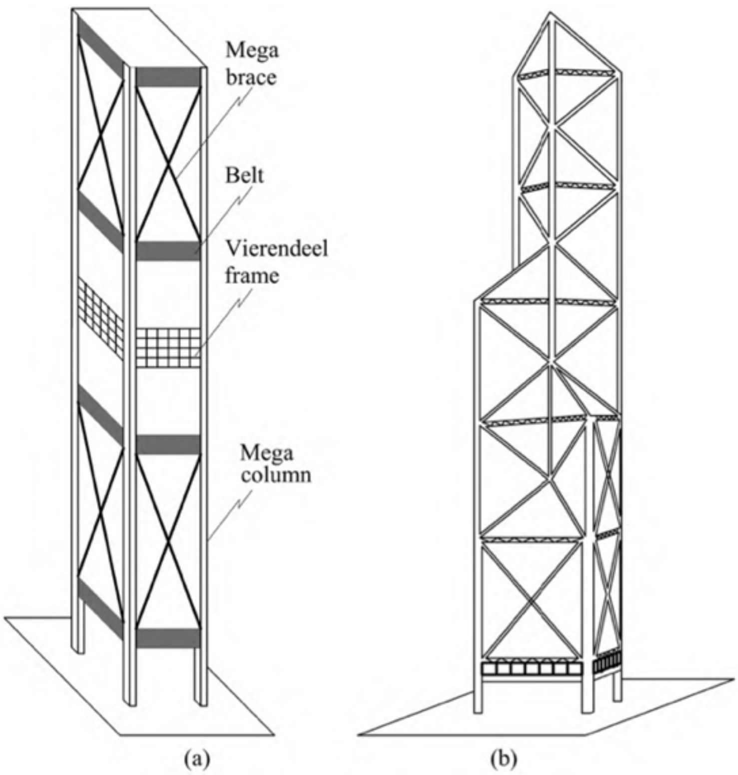


Figure 2.12: Representations of Mega Column Systems: (a) Mega Frame System; (b) Space Truss System (source: [2])

One of the biggest concerns in this type of structure is to ensure the connection between vertical elements as to in this system, the floor slabs alone are probably not rigid enough to act as floor diaphragms. In this way, to restrain the columns or walls laterally in order for them to act and deform as one, belts and Vierendeel frames are normally used, as can be seen in Figure 2.12 (a).

These elements are horizontal shear trusses or shear walls of at least one floor deep that connect the several vertical elements in its floor. Belts are usually located around the perimeter of the building. A good example of a building with belt trusses is the 283m high Cheung Kong Centre, completed in 1999 in Hong Kong, where the belt is evident in Figure 2.13.

Vierendeel frames can go through the building and act also as a transfer structure, since the columns are usually discontinued in the frame. The 259m high Commerzbank Tower (completed in 1997 in Frankfurt) is a good example of a structural system with Vierendeel frames. The building has a triangular floor plan shape with 6 mega shear walls connected together by Vierendeel frames, as can be seen in Figure 2.14.

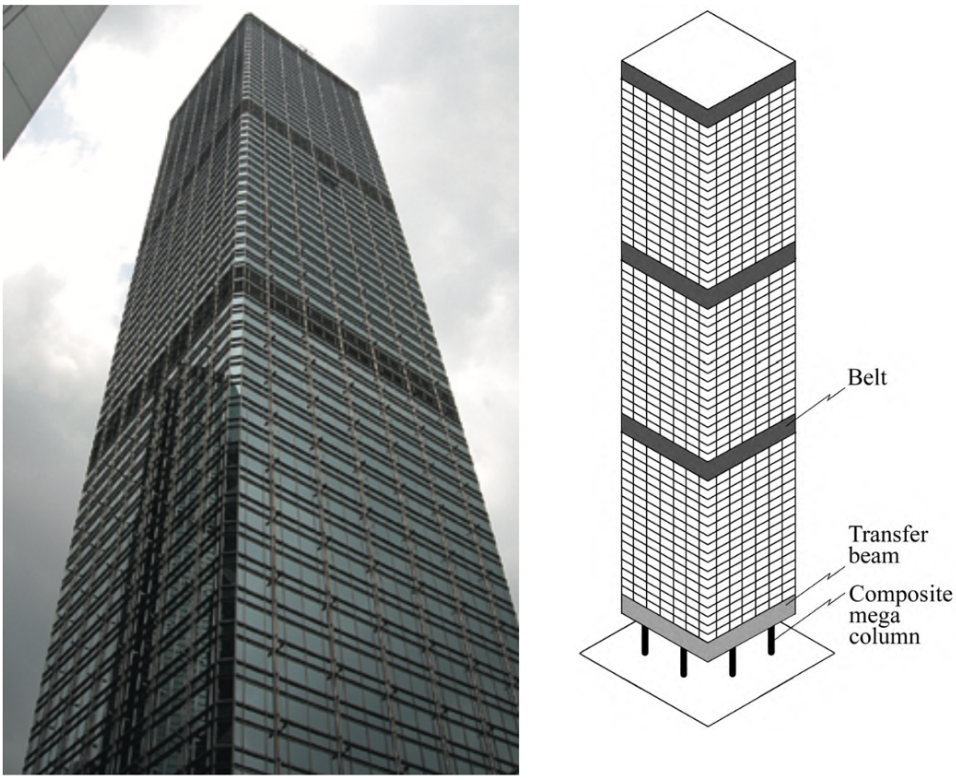


Figure 2.13: Cheung Kong Centre, Hong Kong

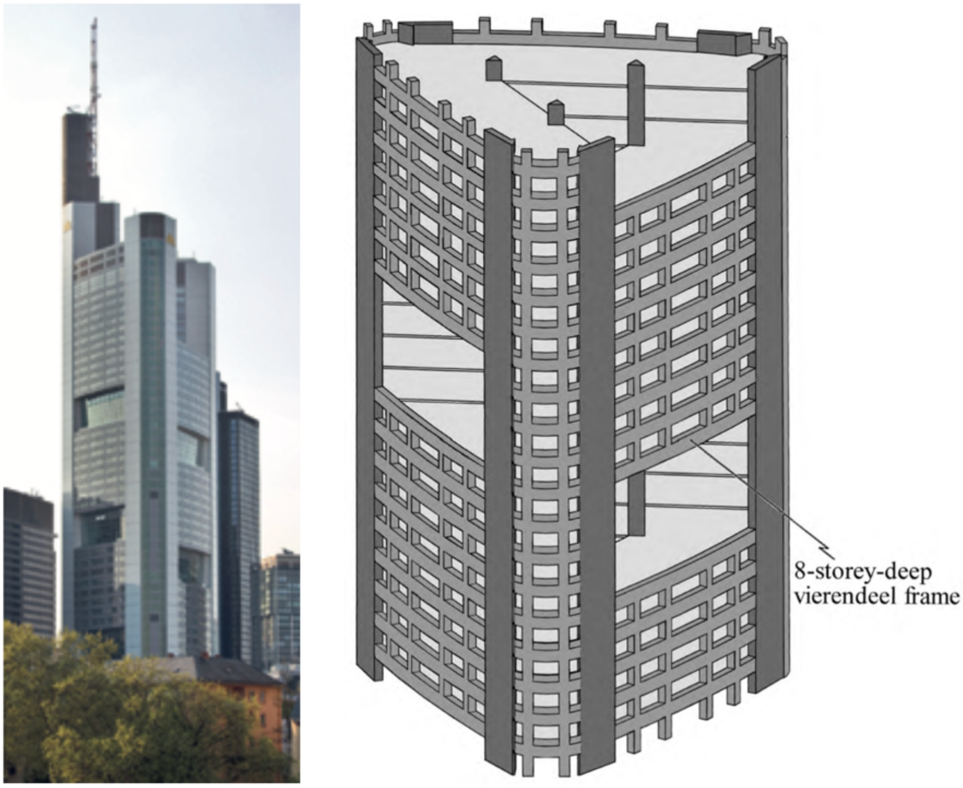


Figure 2.14: Commerzbank Tower, Frankfurt

In some cases, to ensure the lateral connections of the mega column buildings, mega braces are used instead of belts or Vierendeel frames (Figure 2.12 (b)). Even though these braces have the same purpose of the belts and Vierendeel frames, they act in a different way. They also restrain the vertical elements laterally in order for them to act as one, but they don't do it in a single floor. Instead, they restrain them throughout the building's height and, at the same time, they contribute to the building's lateral stiffness with their axial strength and diagonal nature. They are just like normal braces but with a bigger cross section as well and connecting longer spans. They can be located around the building's perimeter and through the building. A good example is the 386m high Bank of China Tower, completed in 1990 in Hong Kong, shown in figure 2.15 with several mega columns that are connected to each other by mega braces that go around the perimeter of the building and through it as well.

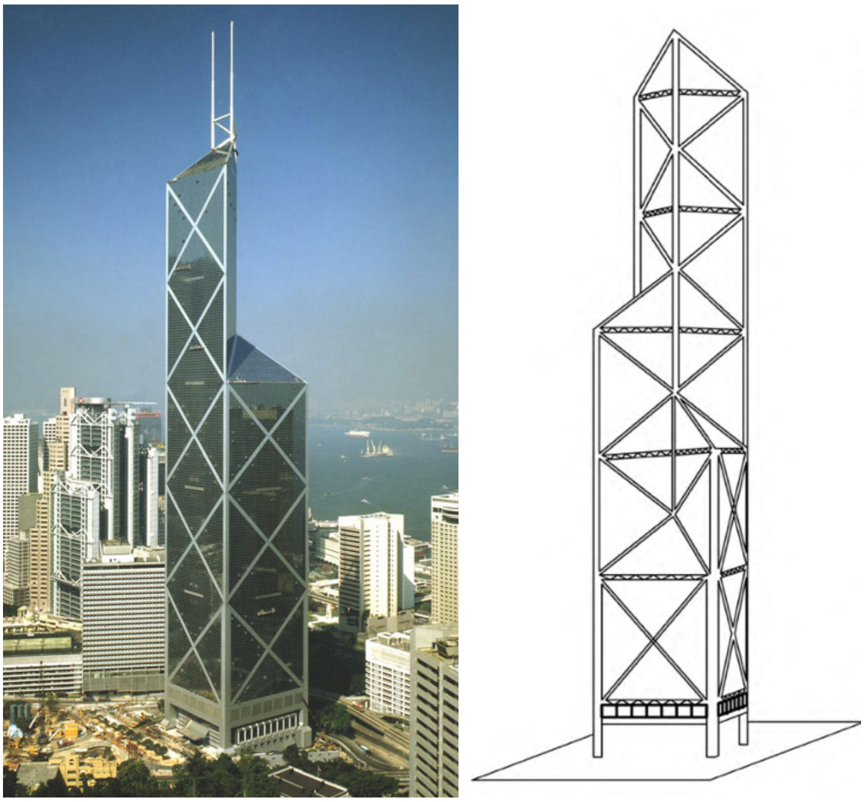
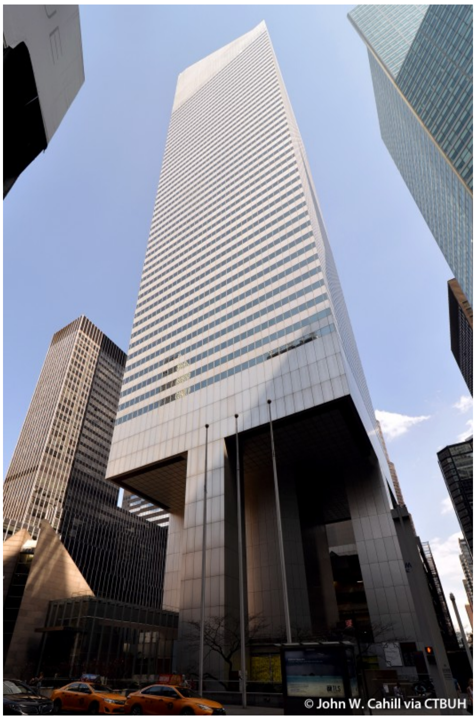


Figure 2.15: Bank of China Tower, Hong Kong

For buildings with mega columns and belts or Vierendeel frames connecting them, the structure can act as a big frame and the belts or Vierendeel frames as girders. This type of structural systems can be called Mega frame systems. For systems with mega columns and mega braces connecting them, they can be called space truss systems since they resemble a vertical tridimensional truss. A Space truss can often be described as similar to a braced tube but instead of having braced frames solely on the perimeter, there are also frames penetrating the building.

Both these systems, the mega frame and the space truss, can efficiently and economically resist lateral loads in buildings with more than 40 stories, reaching 150 stories or even more.

Mega columns can also be found at the base of tall buildings. This solution is used in buildings to open the entrance lobbies and have greater spaces in order to better accommodate the local surroundings and create a comfortable and welcoming environment. In some cases, these mega columns continue until the top of the building creating a mega frame system or a space truss system, but in other cases the rest of the building is composed by several columns with a regular sized cross-section and the mega columns at the base are just larger to compensate being in smaller number. In these last cases, the structural system cannot be categorized as a mega column system (or any of the derivate systems) since the lateral loads are not resisted by the greater stiffness of the mega columns but by other structural elements. This can often be seen in structures with an outrigger frame system or a tubular system. For tube systems for example, many of the columns that form the perimeter of the building aren't supported directly by the foundations. Instead, they begin on an horizontal element called "transfer structure" which transfers the load from that column to the elements that supports the transfer structure. Thus, the columns below the transfer structures have to carry the load of several columns and that is why they sometimes need a bigger cross-section. In the case that they have the dimensions of a mega column, they can be called mega columns but the system cannot be categorized as a mega column structural system. Figure 2.16 depicts the 279m high Citigroup Center (completed in 1977 in New York City) in which is possible to see the detail of the structure at the bottom, where the entire building sits on a transfer structure supported by a reinforced concrete core and 4 mega steel columns. Another example is the 145m high Brunswick Building, completed in 1964 in Chicago, in Figure 2.17 that shows the external frame of the building unloading on a transfer structure supported by mega reinforced concrete columns opening the lobby to the external environment.



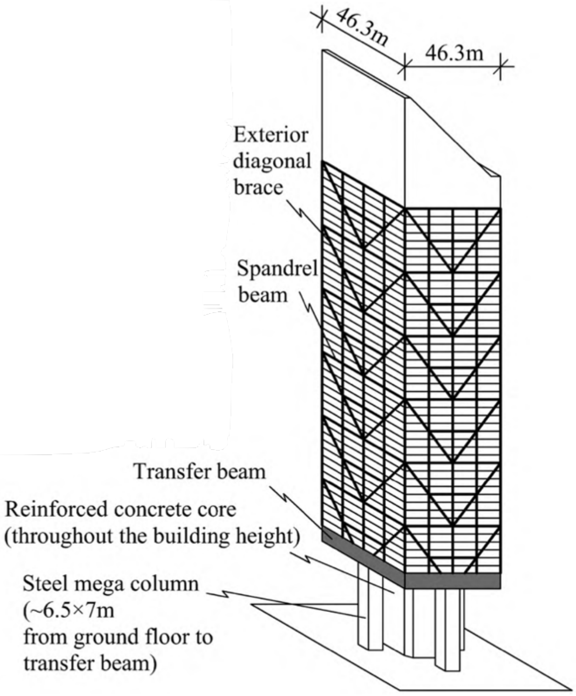


Figure 2.16: Citigroup Center, New York City

Figure 2.17: Brunswick Building, Chicago

Mega cores are reinforced concrete or composite cores that, likewise mega columns, have much greater dimensions than regular cores. These cores can also resist all vertical and horizontal loads and, because of that, they don't need external columns or walls. Moreover, in the same way as core systems, the structures with mega cores can also have external discontinuous columns to help support the floor slabs or even external columns that unload on the foundation and stiffen the structure. Mega cores can alone resist effectively and economically lateral loads in buildings with more than 40 stories [2].

2.5. Outrigger Frame systems

Outriggers were historically used in naval construction. They were the spreaders that connected the sailing ship to the outer stays in order to stabilize the sailing ship and to help resist wind forces in the sail. There is an analogy between sailing ships and tall building where the tall and slender mast is the core of the building, the stays that help stabilize the ship are the external columns and the outriggers have the same function in ships as they have in the buildings where on one side, they help stabilize the ship connecting it to the stays, on the other they help transferring the acting moment on the core as an axial force in the external columns. The Figure 2.18 shows an outrigger canoe. Even though this canoe doesn't have a mast and only has one outrigger it can be evident the presence of it, its connections and functioning as it serves the same purpose as the one mentioned before.

Figure 2.18: Outrigger Canoe

Outriggers are widely used in design and construction of supertall buildings nowadays [1]. They are usually utilized in buildings with a shear-frame system with shear walls concentrated in the core (core-frame systems). In these systems, the cantilever behavior is assured by the core and assisted in the upper stories by the rigid frame and the outrigger acts as a knee helping the structure by stiffening it and significantly minimizing the movement at the top. Usually the core is located at the center of the building's floor plan and the outriggers are spread to the exteriors but the core can also be located at one side of the building and the outriggers spread to the other side [5].

Outriggers in steel structures are commonly represented by an horizontal steel truss and in reinforced concrete structures by an horizontal sheer wall, but it can be found in all structural materials, either steel, reinforced concrete or composite. They have a depth of at least one floor to ensure sufficient flexure and shear stiffness for its purpose and can be in the shape of a shear truss, shear wall or deep beam. These elements are normally connected rigidly to the core and by hinges to the external columns for the moment to be transferred from the core to the outriggers but not to the columns [2]. This way, the columns have mostly axial tension or compression.

In the outrigger level there are also belt trusses that are elements that extend around the building's perimeter. The belt trusses are very similar to the outriggers in the way that they can be either a truss, shear wall or a deep beam and have to have a depth of at least one floor. These elements however are not connected to the core and do not transfer moments. Instead they are connected to every perimeter column and help distribute the axial stress coming from the outrigger to a larger number of columns. This way they minimize the differential elongation and shortening of columns. This is why they need to be deep enough to ensure sufficient stiffness to transfer the load to a large number of columns and assure that all the columns have similar axial deformation. Figure 2.19 represents one example of outrigger frame systems with all its components, namely the shear core, the outriggers and the belt walls. There can be seen that the belts have the same configuration and depth as the outrigger itself and that they are connected to every external column.

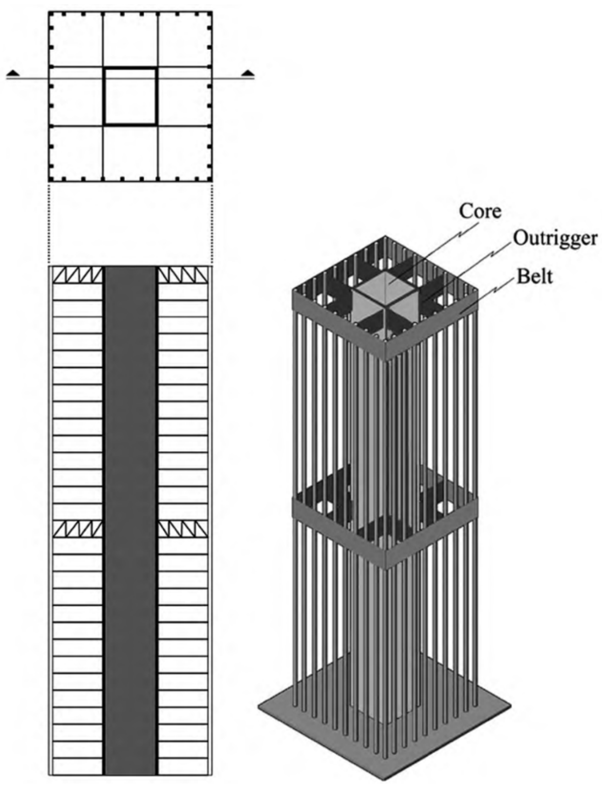


Figure 2.19: Outrigger frame system and all its components (shear core, outrigger and belt)

Belt trusses can also be used in the virtual outrigger systems. These systems replace the need of the actual outrigger and are formed only by the belts and floor slabs. The main idea is to take advantage of the floor diaphragms and eliminate the direct connection of the core to the perimeter columns by the conventional outriggers. This system is as efficient as the stiffness of the belts and floor slabs (specially on the floors where the belts are located). The main advantage of this systems is to free the internal space used by the conventional outriggers. It was introduced in the design of the Plaza Rakyat Office Tower that was going to be built in Kuala Lumpur, Malaysia but, due to financial problems, the project was stopped, and the building was never completed. It was also used in the 264m high Tower Palace Three in Seoul completed in 2004 (Figure 2.20).

Figure 2.20: Tower Palace Three, Seoul

As said before, shear-frame systems are capable of efficiently and economically resisting lateral loads on buildings up to 70 stories high, but for higher buildings the core of the shear-frame system doesn´t have enough strength to resist the bending moments caused by lateral loads. The slenderness of the core and its short width, along with the great amount of stress at the base of the building, can create tension forces. These forces can cause the foundation of the core to uplift.

Introducing outriggers in shear-frame systems not only diminishes significantly the bending moment at the base of the building but also spreads the foundation to a larger size, reducing tensile and compressive forces. In Figure 2.21 can be seen the reduction of the bending moment in the core of the building and it is evident the contribution of the external columns to the spread of the foundation, thus minimizing the excessive stresses and eventual uplifting forces.

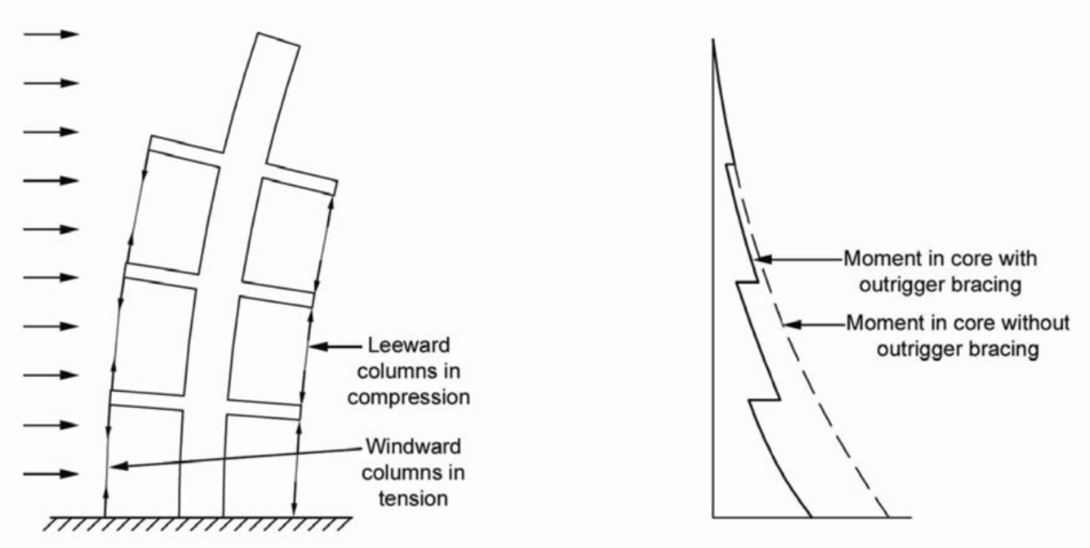


Figure 2.21: Outriggers acting (on the left); bending moment reduction on the core (on the right)

The advantages of the outrigger frame system, besides stiffening the structure, alleviating the stresses in the core and diminishing the sway at the top of the building, are that the building's exterior has a greater aesthetic freedom and the perimeter framing can have a simple beam-column connection without the need for rigid-frame-type connections. This can make the column spacing a lot wider, especially if connecting the outriggers to mega-columns and thus frees the exterior to have greater architectural potential.

The main disadvantages of the outriggers are the space occupied by them, which being an interior structure with very voluminous elements can be a great obstacle, and the fact that they interfere with the repetitive nature of a tall building. This last disadvantage can influence negatively the speed of construction and the erection process of the building. However, it can be solved by a carefully planned project where the elements are placed on technical floors so that their space doesn't interfere with the occupied useful space. Also, having very well planned steps of construction can allow outriggers assemblance to be faster. The space obstacle can also be solved by the use of a virtual outrigger system which, as said before, is the main advantage of this system.

The number of outriggers used in an outrigger frame system and their location in height interfere greatly in the system's stiffness. There is also an expression that approximates the number of outriggers used and their optimal location presented by Smith and Coul in 1991 that will be analyzed later in the text. The number of outriggers used in an outrigger frame system interfere with the system's stiffness in the way that the addition on another outrigger to a system always improves the system's stiffness but improves less than the addition of the previous outrigger.

