TÉCNICO LISBOA

Evaluation of the quality and access to health care in the Portuguese public hospitals: A multicriteria approach

Alexandre Miguel Vilar Ricardo

Dissertation to obtain the Master of Science Degree in

Engineering and Management

Supervisors: Prof. José Rui de Matos Figueira
Prof. Ana Sara Silva Rodrigues Costa

Examination Committee

Chairperson: Prof. Margarida Catalão Lopes
Supervisor: Prof. José Rui de Matos Figueira
Member of the committee: Prof. Nelson Carriço

November 2021

“No thief, however skillful, can rob one of knowledge, and that is why knowledge is the best and safest treasure to acquire.”

L. Frank Baum

Declaration

I declare that this document is an original work of my own authorship and that fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Preface

The work presented in this dissertation has been performed at Centre of Management Studies of Instituto Superior Técnico (CEG-IST) and Civil Engineering Research and Innovation for Sustainability (CERIS), under the supervision of Professor José Rui de Matos Figueira and Professor Ana Sara Silva Rodrigues da Costa, and within the frame of the hSNS FCT - Research Project (PTDC/EGEOGE/30546/2017): "Portuguese public hospital performance assessment using a multicriteria decision analysis framework".

Acknowledgements

In my personal point of view, knowledge is the ultimate gift bestowed upon mankind. This idea blossomed inside me even more during the development of this dissertation. It has been an honor to have the opportunity of acquiring knowledge from all the amazing people that I was fortunate to encounter in my life. This knowledge and the values transmitted are the main reasons for me being who I am today, an imperfect human being with multiple ambitions and the hunger for challenges such as the one presented to me during this phase of my life.

The work developed in these last months to present this dissertation was immensely satisfying, particularly due to the two supervisors who accompanied me, Prof. José Rui Figueira and Prof. Ana Sara Costa. Your hard work and dedication were indescribable and an inspiration to me, thank you for everything!

I would also like to thank my friends and colleagues for all the amazing moments and advice shared during the last few years. In this group, there is a particular person that deserves all my gratitude, my close friend and colleague Alexandre Jesus. It is a rare and amazing feeling to have someone in our lives, who is not family, but whose success is our success. Sharing this academic journey with you was, and will continue to be, a major honor and a reason to continue going forward. Thank you for your friendship!

Our accomplishments in life have much lesser value when we cannot share it with those who are dearest to us, and nobody is dearest to me than my family. To my father and mother, thank you for always believing in me and giving me all the tools needed to follow my dreams. You are my idols, and my love for you cannot be put into words. To my sister and brother, thank you for showing me the true meaning of sharing, friendship and love. Your happiness is my happiness, and your problems are my problems, and it will always be that way. To my girlfriend Ana, thank you for these amazing six years and for being present in all the good and bad moments. Your support means the world to me, just as you do.

In conclusion, it was an amazing experience to have developed this dissertation and to be a student of Instituto Superior Técnico, to which I am also very grateful.

Abstract

The Portuguese healthcare sector is characterized by having a National Health Service (NHS) that ensures access to health care to the population. The healthcare policy reforms that have occurred in Portugal during recent years, particularly those made during the financial crisis, which led to the intervention of foreign aid (2011-2015), have had a significant impact on the efficiency of health care provided by the NHS. The disinvestment in infrastructures and the reduction of human resources compromised the objectives of improving the system's efficiency and its quality. Since then, some measures have been taken to improve the quality and access to health care providers, which enhances the importance of evaluating these dimensions in those care providers. The hospitals' quality and access assessment should be made considering all criteria involved in the situation, but also only specific subsets of them. To make this evaluation, the ELECTRE TRI-nC method with Multiple Criteria Hierarchy Process (MCHP) is implemented in *phyton* and applied to a data set. ELECTRE TRI-nC is a multicriteria sorting method that assigns each action (in this case, a Portuguese public hospital) to pre-defined and ordered categories (representing levels of quality and accessibility). The hospitals are evaluated according to several criteria organized in a hierarchical structure, enabling the use of MCHP to access each hospitals' evaluation considering specific criteria or the comprehensive level. This study highlights the potential of using the proposed method in the healthcare sector as a tool for increasing the NHS' efficiency and ensuring a sustainable system.

Keywords: Multiple Criteria Decision Aiding, ELECTRE TRI-nC, Multiple Criteria Hierarchy Process, Quality assessment, Hospitals, National Health Service

Resumo

O sistema de saúde português é caracterizado pela existência do Sistema Nacional de Saúde (SNS) que assegura o acesso aos cuidados de saúde à população. As reformas na área da saúde ocorridas em Portugal nos últimos anos, nomeadamente aquelas que ocorreram durante a crise financeira que levou à intervenção de ajuda externa (2011-2015), tiveram um impacto significativo na eficiência dos cuidados de saúde prestados pelo SNS. O desinvestimento em infraestruturas e a redução de recursos humanos comprometeram os objetivos de melhorar a eficiência e a qualidade do sistema. Desde então, algumas medidas têm sido adotadas para aumentar a qualidade e o acesso aos cuidados de saúde, evidenciando a importância de avaliar estas dimensões nos prestadores desses serviços. A avaliação da qualidade e acesso aos hospitais públicos Portugueses deve ser feita considerando todos os critérios envolvidos, mas também subconjuntos específicos. Para fazer esta avaliação, foi implementado o método ELECTRE TRI-nC com *Multiple Criteria Hierarchy Process* (MCHP) em *python*, e aplicado a um conjunto de dados. O ELECTRE TRI-nC é um método multicritério de classificação que afeta cada ação (neste caso, hospital público Português) a categorias (que representam níveis de qualidade e acesso) previamente definidas e ordenadas. Os hospitais são avaliados segundo vários critérios organizados numa estrutura hierárquica, possibilitando assim o uso do MCHP para a sua avaliação, considerando critérios específicos ou todos eles juntos. Este estudo evidencia a potencialidade do método proposto no setor da saúde, servindo de ferramenta para aumentar a eficiência do SNS e assegurar a sua sustentabilidade.

Palavras-chave: Abordagem Multicritério de Apoio à Decisão, ELECTRE TRI-nC, Processo Hierárquico com Múltiplos Critérios, Avaliação da qualidade, Hospitais, Serviço Nacional de Saúde

Contents

List of Tables

List of Figures

Nomenclature

Greek symbols

ϕ	Elementary concordance index
σ	Credibility index
ρ	Selecting function
λ	Credibility level
Γ	Range of categories assigned to an action

Subscripts

a	Action
A	Set of actions
C	Category
B	Subset of reference actions
b	Reference action
p	Preference threshold
q	Indifference threshold
v	Veto threshold
g	Criteria
Ğ	Set of all criteria at all considered levels
m	Number of first level criteria
l	Number of levels in the hierarchy tree of criteria
EL	Set of indices of all elementary subcriteria
LBO	Set of indices of all subcriteria located in the last but one level of the hierarchy
$L_{Ğ}$	Set of indices of specific criteria that represents the position of criteria in the hierarchy

Glossary

ACSS	Administration of the Health System
CP	Computational Program
CVC	Catheter-Related Bloodstream Infection
DGH	Directorate-General of Health
DM	Decision Maker
DRG	Diagnosis-Related Group
EHCI	Euro Health Consumer Index
ELECTRE	*ELimination Et Choix Traduisant la Realité*
GDP	Gross Domestic Product
GRIP	Generalized Regression with Intensities of Preference
HCC	Healthcare Centers
INE	National Institute of Statistics
LHUs	Local Health Units
MARSAN	Method for Analysis Research and Selecting of New Activities
MAUT	Multiattribute Utility Theory method
MCDA	Multicriteria Decision Aiding
MCHP	Multiple Criteria Hierarchy Process
MH	Ministry of Health
MHCS	Management Health System Center
MoU	Memorandum of Understanding
NHS	National Health Service
PHCG	Primary Healthcare Center groups
PPPs	Public-Private Partnership
PROMETHEE	Preference Ranking Organization Method for Enrichment of Evaluations
RHA	Regional Health Administration
ROR	Robust Ordinal Regression
SEMA	*Société d´Economie et de Mathématiques*
SMAA	Stochastic Multiobjective Acceptability Analysis
SNS	*Sistema Nacional de Saúde*
SRF	Simos Roy Figueira Procedure
UCFTP	Unifetal, Cephalic and Full-Term Pregnancy
WHO	World Health Organization

Chapter 1: Introduction

1.1 Motivation

If we try to observe the concept of life, its meaning, and the most important aspects related to it, we will find a grey area where religion, science and self-belief collide. However, it seems safe to consider that the most important thing in life is life itself. It is very common to forget this basic concept when we are healthy and dealing on a daily basis with all the aspects underlying life in the current century. Usually, it is only when we, or someone we know, has a health problem, that we realize that life could end at any instance. It is only in these specific moments that we truly realize that without health there is no life, enhancing the importance of each citizen in the world having access to quality health care services, regardless of their social and financial status.

In Portugal, the healthcare system is characterized by having a National Health Service (NHS), financed by the state budget, that grants universal coverage for all health needs, to the entire Portuguese population (Nunes, 2018b). Portugal has gone through some health reforms in recent years due to political changes and more recently due to the financial crisis from which Portugal is still recovering. One of the consequential measures implemented was to improve the healthcare system's efficiency, however, it is important to note that in terms of health, improving efficiency should not compromise the level of quality and access to said system, but rather improving it if possible. Although there are some positive indicators the main goal has not yet been completely achieved, which is reflected in the unsustainable growth of the NHS' expenses in percentage of the nominal Gross Domestic Product (GDP). For instance, the growth in 2018, 2019 and 2020, has been respectively, 6.2%, 5.2% and 6.8%, which in comparison to 2015 and 2016, were considerably higher (1.7% in 2015, 2.4% in 2016) (Portuguese Public Finance Council, Budgetary Evolution of the NHS, Nº 06/2021). The prevalence of the structural debt imposes challenges to the financial sustainability, factor that must be considered in the context of the future definition of health policies and reforms. In order to establish future measures, it is important to firstly assess the actual level of important parameters regarding public health. The public hospitals are the main providers of differentiated care to the Portuguese citizen, which highlights the importance of evaluating the quality and access to them through a multicriteria hierarchy approach.

The method used in this dissertation is the Electre Tri-nC with a Multiple Criteria Hierarchy Process (MCHP), which enables the creation of a hierarchy tree of criteria to evaluate the hospitals at the comprehensive level (considering all criteria), but also at intermediate levels. Hopefully, the use of this method highlights the importance that it may have as a tool to be used constantly in the healthcare sector and beyond.

The COVID-19 pandemic caused an unprecedented worldwide health crisis, which intensively overburdened the NHS, proving the fragile capacity of it to respond to this emergency without jeopardizing the access to treatment of other critical and non-critical medical conditions, as well as the financial sustainability of the system. This fact enhances even more the importance of having adequate tools to evaluate that system, and ideally to serve as a reference to improve it.

1.2 Objectives

The main goal of this dissertation is the evaluation of the quality and access to health care in the Portuguese public hospitals. The concept of quality related to health care is a very subjective theme, which leads to the creation of a specific model depending on the individual perception of the person who is constructing it. Those perceptions are mainly reflected in the definition of the family of criteria, that should include all the indicators considered relevant by the Decision Maker (DM), in order to evaluate the quality and access to its fullest extent. Almeida-Dias et al. (2010) reported that "the term decision maker represents those in whose name or whom the decision aiding must be given". Those criteria should be organized in a hierarchical structure to allow the evaluation with respect to different subsets of criteria that represent specific segments of the two major dimensions. The ELECTRE TRI-nC method, which uses a Multiple Criteria Decision Aiding (MCDA) approach, was used in conjunction with MCHP, to develop an evaluation method, that represents the DM's preferences, through a co-construction interactive process with an analyst. This outranking method enables the sorting of the Portuguese public hospitals into different categories, that represent a specific level of quality and access, at the comprehensive level, but also at intermediate ones. Thus, the objectives of this dissertation are the implementation of the ELECTRE TRI-nC method with a MCHP into a computational program, and the application of a model to evaluate the quality and access to health care in the Portuguese public hospitals. That program must have a general format and a user-friendly framework to facilitate the interaction procedure between an analyst and the DM, thus enabling its use in other real-life problems. A comparison between the method used and the results obtained in this work with Rocha et al. (2021) is also intended.

1.3 Dissertation structure

The structure used is divided into six different sections, which are intended to exploit different components of the project, in the following way:

- Chapter 2: Contextualization of the Portuguese healthcare system and definition of the problem;

- Chapter 3: Literature review based on the MCDA approach, including the description of the development of the outranking methods and all the concepts related to them, and more important, the breakdown of the family ELECTRE of methods and the MCHP;

- Chapter 4: The project methodology, that consists in a full description of the ELECTRE TRI-nC method, the Multiple Criteria Hierarchy Process, and the joining of both;

- Chapter 5: Description of the computational program developed to apply the ELECTRE TRI-nC with MCHP in any model defined.

