Power System Analysis by Direct Three-Phase Representation

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Electrical and Computer Engineering

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Declaration

I declare that this document is an original work of my own authorship and that it fulfils all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Acknowledgments

This thesis is dedicated to the memory of my father, António Patrício Rodrigues Correia Gomes.

I want to thank my supervisors Professor Luís Marcelino Ferreira and Professor Célia Maria Santos Cardoso de Jesus for all the assistance in carrying out this dissertation.

I also want to thank my family for all the encouragement and support, especially in the most difficult times during this process.

Finally, I want to thank to all my friends for all the camaraderie during this course.

Abstract

A direct three-phase representation in power system analysis is advantageous when dealing with unbalanced networks under complex fault situations. The symmetrical components method, which considers that under certain conditions an unbalanced system may be represented as a superposition of three balanced systems, is a satisfactory solution for simple cases but has several limitations regarding complex fault analysis. Conversely, a direct three-phase approach can handle all kinds of fault situations, regardless of the additional computational complexity. Furthermore, with nowadays computational means, and considering the necessity of a more meticulous and complete analysis, the obvious choice is to handle all fault calculations directly in the three-phase domain.

The work developed in this dissertation consists in a direct approach to fault calculations in the phase domain supported by computational tool MATLAB. First, the three-phase admittance matrix of the system is built directly, supported by the well-known three-phase models regarding each component. Then, according to the type of fault and considering its boundary conditions, the subsequent calculations are performed directly in the phase domain. This three-phase approach in fault calculations is a substantial improvement regarding power system analysis specially when dealing with complex networks and large number of buses. The results obtained in this work demonstrate that a direct three-phase method is the proper approach in fault analysis. This method, when supported by computational tools, can solve with less effort and more accurate results all the straightforward situations solvable by the symmetrical components method and countless ones impossible to handle by the traditional approach.

Keywords

Direct Three-phase Representation; Fault Analysis; Fault Calculations in the Phase Domain; Power System Analysis; Symmetrical Components Method; Three-phase Admittance Matrix; Three-phase Models; Unbalanced Networks.

Resumo

Uma representação trifásica direta na análise de sistemas de energia é vantajosa ao lidar com redes desequilibradas em situações de defeito complexas. O método tradicional, que considera que sob certas condições um sistema desequilibrado pode ser representado como uma sobreposição de três sistemas equilibrados, é uma solução satisfatória para casos simples, porém acarreta várias limitações em relação a situações de maior complexidade. No entanto, uma abordagem trifásica direta consegue lidar com todos os tipos de defeito, acrescendo uma elevada complexidade computacional. Além disso, com os atuais meios computacionais, e considerando a necessidade de uma análise mais rigorosa e completa, a escolha óbvia é tratar todas as situações de defeito diretamente no domínio trifásico.

O trabalho desenvolvido nesta dissertação consiste numa abordagem direta aos cálculos relacionados com a análise de defeitos no domínio trifásico com recurso ao MATLAB. Inicialmente, a matriz de admitância trifásica do sistema é construída recorrendo aos modelos trifásicos de cada componente. Em seguida, e de acordo com o tipo de defeito, os cálculos que se impõem são realizados diretamente no domínio trifásico. Esta abordagem no cálculo de defeitos é uma ferramenta essencial quando se lida com sistemas mais complexos. Os resultados obtidos neste trabalho demonstram que uma abordagem trifásica direta é mais adequada no estudo de defeitos. Este método, quando suportado por uma ferramenta computacional, consegue lidar com pouco esforço e resultados mais precisos todas as situações simples que podem ser resolvidas pelo método das componentes simétricas e ainda inúmeras situações impossíveis de resolver pela abordagem tradicional.

Palavras-Chave

Análise de Defeitos; Análise de Defeitos no Domínio Trifásico; Análise de Sistemas de Energia; Redes Desequilibradas; Matriz de Admitância Trifásica; Método das Componentes Simétricas; Modelos Trifásicos; Representação Trifásica Direta.

Contents

List of Figures

List of Tables

Acronyms

$\mathbf{3\phi}$ – Three-phase

LG – Line-to-Ground

LL – Line-to-Line

LLG – Double Line-to-Ground

OCO – One-Conductor Open

TCO – Two-Conductor Open

Y – Wye Transformer Configuration

YG – Wye-Ground Transformer Configuration

Δ – Delta Transformer Configuration

1. Introduction

1.1. Motivation

When one analyses a certain power system, that is supposed to keep continuous its power supply through all buses, we must consider that several events may occur and disturb the network. These events may have different natures, such as physical accidents, wind, lightnings, equipment failures, and so on. The main effect of this unpredictable happenings is a short circuit fault caused by a lightning, and in this case, one knows it usually is a temporary fault. A short circuit fault happens when one phase wire of the transmission line touches the ground or when two phase wires touch each other. Also, when a conductor opens, we have an open conductor fault. Open conductor faults are series faults and short circuit faults are parallel faults. Although both types of faults have different probabilities of occurrence, it is important to consider both in order to be able to perform a complete fault analysis in any power system.

The main purpose of this work is to demonstrate that the traditional method used for fault analysis, which is the symmetrical components method, is not appropriate to solve all the situations that one may find. On the other hand, the direct three-phase representation method will solve all the problems that the traditional method already solves, but we can go further and solve situations that without this method we would not be able to.

In order to understand the traditional method, one must consider that it depends on turning the three phasors of the system into a new kind of components, which are the positive, negative, and zero sequences. The main advantage of this kind of analysis is that after this transformation we can analyse each of the sequences independently, which simplifies all the calculations. The downside is that when there is mutual coupling between transmission lines, and the short circuit fault occurs in one of these lines, it will have common elements in the power system. In order to include the coupled elements in our calculations, we will no longer have the easy to solve diagonal matrices that the symmetrical components method provides, but we will have more complicated matrices to handle instead. In such situation, the direct three-phase representation method, that can be slightly more complicated to apply in simple situations, will take as much effort to solve this kind of problems as it would take in some easier to solve scenario for the traditional method.

Through the years, the main obstacle regarding the three-phase system analysis was the fact for large systems the calculations were too much to handle for a computer in the past. For instances, for a system with n buses, the sequence method will generate three $[n \times n]$ matrices for the fault analysis, while the three-phase method would have to handle a $[3n \times 3n]$ matrix. Nowadays, with the increasing computational capacity of our computers, we can perform all these calculations without much effort, even for complicated situations. Those situations, that were too difficult to handle with the traditional method, are now easily solvable by applying the direct three-phase representation method to the faulted power system.

1.2. Objectives

Considering all the situations described above, and regarding the existing work advances already made in this area of power system analysis under fault conditions, the main purpose of this dissertation is to directly apply the three-phase method to the whole system under analysis and test its results against traditional results obtained through the symmetrical components method. Also, these two methods will be tested for all transformer configurations, and regarding all types of faults and line openings, in order two verify the accuracy of the results obtained without symmetrical assumptions by the direct three-phase representation method.

1.3. Organization of the Document

This dissertation is organized in six chapters, being all easily displayed according to the most relevant concepts regarding this dissertation topic. In a first approach to this document, one can say that the content chapters are displayed in a way that makes the reader easily understand all the relevant concepts on this matter and how they relate between themselves.

This first chapter, as the title suggests, is an introduction on the subject under analysis, briefly supported by some general concepts that will be approached below in this document in order to pursuit the objectives previously defined for this dissertation.

Chapter 2 is about the symmetrical components concept, which is a very important tool in power system analysis, and specially in fault analysis. This chapter, despite its theoretical and dense appearance, is the foundation for all the concepts that follow in the next chapter.

Chapter 3 displays a step-by-step method regarding unsymmetrical fault calculations, with all relevant types of faults explained, in order to make an easier comparison between the traditional method and the direct three-phase representation. It also includes an example and a step-by-step solution according to all the steps mentioned before, in order to achieve both theoretical and practice understanding on the matter.

Chapter 4 is all about the main topic of this dissertation, the direct three-phase representation method for fault analysis regarding power systems. It has a very similar organization to the one presented in the chapter above. However, since it is a less known method, it is presented with more detail and theoretical support. It includes all the three-phase models regarding each power system component explained, that allows us to represent the whole system, and explains how to approach all the relevant types of faults.

In chapter 5 one can compare the experimental results between the two methods considered in this dissertation regarding all the relevant types of faults and considering different transformer connections. This chapter includes a step-by-step solution, using the direct three-phase representation method, to solve the same problem considered in chapter 3 for an easy comparison between them. It also includes experimental results for different data, regarding both methods, for the other types of faults described in this dissertation. These results were obtained using the MATLAB code files annexed in this document.

Finally, the last chapter of this dissertation talks about all the work done and the results presented. All in all, chapter 6 is a discussion about the objectives of this work and how the results obtained may confirm the initial theory behind this direct approach for fault analysis as well as a reflexion about future achievements regarding this subject.

2. Symmetrical Components

The symmetrical components method is currently a very important tool in the analysis of three-phase power systems when these are unbalanced. It can also be very useful to perform calculations regarding unsymmetrical fault currents [2].

When a fault occurs in a three-phase power system it unbalances the system. If it occurs in an initial balanced power system, the analysis becomes easier. It is only needed to perform the calculations for one of the three phases because the other two are just phase displaced. However, for an unbalanced system this single-phase approach is not valid. In these cases, which are most common, the symmetrical components method is a good approach to perform our analysis. This method converts the unbalanced three-phase currents and voltages of the system into three sets of balanced ones. These three new balanced systems, the so-called symmetrical components, are represented by a positive, negative, and zero sequence networks. This method also allows us to decouple the impedances of the system from each other, which simplifies all the calculations.

The generators of a symmetric three-phase balanced system produce balanced voltages with its phases displaced by $2\pi/3 = 120°$ from each other, as shown in Fig. 2.1.

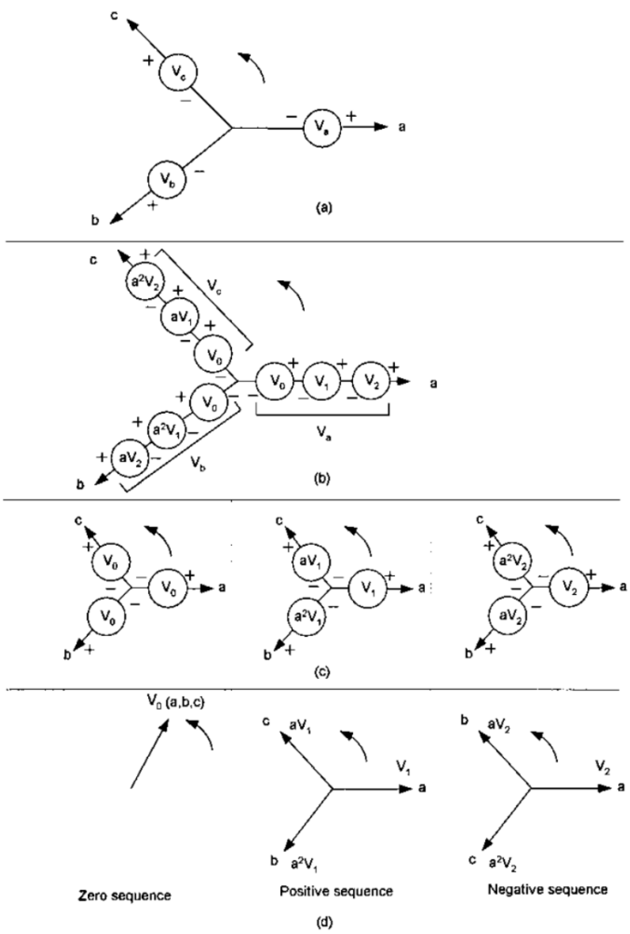

Figure 2.1: (a), (b), (c), and (d) Progressive resolution of voltage vectors into sequence components [2].

For a three-phase system, the relation between each phase component of the voltage and its sequence components is given by

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \tag{2.1}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

where $V_{a0}, V_{b0},$ and $V_{c0},$ are the zero sequence voltages for each phase, $V_{a1}, V_{b1},$ and $V_{c1},$ are the positive sequence voltages, and $V_{a2}, V_{b2},$ and $V_{c2},$ are the negative sequence voltages. Considering each positive sequence voltage as the reference vector, the relation between the nine voltage vectors obtained from the three original unbalanced ones, is given by

$$V_{a0} = V_{b0} = V_{c0}$$
$$V_{b1} = a^2 V_{a1}, \qquad V_{c1} = aV_{a1} \tag{2.2}$$
$$V_{b2} = aV_{a2}, \qquad V_{c2} = a^2 V_{a2}$$

These relations can be rewritten in a matrix form, as follows

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{2.3}$$

Which is equivalent to the following compact expression

$$V^{abc} = T\, V^{012} \tag{2.4}$$

where T represents the Fortscue's matrix. Notice that the reverse transformation can also be applied by inverting the transformation matrix.

2.1. Decoupling a Three-Phase Symmetrical System

In order to decouple a three-phase transmission line section, one must consider that each phase has a mutual coupling with respect to ground, which is illustrated in Fig. 2.2 (a). The impedance matrix of the three-phase transmission line is shown below

$$Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{2.5}$$

where Z_{aa}, Z_{bb}, and Z_{cc} represent the self-impedances of the phases $a, b,$ and c, and the mutual impedances between two phases are represented by Z_{ab}, Z_{ba}, Z_{ac}, Z_{ca}, Z_{bc}, and Z_{cb}.

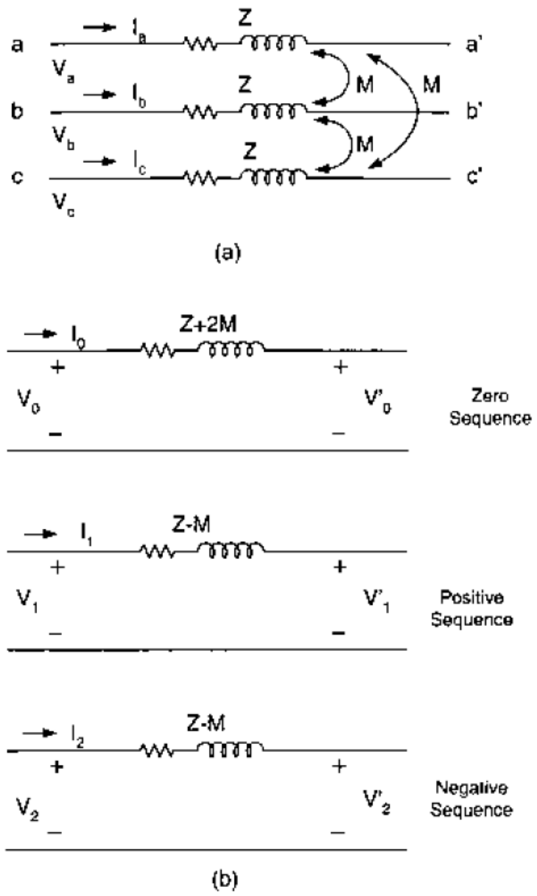

Figure 2.2: (a) Impedances in a three-phase transmission line with mutual coupling between phases; (b) Resolution into symmetrical component impedances [2].

Consider that all the mutual impedances are equal to M, and all the self-impedances equal to Z, which means that the line is being considered as perfectly symmetrical, i.e., $Z_{ab} = Z_{ba} = Z_{ac} = Z_{ca} = Z_{bc} = Z_{cb} = M$, and $Z_{aa} = Z_{bb} = Z_{cc} = Z$. The impedance matrix can be rewritten as

$$Z_{abc} = \begin{bmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{bmatrix} \tag{2.6}$$

A symmetrical component transformation matrix, and its inverse, can be written as follows

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{2.7}$$

$$T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{2.8}$$

For the transformation of impedance, one must derive its expression. Let us start by writing the expressions for the transformation of currents and voltages, as follows

$$I_{abc} = TI_{012} \tag{2.9}$$

$$V_{abc} = TV_{012} \tag{2.10}$$

The reverse transformations, regarding both sequence currents and voltages, are given by

$$I_{012} = T^{-1}I_{abc} \tag{2.11}$$

$$V_{012} = T^{-1}V_{abc} \tag{2.12}$$

Thus, let us derive the expression for the transformation of impedance, as follows

$$\begin{gathered} V_{abc} = Z_{abc}I_{abc} \\ TV_{012} = Z_{abc}TI_{012} \\ V_{012} = T^{-1}Z_{abc}TI_{012} = Z_{012}I_{012} \end{gathered} \tag{2.13}$$

The transformation of impedance, and its inverse, are given by

$$Z_{012} = T^{-1}Z_{abc}T \tag{2.14}$$

$$Z_{abc} = TZ_{012}T^{-1} \tag{2.15}$$

Therefore, the sequence impedance matrix, by means of this transformation, can be computed as follows

$$Z_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = \begin{bmatrix} Z+2M & 0 & 0 \\ 0 & Z-M & 0 \\ 0 & 0 & Z-M \end{bmatrix} \tag{2.16}$$

The sequence impedance matrix, with respect to the initial three-phase coupled system, is diagonal with all its off-diagonal terms being equal to zero. This means that one can transform a three-phase coupled system, through the symmetrical components transformation, into a system with no coupling between its positive, negative, and zero sequence components. The decoupled sequence networks are shown in Fig. 2.2 (b).

2.2. Decoupling a Three-Phase Unsymmetrical System

Let us consider the same three-phase system as before, but now it is unbalanced. Therefore, assume that the phase self-impedances are $Z_1 \neq Z_2 \neq Z_3$, and ignore the mutual impedances. The three-phase impedance matrix is shown below

$$Z_{abc} = \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{bmatrix} \tag{2.17}$$

The symmetrical components transformation, applied to this matrix, is given by

$$Z_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$
$$= \frac{1}{3}\begin{bmatrix} Z_1 + Z_2 + Z_3 & Z_1 + a^2 Z_2 + aZ_3 & Z_1 + aZ_2 + aZ_3 \\ Z_1 + aZ_2 + aZ_3 & Z_1 + Z_2 + Z_3 & Z_1 + a^2 Z_2 + aZ_3 \\ Z_1 + a^2 Z_2 + aZ_3 & Z_1 + aZ_2 + aZ_3 & Z_1 + Z_2 + Z_3 \end{bmatrix} \quad (2.18)$$

In this case, as we can see by means of the transformation performed above, the initial unbalanced system is not decoupled. Since power systems are not perfectly balanced, and due to complexity in computations for the symmetrical components method, the asymmetry of the system can be ignored. The error introduced by means of this approach usually is small, however, in highly unbalanced systems this may not be true.

2.3. Power Invariance in Symmetrical Components Transformation

To show that symmetrical component transformation is power invariant, let us look at the complex power expression for a three-phase circuit, which is shown below

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^* = V'_{abc} I^*_{abc} \quad (2.19)$$

where I_a^* represents the complex conjugate of I_a. Then, the expression above can be rewritten as follows

$$S = [TV_{012}]T^* I^*_{012} = V'_{012} T' T^* I^*_{012} \quad (2.20)$$

The product TT^* is given by

$$TT^* = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.21)$$

Therefore, one can compute the complex power through symmetrical components method, as shown below

$$S = 3V_0 I_0^* + 3V_1 I_1^* + 3V_2 I_2^* \quad (2.22)$$

3. Unsymmetrical Fault Calculations

In this chapter each type of fault is going to be analysed through the sequence method. This approach depends on the construction of three distinct sequence networks seen from the faulted point [2]. The first step of this procedure consists in the reduction of the zero, positive, and negative sequence networks into a single Thèvenin sequence impedance. This approach relies on the fact that only the positive sequence network has a voltage source, that corresponds to the pre-fault voltage, being the only active network. In the following sections it is explained how one can manage these three separate networks, in order to perform unsymmetrical fault calculations, by connecting them in a certain way. This procedure depends on the type of fault.

3.1. Sequence Admittance/Impedance Matrices

Considering the three sequence networks, and neglecting the reference node, which is always at ground potential, one can apply the following equation to build each admittance matrix Y

$$I = YV \tag{3.1}$$

where V is the node voltage vector and I represents the node injected current vector. It is most common to define each current flow as positive when it goes toward the bus, and as negative when flows away from the bus. Also, the node voltage vector V represents the bus voltages measured from the reference node. Finally, Y is the bus admittance matrix. Rewriting Eq. (3.1) in its matrixial form, one gets the following expression

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{3.2}$$

Notice that Y is a square matrix of dimensions $n \times n$, where n represents the number of buses in the system, without the reference bus. Its inverse matrix will be

$$Z = Y^{-1} \tag{3.3}$$

where Z represents the bus impedance matrix, which can be formed simply by inverting the admittance matrix. This matrix is also square and of dimensions $n \times n$. Therefore, Eq. (3.1) can also be written as follows

$$V = ZI \tag{3.4}$$

3.2. Fault Current

In order to do fault calculations, one must connect the faulted bus k to the ground through the fault impedance Z_f. Considering the Eq. (3.4), regardless the fault current, all the remaining node currents will be zero. We can write two equations regarding the faulted bus k,

$$I_k = \frac{V_k}{Z_{kk}} = -I_{f_k} \tag{3.5}$$

$$V_k = Z_f I_{f_k} - V_a \tag{3.6}$$

where V_a represents the Thevenin voltage of bus k before the fault.

The fault current on Eq. (3.5) has the negative sign because it goes in direction do the ground, while the injected current goes towards the node.

From Eq. (3.5) and (3.6), and considering $Z_{Th} = Z_{kk} + Z_f$, the fault current expression is given by

$$I_{f_k} = \frac{V_a}{Z_{Th}} \tag{3.7}$$

3.2.1. Three-Phase Fault

For a three-phase fault, all phases are short-circuited through equal fault impedances in series with a ground impedance, Z_f and Z_g, respectively, where F is the faulted point, as represented in Fig. 3.1. Also, the sequence networks interconnection diagram is shown in Fig. 3.2.

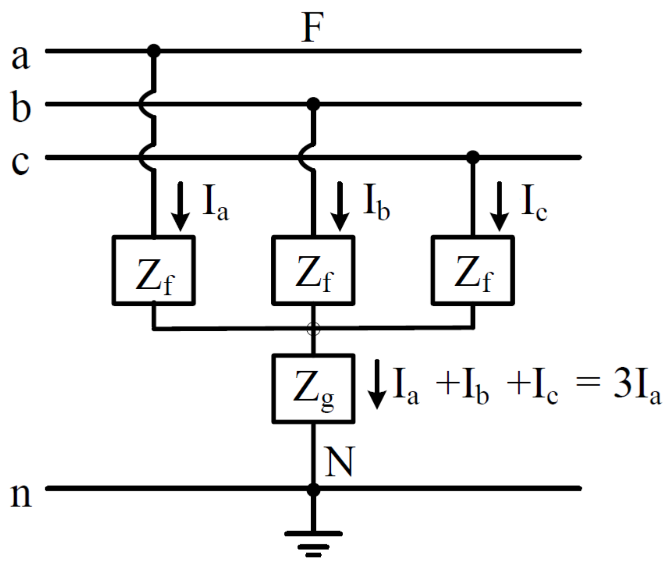

Figure 3.1: General representation of a balanced three-phase fault [13].

This is a symmetrical fault, meaning that the vectorial sum of fault currents is three times the current in each phase.

$$I_a + I_b + I_c = 3I_a \tag{3.8}$$

Since a three-phase fault is symmetrical,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_f + Z_g & Z_g & Z_g \\ Z_g & Z_f + Z_g & Z_g \\ Z_g & Z_g & Z_f + Z_g \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{3.9}$$

Then, the sequence voltages are given by

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = T^{-1} \begin{bmatrix} Z_f + Z_g & Z_g & Z_g \\ Z_g & Z_f + Z_g & Z_g \\ Z_g & Z_g & Z_f + Z_g \end{bmatrix} T \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_f + 3Z_g & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{3.10}$$

Therefore, the fault current can be calculated by

$$I_a = I_1 = \frac{V_a}{Z_1 + Z_f}$$

$$I_b = a^2 I_1 \tag{3.11}$$

$$I_c = aI_1$$

where k represents the faulted bus, and $Z_{Th} = Z_1 + Z_f$.

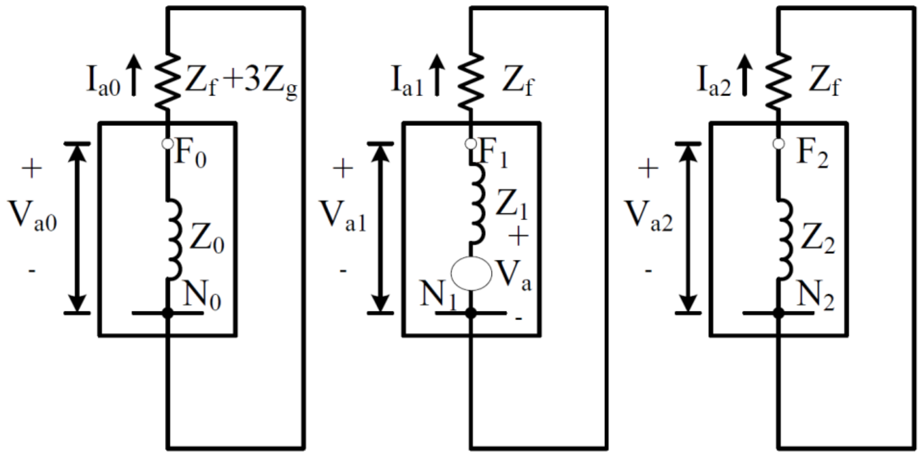

Figure 3.2: Sequence network diagram of a balanced three-phase fault [13].

