Power System Analysis by Direct Three-Phase Representation

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***Abstract* — A direct three-phase representation in power system analysis is advantageous when dealing with unbalanced networks under complex fault situations. The symmetrical components method, which considers that under certain conditions an unbalanced system may be represented as a superposition of three balanced systems, is a satisfactory solution for simple cases but has several limitations regarding complex fault analysis. Conversely, a direct three-phase approach can handle all kinds of fault situations, regardless of the additional computational complexity. Furthermore, with nowadays advanced computational means, and considering the necessity of a more meticulous and complete analysis, the obvious choice is to handle all fault calculations directly in the three-phase domain. The work developed in this dissertation consists in a direct approach to fault calculations in the phase domain supported by computational tool MATLAB. First, the three-phase admittance matrix of the system is built directly, supported by the well-known three-phase models regarding each component. Then, according to the type of fault and considering its boundary conditions, the subsequent calculations are performed directly in the phase domain. This three-phase approach in fault calculations is a substantial improvement regarding power system analysis specially when dealing with complex networks and large number of buses. The results obtained in this work demonstrate that a direct three-phase approach is most adequate in fault analysis. This method, when supported by computational tools, can solve with less effort and more accurate results all the straightforward situations solvable by the symmetrical components method and countless situations impossible to handle by the traditional approach.**

***Keywords* — Power System Analysis; Direct Three-phase Representation; Unbalanced Networks; Symmetrical Components Method; Fault Analysis; Fault Calculations in the Phase Domain; Three-phase Admittance Matrix; Three-phase Models.**

1. Introduction

When we analyse a certain power system, that is supposed to keep continuous its power supply through all buses, we must consider that several events may occur and disturb the network. These events may have different natures, such as physical accidents, wind, lightnings, equipment failures, and so on. The main effect of this unpredictable happenings is a short circuit fault caused by a lightning and, in this case, we know it is a temporary fault. A short circuit fault happens when one phase wire of the transmission line touches the ground or when two phase wires touch each other. Also, when a conductor opens, we have an open conductor fault. Open conductor faults are series faults and short circuit faults are parallel faults. Although both types of faults have different probabilities of occurrence, it is important to consider both in order to be able to perform a complete fault analysis in any power system. The main purpose of this work is to demonstrate that the traditional method used for fault analysis, which is the symmetrical components method, is not appropriate to solve all the situations that we may find. On the other hand, the direct three-phase representation method will solve all the problems that the traditional method already solves, but we can go further and solve situations that without this method we would not be able to. In order to understand the traditional method, one must consider that it depends on turning the three phasors of the system into a new kind of components, which are the positive, negative, and zero sequences. Through the years, the main obstacle regarding the three-phase system analysis was the fact for large systems the calculations were too much to handle for a computer in the past. For instances, for a system with n buses, the sequence method will generate three [n x n] matrices for our fault analysis, while the three-phase method would have to handle a [3n x 3n] matrix. Nowadays, with the increasing computational capacity of our computers, we can perform all these calculations without much effort, even for complicated situations. These two methods will be tested for all transformer configurations, and regarding all types of faults and line openings, in order two verify the accuracy of the results obtained without symmetrical assumptions by the direct three-phase representation method.

2. Symmetrical Components

The symmetrical components method is currently very important tool in the analysis of unbalanced power systems. When a fault occurs in a three-phase power system it unbalances the system. If it occurs in an initial balanced power system, our analysis gets a lot easier. It is only needed to perform the calculations for one of the three phases because the other two are just phase displaced. However, for an unbalanced system this single-phase approach is not valid. In these cases, which are most common, the symmetrical components method is a good approach to perform our analysis. This method converts the unbalanced three-phase currents and voltages of the system into three sets of balanced ones. These three new balanced systems, the so-called symmetrical components, are represented by a positive, negative, and zero sequence networks. This method also allows us to decouple the impedances of the system from each other, which simplifies all the calculations.