The outrigger frame system resists efficiently and economically lateral loads on buildings with more than 40 stories [2]. Since it is a system that increases significantly the stiffness of the building and that can be spacious and expensive to install it is only worth doing so in very tall buildings. It has been proven to be sufficient in buildings with 150 stories and possibly more [2]. For this reason, it has been recurrently applicable in super-tall buildings lately. It was first used in 1965 in Montreal and again in 1973 in Milwaukee, but, later on, it was abundantly used in renown tall buildings like the 421m high Jin Mao Tower in Shanghai, the 508m high Taipei 101 in Taipei and the 828m high Burj Khalifa in Dubai (completed in 1999, 2004 and 2010), presented in Figures 2.22, 2.23 and 2.24 respectively. The last two even held the title of the world's tallest building in their time with the last being still the tallest today.

Figure 2.24: Jin Mao Tower, Shanghai

Figure 2.22: Taipei 101, Taipei

Figure 2.23: Burj Khalifa, Dubai

2.6. Tube systems

Tube systems are tridimensional systems that use the entire perimeter of the building to resist the lateral loads. They were invented by the famous engineer and architect Fazlur Rahman Khan who is considered "the father of tubular designs". Khan invented the tube systems and all its variations that were very revolutionary for its new way of conceiving the structural design. They were first used in the 120m high The Plaza on Dewitt in Chicago in 1966 (Figure 2.25), but later were used in more iconic buildings like the 417m high World Trade Center Twin Towers in New York (completed in 1972 and sadly destroyed in 2001 by the terrorist attacks), the 344m high John Hancock Center (completed in 1969) and the 442m high Willis Tower formerly known as Sears Tower (completed in 1974) both in Chicago.

Figure 2.25: The Plaza on Dewitt, Chicago

Tubular systems were revolutionary in their time because they conceive in a tridimensional way the structural resistance of the rigid frame that is a planar grid resisting lateral loads through the resistance of the columns and beams and the connection of them. In other words, it is a tridimensional rigid frame around the perimeter of the entire building that is capable to resist the lateral loads just with its façade. This system frees the internal space of the building, since it is an exterior structural system. This way, the interior of the building can be planned for any type of use, with any form the architect wants. It can also be placed in the interior of the building a core or even another tube to increase the structural stiffness of the building, redistributing the lateral loads through both systems. The form of the tube, thus the floor plan, can also be in any form and shape the architect wants, being the most common the rectangular and circular shapes, in order to fit the surrounding environment.

Tubular systems can have several types depending on the connection of the elements which can lead to different structural efficiencies. The main types in which tube systems can be divided are: Framed-tube systems; Trussed-tube (Braced-tube) systems; Bundled-tube systems; and Tube-in-tube systems.

2.6.1. Framed-tube Systems

Also known as the Vierendeel tube system or the Perforated tube system, it is a basic tubular system form. It can be described as an evolution of the rigid frame systems and an alternative to the shear-frame systems. It consists of closely spaced perimeter columns, about 1,5m to 4,5m apart, connected together by thick beams with 0,6m to 1,2m of depth [1]. As the column space increases, the cross-section of both the beams and the columns themselves increase as well.

As a structural system it behaves as a vertical cantilever, like all tube systems with a tubular cross-section, but with a great influence of the shear lag effect. This effect happens because the columns and beams have limited stiffness, thus when these elements bend in their frame, the external columns, closer to the webs of the tube, undergo greater loads than the internal columns [33]. Figure 2.26 represents this effect and there it can be seen that the actual stress of the columns is less in the middle perimeter columns than in the corner columns, when compared to the supposed stress of a tubular cantilever represented by the dashed line. As this effect takes place, when analyzing a tridimensional frame such as a tube-frame, the behavior of the system itself is somewhere in between the pure cantilever and the rigid frame behavior and the corner columns' loads are enhanced more than they would be if the shear lag effect wouldn't occur.

To minimize the shear lag effect in a framed-tube system, the columns need to be closer to each other and the beams need to be deeper so that the rigid frame's stiffness increases and the deformation in its plane is minimized. This way the tridimensional systems behave closer to a vertical cantilever even if never as a pure cantilever because the shear lag effect can never be disregarded in a framed-tube system.

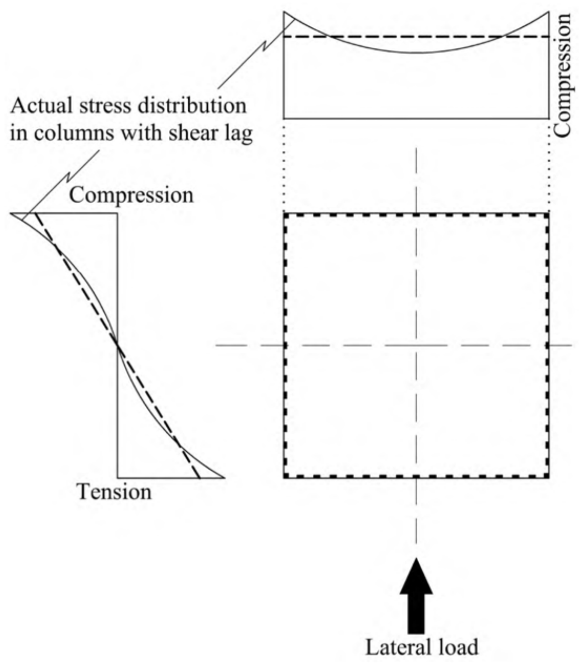


Figure 2.26: Distribution of stress in a tubular cantilever and a framed-tube system affected by the shear lag

As said before, an effective way to reduce the shear lag effect and enhance the efficiency of the structural system in a framed-tube is to approximate the external columns but this obstructs the external sights and, in the ground floor, can hinder the hospitality of the entrance lobby and creation of an inviting ambience. In order to avoid this, some of the external columns may have to be interrupted before they onload on the foundation. Thus, transfer structures are added to the lower floor to transfer the loads of the interrupted columns to the other columns. These other columns, in some cases, can have larger cross-sections in order to resist the loads of several columns but were largely spaced so that the ground floors could be more open to the exterior. The transfer structures are usually in the shape of arches, big beams or horizontal trusses. Some examples of buildings with transfer structures at the lower floors are the 84m high IBM Building (completed in 1964 in Seattle) and the 120m high The Plaza on Dewitt (Figure 2.25). The Figure 2.27 shows the IBM Building in Seattle, evidencing the transfer structure at the bottom with the shape of an arch, along with another figure enhancing the detail of it. In some cases, the use of transfer structures is not necessary because the columns themselves are branching from the bottom to multiple columns. In this case, the columns act as inclined strut transfer structures. One good example of a tall building with a framed-tube system that uses branching columns in the lower floors is the 417m high World Trade Center seen in the Figure 2.28 along with the figure enhancing the detail of the branched columns.

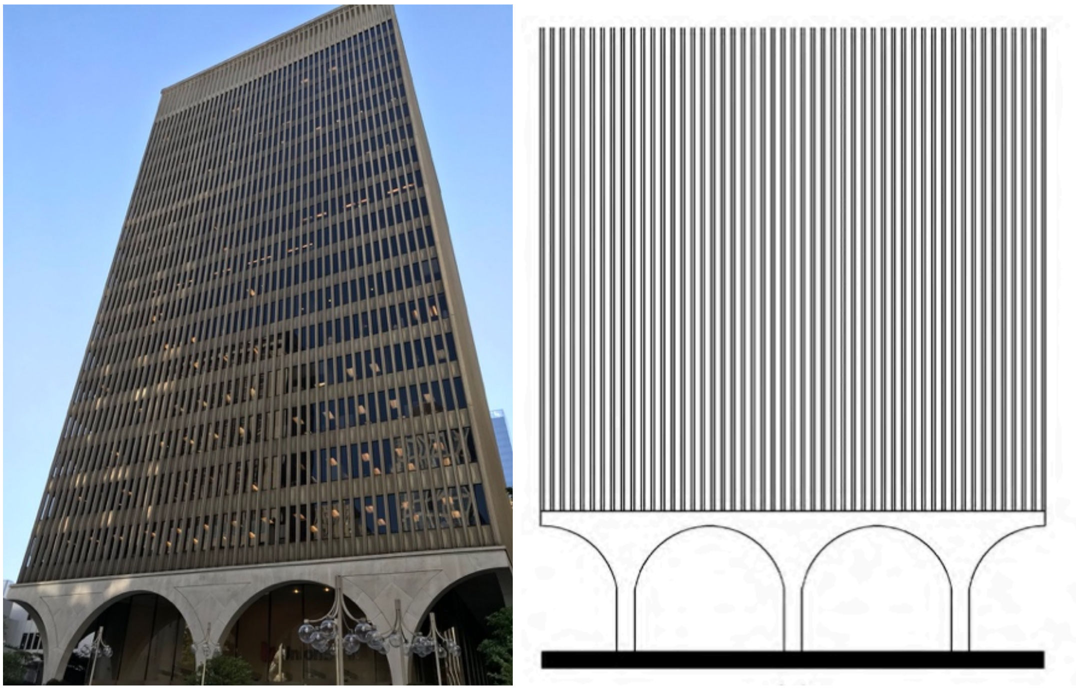


Figure 2.27: IBM Building, Seattle (on the left); detail of the transfer structure (on the right)

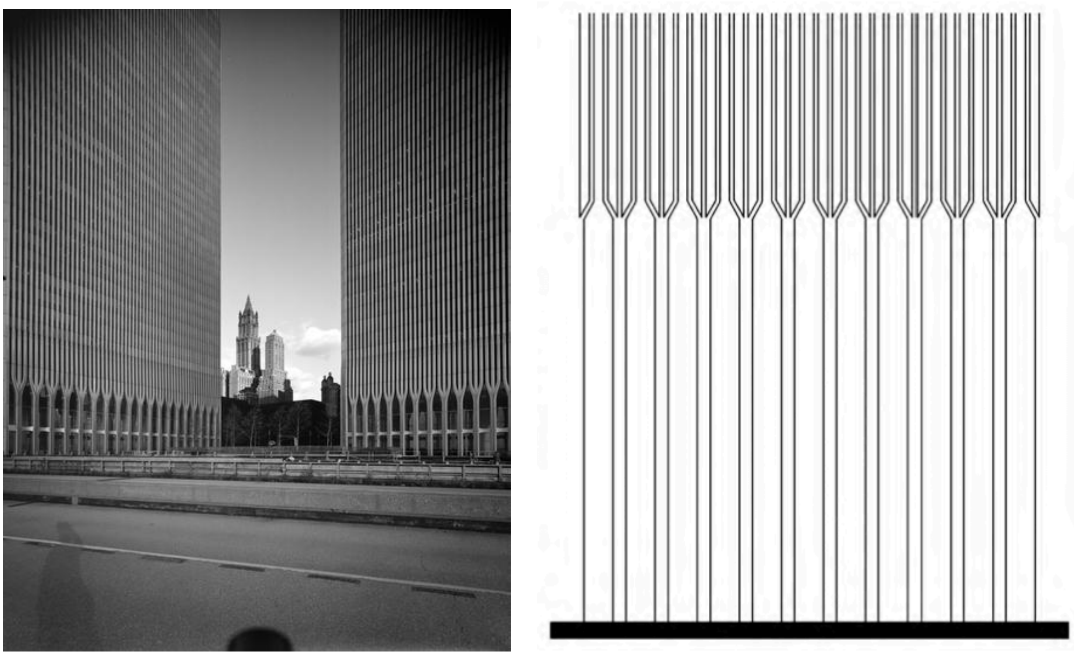


Figure 2.28: World trade Center, New York City (on the left); detail of the branched columns (on the right)

2.6.2. Trussed-tube System

It is a variation of the framed-tube system that consists of stiffening the perimeter rigid frames of the framed-tube with braces. For this reason, trussed-tube systems can also be called the braced-tube systems. The external braces tend to obstruct the external view from inside the building but since the frames of the tube are stiffened by them, the columns can be more spaced from each other creating greater windows embracing more light and improving the architectural quality of the building. The braces also affect the behavior of the frame in the way of making it more wall like thus greatly reducing the shear lag effect.

Framed-tubes became somehow inefficient for buildings with more than 60 stories high [2] because the frames parallel to the direction of the wind act as conventional rigid frames in the way that their deformation is caused by the flexibility on the spandrel beams and the effect of shear lag is aggravated [1]. For this reason and because the vertical and horizontal elements of the frame are designed to resist bending moments, the resulting cross-section is too big from 60 stories upwards. The system also loses the cantilever behavior.

Introducing braces to the frames stiffens the frame in its plane to better resist lateral loads and their resulting bending moments, because instead of resisting solely through the bending resistance of the columns and beams, this system adds the axial resistance of the diagonals.

The diagonal of the braces can also support vertical loads acting as inclined struts and helping the system distribute the loads through the columns and achieving an almost homogeneous vertical load stress on them.

Summing up, the diagonals on the braces can help resisting both the lateral loads — through their axial deformation, and the vertical loads — by supporting and distributing them, allowing the cross-section of the columns and beams of the frames to be smaller and being located more apart from each other creating more spacious windows. Column spacing in framed-tube systems usually is, as said before, between 1,5m and 4,5m and the column spacing on a trussed-tube system can be higher than 10m, as is the case of the 344m high John Hancock Center which has a space between the perimeter columns of 13,5m (Figure 2.29).

Figure 2.29: 875 North Michigan Avenue (also known as John Hancock Center), Chicago

2.6.3. Bundled-Tube Systems

Bundled-tubes are a set of several framed-tubes or trussed-tubes connected to each other working together as a single tube. The internal tubes can be either framed or trussed tubes. It is normally used in very tall buildings where the base's width and length are not large enough for the building's height and it becomes too slender. As the height increases, the base dimensions should increase as well so that a good slenderness ratio is kept in order to control and minimize the sway on the top, but a tall and slender tube can also be stiffened for the same purpose with extra frames inside it so that more tubes are created and connected to each other.

Besides the fact that this system has interior structure features, the internal and external columns can be spaced enough and so it does not affect largely the internal space and floor plan of the building. The system provides great architecture freedom not only by not restraining internal space but also allowing for each tube to have its own geometrical shape and end at different height. The fact that the height of each tube is independent from the others is a great advantage that the other tube systems do not have allowing not only the creation of distinct buildings but also, structurally speaking, it enhances the control of the buildings slenderness ratio.

One good example of a tall building with a bundled-tube system is the 442m high Willis Tower, formerly known as the Sears Tower, that held the title of the world's tallest building in its time. This building is composed by nine framed tubes connected together that finish at different heights with two going from the bottom to the top, as can be seen in Figure 2.30.

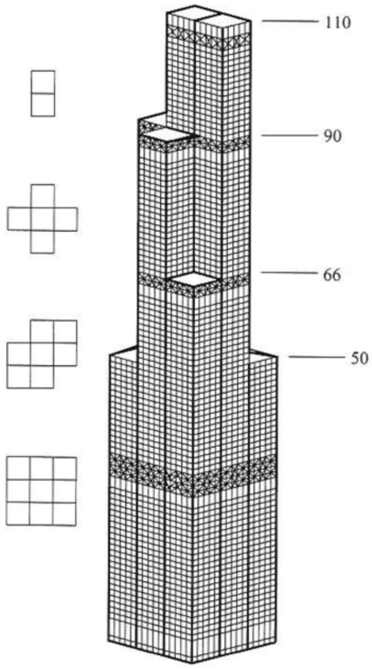


Figure 2.30: Sears Tower, Chicago

2.6.4. Tube-in-Tube Systems

This system can be considered a variation of the bundled tube in the way that it is the method of stiffening a tube with internal structures.

The tube-in-tube system is an external tube stiffened by an internal tube or a core. The structure can even have more than one inner tube to help resist the loads, so in this way it can be both an internal tube and a core inside that internal tube. External and internal tubes can be either framed or trussed and the connection of the tubes is assured by the floor diaphragms which ensure that the lateral loads are distributed to both the external and internal tubes.

2.7. Diagrids

Diagrid systems are a special kind of exterior structure, very close to a tube system, but it has a stronger architectural expression that defines it and it has been frequently used in this era of pluralistic styles for this reason.

Since the beginning of the design of tall buildings and when the primary material for high-rise construction was still structural steel it has been identified the positive effect of diagonals in the resistance and stability of lateral loads but, when in use, they were usually hidden way encasing them in the cores of buildings because they would obstruct the exterior sight.

This changed with the John Hancock Center in Chicago (Figure 2.29). When trussed-tube systems came to play a big role in the new era where the aesthetic statement of the exterior frames' diagonal were solicited. Besides the architectural trend involved, the placement of the diagonals on the exterior of the building would enhance their structural behavior rather than being hidden away in narrower elements as are the cores.

Diagrid systems are somehow similar to tube systems and can be considered somewhere between the framed-tube systems and the trussed-tube systems. As framed-tubes have tubular shapes with closely spaced linear elements in two directions crossing and creating a grid-like frame, so do diagrids, but only with the exception that instead of vertical and horizontal directions they have up right and up left diagonals, seen in Figure 2.31. This difference allows diagrids to have a better resistance against shear lag because instead of resisting lateral loads with the flexural strength of the elements it resists with the axial strength and, as said before, the axial stiffness of elements is much greater that the bending stiffness [6].

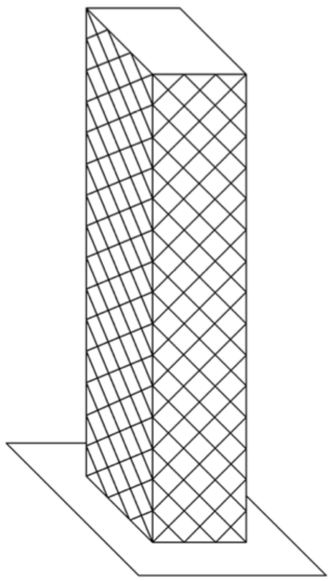


Figure 2.31: Diagrid tube systems

Diagrids are very similar to trussed-tube systems as well by having both tubular shapes formed with diagonals, but trussed-tubes are made of frames reinforced with braces and diagrids are already a diagonal frame and so they do not have vertical elements. This characteristic is very important because the load path of vertical forces is usually supported by the vertical columns and in the case of trussed-tube it is only redistributed by the braces in order to have a homogeneous stress but in the case of diagrids, the diagonals are already the only path to support the vertical loads. In this way, the diagonals forming the diagrid system are the only structure resisting both the vertical and horizontal loads, dismissing any other additional structure to the system.

Diagrids are very effective and can be as stiff as outriggers resisting bending moments resulting from lateral loads. However, outriggers need the presence of a core to resist shear forces since itself doesn't have enough stiffness against shear whereas the diagrids can do so through the axial strength of its diagonals. Nevertheless in very tall buildings, diagrids can have an additional core that, redistributing forces, can stiffen the system helping it resist bending and shear forces and assisting it with the support of the vertical loads. In this case, the diagrid system can be considered to have a tube-in-tube behavior [1].

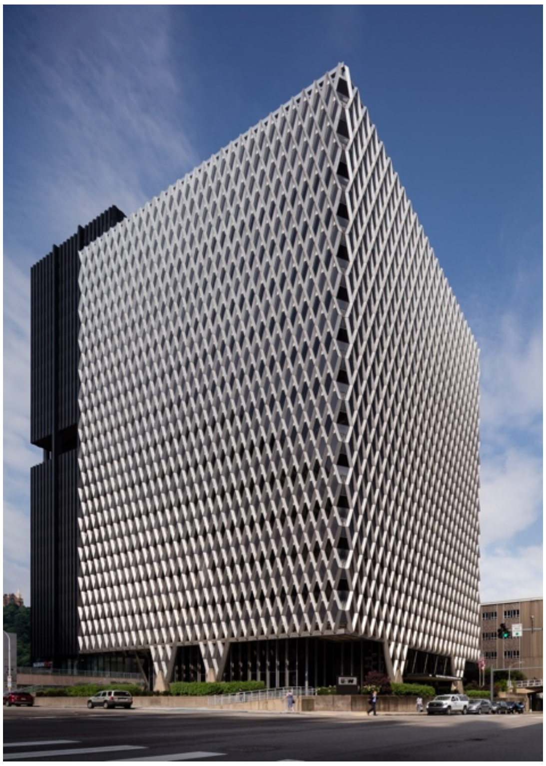


Figure 2.32: IBM Building, Pittsburgh

The first building to be made with a diagrid system was the IBM Building (completed in 1963 in Pittsburgh) which is not considered a tall building, but the exterior perimeter of the building was the first to be made entirely of diagonals and without vertical columns to support it (Figure 2.32). Later on, the diagrids became more popular. Two of the most iconic buildings nowadays with diagrid structural systems are the 180m high Swiss Re Building, completed in London in 2004, and the 182m high Hearst Tower, completed in New York in 2006, both designed by the British architect Sir Norman Foster (Figures 2.33 and 2.34, respectively).

Figure 2.34: Swiss Re Building, London

Figure 2.33: Hearst Building, New York City

Although these examples are made of structural steel, diagrid systems can also be designed in reinforced concrete. Diagrids made of structural steel and reinforced concrete have very different aesthetics but they function in the same way as the diagrid frame resists vertical and lateral loads and offers shear stiffness without the need of other elements. Structures with reinforced concrete diagrids can have more irregular and more fluid diagrid patterns than steel structures which expresses strongly in the building's façade expression [1], but they don't have the ability to have the same slender elements. Some examples of reinforced concrete buildings with diagrid structural systems are COR Building in Miami and the 106m high O-14 Building (completed in Dubai in 2010) in Figure 2.35.

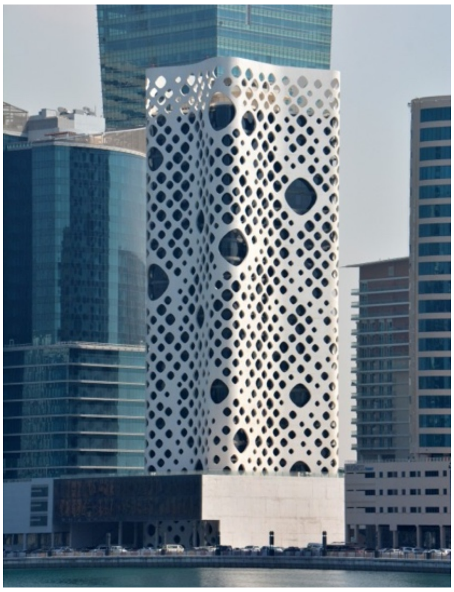


Figure 2.35: O-14 Building, Dubai

2.8. Concluding Remarks

To conclude and gather all the different systems presented, the following chart in Figure 2.36 represents them and their efficient heights in a comparatively mean. It is noted that the chart is represented as a guideline and that each structural system has a wide range of height applications depending upon other criteria like the building shape and stability, aspect ratio, load conditions, site constraints or architectural function. The chart was based on a paper by Ali and Moon [1] and slightly modified to adjust the order presented here. The author referred that, because of the factors presented before, the height limits are only presumptive and are based on experience and the authors' prediction within an acceptable range of aspect ratio of the buildings (between 6 and 8). Even though the height limits are not necessarily correct, it illustrates and serves the purpose of this text as it is only to compare the different systems to each other.

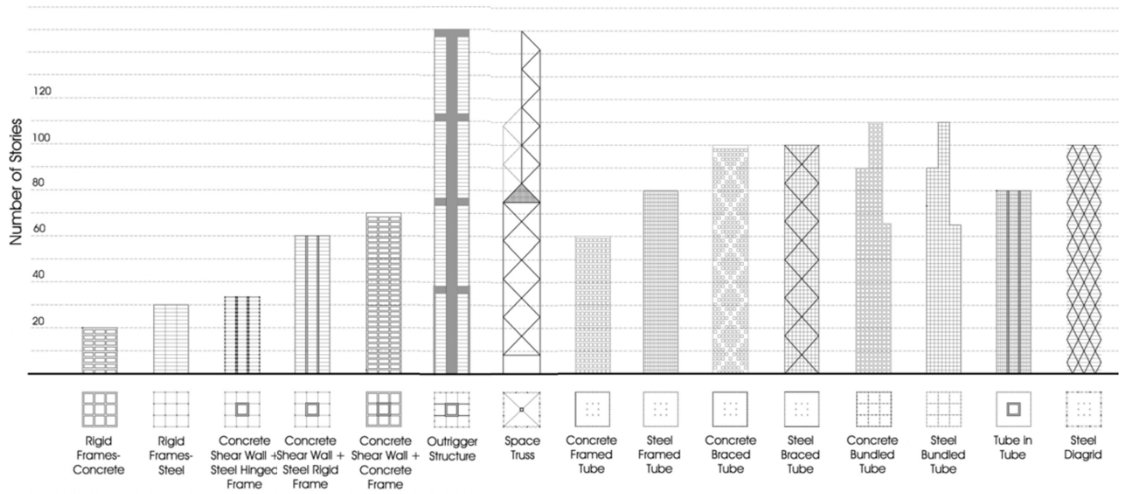


Figure 2.36: Lateral-load resisting structural systems (adapted from [1])

Even though in some cases the material choice has little effect on the height limit of the structure, as is the case for the braced tubes or the bundled tubes, it is interesting to note that in some systems, the material can allow for higher limits and the choice of material depends on the type of system. For example, for rigid frames and for framed tubes, structural steel allows for taller structures but for shear-frame systems it is the other way around.