3.2.2. Line-To-Ground Fault

Let us assume that the fault occurs in phase a, as shown in Fig. 3.3. Since the load current is neglected, the currents for both phases b and c are zero.

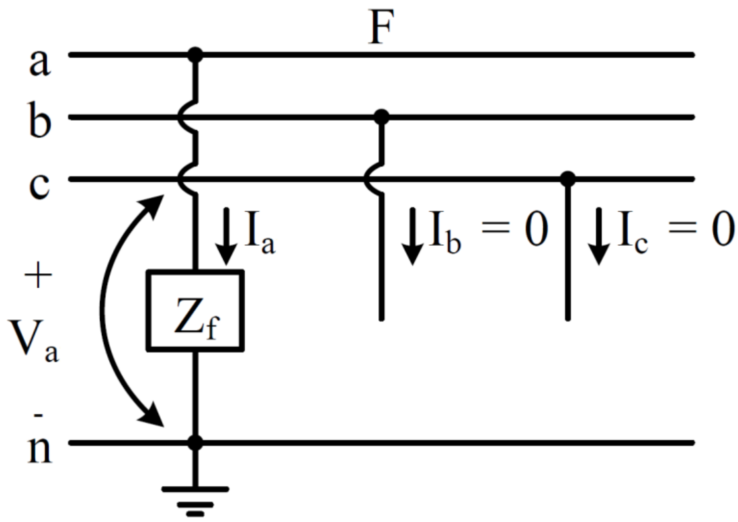

Figure 3.3: General representation of a line-to-ground fault [13].

The following expression gives us the voltage at the fault point

$$V_a = Z_f I_a \tag{3.12}$$

The sequence components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix} \tag{3.13}$$

From (3.6), comes the following

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \tag{3.14}$$

$$3I_{a0}Z_f = V_{a0} + V_{a1} + V_{a2} = -I_{a0}Z_0 + (V_a - I_{a1}Z_1) - I_{a2}Z_2 \tag{3.15}$$

The fault current can now be calculated by

$$I_{a0} = I_{a1} = I_{a2} = \frac{V_a}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{3.16}$$

and $Z_{Th} = Z_0 + Z_1 + Z_2 + 3Z_f$.

And the total fault current is given by

$$I_a = 3I_{a0} = \frac{3V_a}{Z_{Th}} \tag{3.17}$$

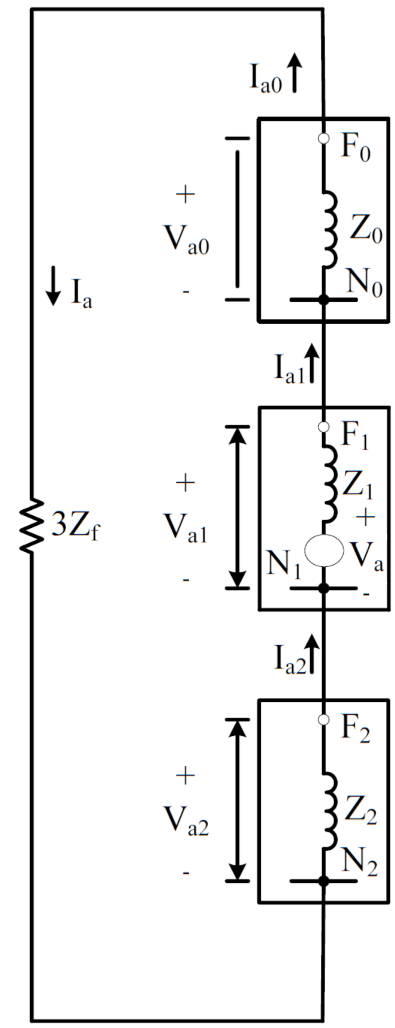

Figure 3.4: Sequence network diagram of a line-to-ground fault [13].

3.2.3. Line-To-Line Fault

For a line-to-line fault, let us assume that the fault occurs between phases b and c, through a fault impedance Z_f, as shown in Fig. 3.5. Therefore, the fault current only circulates through the faulted phases, from phase b to phase c.

$$I_a = 0$$

$$I_b = -I_c \tag{3.18}$$

$$V_b - V_c = Z_f I_b$$

The sequence components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ -a + a^2 \\ -a^2 + a \end{bmatrix} \tag{3.19}$$

From Eq. (3.19), $I_{a0} = 0$ and $I_{a1} = -I_{a2}$.

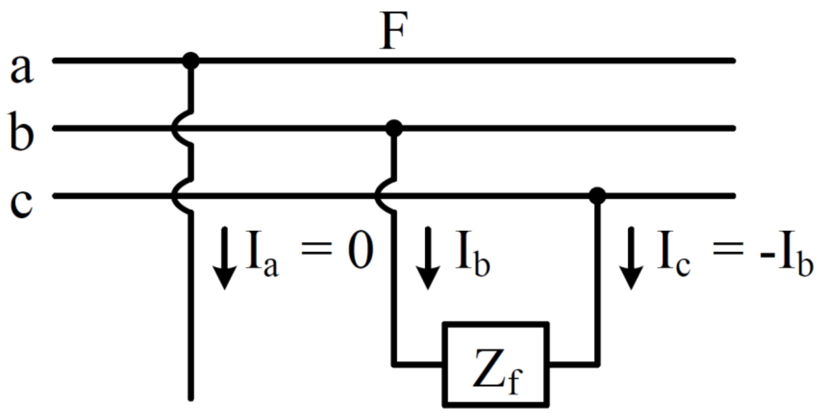

Figure 3.5: General representation of a line-to-line fault [13].

$$V_b - V_c = [0 \quad 1 \quad -1]\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [0 \quad 1 \quad -1]\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$
$$= [0 \quad a^2 - a \quad a - a^2]\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{3.20}$$

Thus,

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_{a1} - V_{a2}) \\ &= (a^2 I_{a1} + a I_{a2}) Z_f \\ &= (a^2 - a) I_{a1} Z_f \end{aligned} \tag{3.21}$$

Which gives

$$(V_{a1} - V_{a2}) = I_{a1} Z_f \tag{3.22}$$

As represented in the equivalent circuit from Fig. 3.6.

Also

$$I_b = (a^2 - a) I_{a1} = -j\sqrt{3} I_{a1} \tag{3.23}$$

The sequence fault current can now be calculated by

$$I_{a1} = -I_{a2} = \frac{V_a}{Z_1 + Z_2 + Z_f} \tag{3.24}$$

and $Z_{Th} = Z_1 + Z_2 + Z_f$.

The fault current is given by

$$I_b = -I_c = -j\sqrt{3}\frac{V_a}{Z_{Th}} \tag{3.25}$$

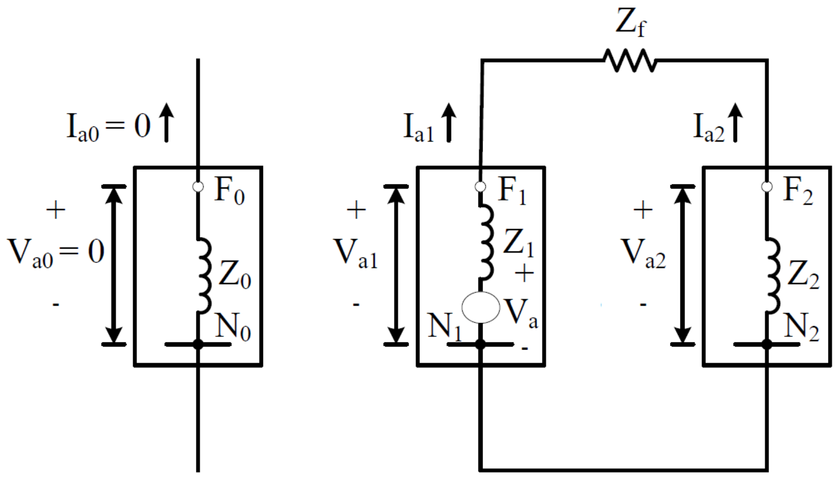

Figure 3.6: Sequence network diagram of a line-to-line fault [13].

3.2.4. Double-Line-To-Ground Fault

For a double line-to-ground fault, let us assume that phases b and c go to ground through two fault impedances Z_f, and a ground impedance Z_g, as shown in Figure 3.7. Therefore, $I_a = 0$, which implies $I_{a0} + I_{a1} + I_{a2} = 0$.

The voltage at the faulted point is given by

$$V_b = \left(Z_f + Z_g\right)I_b + Z_g I_c \tag{3.26}$$

$$V_c = (Z_f + Z_g)I_c + Z_g I_b \tag{3.27}$$

And the sequence components of the voltages can be written as follows

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_a + V_b + V_c \\ V_a + (a + a^2)V_b \\ V_a + +(a + a^2)V_b \end{bmatrix} \tag{3.28}$$

which gives $V_1 = V_2$, and

$$V_0 = \frac{1}{3}(V_a + 2V_b)$$

$$= \frac{1}{3}\left[(V_0 + V_1 + V_2) + 2(I_b + I_c)Z_f\right]$$

$$= \frac{1}{3}\left[(V_0 + 2V_1) + 2(3I_0)Z_f\right]$$

$$= V_1 + 3I_0 Z_f \tag{3.29}$$

This expression gives the equivalent circuit shown in Fig. 3.8.

The fault current can now be calculated by

$$I_{a1} = \frac{V_a}{(Z_1 + Z_f) + [(Z_2 + Z_f)||(Z_0 + Z_f + 3Z_g)]}$$
$$= \frac{V_a}{(Z_1 + Z_f) + \frac{(Z_2 + Z_f)(Z_0 + Z_f + 3Z_g)}{(Z_2 + Z_f) + (Z_0 + Z_f + 3Z_g)}} \quad (3.30)$$

and $Z_{Th} = (Z_1 + Z_f) + \frac{(Z_2+Z_f)(Z_0+Z_f+3Z_g)}{(Z_2+Z_f)+(Z_0+Z_f+3Z_g)}$.

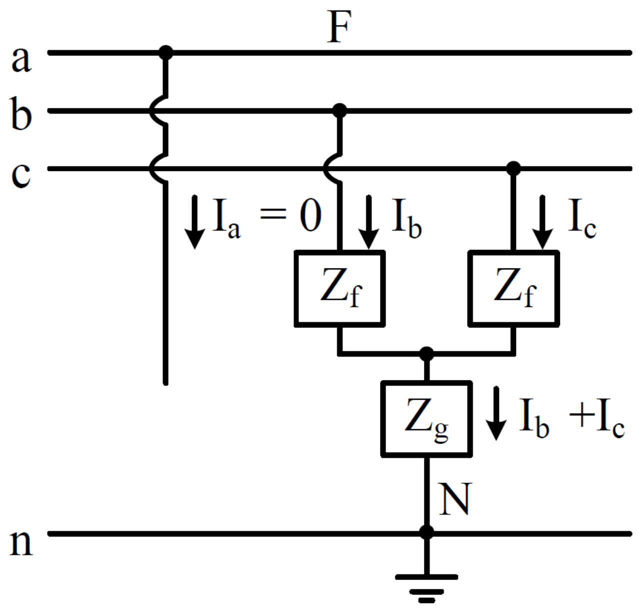

Figure 3.7: General representation of a double line-to-ground fault [13].

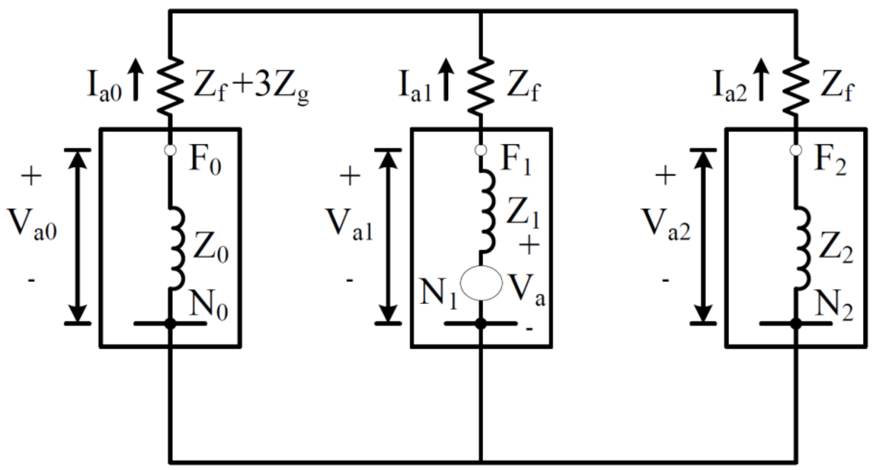

Figure 3.8: Sequence network diagram of a double line-to-ground fault [13].

3.2.5. Open Conductor Faults

Regarding balanced three-phase power systems, when one or two phases of a line opens, as shown in Fig. 3.9, it creates a flow of unbalanced currents, resulting in an unbalanced system. These are called series faults since the faults are in series with the line and may occur due to mechanical damage or by operation of fuses in unsymmetrical faults. In order to study this type of faults, the so-called open conductor faults, the symmetrical components method can also be applied [19].

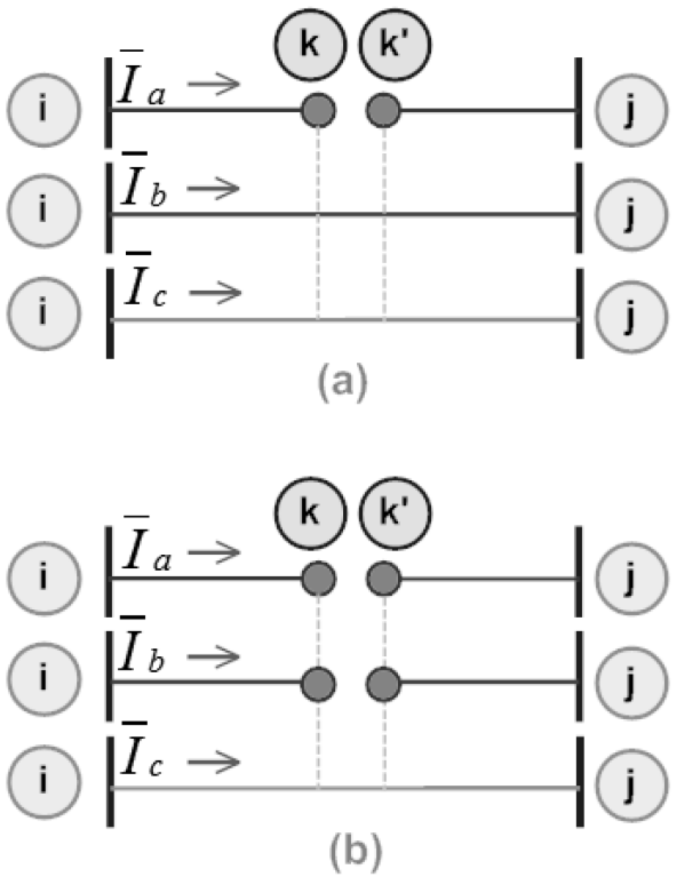

Figure 3.9: (a) One-conductor open series fault; (b) Two-conductors open series fault [19].

Considering that the open conductor fault occurs between buses i and j, the open circuit voltage between these two buses is given by $(V_i^0 - V_j^0)$, as shown in Fig. 3.10.

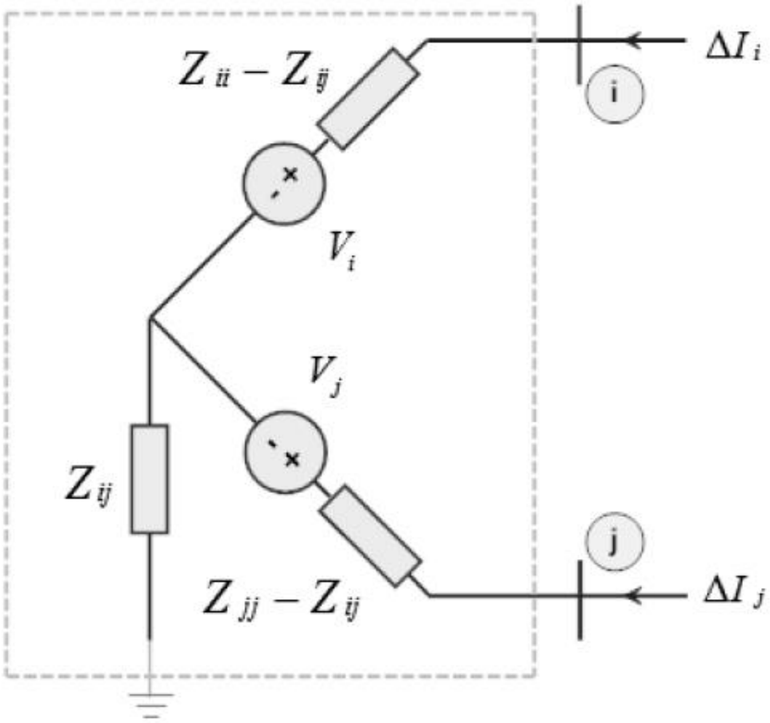

Figure 3.10: Thèvenin's equivalent of the original network [19].

In order to compute the open circuit impedance between the faulted buses, the initial voltages V_i^0 and V_j^0 are set equal to zero and one must connect an ideal current source I between the two buses. Regarding that $\Delta I_i = I$ and $\Delta I_j = -I$, the resulting voltages are given by

$$\bar{V}_i = (\bar{Z}_{ii} - \bar{Z}_{ij})\bar{I} \tag{3.31}$$

$$\bar{V}_j = (\bar{Z}_{jj} - \bar{Z}_{ji})(-\bar{I}) \tag{3.32}$$

From these equations, the voltage difference between buses i and j is computed as follows

$$\Delta\bar{V}_{ij} = \bar{V}_i - \bar{V}_j = (\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij})\bar{I} \tag{3.33}$$

Finally, the open circuit impedance is given by

$$\bar{Z}_{Th,ij} = \frac{\Delta\bar{V}_{ij}}{\bar{I}} = (\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij}) \tag{3.34}$$

Considering x as the fractional length of the broken line from bus i to the break point k, where $0 \le x \le 1$, both positive sequence impedances of the conductor segment, between bus i and the break point k, and from k to the bus j, are given by $xz_{ij}^{(1)}$ and $(1-x)z_{ij}^{(1)}$, respectively. These impedances are considered in the calculations to represent the broken conductor.

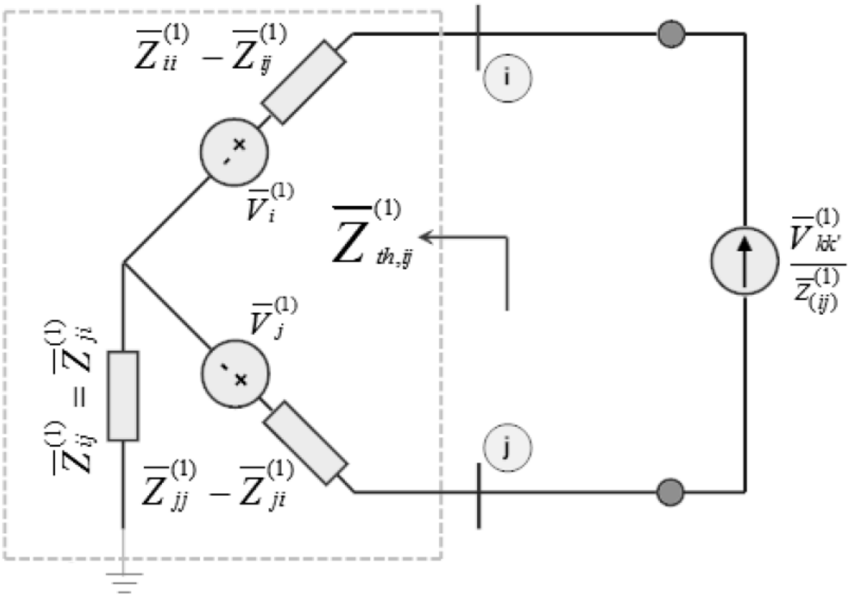

Figure 3.11: Positive sequence Thèvenin's equivalent circuit [19].

After some simplifications in the circuit, the final positive, negative, and zero sequence Thèvenin's equivalent circuit are shown in Fig. 3.11 and Fig. 3.12.

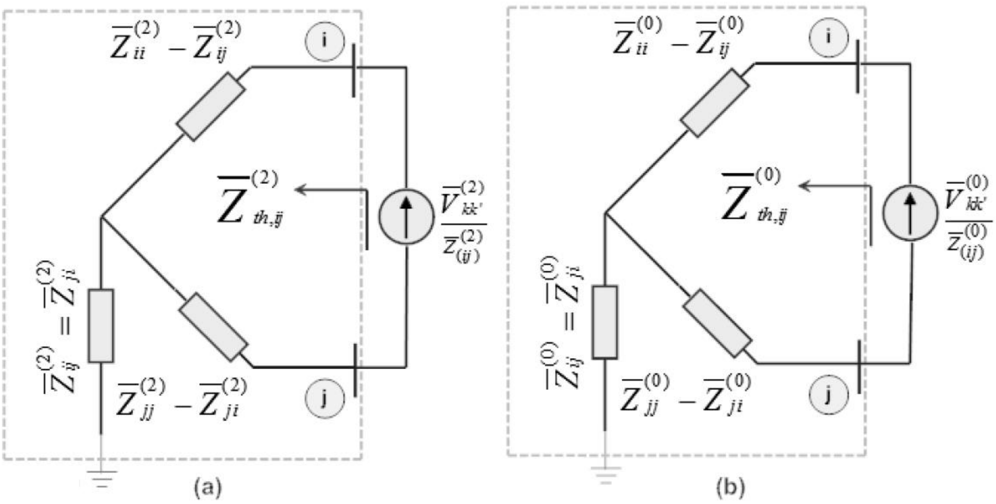

Figure 3.12: (a) Negative sequence; (b) Zero sequence Thèvenin's equivalent circuit [19].

Regarding any bus n, the sequence voltage drops due to the current injections at buses i and j can be computed as follows

$$\Delta\bar{V}_n^{(0)} = \frac{(\bar{Z}_{ni}^{(0)} - \bar{Z}_{nj}^{(0)})}{\bar{z}_{ij}^{(0)}}\bar{V}_{kk'}^{(0)}$$

$$\Delta\bar{V}_n^{(1)} = \frac{\left(\bar{Z}_{ni}^{(1)} - \bar{Z}_{nj}^{(1)}\right)}{\bar{z}_{ij}^{(1)}}\bar{V}_{kk'}^{(1)} \tag{3.35}$$

$$\Delta\bar{V}_n^{(2)} = \frac{(\bar{Z}_{ni}^{(2)} - \bar{Z}_{nj}^{(2)})}{\bar{z}_{ij}^{(2)}}\bar{V}_{kk'}^{(2)}$$

Then, the Thèvenin's equivalent impedances for each sequence network, as seen from the buses k and k', are given by

$$\bar{Z}_{kk'}^{(1)} = x\bar{z}_{ij}^{(1)} + \frac{\bar{Z}_{Th,ij}^{(1)}(-\bar{z}_{ij}^{(1)})}{\bar{Z}_{Th,ij}^{(1)} + (-\bar{z}_{ij}^{(1)})} + (1-x)\bar{z}_{ij}^{(1)}$$

$$\bar{Z}_{kk'}^{(1)} = \frac{-\left(\bar{z}_{ij}^{(1)}\right)^2}{\bar{Z}_{Th,ij}^{(1)} - \bar{z}_{ij}^{(1)}} \tag{3.36}$$

$$\bar{Z}_{kk'}^{(2)} = \frac{-\left(\bar{z}_{ij}^{(2)}\right)^2}{\bar{Z}_{Th,ij}^{(2)} - \bar{z}_{ij}^{(2)}} \tag{3.37}$$

$$\bar{Z}_{kk'}^{(0)} = \frac{-\left(\bar{z}_{ij}^{(0)}\right)^2}{\bar{Z}_{Th,ij}^{(0)} - \bar{z}_{ij}^{(0)}} \tag{3.38}$$

Next, the open-circuit voltage is given by

$$\bar{V}_{Th,kk'}^{(1)} = \frac{-\left(\bar{z}_{ij}^{(1)}\right)^2}{\bar{Z}_{Th,ij}^{(1)} - \bar{z}_{ij}^{(1)}}(V_i^{(1)} - \bar{V}_j^{(1)}) \tag{3.39}$$

and replacing $\frac{\bar{Z}_{kk'}^{(1)}}{\bar{z}_{ij}^{(1)}} = \frac{-\bar{z}_{ij}^{(1)}}{\bar{Z}_{Th,ij}^{(1)} - \bar{z}_{ij}^{(1)}}$ in Eq. (3.39), the final expression for the open-circuit voltage is given by

$$\bar{V}_{Th,kk'}^{(1)} = \frac{\bar{Z}_{kk'}^{(1)}}{\bar{z}_{ij}^{(1)}}\left(V_i^{(1)} - \bar{V}_j^{(1)}\right) \tag{3.40}$$

Before an open-conductor fault occurs, the current flowing from bus i to bus j, in phase a, is the positive sequence component, and is given by

$$\bar{I}_{ij}^{(1)} = \frac{\left(V_i^{(1)} - \bar{V}_j^{(1)}\right)}{\bar{z}_{ij}^{(1)}} \tag{3.41}$$

Also, substituting Eq. (3.41) in Eq. (3.40), the resulting expression is given by

$$\bar{V}_{Th,kk'}^{(1)} = \bar{Z}_{kk'}^{(1)}\bar{I}_{ij}^{(1)} \tag{3.42}$$

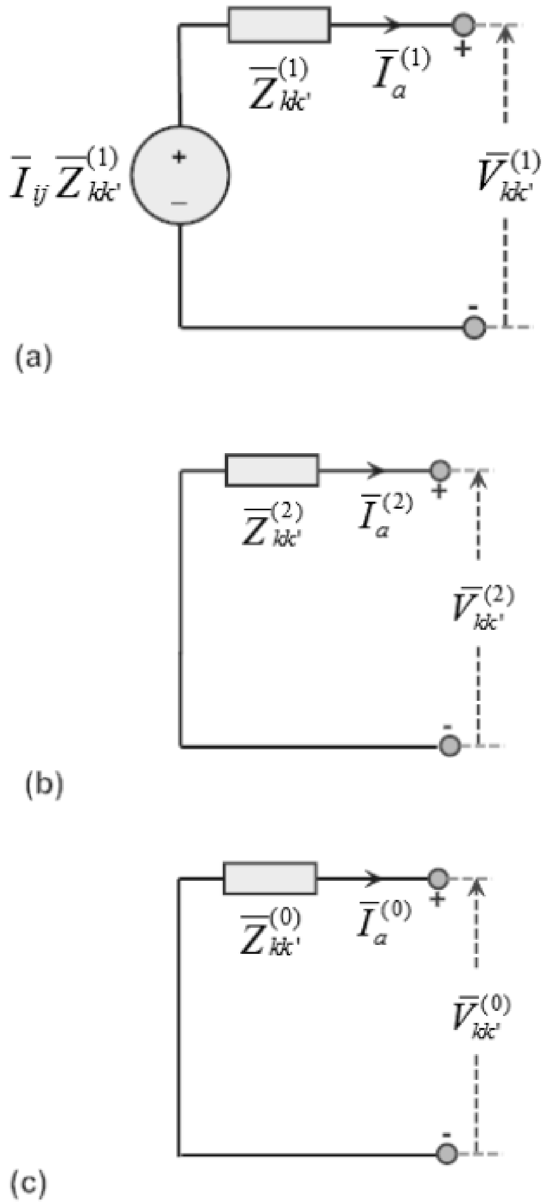

Figure 3.13: (a) Positive sequence; (b) Negative sequence; (c) Zero sequence Thèvenin's equivalent network as seen from k and k' [19].