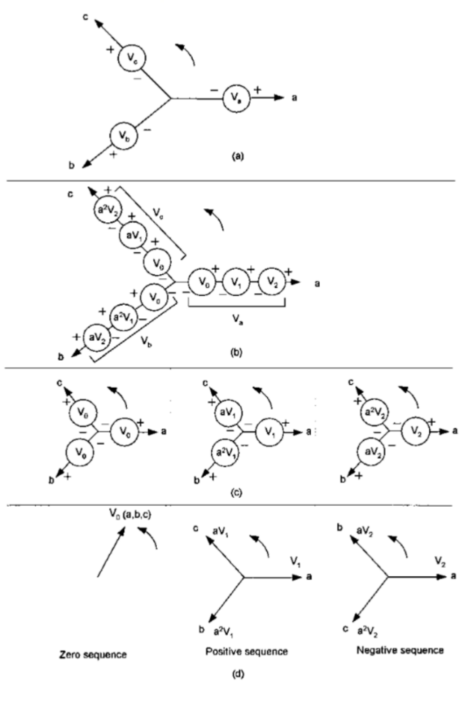

Figure 1: (a), (b), (c), and (d) Progressive resolution of voltage vectors into sequence components.

It is known that the generators of a symmetric three-phase balanced system produce balanced voltages, with its phases displaced by $2\pi/3 = 120°$ from each other. For a three-phase system, the relation between phase components and sequence components of the voltage is given by

$$V^{abc} = T\,V^{012} \tag{2.1}$$

where T represents the Fortscue's matrix. Notice that the reverse transformation can also be applied by inverting the transformation matrix.

3. Unsymmetrical Fault Calculations

In this section each type of fault is going to be analysed through the sequence method. This approach depends on the construction of three distinct sequence networks seen from the faulted point. The first step of this procedure consists in the reduction of the zero, positive, and negative sequence networks into a single Thèvenin sequence impedance. This approach relies on the fact that only the positive sequence network has a voltage source, that corresponds to the pre-fault voltage, being the only active network. In the following sections it is explained how one can manage these three separate networks, in order to perform unsymmetrical fault calculations, by connecting them in a certain way.

3.1. Sequence Admittance Matrix

Considering the three sequence networks, and neglecting the reference node, which is always at ground potential, one can apply the following equation to build each admittance matrix Y

$$I = YV \tag{1}$$

where V is the node voltage vector and I represents the node injected current vector. It is most common to define each current flow as positive when it goes toward the bus, and as negative when flows away from the bus. Also, the node voltage vector V represents the bus voltages measured from the reference node. Finally, Y is the bus admittance matrix. Rewriting Eq. (1) in its matrixial form, one gets the following expression

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{2}$$

Notice that Y is a square matrix of dimensions $n \times n$, where n represents the number of buses in the system, without the reference bus. Its inverse matrix will be

$$Z = Y^{-1} \tag{3}$$

where Z represents the bus impedance matrix, which can be formed simply by inverting the admittance matrix. This matrix is also square and of dimensions $n \times n$. Therefore, Eq. (1) can also be written as follows

$$V = ZI \tag{4}$$

3.2. Fault Current

In order to do fault calculations, one must connect the faulted bus k to the ground through the fault impedance Z_f. Considering the Eq. (4), regardless the fault current, all the remaining node currents will be zero. We can write two equations regarding the faulted bus k,

$$I_k = \frac{V_k}{Z_{kk}} = -I_{f_k} \tag{5}$$

$$V_k = Z_f I_{f_k} - V_a \tag{6}$$

The fault current on Eq. (5) has the negative sign because it goes in direction do the ground, while the injected current goes towards the node. From Eq. (5) and (6), the fault current expression is given by

$$I_{f_k} = \frac{V_a}{Z_{Th}} \tag{7}$$

where V_a represents the Thevenin voltage of bus k before the fault and $Z_{Th} = Z_{kk} + Z_f$.

3.2.1. Three-Phase Fault

For a three-phase fault, all phases are short-circuited through equal fault impedances in series with a ground impedance, Z_f and Z_g. This is a symmetrical fault, meaning that the vectorial sum of fault currents is three times the current in each phase.

$$I_a + I_b + I_c = 3I_a \tag{8}$$

Since a three-phase fault is symmetrical,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_f + Z_g & Z_g & Z_g \\ Z_g & Z_f + Z_g & Z_g \\ Z_g & Z_g & Z_f + Z_g \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{9}$$

Then, the sequence voltages are given by

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = T^{-1} \begin{bmatrix} Z_f + Z_g & Z_g & Z_g \\ Z_g & Z_f + Z_g & Z_g \\ Z_g & Z_g & Z_f + Z_g \end{bmatrix} T \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_f + 3Z_g & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{10}$$

Therefore, the fault current can be calculated by

$$\begin{gathered} I_a = I_1 = \frac{V_a}{Z_1 + Z_f} \\ I_b = a^2 I_1 \\ I_c = a I_1 \end{gathered} \tag{11}$$

where k represents the faulted bus, and $Z_{Th} = Z_1 + Z_f$.