These systems are represented alone but they can be combined and strengthened by each other creating stiffer, and consequentially, higher structures. For example, an exterior structure such as a tubular frame, may be combined with an interior structure such as core-supported outriggers and belt trusses [1].

One example of a building with a combination of structural systems to resist lateral loads is the 492m high Shanghai World Financial Center (SWFC) which is a composite building, with 101 stories, completed in Shanghai in 2008 (Figure 2.37).

Figure 2.37: Shanghai World Financial Center, Shanghai

The SWFC was first designed as a shear walled-frame system with 460m height but after the foundations were completed, the owners decided to increase the height of the building by 32m making it a total of 492m. Since the construction of the foundations were completed, and to avoid the insufficient load-carrying capacity of them due to the increase in height, there was a need to reduce the weight of the building by 10% [2]. In order to do so, the solution was to reduce the thickness of the reinforced concrete core wall since it had the largest share of the total weight. The reduction of the core thickness led to a reduction of the resistance of the core against lateral induced loads and this made it necessary to increase the structures lateral stiffness [8]. Thus, the structural system comprises of three main components: the reinforced concrete shear walls; the mega-frame structure which is composed by mega-columns, diagonals and belt trusses; and outrigger trusses assuring the interaction between the shear core and the mega-frame [9]. The diagonals are formed of welded boxes of structural steel that are in-filled with concrete. The mega-columns are of mixed structural steel and reinforced concrete as well and have a pentagonal shape that shortens throughout the height of the building. The Figure 2.38 clearly depicts the five system elements presented before.

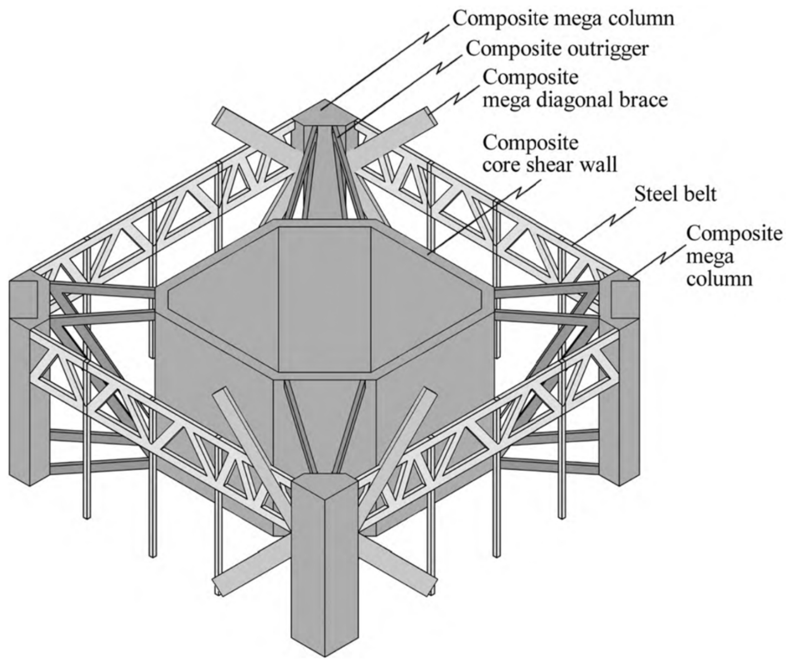


Figure 2.38: Detailed components of the SWFC structural system

3. Outriggers: Concept and Types

The race for higher heights started with the tower of Babel and was built at the time with only brick as a structural material [15]. Nowadays we have much higher constructions with materials like reinforced concrete, steel or composite material of steel and concrete. At earlier times, the main structural system was rigid frames like we saw before, and nowadays we have the means to calculate much more sophisticated systems like the tube frame systems and the outrigger frame systems. The widespread popularity of outrigger systems can be seen as a response to fundamental disadvantages of the tube frame systems. Tube systems have relatively dense exterior frames that resist the lateral loads alone with little or no help from the building core and the lateral resistance of any structural system increases if the perimeter couples with the core and the deeper the beams that connect the exterior to the interior structures, the stiffer the system. While structurally efficient, the tube systems also have a strong presence on the building exterior with limitations for architectural aesthetic freedom and the core-and-outrigger system offers far more perimeter flexibility and openness. Spandrel beams in outriggers are sized for gravity loads alone thus can be relatively shallow and column spacing can be adjusted to meet architectural requirements [11]. Also, compared to tube buildings, outrigger buildings tend to reveal very little of their underlying structural logic from the exterior.

In Figure 3.2, it can be seen that the 223m high Olympia Centre (Chicago, 1986), which has a framed-tube structural system, has very closely spaced columns that inhibit the external view from the inside [24], has a strong presence on the architecture of the building and exhibits easily its structural logic when compared to the building in Figure 3.1 which is the 492m high Shanghai World Financial Center (Shanghai, 2008). This last one has an outrigger frame structural system that allows it to have a bold aesthetic and free architecture planning and, unlike the Olympia Centre, doesn’t reveal anything of its underlying structural logic.

Figure 3.2: Olympia Centre, Chicago

Figure 3.1: Shanghai World Financial Center, Shanghai

Although the use of outriggers became very popular in tall-buildings, research is still very limited. Some studies focus on optimum location and overall efficiency in controlling drifts [11, 25] even though possible locations are limited by usage and building layout. Because outriggers occupy quite some space, they are usually installed in mechanical floors and refugee floors instead of optimum locations to not interfere with usable and rentable space, and in some countries these floors are determined in the codes, as is the case of China for example, that requires tall buildings to have a refugee floor every 15 floors. Thus, the locations for installation of outriggers are determined not by the engineer but by the program of the building.

With slender and taller buildings, the deep beam that once connected the exterior to the interior structures is now not enough and once the building height increases, it is very difficult to adopt the deep beam concept as the depth of this beam will be more like a wall or a single or double floor steel truss. Additionally, it is ideal that the outrigger can be as deep as possible, and engineers try to request at least two-story height but in some buildings the floor space is limited to one story height to increase the floor space usage and rentability. Because the outriggers' location is limited to the available locations instead of the optimum locations and because the height available for installation is also limited, the study of outriggers is preferable and more practical to be focused on the optimum topology instead of optimum location. This affects the effectiveness of the outrigger element and its stiffness with the restrictive height instead of looking at the structure as a whole and locating the best locations.

3.1. Concept

The outrigger concept has been used for half a century in the design and construction of high-rise buildings but it has been used for more than a thousand years in boat construction (Figure 2.18). The first outriggers used, as seen before, was in a Polynesian oceangoing boat that connected the main boat, with a canoe-shaped hull, to outer stabilizing floats (or amas). This concept was observed in the boat factoring industry and brought to the structural design of buildings, and there can be seen some similarities between the two: the outriggers help resist the lateral induced overturning forces that can lead the boats to capsize and buildings to have other important issues; outriggers help to stabilize both the boats and buildings by shorten the period of their sway creating more comfort for the users; and outriggers can be seen in one side or in two sides of the structure of both boats and buildings, although it is more common for boats to have outriggers on just one side and buildings to have outriggers on both sides.

The main idea is to couple the perimeter and the internal structure as a whole. If uncoupled, they both work as a pure cantilever [15] and the lateral stiffness of the system is about the same as the stiffer structure, either interior or exterior. With deep beams connecting interior with exterior structures the stiffness of the system increases a lot, but since the typical span goes from 9 to 15m it is quite difficult to provide beams deep enough to ensure the connection and rigidity without compromising space. Therefore, and since all tall buildings have refugee floors and mechanical floors, this provides an opportunity for engineers to use these spaces and appropriating all their available height to stiffen the structure with the walls or trusses required creating the outriggers. The main behavior of outriggers is simple: they are rigidly attached to the core, engaging the outer columns, and when the lateral load induced moments acting on the core forces it to rotate, the outrigger tips at the end move upwards and downwards following the rotation of the core and at this point, as the outriggers are connected to the perimeter columns, these columns restrain this movement creating an opposing force that will be then transferred to the core to help resist the overturning moment. This is exemplified in Figure 3.3.

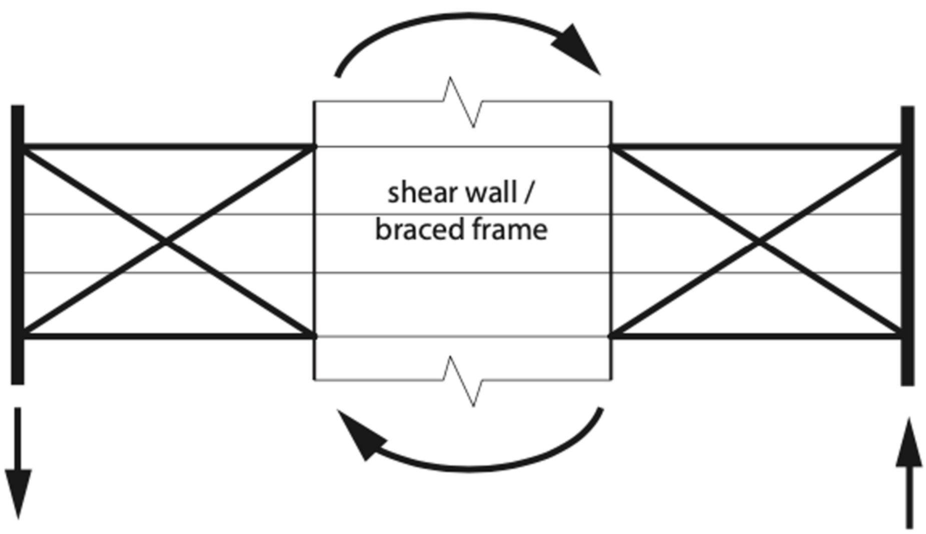


Figure 3.3: Force transfers in conventional outrigger system (Source: [11])

Although their behavior is simple, their design and analysis are not that simple because the distribution of forces depends on the relative stiffness of each element and it cannot be arbitrarily assigned overturning forces to the core and outrigger columns. On one hand, while it is known that bringing outrigger columns and core together as one lateral load resisting system improves the stiffness of the structure and helps resist overturning moments, it doesn’t help against core horizontal story shear and in fact it can actually increase the shear in the core and change direction at outrigger levels. But on the other hand, core and outrigger systems are still popular and frequently selected for tall and slender buildings because the overturning moment is larger when compared to horizontal story shear, and flexural deformations are a major contribution to lateral deflection.

Outrigger systems are very popular due to some benefits they present, but they are not a solution that fits all cases. There are some situations favorable for the application and some that are less suitable.

3.2. Benefits

The most obvious benefit, and probably the most important of them all is the reduction of deformation. A building with a core system and outriggers engaging perimeter columns can experience a reduction in overturning moment of 40% in the core when compared to a system of pure cantilever of the same core [10]. This reduction in overturning moment will then be manifested in reduction of drift and building deformation but this depends on the relative stiffness of the core and outriggers elements. For supertall buildings with mega columns designed for drift control, the reduction of core moment can go up to 60%.

Another benefit is the efficiency of the use of material towards the increase of stiffness. Firstly, the columns that are already sized for gravity loads may be capable of resisting outrigger loads with minimal adjustments as different load factors apply to different design combinations. Then, if additional overall flexural stiffness is required, the additional material for the outrigger lever arm or for a belt is more efficient than the additional material in the core for extra flexural stiffness. At last, by decreasing the building’s overturning moment that must be resisted by the core and walls, the quantities of material in these elements can be reduced since they do not need the same amount of stiffness and, even though the quantities of materials for outriggers and belts are added and for columns are increased, it is by a smaller amount thus, the overall quantities of material are reduced.

Outriggers also help to effectively distribute overturning loads on foundations. A core-only lateral system applying large local forces from overturning moments can generate large shear and flexural demands on foundations that the design becomes uneconomical or impractical. Outriggers can solve these large demands not only by reducing the overturning moment in the core, alleviating the core foundation, but also by spreading the loads through the entire footprint of the building. Reducing these large demands may also help reducing the variations in sub-grade stresses or pile loads under the core which will in turn reduce the foundation rotations helping with the overall and inter-story drift but on the other hand, outriggers may also change other aspects in the foundation design which must be checked for all load combinations.

Outriggers and belt trusses can help reduce differential axial shortening of vertical elements such as core, walls or columns by gravity force transfers. Shortening occurs due to shrinkage, creep and thermal changes and if there is differential shortening of vertical elements, floor slopes can increase to an uncomfortable point and this can be worsened throughout the height of the building. The reduction is achieved by transferring gravity forces between columns through belts or between column and core through the outrigger element, but this comes at a cost as the force transfers can create some locked-in forces in the outrigger and belt elements and these can be as high as the ones resulting from the lateral load resistance. Balancing benefits and costs require a solid understanding of the phenomenon.

Another advantage of the load transfer properties of the outriggers is that it can create an alternative load path in case of a sudden loss of member capacity or connection. This means that if a perimeter column fails, the floors above it can "hang" from the tensioned column left until the upper belt truss that redirects the load to the other unbroken columns [11]. In case of an absence of belt truss, it can be redirected through the outrigger to the core. This can happen in both ways where in the case of a failed core column, the load can be transferred to the perimeter columns through the outrigger. Of course, in the event that this happens, the design must be checked to confirm that the alternate load paths can resist the resulting forces instead of leading to further failures but for this design check the load factors are smaller and the capacities of the elements are larger than those used for the basic design.

Belts can also improve the torsional stiffness of the system. A core-only structure has a limited torsional stiffness compared with a framed tube because of its small width and therefore small distance between elements. An outrigger, while reaching out to a perimeter column connecting it to the core, doesn't improve much the torsional stiffness but a belt truss can force the perimeter columns to act and resist the torsion as one. Even if not as high as a continuous framed tube, belts can significantly improve the torsional stiffness.

Core and outriggers systems also permit for greater architectural freedom as external column spacing can be adjustable to satisfy aesthetical goals and specific functional requirements. In supertall buildings with mega-columns this can be further enhanced as the mega columns permit the façade to be more open. This overcomes one main disadvantage of tubular forms, as said before, which is the close column spacing.

3.3. Less suitable conditions

Structural systems governed by story shear deformations, such as rigid frames, would not benefit enough from outriggers to justify their cost as outriggers are efficient at reducing the overturning moment but do not contribute much to increase shear stiffness. Additionally, cores that are already comparatively stiff in flexure, with a low aspect ratio (i.e., building height/core width), do not need adding an outrigger and columns since outriggers systems interact with cores based on the relative stiffness. The size for the outrigger and columns would be larger than the size needed for strength requirements. For this reason, outriggers in office buildings that have generally more elevators, stairwells and mechanical rooms and thus wider cores, are only worth adding if the height of the building justifies. On the other hand, residential buildings tend to have smaller and narrower cores when compared to office buildings, since they usually only incorporate stairwells and less elevators, and consequentially the justification for the outrigger elements comes at a smaller height.

While in the subject of core, if these are eccentrically located to the center of the floor plan of a building, the lateral loads tend to create torsional forces and torsional deformations in the building. If the design is controlled by the torsional forces, a conventional outrigger system is not effective as it doesn't add enough torsional stiffness, especially if it is an outrigger system without a belt. In this case, it would be a better option to have a tubular system like a framed-tube or a braced-tube.

Buildings with an unsymmetrical floor plan also would not be the appropriate choice to install an outrigger system. In a building where an outrigger system is symmetrically installed, lateral loads are resisted by a couple vertical forces in the perimeter columns, but in the case of unsymmetrically distributed outriggers, when resisting lateral loads, it generates an axial force in the core as well as the couple forces in the perimeter columns. There are also problems associated with the locked-in forces induced by the differential shortening of the vertical elements. In a symmetrically distributed outrigger system, the building will deform downwards due to differential shortening. In the case of unsymmetrically distributed outriggers, the differential shortening will create a locked-in moment in the core that leads to lateral displacement. This means that the building deforms laterally due to gravity loads alone. Still, there are some examples of successful unsymmetrically distributed outrigger systems, proving that it can be done and that it is possible if the above-mentioned concerns are addressed in design.

While steel shortening is elastic and well defined, concrete long-term shortening is dependent on the construction process, time dependable, larger and difficult to predict. For buildings with a steel core and steel perimeter columns this problem doesn't take place and for a building with concrete core and concrete columns it can be comparatively small but there is a problem when different materials are used for the different vertical elements. In a building with a concrete core and steel perimeter columns the difference between steel and concrete shortening can be quite large and may grow with time after construction is completed. In the case of different materials, outrigger installation must be carefully addressed, and the designer must have a deep understanding of the shortening effect.

Both conventional and virtual outriggers require big space and if the mechanical floor design is already too tight or if it only exists in the lower floors, then the addition of the outriggers may not be worthwhile for the project overall as it creates other issues that might not be worth dealing with.

3.4. Conventional Outrigger Systems

Although these conditions and benefits generally apply to most cases, there are different types of outriggers with distinct forms, ways of functioning, features and qualities. As presented before, the two main types of outriggers are the direct or conventional outriggers and the indirect or virtual outriggers.

Direct or conventional outriggers are stiff trusses or walls oriented in a vertical plane that connect the shear core to the columns at the perimeter of the building. Lateral loads causing overturning moment and rotation of the core at outrigger levels will try to move outrigger truss tips up and down and at this point the columns will restrain this movement generating opposing forces.

The most common conventional outrigger system has a concrete core with concrete outrigger walls or steel outrigger trusses projecting to the perimeter columns. The core can be also made of steel (either steel braced frame or steel plate shear wall) but in that case they typically engage steel outrigger trusses. The columns can be made of concrete, steel or composite, but the material should be the same as the one used in the core because using different materials can lead to large levels of differential vertical shortening. The selection of materials, besides shortening, depends on various other factors like the required design strength and stiffness, the connection forces and details, space limitations, material availability, construction methodology and schedule.

The first building to use outriggers was the 190m high Tour-de-la-Bourse (Montreal, 1965) in Figure 3.4. It is a concrete structural system. The principle was to use fewer but larger elements to support the dead loads keeping them under compression regardless of the load cases because the axial force is more concentrated in fewer elements [26]. The building is composed of a concrete core and four large columns at the corners engaged by four levels of X-braced outrigger trusses. One example of the floor plan can be seen in Figure 3.5 and in it there are also represented eight lateral columns that are independent from the primary structural system. They constitute the secondary structural system, and their function is only to support the floor.

Prior to this project, essentially all tall buildings were made of steel frames and not long after the Tour-de-la-Bourse, two other buildings followed and they were a core-and-outrigger system inside steel frames [27]. These two were the 153m high BHP House (Melbourne, 1972) and the 183m high US Bank (Milwaukee, 1974). Designs have evolved since these pioneering projects in order to build the enormous towers of today like the Burj Khalifa and the greatest changes probably resulted from the development of high-strength concrete. High-strength concrete is a concrete with a compressive strength higher than 50MPa and the concrete used in the Burj Khalifa has a compressive strength of 80Mpa.

Figure 3.4: Tour-de-la-Bourse (a.k.a. Montreal Stock Exchange Tower), Montreal

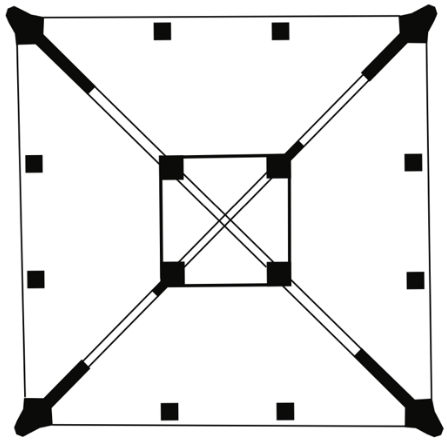


Figure 3.5: Floor Plan of the Montreal Stock Exchange Tower exhibiting the primary structural system

The main purpose of an outrigger system is to reduce the moment of the core walls. The opposing forces generated by the columns create an opposing moment that is then transferred to the core by the outriggers. This reduces the overturning moment in the core minimizing the lateral drift, that is a direct result of the moment, but also lightens the moment stresses at the base of the building. The total moment of the base in the core can be reduced by the generated restraining moment of each outrigger. The moment at the base at the core can be further reduced by increasing the number of outriggers or by increasing the magnitude of the restraining moment of each outrigger [10]. It is more efficient to increase the magnitude of the restraining moment rather than the number of outriggers because if the magnitude is small, then the moment at the base is still large even if there are a large number of outriggers.

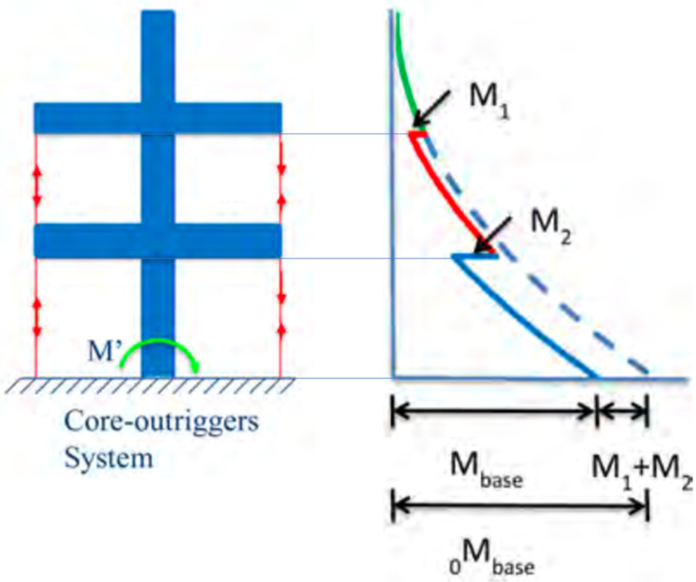


Figure 3.6: Diagram of the overturning moment in the core of a building throughout its height (Source: [10])

The Figure 3.6 demonstrates the moment in the core reduced at a base of a building, previously discussed. To the initial diagram of the overturning moment, it is reduced the magnitude of the two outriggers (M1 and M2) making its amount smaller at the base of the building. This amount can be further reduced by increasing the number of Mi or their magnitude, but it is recommended to increase their magnitude because the height of the building is limited, thus is the number of outriggers possible and reasonable to install and if their stiffness is small, little help can they be to the overall overturning moment of the building.

The importance of an efficient topology of outriggers arises, not only for its magnitude to be as high as possible but also because of the limitations of where to place it. In practice, it is of more importance the form and efficiency of the outrigger rather than the placement of it because this one is limited to the program of the building or the codes of the country as said before, but there is not much information in literature about the topology especially when compared to the ones about the overall efficiency in controlling drifts and optimum locations. Ho [10] is one of the few reports on the comparison of several forms of outrigger trusses with different levels of strength and stiffness.

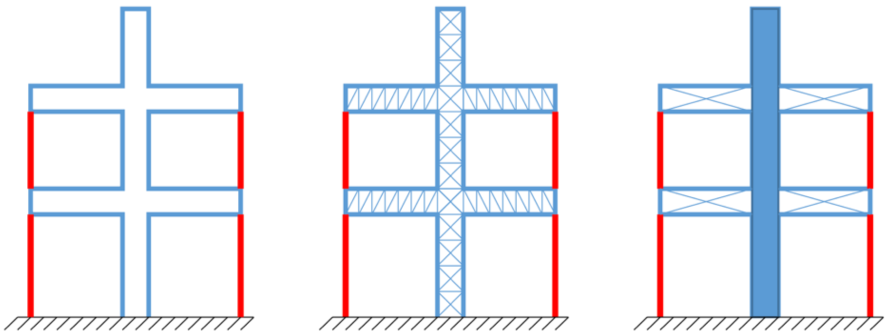


Figure 3.7: Typical diagrammatic drawings for outriggers. On the left, concrete core with concrete outriggers; in the middle, steel braced core with steel truss outriggers; on the right, concrete core with steel truss outriggers (Source: [10])

Ho pointed out that, in reference to most textbooks, the typical diagrams for outriggers are as in Figure 3.7, but from a practical engineering point of view they are neither practical nor efficient. His study on the other hand focused on the most practical and common topologies of outriggers (Figure 3.8) made with two different materials for comparison purposes. His intent was to test different forms of outriggers with the same material to ascertain their behavior and also to test different materials in the same form of outriggers to see if there is a significant difference in strength and stiffness with different member elements.