3.2.5.1. One-Conductor Open Fault

Let us consider that the conductor with respect to phase a is open.

$$\bar{I}_a = 0 \tag{3.31}$$

$$\bar{I}_a^{(0)} + \bar{I}_a^{(1)} + \bar{I}_a^{(2)} = 0 \tag{3.32}$$

$$\bar{V}_{kk',b} = 0, \qquad \bar{V}_{kk',c} = 0 \tag{3.33}$$

Since the voltages across the two unbroken phase conductors are zero and the current on the broken phase conductor is also zero at the point of break.

$$\begin{bmatrix} \bar{V}_a^{(0)} \\ \bar{V}_a^{(1)} \\ \bar{V}_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{V}_{kk',a} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \bar{V}_{kk',a} \\ \bar{V}_{kk',a} \\ \bar{V}_{kk',a} \end{bmatrix} \tag{3.34}$$

$$\bar{V}_a^{(0)} = \bar{V}_a^{(1)} = \bar{V}_a^{(2)} = \frac{1}{3}\bar{V}_{kk',a} \tag{3.35}$$

Considering the equations above, one can conclude that the sequence networks can be connected in parallel, as illustrated in Figure 3.14.

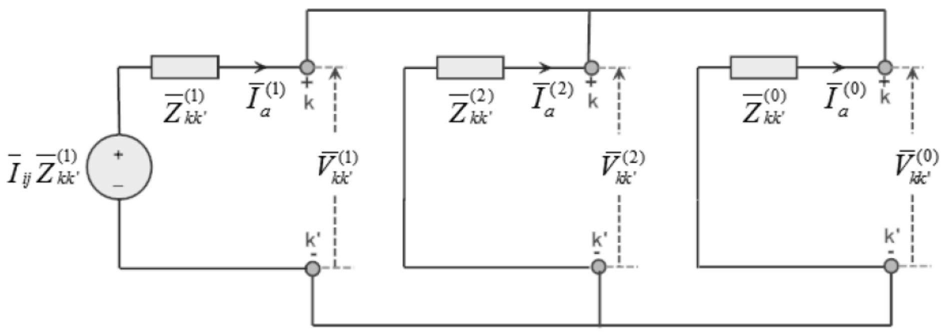

Figure 3.14: Connection of equivalent sequence networks to represent open phase a between k and k' [19].

The positive sequence current, regarding phase a, is given by

$$\bar{I}_a^{(1)} = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}}{\bar{Z}_{kk'}^{(1)} + \dfrac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}}} = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}\left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right)}{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)} + \bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}} \tag{3.36}$$

The sequence voltage drops can be computed as

$$\bar{V}_{kk'}^{(1)} = \bar{I}_a^{(1)}\frac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}} \tag{3.37}$$

Substituting $\bar{I}_a^{(1)}$ from Eq. (3.36), the expression can be simplified as

$$\bar{V}_{kk'}^{(0)} = \bar{V}_{kk'}^{(1)} = \bar{V}_{kk'}^{(2)} = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)} + \bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}} \tag{3.38}$$

where $\bar{I}_{ij}$ is the pre-fault current in phase a between buses i and j.

3.2.5.2. Two-Conductor Open Fault

Now, let us assume that the conductors of phases b and c are open-circuited.

$$\bar{V}_{kk'a}^{(1)} = \bar{V}_a^{(0)} + \bar{V}_a^{(1)} + \bar{V}_a^{(2)} = 0$$

$$\bar{I}_b = 0 \tag{3.39}$$

$$\bar{I}_c = 0$$

Since the voltage across the unbroken phase conductor is zero and the currents on the broken phase conductors are also zero at the point of break.

$$\begin{bmatrix} \bar{I}_a^{(0)} \\ \bar{I}_a^{(1)} \\ \bar{I}_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{I}_a \\ 0 \\ 0 \end{bmatrix} \tag{3.40}$$

$$\bar{I}_a^{(0)} = \bar{I}_a^{(1)} = \bar{I}_a^{(2)} = \frac{1}{3}\bar{I}_a \tag{3.41}$$

Considering the equations shown above one can conclude that the sequence networks can be connected in series as illustrated in Fig. 3.15.

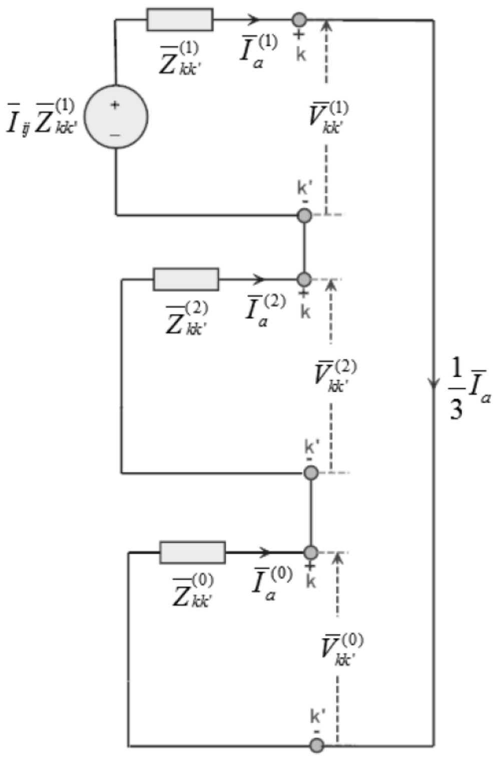

Figure 3.15: Connection of equivalent sequence networks to represent open phases b and c between k and k' [19].

Also, from the equivalent circuit represented in Fig. 3.15, one can compute the sequence currents as follows

$$\bar{I}_a^{(0)} = \bar{I}_a^{(1)} = \bar{I}_a^{(2)} = \bar{I}_{ij} \frac{\bar{Z}_{kk'}^{(1)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}} \tag{3.42}$$

where $\bar{I}_{ij}$ is the pre-fault current in phase a between buses i and j.

Then, the sequence voltages are given by

$$\bar{V}_{kk'}^{(0)} = -\bar{I}_a^{(0)} \bar{Z}_{kk'}^{(0)} = -\bar{I}_{ij} \frac{\bar{Z}_{kk'}^{(1)} \bar{Z}_{kk'}^{(0)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}}$$

$$\bar{V}_{kk'}^{(1)} = \bar{I}_a^{(1)} \left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right) = \bar{I}_{ij} \frac{\bar{Z}_{kk'}^{(1)} \left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right)}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}} \tag{3.43}$$

$$\bar{V}_{kk'}^{(2)} = -\bar{I}_a^{(2)} \bar{Z}_{kk'}^{(2)} = -\bar{I}_{ij} \frac{\bar{Z}_{kk'}^{(1)} \bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}}$$

3.3. Post-Fault Voltages

The post-fault voltages for the zero, positive, and negative sequence networks are given by

$$V_i^{012} = V_{pre}^{012} + \Delta V_i^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix} + \begin{bmatrix} -Z_{ik}^{(0)} I_{fk}^{(0)} \\ -Z_{ik}^{(1)} I_{fk}^{(1)} \\ -Z_{ik}^{(2)} I_{fk}^{(2)} \end{bmatrix}, \qquad i = 1,2,\dots,n \tag{3.44}$$

where n represents the number of buses.

3.4. Post-Fault Line Currents

The post-fault voltages for the zero, positive, and negative sequence networks are given by

$$I_{ij}^{012} = \frac{V_i^{012} - V_j^{012}}{z_{ij}^{012}} = \begin{bmatrix} \dfrac{V_i^{(0)} - V_j^{(0)}}{z_{ij}^{(0)}} \\ \dfrac{V_i^{(1)} - V_j^{(1)}}{z_{ij}^{(1)}} \\ \dfrac{V_i^{(2)} - V_j^{(2)}}{z_{ij}^{(2)}} \end{bmatrix} \tag{3.45}$$

where z_{ij} corresponds to the primitive impedance between buses i and j.

3.5. Example

In this example, let us consider the 5-Bus Power System [17], represented in Fig. 3.16.

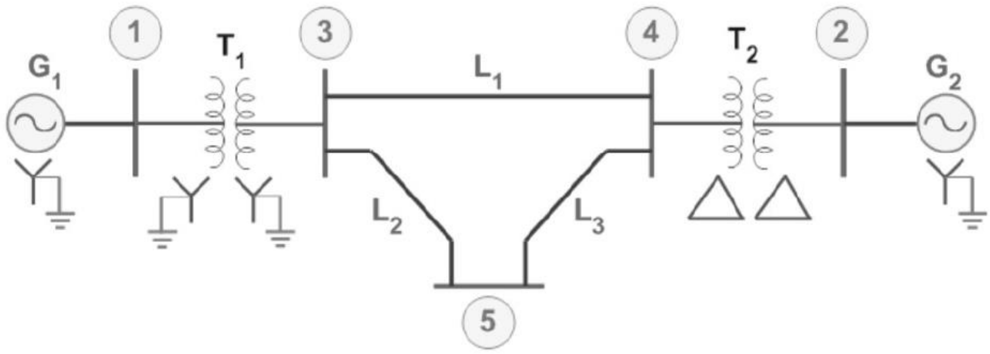

Figure 3.16: 5-Bus system [17].

For the purposes of this example lets consider the following data regarding the power system shown above. The pre-fault voltages for each bus are defined by its module (V_i) and angle (δ_i), all the impedances and generator connections are also defined, as well as the transformers impedances and connections. The line impedances defined, and the fault data is also represented, since there are going to be tested several types of fault in different buses.

$Bus\ \#$	$V_i(p.u.)$	$\delta_i(°)$
1	1.0	0
2	1.0	0
3	0.9165	−8.7552
4	0.9152	−10.1005
5	0.8858	−12.9631

Table 3.1: Pre-Fault Voltages Data (5-Bus System)

Bus #	Generator Connection	Generator Impedance $(p.u.)$		
		$Zero$	$Positive$	$Negative$
1	Y-Grounded	j0.05	j0.20	j0.20
2	Y-Grounded	j0.05	j0.20	j0.20

Table 3.2: Generator Data (5-Bus System)

Line #	End Bus	Type	Line Impedance $(p.u.)$			Line Shunt Admittance $(p.u.)$		
			$Zero$	$Positive$	$Negative$	$Zero$	$Positive$	$Negative$
1	1-3	Transf. (YG / YG)	j0.05	j0.05	j0.05	–	–	–
2	2-4	Transf. (Δ / Δ)	j0.05	j0.05	j0.05	–	–	–
3	3-4	Line	j0.30	j0.10	j0.10	0	0	0
4	3-5	Line	j0.30	j0.10	j0.10	0	0	0
5	4-5	Line	j0.30	j0.10	j0.10	0	0	0

Table 3.3: Line Data (5-Bus System)

Fault Type	Fault Bus	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$	$z_g\ (p.u.)$
Three-Phase Fault	5	0	0	0	0

Table 3.4: Fault Data (5-Bus System)

3.5.1. Sequence Impedance Matrices

First, one must build the system nodal impedance matrices:

$$Y^{(0)} = j\begin{bmatrix} -40.0000 & 0 & 20.0000 & 0 & 0 \\ 0 & -20.0000 & 0 & 0 & 0 \\ 20.0000 & 0 & -26.6667 & 3.3333 & 3.3333 \\ 0 & 0 & 3.3333 & -6.6667 & 3.3333 \\ 0 & 0 & 3.3333 & 3.3333 & -6.6667 \end{bmatrix}$$

$$Y^{(1)} = Y^{(2)} = j\begin{bmatrix} -25 & 0 & 20 & 0 & 0 \\ 0 & -25 & 0 & 20 & 0 \\ 20 & 0 & -40 & -40 & 10 \\ 0 & 20 & 10 & 10 & 10 \\ 0 & 0 & 10 & 10 & -20 \end{bmatrix}$$

$$Z^{(0)} = [Y^{(0)}]^{-1} = j\begin{bmatrix} 0.05 & 0 & 0.05 & 0.05 & 0.05 \\ 0 & 0.05 & 0 & 0 & 0 \\ 0.05 & 0 & 0.10 & 0.10 & 0.10 \\ 0.05 & 0 & 0.10 & 0.30 & 0.20 \\ 0.05 & 0 & 0.10 & 0.20 & 0.30 \end{bmatrix}$$

$$Z^{(1)} = Z^{(2)} = [Y^{(1)}]^{-1} = j\begin{bmatrix} 0.1294 & 0.0706 & 0.1118 & 0.0882 & 0.1000 \\ 0.0706 & 0.1294 & 0.0882 & 0.1118 & 0.1000 \\ 0.1118 & 0.0882 & 0.1397 & 0.1103 & 0.1250 \\ 0.0882 & 0.1118 & 0.1103 & 0.1397 & 0.1250 \\ 0.1000 & 0.1000 & 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

3.5.2. Fault Current

The fault current for a Three-Phase fault at bus 5 is given by

$$I_{5f}^{012} = \frac{V_5}{Z_{55}^{(1)} + Z_f} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.0616\angle -102.9631° \\ 0 \end{bmatrix}$$

$$I_{5f}^{abc} = T_s \cdot I_{5f}^{012} = \begin{bmatrix} 5.0616\angle -102.9631° \\ 5.0616\angle 137.0369° \\ 5.0616\angle 17.0369° \end{bmatrix}$$

3.5.3. Post-Fault Voltages

Then, the post-fault voltages are computed by

$$V_1^{012}{}_{new} = \begin{bmatrix} 0 \\ V_1 \\ 0 \end{bmatrix} - Z_{51}^{(1)} I_{5f}^{012} = \begin{bmatrix} 0 \\ 0.5193\angle 12.6293° \\ 0 \end{bmatrix}$$

$$V_2^{012}{}_{new} = \begin{bmatrix} 0 \\ V_2 \\ 0 \end{bmatrix} - Z_{52}^{(1)} I_{5f}^{012} = \begin{bmatrix} 0 \\ 0.5193\angle 12.6293° \\ 0 \end{bmatrix}$$

$$V_3^{012}{}_{new} = \begin{bmatrix} 0 \\ V_3 \\ 0 \end{bmatrix} - Z_{53}^{(1)} I_{5f}^{012} = \begin{bmatrix} 0 \\ 0.2892\angle 0.4811° \\ 0 \end{bmatrix}$$

$$V_4^{012}{}_{new} = \begin{bmatrix} 0 \\ V_4 \\ 0 \end{bmatrix} - Z_{54}^{(1)} \cdot I_{5f}^{012} = \begin{bmatrix} 0 \\ 0.2850\angle -3.7358° \\ 0 \end{bmatrix}$$

$$V_5^{012}{}_{new} = \begin{bmatrix} 0 \\ V_5 \\ 0 \end{bmatrix} - Z_{55}^{(1)} I_{5f}^{012} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, one must apply the Fortescue's transform in order to compute the three-phase corresponding voltages.

$$V_1^{abc}{}_{new} = TV_1^{012}{}_{new} = \begin{bmatrix} 0.5193\angle 12.6293° \\ 0.5193\angle -107.3707° \\ 0.5193\angle 132.6293° \end{bmatrix}$$

$$V_2^{abc}{}_{new} = TV_2^{012}{}_{new} = \begin{bmatrix} 0.5193\angle 12.6293° \\ 0.5193\angle -107.3707° \\ 0.5193\angle 132.6293° \end{bmatrix}$$

$$V_3^{abc}{}_{new} = TV_3^{012}{}_{new} = \begin{bmatrix} 0.2892\angle 0.4811° \\ 0.2892\angle -119.5189° \\ 0.2892\angle 120.4811° \end{bmatrix}$$

$$V_4^{abc}{}_{new} = TV_4^{012}{}_{new} = \begin{bmatrix} 0.2850\angle -3.7358° \\ 0.2850\angle -123.7358° \\ 0.2850\angle 116.2642° \end{bmatrix}$$

$$V_5^{abc}{}_{new} = TV_5^{012}{}_{new} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3.5.4. Post-Fault Line Currents

Finally, the post-fault line currents are computed as follows

$$I_{13}^{012} = \begin{bmatrix} \dfrac{V_1^{(0)} - V_3^{(0)}}{z_{13}^{(0)}} \\ \dfrac{V_1^{(1)} - V_3^{(1)}}{z_{13}^{(1)}} \\ \dfrac{V_1^{(2)} - V_3^{(2)}}{z_{13}^{(2)}} \end{bmatrix} = \begin{bmatrix} 0 \\ 4.8850\angle -62.9404° \\ 0 \end{bmatrix}$$

$$I_{24}^{012} = \begin{bmatrix} \dfrac{V_2^{(0)} - V_4^{(0)}}{z_{24}^{(0)}} \\ \dfrac{V_2^{(1)} - V_4^{(1)}}{z_{24}^{(1)}} \\ \dfrac{V_2^{(2)} - V_4^{(2)}}{z_{24}^{(2)}} \end{bmatrix} = \begin{bmatrix} 0 \\ 5.1721\angle -59.2783° \\ 0 \end{bmatrix}$$

$$I_{34}^{012} = \begin{bmatrix} \dfrac{V_3^{(0)} - V_4^{(0)}}{z_{34}^{(0)}} \\ \dfrac{V_3^{(1)} - V_4^{(1)}}{z_{34}^{(1)}} \\ \dfrac{V_3^{(2)} - V_4^{(2)}}{z_{34}^{(2)}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.2154\angle -12.8750° \\ 0 \end{bmatrix}$$

$$I_{35}^{012} = \begin{bmatrix} \dfrac{V_3^{(0)} - V_5^{(0)}}{z_{35}^{(0)}} \\ \dfrac{V_3^{(1)} - V_5^{(1)}}{z_{35}^{(1)}} \\ \dfrac{V_3^{(2)} - V_5^{(2)}}{z_{35}^{(2)}} \end{bmatrix} = \begin{bmatrix} 0 \\ 2.8924\angle -89.5189° \\ 0 \end{bmatrix}$$

$$I_{45}^{012} = \begin{bmatrix} \dfrac{V_4^{(0)} - V_5^{(0)}}{z_{45}^{(0)}} \\ \dfrac{V_4^{(1)} - V_5^{(1)}}{z_{45}^{(1)}} \\ \dfrac{V_4^{(2)} - V_5^{(2)}}{z_{45}^{(2)}} \end{bmatrix} = \begin{bmatrix} 0 \\ 2.8503\angle -93.7358° \\ 0 \end{bmatrix}$$

Now, like in the previous step, one must apply the Fortescue's transform in order to compute the three-phase corresponding line currents.

$$I_{13}^{abc} = TI_{13}^{012} = \begin{bmatrix} 4.8850\angle -62.9404° \\ 4.8850\angle 177.0596° \\ 4.8850\angle 57.0596° \end{bmatrix}$$

$$I_{24}^{abc} = TI_{24}^{012} = \begin{bmatrix} 5.1721\angle -59.2783° \\ 5.1721\angle -179.2783° \\ 5.1721\angle 60.7217° \end{bmatrix}$$

$$I_{34}^{abc} = TI_{34}^{012} = \begin{bmatrix} 0.2154\angle -12.8750° \\ 0.2154\angle -132.8750° \\ 0.2154\angle 107.1250° \end{bmatrix}$$

$$I_{35}^{abc} = TI_{35}^{012} = \begin{bmatrix} 2.8924\angle -89.5189° \\ 2.8924\angle 150.4811° \\ 2.8924\angle 30.4811° \end{bmatrix}$$

$$I_{45}^{abc} = TI_{45}^{012} = \begin{bmatrix} 2.8503\angle -93.7358° \\ 2.8503\angle 146.2642° \\ 2.8503\angle 26.2642° \end{bmatrix}$$

4. Direct Three-Phase Representation

4.1. Three-Phase Models

This section shows how one can model the components in a three-phase form in order to apply the direct three-phase representation method for fault analysis in a power system.

4.1.1. Conductors

In Fig. 4.1 (a) and (b) is represented the equivalent circuit of a three-phase conductor and its mutual couplings between phases and ground wires.

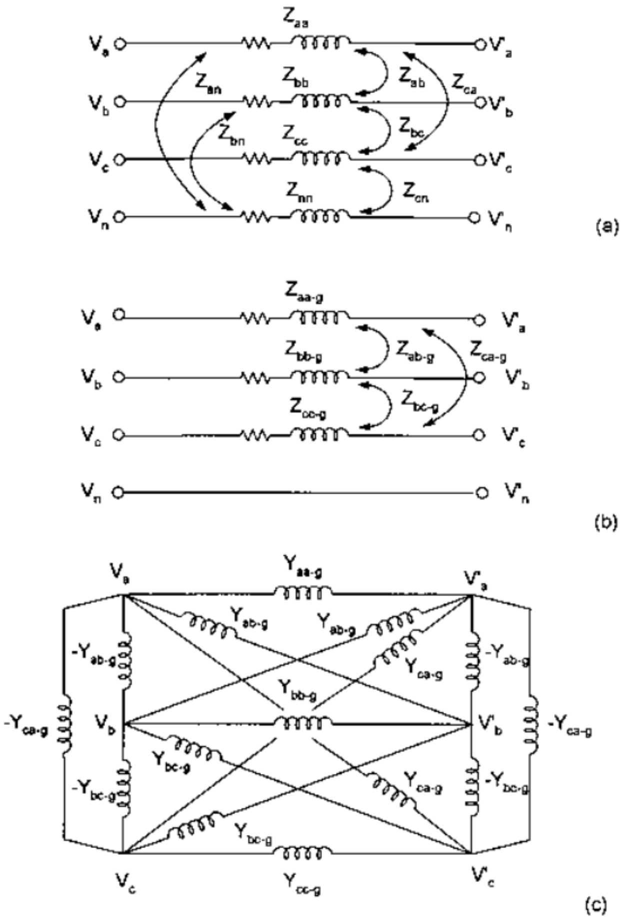

Figure 4.1: (a) Mutual couplings between a line section with ground wire in the impedance form; (b) Transformed network in impedance form; (c) Equivalent admittance network of a series line section [2].

From Fig. 4.1, one can build the expression that relates the line voltages and currents for a three-phase conductor, as follows.

$$\begin{bmatrix} V_a - V_a' \\ V_b - V_b' \\ V_c - V_c' \end{bmatrix} = \begin{bmatrix} Z_{aa-g} & Z_{ab-g} & Z_{ac-g} \\ Z_{ba-g} & Z_{bb-g} & Z_{bc-g} \\ Z_{ca-g} & Z_{cb-g} & Z_{cc-g} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.1}$$

It can also be written in the admittance form as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} Y_{aa-g} & Y_{ab-g} & Y_{ac-g} \\ Y_{ba-g} & Y_{bb-g} & Y_{bc-g} \\ Y_{ca-g} & Y_{cb-g} & Y_{cc-g} \end{bmatrix} \begin{bmatrix} V_a - V_a' \\ V_b - V_b' \\ V_c - V_c' \end{bmatrix} \tag{4.2}$$

Also, in Fig. 4.1 (c), it is shown the equivalent circuit of the line section and its coupling effects. Thus, considering the π model of a transmission line, its three-phase expression is given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_a' \\ I_b' \\ I_c' \end{bmatrix} = \begin{bmatrix} Y^{abc} + \frac{1}{2} Y_{sh} & -Y^{abc} \\ -Y^{abc} & Y^{abc} + \frac{1}{2} Y_{sh} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_a' \\ V_b' \\ V_c' \end{bmatrix} \tag{4.3}$$

where

$$\bar{Y}^{abc} = \bar{Z}^{abc^{-1}} \tag{4.4}$$

The three-phase admittance is built in a similar way as the single-phase admittance matrix, but for the three-phase analysis each element of the single-phase matrix is replaced by a 3×3 matrix.