3.2.2 Line-To-Ground Fault

Let us assume that the fault occurs in phase a. Since the load current is neglected, the currents for both phases b and c are zero. The following expression gives us the voltage at the fault point

$$V_a = Z_f I_a \tag{12}$$

The sequence components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix} \tag{13}$$

From (6), comes the following

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \tag{14}$$

$$\begin{aligned} 3I_{a0} Z_f &= V_{a0} + V_{a1} + V_{a2} \\ &= -I_{a0} Z_0 + (V_a - I_{a1} Z_1) - I_{a2} Z_2 \end{aligned} \tag{15}$$

The fault current can now be calculated by

$$I_{a0} = I_{a1} = I_{a2} = \frac{V_a}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{16}$$

and $Z_{Th} = Z_0 + Z_1 + Z_2 + 3Z_f$.

And the total fault current is given by

$$I_a = 3I_{a0} = \frac{3V_a}{Z_{Th}} \tag{17}$$

3.2.3. Line-To-Line Fault

For a line-to-line fault, let us assume that the fault occurs between phases b and c, through a fault impedance Z_f. Therefore, the fault current only circulates through the faulted phases, from phase b to phase c.

$$\begin{gathered} I_a = 0 \\ I_b = -I_c \\ V_b - V_c = Z_f I_b \end{gathered} \tag{18}$$

The sequence components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ -a + a^2 \\ -a^2 + a \end{bmatrix} \tag{19}$$

From Eq. (19), $I_{a0} = 0$ and $I_{a1} = -I_{a2}$.

$$\begin{aligned} V_b - V_c &= [0 \quad 1 \quad -1] \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [0 \quad 1 \quad -1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \\ &= [0 \quad a^2 - a \quad a - a^2] \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \end{aligned} \tag{20}$$

Thus,

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_{a1} - V_{a2}) \\ &= (a^2 I_{a1} + a I_{a2}) Z_f \\ &= (a^2 - a) I_{a1} Z_f \end{aligned} \tag{21}$$

Which gives

$$(V_{a1} - V_{a2}) = I_{a1} Z_f \tag{22}$$

Also

$$I_b = (a^2 - a) I_{a1} = -j\sqrt{3} I_{a1} \tag{23}$$

The sequence fault current can now be calculated by

$$I_{a1} = -I_{a2} = \frac{V_a}{Z_1 + Z_2 + Z_f} \tag{24}$$

and $Z_{Th} = Z_1 + Z_2 + Z_f$.

The fault current is given by

$$I_b = -I_c = -j\sqrt{3} \frac{V_a}{Z_{Th}} \tag{25}$$

3.2.4. Double-Line-To-Ground Fault

For a double line-to-ground fault, let us assume that phases b and c go to ground through two fault impedances Z_f , and a ground impedance Z_g . Therefore, $I_a = 0$, which implies $I_{a0} + I_{a1} + I_{a2} = 0$. The voltage at the faulted point is given by

$$V_b = \left(Z_f + Z_g\right)I_b + Z_g I_c \quad (26)$$

$$V_c = (Z_f + Z_g)I_c + Z_g I_b \quad (27)$$

And the sequence components of the voltages can be written as follows

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} V_a + V_b + V_c \\ V_a + (a + a^2)V_b \\ V_a + +(a + a^2)V_b \end{bmatrix} \quad (28)$$

which gives $V_1 = V_2$, and

$$V_0 = \frac{1}{3}(V_a + 2V_b)$$

$$= \frac{1}{3}\left[(V_0 + V_1 + V_2) + 2(I_b + I_c)Z_f\right]$$

$$= \frac{1}{3}\left[(V_0 + 2V_1) + 2(3I_0)Z_f\right]$$

$$= V_1 + 3I_0 Z_f \quad (29)$$

The fault current can now be calculated by

$$I_{a1} = \frac{V_a}{(Z_1 + Z_f) + [(Z_2 + Z_f)||(Z_0 + Z_f + 3Z_g)]}$$

$$= \frac{V_a}{(Z_1 + Z_f) + \dfrac{(Z_2 + Z_f)(Z_0 + Z_f + 3Z_g)}{\left(Z_2 + Z_f\right) + (Z_0 + Z_f + 3Z_g)}} \quad (30)$$

and $Z_{Th} = (Z_1 + Z_f) + \frac{(Z_2+Z_f)(Z_0+Z_f+3Z_g)}{(Z_2+Z_f)+(Z_0+Z_f+3Z_g)}$.