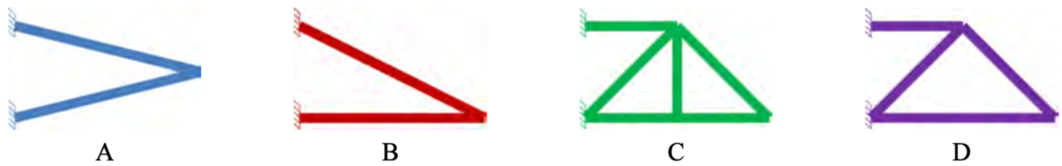


Figure 3.8: Types of outriggers tested in Ho, 2016 (Source: [10])

The same author found that unsymmetrical outriggers have different values of stiffness in cyclic loads due to elements in compression being less stiff than in tension. As outriggers transform lateral loads into vertical compression and tension forces, from an engineering principle, the outriggers should be as symmetric as possible to provide similar responses to upward and downward load and if symmetric topology cannot be used, the designer must be aware of the behavior of outriggers under cyclic loads. Also, for each topology, the outrigger's stiffness varies with the elements' stiffness meaning the behavior changes if the section size changes but the design capacity is not a linear proposition because the behavior for stiff and soft members will be quite different. In general, the higher the stiffness, the less geometric effect and hence higher buckling or critical load. But stiffness is not directly proportional to the ultimate load capacity which is a very important concept specially in cyclic loads like seismic action. Ho concluded that the topology of outriggers should be as simple and symmetrical as possible and if stiffness must be increased, it is suggested to enlarge the member size instead of changing the topology.

There isn't much difference in designing outriggers and simple beam-to-column connections. One key concern of this design is the locked-in forces that were presented before but not explained. As the stiffness of outriggers is very high, a small deflection will induce large forces and this small deflection is the result of differential shortening. Shortening occurs due to elastic deformation, shrinkage and creep and while elastic is predictable by engineers, the others aren't predictable and are time dependent variables which means that they won't appear until the building is complete. These can affect in different ways the core and the columns as the loads are not evenly distributed, thus resulting in the shortening of the elements being in different magnitudes creating this locked-in forces. Engineers must find ways to reduce this effect. For example, to eliminate elastic deformation, engineers introduced delay joints to the system that allow outriggers to be connected to the structure only when this one is nearly completed, and the majority of the dead loads are added. More information on the outrigger connection and the techniques to reduce locked-in forces will be addressed later in the text.

Some of the issues presented may force engineers to take another look at the projects and sometimes come up with other solutions and create alternatives to the conventional outriggers. Offset outriggers are a close alternative to conventional outriggers in the way that they have the same topology and form, and they function in the same way but are located elsewhere other than in the planes of the core walls. This retains some advantages and mitigates other disadvantages.

3.5. Virtual Outrigger Systems

Another alternative that is quite advantageous and frequently used is the indirect or virtual outriggers. They are belt trusses that completely ring the building's perimeter engaging all the exterior columns. The virtual outrigger system provides a similar behavior to the conventional outrigger system but without the outrigger element connecting the core to the perimeter columns. Instead, they count on the floor diaphragms to insure such connection. These diaphragms placed at the top and bottom of the outrigger levels transfer the overturning moment of the core as horizontal forces to the belt truss (Figure 3.9a). The latter structure, in turn, acts as a virtual outrigger and transforms that horizontal force into vertical forces (Figure 3.9b). These vertical forces are then resisted by all exterior columns engaged by the belt truss. The elimination of a direct connection between the core and the columns or belts avoids many of the problems associated with the use of conventional outriggers.

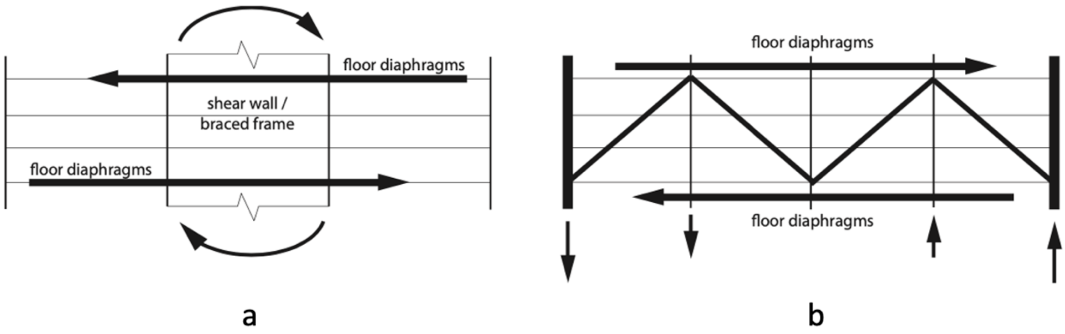


Figure 3.9: Force transfers in Virtual Outrigger Systems: a) from core to the floor diaphragms; b) from floor diaphragms to the columns (through the effect of the belt)(Source: [11])

The floor diaphragms take most importance in the use of virtual outriggers and so does their design. These floors should not be regarded as infinitely stiff, as they are sometimes for convenience and simplification, but instead the design of the floors at the top and bottom of each virtual outrigger should take special attention to the accurate representation of their in-plane stiffness in the analysis. Since the floor slabs that transfer the horizontal forces from the core to the belts will be subjected to in-plane shear, in addition to the vertical load effects, they should be proportioned and strengthened accordingly and, in many cases, it is necessary to use slabs thicker than normal. Their connection to the core and the belts depends on the materials used. If the core is a steel braced frame, the connection between the core and the floor slab is made by shear studs on the core and the same happens if the belt is a steel truss belt only in that case the shear studs are in the belt. If the core or the belt walls are made of reinforced concrete, the connection between these elements and the floor slab is ensured by the concrete-to-concrete monolithic behavior with reinforcing steel extending through the connection.

Virtual outriggers can be seen in two forms: belts and basements. Belt trusses are naturally well suited to be used as virtual outriggers but so are basements. For belts, there can be seen in the shape of steel trusses and concrete wall, in the same way as conventional outriggers. Belts can be concentrated, which is usually the case, creating a truss or wall that completely rings the building in a certain height, but the concept of the virtual outrigger effect can be extended further by distributing individual belt walls along the height of the building. In Figure 3.10, it can be seen an example of two sets of distributed belt walls beside an example of a conventional outrigger system with concentrated belt walls. Distributing the walls along the height of the building can come with some advantages over the concentrated belt walls. For instance, there are some architectural restrictions on the floors where the concentrated belt walls are located while, with the distributed belts, such thing doesn't happen because only a portion of the walls are used. Also, the floors are subjected to large values of in-plane shear force in the case of a concentrated belt while when distributing, the in-plane shear force is also alleviated. The effectiveness of these distributed belt walls in reducing lateral drift of the high-rise building is debatable but, in some studies [28], it can be as high as the concentrated belt walls or even the conventional outriggers acting alone. This effectiveness depends of course on the number and arrangement of the portions of belt walls.

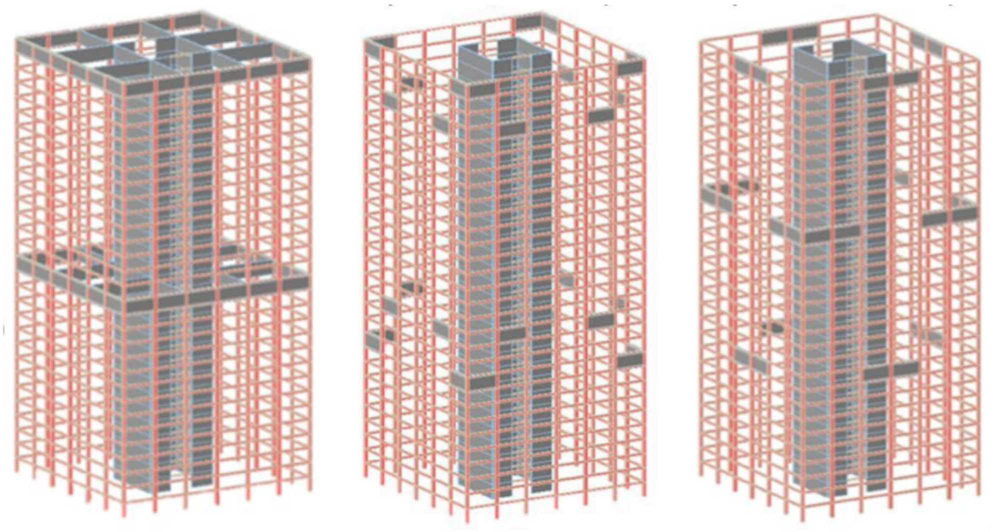


Figure 3.10: Distribution of belts in a building. Outriggers with concentrated belts on the right and distributed belts on the middle and left

The basement of a tall building can also serve as a virtual outrigger. It expands the base creating a greater effective width for resisting overturning which can in turn, reduce lateral load-induced forces in foundation elements and eliminate uplift. Since the basement walls are already significantly strong and stiff to resist the soil pressure, the additional cost involved to apply this concept is little. The functioning is basically the same as the belt truss in the way that the moment from the core transfers to the floors as coupled horizontal forces and then to belt/basement walls that transform it into vertical forces but these final vertical reactions at the ends of the basement can be supplied by friction or adhesion of soil against the wall surfaces [29] in alternative to the conventional foundation elements under the walls which can also be applied. The effectiveness of the basement as a virtual outrigger is likely to be greatest when the core has a “soft” support, such as footings on soil that allows the structure to move just enough so that the moment from the core can go to the outrigger system unlike the hard supports, such as footing directly on rock, that result in most of the overturning moment in the core going down directly into the core foundation. For this reason, and in addition to the floor diaphragms that are already of great importance since it is after all a virtual outrigger, it is very important that the stiffness of the core foundation be modeled with reasonable accuracy. It cannot be assumed both the floor diaphragms and the core foundation to be rigid. This brings up another issue which is the modeling of the basement of tall buildings: there is not a single and generally accepted way of modeling the horizontal restraints of the basement of a building, even when there is no intention of using it as a virtual outrigger. There is little published information about the horizontal restraint conditions of the design of tall buildings and ironically, the concept of using basements as virtual outriggers at the end is simply a matter of realistic three-dimensional modeling of the restraint conditions at the base of the building, along with careful proportioning, design and detailing of all components to maximize the effect of the outriggers.

3.6. Comparison of systems

The whole process of outriggers relies on the relative stiffness between the core and the outrigger-and-column system. The load path is more direct for conventional outriggers and so it provides a better restraint efficiency that the virtual outriggers. Nevertheless, virtual outriggers are in some cases sufficient to meet the needs of the tall building and they stand out in other important issues comparing with the conventional outriggers, making them preferable for certain situations.

The most obvious negative point of the conventional outriggers is the space occupied by them. The diagonals of the outrigger steel trusses are a major constraint on the use of the floors at which the outrigger is located, and outrigger shear walls places the same issue. Even in mechanical floors, the presence of the outriggers can obstruct the correct design and functioning of those floors. The use of virtual outriggers, on the other hand, eliminate this problem since there is no elements connecting the core to the exterior columns. Not having the elements connecting the core to the exterior columns also eliminates the need for the design of the connection of the outrigger to the core. This connection can be very complex and complicated, especially if the core is a concrete shear wall core and the outrigger is a steel truss.

There is also a need, for the conventional outrigger system, to place large outrigger columns on the building's exterior frame where they most conveniently are engaged by the outriggers extending out from the core and resist the vertical loads. This need doesn't occur for the virtual outrigger system because all the columns are engaged together by the belt and there is no need for a large outrigger column that resists all the vertical loads resulting from the outrigger functioning. This can provide virtual outrigger systems with more architectural freedom when compared to the conventional outrigger systems but, this issue can be resolved when a belt truss is employed along with the outrigger. If the conventional outriggers unload on a belt truss that then unloads on the perimeter columns that are all engaged together by the belt, there is no need for the large outrigger column.

Another issue that affects the conventional outrigger system is the differential shortening of the vertical elements and the locked-in forces created by it. Since the core and the outrigger will not shorten equally, especially if they are made from different materials (i.e., if the core is a steel braced frame and the columns are reinforced concrete), the outrigger element which needs to be very stiff to be effective, would be very affected by trying to restrain this differential shortening between core and columns. Although this can be partly fixed by some alternatives on the design of the connections of the outrigger to the core like delayed connections or hybrid solutions, virtual outriggers still present a better alternative than conventional outriggers. Differential shortening between columns is minimal and negligible when compared to differential shortening between core and columns, so it doesn't affect the belts. With regard to the floor diaphragms, which are the connection between the core and the belt and columns, they are not affected by their differential shortening because, even though they present a very high in-plane stiffness, they are very flexible in the vertical out-of-plane direction.

4. Design Considerations

4.1. General Considerations

In tall buildings, the core is usually located in the center of the floor plan. This is not only to free the exterior walls for occupants, since views are a significant part of the intrinsic value in tall buildings, but also because the core represents an important role in the lateral stiffness of the building and in this way it locates the center of lateral stiffness close to the center of lateral wind load and center of mass for lateral seismic loads, minimizing torsional forces. The core may be combined with other elements in order to provide additional torsional stiffness, such as a core and frame or a tube in tube, but the core alone is responsible of resisting a significant part of the overturning moments and controlling drifts. For an aspect ratio of the core higher than eight, the structure is slender enough to consider introducing outriggers. This height is usually smaller for residential buildings than for office buildings since the cores tend to be smaller, as explained before. This is because the drift increases approximately with the cube of the building's height [10], thus, to maintain the drift/height ratio below an acceptable value, as the heigh doubles, core stiffness would have to quadruple [11]. For this reason, and because in some cases thickening core walls would be unpractical, introducing outriggers can reduce the dependence on the core system and maximize usable floor area between the core and exterior columns.

There are ideal locations for outriggers, but the realities of space planning make such considerations unpractical, and the outrigger locations are typically limited to mechanical or refuge floors. This strategy is common since outriggers interfere with usable floor space and mechanical or refugee floors are often required by local codes, but even then, it requires careful coordination with mechanical room layouts, accessibilities requirements and service routes to avoid potential conflicts.

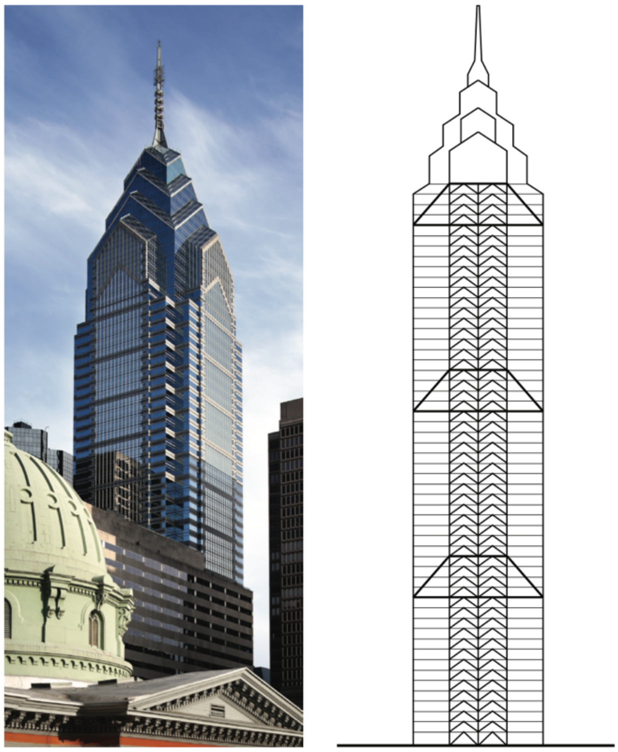


Figure 4.1: One Liberty Place, Philadelphia with the schematic drawing of the super-diagonals on the right

When mechanical or refugee floors are not at appropriate levels, other strategies may be used. One possibility is the "super-diagonal", as in the case of the 288 m high One Liberty Place, in Philadelphia (Figure 4.1). The super-diagonal strategy is a four-story high diagonal that is considered to be an outrigger system and not as a full-building-width braced system, because the diagonal occurs only at certain levels and not throughout the building. This can be advantageous because the outriggers spread through a greater of the building height and therefore it occupies less space on each floor, making it easier to conceal. Another strategy is the before mentioned virtual outriggers which avoid complex connections and locked-in forces in the outrigger elements. However, virtual outriggers can be used together with conventional outriggers to couple the external columns together so that mega outrigger columns are not required and columns can have a regular size.

Whichever systems is used, it needs to be optimized together with the gravity system starting at an early design stage, so that the design of either multiple columns engaged by a belt truss or a mega outrigger column connected with an outrigger arm can be of maximum efficiency.

4.1.1. Locations in elevation

Reduction of building drift due to outrigger depends on their number and location. Location and effectiveness are driven by four issues: number of outrigger sets; outrigger column and truss stiffness; spacing to equalize distances from outriggers to core inflection points and space availability [11].

- ## Number of outrigger sets

The number of outriggers obviously constraints their distribution throughout the height. More outrigger sets provide more opportunities for rotation restraint, but it affects the construction process as it interrupts the workflow compared to a typical floor. On the other hand, fewer outrigger sets result in heavier members, requiring higher capacity to elevate and construct. Costs and benefits must be weighted when defining the number of outrigger sets. Another thing is that the type of outrigger affects the efficiency of the lateral load resisting system and consequently its number. The shorter load path from the core to the column defined by conventional outriggers makes them stiffer and more efficient, and for virtual outriggers to have the same benefit on the overall system they would have to be in higher number.

- ## Outrigger column and truss stiffness

To develop and apply forces counteracting the core overturning moments, outrigger trusses and outrigger columns must be stiff and have high strength. The optimal arrangement of trusses and columns stiffness will largely depend on the pattern of column size changes with height, as it interferes with the element stiffness and placement, and studies have shown [11, 14] that the optimum location for a single outrigger changes from one quarter to two thirds of building height (Figure 4.2). This wide range illustrates the complexity of outrigger design. For example, one outrigger at one quarter height seems to be too low, but has the advantage of having shorter, therefore stiffer, columns. The same can be said about the outrigger trusses that also differ their position according to their stiffness. For example, if one outrigger is located at the top, a second similar outrigger should be located at mid-height, but if it has different stiffness then the location should change. On the same note, if the outriggers location is already defined at a given height, its relative stiffness should be tuned to maximize efficiency.

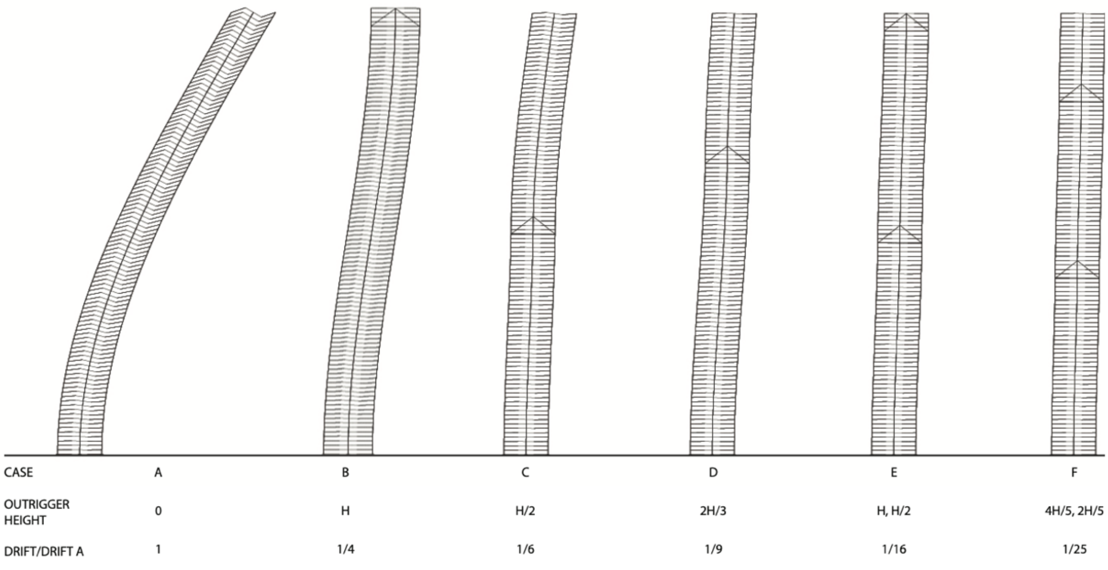


Figure 4.2: Effect of outrigger locations on roof drift from a simplified case of lateral load at roof only (Source: [11, 25])

- **Spacing to equalize distances from outriggers to core inflection points**

If all outriggers are similar, for the same number of outriggers, changing their placement in height can change roof drift by more than 50% [11]. The optimum location for an outrigger in height, according to Smith & Coull [13], is approximately 1/(n+1) up to n/(n+1) [13]. It basically tries to divide the structure in equal parts in order to equalize distances from outriggers to core inflection points. So, if there is only one outrigger it should be located at mid-height, if there are two outriggers they should be located at one third and two thirds' height, and so on. If one outrigger is located at the top, which is usually the case even though it is not optimal, then the same formula could be applied with one less outrigger. Günel and Ilgin [2] analyzed the behavior of one structure with: a) one outrigger at the top; b) one outrigger at optimum location; c) two outriggers with one at the top and d) two outriggers at optimum location. They concluded that:

- the contribution of one outrigger at optimum location for the decrease of displacement at the top of the building is 32% higher than the contribution of one outrigger at the top;

- the contribution of two outriggers at optimum location is 12% higher than the contribution of just one outrigger at optimum location.

- and the contribution of two outriggers with one at the top is just 6% higher than the contribution of one outrigger at optimum location, which raises the question for the necessity of adding one more outrigger.

- **Floor space availability**

For most buildings, the defining issue is the floor space availability, as outriggers consume a great amount of rentable space and therefore are almost always located at mechanical or refugee floors. Designs usually include mechanical floors near the top where outriggers are preferably placed, even if not optimal. The other locations occur every 12 to 25 floors [14] and which floors to use under these conditions, if floors are closely spaced, should be determined by the other constraints.

4.1.2. Diaphragms

Diaphragm properties are very important for outrigger design. They are important for conventional outriggers, because incorrect modeling of them can report incorrect force values in outrigger chords and incorrect building deformation, and are particularly important for virtual outriggers because they are key elements in the load paths definition that make the system work. Instead of core rotation inducing outriggers to tilt, hence compress and pull the columns, the core rotation in a virtual outrigger system will displace stiff floors connected to the top and bottom chords of the belt, trying to rotate it and inducing the compressive and tensile forces on the columns. Virtual outriggers require special attention on the diaphragm stiffness as the displacement of the stiff floors occur through the axial stiffness of the diaphragms and this is why diaphragm floors are usually thicker at belt levels. However, this effect must not be exaggerated. Improperly modeled

diaphragms will result in misleading behaviors and load paths, and incorrect member design forces for both indirect and direct outrigger systems. Overly optimistic diaphragm stiffness will overestimate outrigger participation and underestimate building drift and core overturning forces. Too-low diaphragm stiffness assumptions will underestimate the forces experienced by the diaphragms, belt trusses, and perimeter columns. Designs should envelope reasonable ranges for diaphragm stiffness. Counting on 100% of gross slab properties while designing would be unrealistic because of cracking of the concrete, so diaphragm floors in virtual outrigger systems should be analyzed with reduced stiffness.
For member and connection strength verifications, different effectiveness values could be used: core forces are greater with low slab effectiveness and slab, belt, and participating perimeter column forces are greater with high slab effectiveness. Where diaphragms alone do not offer sufficient stiffness for effective virtual outriggers, horizontal bracing beneath the floor slab can be provided and this can affect material quantities and construction time, among others.

4.2. Construction Considerations

Construction of a core-and-outrigger building has two key aspects: mitigation of differential shortening and effect on overall construction schedule [11]. Regarding the first aspect, it is an important issue in tall buildings as differential shortening can create force transfers in conventional outriggers.

4.2.1. Differential Shortening

There are three causes that can generate differential shortening. The first cause is gravity shortening that can occur in the form of elastic shortening which is elements shortening with the increase of compression stress due to the increment of weight. Tall buildings generally experience this form of differential vertical shortening between core and perimeter vertical members. In an all-steel buildings, this differential vertical shortening is virtually complete once the building is occupied but, additionally to elastic, in concrete buildings shortening can also occur in the forms of creep and shrinkage. Creep is the continued shortening under constant load and shrinkage is the shortening from concrete drying as it approaches ambient relative humidity.

Another cause for differential shortening is the temperature effect. Although less common and relatively smaller, it is still significant for conventional outriggers connecting members with different temperature like what happens with perimeter columns exposed to weather and the temperature-controlled internal core. The magnitude of temperature differential shortening should consider realistic heat flow paths, including the ratio of surfaces exposed to the exterior and interior, the properties of the materials and the climate conditions of the building site.