Regarding the shunt capacitance of the line, represented in Figure 4.2 (a), one can represent it by current injections, as shown in Figure 4.2 (b).

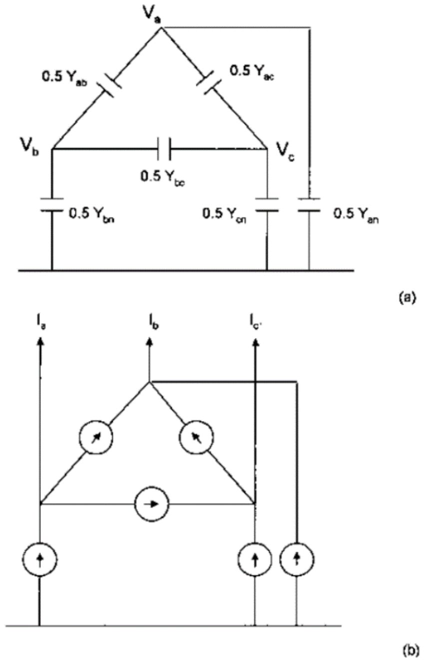

Figure 4.2: (a) Capacitances in a three-phase circuit; (b) Equivalent current injections [2].

The charging currents are given by

$$\begin{aligned} I_a &= -\frac{1}{2}[Y_{ab} + Y_{ac} + Y_{an}]V_a + \frac{Y_{ab}}{2}V_b + \frac{Y_{ac}}{2}V_c \\ I_b &= -\frac{1}{2}[Y_{ab} + Y_{ac} + Y_{an}]V_b + \frac{Y_{ab}}{2}V_a + \frac{Y_{bc}}{2}V_c \\ I_c &= -\frac{1}{2}[Y_{ab} + Y_{ac} + Y_{an}]V_c + \frac{Y_{ac}}{2}V_a + \frac{Y_{bc}}{2}V_b \end{aligned} \tag{4.5}$$

4.1.2. Generators

The model of the generator that one must use for the three-phase analysis is different from the power flow model, which depends on the bus voltage magnitude and its power output, considers an internal voltage behind the generator transient reactance.

First, let us see the three sequence admittances of a generator. The zero sequence admittance is given by

$$Y_0 = \frac{1}{R_0 + jX_0 + 3(R_g + jX_g)} \tag{4.6}$$

where R_0 and X_0 are the zero sequence resistance and reactance, respectively, of the generator; and R_g and X_g represent both resistance and reactance added in the neutral grounding circuit. Notice that for a grounded generator R_g and X_g are zero.

$$Y_1 = \frac{1}{jX'_d} \tag{4.7}$$

$$Y_2 = \frac{1}{jX_2} \tag{4.8}$$

where X'_d is the generator direct axis transient reactance and X_2 is the generator negative sequence reactance, neglecting the generator resistances in both cases. These sequence admittances can be transformed into phase domain, as shown below.

$$\bar{Y}^{abc} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$
$$= \frac{1}{3}\bar{T}_s \bar{Y}^{012} \bar{T}_s^t = \begin{bmatrix} Y_0 + Y_1 + Y_2 & Y_0 + aY_1 + a^2Y_2 & Y_0 + a^2Y_1 + aY_2 \\ Y_0 + a^2Y_1 + aY_2 & Y_0 + Y_1 + Y_2 & Y_0 + aY_1 + a^2Y_2 \\ Y_0 + aY_1 + a^2Y_2 & Y_0 + a^2Y_1 + aY_2 & Y_0 + Y_1 + Y_2 \end{bmatrix} \tag{4.9}$$

Regarding unbalance loading end neutral current flow, the model of the generator can be written as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \boldsymbol{S}^*/E^*{}_1 \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & -Y_1 & -Y_0 \\ Y_{21} & Y_{22} & Y_{23} & -a^2Y_1 & Y_0 \\ Y_{31} & Y_{32} & Y_{33} & -aY_1 & -Y_0 \\ -Y_1 & -aY_1 & -a^2Y_1 & 3Y_1 & 0 \\ -Y_0 & -Y_0 & -Y_0 & 0 & 3Y_0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ E_1 \\ V_n \end{bmatrix} \tag{4.10}$$

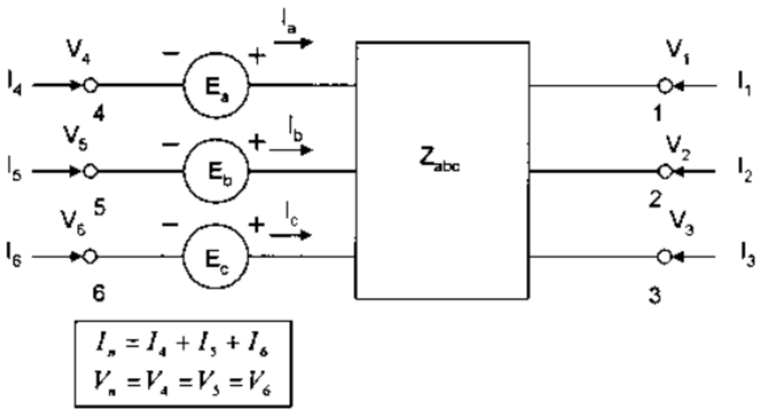

Figure 4.3: Norton equivalent circuit of a generator [2].

$$I_a = \frac{\boldsymbol{S}_1^*}{E_1^*} \quad I_b = \frac{\boldsymbol{S}_2^*}{E_2^*} \quad I_c = \frac{\boldsymbol{S}_3^*}{E_3^*} \tag{4.11}$$

$$I_a = \frac{\boldsymbol{S}_1^*}{E_1^*} \quad I_b = \frac{\boldsymbol{S}_2^*}{aE_1^*} \quad I_c = \frac{\boldsymbol{S}_3^*}{a^2E_1^*} \tag{4.12}$$

$$I_a + I_b + I_c = \frac{\boldsymbol{S}^*}{E_1^*} = \frac{\boldsymbol{S}_1^* + \boldsymbol{S}_2^* + \boldsymbol{S}_3^*}{E_1^*} \tag{4.13}$$

where $\boldsymbol{S}$ stands for the total complex power, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$ represent each phase power, and E_1 is the positive sequence voltage with reference to the transient reactance. Considering that the system is well grounded the neutral voltage is zero. In the expressions above, E_1, E_2, and E_3 stand for the balanced internal machine voltages, and V_1, V_2, and V_3 stand for the terminal voltages. These last ones depend on internal machine impedances and unbalance in machine currents, which are represented by I_a, I_b, and I_c. Notice that, due to the unbalance, each phase power is not equal to on third of the total power. Finally, I_1, I_2, and I_3 represent the injected currents and I_n is the neutral current.

4.1.3. Transformers

In this section are described the three-phase models for transformers, considering its winding connections and turns ratio. In Fig. 4.4 is represented a 12-terminal coupled network, which as three primary windings and three secondary windings mutually coupled through the transformer core. The short-circuit primitive matrix for this network is

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} \tag{4.14}$$

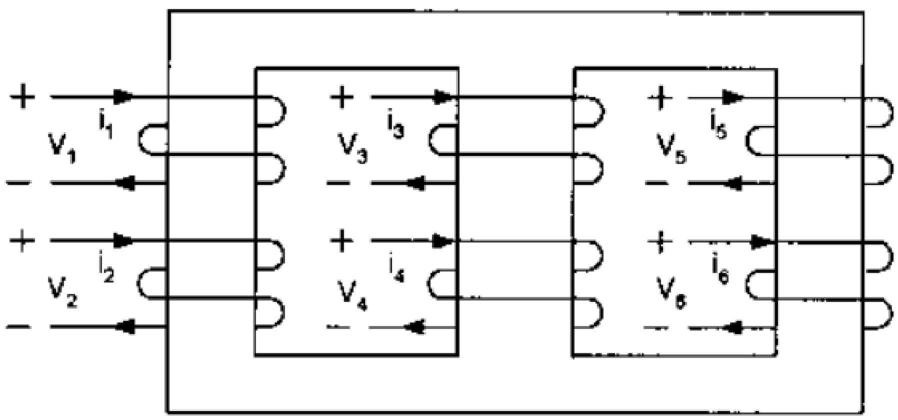

Figure 4.4: Elementary circuit of a three-phase transformer showing 12-terminal coupled primitive network [2].

Let us consider windings 1, 3, and 5 as the primary windings, and windings 2, 4, and 6 as the secondary windings. If there are no mutual couplings the primed elements are equal to zero.

$$\begin{bmatrix} y_p & -y_m & y'_m & y''_m & y'_m & y''_m \\ -y_m & y_s & y''_m & y'''_m & y''_m & y'''_m \\ y'_m & y''_m & y_p & -y_m & y'_m & y''_m \\ y''_m & y'''_m & -y_m & y_s & y''_m & y'''_m \\ y'_m & y''_m & y'_m & y''_m & y_p & -y_m \\ y''_m & y'''_m & y''_m & y'''_m & -y_m & y_s \end{bmatrix} \quad (4.15)$$

In Fig. 4.5 is represented a three-phase transformer, wye-delta connected, which can be related to its branch and node voltages by the connection matrix below:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_A \\ V_B \\ V_C \end{bmatrix} \quad (4.16)$$

which is the same as:

$$\bar{v}_{branch} = \bar{N}\bar{V}_{node} \quad (4.17)$$

Now, one must apply Kron's transformation to the connection matrix N in order to obtain the node admittance matrix:

$$\bar{Y}_{node} = \bar{N}^t \bar{Y}_{prim} \bar{N} \quad (4.18)$$

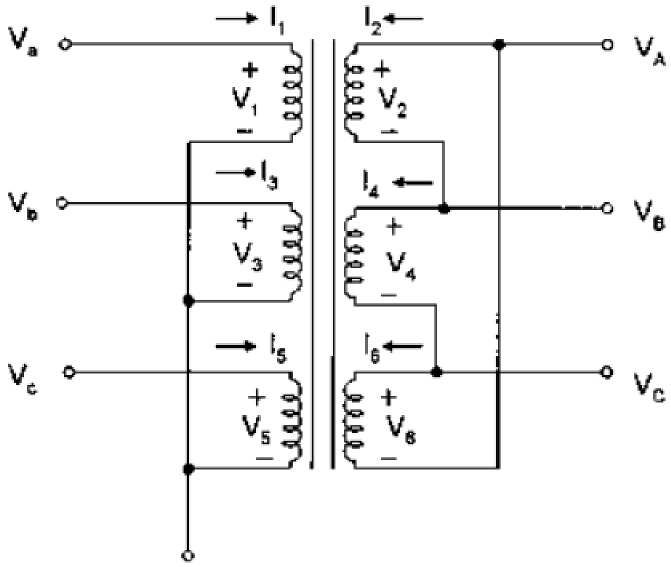

Figure 4.5: Circuit of a grounded wye-delta transformer with voltage and current relations of derivation connection matrix [2].

In phase quantities, and without considering the turns ratio effect, the node admittance matrix is given by

$$\bar{Y}_{node} = \begin{bmatrix} y_s & y'_m & y'_m & -(y_m + y''_m) & (y_m + y''_m) & 0 \\ y'_m & y_s & y'_m & 0 & -(y_m + y''_m) & (y_m + y''_m) \\ y'_m & y'_m & y_s & (y_m + y''_m) & 0 & -(y_m + y''_m) \\ -(y_m + y''_m) & 0 & (y_m + y''_m) & 2(y_s - y'''_m) & -(y_s - y'''_m) & -(y_s - y'''_m) \\ (y_m + y''_m) & -(y_m + y''_m) & 0 & -(y_s - y'''_m) & 2(y_s - y'''_m) & -(y_s - y'''_m) \\ 0 & (y_m + y''_m) & -(y_m + y''_m) & -(y_s - y'''_m) & -(y_s - y'''_m) & 2(y_s - y'''_m) \end{bmatrix} \quad (4.19)$$

The node admittance matrix above can be divided into submatrices as follows:

$$\bar{Y}_{node} = \begin{bmatrix} \bar{Y}_I & \bar{Y}_{II} \\ \bar{Y}_{II}^t & \bar{Y}_{III} \end{bmatrix} \quad (4.20)$$

Where each 3×3 submatrix depends on the winding connections, as shown in Table 4.1. These submatrices are defined as follows:

$$\bar{Y}_I = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix} \quad \bar{Y}_{II} = \frac{1}{3}\begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \quad \bar{Y}_{III} = \frac{1}{\sqrt{3}}\begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix} \quad (4.21)$$

where y_t is the leakage admittance per phase in per unit.

The submatrices above can be modified due to the off-nominal tap ratio between primary and secondary windings of the transformer. Considering that this ratio is given by $\alpha{:}\beta$, where α and β represent the taps on the primary and secondary side, respectively, in per unit.

$$\bar{Y}_{node} = \begin{bmatrix} \frac{\bar{Y}_I}{\alpha^2} & \frac{\bar{Y}_{II}}{\alpha\beta} \\ \frac{\bar{Y}_{II}^t}{\alpha\beta} & \frac{\bar{Y}_{III}}{\beta^2} \end{bmatrix} \tag{4.22}$$

The expression above shows how the submatrices are modified due to the off-nominal tap ratio of the transformer.

Winding connections		Self-admittance		Mutual admittance	
Primary	Secondary	Primary	Secondary	Primary	Secondary
YG	YG	$\bar{Y}_I$	$\bar{Y}_I$	$-\bar{Y}_I$	$-\bar{Y}_I$
YG	Y	$\bar{Y}_{II}$	$\bar{Y}_{II}$	$-\bar{Y}_{II}$	$-\bar{Y}_{II}$
Y	YG	$\bar{Y}_{II}$	$\bar{Y}_{II}$	$-\bar{Y}_{II}$	$-\bar{Y}_{II}$
Y	Y	$\bar{Y}_{II}$	$\bar{Y}_{II}$	$-\bar{Y}_{II}$	$-\bar{Y}_{II}$
YG	Δ	$\bar{Y}_I$	$\bar{Y}_{III}$	$\bar{Y}_{III}$	$\bar{Y}_{III}^t$
Y	Δ	$\bar{Y}_{II}$	$\bar{Y}_{III}$	$\bar{Y}_{III}$	$\bar{Y}_{III}^t$
Δ	Y	$\bar{Y}_{II}$	$\bar{Y}_{III}$	$\bar{Y}_{III}$	$\bar{Y}_{III}^t$
Δ	YG	$\bar{Y}_{II}$	$\bar{Y}_{III}$	$\bar{Y}_{III}$	$\bar{Y}_{III}^t$
Δ	Δ	$\bar{Y}_{II}$	$\bar{Y}_{II}$	$-\bar{Y}_{II}$	$-\bar{Y}_{II}$

Table 4.1: Submatrices of Three-Phase Transformer Connections [2].

4.1.4. Loads

A typical three-phase load, as shown in Fig. 4.6, allows unbalance through load current injections. For the purposes of fault analysis, one can convert a static load into an equivalent impedance if the load currents are considered.

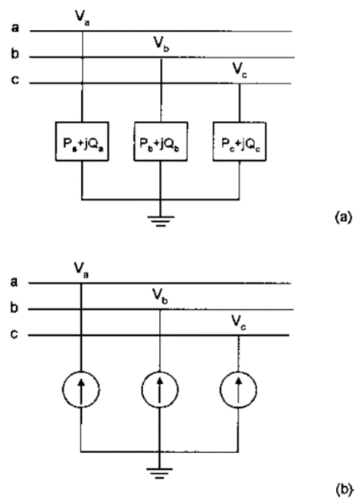

Figure 4.6: (a) Three-phase load representation; (b) Equivalent current injection [2].

4.2. Three-Phase Admittance Matrix

Once all the system components are modelled, one can start building the three sequences admittance matrices. In order to perform a phase domain analysis, one must transform these sequence matrices into three-phase domain matrices.

As an example, let us consider a single bus, bus 1, and its diagonal element of the admittance matrix, which in phase domain is given by

$$Y_{11}^{abc} = \begin{bmatrix} Y_{11}^{aa} & Y_{11}^{ab} & Y_{11}^{ac} \\ Y_{11}^{ba} & Y_{11}^{bb} & Y_{11}^{bc} \\ Y_{11}^{ca} & Y_{11}^{cb} & Y_{11}^{cc} \end{bmatrix} = T \begin{bmatrix} Y_{11}^{(0)} & 0 & 0 \\ 0 & Y_{11}^{(1)} & 0 \\ 0 & 0 & Y_{11}^{(2)} \end{bmatrix} T^{-1} \tag{4.23}$$

where T is the Fortescue's matrix.

Then, the nodal branch-to-branch incidence matrix is given by:

$$A = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array} \tag{4.24}$$

where $a_{pq} = \begin{cases} U, & if\ q\ is\ the\ sending-end\ bus\ of\ branch\ p \\ -U, & if\ q\ is\ the\ receiving-end\ bus\ of\ branch\ p \\ Zero, & otherwise \end{cases}$

where U is a $[3x3]$ identity matrix and $Zero$ a $[3x3]$ zero matrix, being m the total number of lines, and n the total number of buses.

Next, one must build the branch series impedance matrix for n elements, without the transformer branch impedances, as follows:

$$z^{abc} = \begin{bmatrix} z_1^{abc} & Zero & \cdots & Zero \\ Zero & z_2^{abc} & \cdots & Zero \\ \vdots & \vdots & \ddots & \vdots \\ Zero & Zero & \cdots & z_n^{abc} \end{bmatrix} \tag{4.25}$$

Then, one can easily obtain the branch series admittance matrix from the expression above, and it is given by:

$$y^{abc} = (z^{abc})^{-1} \tag{4.26}$$

Finally, the entire three-phase admittance matrix for a n-Bus system is given by:

$$Y^{abc} = A^T y^{abc} A = \begin{bmatrix} Y_{11}^{abc} & Y_{12}^{abc} & \cdots & Y_{1n}^{abc} \\ Y_{21}^{abc} & Y_{22}^{abc} & \cdots & Y_{2n}^{abc} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}^{abc} & Y_{n2}^{abc} & \cdots & Y_{nn}^{abc} \end{bmatrix} \tag{4.27}$$

Since this matrix does not include the shunt elements, one must perform some changes to it. First, the generators must be included.

$$Y_{ii}^{abc} = Y_{ii}^{abc} + y_g^{abc} \tag{4.28}$$

Then, one must include all transformers. Let us consider a transformer connected between buses i and j.

$$\begin{aligned} Y_{ii}^{abc} &= Y_{ii}^{abc} + \bar{Y}_{node_{ii}} - U \\ Y_{ij}^{abc} &= Y_{ij}^{abc} + \bar{Y}_{node_{ij}} + U \\ Y_{ji}^{abc} &= Y_{ji}^{abc} + \bar{Y}_{node_{ji}} + U \\ Y_{jj}^{abc} &= Y_{jj}^{abc} + \bar{Y}_{node_{jj}} - U \end{aligned} \tag{4.29}$$

Finally, the three-phase admittance matrix of the system is completed.

$$Y^{abc} = Z^{abc^{-1}} \tag{4.30}$$

4.3. Fault Current

For fault current calculation purposes, let us consider a fault at bus k. The fault impedance, for different types of faults, can be defined as shown in Fig. 4.7. It is represented by the fault impedances, regarding each phase, and by the ground impedance.

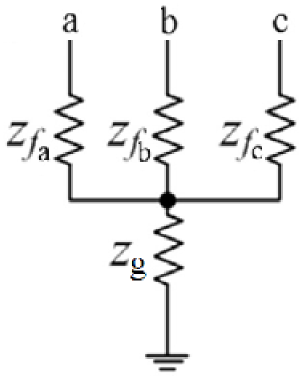

Figure 4.7: Fault impedances [15].

This set of impedances can be considered in a matrix form, given by Z_f^{abc} and Z_g, as shown below.

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \tag{4.31}$$

$$Z_g = \begin{bmatrix} z_g & z_g & z_g \\ z_g & z_g & z_g \\ z_g & z_g & z_g \end{bmatrix} \tag{4.32}$$

where $Z_{fa,b,c}$ represent the fault impedance of each phase, and z_g the ground impedance.

For each type of fault, the fault impedance will be different, meaning that for each case one must understand which of the impedances referred above must be considered in order to compute the fault current.

Let $I_{f_k}^{abc}$ be the fault current vector regarding a shunt fault at bus k, and I_f^{abc} be the total fault current vector. Since the fault occurs at bus k, the fault current will be zero in all buses except for the faulted bus, as shown below.

$$I_f^{abc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{f_k}^{abc} \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.33}$$

Finally, let us see how to compute the fault current, regarding the faulted bus, for different types of faults.

4.3.1. Three-Phase Fault

For a three-phase fault, the total fault impedance includes Z_f^{abc} and Z_g, as shown in Fig. 4.8.

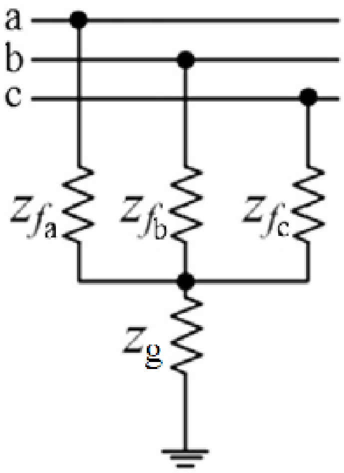

Figure 4.8: Fault impedance for a three-phase fault [15].

The fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = (Z_{kk}^{abc} + Z_f^{abc} + Z_g)^{-1} V_k^{abc} \tag{4.34}$$

4.3.2. Line-To-Ground Fault

For a single line-to-ground fault, the total fault impedance includes only the fault impedance regarding the faulted phase.

Let us consider that the fault occurs in phase a, as shown in Fig. 4.9. Then, $Z_{fb} = Z_{fc} = \infty$, which can be represented for computation purposes by:

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix} \tag{4.35}$$

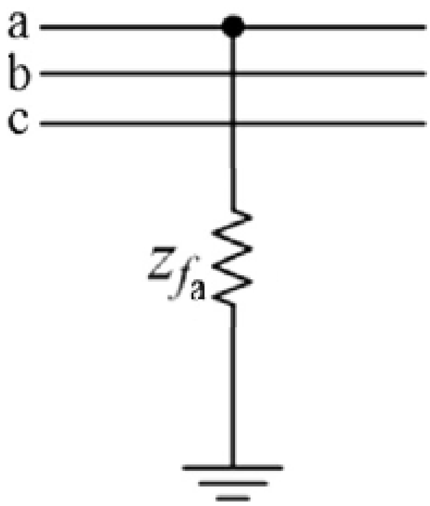

Figure 4.9: Fault impedance for a line-to-ground fault [15].

Thus, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = (Z_{kk}^{abc} + Z_f^{abc})^{-1} V_k^{abc} \tag{4.36}$$

4.3.3. Line-To-Line Fault

For a line-to-line fault, the only fault impedance considered is the impedance between the two faulted phases, represented by Z_f.

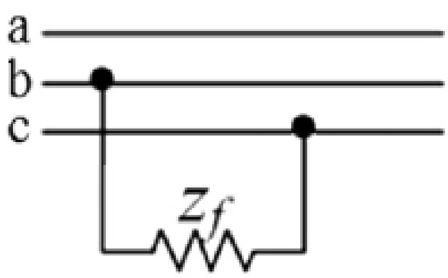

Figure 4.10: Fault impedance for a line-to-line fault [15].

Considering that the fault occurs between phases b and c, as shown in Fig. 4.10, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = \left(Z_{kk}^{bb} + Z_{kk}^{cc} - Z_{kk}^{bc} - Z_{kk}^{cb} + Z_f\right)^{-1} \begin{bmatrix} 0 \\ V_k^b - V_k^c \\ V_k^c - V_k^b \end{bmatrix} \quad (4.37)$$

4.3.4. Double-Line-To-Ground Fault

For a double-line-to-ground fault, the total fault impedance includes the fault impedances regarding both faulted phases and the ground impedance.

Let us consider that the fault occurs in phases b and c, as shown in Fig. 4.11. Then, $Z_{fa} = \infty$, which can be represented by:

$$Z_f^{abc} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \quad (4.38)$$

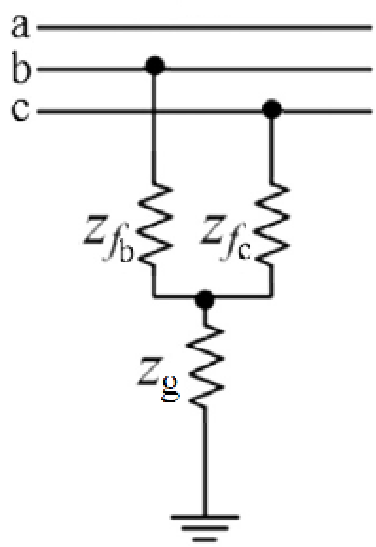

Figure 4.11: Fault impedance for a double-line-to-ground fault [15].