- Chapter 6: Description of the model considered and its application in the computational program developed. Presentation of results and their interpretation. Description of the robustness analysis performed and comparison between the results and the method used in this dissertation with Rocha et al. (2021).

- Chapter 7: Discussion of the results and conclusions withdrawn. Brief description regarding the limitation of the computational program. Proposal of new studies to be developed within the scope of this dissertation.

Chapter 2: Problem definition

This chapter contextualizes the problem, and its main characteristics. In terms of health, a brief description of the Portuguese healthcare sector is presented, followed by the description of its organizational and financial structure and concluding with the contextualization of the health reforms that occurred since the last major financial crisis. Then presented, are the specific characteristics of the problem that will be addressed, and the method proposed to aid in the analysis.

2.1 Portuguese healthcare sector

The importance of the healthcare sector, and its social and financial repercussions, is almost incommensurable. The linkages between this sector and the reduction of poverty, accompanied by economic growth, is more powerful than it is generally recognized (Weil, 2014). In Portugal, as in many other countries, there are approved laws that grant several rights regarding the health area. The Portuguese Constitution, which has been in effect since 1976, despite some modifications during the years, states in Article n.º 64 that "everyone has the right to health protection and the duty of defending and promoting it". That right is accomplished through "a universal and general national health service", that is usually free of charge. To ensure that right, the Portuguese government has to grant several conditions, from which, considering the theme of this dissertation, it is essential to highlight three:

- "Guarantee the access of all citizens to the preventive, curative and rehabilitation medicine care";
- "Guarantee a national and efficient coverage of the entire country in human resources and health units";
- "Discipline and supervise business and private forms of medicine, and their articulating with the national health service, in order to ensure, in public and private health institutions, adequate standards of efficiency and quality".

It is essential to understand these laws to better understand how the healthcare sector works in Portugal, and what is the government's role in it. The evolution in this sector is constant, and it reflects the efforts made to constantly improve the efficiency of health care provided to the Portuguese citizens, especially in the last decades, where there has been multiple transformations in the Portuguese healthcare sector.

2.1.1 Organizational structure

The Portuguese healthcare sector is characterized by a National Health Service (NHS) that was created in 1979 (Nunes, 2018b), with the intention of ensuring the conditions defined in the Portuguese constitution of 1976. The entity that has the task of managing the NHS and the "mission of defining and leading the national politics of health and ensuring a sustainable application and use of the resources and the evaluation of the results" is the Ministry of health (Decree-Law Nº 124/2011 of

29th December from Ministry of Health, 2011). This administrative process is made through the Central Administration of the health system (ACSS), an entity that executes the orders from the Ministry of Health (MH), and five Regional Health Administration (RHAs). In theory, the RHAs would have wider responsibilities then they actually have in practice, including planning operations regarding the hospital care system, however, the majority of those responsibilities is centralized in the MH (Gouveia, 2005). There are several entities that work in a direct, or indirect way with the MH. One of the most important is the Directorate-General of Health (DGH), which is a central service of the MH, and is responsible for the organization of the national public health care services (Simões et al, 2017). It is noteworthy that the autonomous regions of Madeira and Azores are endowed with greater flexibility in the planning and management of health-related matters.

The Portuguese NHS is characterized for having a predominantly public matrix, that allows to offer the provision of health care in three specific segments: Primary care, Secondary care and Tertiary care (Fernandes & Nunes, 2016). The Primary Healthcare Center groups (PHCG) are groups of Healthcare Centers (HCC) spread nationwide, that ensure the majority of primary healthcare delivery to the Portuguese community. Primary care in Portugal is also provided by the private sector, however, there is the possibility of the patients' expenses to be totally, or partially covered by public funding (Simões et al., 2017). The Secondary and Tertiary (differentiated) care are usually provided in hospital establishments, particularly public hospitals. These hospitals are distributed throughout multiple areas of the country according to population density, their health needs, and the number of medical professionals in those areas. According to the National Institute of Statistics (INE), there are 238 hospitals in Portugal, from which 108 are public, 127 are private and three are the result of public-private partnerships (PPPs). Several public hospitals integrate hospital centers groups, which are responsible for administrating various hospital units of a given region. The Local Health Units (LHUs) were established to merge primary care facilities with hospital care providers of certain geographical regions (Fernandes & Nunes, 2016), integrating in this way all the health units (primary and differentiate ones) of that region. It is important to clarify the idea that led to the creation of the LHUs, and the potential impact that they could have in the future of the health sector. The majority of the Portuguese population suffers from at least one chronic condition (Fernandes & Nunes, 2016), like diabetes, chronic obstructive pulmonary disease, between other examples. These individuals are regular users of different units of the healthcare system, which demonstrates the importance of having a single organization that allows facilitating the vertical integration, and the consequential communication, between different levels of healthcare (Nunes, 2018b). This measure could also lead to an improvement in efficiency and quality, and a reduction of cost, mainly due, to a continuous monitoring of the patient, that could avoid possible relapses. Although the majority of health care provision is done in the public sector it is also important to consider the private sector and their contribution (Fernandes & Nunes, 2016).

2.1.2 Financing

There are three distinct systems that coexist to ensure the financing, and the access for citizens, to the Portuguese healthcare system: the NHS, the subsystems of health (social and private insurance schemes for specific segments related with their profession) and the private voluntary health insurance. It is important to also mention the direct payments of some citizen to access health care (PVI). The financing to the healthcare sector in Portugal is made through public funding sources and private ones. Most of the NHS budget is financed through transfers from the government budget, which implies that the tax system imposed to the Portuguese population is the major source of funding of the NHS (Barros, 2012). Nevertheless, despite the low impact that it has on the total budget available, it is important to acknowledge the payment of user fees in specific situations. The budget of the NHS is annually defined by the Ministry of Finance, and it is based on the indications presented by the Ministry of Health and the prevision of expenses. That budget is distributed within the various elements of the NHS, being the Management Health System Center (MHSC) responsible for that distribution (Escoval et al., 2016).

2.1.3 Crisis and post-crisis

At the end of 2009 began a serious financial crisis in Portugal (according to the technical definition of recession, a financial crisis occurs when there are two quarters of negative growth) whose effects are still ongoing (Nunes, 2019). That crisis created the need to resort to an external intervention, made by the International Monetary Fund, the European Commission, and the European Central Bank. All the entities signed a Memorandum of Understanding (MoU), whose purpose was to define a set of measures to be applied by the Portuguese government to meet the payments of the public debt that had been created. The government reforms, influenced by the MoU indications, intended to improve efficiency and reduce the costs of the public services, which had a great impact on the public health sector (Nunes, 2018b; Simões et al., 2017). Those austerity measures have reduced the secondary care expenditures and worsened the access to health care providers (Nunes, 2019). The proposed measures were also defined with the intention of building mechanisms for a better control of health care expenditures in the public sector, and the insertion of new practices to collect and transmit vital information for a sustainable development of the NHS (Barros, 2012).

Following the external intervention period (2011-2015), the new elected government introduced a new strategy into its program to revitalize the NHS, focused on several objectives, such as increasing the access to primary health care or maximizing the hospital response. (Nunes, 2018a). In terms of public hospitals, some investments were made, and a new management model was created to recognize the most efficient hospital and recommend the share of resources between them. Those investments led to an improvement of quality of the health care services provided to the patients.

Several indexes have been created throughout the years to measure the healthcare systems in different countries, considering different perspectives. The Euro Health Consumer Index (EHCI) is used to rank the healthcare systems of different European countries based on several indicators that try to reflect the consumer’s experience. From 2015 to 2018 (newest ranking available) Portugal has increased its ranking position from 20th to 13th place (35 countries were included in the ranking)[1]. This growth could reflect the increase in investment made in the public healthcare sector since 2016, but also the new efficiency oriented practices imposed after the MoU agreement. Despite some good indicators, it is also important to highlight the large increase that occurred since 2017 in the negative deviations from the NHS balance, compared to the objective proposed by the state budget. Between 2014 and 2020, the accumulated budgetary execution of the NHS presented a negative balance of 2865 million of euros (M€), 2018, and 2019, were the years with the highest negative deviations, respectively, 732 M€ and 628 M€. It is also crucial to acknowledge the value of the debt to the NHS external suppliers, that in the end of 2020 was 1516 M€ (Portuguese Public Finance Council, Budgetary Evolution of the NHS, Nº06/2021). These data reveal the inability to keep pace with the continuous growth of the public health expenses, which goes against the idea of having a sustainable NHS.

Considering all these aspects, it is crucial to continue improving the efficiency of the NHS without compromising the quality and access to health care providers, namely, the public hospitals, hence the importance of studying the actual level of both these characteristics in those hospitals.

2.2 Problem characteristics

The main goal of this dissertation is to use a MCDA method that allows to make the assessment of the quality and access to health care in the Portuguese public hospitals. Since the Portuguese health policy is very focused on improving efficiency without compromising the quality and access to health care, the importance of measuring their actual level is fundamental, because, as Lord Kelvin stated, “if you cannot measure it, you cannot improve it” (Carlon & Combs, 2005). The decisions made in the health care area have huge repercussions in the society, an idea that is easily proven considering all responsibilities attributed to all entities linked to health, during the ongoing pandemic situation. This is one of the main reasons that highlights the need for the NHS to have a method capable of correctly measuring the actual level of several resources, in order to fulfill its mission.

In general, it is hard to define quality, especially when the definition does not mention specific areas to focus on, that together help to define it as a whole. For instance, when people comment that a certain vehicle has a given level of quality, they are probably considering the quality of said vehicle in different areas, such as: aerodynamics, security, aesthetics, among others. The quality of the vehicle in those segments defines the general quality. In the Portuguese public hospitals, quality is even harder to gauge. This difficulty is related to the large number of criteria that can be considered to evaluate quality as a whole, and with the fact that these criteria are often interconnected in a hierarchical way. Ferreira and Marques (2020) reported that “quality of delivered health care services is related to the infrastructure, the available technology and equipment, the actions undertaken by clinical and non-clinical staff within the healing process, the patient’s safety, and the final outcomes”. This definition represents a specific point of view regarding the meaning of quality of delivered healthcare services, however, in this specific subjective issue, there is a general different perception between different segments of people, and also within the segments, which will lead to different points

[1] https://healthpowerhouse.com/publications/

of view. In several papers published throughout the years, there is the idea that in terms of health care there are two major perceptions, the health care professionals' perspective and the patient's perspective. The first one is usually focused on caring for and caring about the patient, while the second one is more concerned with the interpersonal interactions that exist in a health care facility (Williams, 1998). This dissertation considers that there is a third major perception, which is, the ministry of health´s perception. This government department, as already mentioned, has the task of evaluating the results related with the national politics of health, which logically includes the national public hospitals, thus making their perception central to the problem presented. The intrinsic subjectivity of this problem and the complexity of all characteristics considered must be the pillars in the search for a method that allows to conjugate mathematical rigor with flexibility. This flexibility is related with having a model that enables to seize the preferences of the DM, and, at the same time, is user friendly in the process of accessing those preferences.

Following the MCDA approach model, in order to make an overall evaluation of the Portuguese public hospitals, the hospitals should be assigned to a certain quality level. Thus, the model must have the capacity to deal with multiple criteria, organized in a hierarchical way, and a process that allows to sort the hospitals to different categories considering only specific criteria, or all. This will allow to compare the overall performances of hospitals, but also the specific performance of those hospitals in specific areas. In this way, it will be possible to make analyses of investments and the study of different behaviors in different hospitals. If a certain area in a given hospital has a high performance with a lower budget, it will be interesting to study the specific behaviors practiced in that area and share that information with other hospitals. Since there is a finite set of resources to be allocated within the Portuguese Public hospitals, this last idea could be a great opportunity to reduce costs and improve quality at the same time. A model that produces results that allow to make different types of comparisons, will provide the NHS, an efficient tool to establish which resources should be allocated to each hospital, in different areas, or even to modify the parameters of accessibility of the Portuguese population to the public hospitals. One of the tools currently used in the Portuguese public hospitals and PPPs is the ACSS benchmarking process[2]. This process evaluates the performances of several indicators of specific dimensions within those facilities. Two of those dimensions are the access and the quality. This initiative, which started in 2011, has several objectives[3]:

- Improve the access and quality of the public hospitals;

- Evaluate the potential for improvement of each hospital in each of the main areas;

- Increase the transparency and share of information with the community.