The last cause for differential shortening isn’t actual shortening but instead it results from support settlement. A building’s core supports a great amount of the total weight of the building and adding to that, in a concrete building, the core itself represents a great fraction of the total weight. This means that the foundation under the core is in greater stress which tends to settle more than the other elements. Not only that but the settlement is typically greater in the center of an area than in its edges because of the way that stresses and strains spread throughout the foundation soil or rock. Load concentration and soil behavior act together to cause the core to displace downward relative to the perimeter columns. Although this is a different phenomenon than differential shortening, the result is the same as it induces force transfers in conventional outriggers, and because it isn't about the shortening of the vertical members but about the vertical displacement of the different elements of the foundation, it is independent from element shortening which means that it can add on to the differential shortening, and consequently the force transfers, or it can reduce it.

4.2.2. Shortening Through Time

The periods in which the previously mentioned effects can happen are during construction, with a special attention to the time when the outriggers are connected to the structure, and after construction. During construction, elastic shortening will occur under gravity loads as construction advances and the loads are applied onto the core and columns. Differential shortening will be affected by the sequence of construction, for example, a core may temporarily experience higher gravity strains than perimeter columns if the core is advancing faster. This phenomenon is independent from the settlements and shortening of elements, thus can conteract or add on the effect of the transfer forces. Another effect to happen during construction is the dishing of the foundation. Ground dishing from rock deformation or pile shortening will increase as building construction proceeds and in some cases, if the sub-grade is subject to consolidation settlement such as clay, dishing may continue to grow for years.

The time at which the outrigger is connected to the structure also has a big relevance in the force transfers presented in the outrigger because this can establish how much of the total differential shortening has already occurred, and how much has yet to occur. Some buildings can delay the connection of the outrigger to the perimeter columns until the structure is nearly complete and this is a good thing because the elastic shortening will be greater, thus reducing force transfer affecting the outrigger.

After construction, in concrete or composite buildings, shortening will occur due to creep and shrinkage as explained before. This phenomenon surpasses the strains from elastic shortening, and it is more difficult to predict with adequate accuracy. It is controlled by the mixture used in the concrete, the ambient humidity, member volume/surface ratio and the reinforcing ratio. Post-construction shortening will more relevantly affect concrete and this is why time dependent differential shortening can be greater when different materials are used for the core and columns.

These otherwise small differences in strain between adjacent columns, or between columns and the core, accumulate, resulting in significant differences in axial shortening over a building's height, resulting in very large forces within the outriggers, transferring a portion of gravity loads. When the columns displace downward relative to the core, a portion of the gravity loads are transferred from the columns to the core as the building tries to equalize strains, and these portions are transferred through the direct connection between the core and the columns which is the conventional outrigger. For this reason, outriggers are then under a constant state of stress which can be as high as the design loads from the lateral forces and that relativizes the purpose of it.

4.2.3. Connections to Columns

Ideally the gravity system is coordinated with the lateral load system so that members of similar materials are used and axial stress levels under gravity are similar for all vertical members. Another aspect is the construction sequence: while for most buildings it is considered means and methods [11], for outrigger system buildings it needs to be considered in the design phase. Both these aspects — the coordination of gravity and lateral load systems and the construction sequence, can help to mitigate and minimize differential column shortening even though not completely eliminate it. To eliminate transfer forces between core and columns, one must avoid direct connections altogether, even though it can create sloping or warping of floors from differential shortening. Thus, virtual outriggers offer a big advantage in this case. It should be noted that virtual outriggers are still subjected to differential shortening between perimeter columns, but this is significantly smaller.

Other methods of mitigating or minimizing the transfer loads are delaying the outrigger connections to the columns or in some cases even adjusting during construction but even though delaying the connections may eliminate most of the elastic shortening, post-top-out differential axial shortening of core and columns will still occur. Concrete core walls and concrete columns will shorten due to creep and shrinkage which will affect the outriggers. Not only that but also some buildings need the lateral strength that the outriggers provide during construction as is the case of the Cheung Kong Center since it is in a typhoon affected area. To best solve both these issues, some methods and connections of outriggers to columns were developed that allow for later adjustment. Some examples of these methods and connections are the Shim Plate Correction Method, the Oil Jack Outrigger Joint System and the Cross Connected Jack System [11]. The Shim Plate Correction Method is inserting steel plates at the top and bottom of outrigger connections to adjust outrigger level allowing for direct connection and transfer of loads between the outrigger and columns and the plates can also be replaced or added, adjusting the level of the outriggers throughout the time. The Oil Jack Outrigger Joint System is cylinder jacks that are installed at the top and bottom of an outrigger connection, filled with oil connected through a pipe and a small orifice offering resistance to the flow of oil and this resistance is proportional to velocity. Finally, the Cross Connected Jack System is also oil filled hydraulic jacks that are installed at the top and bottom of an outrigger with the particularity that the hydraulic jack on the top of one outrigger end is connected to the one on the bottom of the outrigger end on the other side of the building assuring in this way the same behavior and avoiding the need for pressure control under a small orifice.

4.2.4. Connections to Core

The connections of the outriggers to the columns are of great importance because they are key elements on mitigating the differential shortening and transfer forces but the connections of outriggers to the core are very important as well. Because outriggers are of limited number in a tall building, they will experience forces that are large, varying, and usually reversible and they must be transmitted to and distributed within the core of the building thus, these connections require to be large, stiff, and complex. They vary with the materials of the building and, once again, they are more difficult in a composite or mixed material building, for example with a concrete core and a steel outrigger truss. The most appropriate solution will depend on the forces involved, materials and space available, erection equipment and local construction preferences.

For an all-steel building, connections use bolted plates or field welded joints or even both. Even though they must be large and reinforced, they can be conventional. For an all-concrete building, connections can be more complex as they depend on the outrigger geometry, and they require sufficient room for reinforcing bars to pass through the connection without compromising effective concrete placement [34]. Transitioning diagonal into vertical and horizontal reinforcement and other issues must be addressed. The challenge is with mixed materials and ensuring an appropriate load path requires study and creativity. Some alternatives of geometries and solutions to ensure a proper load path from a steel outrigger to a concrete core wall are: an embedded plate (Figure 4.3) – where it can be used composite shear connectors on the plate to resist the vertical component of the force and long horizontal bolts developed within the wall to resist the horizontal force; continuous or partial height embedded steel members – as it can permit more conventional and direct steel-to-steel connections, although it is more complicated for concrete construction when working around heavy steel members and for accuracy of steel placement which can be affected by concrete encasement; localized short steel stub members (Figure 4.4) – that permits conventional steel-to-steel connections while limiting the impact on concrete construction.

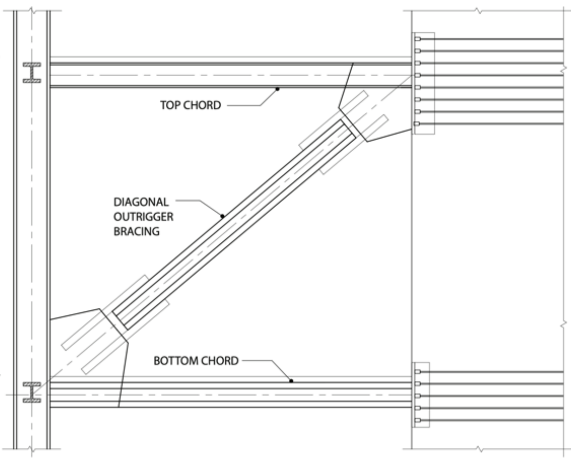


Figure 4.3: Embedded Plates

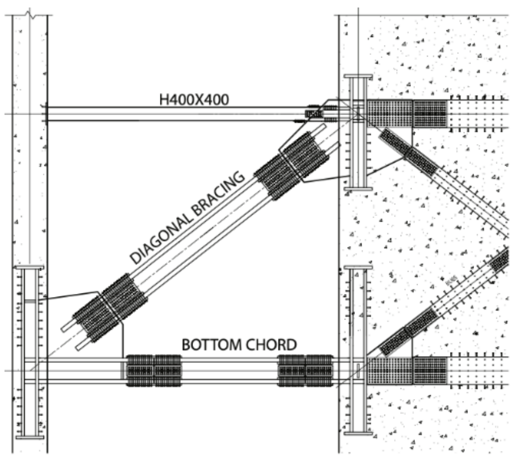


Figure 4.4: Localized Short Steel Stubs

4.3. Seismic considerations

Unlike the other lateral resisting structural systems, outrigger framed systems are not included in the building codes. This is not surprising because there is not a single design approach suitable for all outrigger situations. Seismic design approaches in the codes are based on distributed stiffness and strength while outriggers provide great stiffness and strength at discrete locations, and they may apply forces large enough to the point of damage and non-ductile behavior of other elements. Some examples of provisions in the codes based on this distributed strength and stiffness that are misapplied to outrigger frames are soft-story seismic provisions and strong-column weak-beam provision.

- **Soft-Story Seismic Provision**

Codes discourage having a stiffer or stronger story above a softer or weaker story. This intend is to guard against a uniformly stiff or strong building having a soft or weak story where deformations would be concentrated leading the floor and building to collapse, as may occur at a lobby or other non-typical level. Outriggers, as non-typical levels, can be considered as stiff floors according to codes, which makes the stories immediately below to be soft stories. Some researchers recommend minimizing outrigger stiffness in seismic regions as this could be an issue according to the codes, but the code writers' intention was the concentration of the deformation in one weak story which is not the case in outrigger systems because the shear can be well distributed among the many other similar floors. Also, the calculated story stiffness is based on core shear force rather than story shear force therefore, the stiffness in outrigger floors is not that different from other floors. Intentionally softening outriggers and stiffening perimeter columns to maintain a more uniform stiffness, as some researchers recommend, even though theoretically possible, it is impractical since outriggers soft enough to avoid the stiffness jump may not be stiff enough to provide effective reductions in drift and core overturning moments.

- **Strong-Column Weak-Beam Provision**

As with soft-story and weak-story provisions, strong-column weak-beam provision can be misapplied to outrigger systems in tall buildings. This provision is intended to avoid all columns in a single floor to form hinges at the top and bottom causing the floor as well as the building to collapse. This is assured by checking that lateral loads will cause, at each connection, yielding in beams rather than in columns. By requiring column flexural strength to be greater than beam strength at each connection, the provision aims for columns to act as continuous spines and beams must yield and form hinges, absorbing a large amount of seismic energy before collapse can occur. This provision is misapplied to outriggers because the core of the building already functions as a continuous spine which reliefs that purpose from the perimeter columns and leaves them to bend and yield freely without compromising the verticality of the building. With the core serving as a spine, it is not necessary to apply this provision to outrigger systems. Also, because of this reason, any realistic outrigger will yield before the column and aiming to do so is very counterproductive. Outriggers with enough stiffness and strength to be effective will provide a force couple greater than column flexural capacity. This provision could be however valid if applied to the connection of the core with the outrigger viewing the core as a column and the outrigger as a beam.

4.3.1. Design Solutions

Seismic design approaches successfully used for outrigger systems include performance-based design (PBD) which uses scaled seismic time histories and non-linear building models to demonstrate that the building performs well, and capacity-based design (CBD) which is based on predictable ductile behavior of members that avoid overloading other elements. Current seismic design provisions in building codes like the IBC or the Eurocode, were not developed for applications to tall buildings. Prescriptive seismic design provisions in these building codes do not sufficiently address many aspects of seismic design of tall buildings and, in addition, many building codes have height limitations on many practical and popular seismic force resisting systems.

The most appropriate approach in the design of tall buildings is performance-based design and it is permitted in the codes as an alternative to prescriptive design. It offers many benefits for achieving better tall building design such as clearly defined performance objectives, procedures for selecting and scaling earthquake ground motions for design, nonlinear modeling methods that produce reliable estimates, acceptance criteria for calculated demands and others [17]. A performance-based design is highly recommended for outrigger system buildings when looking at responses to realistic seismic events.

Capacity-based design is also a good approach, and it is very useful for limiting the capacity of certain elements because maintaining capacity in the event of overload may be impractical. For high-strength mega concrete columns, the amount of transverse reinforcement needed to confine them would be intimidating and for steel columns, squashing controlled rather than buckling would require a slenderness too low and consequently very thick plates. This would require such heavy elements that it would be likely for other elements to yield first. In turn, instead of reassuring that the columns resist high load demands, a more suitable approach for a practical design would be that these demands, from seismic events, could be limited relying on having non-column members to yield first. For this reason, outrigger members sometimes are established to be small enough so they could act as fuses, dissipating the energy and limiting the load transfer to the columns. This is achievable but only by optimization because core and outrigger systems are indetermined and changing the outrigger stiffness may change the forces they attract thus, several design cycles may be required to achieve simultaneously the required stiffness and a hierarchy of strength.

4.3.2. Stiffness Reduction Strategies

When limiting the strength of an outrigger is not an option, other strategies are used. Some designs include connections of outriggers to columns that can slip at defined values. Another strategy that can be adopted when using oil jack outriggers joints (presented before) in the connection of outriggers to columns is to adjust the resistance orifices so that they would allow oil to bypass at a certain pressure to allow for outrigger movement. The most direct approach and the strategy that is mostly used is to make the outriggers themselves the fuses with the use of

Buckling Restrained Braces (BRBs). They are inner steel plates of controlled dimensions and material properties making the limit states of the outrigger based on ductile yielding of the inner plates in tension and compression. Some advantages of the BRBs include: the protection of other adjacent elements against unanticipated overload forces, because the capacity of the BRBs are designed, fabricated and tested within tight elastic and plastic behavior limits; the considerable amount of seismic energy that can be absorbed with the BRBs tension and compression yielding; and the easy replacement of the elements after a major event to restore strength and alignment.

5. Case Study: Montreal Stock Exchange Tower

5.1. Description of the building

The chosen building is the Montreal Stock Exchange Tower, in Montreal, Canada. It was designed by engineer Pier Luigi Nervi and architect Luigi Moretti and when completed, on May 1st, 1965, it was the tallest reinforced concrete building in the world, which at that time was clearly distinguished from the original model of the American steel frame skyscraper [18].

Figure 5.1: Montreal Stock Exchange Tower

The building is a bi-symmetric 42 m by 42 m squared floor plan, with 189,9 m height, entirely made of concrete. The main structural system is composed of four corner columns, a central core with crossed walls surrounded by the stairs and elevator shafts, hollowed slabs and for levels of outriggers with four outrigger beams connecting the central core to each corner column. The cross section of every element decreases two times along the height of the building. Along with this main structural system there are two columns at each façade of the building, between the corner columns, as a secondary structural system to help support the vertical loads [19].

This building was chosen because it is entirely made of concrete, it was the first to employ outriggers and it has an obvious and pure outrigger structural system without the use of other elements to assist the lateral loads resistance.

The finite element program used to analyze the structure was ETABS (version 19), through tridimensional linear analysis.

The corner columns start with a squared cross section at the base of the building with 3,02m side, then at 74,9m height (floor 19) it changes to an L section with 2,70m side and 1,95m thickness and later, at 160,2m height (floor40) till the top of the building, it reduces even more to an L shape with 2,70m side and 1,10m thickness.

The core of the building is made of four columns in a squared position spacing 14m from each other with walls connecting them in an X shape and a squared shape. The X shape starts with a thickness of the walls of 56cm until 74,9m height (floor 19). From 74,9m to 160,2m height (floors 19 to 40) it has 43cm thick walls, and from then on it has a thickness of 30cm. The squared shape walls that contour the core are to simulate the effect of the staircases and elevator shafts since they also have that same wall, but unlike the other elements, these walls have the same thickness of 30cm throughout the entire height. The four columns of the core also don't change their cross section, which is a squared section of 0,99m side. The several elements of the core were later combined in the model as one element converted into a pier element.

The outriggers are two story height beams (7,5m of height), with 1,00m side by 1,40m high flanges and a web composed of diagonal frames of squared cross sections with 0,80m side. Near the core and the corner columns, the outriggers have a solid web section with 0,80m side also.

The middle columns on the façade also reduce their cross-section along the height of the building at the same time as the other elements. They start with a rectangular cross-section of 2,70m by 1,40m, then at 74,9m height they have a 1,90m by 1,40m section, and from 160,2m to the top they have a 1,10m by 1,40m section.

The floors are hollow slabs of overall depth 46cm, with a slab thickness of 8cm and 38cm high ribs spacing 1,80m in each direction. Near the vertical elements, the slab is a solid slab. The floor plans with the different dimensions of the cross-sections can be seen in Figure 5.2, and one figure of the model can be seen in Figure 5.3.

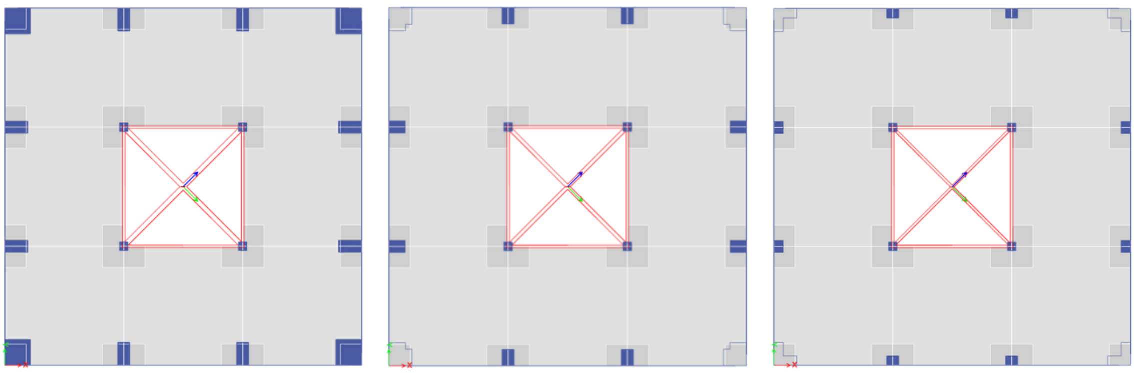


Figure 5.2: Floor plan of the building model (from the base to 74,9m high on the left; from 74,9m to 160,2m in the middle, from 160,2m to the top on the right)

To simulate the loads on the model, the normal loads of an office building, according to the Eurocode, were used [20]. Besides the self-weight of the structure, it was considered other dead loads of 2,5 kN/m^2 on the regular floors and 2,0 kN/m^2 on the roof, and live loads of 3,0 kN/m^2

on the regular floors and 1,0 kN/m^2 on the roof were considered. Besides the regular floors and the roof, it was also applied 3,0 kN/m^2 and 5,0 kN/m^2 to the lobby, respectively for the other dead loads and for the live loads. Besides the vertical loads, it was considered as horizontal loads the seismic action and the wind force. For both these cases the building was considered as if it was situated in Lisbon, Portugal, with the seismic response spectrum and the characteristic values of the wind for Lisbon according to the Eurocodes [21 and 22]. For the seismic analysis, the type of soil considered was type B and the behavior coefficient (q) was considered equal to 3. As for the wind loads, the equivalent forces on each floor were determined and applied as frame loads. For the calculation of the wind forces, the building was considered to be in a Zone A, related to the continental area of Portugal, with a reference value of wind velocity ($v_{b,0}$) of 27m/s and to be in a Zone II, corresponding to a low vegetation area, with a roughness length (z_0) of 0,05m and a minimum height (z_{min}) of 3m.

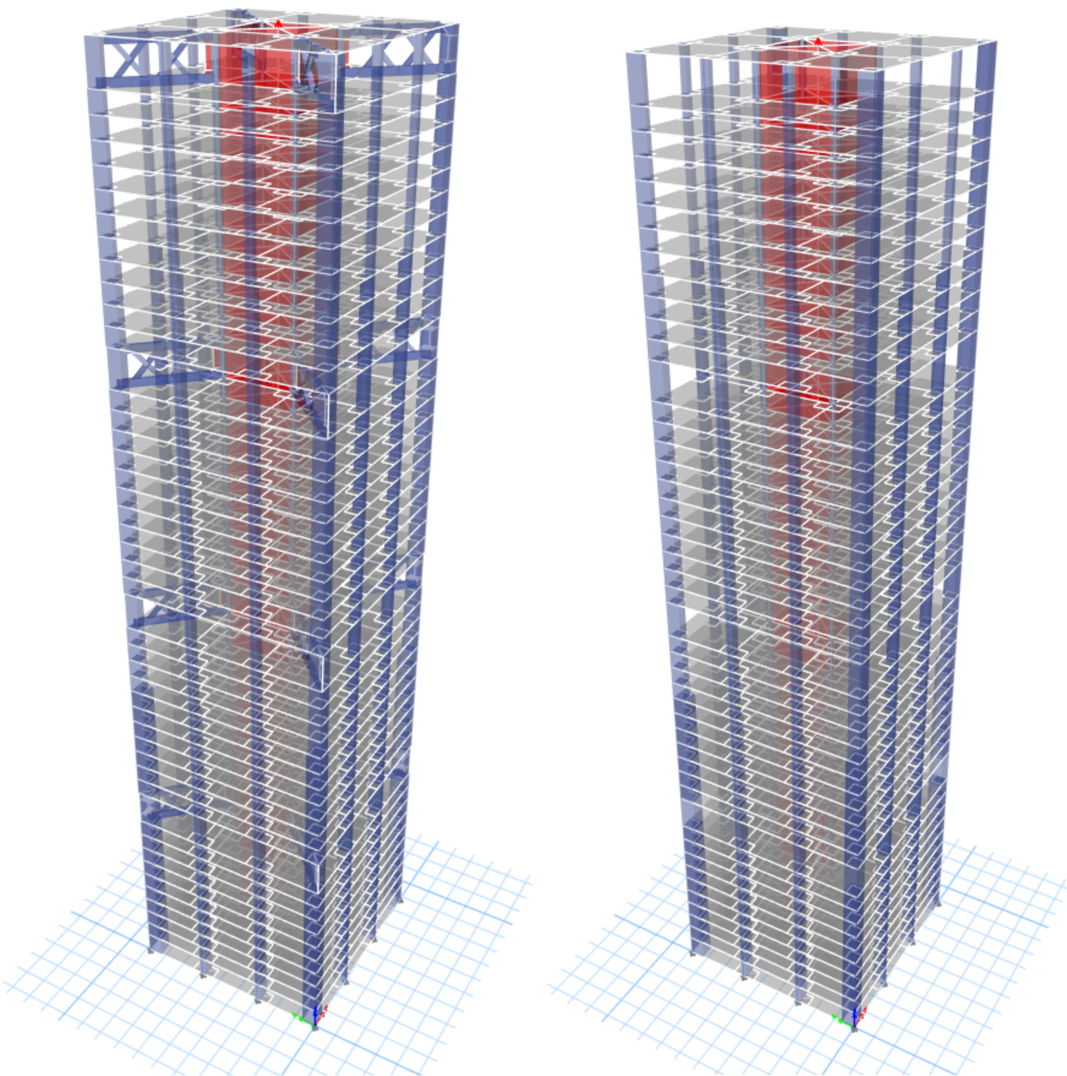


Figure 5.3:Original model (on the left); Simple model (on the right)

5.2. Description of the variants and alternatives

Five different alternative cases were proposed as variations of the solution applied in the original model: Simple; Belts; Distributed 1; Distributed 2; Braced.

Simple: This alternative is composed of the main structure of the building with both the main and the secondary vertical load support systems but without any additional lateral load support system besides the main core (Figure 5.3). So, the only change to the model was to delete the 16 direct outriggers that connect the core to the corner columns, leaving the core by itself to resist the loads. This alternative was considered to evaluate the efficiency of the main structure and its core on resisting the lateral loads when compared to the other alternatives. It was simply for comparison purposes and to evaluate the efficienty of the structure when adding the lateral load resisting systems.

It was observed that the core resisted the main bending moments of the structure as expected, with the perimeter columns resisting a small portion due to their flexural stiffness and the flexural stiffness of the floors that transmit a small portion of the bending moment of the core to the columns to be resisted as axial loads.

The other four alternatives were developed by one criterion, but first, these alternatives are some of the solutions presented in Chapter 2. They are supposed to be alternative solutions to direct or conventional outriggers for resisting the lateral loads. The first three solutions are variants of the outrigger system — Virtual Outrigger System materialized by one simple Belt or by two different sets of a Distributed Belts. The last solution was chosen to check the efficiency of a tube system — Braced Tube, when compared to the outrigger system.

The criterion proposed for the four alternatives was the preservation of the quantity of material used, reflected by the volume of the elements and by the self-weight of the structure. This seems to be a valid criterion for comparison of solutions because it focus the most efficient way of using the same amounts of material and, even though this study is not extended to detailing of the required steel reinforcement, it is considered to be a good indicator of the final price of the structure.