Thus, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = (Z_{kk}^{abc} + Z_f^{abc} + Z_g)^{-1} V_k^{abc} \quad (4.39)$$

4.3.5. Open Conductor Fault

A fault between buses, such as an open conductor fault, is called a series fault [16]. Let us consider that a series fault occurs in the branch between buses i and j. According to this assumption, $Z_{Th,ij}^{abc}$ is the 6×6 impedance matrix, related to the faulted buses, obtained from the total impedance matrix of the system.

$$Z_{Th,ij}^{abc} = \begin{bmatrix} Z_{ii}^{abc} & Z_{ij}^{abc} \\ Z_{ji}^{abc} & Z_{jj}^{abc} \end{bmatrix} \tag{4.40}$$

Also, let the 3×3 primitive admittance of the faulted line change from Y_l^{old} to Y_l^{new}, and Y_l^{new} is given by Eq. (4.41).

$$Y_l^{new} = \left(z_l^{abc} + Z_f^{abc}\right)^{-1} \tag{4.41}$$

where Z_f^{abc} depends on the opening phases.

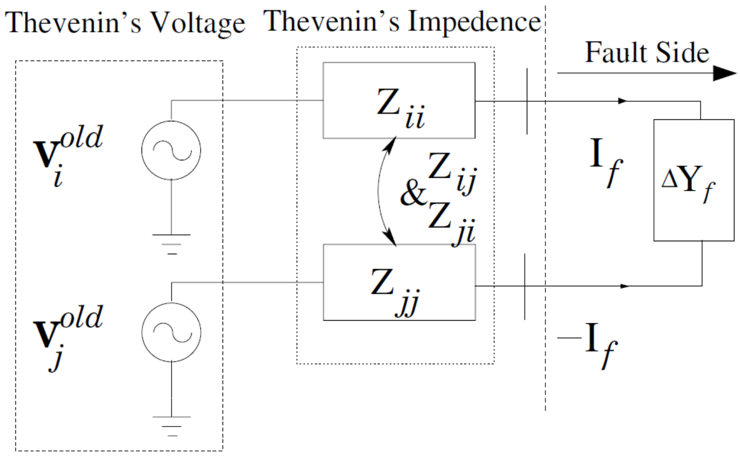

Figure 4.12: Block diagram representation of Thevenin's equivalent circuit for series fault [16].

The previous procedure will result in the change of the four block entries in $Y_{Th,ij}^{old}$. Also, notice that $Y_{Th,ij}^{old} = (Z_{Th,ij}^{old})^{-1}$.

$$\Delta Y_f = Y_l^{new} - Y_l^{old} \tag{4.42}$$

The modification of the admittance matrix will involve addition and subtraction of ΔY_f, depending on the block entry being diagonal or non-diagonal, respectively.

$$Y_{ii}^{new} = Y_{ii}^{old} + \Delta Y_f$$

$$Y_{ij}^{new} = Y_{ij}^{old} - \Delta Y_f \tag{4.43}$$

$$Y_{ji}^{new} = Y_{ji}^{old} - \Delta Y_f$$

$$Y_{jj}^{new} = Y_{jj}^{old} + \Delta Y_f$$

This step can also be represented as follows:

$$Y_{Th,ij}^{new} = Y_{Th,ij}^{old} + [E_i \quad E_j]\, \hat{Y}_f \,[E_i \quad E_j]^T \tag{4.44}$$

where $\hat{Y}_f$ is given by

$$\hat{Y}_f = \begin{bmatrix} \Delta Y_f & -\Delta Y_f \\ -\Delta Y_f & \Delta Y_f \end{bmatrix} \tag{4.45}$$

and E_i is a block vector given by

$$E_i(j) = \begin{cases} O_3, & i \neq j \\ I_3, & i = j \end{cases} \tag{4.46}$$

where O_3 is a 3×3 size matrix of zeros, and I_3 is the identity matrix of the same size.

The post-fault voltages for a series fault are given by:

$$\begin{bmatrix} V_i^{new} \\ V_j^{new} \end{bmatrix} = \left(I_6 + Z_{Th,ij}^{abc} \hat{Y}_f\right)^{-1} \begin{bmatrix} V_i^{old} \\ V_j^{old} \end{bmatrix} \tag{4.47}$$

And the fault currents are given by:

$$I_{f_{ij}}^{abc} = Y_l^{new}\left(V_i^{new} - V_j^{new}\right) \tag{4.48}$$

Notice that, for a series fault, the fault current vector will have to entries with opposite signs, as shown below:

$$I_f^{abc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{f_{ij}}^{abc} \\ -I_{f_{ij}}^{abc} \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.49}$$

where $I_{f_{ij}}^{abc}$ is the fault current from bus i to bus j, and $-I_{f_{ij}}^{abc}$ from bus j to bus i, as illustrated in Fig. 4.12.

4.3.5.1. One-Conductor Open

Considering that only phase a opens, the line impedance regarding the opening phase is $Z_{fa} = \infty$, which results in the following modification to Z_f^{abc}.

$$Z_f^{abc} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \tag{4.50}$$

Then, one can obtain Y_l^{new} and perform all the calculations explained above.

$$Y_l^{new} = \left(z_l^{abc} + Z_f^{abc}\right)^{-1} \tag{4.51}$$

4.3.5.2. Two-Conductors Open

In this case, let us consider that phases b and c are the faulted conductors. Thus, the line impedance regarding the openings phases is $Z_{fb} = Z_{fc} = \infty$, which results in the following modification to Z_f^{abc}.

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix} \tag{4.52}$$

Thus, as in the one-conductor opening case, one can obtain Y_l^{new}, given by Eq.4.51.

4.4. Post-Fault Voltages

The changes in the post-fault bus voltages are given by:

$$\Delta V = -Z^{abc} I_f^{abc} \tag{4.54}$$

Let V_{pre} be the vector that represents all the bus voltages before the fault.

$$V_{pre} = \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_n^{abc} \end{bmatrix} \tag{4.56}$$

Then, the post-fault bus voltage vector is given by:

$$V_{new} = V_{pre} + \Delta V \tag{4.57}$$

4.5. Post-Fault Branch Currents

In order to compute the post-fault branch currents, one must obtain the voltage drops regarding every line of the system. Considering the post-fault bus voltage vector, the line voltage vector is given by:

$$V_{line} = AV_{new} \tag{4.58}$$

where A is the nodal branch-to-branch incidence matrix defined in Eq. 4.25.

Then, the post-fault branch current vector can be computed by:

$$I_{line} = Y_{line}V_{line} \tag{4.59}$$

where Y_{line} is the branch series admittance matrix including the transformer branch impedances.

5. Experimental Results (5-Bus System)

In this chapter, the example presented in the end of Chapter 3 is going to be analysed in detail for several cases of faults regarding all the subjects referred so far in this dissertation. Thus, all the types of faults described so far will be tested in the 5-Bus power system represented below, using both traditional and direct methods in order to compare the results.

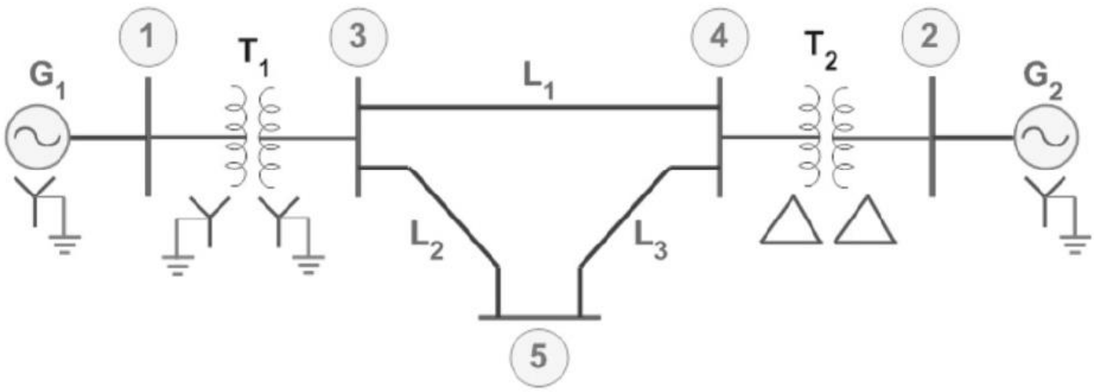

Figure 5.1: 5-Bus system [17].

Bus i		$V_i^{abc}[p.u.]$	$\delta_i^{abc}[°]$
1	a	1.0	0
	b	1.0	−120
	c	1.0	120
2	a	1.0	0
	b	1.0	−120
	c	1.0	120
3	a	0.9165	−8.7552
	b	0.9165	−128.7552
	c	0.9165	111.2448
4	a	0.9152	−10.1005
	b	0.9152	−130.1005
	c	0.9152	109.8995
5	a	0.8858	−12.9631
	b	0.8858	−132.9631
	c	0.8858	107.0369

Table 5.1: Pre-fault Voltages Data (5-Bus System)

Bus #	Generator Connection	Generator Impedance $(p.u.)$		
		$Zero$	$Positive$	$Negative$
1	Y-Grounded	j0.05	j0.20	j0.20
2	Y-Grounded	j0.05	j0.20	j0.20

Table 5.2: Generator Data (5-Bus System)

Line #	End Bus	Type	Line Impedance $(p.u.)$			Line Shunt Admittance $(p.u.)$		
			$Zero$	$Positive$	$Negative$	$Zero$	$Positive$	$Negative$
1	1-3	Transf. (YG / YG)	j0.05	j0.05	j0.05	–	–	–
2	2-4	Transf. (Δ- Δ)	j0.05	j0.05	j0.05	–	–	–
3	3-4	Line	j0.30	j0.10	j0.10	0	0	0
4	3-5	Line	j0.30	j0.10	j0.10	0	0	0
5	4-5	Line	j0.30	j0.10	j0.10	0	0	0

Table 5.3: Line Data (5-Bus System)

5.1. Three-Phase Fault

Fault Type	Faulted Bus	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$	$z_g\ (p.u.)$
Three-Phase Fault	5	–	–	–	0

Table 5.4: Fault Data (5-Bus System)

5.1.1. Admittance Matrices

Let us start by computing the admittance matrices, as seen below

$$y_g^{012} = \begin{bmatrix} 1/z_g^{(0)} & 0 & 0 \\ 0 & 1/z_g^{(1)} & 0 \\ 0 & 0 & 1/z_g^{(2)} \end{bmatrix} = \begin{bmatrix} -j20 & 0 & 0 \\ 0 & -j5 & 0 \\ 0 & 0 & -j5 \end{bmatrix}$$

$$z_t^{012} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$z_l^{012} = \begin{bmatrix} z_l^{(0)} & 0 & 0 \\ 0 & z_l^{(1)} & 0 \\ 0 & 0 & z_l^{(2)} \end{bmatrix} = \begin{bmatrix} j0.3000 & 0 & 0 \\ 0 & j0.1000 & 0 \\ 0 & 0 & j0.1000 \end{bmatrix}$$

$$y_g^{abc} = T^{-1} y_g^{012} T = \begin{bmatrix} -j10 & -j5 & -j5 \\ -j5 & -j10 & -j5 \\ -j5 & -j5 & -j10 \end{bmatrix}$$

$$z_t^{abc} = T^{-1} z_t^{012} T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$z_l^{abc} = T^{-1} z_l^{012} T = \begin{bmatrix} j0.1667 & j0.0667 & j0.0667 \\ j0.0667 & j0.1667 & j0.0667 \\ j0.0667 & j0.0667 & j0.1667 \end{bmatrix}$$

5.1.2. Three-Phase Admittance Matrix

The nodal branch-to-branch incidence matrix of the 5-Bus system is given by:

$$A = \begin{bmatrix} U & Zero & -U & Zero & Zero \\ Zero & U & Zero & -U & Zero \\ Zero & Zero & U & -U & Zero \\ Zero & Zero & U & Zero & -U \\ Zero & Zero & Zero & U & -U \end{bmatrix}$$

where U is the 3×3 identity matrix, and $Zero$ is a 3×3 zero matrix.

Next, one must build the branch series impedance matrix, without the transformer branch impedances, as follows:

$$z^{abc} = \begin{bmatrix} z_t^{abc} & Zero & Zero & Zero & Zero \\ Zero & z_t^{abc} & Zero & Zero & Zero \\ Zero & Zero & z_l^{abc} & Zero & Zero \\ Zero & Zero & Zero & z_l^{abc} & Zero \\ Zero & Zero & Zero & Zero & z_l^{abc} \end{bmatrix}$$

Then, one can easily obtain the branch series admittance matrix from the expression above, and it is given by:

$$y^{abc} = (z^{abc})^{-1}$$

Finally, the entire three-phase admittance matrix for the 5-Bus system is given by:

$$Y_{Bus}^{abc} = A^T y^{abc} A = \begin{bmatrix} Y_{11}^{abc} & Y_{12}^{abc} & Y_{13}^{abc} & Y_{14}^{abc} & Y_{15}^{abc} \\ Y_{21}^{abc} & Y_{22}^{abc} & Y_{23}^{abc} & Y_{24}^{abc} & Y_{25}^{abc} \\ Y_{31}^{abc} & Y_{32}^{abc} & Y_{33}^{abc} & Y_{34}^{abc} & Y_{35}^{abc} \\ Y_{41}^{abc} & Y_{42}^{abc} & Y_{43}^{abc} & Y_{44}^{abc} & Y_{45}^{abc} \\ Y_{51}^{abc} & Y_{52}^{abc} & Y_{53}^{abc} & Y_{54}^{abc} & Y_{55}^{abc} \end{bmatrix}$$

Since this matrix does not include the shunt elements, one must perform some changes to it. First, the two generators must be included.

$$Y_{11}^{abc} = Y_{11}^{abc} + y_g^{abc}$$

$$Y_{22}^{abc} = Y_{22}^{abc} + y_g^{abc}$$

Then, one must include both transformers. The first one, is a YT/YT connected transformer and is connected between buses 1 and 3.

$$\bar{Y}_{node} = \begin{bmatrix} \bar{Y}_I & -\bar{Y}_I \\ -\bar{Y}_I & \bar{Y}_I \end{bmatrix}$$

$$\bar{Y}_{11}^{abc} = \bar{Y}_{11}^{abc} + \bar{Y}_{node_{11}} - U$$

$$\bar{Y}_{13}^{abc} = \bar{Y}_{13}^{abc} + \bar{Y}_{node_{12}} + U$$

$$\bar{Y}_{31}^{abc} = \bar{Y}_{31}^{abc} + \bar{Y}_{node_{21}} + U$$

$$\bar{Y}_{33}^{abc} = \bar{Y}_{33}^{abc} + \bar{Y}_{node_{22}} - U$$

The second one, is a Δ/Δ connected transformer and is connected between buses 2 and 4.

$$\bar{Y}_{node} = \begin{bmatrix} \bar{Y}_{II} & -\bar{Y}_{II} \\ -\bar{Y}_{II} & \bar{Y}_{II} \end{bmatrix}$$

$$\bar{Y}_{22}^{abc} = \bar{Y}_{22}^{abc} + \bar{Y}_{node_{11}} - U$$

$$\bar{Y}_{24}^{abc} = \bar{Y}_{24}^{abc} + \bar{Y}_{node_{12}} + U$$

$$\bar{Y}_{42}^{abc} = \bar{Y}_{42}^{abc} + \bar{Y}_{node_{21}} + U$$

$$\bar{Y}_{44}^{abc} = \bar{Y}_{44}^{abc} + \bar{Y}_{node_{22}} - U$$

Finally, the three-phase admittance matrix of the system is completed.

$$\bar{Y}^{abc} = \bar{Y}_{Bus}^{abc}$$

$$Y^{abc} = j\left[\begin{matrix} -30.0000 & -5.0000 & -5.0000 & 0.0000 & 0.0000 & 0.0000 & 20.0000 & 0.0000 & 0.0000 \\ -5.0000 & -30.0000 & -5.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 20.0000 & 0.0000 \\ -5.0000 & -5.0000 & -30.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 20.0000 \\ 0.0000 & 0.0000 & 0.0000 & -23.3333 & 1.6667 & 1.6667 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.6667 & -23.3333 & 1.6667 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.6667 & 1.6667 & -23.3333 & 0.0000 & 0.0000 & 0.0000 \\ 20.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -35.5556 & 4.4444 & 4.4444 \\ 0.0000 & 20.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 4.4444 & -35.5556 & 4.4444 \\ 0.0000 & 0.0000 & 20.0000 & 0.0000 & 0.0000 & 0.0000 & 4.4444 & 4.4444 & -35.5556 \\ 0.0000 & 0.0000 & 0.0000 & 13.3333 & -6.6667 & -6.6667 & 7.7778 & -2.2222 & -2.2222 \\ 0.0000 & 0.0000 & 0.0000 & -6.6667 & 13.3333 & -6.6667 & -2.2222 & 7.7778 & -2.2222 \\ 0.0000 & 0.0000 & 0.0000 & -6.6667 & -6.6667 & 13.3333 & -2.2222 & -2.2222 & 7.7778 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 7.7778 & -2.2222 & -2.2222 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -2.2222 & 7.7778 & -2.2222 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -2.2222 & -2.2222 & 7.7778 \end{matrix}\right.$$

$$\cdots \left.\begin{matrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 13.3333 & -6.6667 & -6.6667 & 0.0000 & 0.0000 & 0.0000 \\ -6.6667 & 13.3333 & -6.6667 & 0.0000 & 0.0000 & 0.0000 \\ -6.6667 & -6.6667 & 13.3333 & 0.0000 & 0.0000 & 0.0000 \\ 7.7778 & -2.2222 & -2.2222 & 7.7778 & -2.2222 & -2.2222 \\ -2.2222 & 7.7778 & -2.2222 & -2.2222 & 7.7778 & -2.2222 \\ -2.2222 & -2.2222 & 7.7778 & -2.2222 & -2.2222 & 7.7778 \\ -28.8889 & 11.1111 & 11.1111 & 7.7778 & -2.2222 & -2.2222 \\ 11.1111 & -28.8889 & 11.1111 & -2.2222 & 7.7778 & -2.2222 \\ 11.1111 & 11.1111 & -28.8889 & -2.2222 & -2.2222 & 7.7778 \\ 7.7778 & -2.2222 & -2.2222 & -15.5556 & 4.4444 & 4.4444 \\ -2.2222 & 7.7778 & -2.2222 & 4.4444 & -15.5556 & 4.4444 \\ -2.2222 & -2.2222 & 7.7778 & 4.4444 & 4.4444 & -15.5556 \end{matrix}\right]$$

5.1.3. Fault Current

Since bus 5 is the faulted bus, let us consider the following sub-matrix:

$$Z_{55}^{abc} = j\begin{bmatrix} 0.2167 & 0.0417 & 0.0417 \\ 0.0417 & 0.2167 & 0.0417 \\ 0.0417 & 0.0417 & 0.2167 \end{bmatrix}$$

where Z_{55}^{abc} is the impedance sub-matrix, regarding the faulted bus, obtained from $Z^{abc} = (Y^{abc})^{-1}$.

For a Three-Phase Fault, the fault current in the faulted bus is given by:

$$I_{f_5}^{abc} = (Z_{55}^{abc} + Z_f^{abc} + Z_g)^{-1} V_5^{abc} = \begin{bmatrix} 5.0616\angle -102.9631° \\ 5.0616\angle 137.0369° \\ 5.0616\angle 17.0369° \end{bmatrix}$$

And the total fault current vector is shown below:

$$I_f^{abc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5.0616\angle-102.9631° \\ 5.0616\angle137.0369° \\ 5.0616\angle17.0369° \end{bmatrix}$$

5.1.4. Post-Fault Bus Voltages

The changes in the post-fault bus voltages are computed as follows:

$$\Delta V = -Z^{abc} I_f^{abc}$$

And the pre-fault bus voltages vector for the 5-Bus system are given by:

$$V_{pre}^{abc} = \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \\ V_5^{abc} \end{bmatrix}$$

Thus, the post-fault voltages are:

$$V_{new}^{abc} = V_{pre}^{abc} + \Delta V = \begin{bmatrix} 0.5193\angle12.6293° \\ 0.5193\angle-107.3707° \\ 0.5193\angle132.6293° \\ 0.5193\angle12.6293° \\ 0.5193\angle-107.3707° \\ 0.5193\angle132.6293° \\ 0.2892\angle0.4811° \\ 0.2892\angle-119.5189° \\ 0.2892\angle120.4811° \\ 0.2850\angle-3.7358° \\ 0.2850\angle-123.7358° \\ 0.2850\angle116.2642° \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

5.1.5. Post-Fault Branch Currents

Considering the post-fault voltages vector obtained above, the line voltage vector is given by:

$$V_{line} = AV_{new}^{abc} = \begin{bmatrix} 0.2442\angle 27.0582° \\ 0.2442\angle -92.9339° \\ 0.2442\angle 147.0646° \\ 0.2586\angle 30.7206° \\ 0.2586\angle -89.2689° \\ 0.2586\angle 150.7195° \\ 0.0215\angle 77.1250° \\ 0.0215\angle -42.9345° \\ 0.0215\angle -162.9950° \\ 0.2892\angle 0.4755° \\ 0.2892\angle -119.5164° \\ 0.2892\angle 120.4746° \\ 0.2850\angle -3.7419° \\ 0.2850\angle -123.7402° \\ 0.2850\angle 116.2594° \end{bmatrix}$$

Then, one must compute the branch series admittance matrix as shown below:

$$Y_{line} = \begin{bmatrix} -Y_{11}^{abc} & Zero & Zero & Zero & Zero \\ Zero & -Y_{22}^{abc} & Zero & Zero & Zero \\ Zero & Zero & -Y_{33}^{abc} & Zero & Zero \\ Zero & Zero & Zero & -Y_{44}^{abc} & Zero \\ Zero & Zero & Zero & Zero & -Y_{55}^{abc} \end{bmatrix}$$

Finally, the post-fault branch current vector can be easily obtained as follows:

$$I_{line} = Y_{line}V_{line} = \begin{bmatrix} 4.8850\angle -62.9404° \\ 4.8850\angle 177.0596° \\ 4.8850\angle 57.0596° \\ 5.1721\angle -59.2783° \\ 5.1721\angle -179.2783° \\ 5.1721\angle 60.7217° \\ 0.2154\angle -12.8750° \\ 0.2154\angle -132.8750° \\ 0.2154\angle 107.1250° \\ 2.8924\angle -89.5189° \\ 2.8924\angle 150.4811° \\ 2.8924\angle 30.4811° \\ 2.8503\angle -93.7358° \\ 2.8503\angle 146.2642° \\ 2.8503\angle 26.2642° \end{bmatrix}$$

This concludes the step-by-step fault analysis of a three-phase fault in the 5-Bus system, where bus 5 is the faulted bus. Also, the comparison of the results obtained by each method are represented below.