3.2.5. One-Conductor Open Fault

Let us consider that the conductor with respect to phase a is open.

$$\bar{I}_a = 0 \quad (31)$$

$$\bar{I}_a^{(0)} + \bar{I}_a^{(1)} + \bar{I}_a^{(2)} = 0 \quad (32)$$

$$\bar{V}_{kk',b} = 0, \qquad \bar{V}_{kk',c} = 0 \quad (33)$$

The voltages across the two unbroken phase conductors are zero and the current on the broken phase conductor is also zero at the point of break.

$$\begin{bmatrix} \bar{V}_a^{(0)} \\ \bar{V}_a^{(1)} \\ \bar{V}_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{V}_{kk',a} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \bar{V}_{kk',a} \\ \bar{V}_{kk',a} \\ \bar{V}_{kk',a} \end{bmatrix} \quad (34)$$

$$\bar{V}_a^{(0)} = \bar{V}_a^{(1)} = \bar{V}_a^{(2)} = \frac{1}{3}\bar{V}_{kk',a} \quad (35)$$

Considering the equations above, one can conclude that the sequence networks can be connected in parallel. The positive sequence current, regarding phase a, is given by

$$\bar{I}_a^{(1)} = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}}{\bar{Z}_{kk'}^{(1)} + \dfrac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}}}$$

$$= \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}\left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right)}{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)} + \bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}} \quad (36)$$

The sequence voltage drops can be computed as

$$\bar{V}_{kk'}^{(1)} = \bar{I}_a^{(1)}\frac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}} \quad (37)$$

Substituting $\bar{I}_a^{(1)}$ from Eq. (3.36), the expression can be simplified as

$$\bar{V}_{kk'}^{(0)} = \bar{V}_{kk'}^{(1)} = \bar{V}_{kk'}^{(2)}$$

$$= \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)} + \bar{Z}_{kk'}^{(0)}\bar{Z}_{kk'}^{(2)}} \quad (38)$$

where $\bar{I}_{ij}$ is the pre-fault current in phase a between buses i and j.

3.2.6. Two-Conductor Open Fault

Now, let us assume that the conductors of phases b and c are open-circuited.

$$\bar{V}_{kk'a}^{(1)} = \bar{V}_a^{(0)} + \bar{V}_a^{(1)} + \bar{V}_a^{(2)} = 0$$

$$\bar{I}_b = 0 \quad (39)$$

$$\bar{I}_c = 0$$

The voltage across the unbroken phase conductor is zero and the currents on the broken phase conductors are also zero at the point of break.

$$\begin{bmatrix} \bar{I}_a^{(0)} \\ \bar{I}_a^{(1)} \\ \bar{I}_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{I}_a \\ 0 \\ 0 \end{bmatrix} \quad (40)$$

$$\bar{I}_a^{(0)} = \bar{I}_a^{(1)} = \bar{I}_a^{(2)} = \frac{1}{3}\bar{I}_a \quad (41)$$

Considering the equations shown above one can conclude that the sequence networks can be connected in series. The sequence currents can be computed as follows

$$\bar{I}_a^{(0)} = \bar{I}_a^{(1)} = \bar{I}_a^{(2)} = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}} \quad (42)$$

where $\bar{I}_{ij}$ is the pre-fault current in phase a between buses i and j. Then, the sequence voltages are given by

$$\bar{V}_{kk'}^{(0)} = -\bar{I}_a^{(0)}\bar{Z}_{kk'}^{(0)} = -\bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(0)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}}$$

$$\bar{V}_{kk'}^{(1)} = \bar{I}_a^{(1)}\left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right) = \bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}\left(\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(2)}\right)}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}}$$

$$\bar{V}_{kk'}^{(2)} = -\bar{I}_a^{(2)}\bar{Z}_{kk'}^{(2)} = -\bar{I}_{ij}\frac{\bar{Z}_{kk'}^{(1)}\bar{Z}_{kk'}^{(2)}}{\bar{Z}_{kk'}^{(0)} + \bar{Z}_{kk'}^{(1)} + \bar{Z}_{kk'}^{(2)}} \quad (3.43)$$

3.3. Post-Fault Voltages

The post-fault voltages for the zero, positive, and negative sequence networks are given by

$$V_i^{012} = V_{pre}^{012} + \Delta V_i^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix} + \begin{bmatrix} -Z_{ik}^{(0)} I_{fk}^{(0)} \\ -Z_{ik}^{(1)} I_{fk}^{(1)} \\ -Z_{ik}^{(2)} I_{fk}^{(2)} \end{bmatrix} \quad (44)$$

where n represents the number of buses.