Belts: The first alternative is a virtual outrigger materialized by Belts (Figure 5.4.). There are four Belts at the same levels of the conventional outriggers. To calculate the thickness of the belt walls, first a calculation of the volume of each outrigger arm was performed. Then, that same volume was spread throughout one wall, and so an approximate value of the thickness was found to have approximately the same total self-weight and to have a reasonable value of thickness of a wall. The thickness of the wall was then defined as 24cm and the total self-weight calculated and the one given by ETABS were 806,5MN and 803,2MN respectively. The difference in self-weight of this structure to the original is 1,23%.

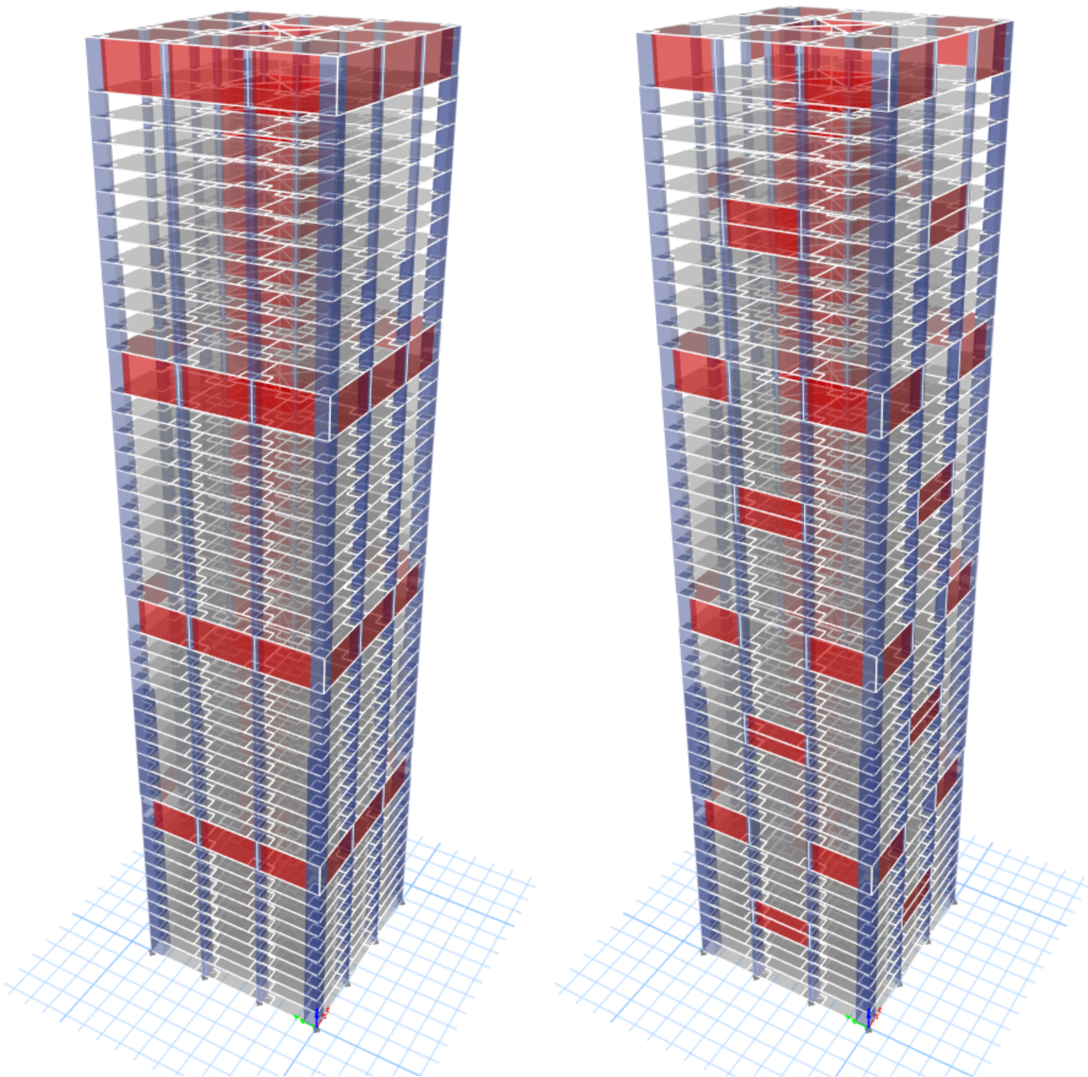


Figure 5.4: Belts model (on the left); Distributed 1 model (on the right)

Distributed 1: Another alternative was a variation of the virtual outrigger system, which consisted in dividing the previous modeled belt wall in 3 parts for each wall of each belt and relocating the middle part to mid-span of the belt levels (Figure 5.4). Since the floor where the outriggers are located is taller than the other regular floors, even a little bit taller than two floors, by 10cm, the total volume and self-weight of the structure didn't suffer a reduction enough to justify adding thickness to the walls, leaving the difference in self-weight of this structure to the original as 1,24%.

Distributed 2: Since it was observed that distributing the belt wall along the height of the building seemed to have a positive impact on the efficiency of the system, the other alternative was yet another variation of the virtual outrigger but only this time it was even more distributed. The principle was to have at any floor one wall in each direction so that the belt was as much distributed as possible (Figure 5.5). The walls had also the height of two floors and opposite sides of the building were combined to have alternated walls. In this way, almost every floor had one wall in the x direction, one in the y direction and no more. It was maintained the concern of symmetry, not in a floor plan because it was now impossible, but in the overall behavior of the structure. The final result was something similar to a combination of distributed belt walls and a braced tube since it was almost along the entire height of the building, and it was in a diagonal shape in the façade. The thickness of the walls was the same as before leaving the self-weight difference even lower than in the previous examples, but still with a difference relative to the original structure of 1,25%.

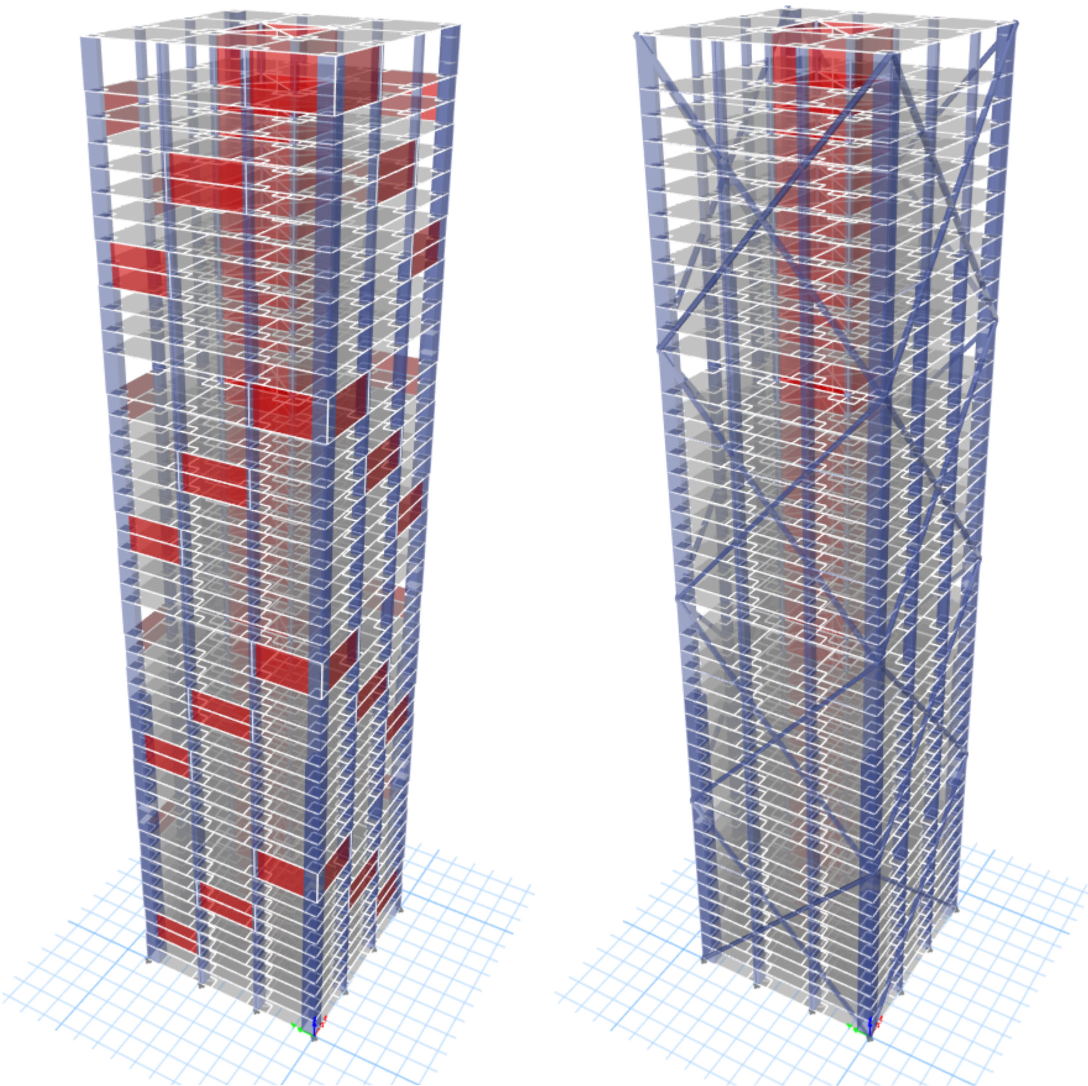


Figure 5.5: Distributed 2 model (on the left); Braced model (on the right)

Braced: At last, the alternative to evaluate a tube system was a Braced Tube (Figure 5.5). For this case, a concentric X shaped brace was introduced in between each outrigger level. The cross-section of each brace was chosen to be a squared section and the side was so that the volume of the outrigger arm would be the same as the volume of both the diagonals of one X shaped brace. The side turned out to be 77cm and the total self-weight error resulted equal to 1,25%.

The differences to the original model were all below 1,25%, which was considered to be enough to consider all the solutions as equivalent according to the criterion presented above.

5.3. Analysis of the results

From the analysis, the data obtained made it possible to assess the reactions and the bending moments at the pier base, for detecting the percentage of moments at the pier and at the perimeter of the building, the modal participation mass ratios, to ensure that the model was symmetric, and the displacements of the center of mass of each floor. Denote that the pier is the ensemble of the elements of the core with their respective forces and bending moments combined.

Table 1: Periods and fundamental frequencies of the different solutions

	Original	Simple	Belts	Dist.1	Dist.2	Braced
Period [s]	4,720	7,921	5,309	4,914	4,987	4,175

The period of the belt system was a little higher than the original solution, as expected, because it doesn't have the direct connection of the conventional outrigger but instead it as a virtual connection materialized by the floor diaphragms, and the direct connection has better efficiency on transposing the moments from the core to the columns.

The distributed belt system 1 had a little lower period than the belt system but still higher than the original system. This is because it is less efficient than a direct connection, but because it is more distributed in height than the belt system, it is more efficient.

Because the distributed belt system 1 showed to be more efficient than the belt system, the distributed belt system 2 was defined to be even more distributed than the system 1, as previously explained. The result was that the period was still lower than that of the belt system but higher than the other distributed system with the different pattern.

Finally, the more rigid solution, with the lowest period, was the braced tube system. This is because it is the most efficient system to function as a continuous tube at the perimeter and a tube in tube considering the braces and the core, and especially because it uses the axial stiffness of the diagonals, which is much greater than the flexural stiffness, to resist the lateral loads.

Distribution of moments: One of the main concerns and goals of the analysis was the assessment of the distribution of the bending moments at the base, from the core to the other vertical elements at the perimeter of the building such as the corner columns. With a system that is efficient on engaging the perimeter columns, connecting them to the core, the forces and bending moment of the core should then be minimized and distributed to the other vertical elements as axial forces. In that way, the first analysis was the total forces at the base of the building and the forces at the pier element. With both these values, for each combination, it was identified the moment that was transmitted to the perimeter columns. The first thing that was verified was that the solutions with the lower period value were the ones with lower percentage of moment in the core and higher in the perimeter columns. In the same way, the building with the expected worst performance, which was the simple solution, had the highest period, corresponding also to the highest percentage of moment in the pier element. This was the expected result and sustains that the better distribution of moments to the perimeter elements, the larger the base to resist them which corresponds to the better performance of the structure. A table with the bending moments at the base of the building, the pier element and the perimeter columns, as well as the percentage, is provided in the Annex A.

Displacements: As expected, the solution with higher displacements was the one with the highest period, and the one with the lowest displacements was the one with the lowest period. Regarding the displacements, since the building is symmetric only one direction was assessed. Also, because the type 1 seismic action was always more relevant to these structures, the type 2 seismic action was disregarded. The displacements of all the solutions under the seismic load and under the wind load can be seen in the following Figures 5.6 and 5.7, respectively.

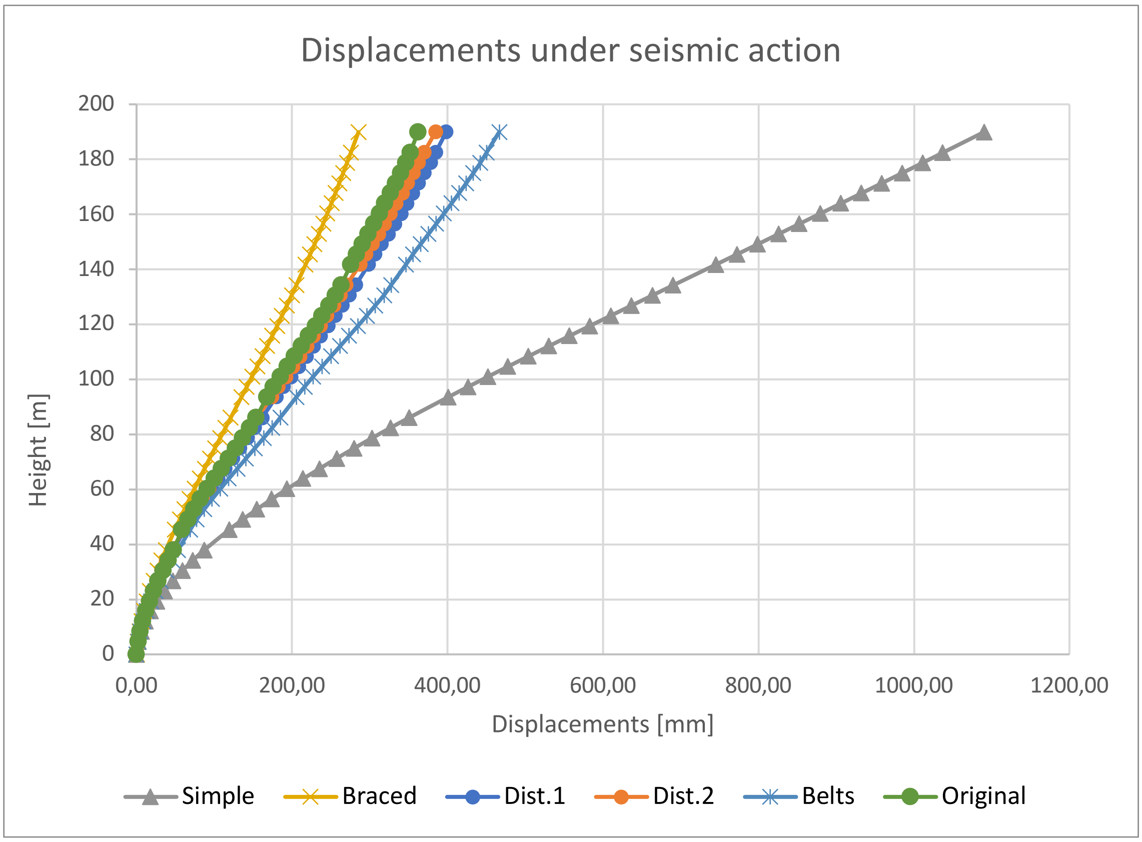


Figure 5.6:Displacements of the floors of the building under seismic loads

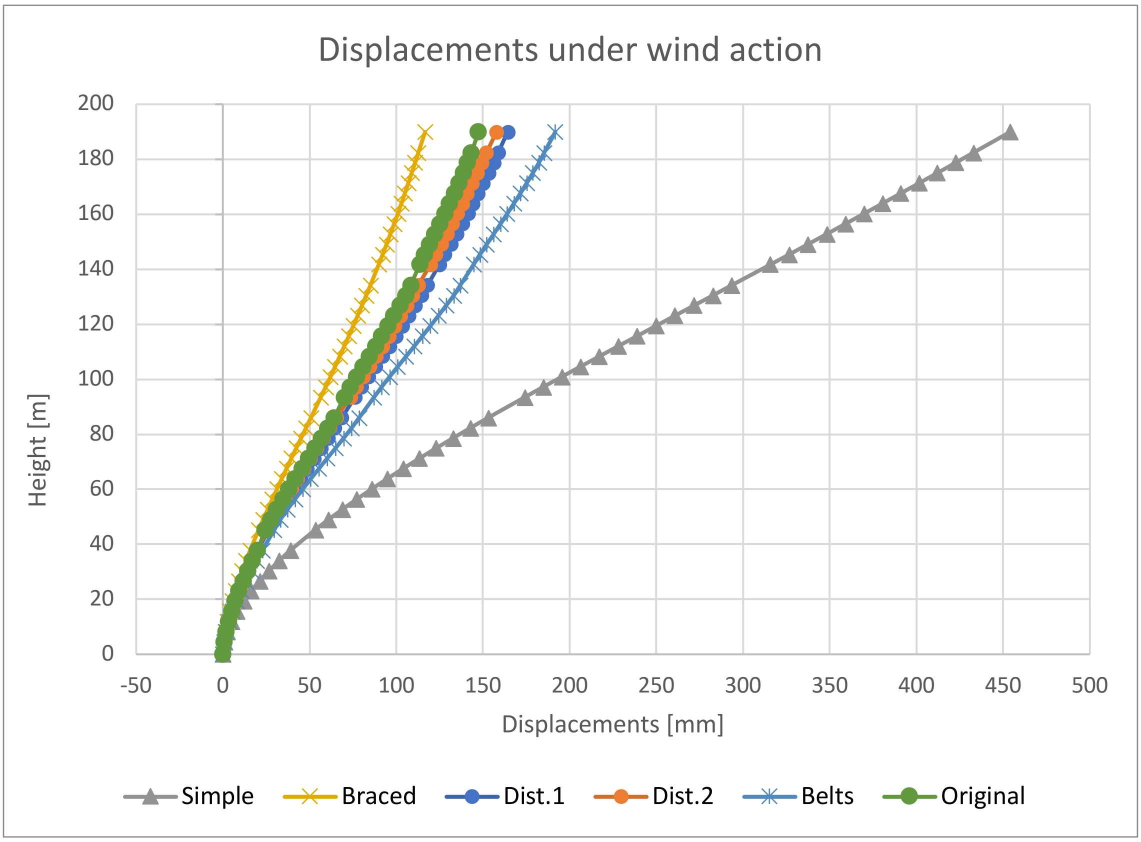


Figure 5.7: Displacements of the floors of the building under wind loads

5.4. Seismic analysis

Mainly two analyses of each solution were performed, corresponding to a seismic analysis and a wind comfort analysis. For the first analysis, the need to considerer geometrical 2nd degree effects was assessed, and the limits of relative displacement of the floors were also verified.

For the verification of the damage limitation state due to relative displacement of the floors under seismic loads, there is a condition given by the Eurocode [21], which is:

$$d_r \cdot v \leq 0{,}005h \tag{5.1}$$

Where:
$d_r = q(d_i - d_{i-1})$
q is the behavior coefficient
d_i is the total displacement of story i
$v = 0{,}40$ for type 1 seismic action
h is the floor height

The purpose of this verification is to assure that the displacement between floors isn't enough to damage the non-structural elements ($0{,}0075h$), or the structural elements ($0{,}005h$). In this analysis, every solution verified these limits, except for the simple solution. This can be observed in Table 2 and in further detail in Annex B. Table 2 is an abbreviation of the Annex B as the annex displays the values for all the floors and, for simplification reasons, it is only displayed in Table 2 the results of approximately every 5 floors.

The verification of the 2nd degree effects is intended to verify if the relative displacement of a building, considering the weight and the shear forces, expressed by a factor of θ, is enough to justify an adjustment to the forces and bending moments of the building. θ is given by:

$$\theta = \frac{P_{tot} \cdot d_r}{V_{tot} \cdot h}$$

If θ is less or equal to 0,1, it can be disregarded; if it is between 0,1 and 0,2 (including 0,2), it can be adjusted by an amplification factor of $(1/1-\theta)$; if it is higher than 0,2, the 2nd degree effects must be considered, and it can't be used a linear analysis and a non-linear finite-element analysis must take place. The value of θ can never be over 0,3. The purpose of this verification, as explained before, is to better calculate the forces and bending moments of the structure in order to define the reinforcement of the cross section of elements but also to design the structural system and to validate it. Since in this analysis it wasn't made any definition of the amount of reinforcing steel in the cross sections, the purpose of this verification is mainly comparative. It was used to check if the solutions were comparable, since one could be so unparalleled that it couldn't be compared in the same terms as the others, and to have another comparison criterion between the solutions to better classify them.

Table 2: Seismic Verifications

Floor	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ
	Original Outriggers				Simple				Belts			
1	1,78	5,19	Verifica!	0,024	2,21	6,47	Verifica!	0,031	1,85	5,41	Verifica!	0,025
5	16,79	14,51	Verifica!	0,077	26,27	24,77	Verifica!	0,137	18,38	16,23	Verifica!	0,087
10	47,75	21,23	Verifica!	0,110	86,55	42,93	Verifica!	0,230	52,93	22,00	Verifica!	0,115
15	90,99	26,03	Verifica!	0,125	193,60	59,45	Não	0,296	107,69	31,74	Verifica!	0,154
20	136,24	27,32	Verifica!	0,119	302,63	69,15	Não	0,307	163,33	33,80	Verifica!	0,147
25	185,09	26,60	Verifica!	0,103	451,57	76,86	Não	0,303	226,95	33,28	Verifica!	0,129
30	230,04	26,60	Verifica!	0,093	582,95	79,85	Não	0,286	284,69	34,39	Verifica!	0,121
35	275,67	38,16	Verifica!	0,058	745,07	164,52	Não	0,255	346,45	57,56	Verifica!	0,089
40	312,41	21,86	Verifica!	0,047	879,08	80,14	Não	0,183	395,33	29,69	Verifica!	0,065
45	346,18	19,00	Verifica!	0,031	1011,01	78,62	Não	0,122	442,25	26,92	Verifica!	0,043
47	361,94	30,10	Verifica!	0,028	1090,01	158,95	Não	0,123	466,42	48,85	Verifica!	0,044
	Distributed 1				Distributed 2				Braced			
1	1,765	5,154	Verifica!	0,024	1,572	4,56	Verifica!	0,021	1,388	3,996	Verifica!	0,018
5	16,229	14,064	Verifica!	0,075	15,269	13,788	Verifica!	0,073	13,052	11,001	Verifica!	0,057
10	47,094	20,442	Verifica!	0,106	44,975	20,523	Verifica!	0,106	37,437	16,545	Verifica!	0,083
15	96,257	27,564	Verifica!	0,132	92,892	23,991	Verifica!	0,115	74,335	19,077	Verifica!	0,090
20	142,148	28,731	Verifica!	0,124	136,399	27,642	Verifica!	0,119	107,466	20,382	Verifica!	0,087
25	198,345	29,025	Verifica!	0,112	191,652	27,978	Verifica!	0,108	148,336	20,103	Verifica!	0,077
30	246,055	27,9	Verifica!	0,098	236,678	28,308	Verifica!	0,099	180,781	19,017	Verifica!	0,065
35	298,468	50,103	Verifica!	0,075	287,448	53,067	Verifica!	0,078	217,441	36,06	Verifica!	0,051
40	340,171	24,585	Verifica!	0,053	326,313	22,056	Verifica!	0,047	245,308	16,311	Verifica!	0,034
45	377,704	22,611	Verifica!	0,035	362,523	21,078	Verifica!	0,032	270,706	14,997	Verifica!	0,023
47	398,23	40,818	Verifica!	0,034	384,5	44,943	Verifica!	0,035	285,47	29,7	Verifica!	0,024

As can be seen by the results showed in Table 2, the results of all solutions were relatively low. Not counting the simple solution, which is an example of a solution that cannot be compared with the others, in which case the results of θ were over 0,31, the worst case of the value of θ for the other systems is 0,14 and it is only in 2 floors of the Distributed 1 and the Distributed 2 systems. The results of the original solution are a maximum of 0,12 and for the braced tube the results are lower than 0,1 which not only assures that the 2nd degree effects can be disregarded but it also confirms the braced tube as the best system to resist the lateral loads.