	Fault Current $(p.u.)$		
	Phase A	Phase B	Phase C
012	$5.0616\angle-102.9631°$	$5.0616\angle137.0369°$	$5.0616\angle17.0369°$
abc	$5.0616\angle-102.9631°$	$5.0616\angle137.0369°$	$5.0616\angle17.0369°$

Table 5.5: Fault Current (3ϕ fault)

		Post − Fault Bus Voltages $(p.u.)$		
	Bus	Phase A	Phase B	Phase C
012	1	$0.5193\angle12.6293°$	$0.5193\angle-107.3707°$	$0.5193\angle132.6293°$
	2	$0.5193\angle12.6293°$	$0.5193\angle-107.3707°$	$0.5193\angle132.6293°$
	3	$0.2892\angle0.4811°$	$0.2892\angle-119.5189°$	$0.2892\angle120.4811°$
	4	$0.2850\angle-3.7358°$	$0.2850\angle-123.7358°$	$0.2850\angle116.2642°$
	5	0.0	0.0	0.0
abc	1	$0.5193\angle12.6293°$	$0.5193\angle-107.3707°$	$0.5193\angle132.6293°$
	2	$0.5193\angle12.6293°$	$0.5193\angle-107.3707°$	$0.5193\angle132.6293°$
	3	$0.2892\angle0.4811°$	$0.2892\angle-119.5189°$	$0.2892\angle120.4811°$
	4	$0.2850\angle-3.7358°$	$0.2850\angle-123.7358°$	$0.2850\angle116.2642°$
	5	0.0	0.0	0.0

Table 5.6: Post-Fault Bus Voltages (3ϕ fault)

		Post − Fault Branch Currents $(p.u.)$		
	Line	Phase A	Phase B	Phase C
012	1 − 3	$4.8850\angle-62.9404°$	$4.8850\angle177.0596°$	$4.8850\angle57.0596°$
	2 − 4	$5.1721\angle-59.2783°$	$5.1721\angle-179.2783°$	$5.1721\angle60.7217°$
	3 − 4	$0.2154\angle-12.8750°$	$0.2154\angle-132.8750°$	$0.2154\angle107.1250°$
	3 − 5	$2.8924\angle-89.5189°$	$2.8924\angle150.4811°$	$2.8924\angle30.4811°$
	4 − 5	$2.8503\angle-93.7358°$	$2.8503\angle146.2642°$	$2.8503\angle26.2642°$
abc	1 − 3	$4.8850\angle-62.9404°$	$4.8850\angle177.0596°$	$4.8850\angle57.0596°$
	2 − 4	$5.1721\angle-59.2783°$	$5.1721\angle-179.2783°$	$5.1721\angle60.7217°$
	3 − 4	$0.2154\angle-12.8750°$	$0.2154\angle-132.8750°$	$0.2154\angle107.1250°$
	3 − 5	$2.8924\angle-89.5189°$	$2.8924\angle150.4811°$	$2.8924\angle30.4811°$
	4 − 5	$2.8503\angle-93.7358°$	$2.8503\angle146.2642°$	$2.8503\angle26.2642°$

Table 5.7: Post-Fault Branch Currents (3ϕ fault)

5.2. Line-To-Ground Fault

Fault Type	Faulted Bus	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$	$z_g\ (p.u.)$
Line-To-Ground Fault	5	0	∞	∞	0

Table 5.8: Fault Data (5-Bus System)

	$Fault\ Current\ (p.u.)$		
	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	$4.0882\angle -102.9631°$	0.0	0.0
abc	$4.0882\angle -102.9631°$	0.0	0.0

Table 5.9: Fault Current (LG fault)

		$Post-Fault\ Bus\ Voltages\ (p.u.)$		
	Bus	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	1	$0.6724\angle 6.5266°$	$0.9822\angle -116.1969°$	$0.9549\angle 117.0067°$
	2	$0.7369\angle 4.7589°$	$0.9689\angle -112.2715°$	$0.9126\angle 113.7265°$
	3	$0.4422\angle -4.2159°$	$0.9022\angle -126.8067°$	$0.8978\angle 109.4464°$
	4	$0.3043\angle -4.3236°$	$0.9661\angle -135.5026°$	$0.9744\angle 114.9540°$
	5	0.0	$0.9821\angle -141.6022°$	$0.9821\angle 151.6760°$
abc	1	$0.6724\angle 6.5266°$	$0.9822\angle -116.1969°$	$0.9549\angle 117.0067°$
	2	$0.7369\angle 4.7589°$	$0.9689\angle -112.2715°$	$0.9126\angle 113.7265°$
	3	$0.4422\angle -4.2159°$	$0.9022\angle -126.8067°$	$0.8978\angle 109.4464°$
	4	$0.3043\angle -4.3236°$	$0.9661\angle -135.5026°$	$0.9744\angle 114.9540°$
	5	0.0	$0.9821\angle -141.6022°$	$0.9821\angle 151.6760°$

Table 5.10: Post-Fault Bus Voltages (LG fault)

		$Post-Fault\ Branch\ Currents\ (p.u.)$		
	$Line$	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	$1-3$	$5.0357\angle -64.3650°$	$3.8315\angle -146.0787°$	$2.6955\angle 88.2196°$
	$2-4$	$4.4020\angle -48.7180°$	$3.3611\angle -160.4279°$	$4.4417\angle 86.6109°$
	$3-4$	$0.5024\angle -77.5752°$	$0.6499\angle -112.4770°$	$0.2888\angle -124.9217°$
	$3-5$	$2.6406\angle -88.2094°$	$0.8854\angle -143.9526°$	$0.5048\angle 87.5048°$
	$4-5$	$2.1488\angle -90.6823°$	$0.4741\angle 170.3457°$	$0.7645\angle 75.8171°$
	$1-3$	$5.0357\angle -64.3650°$	$3.8315\angle -146.0787°$	$2.6955\angle 88.2196°$
	$2-4$	$4.4020\angle -48.7180°$	$3.3611\angle -160.4279°$	$4.4417\angle 86.6109°$

abc	$3-4$	$0.5024\angle-77.5752°$	$0.6499\angle-112.4770°$	$0.2888\angle-124.9217°$
	$3-5$	$2.6406\angle-88.2094°$	$0.8854\angle-143.9526°$	$0.5048\angle 87.5048°$
	$4-5$	$2.1488\angle-90.6823°$	$0.4741\angle 170.3457°$	$0.7645\angle 75.8171°$

Table 5.11: Post-Fault Branch Currents (LG fault)

5.3. Line-To-Line Fault

Fault Type	Faulted Bus	$z_f(p.u.)$
Line-To-Line Fault	4	$-$

Table 5.12: Fault Data (5-Bus System)

	$Fault\ Current\ (p.u.)$		
	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	0.0	$5.6731\angle 169.8995°$	$5.6731\angle-10.1005°$
abc	0.0	$5.6731\angle 169.8995°$	$5.6731\angle-10.1005°$

Table 5.13: Fault Current (LL fault)

		$Post-Fault\ Bus\ Voltages\ (p.u.)$		
	Bus	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	1	$1.0\angle 0.0°$	$0.5561\angle-137.8427°$	$0.6963\angle 147.5868°$
	2	$1.0\angle 0.0°$	$0.4579\angle-148.1227°$	$0.6573\angle 158.4159°$
	3	$0.9165\angle-8.7552°$	$0.4744\angle-167.9933°$	$0.5019\angle 151.6718°$
	4	$0.9152\angle-10.1005°$	$0.4576\angle 169.8995°$	$0.4576\angle 169.8995°$
	5	$0.8858\angle-12.9631°$	$0.4819\angle 174.0506°$	$0.4117\angle 158.8194°$
abc	1	$1.0\angle 0.0°$	$0.5561\angle-137.8427°$	$0.6963\angle 147.5868°$
	2	$1.0\angle 0.0°$	$0.4579\angle-148.1227°$	$0.6573\angle 158.4159°$
	3	$0.9165\angle-8.7552°$	$0.4744\angle-167.9933°$	$0.5019\angle 151.6718°$
	4	$0.9152\angle-10.1005°$	$0.4576\angle 169.8995°$	$0.4576\angle 169.8995°$
	5	$0.8858\angle-12.9631°$	$0.4819\angle 174.0506°$	$0.4117\angle 158.8194°$

Table 5.14: Post-Fault Bus Voltages (LL fault)

		$Post - Fault\ Branch\ Currents\ (p.u.)$		
	$Line$	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	$1-3$	$3.3665\angle -34.0299°$	$5.5874\angle -169.3222°$	$3.9769\angle 47.2276°$
	$2-4$	$3.7715\angle -31.6672°$	$6.5580\angle -169.1546°$	$4.552\angle 44.8495°$
	$3-4$	$0.2154\angle -12.8750°$	$1.7944\angle 175.6929°$	$1.5817\angle -3.1445°$
	$3-5$	$0.7293\angle -35.7386°$	$1.4942\angle -174.1083°$	$1.0656\angle 32.9378°$
	$4-5$	$0.5374\angle -44.6984°$	$0.4181\angle -43.5595°$	$0.9555\angle 135.7999°$
abc	$1-3$	$3.3665\angle -34.0299°$	$5.5874\angle -169.3222°$	$3.9769\angle 47.2276°$
	$2-4$	$3.7715\angle -31.6672°$	$6.5580\angle -169.1546°$	$4.552\angle 44.8495°$
	$3-4$	$0.2154\angle -12.8750°$	$1.7944\angle 175.6929°$	$1.5817\angle -3.1445°$
	$3-5$	$0.7293\angle -35.7386°$	$1.4942\angle -174.1083°$	$1.0656\angle 32.9378°$
	$4-5$	$0.5374\angle -44.6984°$	$0.4181\angle -43.5595°$	$0.9555\angle 135.7999°$

Table 5.15: Post-Fault Branch Currents (LL fault)

5.4. Double-Line-To-Ground Fault

Fault Type	Faulted Bus	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$	$z_g\ (p.u.)$
Double-Line-To-Ground Fault	4	∞	0	0	$j0.1$

Table 5.16: Fault Data (5-Bus System)

	$Fault\ Current\ (p.u.)$		
	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	0.0	$5.7649\angle 159.6612°$	$5.7649\angle 0.1379°$
abc	0.0	$5.7649\angle 159.6612°$	$5.7649\angle 0.1379°$

Table 5.17: Fault Current (LLG fault)

		$Post - Fault\ Bus\ Voltages\ (p.u.)$		
	Bus	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	1	$0.9743\angle 0.2694°$	$0.5192\angle -132.2232°$	$0.6372\angle 145.3915°$
	2	$0.9249\angle 0.8295°$	$0.4302\angle -144.7204°$	$0.6199\angle 157.7134°$
	3	$0.9094\angle -8.7448°$	$0.3783\angle -161.9405°$	$0.4026\angle 146.9489°$
	4	$1.0247\angle -10.1005°$	$0.2049\angle 169.8995°$	$0.2049\angle 169.8995°$
	5	$0.9369\angle -12.8066°$	$0.3033\angle 176.5031°$	$0.2382\angle 150.5019°$
	1	$0.9743\angle 0.2694°$	$0.5192\angle -132.2232°$	$0.6372\angle 145.3915°$

abc	2	$0.9249\angle 0.8295°$	$0.4302\angle -144.7204°$	$0.6199\angle 157.7134°$
	3	$0.9094\angle -8.7448°$	$0.3783\angle -161.9405°$	$0.4026\angle 146.9489°$
	4	$1.0247\angle -10.1005°$	$0.2049\angle 169.8995°$	$0.2049\angle 169.8995°$
	5	$0.9369\angle -12.8066°$	$0.3033\angle 176.5031°$	$0.2382\angle 150.5019°$

Table 5.18: Post-Fault Bus Voltages (LLG fault)

		$Post-Fault\ Branch\ Currents\ (p.u.)$		
	$Line$	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	$1-3$	$3.2306\angle -27.8294°$	$5.3487\angle -177.7029°$	$4.7004\angle 52.7228°$
	$2-4$	$3.9279\angle -36.8529°$	$6.4922\angle -170.7280°$	$4.7147\angle 46.1817°$
	$3-4$	$0.3254\angle 38.5012°$	$1.8241\angle 158.0440°$	$1.7389\angle 15.3568°$
	$3-5$	$0.6840\angle -26.0830°$	$1.4426\angle 175.2181°$	$1.2716\angle 42.1306°$
	$4-5$	$0.6186\angle -54.4472°$	$0.6186\angle -65.6484°$	$0.8321\angle 151.8561°$
abc	$1-3$	$3.2306\angle -27.8294°$	$5.3487\angle -177.7029°$	$4.7004\angle 52.7228°$
	$2-4$	$3.9279\angle -36.8529°$	$6.4922\angle -170.7280°$	$4.7147\angle 46.1817°$
	$3-4$	$0.3254\angle 38.5012°$	$1.8241\angle 158.0440°$	$1.7389\angle 15.3568°$
	$3-5$	$0.6840\angle -26.0830°$	$1.4426\angle 175.2181°$	$1.2716\angle 42.1306°$
	$4-5$	$0.6186\angle -54.4472°$	$0.6186\angle -65.6484°$	$0.8321\angle 151.8561°$

Table 5.19: Post-Fault Branch Currents (LLG fault)

5.5. Open Conductor Faults

5.5.1. One Conductor Open (Phase a)

Fault Type	Faulted Line	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$
One Conductor Open	$4-5$	∞	0	0

Table 5.20: Fault Data (5-Bus System)

	$Fault\ Current\ (p.u.)$		
	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	0.0	$0.4781\angle -147.9533°$	$0.4781\angle 58.5564°$
abc	0.0	$0.4781\angle -147.9533°$	$0.4781\angle 58.5564°$

Table 5.21: Fault Current (OCO fault)

	Bus	$Post - Fault\ Bus\ Voltages\ (p.u.)$		
		$Phase\ A$	$Phase\ B$	$Phase\ C$
012	1	$1.0006\angle 0.0350°$	$1.0018\angle -119.7033°$	$1.0052\angle 120.1060°$
	2	$0.9994\angle -0.0350°$	$0.9982\angle -120.2977°$	$0.9948\angle 119.8929°$
	3	$0.9171\angle -8.7008°$	$0.9177\angle -128.3337°$	$0.9225\angle 111.4486°$
	4	$0.9187\angle -9.7804°$	$0.9068\angle -130.4918°$	$0.9124\angle 109.2701°$
	5	$0.8839\angle -13.1646°$	$0.8959\angle -131.4931°$	$0.9021\angle 108.2479°$
abc	1	$1.0006\angle 0.0350°$	$1.0018\angle -119.7033°$	$1.0052\angle 120.1060°$
	2	$0.9994\angle -0.0350°$	$0.9982\angle -120.2977°$	$0.9948\angle 119.8929°$
	3	$0.9171\angle -8.7008°$	$0.9177\angle -128.3337°$	$0.9225\angle 111.4486°$
	4	$0.9187\angle -9.7804°$	$0.9068\angle -130.4918°$	$0.9124\angle 109.2701°$
	5	$0.8839\angle -13.1646°$	$0.8959\angle -131.4931°$	$0.9021\angle 108.2479°$

Table 5.22: Post-Fault Bus Voltages (OCO fault)

5.5.2. Two Conductors Open (Phases b and c)

Fault Type	Faulted Line	$z_f^a\ (p.u.)$	$z_f^b\ (p.u.)$	$z_f^c\ (p.u.)$
Two Conductors Open	$4-5$	0	∞	∞

Table 5.23: Fault Data (5-Bus System)

	$Fault\ Current\ (p.u.)$		
	$Phase\ A$	$Phase\ B$	$Phase\ C$
012	$0.3115\angle -44.6984°$	0.0	0.0
abc	$0.3115\angle -44.6984°$	0.0	0.0

Table 5.24: Fault Current (TCO fault)

	Bus	$Post - Fault\ Bus\ Voltages\ (p.u.)$		
		$Phase\ A$	$Phase\ B$	$Phase\ C$
012	1	$1.0017\angle 0.0993°$	$1.0012\angle -120.0177°$	$0.9997\angle 120.0675°$
	2	$0.9983\angle -0.0997°$	$0.9988\angle -119.9822°$	$1.0003\angle 119.9325°$
	3	$0.9183\angle -8.6009°$	$0.9180\angle -128.7651°$	$0.9159\angle 111.3320°$
	4	$0.9076\angle -10.7986°$	$0.9240\angle -130.1445°$	$0.9114\angle 110.4024°$
	5	$0.8969\angle -11.8349°$	$0.8806\angle -132.9529°$	$0.8882\angle 106.7420°$
	1	$1.0017\angle 0.0993°$	$1.0012\angle -120.0177°$	$0.9997\angle 120.0675°$

abc	2	$0.9983\angle -0.0997°$	$0.9988\angle -119.9822°$	$1.0003\angle 119.9325°$
	3	$0.9183\angle -8.6009°$	$0.9180\angle -128.7651°$	$0.9159\angle 111.3320°$
	4	$0.9076\angle -10.7986°$	$0.9240\angle -130.1445°$	$0.9114\angle 110.4024°$
	5	$0.8969\angle -11.8349°$	$0.8806\angle -132.9529°$	$0.8882\angle 106.7420°$

Table 5.25: Post-Fault Bus Voltages (TCO fault)

6. Conclusions and Future Work

6.1. Conclusions

The objective of this dissertation was to apply a direct three-phase method in power system fault analysis and compare the results with the ones obtained through the traditional way, the so-called symmetrical components method. The goal was to achieve the same results and to demonstrate that the three-phase method could solve the same problems with more efficiency and less effort. Also, based on the demonstrations one can verify that the direct three-phase method, when applied to more complex cases, would solve them without many complications.

In order to achieve this work purposes an initial approach was made to the symmetrical components method, considering all its limitations due to the assumption that all the system components are symmetric with respect to all phases and that both generated and load power are equally valued for all phases. As a starting point for this study, a typical example of a power system was considered, and the fault analysis was made for all relevant types of faults. This well-known method starts by computing the three sequence admittance matrices, followed by the determination of fault current and both post-fault voltages and line currents. In general, it is a very simple procedure, however, the calculations regarding the fault current differ between every type of fault and get particularly complicated when it comes to line opening faults, also known as shunt faults. In addition, if we try to analyse a power system with a larger number of buses it will get much more complex to solve and impossible to solve for most real-life situations.

Regarding the direct three-phase method, the first step was to establish the three-phase models for all power system components so that the admittance matrix for the system could be build. In this step one can immediately notice that the matrix complexity is three times higher than in the symmetrical components method, which make the computations more difficult to perform without a computer. However, and considering the same power system example, the calculations made with matlab were much easier to perform, as expected. Now, when considering fault current calculations, there are less differences in calculations between the different types of series fault, which is an advantage when compared to the traditional method. Also, regarding the shunt faults calculations, and despite the small difference and a slightly difficulty increase in calculations, it is a straightforward method that can be applied to every type of power system with less effort and more accurate results than through the traditional way. Finally, both post-fault voltages and line currents computations are very direct and simple as they were in the symmetrical components method, being the main advantage the fact that it is as easier for a small system as it is for a larger one, no matter the number of buses in it.

Considering now both methods and the results obtained, one can say that these results were the ones expected since they are practically the same. Also, they demonstrate that not only the symmetrical components method is a good approach in certain cases but that the direct three-phase representation method is a much reliable method that requires only a computer with the ability to perform all the calculations.

The method proposed in this dissertation articulates several progresses made in this specific area of fault analysis regarding power systems, despite the few paperwork on the matter so far. This work assembles some concepts already explored on their own on the subject and makes an important connection between them in order to structure a valid and appropriate method to perform all kinds of fault calculations with resource to computational tools that can support the computational complexity implied.

In conclusion, considering the accurate results obtained in this dissertation and all the theoretical support behind the calculations and all the procedures, it is fair to say that this method is most appropriated when it comes to fault calculations regarding power system analysis.

6.2. Future Work

Further developments in this matter could be achieved and this dissertation could be a good starting point. Since all the computations regarding the direct three-phase method for fault analysis in power systems have additional computational complexity when compared to the traditional method, a good development to this work would be a more rigorous computer program with a proper interface to allow any person to perform all the calculations needed regarding any type of fault and any kind of power system no matter the number of buses and components in it. This would be a very helpful tool in this field of study and a very interesting challenge. Also, several additional tests may be required to fully validate this method regarding any real-life situation, which may be a lot easier with access to a computational tool such as the one described before.

Bibliography

[1] J. P. S. Paiva, *Redes de Energia Eléctrica: Uma Análise Sistémica*, 4th ed. IST Press, 2015.

[2] J. C. Das, *Power System Analysis: Short-Circuit Load Flow and Harmonics*, 2nd ed. CRC Press, 2012.

[3] Stagg and El-Abiad, *Computer Methods in Power System Analysis*, 12th ed. McGraw-Hill International Editions, 1987.

[4] Y. Hase, *Handbook of Power System Engineering*, John Wiley & Sons, 2007.

[5] N. D. Tleis, *Power Systems Modelling and Fault Analysis: Theory and Practice*, Oxford University Press, 2008.

[6] P. M. Anderson, *Analysis of Faulted Power Systems*, IEEE Press Series on Power Engineering, 1995.

[7] J. J. Grainger and W. D. Stevenson, Jr., *Power System Analysis*, McGraw-Hill International Editions, 1994.

[8] A. R. Bergen and V. Vittal, *Power System Analysis*, 2nd ed. Prentice Hall, 2000.

[9] J. J. Winder, Jr., *Power Transformers: Principles and Applications*, Marcel Dekker, 2002.

[10] J. D. Glover, T. J. Overbye, M. S. Sarma, *Power System Analysis and Design*, 6th ed. Cengage Learning, 2017.

[11] P. S. R. Murthy, *Power System Analysis*, BS Publications, 2007.

[12] T. K. Nagsarkar and M. S. Sukhija, *Power System Analysis*, 2nd ed. India: Oxford University Press, 2014.

[13] J. Santamaria, “Analysis of Power Systems Under Fault Conditions”, Master’s thesis, Dept. Electrical and Electronic Eng., California State Univ., Sacramento, CA, 2011.

[14] J. Zhu, “Analysis of Transmission System Faults in the Phase Domain”, Master’s thesis, Dept. Elect. Eng., Texas A&M Univ., College Station, TX, 2004.

[15] A. Eslami, "A three-phase comprehensive methodology to analyse short circuits, open circuits and internal faults of transformers based on the compensation theorem", *International Journal of Electrical Power & Energy Systems*, vol. 96, pp. 238-252, Mar2018.

[16] R. K. Gajbhiye, P. Kulkarni, S. A. Soman, "Analysis of faulted power systems in three phase coordinates – a generic approach", in *2005 International Power Engineering Conference*,

Singapore, 2005, pp.1052-1057.

[17] “4.11 Example of fault calculation for three phase and LG faults in power system network”, https://studylib.net/doc/18577420/4.11-example-of-fault-calculation-for-three-phase-and-lg, Accessed: 2021-02-10.

[18] “4.12 Example of fault calculation for LL and LLG fault in power system network”, https://www.coursehero.com/file/49120729/EE330-lecture11pdf/, Accessed: 2021-02-10.

[19] “4.13 Open conductor faults”, https://www.coursehero.com/file/49120733/EE330-lecture12pdf/, Accessed: 2021-02-10.

[20] “4.14 Example of fault calculation for open conductor fault in power system network”, https://www.coursehero.com/file/49120710/EE330-lecture13pdf/, Accessed: 2021-02-10.

Appendix A

MATLAB Code - Symmetrical Components Method

```
%% Fault Analysis – 012 Analysis

% Cleanning
clc;
clear;

% Generators G1 and G2
xg_0 = 0.05; xg_1 = 0.2; xg_2 = xg_1;
zg_0 = 1j * xg_0;
zg_1 = 1j * xg_1;
zg_2 = 1j * xg_2;

% Transformers T1 and T2
xt_0 = 0.05; xt_1 = xt_0; xt_2 = xt_0;
zt_0 = 1j * xt_0;
zt_1 = 1j * xt_1;
zt_2 = 1j * xt_2;

% Transmission Lines L1, L2, and L3
xl_0 = 0.3; xl_1 = 0.1; xl_2 = xl_1;
zl_0 = 1j * xl_0;
zl_1 = 1j * xl_1;
zl_2 = 1j * xl_2;

% Pre – Fault Bus Voltages
v1 = 1; v2 = v1;
v3 = 0.9058 – 1j * 0.1395;
v4 = 0.9010 – 1j * 0.1605;
v5 = 0.8632 – 1j * 0.1987;

V_pre = [v1; v2; v3; v4; v5];

disp('Pre – Fault Bus Voltages:');
```

```
disp([abs(V_pre) angle(V_pre) * 180/pi]);


% Fortescue Matrix (T)
a = 1 * exp(1j * 2 * pi/3);

T = [ 1   1   1
   1  a^2   a
   1   a   a^2
  ];

% A. Admittance Matrices
% Sequence Admittance Matrices
Y_0 = [1/zg_0 + 1/zt_0    0      − 1/zt_0      0      0
       0     1/zg_0     0        0       0
     −1/zt_0    0    1/zt_0 + 1/zl_0 + 1/zl_0   − 1/zl_0    − 1/zl_0
       0      0     − 1/zl_0     1/zl_0 + 1/zl_0   − 1/zl_0
       0      0     − 1/zl_0      − 1/zl_0   1/zl_0 + 1/zl_0
   ];

Y_1 = [1/zg_1 + 1/zt_1     0       − 1/zt_1       0       0
       0     1/zg_1 + 1/zt_1     0        − 1/zt_1      0
     −1/zt_1      0     1/zt_1 + 1/zl_1 + 1/zl_1    − 1/zl_1      − 1/zl_1
       0      − 1/zt_1      − 1/zl_1     1/zt_1 + 1/zl_1 + 1/zl_1   − 1/zl_1
       0       0       − 1/zl_1       − 1/zl_1     1/zl_1 + 1/zl_1
   ];

Y_2 = [1/zg_2 + 1/zt_2     0       − 1/zt_2       0       0
       0     1/zg_2 + 1/zt_2     0        − 1/zt_2      0
     −1/zt_2      0     1/zt_2 + 1/zl_2 + 1/zl_2    − 1/zl_2      − 1/zl_2
       0      − 1/zt_2      − 1/zl_2     1/zt_2 + 1/zl_2 + 1/zl_2   − 1/zl_2
       0       0       − 1/zl_2       − 1/zl_2     1/zl_2 + 1/zl_2
   ];

Z_0 = inv(Y_0);
Z_1 = inv(Y_1);
Z_2 = inv(Y_2);
```