3.3. Post-Fault Line Currents

The post-fault voltages for the zero, positive, and negative sequence networks are given by

$$I_{ij}^{012} = \frac{V_i^{012} - V_j^{012}}{z_{ij}^{012}} = \begin{bmatrix} \dfrac{V_i^{(0)} - V_j^{(0)}}{z_{ij}^{(0)}} \\ \dfrac{V_i^{(1)} - V_j^{(1)}}{z_{ij}^{(1)}} \\ \dfrac{V_i^{(2)} - V_j^{(2)}}{z_{ij}^{(2)}} \end{bmatrix} \quad (45)$$

where $\bar{z}_{ij}$ corresponds to the primitive impedance between buses i and j.

4. Direct Three-Phase Representation

4.1. Three-Phase Models

This section shows how one can model the components in a three-phase form in order to apply the direct three-phase representation method for fault analysis in a power system.

4.2. Three-Phase Admittance Matrix

Once all the system components are modelled, one can start building the three sequences admittance matrices. In order to perform a phase domain analysis, one must transform these sequence matrices into three-phase domain matrices. Then, the nodal branch-to-branch incidence matrix is given by:

$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & \dots & n \end{array} \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array} \quad (46)$$

Where $a_{pq} =$
$$\begin{cases} U, & if\ q\ is\ the\ sending - end\ bus\ of\ branch\ p \\ -U, & if\ q\ is\ the\ receiving - end\ bus\ of\ branch\ p \\ Zero, & otherwise \end{cases}$$

where U is a $[3x3]$ identity matrix and $Zero$ a $[3x3]$ zero matrix, being m the total number of lines, and n the total number of buses. Next, one must build the branch series impedance matrix for n elements, without the transformer branch impedances, as follows:

$$z^{abc} = \begin{bmatrix} z_1^{abc} & Zero & \cdots & Zero \\ Zero & z_2^{abc} & \cdots & Zero \\ \vdots & \vdots & \ddots & \vdots \\ Zero & Zero & \cdots & z_n^{abc} \end{bmatrix} \quad (47)$$

Then, one can easily obtain the branch series admittance matrix from the expression above, and it is given by:

$$y^{abc} = (z^{abc})^{-1} \quad (48)$$

Finally, the entire three-phase admittance matrix for a n-Bus system is given by:

$$Y^{abc} = A^T y^{abc} A = \begin{bmatrix} Y_{11}^{abc} & Y_{12}^{abc} & \cdots & Y_{1n}^{abc} \\ Y_{21}^{abc} & Y_{22}^{abc} & \cdots & Y_{2n}^{abc} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}^{abc} & Y_{n2}^{abc} & \cdots & Y_{nn}^{abc} \end{bmatrix} \quad (49)$$

Since this matrix does not include the shunt elements, one must perform some changes to it. First, the generators must be included.

$$Y_{ii}^{abc} = Y_{ii}^{abc} + y_g^{abc} \quad (50)$$

Then, one must include all transformers. Let us consider one connected between buses i and j.

$$\begin{aligned} Y_{ii}^{abc} &= Y_{ii}^{abc} + \bar{Y}_{node\,ii} - U \\ Y_{ij}^{abc} &= Y_{ij}^{abc} + \bar{Y}_{node\,ij} + U \\ Y_{ji}^{abc} &= Y_{ji}^{abc} + \bar{Y}_{node\,ji} + U \\ Y_{jj}^{abc} &= Y_{jj}^{abc} + \bar{Y}_{node\,jj} - U \end{aligned} \quad (51)$$

Finally, the three-phase admittance matrix of the system is completed.

$$Y^{abc} = {Z^{abc}}^{-1} \quad (52)$$

4.3. Fault Current

The fault impedance, for different types of faults, can be defined as shown in Fig. 2. It is represented by the fault impedances, regarding each phase, and by the ground impedance.

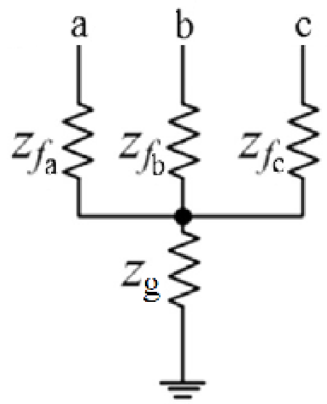

Figure 2: Fault impedances.

This set of impedances can be considered in a matrix form, given by Z_f^{abc} and Z_g, as shown below.