The indicators used in the benchmarking process to evaluate the access and quality dimensions can be representative references of the type of criteria usually considered to evaluate them. Nevertheless, the decision of incorporating those indicators in the model is the sole responsibility of the DM, which has the task of defining all the criteria that will be organized according to a hierarchical structure.

---

[2] https://diretiva.min-saude.pt/monitorizacao-e-avaliacao/

[3] https://benchmarking-acss.min-saude.pt/BH_Enquadramento/Objetivos

The MCDA approach could be one of the most efficient ways to deal with this type of problem. The methods developed within this paradigm present several aspects related to the specific situation presented above. The one proposed in this dissertation is a conjunction, with the necessary adjustments, of the method ELECTRE TRI-nC with MCHP.

2.3 Summary

This chapter presented a characterization of the Portuguese healthcare public sector, particularly of the NHS. The description of how this service works, the health care providers that belong to it and the financing structure, allowed to withdraw some conclusions. Expenditure on health care has been increasing in recent years at a rate that makes it difficult to have a sustainable system, hence the importance of improving its efficiency. The focus should be on never compromising the quality level of the health care providers, what creates the obligation to have different tools to actually measure this dimension. However, that measurement should not be made only in general levels, because having that type of information about specific areas of a broader dimension is what allows to know where the problem is, and what measures should be implemented. The integration of all this information between all the health care providers (even in the private sector) would enable the construction of a healthcare reform based on the collection of information, the sharing of it and the focus on improving the health care provided to the Portuguese population. This dissertation aims to contribute positively to the healthcare sector, through the evaluation of the public hospitals, and the introduction of a new method that could become an efficient tool for the NHS.

Chapter 3: Literature Revision

This chapter describes the literature review of the MCDA approach, focusing on the method proposed in this dissertation. Firstly, a brief historical description of some developments that were the basis for the formal beginning of the MCDA is presented, followed by the introduction of the outranking methods and the presentation of its main ideas. Secondly, a description of the evolution of the ELECTRE family of methods is made, and also a presentation of other relevant outranking methods. Lastly, the Multiple Criteria Hierarchy Process, and its use in different MCDA methods, is described.

3.1 Early history of MCDA

Since the beginning of human history, there has been always the need to make decisions. Most of the problems, where a decision must be found, involve multiple criteria. Despite the fact that our early ancestors had to make decisions of this type, they did not have the ability to define it, characterize it, and especially, to analyze it. In this sense, the MCDA field could be considered to be as old, as the existence of decisions where trade-offs between objectives/criteria happen, that is, as old as human beings exist. Although there are no consensus regarding the formal beginning of MCDA, it is often considered, that the method used by Benjamin Franklin, to decide if a certain decision should or should not be taken, was one of the first approaches to the theme. Franklin wrote a letter, in 1772, to his friend Joseph Priestly, where he has explained the method, which consisted in dividing a sheet of paper in two and writing the “Pros” of a decision on one side and the “Cons” on the other. Then, it was necessary to estimate the weights of the arguments of both sides, and by conceptualizing the concept of balance, observe if the benefits of that decision were higher than the disadvantages (Dawes and Corrigan, 1974). This method was one of the first approaches to several concepts that are fundamental in the MCDA approach, like the concept of weight, that was used by Benjamin to compare the “Pros” and “Cons” of a decision. There is no consensus between the scientific community on the meaning of criteria weights (Schoner & Wedley, 1989), which enhances the importance of defining its purpose, consering the method used, for the DM to understand its true meaning (Choo et al., 1999). In general, different criterions have different importances for the DM regarding the final decision that he must take, and, if a certain criterion impact the final result in a stronger way, then its weight should reflect it.

In the following two centuries, several studies, that in a direct, or indirect way are related to MCDA, were developed. Among these works, the publications made by Vifredo Pareto (1814-1923), a well-known economist, were particularly important to the development of several concepts used in this area. He was one of the first to analyze the aggregation of conflicting criteria into a single composite index, and the concept of efficiency, which is also known as Pareto-optimality (Stiglitz, 1981). He concluded that, in a given situation, it is possible to achieve an efficient solution, when it is not possible to improve any preference criterion without worsening another one. This concept is also related with the work of Francis Edgeworth (1845-1926), who introduced the indifference curve theory. In terms of multiple criteria problems, and considering only two criteria, an indifference curve is a representation on a graph of the possible combinations of performances in two different criteria, that give the DM the same benefit (Allen, 1934). This curve is consistent with the efficiency concept presented by Pareto, since each possible combination in it, is an efficient solution (Köksalan et al., 2016). Another important personality in the MCDA evolution was Georg Cantor (1845-1926), a German mathematician, that studied the mathematical concept of infinity, and established the first ideas regarding the set theory and number theory, which have impact in the mathematical concepts used in MCDA (Köksalan et al., 2016).

Since the beginning of the twentieth century, multiple authors, from different fields of science, studied the possibility of characterizing, in mathematical terms, the choice behavior. This framework led to multiple theories that are fundamental in Economics, as the case of the revealed Preference theory, (Samuelson, 1938). It seems innocuous to mention the revealed preference theory without mentioning the utility function, that has developed by Neumann and Morgenstern (1944). In general, the purpose of the utility function is to measure the satisfaction/benefit of the consumer´s decision to him/her (Rader, 1963). This background led to the formal beginning of the MCDA during the 1960´s, with the outranking methods has a critical point in its evolution.

3.2 Outranking methods

The MCDA approach involves aiding in decisions' problems where there are multiple, usually conflicting criteria (Zanakis, 1998). A chronicle problem in the MCDA area is the construction of a preference relation, that has the capacity to represent the preferences of the DM, regarding multiple actions/alternatives evaluated in multiple criteria, allowing to recommend a solution based on his/her judgements. One way of solving this situation, is to develop a value function to represent the DM's preferences, which is different from using outranking relations (Roy and Bouyssou, 1986). The value functions can be very different, even in terms of complexity, and some require all actions to be comparable, which it is not common in MCDA problems. The first outranking method proposed was even created because of problems related to the value functions, introducing a new concept that is crucial in this area, the concept of outranking. The outranking approach is different from the value function one, in the sense that the output of an analysis of different actions in the outranking approach is not a value for each one of them, contrarily to the value function one (Belton & Stewart, 2002). There are a huge variety of methods that deal with DM's preferences in a different way, the process used to access and modelling them, is a perfect example of how to distinguish different groups of methods. For instance, there are methods that model those preferences through an aggregation value function, while others model them by considering different preference relations (Guitouni & Martel, 1998). The first ones are performance aggregation oriented, as in the case of the Multiattribute Utility Theory method (MAUT), and the second ones are preference aggregation based. The method presented in this dissertation fits into this last family of methods.

3.2.1 Origin of outranking methods

During the mid-1960s, a consulting company, SEMA (Société d´Economie et de Mathématiques), was working in developing a method that would allow firms to choose one object, among a set of possible new objects, considering their performances in multiple criteria. They were trying to use a method called MARSAN (Method for Analysis Research and Selecting of New Activities) to solve it. This method uses a weighted sum procedure to obtain a comprehensive score that aggregates the performances in all criteria (Laffy, 1966). They were experiencing some difficulties with the use of weighted sums because it could lead to the compensation dilemma, which means that there was the possibility of assigning a higher score, at the comprehensive level, to an action a in comparison with the score of action b, even if, there is a criterion in which action a is much worse than b. The compensation occurs because in many of the criteria considered action a is preferred to b, which overtakes the fact that a is much worse than b in one criterion (Roy, 1996). Frequently, in real-life problems, this compensation does not occur, because if a certain action a is much worse than b in one criterion, then it is impossible that action a has a higher score than b at the comprehensive level. Bernard Roy, who was approached by this company, joined the group that was working on the project to help solving the various limitations of MARSAN. Together they developed an outranking method called ELECTRE, which stands for "ELimination Et Choix Traduisant la Realité" (Roy, 1968). Several ideas presented by Bernard Roy in the first known outranking method, ELECTRE, are transversal to all outranking methods.

3.2.2 Outranking methods' problem context

In order to understand the outranking methods, and consequently, the ELECTRE family, it is crucial to understand in what contexts they are relevant. The DM must include in the model at least three attributes/criteria to represent the situation, to ensure the concept of concordance (Figueira et al., 2012). The DM has to establish the set of potential actions that are the object of study. In some outranking methods there is the need to also establish the intrinsic weight of the criteria. In the ELECTRE methods, weights are seen as the relative importance of a certain criterion in the overall analysis of an action, that is, the power of vote of that criterion (Figueira et al., 2010). In conjunction with this requisites, there are also specific difficulties inherent to some problems that justify the use of an outranking method, and at least one of the following four must exist (Figueira et al., 2016):

1) When defining the performances of the potential actions in the criteria, at least one of those actions is evaluated on an interval scale or an ordinal scale, which makes inadequate to compare differences between the performances.

2) The nature of the scales associated with the criteria is also associated with the heterogeneity phenom, which makes it harder to define a unique and common scale that could be used to represent the original ones;

3) The DM does not pretend that the method is a compensatory one, which means that the aggregation procedure should not allow a compensation of a poor performance on a certain criterion with a good performance in another one;

4) For at least one criterion the small differences of preferences must not be judged as significant, which requires the use of discriminating thresholds.

3.2.3 Outranking relations

The preference relation built in outranking methods is commonly called an outranking relation, it is possible to verify that the idea of having a preference relation is the same as in the value function approach. The most common definition of an outranking relation is “action a outranks action b if a is at least as good as b”, which is equal to “a not being worse than b” (Roy, 1990). This is a binary relation designated by aSb, which happens when there are sufficient arguments to conclude that a is at least as good as b, and there is no arguments sufficiently strong to refute that conclusion. This way of modeling preferences, with binary outranking relations, makes it possible to establish four different situations, when comparing two actions, a and b (Figueira et al., 2016):

1) aSb and $not(bSa) \Leftrightarrow a$ is preferred to b;

2) bSa and $not(aSb) \Leftrightarrow b$ is preferred to a;

3) aSb and $bSa \Leftrightarrow a$ is indifferent to b;

4) $not(aSb)$ and $not(bSa) \Leftrightarrow a$ is incomparable to b. This situation was introduced by the use of outranking relations, and it is extremely useful in situations where a comparison between two actions cannot be made.

Usually, to develop an outranking relation it is required a pairwise comparison between actions. In the majority of the outranking methods, that comparison is made following a principle of concordance and discordance. In a simple manner, it is concluded that “a is at least as good as b” if the concordance condition, and the non-discordance condition are respected. This means, respectively, that the majority of the criteria supports the idea of “a being at least as good as b", and the rest of criteria that do not support this idea, are not strong enough (Roy, 1990). There are two main procedures used in ELECTRE methods to achieve results that aid the DM, a multiple criteria aggregation procedure, whose purpose is to construct several outranking relations allowing to compare actions in a comprehensive way, and the exploitation procedure that produce results oriented to the problematic nature of the problem.

3.3 ELECTRE methods

Since the creation of the ELECTRE method (that later was called ELECTRE I), several methods within the family were developed, the majority of them were created in response to a particular real-word problem. In order to better understand the ELECTRE methods, they are presented by the three major MCDA problematics (choice, ranking and sorting) (Govindan & Jepsen, 2016):

3.3.1 Choice problematic

The objective of the methods created to deal with this problematic is to aid the DM in choosing the best possible action between a set of multiple ones. One way of solving this problem is the selection of a small subset of actions that has in it, the best possible action (Figueira et al., 2016). The first ELECTRE method ever created is nowadays considered to have small interest in practical terms, however its creation led to an explosion in the MCDA problematic.

The following notation will be transversal to all the methods presented within the ELECTRE family:

- $A = \{a_1, a_2, a_3, \dots, a_i, \dots\}$ denotes the set of actions;
- $F = \{g_1, g_2, \dots, g_j, \dots, g_n\}$, with $n \geq 3$, denote a coherent family of multiple attributes/criteria;
- w_j, with $j = 1, \dots, n$ and $w_j > 0$, denote the weight that is assigned to each attribute.