$\%imag(Y\_0); imag(Y\_1); imag(Y\_2);$

$\%imag(Z\_0); imag(Z\_1); imag(Z\_2);$

$\%\% 1 - Three - phase\ Bolted\ fault\ at\ bus\ 5\ (Zg = 0)$

$Zf = 0;$

$Zg = 0;$

$\%v3 = 1;\ v4 = 1;\ v5 = 1;$

$\%\ B. Fault\ current$

$I\_012\_5f = v5/(Z\_1(5,5) + Zf) * [0;\ 1;\ 0];$

$I\_abc\_5f = T * I\_012\_5f;$

$\%\ C. Post - Fault\ Bus\ Voltages$

$V1\_new\_012 = [0;\ v1;\ 0] - Z\_1(5,1) * I\_012\_5f;$

$V2\_new\_012 = [0;\ v2;\ 0] - Z\_1(5,2) * I\_012\_5f;$

$V3\_new\_012 = [0;\ v3;\ 0] - Z\_1(5,3) * I\_012\_5f;$

$V4\_new\_012 = [0;\ v4;\ 0] - Z\_1(5,4) * I\_012\_5f;$

$V5\_new\_012 = [0;\ v5;\ 0] - Z\_1(5,5) * I\_012\_5f;$

$V1\_new\_abc = T * V1\_new\_012;$

$V2\_new\_abc = T * V2\_new\_012;$

$V3\_new\_abc = T * V3\_new\_012;$

$V4\_new\_abc = T * V4\_new\_012;$

$V5\_new\_abc = T * V5\_new\_012;$

$\%\ D. Post - Fault\ Branch\ Currents$

$I\_012\_13 = [(V1\_new\_012(1) - V3\_new\_012(1))/zt\_0$

$(V1\_new\_012(2) - V3\_new\_012(2))/zt\_1$

$(V1\_new\_012(3) - V3\_new\_012(3))/zt\_2];$

$I\_012\_24 = [(V2\_new\_012(1) - V4\_new\_012(1))/zt\_0$

$(V2\_new\_012(2) - V4\_new\_012(2))/zt\_1$

$(V2\_new\_012(3) - V4\_new\_012(3))/zt\_2];$

$I\_012\_34 = [(V3\_new\_012(1) - V4\_new\_012(1))/zl\_0$

$(V3\_new\_012(2) - V4\_new\_012(2))/zl\_1$

$(V3\_new\_012(3) - V4\_new\_012(3))/zl\_2];$

$I\_012\_35 = [(V3\_new\_012(1) - V5\_new\_012(1))/zl\_0$

$(V3\_new\_012(2) - V5\_new\_012(2))/zl\_1$

$(V3\_new\_012(3) - V5\_new\_012(3))/zl\_2];$

```
I_012_45 = [(V4_new_012(1) − V5_new_012(1))/zl_0
        (V4_new_012(2) − V5_new_012(2))/zl_1
        (V4_new_012(3) − V5_new_012(3))/zl_2];


I_abc_13 = T ∗ I_012_13;
I_abc_24 = T ∗ I_012_24;
I_abc_34 = T ∗ I_012_34;
I_abc_35 = T ∗ I_012_35;
I_abc_45 = T ∗ I_012_45;


% Results
disp('1 − Three − Phase Bolted Fault at Bus 5'); disp(' ');
disp('Fault Current:');
disp([abs(I_abc_5f) angle(I_abc_5f) ∗ 180/pi]);
disp('Bus Voltages:');
disp([abs(V1_new_abc) angle(V1_new_abc) ∗ 180/pi; abs(V2_new_abc) angle(V2_new_abc)
        ∗ 180/pi; abs(V3_new_abc) angle(V3_new_abc)
        ∗ 180/pi; abs(V4_new_abc) angle(V4_new_abc)
        ∗ 180/pi; abs(V5_new_abc) angle(V5_new_abc) ∗ 180/pi]);
disp('Line Currents:');
disp([abs(I_abc_13) angle(I_abc_13) ∗ 180/pi; abs(I_abc_24) angle(I_abc_24)
        ∗ 180/pi; abs(I_abc_34) angle(I_abc_34) ∗ 180/pi; abs(I_abc_35) angle(I_abc_35)
        ∗ 180/pi; abs(I_abc_45) angle(I_abc_45) ∗ 180/pi]);


%% 2 − Line − to − ground fault at bus 5 (Zf = 0)
Zf = 0;
%v3 = 1; v4 = 1; v5 = 1;


% B. Fault current
I_012_5f = (v5/(Z_0(5,5) + Z_1(5,5) + Z_2(5,5) + 3 ∗ Zf)) ∗ [1; 1; 1];


I_abc_5f = T ∗ I_012_5f;


% C. Post − Fault Bus Voltages
V1_new_012 = [0; v1; 0] − [Z_0(5,1); Z_1(5,1); Z_2(5,1)].∗ I_012_5f;
V2_new_012 = [0; v2; 0] − [Z_0(5,2); Z_1(5,2); Z_2(5,2)].∗ I_012_5f;
V3_new_012 = [0; v3; 0] − [Z_0(5,3); Z_1(5,3); Z_2(5,3)].∗ I_012_5f;
V4_new_012 = [0; v4; 0] − [Z_0(5,4); Z_1(5,4); Z_2(5,4)].∗ I_012_5f;
V5_new_012 = [0; v5; 0] − [Z_0(5,5); Z_1(5,5); Z_2(5,5)].∗ I_012_5f;
```

```
V1_new_abc = T * V1_new_012;
V2_new_abc = T * V2_new_012;
V3_new_abc = T * V3_new_012;
V4_new_abc = T * V4_new_012;
V5_new_abc = T * V5_new_012;

% D.Post - Fault Branch Currents
I_012_13 = [(V1_new_012(1) - V3_new_012(1))/zt_0
    (V1_new_012(2) - V3_new_012(2))/zt_1
    (V1_new_012(3) - V3_new_012(3))/zt_2];
I_012_24 = [(V2_new_012(1) - V4_new_012(1)) * 0
    (V2_new_012(2) - V4_new_012(2))/zt_1
    (V2_new_012(3) - V4_new_012(3))/zt_2];
%I_012_24(1) = 0;
I_012_34 = [(V3_new_012(1) - V4_new_012(1))/zl_0
    (V3_new_012(2) - V4_new_012(2))/zl_1
    (V3_new_012(3) - V4_new_012(3))/zl_2];
I_012_35 = [(V3_new_012(1) - V5_new_012(1))/zl_0
    (V3_new_012(2) - V5_new_012(2))/zl_1
    (V3_new_012(3) - V5_new_012(3))/zl_2];
I_012_45 = [(V4_new_012(1) - V5_new_012(1))/zl_0
    (V4_new_012(2) - V5_new_012(2))/zl_1
    (V4_new_012(3) - V5_new_012(3))/zl_2];

I_abc_13 = T * I_012_13;
I_abc_24 = T * I_012_24;
I_abc_34 = T * I_012_34;
I_abc_35 = T * I_012_35;
I_abc_45 = T * I_012_45;

% Results
disp('2 - Line - to - Ground Fault at Bus 5 (Zf = 0)'); disp(' ');
disp('Fault Current:');
disp([abs(I_abc_5f) angle(I_abc_5f) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V1_new_abc) angle(V1_new_abc) * 180/pi; abs(V2_new_abc) angle(V2_new_abc)
            * 180/pi; abs(V3_new_abc) angle(V3_new_abc)
            * 180/pi; abs(V4_new_abc) angle(V4_new_abc)
            * 180/pi; abs(V5_new_abc) angle(V5_new_abc) * 180/pi]);
```

$disp('Line\ Currents:');$

$disp([abs(I\_abc\_13)\ angle(I\_abc\_13)*180/pi;\ abs(I\_abc\_24)\ angle(I\_abc\_24)*180/pi;\ abs(I\_abc\_34)\ angle(I\_abc\_34)*180/pi;\ abs(I\_abc\_35)\ angle(I\_abc\_35)*180/pi;\ abs(I\_abc\_45)\ angle(I\_abc\_45)*180/pi]);$

$\%\%\ 3\ -\ Line-to-Line\ fault\ at\ bus\ 4$

$\%v3=1;\ v4=1;\ v5=1;$

$Zf=0;$

$\%\ B. Fault\ current$

$I\_012\_4f=(v4/(Z\_1(4,4)+Z\_2(4,4)+Zf))*[0;1;-1];$

$I\_abc\_4f=T*I\_012\_4f;$

$\%\ C. Post-Fault\ Bus\ Voltages$

$V1\_new\_012=[0;\ v1;\ 0]-[0;\ Z\_1(4,1);\ Z\_2(4,1)].*I\_012\_4f;$

$V2\_new\_012=[0;\ v2;\ 0]-[0;\ Z\_1(4,2);\ Z\_2(4,2)].*I\_012\_4f;$

$V3\_new\_012=[0;\ v3;\ 0]-[0;\ Z\_1(4,3);\ Z\_2(4,3)].*I\_012\_4f;$

$V4\_new\_012=[0;\ v4;\ 0]-[0;\ Z\_1(4,4);\ Z\_2(4,4)].*I\_012\_4f;$

$V5\_new\_012=[0;\ v5;\ 0]-[0;\ Z\_1(4,5);\ Z\_2(4,5)].*I\_012\_4f;$

$V1\_new\_abc=T*V1\_new\_012;$

$V2\_new\_abc=T*V2\_new\_012;$

$V3\_new\_abc=T*V3\_new\_012;$

$V4\_new\_abc=T*V4\_new\_012;$

$V5\_new\_abc=T*V5\_new\_012;$

$\%\ D. Post-Fault\ Branch\ Currents$

$I\_012\_13=[(V1\_new\_012(1)-V3\_new\_012(1))/zt\_0$

$(V1\_new\_012(2)-V3\_new\_012(2))/zt\_1$

$(V1\_new\_012(3)-V3\_new\_012(3))/zt\_2];$

$I\_012\_24=[(V2\_new\_012(1)-V4\_new\_012(1))/zt\_0$

$(V2\_new\_012(2)-V4\_new\_012(2))/zt\_1$

$(V2\_new\_012(3)-V4\_new\_012(3))/zt\_2];$

$I\_012\_34=[(V3\_new\_012(1)-V4\_new\_012(1))/zl\_0$

$(V3\_new\_012(2)-V4\_new\_012(2))/zl\_1$

$(V3\_new\_012(3)-V4\_new\_012(3))/zl\_2];$

$I\_012\_35=[(V3\_new\_012(1)-V5\_new\_012(1))/zl\_0$

$(V3\_new\_012(2)-V5\_new\_012(2))/zl\_1$

$(V3\_new\_012(3) - V5\_new\_012(3))/zl\_2];$

$I\_012\_45 = [(V4\_new\_012(1) - V5\_new\_012(1))/zl\_0$

$(V4\_new\_012(2) - V5\_new\_012(2))/zl\_1$

$(V4\_new\_012(3) - V5\_new\_012(3))/zl\_2];$

$I\_abc\_13 = T * I\_012\_13;$

$I\_abc\_24 = T * I\_012\_24;$

$I\_abc\_34 = T * I\_012\_34;$

$I\_abc\_35 = T * I\_012\_35;$

$I\_abc\_45 = T * I\_012\_45;$

$\%\ Results$

$disp('3\ -\ Line - to - Line\ Fault\ at\ Bus\ 4');\ disp('\ ');$

$disp('Fault\ Current:');$

$disp([abs(I\_abc\_4f)\ angle(I\_abc\_4f) * 180/pi]);$

$disp('Bus\ Voltages:');$

$$disp([abs(V1\_new\_abc)\ angle(V1\_new\_abc) * 180/pi;\ abs(V2\_new\_abc)\ angle(V2\_new\_abc) * 180/pi;\ abs(V3\_new\_abc)\ angle(V3\_new\_abc) * 180/pi;\ abs(V4\_new\_abc)\ angle(V4\_new\_abc) * 180/pi;\ abs(V5\_new\_abc)\ angle(V5\_new\_abc) * 180/pi]);$$

$disp('Line\ Currents:');$

$$disp([abs(I\_abc\_13)\ angle(I\_abc\_13) * 180/pi;\ abs(I\_abc\_24)\ angle(I\_abc\_24) * 180/pi;\ abs(I\_abc\_34)\ angle(I\_abc\_34) * 180/pi;\ abs(I\_abc\_35)\ angle(I\_abc\_35) * 180/pi;\ abs(I\_abc\_45)\ angle(I\_abc\_45) * 180/pi]);$$

$\%\%\ 4\ -\ Double - Phase - to - Ground\ fault\ at\ bus\ 4\ (Zf\ = j0.1)$

$Zf\ = 0.1i;$

$Zg\ = 0;$

$\%v3 = 1;\ v4 = 1;\ v5 = 1;$

$\%\ B. Fault\ current$

$$I\_012\_4f = v4 * [-(Z\_2(4,4) + Zf)/((Z\_1(4,4) + Zf) * (Z\_2(4,4) + Zf) + (Z\_1(4,4) + Zf) * (Z\_0(4,4) + 3 * Zg + Zf) + Z\_2(4,4) * (Z\_0(4,4) + 3 * Zg + Zf))$$

$$((Z\_0(4,4) + 3 * Zg + Zf) + (Z\_2(4,4) + Zf))/((Z\_1(4,4) + Zf) * (Z\_2(4,4) + Zf) + (Z\_1(4,4) + Zf) * (Z\_0(4,4) + 3 * Zg + Zf) + (Z\_2(4,4) + Zf) * (Z\_0(4,4) + 3 * Zg + Zf))$$

$$-(Z\_0(4,4) + 3 * Zg + Zf)/((Z\_1(4,4) + Zf) * (Z\_2(4,4) + Zf) + (Z\_1(4,4) + Zf) * (Z\_0(4,4) + 3 * Zg + Zf) + (Z\_2(4,4) + Zf) * (Z\_0(4,4) + 3 * Zg + Zf))$$

$];$

$I\_abc\_4f = T * I\_012\_4f;$

$\% \ C.Post - Fault\ Bus\ Voltages$

$V1\_new\_012 = [0;\ v1;\ 0] - [Z\_0(4,1);\ Z\_1(4,1);\ Z\_2(4,1)].* I\_012\_4f;$

$V2\_new\_012 = [0;\ v2;\ 0] - [Z\_0(4,2);\ Z\_1(4,2);\ Z\_2(4,2)].* I\_012\_4f;$

$V3\_new\_012 = [0;\ v3;\ 0] - [Z\_0(4,3);\ Z\_1(4,3);\ Z\_2(4,3)].* I\_012\_4f;$

$V4\_new\_012 = [0;\ v4;\ 0] - [Z\_0(4,4);\ Z\_1(4,4);\ Z\_2(4,4)].* I\_012\_4f;$

$V5\_new\_012 = [0;\ v5;\ 0] - [Z\_0(4,5);\ Z\_1(4,5);\ Z\_2(4,5)].* I\_012\_4f;$

$V1\_new\_abc = T * V1\_new\_012;$

$V2\_new\_abc = T * V2\_new\_012;$

$V3\_new\_abc = T * V3\_new\_012;$

$V4\_new\_abc = T * V4\_new\_012;$

$V5\_new\_abc = T * V5\_new\_012;$

$\% \ D.Post - Fault\ Branch\ Currents$

$I\_012\_13 = [(V1\_new\_012(1) - V3\_new\_012(1))/zt\_0$

$(V1\_new\_012(2) - V3\_new\_012(2))/zt\_1$

$(V1\_new\_012(3) - V3\_new\_012(3))/zt\_2];$

$I\_012\_24 = [(V2\_new\_012(1) - V4\_new\_012(1)) * 0$

$(V2\_new\_012(2) - V4\_new\_012(2))/zt\_1$

$(V2\_new\_012(3) - V4\_new\_012(3))/zt\_2];$

$I\_012\_34 = [(V3\_new\_012(1) - V4\_new\_012(1))/zl\_0$

$(V3\_new\_012(2) - V4\_new\_012(2))/zl\_1$

$(V3\_new\_012(3) - V4\_new\_012(3))/zl\_2];$

$I\_012\_35 = [(V3\_new\_012(1) - V5\_new\_012(1))/zl\_0$

$(V3\_new\_012(2) - V5\_new\_012(2))/zl\_1$

$(V3\_new\_012(3) - V5\_new\_012(3))/zl\_2];$

$I\_012\_45 = [(V4\_new\_012(1) - V5\_new\_012(1))/zl\_0$

$(V4\_new\_012(2) - V5\_new\_012(2))/zl\_1$

$(V4\_new\_012(3) - V5\_new\_012(3))/zl\_2];$

$I\_abc\_13 = T * I\_012\_13;$

$I\_abc\_24 = T * I\_012\_24;$

$I\_abc\_34 = T * I\_012\_34;$

$I\_abc\_35 = T * I\_012\_35;$

$I\_abc\_45 = T * I\_012\_45;$

$\% \ Results$

$disp('4\ -\ Double - Phase - to - Ground\ fault\ at\ bus\ 4\ (Zf = j0.1)');\ disp('\ ');$

```
disp('Fault Current:');

disp([abs(I_abc_4f) angle(I_abc_4f) * 180/pi]);

disp('Bus Voltages:');

disp([abs(V1_new_abc) angle(V1_new_abc) * 180/pi; abs(V2_new_abc) angle(V2_new_abc)
        * 180/pi; abs(V3_new_abc) angle(V3_new_abc)
        * 180/pi; abs(V4_new_abc) angle(V4_new_abc)
        * 180/pi; abs(V5_new_abc) angle(V5_new_abc) * 180/pi]);

disp('Line Currents:');

disp([abs(I_abc_13) angle(I_abc_13) * 180/pi; abs(I_abc_24) angle(I_abc_24)
        * 180/pi; abs(I_abc_34) angle(I_abc_34) * 180/pi; abs(I_abc_35) angle(I_abc_35)
        * 180/pi; abs(I_abc_45) angle(I_abc_45) * 180/pi]);


%% 5 − Open Conductor Fault (between buses 4 and 5)

% Pre − Fault Bus Voltages

v3 = 0.9058 − 1j * 0.1395;

v4 = 0.9010 − 1j * 0.1605;

v5 = 0.8632 − 1j * 0.1987;


% Pre − Fault Current in phase a of the Faulted Line

I_45 = (v4 − v5)/zl_1;


% Thevenin's Impedance of the Network as Seen From Buses 4 and 5

Zth_45_0 = Z_0(4,4) + Z_0(5,5) − 2 * Z_0(4,5);

Zth_45_1 = Z_1(4,4) + Z_1(5,5) − 2 * Z_1(4,5);

Zth_45_2 = Z_2(4,4) + Z_2(5,5) − 2 * Z_2(4,5);


% Thevenin's Equivalent Impedances as Seen From Buses k and k'

Z_kk_0 = −(zl_0)^2/(Zth_45_0 − zl_0);

Z_kk_1 = −(zl_1)^2/(Zth_45_1 − zl_1);

Z_kk_2 = −(zl_2)^2/(Zth_45_2 − zl_2);


%% 5.1 − One Conductor Open (phase a)

% Sequence Currents for Phase a

Ia_1 = I_45 * (Z_kk_1 * (Z_kk_0 + Z_kk_2)/(Z_kk_0 * Z_kk_1 + Z_kk_1 * Z_kk_2 + Z_kk_2 * Z_kk_0));


% Sequence Voltage Drops

V_kk_1 = Ia_1 * Z_kk_0 * Z_kk_2/(Z_kk_0 + Z_kk_2);


%V_kk_1 = I_45 * Z_kk_0 * Z_kk_1 * Z_kk_2/(Z_kk_0 * Z_kk_1 + Z_kk_1 * Z_kk_2 + Z_kk_2 * Z_kk_0);

V_kk_2 = V_kk_1;
```

$V\_kk\_0 = V\_kk\_1;$

$V\_kk\_012 = [V\_kk\_0;\ V\_kk\_1;\ V\_kk\_2];$

$\%\ Three - Phase\ Voltage\ Drops$

$V\_kk\_abc = T * V\_kk\_012;$

$\%\ As\ a\ check\ let\ us\ compute\ Ia2\ and\ Ia0$

$Ia\_2 = -V\_kk\_2/Z\_kk\_2;$

$Ia\_0 = -V\_kk\_0/Z\_kk\_0;$

$\%\ Total\ Current\ for\ Phase\ a$

$Ia\_012 = [Ia\_0;\ Ia\_1;\ Ia\_2];$

$Ia = Ia\_0 + Ia\_1 + Ia\_2;$

$\%\ Changes\ in\ Bus\ Voltages$

$\%\ Bus\ 1$

$dV1\_012 = [(Z\_0(1{,}4) - Z\_0(1{,}5)) * V\_kk\_0/zl\_0$

$(Z\_1(1{,}4) - Z\_1(1{,}5)) * V\_kk\_1/zl\_1$

$(Z\_2(1{,}4) - Z\_2(1{,}5)) * V\_kk\_2/zl\_2$

$];$

$\%\ Bus\ 2$

$dV2\_012 = [(Z\_0(2{,}4) - Z\_0(2{,}5)) * V\_kk\_0/zl\_0$

$(Z\_1(2{,}4) - Z\_1(2{,}5)) * V\_kk\_1/zl\_1$

$(Z\_2(2{,}4) - Z\_2(2{,}5)) * V\_kk\_2/zl\_2$

$];$

$\%\ Bus\ 3$

$dV3\_012 = [(Z\_0(3{,}4) - Z\_0(3{,}5)) * V\_kk\_0/zl\_0$

$(Z\_1(3{,}4) - Z\_1(3{,}5)) * V\_kk\_1/zl\_1$

$(Z\_2(3{,}4) - Z\_2(3{,}5)) * V\_kk\_2/zl\_2$

$];$

$\%\ Bus\ 4$

$dV4\_012 = [(Z\_0(4{,}4) - Z\_0(4{,}5)) * V\_kk\_0/zl\_0$

$(Z\_1(4{,}4) - Z\_1(4{,}5)) * V\_kk\_1/zl\_1$

$(Z\_2(4{,}4) - Z\_2(4{,}5)) * V\_kk\_2/zl\_2$

$];$

$\%\ Bus\ 5$

$dV5\_012 = [(Z\_0(5{,}4) - Z\_0(5{,}5)) * V\_kk\_0/zl\_0$

$(Z\_1(5{,}4) - Z\_1(5{,}5)) * V\_kk\_1/zl\_1$

$(Z\_2(5,4) - Z\_2(5,5)) * V\_kk\_2/zl\_2$

];

% $Total$

$dV1 = dV1\_012(1) + dV1\_012(2) + dV1\_012(3);$

$dV2 = dV2\_012(1) + dV2\_012(2) + dV2\_012(3);$

$dV3 = dV3\_012(1) + dV3\_012(2) + dV3\_012(3);$

$dV4 = dV4\_012(1) + dV4\_012(2) + dV4\_012(3);$

$dV5 = dV5\_012(1) + dV5\_012(2) + dV5\_012(3);$

% $Bus\ Voltages\ During\ Fault$

$V1\_df = v1 + dV1;$

$V2\_df = v2 + dV2;$

$V3\_df = v3 + dV3;$

$V4\_df = v4 + dV4;$

$V5\_df = v5 + dV5;$

$V1\_df\_012 = [0; v1; 0] + dV1\_012;$

$V2\_df\_012 = [0; v2; 0] + dV2\_012;$

$V3\_df\_012 = [0; v3; 0] + dV3\_012;$

$V4\_df\_012 = [0; v4; 0] + dV4\_012;$

$V5\_df\_012 = [0; v5; 0] + dV5\_012;$

% $Current\ in\ Line\ 4 - 5$

$I\_012\_45 = Ia\_012;$

$I\_abc\_45 = T * I\_012\_45;$

% $Post - Fault\ Bus\ Voltages$

$V1\_new\_012 = [0;\ v1;\ 0] - [Z\_0(4,1) - Z\_0(5,1);\ Z\_1(4,1) - Z\_1(5,1);\ Z\_2(4,1) - Z\_2(5,1)].* I\_012\_45;$

$V2\_new\_012 = [0;\ v2;\ 0] - [Z\_0(4,2) - Z\_0(5,2);\ Z\_1(4,2) - Z\_1(5,2);\ Z\_2(4,2) - Z\_2(5,2)].* I\_012\_45;$

$V3\_new\_012 = [0;\ v3;\ 0] - [Z\_0(4,3) - Z\_0(5,3);\ Z\_1(4,3) - Z\_1(5,3);\ Z\_2(4,3) - Z\_2(5,3)].* I\_012\_45;$

$V4\_new\_012 = [0;\ v4;\ 0] - [Z\_0(4,4) - Z\_0(5,4);\ Z\_1(4,4) - Z\_1(5,4);\ Z\_2(4,4) - Z\_2(5,4)].* I\_012\_45;$

$V5\_new\_012 = [0;\ v5;\ 0] - [Z\_0(4,5) - Z\_0(5,5);\ Z\_1(4,5) - Z\_1(5,5);\ Z\_2(4,5) - Z\_2(5,5)].* I\_012\_45;$

$V1\_new\_abc = T * V1\_new\_012;$

$V2\_new\_abc = T * V2\_new\_012;$

$V3\_new\_abc = T * V3\_new\_012;$

$V4\_new\_abc = T * V4\_new\_012;$

$V5\_new\_abc = T * V5\_new\_012;$

```
% Results
disp('5.1 − One Conductor Open (phase a)'); disp(' ');
disp('Sequence Currents for Phase a:');
%disp(Ia_012);
disp([abs(Ia_012) angle(Ia_012) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V1_new_abc) angle(V1_new_abc) * 180/pi; abs(V2_new_abc) angle(V2_new_abc)
        * 180/pi; abs(V3_new_abc) angle(V3_new_abc)
        * 180/pi; abs(V4_new_abc) angle(V4_new_abc)
        * 180/pi; abs(V5_new_abc) angle(V5_new_abc) * 180/pi]);
disp('Line Fault Current:');
disp([abs(I_abc_45) angle(I_abc_45) * 180/pi]);


%% 5.2 − Two Conductors Open (phases b and c)
% Sequence Currents for Phase a
Ia_1 = I_45 * (Z_kk_1)/(Z_kk_0 + Z_kk_1 + Z_kk_2);


% As a check let us compute Ia2 and Ia0
Ia_2 = Ia_1;
Ia_0 = Ia_1;


% Total Current for Phase a
Ia_012 = [Ia_0; Ia_1; Ia_2];
Ia = Ia_0 + Ia_1 + Ia_2;


% Sequence Voltage Drops
%V_kk_1 = I_45 * Z_kk_1/(Z_kk_0 + Z_kk_1 + Z_kk_2) * (Z_kk_0 + Z_kk_2);


V_kk_1 = Ia_1 * (Z_kk_0 + Z_kk_2);
V_kk_2 = −Ia_2 * Z_kk_2;
V_kk_0 = −Ia_0 * Z_kk_0;
V_kk_012 = [V_kk_0; V_kk_1; V_kk_2];


% Three − Phase Voltage Drops
V_kk_abc = T * V_kk_012;


% Changes in Bus Voltages
% Bus 1
```