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \quad (53)$$

$$Z_g = \begin{bmatrix} z_g & z_g & z_g \\ z_g & z_g & z_g \\ z_g & z_g & z_g \end{bmatrix} \quad (54)$$

where $Z_{fa,b,c}$ represent the fault impedance of each phase, and z_g the ground impedance. For each type of fault, the fault impedance will be different, meaning that for each case one must understand which of the impedances referred above must be considered in order to compute the fault current. Let $I_{f_k}^{abc}$ be the fault current vector regarding a shunt fault at bus k, and I_f^{abc} be the total fault current vector. Since the fault occurs at bus k, the fault current will be zero in all buses except for the faulted bus, as shown below.

$$I_f^{abc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{f_k}^{abc} \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (55)$$

Finally, let us see how to compute the fault current, regarding the faulted bus, for different types of faults.

4.3.1. Three-Phase Fault

For a three-phase fault, the total fault impedance includes Z_f^{abc} and Z_g, as shown in Fig. 3.

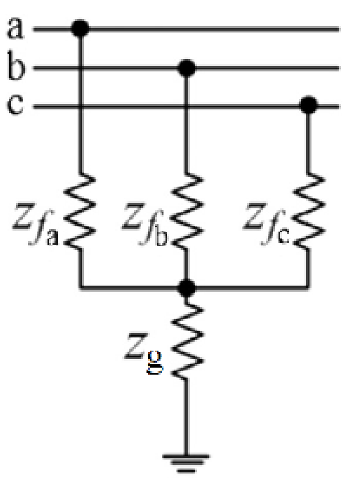

Figure 3: Fault impedance for a three-phase fault.

The fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = \left(Z_{kk}^{abc} + Z_f^{abc} + Z_g\right)^{-1} V_k^{abc} \quad (56)$$

4.3.2. Line-To-Ground Fault

For a single line-to-ground fault, the total fault impedance includes only the fault impedance regarding the faulted phase. Let us consider that the fault occurs in phase a, as shown in Fig. 4. Then, $Z_{fb} = Z_{fc} = \infty$, which can be represented for computation purposes by:

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix} \quad (57)$$

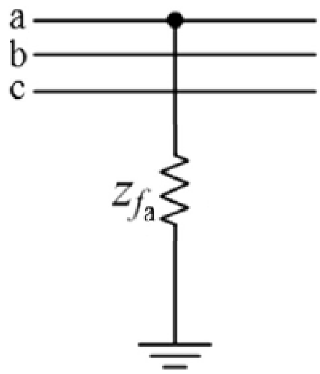

Figure 4: Fault impedance for a line-to-ground fault.

Thus, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = (Z_{kk}^{abc} + Z_f^{abc})^{-1} V_k^{abc} \quad (58)$$

4.3.3. Line-To-Line Fault

For a line-to-line fault, the only fault impedance considered is the impedance between the two faulted phases, represented by Z_f.

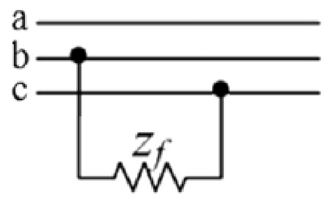

Figure 5: Fault impedance for a line-to-line fault.

Considering that the fault occurs between phases b and c, as shown in Fig. 5, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = \left(Z_{kk}^{bb} + Z_{kk}^{cc} - Z_{kk}^{bc} - Z_{kk}^{cb} + Z_f\right)^{-1} \begin{bmatrix} 0 \\ V_k^b - V_k^c \\ V_k^c - V_k^b \end{bmatrix} \quad (59)$$

4.3.4. Double-Line-To-Ground Fault

For a double-line-to-ground fault, the total fault impedance includes the fault impedances regarding both faulted phases and the ground impedance. Let us consider that the fault occurs in phases b and c, as shown in Fig. 6. Then, $Z_{fa} = \infty$, which can be represented by:

$$Z_f^{abc} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \qquad (60)$$

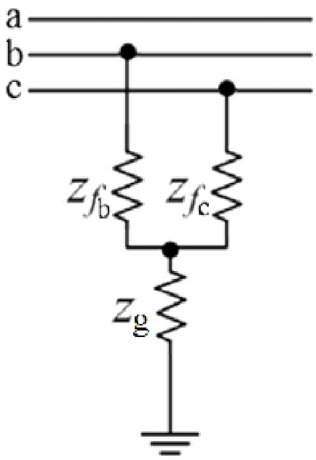

Figure 6: Fault impedance for a double-line-to-ground fault.