This method requires a pairwise comparison of all actions, on each criterion. That comparison will define if, in a certain criterion j, an action a outranks b, that is aS_jb. To make this conclusion Roy (1968) established that $g_j(a) \geq g_j(b)$, which means that "a is at least as good as b on criterion j". This particular method uses true criteria in its process, which means that there is no thresholds associated with the criteria. This enables to determine the complete preorder of the ranking structure on one criterion, because, since there is no thresholds associated with it, the smallest increase in the differences of performances between some actions is considered and helps create a rank order for that criterion. Notice, that there is the possibility of existing a tie between options with the same ranking, that is why the preference structure in true criteria is considered to be a complete preorder instead of a complete order (Rogers et al., 2000). At the comprehensive level (considering the whole set of criteria), to conclude that aSb, it was created the concordance and discordance concept. They could be interpreted, respectively, has the arguments in favor and against the outranking relation (Figueira et al., 2010). Two rules to verify these concepts were defined:

(1) The sum of the weights associated with the criteria that supports that coalition (concordance) must be greater than, or equal to, a certain concordance level, s, whose value is usually between the range $[0.5, 1 - \min_{j \in F} w_j]$. The concordance index is used to represent that sum, and it is defined as follows:

$$c(aSb) = \sum_{\{j : g_j(a) \geq g_j(b)\}} w_j \tag{3.1}$$

where $\{j : g_j(a) \geq g_j(b)\}$ represents the set of indices of the criteria that supports aSb. Logically, to respect this condition:

$$c(aSb) \geq s. \tag{3.2}$$

(2) The discordance against the assertation "a outranks b" cannot be equal or greater than a certain discordance level, v. That discordance is represented by a discordance level defined as follows:

$$d(aSb) = \max_{\{j : g_j(a) < g_j(b)\}} \{g_j(b) - g_j(a)\} \tag{3.3}$$

Logically, to respect this condition:

$$d(aSb) \leq v. \tag{3.4}$$

The concordance and discordance indices must be computed to every pair of actions that belong to set A, where $a \neq b$. The presented procedure leads to a binary relation in comprehensive terms (Figueira et al., 2016).This procedure reflects the first part of the structure of this ELECTRE method. The second part concerns the way of achieving the smallest possible subset of actions, that has in it the best possible action. The main idea of the creation of this type of subsets is that any action who is not in it, is outranked for at least one action belonging to it. ELECTRE I, has also introduced the graph kernel concept, K_G. The concept of kernel is that any action that is outranked from at least one action that belongs to the kernel is excluded from it. After the analysis of all actions, the subset that in the end still belongs in the kernel contains the best action (Figueira et al., 2016; Roy & Boyssou, 1993).

It is important to observe that it is required to access the value function g_j. However, in order to make a comparison between actions, the scales of the criteria must be equal, and in real life problems there is the possibility of having qualitative and quantitative elements, which leads to ordinal and numerical scales. The process of transforming each scale associated with a criterion, in a common scale that allows to compare actions, and to justify the use of a max operator in the discordance index, is not an easy one. The heterogeneity related with the scales is, in this way, a serious problem (Figueira et al., 2016). Another drawback is a common difficulty associated with MCDA problems, which considers that a small difference of performances between two actions, in at least one criterion of the model, should not be judged as significant, introducing in this way the need to establish discriminating thresholds (indifference and preferences).The imperfect knowledge is also an important drawback. There are several reasons associated with this concept: the imperfect character of the data from the computation of the performances of actions in the criteria, the ambiguity related with the choice of the criteria, the incapacity of the analyst to fully understand the preferences of the DM, etc. (Roy et al., 2014).

ELECTRE Iv (ELECTRE I with veto threshold) was proposed by Roy and Bouyssou (1993) and introduced a new concept that has proven to be very useful in the MCDA paradigm, the veto threshold. This method is very similar to the previous one (the exploitation procedure is equal), the main differences is how to deal with the discordance between the outranking relationships, and how to consider the importance of each criterion. In this method, no w_j is associated with the criteria, which can be proven to be very helpful in situations where the DM does not want, or have the capacity, to define it. Nevertheless, it is important to state that the method does not consider that each criterion has the exactly same importance than the others (Roy, 1990). Contrarily to ELECTRE I, the discordance condition (now designated no veto condition), when analyzing the hypothesis of "a outranks b " is defined as follows:

$$g_j(a) + v_j g_j(a) \geq g_j(b), \ \forall\, j : g_j(a) < g_j(b) \tag{3.5}$$

The variable v_j is the veto threshold associated with criterion j. All the criteria that do not agree with the outranking relation under study have to be tested, if the differences in performances between two actions is greater than the intrinsic veto threshold the outranking relation is rejected. One of the greatest advantages of the veto, is the fact that it is related to the differences in performances, instead of the scale of the criteria, which helps dealing with the heterogeneity phenom associated with the scales. One of the disadvantages of the method is the incapacity to deal with the imperfect knowledge phenom.

ELECTRE IS was created with the main purpose of having a MCDA method that efficiently deals with the problem of imperfect knowledge. In terms of differences, this method introduced: the concept of pseudo-criteria, a different way of establishing the concordance condition and the no veto condition, and a different version of the exploitation procedure (Roy & Skalka, 1987). A pseudo-criterion is a criterion that has thresholds associated with it. These thresholds were created to cope with the imperfect character of the data, and the difficulty of the DM and establishing the exact performances of actions in the criteria. If in a criterion j, the DM considers that a small difference in the performances is irrelevant regarding one action being better than other, then it is required to associate an indifference threshold, q_j, whose value is equal to the maximum differences in performances on criterion j that the DM considers that can be neglected. It is also possible to create a preference threshold, p_j, whose value is equal to the minimum differences in performances on criterion j that makes possible to establish that a certain action is preferred to another one. Logically, $p_j > q_j > 0$. The concordance and no veto conditions were modified to empower the use of the veto effect, giving it more meaning when there is a decrease in the power of the arguments that justify an action outranking another at the comprehensive level. In the exploitation procedure the major novelty is the consideration of the degree of robustness concerning the outranking relation of action a over b (Figueira et al., 2016). ELECTRE IS method is still used nowadays to deal with MCDA choice problems.

3.3.2 Ranking problematic

The objective in the ranking problems, is to rank all the actions (that belong to a given set), from the best to the worst, considering the preferences of the DM. ELECTRE II was the first ELECTRE method created to deal with ranking problems, and it was developed by Roy and Bertier (1971). This method has presented a new important technique that had highlighted the fact that there is different outranking relations, being some stronger than the others. Modulating the credibility of the outranking relation was introduced considering two different relations, the strong outranking relation and the weak one (Roy & Bertier, 1971). The existence of different relations enables the construction of an embedded outranking relation sequence, which is the base of the technique developed in this method. To establish which type of outranking relation one action should have over another one, two concordance levels were defined, cl^1 and cl^2, with $cl^1 > cl^2$. Both these values belong in the same interval defined for ELECTRE I method $[0.5, 1 - \min_{j \in F} k_j]$. The concordance condition of action a outranking b is defined as follows: $c(aSb) \geq cl^r$ and $c(aSb) \geq c(bSa)$, for $r = 1, 2$. Logically, if this condition is valid for $r = 1 \wedge \mathrm{r} = 2$, then the outranking relation of action a over b is considered strong, if the condition is only valid for $\mathrm{r} = 2$, then the outranking relation of a over b is considered weak. The exploitation procedure considers this new condition to recommend a possible ranking of the actions.

The next ELECTRE method created within the ranking problematic was the ELECTRE III (Roy, 1978). It was created with the intention of improving the limitations of ELECTRE II in dealing with the intrinsic aspects related with collecting the necessary data, namely its imprecision, uncertainty or ill-determination. This new method uses two crucial ideas in the ELECTRE family, the use of pseudo-criteria instead of true-criteria, and the construction of a credibility index for the outranking relation of action a over b. The thresholds associated with the criteria have proved to be crucial in dealing with the limitative aspects of collecting data from a DM. Regarding the use of a credibility index for checking the outranking of action a over b, the main feature is to use conjointly the concordance index used in ELECTRE IS, and a new index that measures how much a specific criterion discord, or not, with the assertation a aSb. This index is known as the discordance index, $d_j(aSb)$. If a certain criterion is not discordant with the outranking relation aSb, the discordant index is equal to 0 (minimum), but, when a criterion is completely discordant with that relation, its value become equal to 1 (maximum). This last situation represents a difference in performances between b and a in that criterion, higher than the veto threshold associated with it. If neither of these last two situations occur than the discordant level will have a value in the range $]0,1[$, that increases in proportion to the difference in performances between b and a in a given criterion. The credibility index, as the name indicates, measures the credibility of the assertation aSb. It is more realistic than the concordant index, $c(aSb)$ because it considers both the reasons that support that assertation and the reasons against (Figueira et al., 2016). The exploitation procedure is very similar to the one used in the ELECTRE II method.

ELECTRE Iv was the third method created to deal with ranking problems. Its structure and fundamental concepts are very similar to the previous method presented, however, instead of using an embedded outranking relation sequence based on two outranking relations, it was based in five (Roy & Hugonnard, 1982). The exploitation procedure is exactly equal to the used in the ELECTRE III.

3.3.3 Sorting problematic

The objective in sorting problems, is to assign actions to categories. The set of categories must be defined a priori, and each action is independent from the other existent actions. Consider that $C = \{C_1, \dots, C_h, \dots, C_k\}$ represent the set of pre-ordered categories. In order to assign each action to a certain category, or multiple categories, they have to be compared with profiles, norms or references, which in their turn define the categories. A certain action is compared with the elements that define a certain category to observe if it should be assigned to that category or not. Nevertheless, it is crucial that the categories have norms that prevent them from being influenced by the assignment of a certain action to them. This idea grants that each action is judged independently, and that the assignment of an action to a specific category does not have any effect in the assignment of others.

ELECTRE TRI, that is also known as ELECTRE TRI-B (Almeida-Dias et al., 2010), was developed to deal with sorting problems and it was firstly presented in the doctoral dissertation of Yu (1992), and subsequently scrutinized in the book written by Roy and Bouyssou (1993) (Bouyssou & Marchant, 2015). In this method a set of categories is established a priori, and they are ordered from the worst category to the best one. They are characterized by two profiles, one representing the lower limit of the category and the other the upper limit. Logically, the upper limit of a given category is equal to the lower limit of the next better category in the order pre-defined, that is b_h (which represents a profile action) is the upper limit of C_h, and the lower limit of C_{h+1} (Almeida-Dias et al., 2010). A family of criteria is also defined, and these criteria are presented in a flat structure (all at the same level). The performances of the profiles and actions in each criterion are defined to allow to compare each action with the limit profiles of the categories. The assignment of a certain action to a category is based on the credibility index used in ELECTRE III, that measures the credibility of the outranking relation of that action over the upper profile of the category and vice-versa. The fuzzy relation created has then to be converted into a crisp relation. This conversion is made through the definition of a cutting level that represents the minimal value possible consistent with the outranking of the action over the reference limit profile. In terms of preferences models, a fuzzy relation is used when it is difficult to make direct conclusions regarding the preference of an action over another one (in this case an action over a reference limit profile). To help in this process, it is computed a value between zero and one, that represents the degree of the preference (Orlovsky, 1978). The credibility index represents this specific situation, that is why the fuzzy outranking relations occupied a very important place in several MCDA methods. The use of fuzzy preference relations has often the intention of helping a DM in expressing its preferences, and, at the same time, increasing the mathematical tractability and physical plausibility of the preference model being created (Bezdek et al., 1978). A crisp outranking relation helps to define the interval where the outranking of an action over another is considered to be valid. After the definition of all crisp relations, an exploiting procedure is made with the objective of assigning an action to one or more categories. The four binary relations mentioned in Section **3.2.3** are one of the pillars of this procedure. There are also two important rules that are used to define in which category, or categories, a certain action should be assigned to, the conjunctive rule and the disjunctive rule. The conjunctive rule states that an action a can be assigned to category C_h, if the evaluation of a, on a sufficient majority of criteria, is at least as good as b_h, and the differences in performances in each criterion, of the remaining ones, are not higher than the specific veto threshold associated with them. The disjunctive is similar, the only difference is that instead of considering that a must be at least as good as b_h on a sufficient majority of criteria, it only requires a sufficient minority of criteria, with the veto rule still applying equally. In both rules, the category in which an action a is assigned to is the highest category that fulfills the previously defined conditions. In this way, each rule will define a unique category for the assignment (Roy & Bouyssou, 1993). Considering the differences between the rules, it is easy to conclude that the assignment of an action in the disjunctive rule is frequently to a higher category than in the conjunctive rule. The exploitation procedure will then lead to two possible scenarios for the assignment of actions, one which is considered to be an optimistic one, and another which is the pessimistic (Almeida-Dias et al., 2010). The DM has the possibility of deciding which type of scenario he/she prefers. One of the disadvantages of this method is the difficulty of the DM in establishing refence profile actions to define the exact boundary between one category and the other.