5.5. Wind comfort analysis

The wind analysis is mainly due to the comfort of the users of the building and so it is under Serviceability Limit States. For this analysis, three verifications were made which are the maximum displacement at the top of the building, the relevance of vibration for the direction transversal to the wind due to the vortex shedding effect, and the vibration of the building in the direction of the wind due to the wind force.

The maximum displacement at the top of the building due to the wind forces corresponds to the amplitude of the movement. For this, the Eurocode gives a maximum value of H/500 (H being the total height of the building) [22]. All the systems checked this requirement, except for the simple system, and the one that has the most tolerance is the braced tube as can be seen in the following table.

Table 3: Maximum displacement verification

	Original	Simple	Belts	Dist.1	Dist.2	Braced	H/500 [mm]
dmax [mm]	147,41	454,019	191,705	164,23	157,812	116,631	
d<H/500	Verifies!	Doesn't	Verifies!	Verifies!	Verifies!	Verifies!	397,6

Eurocode also gives a minimum requirement to dismiss the consideration of the effects of vortex shedding which can stimulate the vibration of the building in the direction transversal to the wind. This minimum requirement is expressed by the following formula given by the Eurocode [22]:

$$v_{crit,i} > 1{,}25 \cdot v_m$$

Where:

$v_{crit,i}$ is the critical wind velocity for *i* mode

v_m is the characteristic average wind velocity

Again, all solutions checked this requirement which means that the effect of vortex shedding can be disregarded. This can be seen in Table 5 and in Annex C. Table 4 is an abbreviation of the Annex C as the annex displays the values for all the floors and, for simplification reasons, it is only displayed in Table 2 the results of approximately every 5 floors.

The vibration of the building in the direction of the wind due to the wind force depends on the wind peak acceleration, which is the multiplication of the wind peak factor (k_p) with the standard deviation of the characteristic acceleration ($\sigma_{a,x}$). The wind peak factor (k_p) and the standard deviation of the characteristic acceleration ($\sigma_{a,x}$) are expressed by the following formulas, respectively [22]:

$$k_p = \sqrt{2 \cdot \ln(v \cdot T)} + \frac{0{,}6}{\sqrt{2 \cdot \ln(v \cdot T)}}$$

and

$$\sigma_{a,x}(z) = \frac{c_f \cdot \rho \cdot b \cdot I_V(z_s) \cdot v_m^2(z_s)}{m_{1,x}} \cdot R \cdot K_x \cdot \Phi_{1,x}(\mathrm{z})$$

The Eurocode doesn't give standard values of acceptability for the wind characteristic acceleration but instead it says that it must be agreed with the owner/promoter of the building. *fib* bulletin [23] presents acceptance values according to human perception of movement, vibration and acceleration.

Table 4: Human perception of movement (Source: [23])

Perception	**Acceleration limits**
Imperceptible	a < 0,005g
Perceptible	0,005g < a < 0,015g
Annoying	0,015g < a < 0,05g
Very Annoying	0,05g < a < 0,15g
Intolerable	0,15g < a

According to these values, as can be seen in Table 5 and in more detail in Annex C, for all considered solutions the obtained results are imperceptible. This parameter is intended as a verification regarding comfort, but since the detailing of the reinforcement wasn't made, it is also intended to be a comparative criterion for the different solutions, as is the case of θ for the 2nd degree effects. It is used to check if the solutions are comparable and to see which solution has the most clearance and the lowest values. It is seen that the braced tube system is the one with the lowest values of the wind characteristic acceleration with 0,023 m/s^2. Other than that, it is also seen that even for the least robust structures the wind comfort requirements are met as the wind characteristic acceleration is classified as imperceptible for all cases.

Table 5: Wind verifications

Floor	$v_m(z)$	Displac. [mm]	$v_{crit,x} > 1,25 \cdot v_m(z)$	Peak acceleration $[m/s^2]$	Perception	Displac. [mm]	$v_{crit,x} > 1,25 \cdot v_m(z)$	Peak acceleration $[m/s^2]$	Perception	Displac. [mm]	$v_{crit,x} > 1,25 \cdot v_m(z)$	Peak acceleration $[m/s^2]$	Perception
		Original Outriggers $(v_{crit,x} = 102,58\ m/s)$				Simple $(v_{crit,x} = 62,47\ m/s)$				Belts $(v_{crit,x} = 91,36\ m/s)$			
1	23,08	0,74	Verifies!	0,0001	Imperceptible	1,016	Verifies!	0,0001	Imperceptible	0,794	Verifies!	0,0001	Imperceptible
5	30,55	7,06	Verifies!	0,0010	Imperceptible	11,991	Verifies!	0,0010	Imperceptible	7,937	Verifies!	0,0010	Imperceptible
10	34,00	20,05	Verifies!	0,0029	Imperceptible	38,968	Verifies!	0,0035	Imperceptible	22,791	Verifies!	0,0029	Imperceptible
15	36,38	38,14	Verifies!	0,0058	Imperceptible	85,903	Verifies!	0,0079	Imperceptible	46,125	Verifies!	0,0061	Imperceptible
20	37,76	56,96	Verifies!	0,0088	Imperceptible	132,755	Verifies!	0,0123	Imperceptible	69,646	Verifies!	0,0095	Imperceptible
25	39,04	77,07	Verifies!	0,0122	Imperceptible	195,465	Verifies!	0,0185	Imperceptible	96,147	Verifies!	0,0134	Imperceptible
30	39,90	95,19	Verifies!	0,0153	Imperceptible	249,746	Verifies!	0,0239	Imperceptible	119,722	Verifies!	0,0169	Imperceptible
35	40,78	113,50	Verifies!	0,0185	Imperceptible	315,729	Verifies!	0,0307	Imperceptible	144,609	Verifies!	0,0208	Imperceptible
40	41,41	128,01	Verifies!	0,0211	Imperceptible	369,697	Verifies!	0,0362	Imperceptible	164,012	Verifies!	0,0238	Imperceptible
45	41,97	141,12	Verifies!	0,0235	Imperceptible	422,49	Verifies!	0,0416	Imperceptible	182,257	Verifies!	0,0266	Imperceptible
47	42,28	147,41	Verifies!	0,0246	Imperceptible	454,019	Verifies!	0,0448	Imperceptible	191,705	Verifies!	0,0281	Imperceptible
		Distributed 1 $(v_{crit,x} = 98,67\ m/s)$				Distributed 2 $(v_{crit,x} = 97,5\ m/s)$				Braced $(v_{crit,x} = 116,9\ m/s)$			
1	23,08	0,749	Verifies!	0,0001	Imperceptible	0,667	Verifies!	0,0001	Imperceptible	0,571	Verifies!	0,0001	Imperceptible
5	30,55	6,954	Verifies!	0,0009	Imperceptible	6,532	Verifies!	0,0010	Imperceptible	5,462	Verifies!	0,0009	Imperceptible
10	34,00	20,166	Verifies!	0,0029	Imperceptible	19,28	Verifies!	0,0029	Imperceptible	15,712	Verifies!	0,0027	Imperceptible
15	36,38	41,058	Verifies!	0,0060	Imperceptible	39,563	Verifies!	0,0062	Imperceptible	31,225	Verifies!	0,0056	Imperceptible
20	37,76	60,511	Verifies!	0,0091	Imperceptible	57,847	Verifies!	0,0094	Imperceptible	45,112	Verifies!	0,0083	Imperceptible
25	39,04	83,989	Verifies!	0,0129	Imperceptible	80,826	Verifies!	0,0134	Imperceptible	62,061	Verifies!	0,0117	Imperceptible
30	39,90	103,363	Verifies!	0,0162	Imperceptible	99,134	Verifies!	0,0168	Imperceptible	75,185	Verifies!	0,0144	Imperceptible
35	40,78	124,79	Verifies!	0,0198	Imperceptible	119,733	Verifies!	0,0207	Imperceptible	89,888	Verifies!	0,0175	Imperceptible
40	41,41	141,39	Verifies!	0,0227	Imperceptible	135,107	Verifies!	0,0236	Imperceptible	100,93	Verifies!	0,0199	Imperceptible
45	41,97	156,108	Verifies!	0,0252	Imperceptible	149,263	Verifies!	0,0263	Imperceptible	110,853	Verifies!	0,0220	Imperceptible
47	42,28	164,23	Verifies!	0,0266	Imperceptible	157,812	Verifies!	0,0280	Imperceptible	116,631	Verifies!	0,0232	Imperceptible

5.6. Concluding Remarks

From the original structure employed in the Montreal Stock Exchange Tower, various alternative solutions were presented in order to assess the efficiency of the structure. At first, an evaluation of the structure without the outriggers was made to see the benefits that these elements add to the system. The increment of the quantity of material was less than 5% and the period decreased more than 40% of the original value. As for the displacements, its decrease at the top of the building due to the same seismic action or the same wind loads were more than one third of the total value in each case.

Regarding the other solutions, they were all developed with approximately the same amount of material, thus it can be assumed that that the cost of materials is roughly the same on all these alternatives. It was verified also that all solutions could be employed with some adjustments. Since some alternatives have more clearance with the safety and comfort verifications, their cross sections could be reduced and therefore their quantities of materials and final cost could be lower than the other solutions. This applies mainly to the braced tube solution which can be assumed to be the most efficient system of the alternatives studied.

6. Conclusions and future prospects

6.1. Conclusion

Outrigger frame systems provide strong and stiff structures for high-rise buildings that connect the core of a building with the perimeter columns to spread the moment resisting elements along a greater area of the foundation. They also achieve this without strongly interfering with the perimeter frame of the building allowing for greater architectural freedom than the other lateral load resisting systems.

The other lateral load resisting systems can be either exterior structures or interior structures. The systems presented are also arranged in a chart according to their efficient height. Some can only be used until certain height without the aid of other elements or systems to help resist the lateral loads, like the case of rigid frames, shear walls or core systems. Other systems are more efficient at resisting lateral loads and can be applied in taller structures but they either are costly to be applied in smaller structures like the outrigger frame systems, or are very intrusive to the structures and don't allow them to have freedom of element placement like the tube systems. The lateral load resisting systems can also be combined in order to have a stronger stiffer system to allow even taller structures. Such is the case of the SWFC.

The concept of the outriggers was brought from the naval construction. The stays connected the ship to side floaters that helped the main ship stabilize. For a building application, the outrigger spreads the moment of the core through direct elements which are the outrigger arms, or through indirect means such as the slabs, to the perimeter columns. The bending moment of the core is then resisted by axial force of the perimeter columns. This additional axial load on the perimeter columns is easily attended by redefining the cross section of the columns. This eventual increase in material of the cross section is small when compared to the material used in the core to resist the moments. There are two different types of outriggers systems, the direct or conventional outriggers and the indirect or virtual outriggers.

Even though the system may seem elegant and simple, designing outriggers and outrigger systems is far from a simple task. Several design and construction considerations must be considered as for outriggers are very stiff and heavy elements that interact with columns, core and floor diaphragms and are very sensitive to differential vertical shortening and, of course, lateral loads.

At last, a numerical model of a case study was constructed and tested for seismic action and wind loads. The building chosen was the Montreal Stock Exchange Tower in Montreal, Canada, but it was assumed to be in Lisbon with the lateral loads being considered accordingly. For purposes of analysis and comparison of solutions, five other alternative systems were considered, based on the systems previously presented, and with the same quantity of concrete as the original structure. After being subjected to the same loads, the results obtained for all considered solutions were related to the fundamental frequencies of the structural systems and their distribution of moments at the base, the displacements of each floor, the seismic verifications, and the wind comfort verifications. It was observed that all the solutions met the safety and comfort verifications, but some had more clearance and could use less material and be more competitive than others, namely the braced tube system.

Even though the best solution proved to be the braced tube solution, all the others were also viable for this situation. That in mind, they all present valid alternatives, other than the braced tubes, that can have greater architectural freedom. This means that there can be other options than having the diagonals of the braces on the façade of the building which can be inconvenient and obstruct the views from inside the building, standing out as an aesthetic expression. Also, all the alternatives for the original outrigger system were solutions that didn't compromise nor did they intervene with the interior space of the structure. This is a great advantage since the value of the rentable area, in densely populated cities where the high-rise buildings are usually located, is very high and these solutions can present an interesting alternative since they present, for the same building, more floor space to be occupied.

As for the quality of the interior space, any space that can have the greatest amount of opening for the external views from inside and for entrance of light is more valuable. Since the views are a considerable part of the intrinsic value of a high-rise building and the entrance of natural light improves the quality of the space, the distributed belt walls also present a very good alternative to the virtual outrigger system with belt walls since the floors where the belts are situated do not have any opening. Even though they can be occupied and used, they don't have the same value as if those belts would be distributed.

6.2. Future developments

This thesis provides an overall clear comparison of outrigger frame systems, either direct outriggers, belt virtual outriggers or distributed wall virtual outriggers. It also compares these structural solutions to a core system and a braced tube system. Since the comparison of outrigger systems and tube systems was only made through braced tubes, it would be useful to evaluate in further detail the other tubular systems as well.

One thing that was not taken into consideration was the value of the usable area and the exposition of light, directly related to the architectural freedom. Since direct outriggers interfere with rentable space of floors, and belts or distributed walls can obstruct the entrance of light, the rentable value of those floors decrease but it may increase the value of other floors. It would be interesting to cross the information of the structural efficiency of the building with the architectural value of space, to check the gains and losses of applying the different solutions and to try to quantify the value of the benefits of having more or less light exposure, for example.

Another alternative study that could be useful is the quantification of the material that could be reduced in each solution. Since some solutions presented here have more clearance with the safety and comfort verifications, like the braced tube, it would be interesting to start reducing the quantity of material without compromising the quality and efficiency of the structure, and at the end quantify the quantity of material and the cost that could be saved for employing different solutions.

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Annex A – Percentage of bending moment in Pier and Columns

		Original		Simple		Belts		Dist.1		Dist.2		Braced	
		kNm	[%]	kNm	[%]	kNm	[%]	kNm	[%]	kNm	[%]	kNm	[%]
Seismic Action	Base Moment	3993609,1	100 %	3639048,7	100 %	3906591,0	100 %	3911261,6	100 %	3896590,5	100 %	3928026,5	100 %
	Pier Moment	1983523,5	49,67 %	3415740,3	93,86 %	2225187,9	56,96 %	1985855,8	50,77 %	1869072,8	47,97 %	1573936,6	40,07 %
	Columns Moment	2010085,6	50,33 %	223308,4	6,14 %	1681403,1	43,04 %	1925405,8	49,23 %	2027517,7	52,03 %	2354090,0	59,93 %
Wind Action	Base Moment	3252622,0	100 %	3252622,0	100 %	3252622,0	100 %	3252622,0	100 %	3252622,0	100 %	3252622,0	100 %
	Pier Moment	1172606,1	36,05 %	2192703,7	67,41 %	1346917,8	41,41 %	1196283,6	36,78 %	1127686,0	34,67 %	932619,4	28,67 %
	Columns Moment	2080015,9	63,95 %	1059918,3	32,59 %	1905704,2	58,59 %	2056338,4	63,22 %	2124936,0	65,33 %	2320002,6	71,33 %

Annex B – Seismic verifications

		Original Outriggers				Simple			
Floor	Height [m]	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ
1	4,5	1,78	5,19	Verifica!	0,024	2,21	6,47	Verifica!	0,031
2	8,2	4,35	7,71	Verifica!	0,042	5,90	11,09	Verifica!	0,063
3	11,9	7,77	10,28	Verifica!	0,056	11,19	15,87	Verifica!	0,089
4	15,6	11,96	12,55	Verifica!	0,067	18,01	20,45	Verifica!	0,114
5	19,3	16,79	14,51	Verifica!	0,077	26,27	24,77	Verifica!	0,137
6	23	22,19	16,18	Verifica!	0,085	35,89	28,86	Verifica!	0,158
7	26,7	28,03	17,53	Verifica!	0,092	46,80	32,74	Verifica!	0,179
8	30,4	34,22	18,58	Verifica!	0,097	58,94	36,42	Verifica!	0,198
9	34,1	40,67	19,36	Verifica!	0,101	72,25	39,91	Verifica!	0,216
10	37,8	47,75	21,23	Verifica!	0,110	86,55	42,93	Verifica!	0,230
11	45,3	58,20	31,35	Verifica!	0,080	119,14	97,75	Não	0,257
12	49	66,05	23,54	Verifica!	0,118	136,47	51,99	Não	0,271
13	52,7	73,95	23,72	Verifica!	0,117	154,73	54,79	Não	0,281
14	56,4	82,31	25,07	Verifica!	0,122	173,78	57,15	Não	0,289
15	60,1	90,99	26,03	Verifica!	0,125	193,60	59,45	Não	0,296
16	63,8	99,87	26,66	Verifica!	0,125	214,13	61,59	Não	0,300
17	67,5	108,90	27,09	Verifica!	0,125	235,32	63,58	Não	0,303
18	71,2	117,98	27,22	Verifica!	0,123	257,12	65,40	Não	0,305
19	74,9	127,13	27,47	Verifica!	0,121	279,58	67,36	Não	0,306
20	78,6	136,24	27,32	Verifica!	0,119	302,63	69,15	Não	0,307
21	82,3	145,19	26,86	Verifica!	0,115	326,23	70,81	Não	0,307
22	86	153,65	25,38	Verifica!	0,106	350,29	72,17	Não	0,306
23	93,5	167,73	42,24	Verifica!	0,086	400,68	151,19	Não	0,310
24	97,2	176,22	25,46	Verifica!	0,100	425,95	75,80	Não	0,304
25	100,9	185,09	26,60	Verifica!	0,103	451,57	76,86	Não	0,303
26	104,6	194,08	26,99	Verifica!	0,102	477,45	77,63	Não	0,300
27	108,3	203,14	27,17	Verifica!	0,101	503,55	78,32	Não	0,297
28	112	212,19	27,15	Verifica!	0,099	529,86	78,92	Não	0,294
29	115,7	221,17	26,95	Verifica!	0,096	556,34	79,42	Não	0,290
30	119,4	230,04	26,60	Verifica!	0,093	582,95	79,85	Não	0,286
31	123,1	238,73	26,07	Verifica!	0,089	609,67	80,17	Não	0,281
32	126,8	247,19	25,38	Verifica!	0,085	636,48	80,42	Não	0,276
33	130,5	255,50	24,92	Verifica!	0,081	663,39	80,74	Não	0,270
34	134,2	262,95	22,35	Verifica!	0,071	690,23	80,52	Não	0,261
35	141,7	275,67	38,16	Verifica!	0,058	745,07	164,52	Não	0,255
36	145,4	282,90	21,70	Verifica!	0,060	772,02	80,84	Não	0,237
37	149,1	290,31	22,22	Verifica!	0,058	798,88	80,60	Não	0,225
38	152,8	297,74	22,31	Verifica!	0,055	825,67	80,35	Não	0,212
39	156,5	305,12	22,14	Verifica!	0,051	852,37	80,11	Não	0,198
40	160,2	312,41	21,86	Verifica!	0,047	879,08	80,14	Não	0,183
41	163,9	319,52	21,35	Verifica!	0,043	905,65	79,69	Não	0,169
42	167,6	326,48	20,87	Verifica!	0,040	932,12	79,43	Não	0,155
43	171,3	333,26	20,35	Verifica!	0,036	958,51	79,17	Não	0,143
44	175	339,84	19,73	Verifica!	0,033	984,80	78,87	Não	0,131
45	178,7	346,18	19,00	Verifica!	0,031	1011,01	78,62	Não	0,122
46	182,4	351,90	17,18	Verifica!	0,028	1037,02	78,04	Não	0,115
47	189,9	361,94	30,10	Verifica!	0,028	1090,01	158,95	Não	0,123

	$\theta < 0{,}1$
	$0{,}1 < \theta < 0{,}2$
	$0{,}2 < \theta < 0{,}3$
	$0{,}3 < \theta$

		Belts				Distributed 1			
Floor	Height [m]	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ
1	4,5	1,85	5,41	Verifica!	0,025	1,765	5,154	Verifica!	0,024
2	8,2	4,61	8,28	Verifica!	0,046	4,298	7,599	Verifica!	0,042
3	11,9	8,35	11,21	Verifica!	0,061	7,615	9,951	Verifica!	0,054
4	15,6	12,97	13,88	Verifica!	0,075	11,541	11,778	Verifica!	0,063
5	19,3	18,38	16,23	Verifica!	0,087	16,229	14,064	Verifica!	0,075
6	23	24,48	18,29	Verifica!	0,097	21,543	15,942	Verifica!	0,084
7	26,7	31,15	20,01	Verifica!	0,106	27,278	17,205	Verifica!	0,090
8	30,4	38,26	21,32	Verifica!	0,113	33,605	18,981	Verifica!	0,099
9	34,1	45,60	22,02	Verifica!	0,116	40,28	20,025	Verifica!	0,104
10	37,8	52,93	22,00	Verifica!	0,115	47,094	20,442	Verifica!	0,106
11	45,3	69,05	48,37	Verifica!	0,124	61,802	44,124	Verifica!	0,112
12	49	77,26	24,62	Verifica!	0,125	69,463	22,983	Verifica!	0,115
13	52,7	86,90	28,94	Verifica!	0,144	78,055	25,776	Verifica!	0,128
14	56,4	97,11	30,62	Verifica!	0,151	87,069	27,042	Verifica!	0,132
15	60,1	107,69	31,74	Verifica!	0,154	96,257	27,564	Verifica!	0,132
16	63,8	118,53	32,51	Verifica!	0,154	104,821	25,692	Verifica!	0,121
17	67,5	129,57	33,12	Verifica!	0,154	114,057	27,708	Verifica!	0,128
18	71,2	140,75	33,55	Verifica!	0,153	123,323	27,798	Verifica!	0,126
19	74,9	152,07	33,95	Verifica!	0,151	132,571	27,744	Verifica!	0,122
20	78,6	163,33	33,80	Verifica!	0,147	142,148	28,731	Verifica!	0,124
21	82,3	174,30	32,90	Verifica!	0,141	151,708	28,68	Verifica!	0,121
22	86	184,63	30,98	Verifica!	0,130	161,359	28,953	Verifica!	0,120
23	93,5	205,46	62,49	Verifica!	0,128	179,4	54,123	Verifica!	0,109
24	97,2	215,86	31,20	Verifica!	0,124	188,67	27,81	Verifica!	0,109
25	100,9	226,95	33,28	Verifica!	0,129	198,345	29,025	Verifica!	0,112
26	104,6	238,42	34,40	Verifica!	0,131	208,191	29,538	Verifica!	0,112
27	108,3	250,03	34,84	Verifica!	0,130	217,942	29,253	Verifica!	0,108
28	112	261,66	34,89	Verifica!	0,128	227,336	28,182	Verifica!	0,103
29	115,7	273,23	34,72	Verifica!	0,125	236,755	28,257	Verifica!	0,101
30	119,4	284,69	34,39	Verifica!	0,121	246,055	27,9	Verifica!	0,098
31	123,1	295,98	33,85	Verifica!	0,117	255,1	27,135	Verifica!	0,093
32	126,8	306,98	32,99	Verifica!	0,112	264,254	27,462	Verifica!	0,091
33	130,5	318,01	33,09	Verifica!	0,109	273,487	27,699	Verifica!	0,090
34	134,2	327,26	27,77	Verifica!	0,089	281,767	24,84	Verifica!	0,078
35	141,7	346,45	57,56	Verifica!	0,089	298,468	50,103	Verifica!	0,075
36	145,4	355,36	26,74	Verifica!	0,075	306,451	23,949	Verifica!	0,066
37	149,1	365,37	30,02	Verifica!	0,080	314,938	25,461	Verifica!	0,067
38	152,8	375,44	30,22	Verifica!	0,076	323,491	25,659	Verifica!	0,063
39	156,5	385,44	29,98	Verifica!	0,071	331,976	25,455	Verifica!	0,059
40	160,2	395,33	29,69	Verifica!	0,065	340,171	24,585	Verifica!	0,053
41	163,9	405,07	29,21	Verifica!	0,060	347,484	21,939	Verifica!	0,044
42	167,6	414,66	28,75	Verifica!	0,055	354,67	21,558	Verifica!	0,041
43	171,3	424,06	28,22	Verifica!	0,051	362,394	23,172	Verifica!	0,040
44	175	433,27	27,63	Verifica!	0,047	370,167	23,319	Verifica!	0,038
45	178,7	442,25	26,92	Verifica!	0,043	377,704	22,611	Verifica!	0,035
46	182,4	450,13	23,67	Verifica!	0,037	384,624	20,76	Verifica!	0,031
47	189,9	466,42	48,85	Verifica!	0,044	398,23	40,818	Verifica!	0,034