Thus, the fault current at the faulted bus is given by:

$$I_{f_k}^{abc} = (Z_{kk}^{abc} + Z_f^{abc} + Z_g)^{-1} V_k^{abc} \qquad (61)$$

4.3.5. Open Conductor Fault

A fault between buses, such as an open conductor fault, is called a series fault. Let us consider that a series fault occurs in the branch between buses i and j. According to this assumption, $Z_{Th,ij}^{abc}$ is the 6×6 impedance matrix, related to the faulted buses, obtained from the total impedance matrix of the system.

$$Z_{Th,ij}^{abc} = \begin{bmatrix} Z_{ii}^{abc} & Z_{ij}^{abc} \\ Z_{ji}^{abc} & Z_{jj}^{abc} \end{bmatrix} \qquad (62)$$

Also, let the 3×3 primitive admittance of the faulted line change from Y_l^{old} to Y_l^{new}, and Y_l^{new} is given by Eq. (63).

$$Y_l^{new} = \left(z_l^{abc} + Z_f^{abc}\right)^{-1} \qquad (63)$$

where Z_f^{abc} depends on the opening phases.

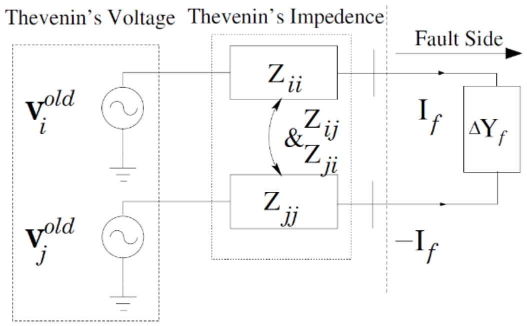

Figure 7: Block diagram representation of Thevenin's equivalent circuit for series fault.

The previous procedure will result in the change of the four block entries in $Y_{Th,ij}^{old}$. Also, notice that $Y_{Th,ij}^{old} = (Z_{Th,ij}^{old})^{-1}$.

$$\Delta Y_f = Y_l^{new} - Y_l^{old} \qquad (64)$$

The modification of the admittance matrix will involve addition and subtraction of ΔY_f, depending on the block entry being diagonal or non-diagonal, respectively.

$$\begin{aligned} Y_{ii}^{new} &= Y_{ii}^{old} + \Delta Y_f \\ Y_{ij}^{new} &= Y_{ij}^{old} - \Delta Y_f \\ Y_{ji}^{new} &= Y_{ji}^{old} - \Delta Y_f \\ Y_{jj}^{new} &= Y_{jj}^{old} + \Delta Y_f \end{aligned} \qquad (65)$$

This step can also be represented as follows:

$$Y_{Th,ij}^{new} = Y_{Th,ij}^{old} + [E_i \quad E_j]\, \hat{Y}_f\, [E_i \quad E_j]^T \qquad (66)$$

where $\hat{Y}_f$ is given by

$$\hat{Y}_f = \begin{bmatrix} \Delta Y_f & -\Delta Y_f \\ -\Delta Y_f & \Delta Y_f \end{bmatrix} \qquad (67)$$

and E_i is a block vector given by

$$E_i(j) = \begin{cases} O_3, & i \neq j \\ I_3, & i = j \end{cases} \qquad (68)$$

where O_3 is a 3×3 size matrix of zeros, and I_3 is the identity matrix of the same size.

The post-fault voltages for a series fault are given by:

$$\begin{bmatrix} V_i^{new} \\ V_j^{new} \end{bmatrix} = \left(I_6 + Z_{Th,ij}^{abc} \hat{Y}_f\right)^{-1} \begin{bmatrix} V_i^{old} \\ V_j^{old} \end{bmatrix} \qquad (69)$$

And the fault currents are given by:

$$I_{f_{ij}}^{abc} = Y_l^{new}\left(V_i^{new} - V_j^{new}\right) \qquad (70)$$

Notice that, for a series fault, the fault current vector will have to entries with opposite signs, as shown below:

$$I_f^{abc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_{f_{ij}}^{abc} \\ -I_{f_{ij}}^{abc} \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (71)$$

where $I_{f_{ij}}^{abc}$ is the fault current from bus i to bus j, and $-I_{f_{ij}}^{abc}$ from bus j to bus i, as illustrated in Fig. 7.