ELECTRE TRI-C is a multiple criteria sorting method based on characteristic reference actions (Almeida-Dias et al., 2010). This method is used to aid the DM in situations where each category, that belong to the set of pre-ordered categories, is defined using a single characteristic reference action, that the DM considers to be the most representative one of that category. Instead of defining a profile action which coincides with the boundaries between two categories, as it is required in the previous mentioned method, the DM only has to define a unique characteristic reference action to assign to each category. They are established, and its performances, through a co-construction interactive process between an analyst and a DM. There are four structural requirements that must be respected, and the method was developed to verify them: conformity, homogeneity, monotonicity and stability. In terms of the exploitation procedure, the ELECTRE TRI-C method uses two joint rules, the ascending and descending rule, that must be used conjointly to define a range of possible categories to assign a certain action (Almeida-Dias et al., 2010). The use of a selecting function that considers conjointly the credibility of the outranking of the action over the characteristic reference action and the opposite, is the basis for selecting, in each rule, a specific category. The range of possible categories to assign one action is the interval between the specific categories resulting from the application of the two rules.

In 2012, a new method was developed, and it is a generalization of the previous one. This method is the ELECTRE TRI-nC, and its main feature is that each category is characterized by one or more reference actions. The possibility of defining multiple reference actions to characterize each category gives more flexibility to the DM, which will not have to choose only one action between a set that he/she considers to represent well the category that is being characterized (Almeida-Dias et al., 2012). The rest of the method is equal to the ELECTRE TRI-C, including the exploitation procedure.

It is important to notice, that due to the undeniable success of the ELECTRE TRI methods, several papers have been released to deeper study the characteristics of the different methods. For instance, a study about the interactions between criteria and their effect in the relative importance of those criteria in the ELECTRE methods, was developed. (Figueira et al., 2009a). It presented a procedure that allows to consider the effects of possible interactions between the criteria in the assignment procedure, thus trying to take even more into account the complexity underlying real-life problems.

3.3.4 Other outranking methods

Multiple outranking methods have been developed since the creation of the original ELECTRE (1966). Within this family, the PROMETHEE (Preference Ranking Organization Method for Enrichment of Evaluations) methods have proven to be successful in many different situations. PROMETHEE I and PROMETHEE II were the first PROMETHEE methods created (Brans, 1982). The first one was developed to deal with partial ranking and the second one to deal with complete ranking. The purpose of the majority of the PROMETHEE methods created throughout the last decades was the ranking of a set of actions, which, as already mentioned, is a particular problematic within the MCDA paradigm. The main idea of the method is to make pairwise comparisons between the actions, which requires that the DM provides some preference information on the parameters involved, as the weights of criteria and thresholds involved (Brans & Smet, 2016). The DM could provide this information in a direct or indirect way. When the DM do not have the capacity, or do not feel comfortable, to give exact values regarding the parameters required in the method (direct preferences), he/she will have to give indirect preferences, that is, preferences between the actions. These preferences will then be the inputs in a process whose output is to produce compatible preference parameters. To accomplish that, there are several methodologies that can be used, particularly the Robust Ordinal Regression (ROR) and the Stochastic Multiobjective Acceptability Analysis (SMAA) (Corrente et al., 2014). There are others that can be applied to deduce the preference parameters, however some of them do not have the capacity to overpass the plurality phenom that could occur in this type of process. That phenom happens when the method used finds multiple compatible preference parameters to represent the preferences of the DM. The use of one of the possible preference parameters over another one will influence the results of the model. To overcome this type of situation the ROR methodology was created. ROR is a family of MCDA methods that is based on two different preference relations, a necessary preference relation and a sufficient preference one. The first one states that "action a is necessary preferred to b" if "a is at least as good as b" for all compatible sets of parameters, while the second one considers that "action a is possibly preferred to b" if "a is at least as good as b" for at least one compatible set of parameters (Corrente et al., 2013). This method considers all compatible value function simultaneously by using the two preference relations previously explained. The concept of ROR has been used in several MCDA methods, and in different problematic natures. SMAA is also a family of MCDA methods that considers the lack of data and imprecision intrinsic to it, through the use of probability distributions related with the weights of the criteria and the evaluation of the actions. After inferring the preference parameters, and in conjunction with the exploitation procedure, the PROMETHEE method gives a ranking of the actions.

3.4 Multiple Criteria Hierarchy Process (MCHP)

In the previously mentioned methods, the evaluation criteria are all considered to be at the same level, which represents a flat structure. However, in real life situations, the majority of multiple criteria problems is better represented in a hierarchical structure of criteria. It is even easier for a DM to represent a specific situation when using this type of structure, decomposing a complex problem into less complex segments. This idea led to the creation of a MCHP, developed by Corrente et al. (2012). This methodology aims at defining a new process that has the capacity to formulate a hierarchical structure of criteria to represent a problem with multiple criteria. Instead of a flat structure, this process will lead to a hierarchy tree of criteria, with multiple nodes, that are criteria, dividing themself into other nodes, until decomposing all the complex criteria into simple ones, resulting in multiple levels in the hierarchy tree. The main idea of MCHP is to consider preference relations at each node of the hierarchy tree. This is made in two different phases, the first one is when the DM is expressing its preferences regarding some reference actions, which is known as the phase of eliciting preference information, and the other is the final recommendations phase. Investigating the preference relations in each node is very useful in complex problems because it allows to observe the preferences of the DM in particular segments of the problem which results in a well justified overall recommendation. For instance, if action a is preferred to action b at the comprehensive level, it is interesting to observe the existent preference relation in some subcriterion that are crucial for the DM (Corrente et al., 2012). Preference relations can be built within any MCDA method, enabling the use of MCHP in any of them.

In recent years, this new process has been proposed to be used in various MCDA methods (Corrente et al., 2012, 2013, 2017a, 2017b), being the first one the ROR (more precisely the GRIP method). The application of MCHP to ROR allows to define the two preference relations used in this method for all the existing nodes in the hierarchy tree. The first method that used the ROR methodology was the UTA^{GMS}, which ranks actions by using a set of additive value functions that were the result of an ordinal regression process that elucidates the DM's preferences on some pairwise actions (Greco et al., 2008). A generalization of this method was then purposed. The main novelty of this new method, the Generalized Regression with Intensities of Preference (GRIP), was the introduction of the intensities of preferences for pairs of actions which the DM decides to give preference information about. The GRIP method, as the UTA^{GMS} method, constructs additive value functions compatible with preference information elicited by the DM, however, it also requires the intensities of that preferences. It is interesting to observe that both these methods used two different methodologies conjointly, the outranking approach (pairwise comparison of actions) and the idea of assigning a score to each action through the use of value functions (as in the case of MAUT) (Figueira et al., 2009b). The use of MCHP in GRIP begins with the definition of the hierarchy tree of criteria, followed by the collection of preference information given by the DM on a pairwise comparison of some reference actions in some criteria of the hierarchy tree (including the comprehensive level possibly). Then, it is computed that information, and through a process of ordinal regression, an additive value function that is compatible with that information is defined. However, in some cases, it is possible that the ordinal regression process finds multiple additive value function that are compatible. In this case, ROR uses all the functions simultaneously, which requires the use of properties of necessary and possible preference relations (Figueira et al., 2009b). The intensity of preference is defined through four different preference relations, two of those regarding the comprehensive level and the other two about a certain subcriterion. The process of establishing those preference relations is made, not only at the comprehensive level, but also in all subcriteria existent in the tree. The same occurs with the value functions defined, because the DM could have interest in having value functions at intermediate levels. This will allow to study the ranking of actions, or even their score, in specific subcriterion in different levels of the hierarchy, which is usually very useful to the DM.

After the implementation of the MCHP in ROR, it was suggested its application in the method ELECTRE and PROMETHEE (Corrente et al., 2013). The ROR concept has already been proposed to be applied in some outranking methods, more precisely in the ELECTRE method and in the PROMETHEE one. In ELECTRE^{GKMS} (ELECTRE with ROR), the DM, besides giving preference information in the form of the outranking veracity between some pair of actions (fictitious or real ones), also defines the ranges of variation of comparison thresholds in the pseudo-criteria. The ROR concept is than used to build a set of values within those ranges that maintain the DM elicited preferences. The necessary and possible outranking relations are then explored in order to make the final recommendations (Greco et al., 2011). In PROMETHEE^{GKS} (PROMETHEE with ROR), the set of compatible outranking models is constructed with the use of the ROR process, contrarily to the original PROMETHEE method that only uses ordinal regression (Greco & Roman, 2012). The proposed method by Corrente et al. (2013) combines two important features that give more freedom to the DM when defining the model, the possibility of establishing a range of values to define the thresholds associated with the pseudo-criteria, and the opportunity to create a hierarchical tree of criteria. The MCHP was then proposed to be applied in the sorting methods without the ROR methodology. The ELECTRE TRI methods were the first family of methods chosen to apply a hierarchy structure of criteria, namely the ELECTRE TRI-B, ELECTRE TRI-C and ELECTRE TRI-nC. The paper that explore this theme has applied the MCHP in a way that allows to sort the actions at the comprehensive level, but also in all the nodes of the hierarchy tree with exception to the last but one level (Corrente et al., 2016). It also introduced a generalization of the Simos-Roy-Figueira (SRF) method to define the values of the weights of criteria organized in a hierarchical way, and a methodology to deal with different types of interactions between criteria, namely, possible existent synergies, redundancy, and antagonistic effects between them (Figueira & Roy, 2002; Figueira et al. 2009, Corrente et al., 2016). The SRF weighing procedure is a method based on the Simos' procedure, proposed by Simos (1990), that defines a numerical value to the criteria's weights compatible with the relative importance that the DM considers that they have in his/her decision. The SRF method uses a co-construction interactive process that is based on a procedure that uses a set of cards to create intervals between criteria, which are interpreted in a way that allows to estimate values to the weights (Figueira & Roy, 2002). The use of a physical ranking of cards (criteria), makes the understanding of the procedure's purpose very intuitive for the DM (Simos, 1990a,b). The MCHP in ELECTRE TRI methods assumes that the DM has the capacity to give all the necessary direct preference information needed to the model. It also establish two coherence properties, and conditions to ensure that those properties are respected, regarding the existence of subcriteria in the structure of criteria (Corrente et al., 2016). MCHP was later proposed to be applied in the ELECTRE III ranking method (Corrente et al., 2017).

3.5 Summary

This chapter introduced the concepts related to the MCDA approach and the outranking methods, and a description of the ELECTRE family of methods, that includes the method proposed in this dissertation, the ELECTRE TRI-nC. The original flat structure of this method regarding the criteria is changed by the introduction of the MCHP, that is also presented in this chapter. Several other outranking methods and their use with the MCHP were also addressed.

Chapter 4: Methodology

This chapter presents the method that is going to be used to evaluate the quality and access to health care in the Portuguese public hospitals. The inherent difficulty on evaluating two characteristics as complex and subjective as the quality and access led to the choice of a sorting method, whose model definition is made through an interactive construction process with a DM. The proposed method is the ELECTRE TRI-nC, however, the original one considers a flat structure of criteria, which does not illustrate well how the criteria are organized in this type of situation. Hence the choice to incorporate the MCHP in this method, thus enabling the construction of a hierarchy tree of criteria to represent as closely as possible to reality the health care problem that is being addressed. The first part of this chapter describes the ELECTRE TRI-nC method and the MCHP, individually. The last part describes the incorporation of the MCHP into the ELECTRE TRI-nC method and the SRF weighing procedure.

4.1 ELECTRE TRI-nC method

ELECTRE TRI-nC is an outranking method that was developed to aid the DM in MCDA problems where his/her objective is to sort the objects of a decision (actions) to a set of categories, taking into consideration their evaluation in multiple criteria (Almeida-Dias et al., 2012). This method has several similar assumptions and properties to other methods in the ELECTRE family. However, it has a new assumption that concerns the characterization of a defined set of categories.

In this method, there are three assumptions that must be taken into consideration:

Assumption 1: The actions must be assigned to a set of categories that are completely ordered from the lowest extreme to the highest extreme of a specific aspect (for example, from the least harmful to the most harmful, from the least profitable to the most profitable, etc.).

Assumption 2: The actions will be assigned to categories that were established a priori. The process of establishing those categories and the definition of their order could be done simultaneously.

Assumption 3: The characterization of each category is made through the assignment of a subset of reference actions, that the DM considers exemplary of the ones that should be assigned to that category.

The MCDA methods develop processes that help the DM to structure the problem and to identify his/her preferences (Cinelli et al., 2020). That is why assumption 3 has been modified in several ELECTRE methods to give more freedom to the DM in the process of characterizing the categories. As mentioned in Section **3.3.3**, ELECTRE TRI-nC uses a set of unlimited reference actions to characterize them.