	$\theta < 0,1$
	$0,1 < \theta < 0,2$
	$0,2 < \theta < 0,3$
	$0,3 < \theta$

		Distributed 2				Braced			
Floor	Height [m]	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ	Displac. [mm]	dr [mm]	dr *v< 0,005h	θ
1	4,5	1,572	4,56	Verifica!	0,021	1,388	3,996	Verifica!	0,018
2	8,2	4,037	7,395	Verifica!	0,041	3,397	6,027	Verifica!	0,032
3	11,9	6,978	8,823	Verifica!	0,048	6,068	8,013	Verifica!	0,042
4	15,6	10,673	11,085	Verifica!	0,059	9,385	9,951	Verifica!	0,052
5	19,3	15,269	13,788	Verifica!	0,073	13,052	11,001	Verifica!	0,057
6	23	20,483	15,642	Verifica!	0,082	17,235	12,549	Verifica!	0,065
7	26,7	25,738	15,765	Verifica!	0,083	21,797	13,686	Verifica!	0,070
8	30,4	31,813	18,225	Verifica!	0,095	26,698	14,703	Verifica!	0,075
9	34,1	38,134	18,963	Verifica!	0,098	31,922	15,672	Verifica!	0,080
10	37,8	44,975	20,523	Verifica!	0,106	37,437	16,545	Verifica!	0,083
11	45,3	60,718	47,229	Verifica!	0,119	49,354	35,751	Verifica!	0,088
12	49	67,445	20,181	Verifica!	0,101	55,411	18,171	Verifica!	0,089
13	52,7	76,077	25,896	Verifica!	0,128	61,632	18,663	Verifica!	0,091
14	56,4	84,895	26,454	Verifica!	0,129	67,976	19,032	Verifica!	0,091
15	60,1	92,892	23,991	Verifica!	0,115	74,335	19,077	Verifica!	0,090
16	63,8	101,313	25,263	Verifica!	0,119	80,835	19,5	Verifica!	0,091
17	67,5	109,98	26,001	Verifica!	0,120	87,452	19,851	Verifica!	0,091
18	71,2	118,946	26,898	Verifica!	0,122	94,027	19,725	Verifica!	0,088
19	74,9	127,185	24,717	Verifica!	0,109	100,672	19,935	Verifica!	0,087
20	78,6	136,399	27,642	Verifica!	0,119	107,466	20,382	Verifica!	0,087
21	82,3	145,158	26,277	Verifica!	0,111	114,257	20,373	Verifica!	0,086
22	86	154,19	27,096	Verifica!	0,112	121,06	20,409	Verifica!	0,084
23	93,5	173,896	59,118	Verifica!	0,118	134,901	41,523	Verifica!	0,083
24	97,2	182,326	25,29	Verifica!	0,099	141,635	20,202	Verifica!	0,079
25	100,9	191,652	27,978	Verifica!	0,108	148,336	20,103	Verifica!	0,077
26	104,6	201,072	28,26	Verifica!	0,107	154,969	19,899	Verifica!	0,075
27	108,3	209,925	26,559	Verifica!	0,098	161,541	19,716	Verifica!	0,072
28	112	218,864	26,817	Verifica!	0,098	168,028	19,461	Verifica!	0,070
29	115,7	227,242	25,134	Verifica!	0,090	174,442	19,242	Verifica!	0,068
30	119,4	236,678	28,308	Verifica!	0,099	180,781	19,017	Verifica!	0,065
31	123,1	244,743	24,195	Verifica!	0,082	187,02	18,717	Verifica!	0,062
32	126,8	253,54	26,391	Verifica!	0,087	193,252	18,696	Verifica!	0,060
33	130,5	261,765	24,675	Verifica!	0,079	199,406	18,462	Verifica!	0,057
34	134,2	269,759	23,982	Verifica!	0,074	205,421	18,045	Verifica!	0,054
35	141,7	287,448	53,067	Verifica!	0,078	217,441	36,06	Verifica!	0,051
36	145,4	294,709	21,783	Verifica!	0,060	223,261	17,46	Verifica!	0,046
37	149,1	303,26	25,653	Verifica!	0,067	228,917	16,968	Verifica!	0,042
38	152,8	311,644	25,152	Verifica!	0,062	234,475	16,674	Verifica!	0,039
39	156,5	318,961	21,951	Verifica!	0,051	239,871	16,188	Verifica!	0,036
40	160,2	326,313	22,056	Verifica!	0,047	245,308	16,311	Verifica!	0,034
41	163,9	333,874	22,683	Verifica!	0,045	250,504	15,588	Verifica!	0,030
42	167,6	341,466	22,776	Verifica!	0,042	255,64	15,408	Verifica!	0,028
43	171,3	348,325	20,577	Verifica!	0,035	260,685	15,135	Verifica!	0,025
44	175	355,497	21,516	Verifica!	0,035	265,707	15,066	Verifica!	0,024
45	178,7	362,523	21,078	Verifica!	0,032	270,706	14,997	Verifica!	0,023
46	182,4	369,519	20,988	Verifica!	0,030	275,57	14,592	Verifica!	0,021
47	189,9	384,5	44,943	Verifica!	0,035	285,47	29,7	Verifica!	0,024

	$\theta < 0,1$
	$0,1 < \theta < 0,2$
	$0,2 < \theta < 0,3$
	$0,3 < \theta$

Annex C – Wind verifications

			Original Outriggers				Simple			
Floor	Height [m]	$v_m(z)$	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception
1	4,5	23,08402	0,74	Verifies!	0,0001	Imperceptible	1,016	Verifies!	0,0001	Imperceptible
2	8,2	26,16231	1,82	Verifies!	0,0002	Imperceptible	2,72	Verifies!	0,0002	Imperceptible
3	11,9	28,07275	3,26	Verifies!	0,0004	Imperceptible	5,144	Verifies!	0,0004	Imperceptible
4	15,6	29,46161	5,02	Verifies!	0,0007	Imperceptible	8,248	Verifies!	0,0007	Imperceptible
5	19,3	30,55345	7,06	Verifies!	0,0010	Imperceptible	11,991	Verifies!	0,0010	Imperceptible
6	23	31,45319	9,32	Verifies!	0,0013	Imperceptible	16,334	Verifies!	0,0014	Imperceptible
7	26,7	32,21843	11,78	Verifies!	0,0017	Imperceptible	21,241	Verifies!	0,0019	Imperceptible
8	30,4	32,8842	14,38	Verifies!	0,0021	Imperceptible	26,677	Verifies!	0,0023	Imperceptible
9	34,1	33,4734	17,09	Verifies!	0,0025	Imperceptible	32,613	Verifies!	0,0029	Imperceptible
10	37,8	34,00185	20,05	Verifies!	0,0029	Imperceptible	38,968	Verifies!	0,0035	Imperceptible
11	45,3	34,93037	24,43	Verifies!	0,0036	Imperceptible	53,368	Verifies!	0,0048	Imperceptible
12	49	35,33314	27,72	Verifies!	0,0041	Imperceptible	60,986	Verifies!	0,0055	Imperceptible
13	52,7	35,70658	31,03	Verifies!	0,0046	Imperceptible	68,981	Verifies!	0,0063	Imperceptible
14	56,4	36,05467	34,52	Verifies!	0,0052	Imperceptible	77,291	Verifies!	0,0070	Imperceptible
15	60,1	36,38064	38,14	Verifies!	0,0058	Imperceptible	85,903	Verifies!	0,0079	Imperceptible
16	63,8	36,68712	41,84	Verifies!	0,0063	Imperceptible	94,791	Verifies!	0,0087	Imperceptible
17	67,5	36,97632	45,59	Verifies!	0,0070	Imperceptible	103,929	Verifies!	0,0096	Imperceptible
18	71,2	37,25008	49,37	Verifies!	0,0076	Imperceptible	113,302	Verifies!	0,0105	Imperceptible
19	74,9	37,50998	53,18	Verifies!	0,0082	Imperceptible	122,922	Verifies!	0,0114	Imperceptible
20	78,6	37,75733	56,96	Verifies!	0,0088	Imperceptible	132,755	Verifies!	0,0123	Imperceptible
21	82,3	37,99331	60,67	Verifies!	0,0094	Imperceptible	142,789	Verifies!	0,0133	Imperceptible
22	86	38,21891	64,15	Verifies!	0,0100	Imperceptible	152,969	Verifies!	0,0143	Imperceptible
23	93,5	38,64785	69,96	Verifies!	0,0110	Imperceptible	174,189	Verifies!	0,0164	Imperceptible
24	97,2	38,84694	73,44	Verifies!	0,0116	Imperceptible	184,777	Verifies!	0,0175	Imperceptible
25	100,9	39,03859	77,07	Verifies!	0,0122	Imperceptible	195,465	Verifies!	0,0185	Imperceptible
26	104,6	39,22334	80,73	Verifies!	0,0128	Imperceptible	206,223	Verifies!	0,0196	Imperceptible
27	108,3	39,40167	84,39	Verifies!	0,0134	Imperceptible	217,042	Verifies!	0,0207	Imperceptible
28	112	39,57401	88,04	Verifies!	0,0141	Imperceptible	227,911	Verifies!	0,0218	Imperceptible
29	115,7	39,74074	91,64	Verifies!	0,0147	Imperceptible	238,815	Verifies!	0,0229	Imperceptible
30	119,4	39,90222	95,19	Verifies!	0,0153	Imperceptible	249,746	Verifies!	0,0239	Imperceptible
31	123,1	40,05878	98,66	Verifies!	0,0159	Imperceptible	260,691	Verifies!	0,0251	Imperceptible
32	126,8	40,2107	102,04	Verifies!	0,0165	Imperceptible	271,643	Verifies!	0,0262	Imperceptible
33	130,5	40,35825	105,40	Verifies!	0,0171	Imperceptible	282,645	Verifies!	0,0273	Imperceptible
34	134,2	40,50168	108,34	Verifies!	0,0176	Imperceptible	293,512	Verifies!	0,0284	Imperceptible
35	141,7	40,78065	113,50	Verifies!	0,0185	Imperceptible	315,729	Verifies!	0,0307	Imperceptible
36	145,4	40,91288	116,42	Verifies!	0,0191	Imperceptible	326,652	Verifies!	0,0318	Imperceptible
37	149,1	41,04179	119,36	Verifies!	0,0196	Imperceptible	337,475	Verifies!	0,0329	Imperceptible
38	152,8	41,16754	122,29	Verifies!	0,0201	Imperceptible	348,251	Verifies!	0,0340	Imperceptible
39	156,5	41,29028	125,18	Verifies!	0,0206	Imperceptible	358,979	Verifies!	0,0351	Imperceptible
40	160,2	41,41016	128,01	Verifies!	0,0211	Imperceptible	369,697	Verifies!	0,0362	Imperceptible
41	163,9	41,52729	130,78	Verifies!	0,0216	Imperceptible	380,341	Verifies!	0,0373	Imperceptible
42	167,6	41,64181	133,47	Verifies!	0,0221	Imperceptible	390,941	Verifies!	0,0383	Imperceptible
43	171,3	41,75383	136,10	Verifies!	0,0226	Imperceptible	401,499	Verifies!	0,0394	Imperceptible
44	175	41,86346	138,66	Verifies!	0,0230	Imperceptible	412,011	Verifies!	0,0405	Imperceptible
45	178,7	41,97079	141,12	Verifies!	0,0235	Imperceptible	422,49	Verifies!	0,0416	Imperceptible
46	182,4	42,07592	143,35	Verifies!	0,0239	Imperceptible	432,847	Verifies!	0,0426	Imperceptible
47	189,9	42,28264	147,41	Verifies!	0,0246	Imperceptible	454,019	Verifies!	0,0448	Imperceptible
H/500 [mm]	379,8		d<H/500 Verifies!	$v_{crit,x}$ 102,58			d<H/500 Não	$v_{crit,x}$ 62,47		

			Belts				Distributed 1			
Floor	Height [m]	$v_m(z)$	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception
1	4,5	23,08402	0,794	Verifies!	0,0001	Imperceptible	0,749	Verifies!	0,0001	Imperceptible
2	8,2	26,16231	1,988	Verifies!	0,0002	Imperceptible	1,834	Verifies!	0,0002	Imperceptible
3	11,9	28,07275	3,605	Verifies!	0,0004	Imperceptible	3,257	Verifies!	0,0004	Imperceptible
4	15,6	29,46161	5,603	Verifies!	0,0007	Imperceptible	4,941	Verifies!	0,0007	Imperceptible
5	19,3	30,55345	7,937	Verifies!	0,0010	Imperceptible	6,954	Verifies!	0,0009	Imperceptible
6	23	31,45319	10,564	Verifies!	0,0013	Imperceptible	9,233	Verifies!	0,0013	Imperceptible
7	26,7	32,21843	13,437	Verifies!	0,0017	Imperceptible	11,693	Verifies!	0,0016	Imperceptible
8	30,4	32,8842	16,493	Verifies!	0,0021	Imperceptible	14,402	Verifies!	0,0020	Imperceptible
9	34,1	33,4734	19,646	Verifies!	0,0025	Imperceptible	17,257	Verifies!	0,0024	Imperceptible
10	37,8	34,00185	22,791	Verifies!	0,0029	Imperceptible	20,166	Verifies!	0,0029	Imperceptible
11	45,3	34,93037	29,682	Verifies!	0,0039	Imperceptible	26,428	Verifies!	0,0038	Imperceptible
12	49	35,33314	33,172	Verifies!	0,0043	Imperceptible	29,67	Verifies!	0,0043	Imperceptible
13	52,7	35,70658	37,287	Verifies!	0,0049	Imperceptible	33,328	Verifies!	0,0049	Imperceptible
14	56,4	36,05467	41,631	Verifies!	0,0055	Imperceptible	37,159	Verifies!	0,0054	Imperceptible
15	60,1	36,38064	46,125	Verifies!	0,0061	Imperceptible	41,058	Verifies!	0,0060	Imperceptible
16	63,8	36,68712	50,717	Verifies!	0,0068	Imperceptible	44,704	Verifies!	0,0066	Imperceptible
17	67,5	36,97632	55,385	Verifies!	0,0074	Imperceptible	48,595	Verifies!	0,0072	Imperceptible
18	71,2	37,25008	60,119	Verifies!	0,0081	Imperceptible	52,536	Verifies!	0,0078	Imperceptible
19	74,9	37,50998	64,898	Verifies!	0,0088	Imperceptible	56,46	Verifies!	0,0085	Imperceptible
20	78,6	37,75733	69,646	Verifies!	0,0095	Imperceptible	60,511	Verifies!	0,0091	Imperceptible
21	82,3	37,99331	74,259	Verifies!	0,0101	Imperceptible	64,546	Verifies!	0,0097	Imperceptible
22	86	38,21891	78,58	Verifies!	0,0108	Imperceptible	68,585	Verifies!	0,0104	Imperceptible
23	93,5	38,64785	87,227	Verifies!	0,0121	Imperceptible	76,108	Verifies!	0,0116	Imperceptible
24	97,2	38,84694	91,557	Verifies!	0,0127	Imperceptible	79,973	Verifies!	0,0123	Imperceptible
25	100,9	39,03859	96,147	Verifies!	0,0134	Imperceptible	83,989	Verifies!	0,0129	Imperceptible
26	104,6	39,22334	100,87	Verifies!	0,0141	Imperceptible	88,061	Verifies!	0,0136	Imperceptible
27	108,3	39,40167	105,63	Verifies!	0,0148	Imperceptible	92,076	Verifies!	0,0143	Imperceptible
28	112	39,57401	110,374	Verifies!	0,0155	Imperceptible	96,138	Verifies!	0,0149	Imperceptible
29	115,7	39,74074	115,077	Verifies!	0,0162	Imperceptible	99,766	Verifies!	0,0156	Imperceptible
30	119,4	39,90222	119,722	Verifies!	0,0169	Imperceptible	103,363	Verifies!	0,0162	Imperceptible
31	123,1	40,05878	124,286	Verifies!	0,0176	Imperceptible	107,244	Verifies!	0,0168	Imperceptible
32	126,8	40,2107	128,728	Verifies!	0,0183	Imperceptible	110,963	Verifies!	0,0175	Imperceptible
33	130,5	40,35825	133,193	Verifies!	0,0190	Imperceptible	114,725	Verifies!	0,0181	Imperceptible
34	134,2	40,50168	136,905	Verifies!	0,0196	Imperceptible	118,053	Verifies!	0,0187	Imperceptible
35	141,7	40,78065	144,609	Verifies!	0,0208	Imperceptible	124,79	Verifies!	0,0198	Imperceptible
36	145,4	40,91288	148,241	Verifies!	0,0213	Imperceptible	128,05	Verifies!	0,0204	Imperceptible
37	149,1	41,04179	152,214	Verifies!	0,0219	Imperceptible	131,424	Verifies!	0,0210	Imperceptible
38	152,8	41,16754	156,198	Verifies!	0,0226	Imperceptible	134,814	Verifies!	0,0215	Imperceptible
39	156,5	41,29028	160,132	Verifies!	0,0232	Imperceptible	138,164	Verifies!	0,0221	Imperceptible
40	160,2	41,41016	164,012	Verifies!	0,0238	Imperceptible	141,39	Verifies!	0,0227	Imperceptible
41	163,9	41,52729	167,815	Verifies!	0,0244	Imperceptible	144,283	Verifies!	0,0232	Imperceptible
42	167,6	41,64181	171,549	Verifies!	0,0250	Imperceptible	147,125	Verifies!	0,0237	Imperceptible
43	171,3	41,75383	175,205	Verifies!	0,0255	Imperceptible	150,147	Verifies!	0,0242	Imperceptible
44	175	41,86346	178,779	Verifies!	0,0261	Imperceptible	153,174	Verifies!	0,0247	Imperceptible
45	178,7	41,97079	182,257	Verifies!	0,0266	Imperceptible	156,108	Verifies!	0,0252	Imperceptible
46	182,4	42,07592	185,309	Verifies!	0,0271	Imperceptible	158,788	Verifies!	0,0257	Imperceptible
47	189,9	42,28264	191,705	Verifies!	0,0281	Imperceptible	164,23	Verifies!	0,0266	Imperceptible
H/500 [mm]	379,8		d<H/500 Verifies!	$v_{crit,x}$	91,36		d<H/500 Verifies!	$v_{crit,x}$	98,67494	

			Distributed 2				Braced			
Floor	Height [m]	$v_m(z)$	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception	Displac. [mm]	$v_{crit,x} > 1{,}25 \cdot v_m(z)$	Peak acceleration [m/s2]	Perception
1	4,5	23,08402	0,667	Verifies!	0,0001	Imperceptible	0,571	Verifies!	0,0001	Imperceptible
2	8,2	26,16231	1,722	Verifies!	0,0002	Imperceptible	1,41	Verifies!	0,0002	Imperceptible
3	11,9	28,07275	3,02	Verifies!	0,0004	Imperceptible	2,512	Verifies!	0,0004	Imperceptible
4	15,6	29,46161	4,566	Verifies!	0,0007	Imperceptible	3,92	Verifies!	0,0006	Imperceptible
5	19,3	30,55345	6,532	Verifies!	0,0010	Imperceptible	5,462	Verifies!	0,0009	Imperceptible
6	23	31,45319	8,758	Verifies!	0,0013	Imperceptible	7,219	Verifies!	0,0012	Imperceptible
7	26,7	32,21843	11,046	Verifies!	0,0016	Imperceptible	9,143	Verifies!	0,0015	Imperceptible
8	30,4	32,8842	13,643	Verifies!	0,0020	Imperceptible	11,205	Verifies!	0,0019	Imperceptible
9	34,1	33,4734	16,307	Verifies!	0,0025	Imperceptible	13,404	Verifies!	0,0023	Imperceptible
10	37,8	34,00185	19,28	Verifies!	0,0029	Imperceptible	15,712	Verifies!	0,0027	Imperceptible
11	45,3	34,93037	25,908	Verifies!	0,0040	Imperceptible	20,721	Verifies!	0,0036	Imperceptible
12	49	35,33314	28,775	Verifies!	0,0044	Imperceptible	23,268	Verifies!	0,0041	Imperceptible
13	52,7	35,70658	32,424	Verifies!	0,0051	Imperceptible	25,878	Verifies!	0,0046	Imperceptible
14	56,4	36,05467	36,146	Verifies!	0,0057	Imperceptible	28,538	Verifies!	0,0051	Imperceptible
15	60,1	36,38064	39,563	Verifies!	0,0062	Imperceptible	31,225	Verifies!	0,0056	Imperceptible
16	63,8	36,68712	43,069	Verifies!	0,0068	Imperceptible	33,949	Verifies!	0,0062	Imperceptible
17	67,5	36,97632	46,722	Verifies!	0,0075	Imperceptible	36,703	Verifies!	0,0067	Imperceptible
18	71,2	37,25008	50,502	Verifies!	0,0081	Imperceptible	39,482	Verifies!	0,0072	Imperceptible
19	74,9	37,50998	53,995	Verifies!	0,0087	Imperceptible	42,29	Verifies!	0,0077	Imperceptible
20	78,6	37,75733	57,847	Verifies!	0,0094	Imperceptible	45,112	Verifies!	0,0083	Imperceptible
21	82,3	37,99331	61,52	Verifies!	0,0100	Imperceptible	47,944	Verifies!	0,0089	Imperceptible
22	86	38,21891	65,361	Verifies!	0,0106	Imperceptible	50,766	Verifies!	0,0094	Imperceptible
23	93,5	38,64785	73,466	Verifies!	0,0122	Imperceptible	56,49	Verifies!	0,0106	Imperceptible
24	97,2	38,84694	76,978	Verifies!	0,0127	Imperceptible	59,269	Verifies!	0,0112	Imperceptible
25	100,9	39,03859	80,826	Verifies!	0,0134	Imperceptible	62,061	Verifies!	0,0117	Imperceptible
26	104,6	39,22334	84,661	Verifies!	0,0142	Imperceptible	64,716	Verifies!	0,0123	Imperceptible
27	108,3	39,40167	88,323	Verifies!	0,0148	Imperceptible	67,386	Verifies!	0,0128	Imperceptible
28	112	39,57401	91,97	Verifies!	0,0154	Imperceptible	70,024	Verifies!	0,0134	Imperceptible
29	115,7	39,74074	95,577	Verifies!	0,0161	Imperceptible	72,624	Verifies!	0,0139	Imperceptible
30	119,4	39,90222	99,134	Verifies!	0,0168	Imperceptible	75,185	Verifies!	0,0144	Imperceptible
31	123,1	40,05878	102,52	Verifies!	0,0174	Imperceptible	77,711	Verifies!	0,0150	Imperceptible
32	126,8	40,2107	106,059	Verifies!	0,0181	Imperceptible	80,218	Verifies!	0,0155	Imperceptible
33	130,5	40,35825	109,391	Verifies!	0,0187	Imperceptible	82,718	Verifies!	0,0160	Imperceptible
34	134,2	40,50168	112,628	Verifies!	0,0193	Imperceptible	85,093	Verifies!	0,0165	Imperceptible
35	141,7	40,78065	119,733	Verifies!	0,0207	Imperceptible	89,888	Verifies!	0,0175	Imperceptible
36	145,4	40,91288	122,602	Verifies!	0,0212	Imperceptible	92,182	Verifies!	0,0180	Imperceptible
37	149,1	41,04179	125,985	Verifies!	0,0219	Imperceptible	94,42	Verifies!	0,0185	Imperceptible
38	152,8	41,16754	129,286	Verifies!	0,0225	Imperceptible	96,611	Verifies!	0,0190	Imperceptible
39	156,5	41,29028	132,228	Verifies!	0,0230	Imperceptible	98,762	Verifies!	0,0194	Imperceptible
40	160,2	41,41016	135,107	Verifies!	0,0236	Imperceptible	100,93	Verifies!	0,0199	Imperceptible
41	163,9	41,52729	138,091	Verifies!	0,0242	Imperceptible	102,959	Verifies!	0,0203	Imperceptible
42	167,6	41,64181	140,919	Verifies!	0,0247	Imperceptible	104,963	Verifies!	0,0207	Imperceptible
43	171,3	41,75383	143,753	Verifies!	0,0252	Imperceptible	106,935	Verifies!	0,0211	Imperceptible
44	175	41,86346	146,55	Verifies!	0,0258	Imperceptible	108,886	Verifies!	0,0215	Imperceptible
45	178,7	41,97079	149,263	Verifies!	0,0263	Imperceptible	110,853	Verifies!	0,0220	Imperceptible
46	182,4	42,07592	152,034	Verifies!	0,0268	Imperceptible	112,716	Verifies!	0,0224	Imperceptible
47	189,9	42,28264	157,812	Verifies!	0,0280	Imperceptible	116,631	Verifies!	0,0232	Imperceptible
H/500 [mm]	379,8		d<H/500 Verifies!	$v_{crit,x}$	97,49304		d<H/500 Verifies!	$v_{crit,x}$	116,9005	