4.3.5.1. One-Conductor Open

Considering that only phase a opens, the line impedance regarding the opening phase is $Z_{fa} = \infty$, which results in the following modification to Z_f^{abc}.

$$Z_f^{abc} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & Z_{fb} & 0 \\ 0 & 0 & Z_{fc} \end{bmatrix} \tag{72}$$

Then, one can obtain Y_l^{new} and perform all the calculations explained above.

$$Y_l^{new} = \left(z_l^{abc} + Z_f^{abc}\right)^{-1} \tag{73}$$

4.3.5.2. Two-Conductor Open

In this case, let us consider that phases b and c are the faulted conductors. Thus, the line impedance regarding the openings phases is $Z_{fb} = Z_{fc} = \infty$, which results in the following modification to Z_f^{abc}.

$$Z_f^{abc} = \begin{bmatrix} Z_{fa} & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix} \tag{74}$$

Thus, as in the one-conductor opening case, one can obtain Y_l^{new}, given by Eq.73.

4.4. Post-Fault Voltages

The changes in the post-fault bus voltages are given by:

$$\Delta V = -Z^{abc} I_f^{abc} \tag{75}$$

Let V_{pre} be the vector that represents all the bus voltages before the fault.

$$V_{pre} = \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_n^{abc} \end{bmatrix} \tag{76}$$

Then, the post-fault bus voltage vector is given by:

$$V_{new} = V_{pre} + \Delta V \tag{77}$$

4.5. Post-Fault Line Currents

In order to compute the post-fault branch currents, one must obtain the voltage drops regarding every line of the system. Considering the post-fault bus voltage vector, the line voltage vector is given by:

$$V_{line} = A V_{new} \tag{78}$$

where A is the nodal branch-to-branch incidence matrix defined in Eq. 46.

Then, the post-fault branch current vector can be computed by:

$$I_{line} = Y_{line} V_{line} \tag{79}$$

where Y_{line} is the branch series admittance matrix including the transformer branch impedances.

5. Experimental Results

In order to compare both symmetrical components and direct three-phase representation methods, the five-bus power system shown in Fig. 8 was considered.

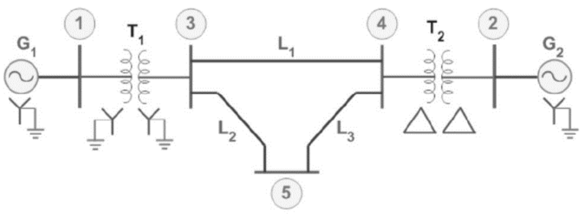

Figure 8: 5-Bus system.

The simulations were performed with a calculation tool, MATLAB. Several cases with different data were considered in order to validate the three-phase method. First, shunt faults were tested, such as three-phase fault, line-to-ground fault, line-to-line fault and double-line-to-ground fault. Then, open conductor faults, also known as series faults, were tested for one-conductor open and two-conductor open cases. In addition to this, other important variables were considered in the fault calculations, such as the faulted bus, fault impedances and transformer configurations. The results obtained for all situations of fault and data, like the ones described above, by the direct three-phase representation method were the ones expected, being precisely the same as the ones obtained by the symmetrical components method.

6. Conclusions

The objective of this dissertation was to apply a direct three-phase method in power system fault analysis and compare the results with the ones obtained through the traditional way, the so-called symmetrical components method. Considering the results obtained, one can say that these results were the ones expected since they are practically the same. Also, they demonstrate that not only the symmetrical components method is a good approach in certain cases but that the direct three-phase representation method is a much reliable method that requires only a computer with the ability to perform all the calculations. The method proposed in this dissertation articulates several progresses made in this specific area of fault analysis regarding power systems, despite the few paperwork on the matter so far. This work assembles some concepts already explored on they own on the subject and makes an important connection between them in order to structure a valid and appropriate method to perform all kinds of fault calculations with resource to computational tools that can support the computational complexity implied. In conclusion, considering the accurate results obtained in this dissertation and all the theoretical support behind the calculations and all the procedures, it is fair to say that this method is most appropriated when it comes to fault calculations regarding power system analysis.

7. Future Work

Further developments in this matter could be achieved and this dissertation could be a good starting point. Since all the computations regarding the direct three-phase method for fault analysis in power systems have additional computational complexity when compared to the traditional method, a good development to this work would be a more rigorous computer program with a proper interface to allow any person to perform all the calculations needed regarding any type of fault and any kind of power system. This would be a very helpful tool in this field of study and a very interesting challenge. Also, several additional tests may be required to fully validate this method regarding any real-life situation.

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