4.1.1 Problem statement

This method is based in a co-construction interactive process between two actors, the DM and the analyst who is aiding him/her. From now on, let $A = \{a_1, a_2, \dots, a_i, \dots,\}$ denote the set of potential actions in the problem. These actions could all be known a priori, or can be added by the DM during the decision aiding process with the analyst. Let $C = \{C_1, C_2, \dots, C_h, \dots, C_q\}$, with $q \geq 2$ (if $q = 1$ there is no sorting problem), denote the set of completely ordered categories, whose purpose is to receive the actions from A. The potential actions a, with $a \in A$, are evaluated in a coherent family of multiple criteria, denoted by $F = \{g_1, g_2, \dots, g_j, \dots, g_n\}$ with $n \geq 3$. Each criterion g_j can have a decreasing, or increasing direction of preference, but in the rest of the document, it is assumed, without loss of generality, that the preferences increase when the performance in the criteria increases also, that is, all criteria $g_j \in F$ are to be maximized. There are two thresholds associated with each criterion g_j: a preference threshold, p_j, and an indifference threshold, q_j, such that $p_j \geq q_j \geq 0$. These thresholds model the imperfect character of the data regarding the computation of the performances $g_j(a)$, for all $a \in A$. As the inequality $p_j \geq q_j \geq 0$ shows, it is possible to have $p_j = q_j = 0$, for all $g_j \in F$, but this is a very particular case, because it means that the DM has the ability in all criteria to distinguish differences in performances that are almost imperceptible, which is not very common in real life problems. Let $B_h = \{b_h^r, r = 1, \dots, m_h\}$ denote a subset of reference actions, indicated by the DM, to characterize category C_h, with $h = 1, \dots, q$ and $m_h \geq 1$, and let $B = \{B_0, B_1, \dots, B_h, \dots, B_q, B_{q+1}\}$ denote the set of subsets of reference actions, where $B_h = \{ b_h^r, r = 1, \dots, m_h\}$, with $h = 1, \dots, q$. Notice that B_0 and B_{q+1} are particular cases, that represent a subset with a reference action, that has the worst, and the best possible performance for all $g_j \in F$, respectively. If combined together, the criteria and thresholds, it is possible to define three binary relations:

(i) $|\boldsymbol{g_j(a) - g_j(b)}| \leq \boldsymbol{q_j}$ describes a non-significant advantage of one of the two actions over the other, that means "a is indifferent to b" and "b is indifferent to a" according to criterion g_j, denoted by aI_jb. Let $C(aIb)$ represent the subset of criteria such that aI_jb.

(ii) $|\boldsymbol{g_j(a) - g_j(b)}| > \boldsymbol{p_j}$ describes a significant advantage of a over b, which indicates that "a is strictly preferred to b" according to criterion g_j, denoted by aP_jb. Let $C(aPb)$ represent the subset of criteria such that aP_jb.

(iii) $\boldsymbol{q_j < g_j(a) - g_j(b) < p_j}$ describes an ambiguity zone. This means that the advantage of a over b is significant enough to not consider it indifferent, however it is not large enough to conclude that a is strictly preferred to b. Since there is apprehension in considering this binary relation equal to one of the above, it will be considered that "a is weakly preferred to b", according to criterion g_j, denoted by aQ_jb. Let $C(aQb)$ represent the subset of criteria such that aQ_jb.

It is also possible to associate a veto threshold to each criterion g_j, denoted by v_j, with $j = 1, \dots, n$ and $v_j > p_j$. This parameter was created to represent situations where the difference in performance between two actions in one criterion is large enough to negate any possibility of an outranking relation, that could be indicated by other criteria (Nowak, 2004).

4.1.2 Outranking

The outranking concept was already mentioned in section **3.2.3** and is meaning is a fundamental one: if "a is at least as good as b" according to criterion g_j, then "a outranks b", denoted by aS_jb. This statement is validated, without ambiguity, if $g_j(a) - g_j(b) \geq -q_j$, nevertheless if $-p_j \leq g_j(a) - g_j(b) \leq -q_j$ there is the possibility of an indifference relation between a and b that should not be neglected. This indifference is increasingly more credible when $g_j(a) - g_j(b)$ moves closer to $-q_j$. In order to conclude, that "a outranks b", at the comprehensive level, all the criteria, and several indices and variables related with them, must be considered. Let w_j denote a single vector of intrinsic weights that is associated with the set of criteria, such that $w_j > 0, j = 1, \ldots, n$. Assume that $\sum_{j=1}^{n} w_j = 1$.

Let $c(a,b)$ denote the comprehensive concordance index, that is defined as follows:

$$c(a,b) = \sum_{j \in C(aPb)} w_j + \sum_{j \in C(aQb)} w_j + \sum_{j \in C(aIb)} w_j + \sum_{j \in C(bQa)} w_j \varphi_j, \tag{4.1}$$

where,

$$\varphi_j = \frac{g_j(a) - g_j(b) + p_j}{p_j - q_j} \in [0,1[. \tag{4.2}$$

Since the intrinsic weights used in the ELECTRE methods are interpreted as the power of vote, the variable φ_j characterizes the manner in which that power of vote decreases according to criterion $g_j \in C(bQa)$.

Let $d_j(a,b)$, $j = 1, \ldots, n$ denote the partial discordance index, defined as follows:

$$d_j(a,b) = \begin{cases} 1 & if \quad g_j(a) - g_j(b) < -v_j, \\ \frac{g_j(a) - g_j(b) + p_j}{p_j - v_j} & if \quad -v_j \leq g_j(a) - g_j(b) < -p_j \\ 0 & if \quad g_j(a) - g_j(b) \geq -p_j \end{cases} \tag{4.3}$$

Combining the two last concepts, it is possible to achieve an index that represents the credibility of the comprehensive outranking of a over b. This index, that considers all the criteria from F, illustrates the strength of the statement "a outranks b", and is defined as follows:

$$\sigma(a,b) = c(a,b) \prod_{j=1}^{n} T_j(a,b) \tag{4.4}$$

where,

$$T_j(a,b) = \begin{cases} \frac{1 - d_j(a,b)}{1 - c(a,b)} & if \ d_j(a,b) > c(a,b) \\ 1 & otherwise \end{cases} \tag{4.5}$$

This index is compared with a credibility level, λ, that represents the minimum degree of credibility that the DM considers to be necessary to validate the statement "a outranks b", taking all the criteria from F into account. Usually, the range of values of this credibility level is $[0.5, 1]$. This comparison allows to establish four different binary relations defined as follows:

(Z1) λ-outranking: $aS^{\lambda}b \Leftrightarrow \sigma(a,b) \geq \lambda$

(Z2) λ-preference: $aP^{\lambda}b \Leftrightarrow \sigma(a,\mathrm{b}) \geq \lambda \wedge \sigma(b,a) < \lambda$

(Z3) λ-indifference: $aI^{\lambda}b \Leftrightarrow \sigma(a,\mathrm{b}) \geq \lambda \wedge \sigma(b,a) \geq \lambda$

(Z4) λ-incomparability: $aR^{\lambda}b \Leftrightarrow \sigma(a,\mathrm{b}) < \lambda \wedge \sigma(b,a) < \lambda$

After the definition of the subset of reference actions $B_h = \{b_h^r, r = 1, \dots, m_h\}$, that define each category C_h, with $h = 1, \dots, q$ and $m_h \geq 1$, it is required to impose two conditions regarding the reference actions of consecutive categories to ensure that they are distinct, the dominance condition and the weak separability condition. However, if the DM considers that the second one is not strong enough to define distinct categories, there is the possibility of replacing it with one of the other two existent, the strict or the hyper-strict separability (Almeida-Dias, 2012). These conditions are described in **Appendix A**.

4.1.3 Assignment procedure

The assignment of an action a, with $a \in A$, to a certain category, C_h, is based, among other rules, in the comparison of that action with the reference actions, B_h. Since each category could be characterized by several reference actions, $b_h^r, r = 1, \dots, m_h$, the comparison of an action a with them, will provide m_h credibility indices of the type $\sigma(b_h^r, a)$ and $\sigma(a, b_h^r)$.

It is useful to find an aggregation operator, that allows to compare an action a with a subset of reference actions, B_h, with respect to one representative credibility index. The max operator is a logical choice, and was already used in multiple MCDA methods (Almeida-Dias et al., 2012). It is defined as follows:

$$\sigma(a, B_h) = \max_{r=1,\dots,m_h} \{\sigma(a, b_h^r)\} \tag{4.6}$$

$$\sigma(B_h, a) = \max_{r=1,\dots,m_h} \{\sigma(b_h^r, a)\} \tag{4.7}$$

The representative credibility indices computed in eq. 4.6 and 4.7, can be designated as the categorical outranking degrees of an action a over the subset of reference actions, B_h, and vice versa. Notice that there are several aggregation operators used in MCDA methods. The ordered weighted aggregation (OWA) operator, proposed by (Yager, 1988) could be an interesting possibility to analyze in the future, since it has the capacity to mix two complementary requirements (all the requirements must be satisfied and at least one of the criteria to be satisfied).

The assignment procedure of the ELECTRE TRI-nC method uses two joint rules, that must be used in conjunction: the ascending rule and the descending rule. A selecting function, $\rho(\mathrm{a}, b_h^r)$ is also required, in order to choose between the two consecutive categories where an action a can be assigned. The selecting function, $\rho(\mathrm{a}, b_h^r)$, must fulfill two properties, in order to maintain the role of the subset of reference actions b_h:

Property 1:

- The selecting function must be a function of $\sigma(a, B_h)$ and $\sigma(B_h, a)$, with B_h representing the subset of reference actions, $h = 1, \dots, \mathrm{q}$.

- If a certain category C_h is pre-selected to assign action a, the condition used to select that category instead of the adjacent one (that have to be considered) must be relevant. The condition chosen is the following one: $\rho(a, B_h) > \rho(a, B_s)$, with $s = h - 1$ or $s = h + 1$, depending on the joint rule used to make the pre-selection.

Property 2:

- Let a and b be two actions that in the pre-selection phase were assigned to the same category. If a strictly dominates b, then $\rho(a, B_h) > \rho(a, B_{h+1})$ implicates that $\rho(b, B_h) > \rho(b, B_{h+1})$.

As in the case of the aggregator operator, there is no unique possibility that fulfills the two properties required, which opens possibilities for future studies, to addressed different functions, and their differences in the results. For now, the selecting function used is the following one:

$$\rho(a, B_h) = \min\{\sigma(a, B_h),\ \sigma(B_h, a)\} \tag{4.8}$$

The first step is to select a credibility level, λ in the range $[0.5, 1]$. Then, it is required to compute the credibility indices of an action a, with the subset of reference actions B_h. This part of the procedure is equal in both rules. However, the rest of the procedure requires a distinction between the two:

Ascending rule:

- Increasing the value of h, from 0, until the first value, k, that respects the following condition: $\sigma(B_k, a) \geq \lambda$. In this way, the category pre-selected, B_k, is the lowest subset of reference actions, that, considering the chosen credibility level, validates the statement "B_k outranks a" (C_k is then called the ascending pre-selected category). Nevertheless, this assignment must be studied, particularly the possibility of assigning action a to the category C_{k-1}. Remind that subset B_{k-1} does not outrank action a with the chosen credibility level, because as previously mentioned, in ELECTRE TRI-nC the subset of reference actions B_{k-1} was not created to act as a subset of lower bounds for the category C_k. When observing the selecting function defined, if $\rho(a, B_k) > \rho(a, B_{k-1})$ then C_k is selected as a possible category to assign a, otherwise assign a to category C_{k-1}. The two particular cases of this rule occur when $\sigma(B_k, a) \geq \lambda$, with $k = 1$ or $k = q + 1$ These situations do not require an investigation of possible assignment of a to the adjacent category, then, respectively, C_1 or C_q, are selected as possible categories to assign action a.

Descending rule:

- Decrease the value of h, from $(q + 1)$ until the first value, t, that respects the following condition: $\sigma(a, B_t) \geq \lambda$ (C_t is then called the descending pre-selected category). Since B_{t+1} was not defined to act as a subset of upper bounds for the category B_t, then "a outranks B_{t+1}" is not validated with the chosen credibility level, λ. Nevertheless, it is necessary to study the possibility of assigning action a to category C_{t+1}. The two particular cases of this rule occur when $\sigma(\{a\}, B_t) \geq \lambda$, with $t = 0$ or $t = q$. These situations do not require an investigation of possible assignment of a to the adjacent category, then, respectively, C_1 or C_q, are selected as possible categories to assign action a.

By using both rules in conjunction, the ELECTRE TRI-nC assignment procedure gives the highest, and the lowest possible categories where an action a should be assigned to, which results in:

- A range of three or more consecutive categories, in which the selected categories are the extremes;
- Two categories, if the two selected categories are consecutive;
- One category, when the two selected categories are equal.