$dV1\_012 = [(Z\_0(1,4) - Z\_0(1,5)) * V\_kk\_0/zl\_0$

$(Z\_1(1,4) - Z\_1(1,5)) * V\_kk\_1/zl\_1$

$(Z\_2(1,4) - Z\_2(1,5)) * V\_kk\_2/zl\_2$

];

% Bus 2

$dV2\_012 = [(Z\_0(2,4) - Z\_0(2,5)) * V\_kk\_0/zl\_0$

$(Z\_1(2,4) - Z\_1(2,5)) * V\_kk\_1/zl\_1$

$(Z\_2(2,4) - Z\_2(2,5)) * V\_kk\_2/zl\_2$

];

% Bus 3

$dV3\_012 = [(Z\_0(3,4) - Z\_0(3,5)) * V\_kk\_0/zl\_0$

$(Z\_1(3,4) - Z\_1(3,5)) * V\_kk\_1/zl\_1$

$(Z\_2(3,4) - Z\_2(3,5)) * V\_kk\_2/zl\_2$

];

% Bus 4

$dV4\_012 = [(Z\_0(4,4) - Z\_0(4,5)) * V\_kk\_0/zl\_0$

$(Z\_1(4,4) - Z\_1(4,5)) * V\_kk\_1/zl\_1$

$(Z\_2(4,4) - Z\_2(4,5)) * V\_kk\_2/zl\_2$

];

% Bus 5

$dV5\_012 = [(Z\_0(5,4) - Z\_0(5,5)) * V\_kk\_0/zl\_0$

$(Z\_1(5,4) - Z\_1(5,5)) * V\_kk\_1/zl\_1$

$(Z\_2(5,4) - Z\_2(5,5)) * V\_kk\_2/zl\_2$

];

% $Total$

$dV1 = dV1\_012(1) + dV1\_012(2) + dV1\_012(3);$

$dV2 = dV2\_012(1) + dV2\_012(2) + dV2\_012(3);$

$dV3 = dV3\_012(1) + dV3\_012(2) + dV3\_012(3);$

$dV4 = dV4\_012(1) + dV4\_012(2) + dV4\_012(3);$

$dV5 = dV5\_012(1) + dV5\_012(2) + dV5\_012(3);$

% $Bus\ Voltages\ During\ Fault$

$V1\_new = v1 + dV1;$

$V2\_new = v2 + dV2;$

$V3\_new = v3 + dV3;$

$V4\_new = v4 + dV4;$

$V5\_new = v5 + dV5;$

```
% Bus Voltages During Fault
V1_df = v1 + dV1;
V2_df = v2 + dV2;
V3_df = v3 + dV3;
V4_df = v4 + dV4;
V5_df = v5 + dV5;

V1_df_012 = [0; v1; 0] + dV1_012;
V2_df_012 = [0; v2; 0] + dV2_012;
V3_df_012 = [0; v3; 0] + dV3_012;
V4_df_012 = [0; v4; 0] + dV4_012;
V5_df_012 = [0; v5; 0] + dV5_012;

% Current in Line 4 - 5
I_012_45 = Ia_012;
I_abc_45 = T * I_012_45;

% Post - Fault Bus Voltages
V1_new_012 = [0; v1; 0] - [Z_0(4,1) - Z_0(5,1); Z_1(4,1) - Z_1(5,1); Z_2(4,1) - Z_2(5,1)].* I_012_45;
V2_new_012 = [0; v2; 0] - [Z_0(4,2) - Z_0(5,2); Z_1(4,2) - Z_1(5,2); Z_2(4,2) - Z_2(5,2)].* I_012_45;
V3_new_012 = [0; v3; 0] - [Z_0(4,3) - Z_0(5,3); Z_1(4,3) - Z_1(5,3); Z_2(4,3) - Z_2(5,3)].* I_012_45;
V4_new_012 = [0; v4; 0] - [Z_0(4,4) - Z_0(5,4); Z_1(4,4) - Z_1(5,4); Z_2(4,4) - Z_2(5,4)].* I_012_45;
V5_new_012 = [0; v5; 0] - [Z_0(4,5) - Z_0(5,5); Z_1(4,5) - Z_1(5,5); Z_2(4,5) - Z_2(5,5)].* I_012_45;

V1_new_abc = T * V1_new_012;
V2_new_abc = T * V2_new_012;
V3_new_abc = T * V3_new_012;
V4_new_abc = T * V4_new_012;
V5_new_abc = T * V5_new_012;

% Results
disp('5.2 - Two Conductors Open (phases b and c)'); disp(' ');
disp('Sequence Currents for Phase a:');
%disp(Ia_012);
disp([abs(Ia_012) angle(Ia_012) * 180/pi]);
disp('Bus Voltages:');
```

$$disp([abs(V1\_new\_abc)\ angle(V1\_new\_abc) * 180/pi;\ abs(V2\_new\_abc)\ angle(V2\_new\_abc) * 180/pi;\ abs(V3\_new\_abc)\ angle(V3\_new\_abc) * 180/pi;\ abs(V4\_new\_abc)\ angle(V4\_new\_abc) * 180/pi;\ abs(V5\_new\_abc)\ angle(V5\_new\_abc) * 180/pi]);$$

$$disp('Line\ Fault\ Current:');$$

$$disp([abs(I\_abc\_45)\ angle(I\_abc\_45) * 180/pi]);$$

Appendix B

MATLAB Code – Direct Three-Phase Representation Method

```
%% Fault Analysis - abc Analysis

% Cleanning
clc;
clear;

% Generators G1 and G2
xg_0 = 0.05; xg_1 = 0.2; xg_2 = xg_1;
zg_0 = 1j * xg_0;
zg_1 = 1j * xg_1;
zg_2 = 1j * xg_2;

y_012_g = [1/zg_0 0 0; 0 1/zg_1 0; 0 0 1/zg_2];

% Transformers T1 and T2
xt_0 = 0.05; xt_1 = xt_0; xt_2 = xt_0;
zt_0 = 1j * xt_0;
zt_1 = 1j * xt_1;
zt_2 = 1j * xt_2;
m = 1;

z_012_t = [1 0 0; 0 1 0; 0 0 1];

% Transmission Lines L1, L2, and L3
xl_0 = 0.3; xl_1 = 0.1; xl_2 = xl_1;
zl_0 = 1j * xl_0;
zl_1 = 1j * xl_1;
zl_2 = 1j * xl_2;

z_012_l = [zl_0 0 0; 0 zl_1 0; 0 0 zl_2];

% Fortescue Matrix (T)
a = 1 * exp(1j * 2 * pi/3);
```

```
T = [ 1   1    1
   1  a^2   a
   1   a   a^2
  ];

% Three − Phase Impedances
y_abc_g = T\y_012_g ∗ T;
z_abc_t = T\z_012_t ∗ T;
z_abc_l = T\z_012_l ∗ T;

% Pre − Fault Bus Voltages
v1 = 1; v2 = 1;
v3 = 0.9058 − 1j ∗ 0.1395; %v3 = 1;
v4 = 0.9010 − 1j ∗ 0.1605; %v4 = 1;
v5 = 0.8632 − 1j ∗ 0.1987; %v5 = 1;

v_abc_1 = T(: ,2) ∗ v1;
v_abc_2 = T(: ,2) ∗ v2;
v_abc_3 = T(: ,2) ∗ v3;
v_abc_4 = T(: ,2) ∗ v4;
v_abc_5 = T(: ,2) ∗ v5;

V_pre = [v_abc_1; v_abc_2; v_abc_3; v_abc_4; v_abc_5];

disp('Pre − Fault Bus Voltages: ');
disp([abs(V_pre) angle(V_pre) ∗ 180/pi]);

%% A. Admittance Matrices
% Nodal Admittance Matrix
U = [1 0 0; 0 1 0; 0 0 1];
Zero = [0 0 0; 0 0 0; 0 0 0];

A = [  U  Zero   − U  Zero Zero
  Zero   U  Zero   − U  Zero
  Zero Zero   U    − U  Zero
  Zero Zero   U  Zero   − U
  Zero Zero Zero   U    − U
```

```
];

z_abc = [z_abc_t  Zero   Zero   Zero   Zero
     Zero   z_abc_t  Zero   Zero   Zero
     Zero   Zero   z_abc_l  Zero   Zero
     Zero   Zero   Zero   z_abc_l  Zero
     Zero   Zero   Zero   Zero   z_abc_l
     ];

%y_abc = inv(z_abc);
Y_abc_Bus = A.'/z_abc * A;

% Add Generators
Y_abc_Bus(1:3,1:3) = Y_abc_Bus(1:3,1:3) + y_abc_g;
Y_abc_Bus(4:6,4:6) = Y_abc_Bus(4:6,4:6) + y_abc_g;

% Add Transformer T1 (YT/YT)
y1_t = 1/zt_1; y0 = y1_t; y_I = [y1_t 0 0; 0 y1_t 0; 0 0 y1_t];
Y_Trf = [y_I/(m^2)  -y_I/conj(m); -y_I/m y_I];
Y_abc_Bus(1:3,1:3) = Y_abc_Bus(1:3,1:3) + Y_Trf(1:3,1:3) - U;
Y_abc_Bus(7:9,1:3) = Y_abc_Bus(7:9,1:3) + Y_Trf(4:6,1:3) + U;
Y_abc_Bus(1:3,7:9) = Y_abc_Bus(1:3,7:9) + Y_Trf(1:3,4:6) + U;
Y_abc_Bus(7:9,7:9) = Y_abc_Bus(7:9,7:9) + Y_Trf(4:6,4:6) - U;

% Add Transformer T2 (D/D)
y1_t = 1/zt_1; y0 = y1_t; y_II = (1/3) * [2 * y1_t  - y1_t  - y1_t; -y1_t 2 * y1_t  - y1_t; -y1_t  - y1_t 2 * y1_t];
Y_Trf = [y_II/(m^2)  - y_II/conj(m); -y_II/m y_II];
Y_abc_Bus(4:6,4:6) = Y_abc_Bus(4:6,4:6) + Y_Trf(1:3,1:3) - U;
Y_abc_Bus(10:12,4:6) = Y_abc_Bus(10:12,4:6) + Y_Trf(4:6,1:3) + U;
Y_abc_Bus(4:6,10:12) = Y_abc_Bus(4:6,10:12) + Y_Trf(1:3,4:6) + U;
Y_abc_Bus(10:12,10:12) = Y_abc_Bus(10:12,10:12) + Y_Trf(4:6,4:6) - U;

% Three - Phase Admittance Matrix
Y_abc = Y_abc_Bus;
Z_abc = inv(Y_abc);

%% 1 - Three - phase fault at bus 5
Zf_n = 0;  %Three - Phase - to - Ground Fault
```

```
Zf_a = 0; Zf_b = 0; Zf_c = 0;
Zn = [Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n];
Zf = [Zf_a 0 0; 0 Zf_b 0; 0 0 Zf_c];

% B.Fault Current
Z_55 = Z_abc(13:15,13:15);

I_abc_5f = (Z_55 + (Zf + Zn))\v_abc_5;
I_abc_f = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; I_abc_5f];

% C.Post - Fault Bus Voltages
delta_V = -Z_abc * I_abc_f;
V_pre = [v_abc_1; v_abc_2; v_abc_3; v_abc_4; v_abc_5];
V_new = V_pre + delta_V;

% D.Post - Fault Branch Currents
V_line = A * V_new;
y_abc_13 = -Y_abc(1:3,7:9);
y_abc_24 = -Y_abc(4:6,10:12);
y_abc_34 = -Y_abc(7:9,10:12);
y_abc_35 = -Y_abc(7:9,13:15);
y_abc_45 = -Y_abc(10:12,13:15);
Y_line = [y_abc_13 Zero Zero Zero Zero
    Zero y_abc_24 Zero Zero Zero
    Zero Zero y_abc_34 Zero Zero
    Zero Zero Zero y_abc_35 Zero
    Zero Zero Zero Zero y_abc_45
    ];

I_line = Y_line * V_line;

% Results
disp('1 - Three - phase Bolted Fault at Bus 5'); disp(' ');
disp('Fault Current:');
disp([abs(I_abc_f) angle(I_abc_f) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V_new) angle(V_new) * 180/pi]);
disp('Line Currents:');
```

```
disp([abs(I_line) angle(I_line) * 180/pi]);

%% 2 - Line-to-ground fault at bus 5 (Zf = 0)
Zf_n = 0;
Zf_a = 0; Zf_b = 10e10; Zf_c = 10e10;
Zn = [Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n];
Zf = [Zf_a 0 0; 0 Zf_b 0; 0 0 Zf_c];

% B. Fault Current
Z_55 = Z_abc(13:15,13:15);

I_abc_5f = (Z_55 + (Zf + Zn))\v_abc_5;
I_abc_f = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; I_abc_5f];

% C. Post-Fault Bus Voltages
delta_V = -Z_abc * I_abc_f;
V_pre = [v_abc_1; v_abc_2; v_abc_3; v_abc_4; v_abc_5];
V_new = V_pre + delta_V;

% D. Post-Fault Branch Currents
V_line = A * V_new;
y_abc_13 = -Y_abc(1:3,7:9);
y_abc_24 = -Y_abc(4:6,10:12);
y_abc_34 = -Y_abc(7:9,10:12);
y_abc_35 = -Y_abc(7:9,13:15);
y_abc_45 = -Y_abc(10:12,13:15);
Y_line = [y_abc_13 Zero Zero Zero Zero
    Zero y_abc_24 Zero Zero Zero
    Zero Zero y_abc_34 Zero Zero
    Zero Zero Zero y_abc_35 Zero
    Zero Zero Zero Zero y_abc_45
    ];

I_line = Y_line * V_line;

% Results
disp('2 - Line-to-Ground Fault at Bus 5 (Zf = 0)'); disp(' ');
disp('Fault Current:');
```

$disp([abs(I\_abc\_f)\ angle(I\_abc\_f)*180/pi]);$

$disp('Bus\ Voltages:');$

$disp([abs(V\_new)\ angle(V\_new)*180/pi]);$

$disp('Line\ Currents:');$

$disp([abs(I\_line)\ angle(I\_line)*180/pi]);$

$\%\%\ 3\ -\ Line-to-Line\ fault\ at\ bus\ 4$

$Zf=0;$

$\%\ B.Fault\ Current$

$Z\_44=Z\_abc(10:12,10:12);$

$\%I\_abc\_4f=(Z\_44+Zf/2)\backslash[0\ 0\ 0;\ 0\ 1\ -1;\ 0\ -1\ 1]*v\_abc\_4/2;$

$\%I\_abc\_4f=((Z\_44(2,2)+Z\_44(3,3)-Z\_44(2,3)-Z\_44(3,2)+Zf)\backslash(v\_abc\_4(2)-v\_abc\_4(3)))*[0;1;-1];$

$I\_abc\_4f=((Z\_44(2,2)+Z\_44(3,3)-Z\_44(2,3)-Z\_44(3,2)+Zf)\backslash 1)*[0;v\_abc\_4(2)-v\_abc\_4(3);v\_abc\_4(3)-v\_abc\_4(2)];$

$I\_abc\_f=[0;0;0;0;0;0;0;0;0;I\_abc\_4f;0;0;0];$

$\%\ C.Post-Fault\ Bus\ Voltages$

$delta\_V=-Z\_abc*I\_abc\_f;$

$V\_pre=[v\_abc\_1;\ v\_abc\_2;\ v\_abc\_3;\ v\_abc\_4;\ v\_abc\_5];$

$V\_new=V\_pre+delta\_V;$

$\%\ D.Post-Fault\ Branch\ Currents$

$V\_line=A*V\_new;$

$y\_abc\_13=-Y\_abc(1:3,7:9);$

$y\_abc\_24=-Y\_abc(4:6,10:12);$

$y\_abc\_34=-Y\_abc(7:9,10:12);$

$y\_abc\_35=-Y\_abc(7:9,13:15);$

$y\_abc\_45=-Y\_abc(10:12,13:15);$

$Y\_line=[y\_abc\_13\ Zero\ Zero\ Zero\ Zero$

$Zero\ y\_abc\_24\ Zero\ Zero\ Zero$

$Zero\ Zero\ y\_abc\_34\ Zero\ Zero$

$Zero\ Zero\ Zero\ y\_abc\_35\ Zero$

$Zero\ Zero\ Zero\ Zero\ y\_abc\_45$

$];$

$I\_line=Y\_line*V\_line;$

```
% Results
disp('3 - Line - to - Line Fault at Bus 4'); disp(' ');
disp('Fault Current: ');
disp([abs(I_abc_f) angle(I_abc_f) * 180/pi]);
disp('Bus Voltages: ');
disp([abs(V_new) angle(V_new) * 180/pi]);
disp('Line Currents: ');
disp([abs(I_line) angle(I_line) * 180/pi]);

%% 4 - Double - Phase - to - Ground fault at bus 4 (Zf = j0.1)
Zf_n = 0.1i;
Zf_a = 10e10; Zf_b = 0; Zf_c = 0;
Zn = [Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n; Zf_n Zf_n Zf_n];
Zf = [Zf_a 0 0; 0 Zf_b 0; 0 0 Zf_c];

% B. Fault Current
Z_44 = Z_abc(10:12,10:12);

I_abc_4f = (Z_44 + (Zf + Zn))\v_abc_4;
I_abc_f = [0; 0; 0; 0; 0; 0; 0; 0; 0; I_abc_4f; 0; 0; 0];

% C. Post - Fault Bus Voltages
%Z_abc(10,10) = 0;
delta_V = -Z_abc * I_abc_f;
V_pre = [v_abc_1; v_abc_2; v_abc_3; v_abc_4; v_abc_5];
V_new = V_pre + delta_V;

% D. Post - Fault Branch Currents
V_line = A * V_new;
y_abc_13 = -Y_abc(1:3,7:9);
y_abc_24 = -Y_abc(4:6,10:12);
y_abc_34 = -Y_abc(7:9,10:12);
y_abc_35 = -Y_abc(7:9,13:15);
y_abc_45 = -Y_abc(10:12,13:15);
Y_line = [y_abc_13 Zero Zero Zero Zero
    Zero y_abc_24 Zero Zero Zero
    Zero Zero y_abc_34 Zero Zero
```

```
    Zero Zero Zero y_abc_35 Zero
    Zero Zero Zero Zero y_abc_45
    ];

I_line = Y_line * V_line;

% Results
disp('4 - Double - Phase - to - Ground fault at bus 4 (Zf = j0.1)'); disp(' ');
disp('Fault Current:');
disp([abs(I_abc_f) angle(I_abc_f) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V_new) angle(V_new) * 180/pi]);
disp('Line Currents:');
disp([abs(I_line) angle(I_line) * 180/pi]);

%% 5 - Open Conductor Fault (between buses 4 and 5)
disp('5 - Open Conductor Fault'); disp(' ');

% Pre - Fault Bus Voltages
v1 = 1; v2 = 1;
v3 = 0.9058 - 1j * 0.1395;
v4 = 0.9010 - 1j * 0.1605;
v5 = 0.8632 - 1j * 0.1987;

v_abc_1 = T(:,2) * v1;
v_abc_2 = T(:,2) * v2;
v_abc_3 = T(:,2) * v3;
v_abc_4 = T(:,2) * v4;
v_abc_5 = T(:,2) * v5;

% Pre - Fault Current in phase a of the Faulted Line
I_abc_45 = z_abc_l\(v_abc_4 - v_abc_5);
disp('Pre - Fault Current:');
disp([abs(I_abc_45) angle(I_abc_45) * 180/pi]);

% Thevenin's Impedance of the Network as Seen From Buses 4 and 5
Zth_45 = Z_abc(10:15,10:15);
Vth_45 = [v_abc_4; v_abc_5];
```

$\%\% \ 5.1 - One\ Conductor\ Open\ (phase\ a)$

$Zf\_a = 10e10;\ Zf\_b = 0;\ Zf\_c = 0;$

$\%\ New\ Admittance\ Matrix$

$Y\_abc\_old = Y\_abc(10{:}15{,}10{:}15);$

$Yl\_old = inv(z\_abc\_l);$

$Yl\_new = inv(z\_abc\_l + eye(3).* [Zf\_a;\ Zf\_b;\ Zf\_c]);$

$DY\_45f = Yl\_new - Yl\_old;$

$Y\_45f = [DY\_45f\ \ -DY\_45f;\ -DY\_45f\ DY\_45f];$

$E4 = [eye(3);\ zeros(3{,}3)];$

$E5 = [zeros(3{,}3);\ eye(3)];$

$Y\_abc\_new = Y\_abc\_old + [E4\ E5] * Y\_45f * [E4\ E5].';$

$Y2\_abc = Y\_abc;\ Y2\_abc(10{:}15{,}10{:}15) = Y\_abc\_new;$

$Z2\_abc = inv(Y2\_abc);$

$\%\ During - Fault\ Bus\ Voltages$

$V\_oc = inv(eye(6) + Zth\_45 * Y\_45f) * Vth\_45;$

$\%\ Fault\ Currents$

$I\_shunt = Yl\_new * (V\_oc(1{:}3) - V\_oc(4{:}6));$

$I\_abc\_f = [0;0;0;0;0;0;0;0;0;I\_shunt;-I\_shunt];$

$\%\ C.Post - Fault\ Bus\ Voltages$

$delta\_V = -Z\_abc * I\_abc\_f;$

$V\_pre = [v\_abc\_1;\ v\_abc\_2;\ v\_abc\_3;\ v\_abc\_4;\ v\_abc\_5];$

$V\_new = V\_pre + delta\_V;$

$\%\ D.Post - Fault\ Branch\ Currents$

$V\_line = A * V\_new;$

$y\_abc\_13 = -Y\_abc(1{:}3{,}7{:}9);$

$y\_abc\_24 = -Y\_abc(4{:}6{,}10{:}12);$

$y\_abc\_34 = -Y\_abc(7{:}9{,}10{:}12);$

$y\_abc\_35 = -Y\_abc(7{:}9{,}13{:}15);$

$y\_abc\_45 = -Y\_abc(10{:}12{,}13{:}15);$

$Y\_line = [y\_abc\_13\ Zero\ Zero\ Zero\ Zero$

```
    Zero y_abc_24 Zero Zero Zero
    Zero Zero y_abc_34 Zero Zero
    Zero Zero Zero y_abc_35 Zero
    Zero Zero Zero Zero y_abc_45
    ];

I_line = Y_line * V_line;



% Results
disp('5.1 - One Conductor Open (phase a)'); disp(' ');
disp('Fault Current:');
disp([abs(I_shunt) angle(I_shunt) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V_new) angle(V_new) * 180/pi]);

%% 5.2 - Two Conductors Open (phases b and c)
Zf_a = 0; Zf_b = 10e10; Zf_c = 10e10;

% New Admittance Matrix
Y_abc_old = Y_abc(10:15,10:15);
Yl_old = inv(z_abc_l);
Yl_new = inv(z_abc_l + eye(3).* [Zf_a; Zf_b; Zf_c]);
DY_45f = Yl_new - Yl_old;
Y_45f = [DY_45f -DY_45f; -DY_45f DY_45f];
E4 = [eye(3); zeros(3,3)];
E5 = [zeros(3,3); eye(3)];

Y_abc_new = Y_abc_old + [E4 E5] * Y_45f * [E4 E5].';
Y2_abc = Y_abc; Y2_abc(10:15,10:15) = Y_abc_new;
Z2_abc = inv(Y2_abc);

% During - Fault Bus Voltages
V_oc = inv(eye(6) + Zth_45 * Y_45f) * Vth_45;

% Fault Currents
I_shunt = Yl_new * (V_oc(1:3) - V_oc(4:6));
I_abc_f = [0; 0; 0; 0; 0; 0; 0; 0; 0; I_shunt; -I_shunt];
```

```
% C. Post - Fault Bus Voltages
delta_V = -Z_abc * I_abc_f;
V_pre = [v_abc_1; v_abc_2; v_abc_3; v_abc_4; v_abc_5];
V_new = V_pre + delta_V;

% D. Post - Fault Branch Currents
V_line = A * V_new;
y_abc_13 = -Y_abc(1:3,7:9);
y_abc_24 = -Y_abc(4:6,10:12);
y_abc_34 = -Y_abc(7:9,10:12);
y_abc_35 = -Y_abc(7:9,13:15);
y_abc_45 = -Y_abc(10:12,13:15);
Y_line = [y_abc_13 Zero Zero Zero Zero
    Zero y_abc_24 Zero Zero Zero
    Zero Zero y_abc_34 Zero Zero
    Zero Zero Zero y_abc_35 Zero
    Zero Zero Zero Zero y_abc_45
    ];

I_line = Y_line * V_line;

% Results
disp('5.2 - Two Conductors Open (phases b and c)'); disp(' ');
disp('Fault Current:');
disp([abs(I_shunt) angle(I_shunt) * 180/pi]);
disp('Bus Voltages:');
disp([abs(V_new) angle(V_new) * 180/pi]);
```