Let $\Gamma(\mathrm{a})$ denote the range of consecutive categories that, according to the ELECTRE TRI-nC procedure, are possible categories to assign a certain action a.

There are four structural requirements that ensure the desirable properties of the ELECTRE TRI-nC method (Almeida-Dias, 2012). Those structural requirements are the following ones:

(a) Conformity: Each reference action b_h^r, with $r = 1, \ldots, m_h$, must be assigned to category C_h, with $h = 1, \ldots, q$.

(b) Homogeneity: If two actions have the same outranking credibility indices, when compared with each one of the reference actions, then they must be assigned to the same category.

(c) Monotonicity: If an action a strictly dominates action b, then, at least, action a is assigned to the same category that b is assigned to.

(d) Stability: When applying a merging or a splitting operation, the actions that were assigned to the non-modified categories will be assigned to the same ones, or, to the new categories, after modification.

In the ELECTRE TRI-nC method it is possible to split consecutive categories, or even to merge them into a new category. However, there are some rules regarding the subset of reference actions to perform these operations and maintain the required stability.

4.1.4 Weighing procedure

In the ELECTRE methods, each criterion has an intrinsic weight associated with it, which means that the weights are independent from the range of the scale and the unit selected in each criterion (Corrente et al., 2016). That reason eliminates some existent methods that do not have the ability to attribute an intrinsic weight to each criterion, for instance the MAUT method (Keeney & Raiffa, 1993) and the Weighted sum procedure. The ELECTRE TRI-nC method uses the SRF weighing procedure to define the values of the weights. The first step of the method is to write each criterion used in one card. Then, the DM has to rank that cards from the least important criterion until the most important one, with the possibility of having more than one card in a certain ranking position, when the criteria present in those cards have the same importance for the DM. After this procedure, the DM is asked to insert one, or more, empty cards between the sets of criteria, the number of empty cards chosen reflects the differences in importance between the two consecutives sets of criteria, that is, more empty cards between sets means a higher difference in importance between them (Figueira & Roy, 2002). It is also possible to not insert any empty card between two adjacent ranking positions. In this case, the differences in importance between the criterion/criteria inserted in those ranking positions are the minimum possible. Recap that $F = \{g_1, \dots, g_j, \dots, g_n\}$ with $n \leq 3$, is the set of considered criteria. Let $L = \{L_1, \dots, L_s, \dots, L_v\}$, with $L \subseteq F \wedge \mathrm{L_w} \cap \mathrm{L_s} = \emptyset\ \forall\ w \neq s, w, s = 1, \dots, v,$ represents the set of sets of criteria that are in the same ranking position, with L_1 being the set of least important criteria and L_v the most important one, and e_s the number of empty cards between L_s and L_{s+1}, with $s = 1, \dots, v-1$. The DM still has to define one more parameter Z, that represents the ratio between the weights of criteria that belong to set L_v and L_1. All this information is used to compute the normalized weights for each criterion $g_j \in F$, in the following way:

$$w_j = \frac{w_j'}{\sum_{i \in F} w_j'}, \tag{4.9}$$

where:

$$w_j' = 1 + \frac{(z-1)\,[l(j) - 1 + \sum_{s=1}^{l(j)-1} e_s]}{v - 1 + \sum_{s=1}^{v-1} e_s}, \tag{4.10}$$

and l_j represents the rank of importance where criterion j belongs ($j \in L_{l(j)}$). The normalizations is made to ensure that $\sum_{j=1}^{n} w_j = 1$. The criteria weights are then available to be used.

4.1.5 Advantages and disadvantages

The ELECTRE TRI-nC method has several advantages in comparison with other outranking methods. The fact that it is user-friendly, in the sense that its process of defining the categories gives more freedom to the DM than the other ELECTRE methods. The categories are also defined considering more information about them, making their characterization a more robust process. It is also a method that is based in a co-construction interactive process, which is very useful when the DM's preferences are not completely clear to him/her. The thresholds associated with the criteria also help to minimize the effects of uncertainty regarding the data provided by the DM when defining the performances of the actions in the criteria, which is completely normal in complex problems. The major disadvantage of the method is the use of a flat structure of criteria. Usually, this type of structure does not represent real life problems well, and, at the same time, does not allow to evaluate the actions in different levels of the structure, which is limitative in some situations. The evaluating problem considered in this dissertation is one of those situations, that is why a new approach to this method is going to be used. This approach will be based in the MCHP, and its implementation in the previously explained method.

4.2 Multiple Criteria Hierarchy Process (MCHP)

The MCHP was created to deal with decision problems that have criteria structured in a hierarchical way, and consequently organized in different levels. This happens, especially, when there are different aspects, that are interconnected at the comprehensive level, but, at the same time, can be seen as independent. The complexity of the problem could lead to some difficulties and hesitations of the DM when structuring it. This process enables the possibility of focusing in "smaller problems" from the major one (Corrente et al., 2012), which could prove to be very effective in helping the DM when developing the model. Naturally, there is the need to aggregate the criteria of different levels (the criteria present in the inferior layer have to be aggregated into one or more criteria of the above layer), in order to establish outranking relations in different criteria, at different levels of the hierarchy. This new feature allows the comparison of actions not only at the comprehensive level, but also at intermediate areas (subcriteria in lower levels), which, in the sorting problematic, represents the possibility of sorting actions into categories by considering specific subcriteria in different levels of the hierarchy.

4.2.1 Notation:

There is a set $\check{G}$ of hierarchically ordered criteria, that are distributed over l different levels. The criterion that represents the comprehensive level is the root criterion one, and it is located in level 0. Any criterion defined in the hierarchical tree descends from it. The criteria that directly descend from the root criterion are located in the first level of the hierarchy, and they can be considered as first level criteria. The criteria that are in level l (last level) are elementary ones. The following notation was defined by Corrente et al. (2012), and it will be used in the rest of the dissertation:

- $\check{G}$ is the set of all criteria at all considered levels;

- m is the number of first level criteria;

- l is the number os levels/layers in the hierarchy of criteria;

- $A = \{a_1, \dots, a_i, \dots\}$ is the finite set of actions;

- $L_{\check{G}}$ is the set of indices of specific criteria that represents the position of criteria in the hierarchy;

- $G_r \in \check{G}$, with $r = (i_1, \dots, i_h) \in L_{\check{G}}$, denotes a subcriterion of the root criterion at level h;

- $G_r = G_0$, represents the entire set of subcriteria, not a particular criterion or subcriterion;

- $n(r)$ is the number of subcriteria of G_r in its subsequent level, that is, the subcriteria of the subsequent level of G_r are $G_{(r,1),\dots,}$ $G_{(r,n(r))}$;

- g_j, with $j = (i_1, \dots, i_l) \in L_{\check{G}}$, is an elementary subcriterion of the first level criterion G_{i_1}, at level l.

- $g_j(a)$, with $a \in A \wedge j = (i_1, \dots, i_l) \in L_{\check{G}}$, is the performance of action a on the elementary subcriterion g_j;

- EL represents the set of indices of all elementary subcriteria:

$$EL = \{j = (i_1, \dots, i_l) \in L_{\check{G}}\} \text{ where } \begin{cases} i_1 = 1, \dots, m \\ i_2 = 1, \dots, n(i_1) \\ \dots \\ i_l = 1, \dots, n(i_1, \dots, i_{l-1}) \end{cases}$$

- $E(G_r)$ is the set of indices of elementary subcriteria descending from G_r:

$$E(G_r) = \{(r, i_{h+1}, \dots, i_l) \in L_{\check{G}}\} \text{ where } \begin{cases} i_{h+1} = 1, \dots, n(r) \\ \dots \\ i_l = 1, \dots, n(r, \dots, i_{l-1}) \end{cases}$$

- Logically, $E(G_0) = EL$;

- LBO represents the set of indices of all subcriteria located in the level $l-1$ (the last but one level of the hierarchy)

- $LB(G_r)$ represents the set of indices, descending from criterion G_r, of all subcriteria located in the level $l-1$. Logically, $LB(G_0) = LBO$;

4.2.2 Basic concepts

There are some assumptions that have to be made to have a logical process that can be adapted for different MCDA methods and situations. These assumptions were constructed in such a way, as not to allow the loss of generality of the method, and they are the following ones (Corrente et al., 2012):

1) When defining the performances of the actions in all the elementary subcriterion, $g_t(a)$, with $t \in EL \wedge a \in A$, it is important that those performances are defined in cardinal numbers, even in if the original scale of a criterion is an ordered qualitative one. This particular case requires a coherent way of transforming that scale in a quantitative one, without losing the DM's preference order. This allows to make conclusions such as: $g_t(a) \geq g_t(b)$ means that "action a outranks b" on criterion g_t, for all $a, b \in A \wedge t \in EL$;

2) All elementary criterion, g_t, with $t \in EL$, have a specific direction of preference;

3) The actions a, with $a \in A$, are only evaluated directly on the elementary subcriteria, g_t, which means, that, in order to make preferences considerations at criterions $G_r \in \check{G}$, first it is required to access the elementary criteria performances;

4) In each criterion $G_r \in \check{G}$, there is a preference relation, $\succsim_r$, regarding the options of set A, in a way that, for all $a, b \in A$, $a \succsim_r b$ means "action a outranks b on subcriterion G_r". To establish this relation, a dominance principle for hierarchy of criteria must be respected, it states that in order to consider that "a outranks b on subcriterion G_r", "a must also outrank b in all subcriteria in the subsequent level of G_r, $G_{(r,j)}$". In mathematical terms this principle can be defined in this way: In a certain criterion G_r, $r \in L_{\check{G}} \backslash EL$, if $a \succsim_{(r,j)} b \ \forall j = 1, \dots, n(r)$, then $a \succsim_r b$. When the subcriterion is one elementary criterion, the preference relation holds if the performance of an a action is equal or greater than action b, that is, when $G_r = g_t$, $t \in EL$, $a \succsim_t b$ holds if $g_t(a) \geq g_t(b)$;

5) There is the need to use an aggregating procedure to aggregate the evaluations of actions regarding the elementary subcriteria, g_t, $t \in EL$.

4.2.3 Application

One of the main advantages of the MCHP is the possibility of being used in different MCDA methods, despite some possible adjustments imposed by the method used. The generality aspects of its structure allow its use in all the major MCDA problematics (choice, ranking and sorting). The ability to produce results in intermediate levels has foment considerable interest within the scientific community. Hence, as mentioned in Section **3.4.1**, the publishment of multiple articles proposing its application in different methods, namely the ELECTRE TRI methods (Corrente et al., 2016).

4.3 ELECTRE TRI-nC method with MCHP

The MCHP was already proposed for ELECTRE TRI methods to eliminate one of the major limitations of this method: its incapacity to deal with problems where not all criteria are considered to be at the same level. The particular case of evaluation type of problems is an interesting example of a situation where criteria are usually considered to be structured in a hierarchical way. In real life problems, when it is necessary to evaluate an action, or multiple ones, at the comprehensive level, the actors commonly consider diferent groups of evaluation, that have a portion of all the criteria defined, to focus.

The specific case of evaluating the quality and access to health care in the Portuguese public hospitals, can be decomposed in different layers, with each layer having different groups, until reaching the elementary criteria. This method gives the DM the ability to make conclusions about the outranking relation between actions, not only at the comprehensive level, but also in all subcriteria presented in different levels. This opens the possibility to not just access the overall quality level of the Portuguese public hospitals, but also the quality of specific criteria considered in the model defined by the DM.

4.3.1 Basic concepts

Since this new model is an extension of the ELECTRE TRI-nC, all the assumptions assumed in this method prevail. Nevertheless, since the criteria structure differs, there are new concepts that will modify its procedure. In the first instance, it is important to recap what is considered to be given initially in the problem, and the notation and rules that have to be respected. A set of actions $A = \{a_1, a_2, \ldots, a_n\}$ will be evaluated in a given set of criteria $\check{G}$, to be assigned to a set of pre-ordered categories $C = \{C_1, C_2, \ldots, C_h, \ldots, C_q\}$, with $q \geq 2$. A set of reference actions is defined to characterize each of the categories, $B_h = \{b_h^r, r = 1, \ldots, m_h\}$ represents this subset. $B = \{B_0, B_1, \ldots, B_h, \ldots, B_q,\ B_{q+1}\}$ represents the set of subsets of reference actions. The particular sets B_0 and B_{q+1}, and the reference actions that define each of them, have the same meaning as the one presented in Section **4.1.1**. A weighting procedure will be used to access the weights of each elementary criterion, g_t. The subcriteria, G_r, have multiple subcriteria that are descending from it, so it is necessary to create an assumption that allows to access the weight of G_r, considering the weights of all g_t that descend from it. As in the original ELECTRE TRI-nC, it is required that the categories are ordered from one extreme to the other (for instance, from the highest quality to the lowest quality), which implies that C_{h+1} is preferred to C_h, for all $h = 1, \dots, q-1$. This method requires the construction of a hierarchical tree concerning the criteria. There are several possibilities in the development of this tree, for instance, the DM could follow a top-to-bottom approach, or the opposite. The top-to-bottom approach consists in choosing first, the first level criteria $\mathrm{G}_{\mathrm{i}_1}$, with $(i_1 = 1, \dots, m)$, then the subcriteria that directly descend from the previous ones, and continue this process until reaching the last level of the hierarchy. In this level, the DM will define as elementary criteria, the attributes which he/she has the capacity to determine its preferences for each action. The nomenclature used in Section **4.2.1** to designate all the aspects related with criteria, and the new way of organizing them, are still valid in this method.

4.3.2 Weights

The weighing procedure used will be the same applied in the original ELECTRE TRI-nC method, the SRF weighing procedure, with the required adjustments to deal with a hierarchy structure of criteria. The MCHP imposes the need to follow a top-to-bottom approach to use this method. Starting from the root criterion G_0, and continuing until reaching the last level of the hierarchy, the DM has to rank, in the same way as explained in Section **4.1.4**, for each criterion $G_r, r \in \mathrm{L}_{\check{\mathrm{G}}} \backslash EL$, all the direct subcriteria descending from it, that is, $G_{(r,1)}, \dots, G_{(r,n(r))}$. Let $L_{(r,1)}$ represent the set of the least important criteria that directly descend from G_r, and $L_{(r,v(r))}$ the most important ones, such that: $L_{(r,w)} \cap \mathrm{L}_{(\mathrm{r,s})} = \emptyset \; \forall \; w \neq s \wedge w, s = 1, \dots, v(r)$. Let also designate by $e_{(r,s)}$ the number of empty cards between sets $L_{(r,s)}$ and $\mathrm{L}_{(\mathrm{r,s+1})}$, with $s = 1, \dots, v(r) - 1$. The variable z_r represents the ratio between the weights of criteria from $L_{(r,v(r))}$ and $L_{(r,1)}$. The locally normalized weights, $w^*_{r,j}$, are then defined to each criterion $G_{r,j}$, with $j = 1, \dots, n(r)$, in the following way:

$$w^*_{r,j} = \frac{w'_{(r,j)}}{\sum_{s=1}^{n(r)} w'_{(r,s)}}, \tag{4.11}$$

where:

$$w'_{(r,j)} = 1 + \frac{(Z_r - 1)[\, l(r,j) - 1 + \sum_{s=1}^{l(r,j)-1} e_{(r,s)}]}{v(r) - 1 + \sum_{s=1}^{v(r)-1} e_{(r,s)}}, \tag{4.12}$$

and $l_{r,s}$ represents the rank of importance where criterion $G_{(r,s)}$ belongs ($(r,s) \in L_{l(r,s)}$). There is a new step, introduced by the MCHP, to access the final normalized weights of all criteria, and it is defined as follows:

- $w_r = w^*_r$ for all G_r in the first level of the hierarchy;
- $w_{(r,s)} = w_r w^*_{(r,s)}, s = 1, \dots, n(r),$ where w_r is the globally normalized weight of criterion G_r.

Notice, that this equations imposes that:

- $\sum_{s=1}^{n(r)} w^*_{(r,s)} = 1 \; \forall \; G_r, r \in \mathrm{L}_{\check{\mathrm{G}}} \; EL, \; \wedge \sum_{t \in EL} w_t = 1.$

4.3.3 Outranking concept

Since the major concept of the method is the outranking one, it is required to establish some indexes to make conclusions about the outranking of an action over another. However, contrarily to ELECTRE TRI-nC, these indexes will also consider the new structure of the criteria, specially the presence of subcriteria, creating, in this way, a new scheme that must be respected. From now on, a and b will be used to denote an action a from set A, and a reference action b_h^r from subset B_h. To make conclusions regarding the possibility of the outranking of an action a over b, there are four indexes that must be considered. First, it has to be computed two elementary concordance indexes to indicate the degree of concordance and discordance with the possibility of "a outranks b on criterion g_t". These indexes are the elementary concordance index $\mathrm{\phi_t}(a,b)$, and the elementary discordance index $\mathrm{d_t}(a,b)$, for each elementary criterion g_t:

$$\mathrm{\phi_t}(a,b) = \begin{cases} 1 & if \quad g_t(b) - g_t(a) \leq q_t, \quad (aS_tb) \\ \dfrac{p_t - [g_t(b) - g_t(a)]}{p_t - q_t} & if \quad q_t \leq g_t(b) - g_t(a) < p_t, \quad (bQ_ta) \\ 0 & if \quad g_t(b) - g_t(a) \geq p_t. \quad (bP_ta) \end{cases} \tag{4.13}$$

$$\mathrm{d_t}(a,b) = \begin{cases} 0 & if \quad g_t(b) - g_t(a) \leq p_t, \\ \dfrac{[g_t(b) - g_t(a)] - p_t}{v_t - p_t} & if \quad p_t < g_t(b) - g_t(a) < v_t, \\ 1 & if \quad g_t(b) - g_t(a) \geq v_t. \end{cases} \tag{4.14}$$

The new hierarchy process creates the need to establish new indexes to check the outranking possibilities on the subcriteria $\mathrm{G_r}$. The first partial index represents the degree of concordance with the possibility of a outranking b on criterion $\mathrm{G_r}$, and it is defined by the partial concordance index $\mathrm{C_r}(a,b)$ for each criterion $\mathrm{G_r}$, $r \in L_{\check{G}} \backslash EL$:

$$C_r(a,b) = \sum_{j \in E(G_r)} w_j \mathrm{\phi}_j(a,b) \tag{4.15}$$

The second partial index considers, simultaneously, the arguments in favor and against the preference of a over b on all the elementary criteria that directly descend from criterion $\mathrm{G_w}$, which represents the credibility of " a outranks b on criterion $\mathrm{G_w}$". It is denoted by the partial credibility index $\sigma_r(a,b)$ for each criterion $\mathrm{G_r}$, $r \in L_{\check{G}} \backslash EL$, and $\overline{F_r} = \{j \in E(G_r): d_j(a,b) > C_r(a,b)\}$.

$$\sigma_r(a,b) = C_r(a,b) \prod_{j \in \overline{F_r}} \frac{1 - d_j(a,b)}{1 - C_r(a,b)} \tag{4.16}$$

There are two important assumptions that must be considered:

- $W_r = \sum_{j \in E(G_r)} w_j$, for each $r \in \mathrm{L_{\check{G}}}$. This assumption considers that the weight of subcriteria $\mathrm{G_r}$ is equal to the sum of the weights of all elementary criteria that descend from it.

- $\lambda_r \in [\frac{w_r}{2}, w_r]$, for all $r \in L_{\check{G}} \setminus EL$.

In the original ELECTRE TRI-nC, the criteria are all considered to be at the same level, thus, it is only required to establish one credibility level for the comprehensive level, λ. Analogously, in this method it is required the creation of a credibility level λ_r to each criterion G_r, $r \in L_{\check{G}} \setminus EL$. There are three possible different outranking relations for each non elementary criterion G_r:

(OR1) $aS_r'b \Leftrightarrow C_r(a,b) \geq \lambda_r$;

(OR2) $aS_r''b \Leftrightarrow C_r(a,b) \geq \lambda_r \wedge g_t(b) - g_t(a) < v_t$, for all $t \in E(G_r)$;

(OR3) $aS_r''b \Leftrightarrow \sigma_r(a,b) \geq \lambda_r$.

Despite all being valid, there is an increase in the level of complexity, from the first until the third one, which in turn, translates into an increasingly argumentative strongly relation of the outranking of action a over b, on criterion G_r. The DM has the power of choosing the level of credibility that he/she wishes to have in the outranking relations. The second and third relations are the most credible ones, which will be reflected in more robust outranking relations. Nevertheless, there will be few of them, because, contrarily to the first outranking relation they consider simultaneously the argument in favor and against that relation. Through the use of any of the binary relations previously established, it is possible to define three binary relations for each non-elementary criterion G_r:

$\boldsymbol{a} \succsim_r \boldsymbol{b}$ (a is preferred to b on criterion G_r) iff aS_rb and $not(bS_ra)$;

$\boldsymbol{a} \sim_r \boldsymbol{b}$ (a is indifferent to b on criterion G_r) iff aS_rb and bS_ra;

$\boldsymbol{a}\, ?_r\, \boldsymbol{b}$ (a is comparable with b on criterion G_r) iff $not(bS_ra)$ and $not(aS_rb)$.

Two important coherence properties were created to impose a logical relation, in terms of outranking relationships, between a criterion G_r, $r \in L_{\check{G}} \setminus \{EL, LBO\}$, and the subcriteria descending from it. Those coherence properties and the conditions to ensure them are presented in **Appendix B**.

4.3.4 Assignment procedure

The incorporation of MCHP in the ELECTRE TRI-nC method does not cause many differences regarding the characterization of the categories and the assignment procedure. As in the original method, there is the need to have conditions to ensure that the subsets of reference actions, that define two consecutive categories, have a certain degree of separability. The separability conditions presented in **Appendix A**, to ensure that degree of separability, were adapted to deal with a hierarchy structure of criteria (for more details, see **Appendix C**).

The assignment procedure of this method is almost equal to the one presented in the original ELECTRE TRI-nC, the only difference is that the assignment of an action to one or more continuous categories, does not happen only at the comprehensive level, but also in all criteria G_r, with $r \in L_{\check{G}} \setminus EL$. There are two assignment rules used for choosing the category/categories where an action should be assigned to, the ascending rule and the descending rule, which are used conjointly. In order to use them, it is required first to access the credibility indices $\sigma_r(\{a\}, B_h)$ and $\sigma_r(B_h, \{a\})$, for all $s = 0, \dots, q +$

1. The max operator continues to be a good solution to overpass the high number of credibility indices formed with this type of process. It is defined by the following two equations:

$$\sigma_r(a, B_h) = \max_{r=1,\dots,m_h} \{ \sigma(a, b_h^r)\} \tag{4.17}$$

$$\sigma_r(B_h, a) = \max_{r=1,\dots,m_h} \{ \sigma(b_h^r, a)\} \tag{4.18}$$

The selecting function used is the same presented in the original ELECTRE TRI-nC:

$$\rho_r(a, b_h) = \min \{\sigma_r(a, b_h), \sigma_r(b_h, a)\} \tag{4. 19}$$

In the new structure of criteria presented, the hierarchy one, both the ascending and descending rules have to be rewritten to consider the assignment of actions to the categories in any criterion, at any level of the hierarchy, excluding the elementary criteria.

Considering a certain criterion G_r, with $r \in L_{\check{G}} \setminus EL$, the two rules are described as follows:

Ascending rule:

- After the definition of the cutting level that is associated with the criterion being analyzed, λ_r, increase the value of h, from 1, until the first value, k, such that $\sigma_r(B_k, a) \geq \lambda_r$:

  - If $k = 1$, assign a to category C_1;

  - $1 < k < q + 1$ and $\rho_r(a, B_k) > \rho_r(a, B_{k-1})$, assign a to category C_k, otherwise assign action a to category C_{k-1};

  - If $k = q + 1$, assign a to category C_q;

Descending rule:

- After the definition of the cutting level that is associated with the criterion being analyzed, λ_r, decrease h from q until the first value k, such that $\sigma_r(a, B_k) \geq \lambda_r$:

  - If k = q, assign a to category C_q;

  - If $0 < k < q + 1$ and $\rho_r(a, B_k) > \rho_r(a, B_{k+1})$, assign a to category C_k, otherwise assign action a to category C_{k+1};

  - If k = 0, assign a to category C_1;

It is possible to make a generalization of the previous rules to the ELECTRE methods that use a flat structure of criteria. It is only required to consider that there is only one criterion, the one representing the comprehensive level, $G_r = G_0$. The four structural requirements defined in section **4.1.3** are still applied to this new method, which implies that this assignment procedure has to respect them in any level of the hierarchy. After using both rules conjointly for each action a_i, with $i = 1, \dots, n$, all the actions will be assigned to one, or more, continuous categories, with $\Gamma(a_i)$ representing the categories where action a_i is assigned to. Notice that the DM has always the final word, however, if the method is perfectly constructed, which means that it respects all the rules required, it will reflect the DM's preferences in the sorting solution presented.

4.4 Summary

This chapter presented an explanation of the ELECTRE TRI-nC method and the MCHP, with focus on the notation, the structural requirements and the procedure used in each one of them. The implementation of the MCHP in the ELECTRE TRI-nC and the SRF weighing procedure was also described, with emphasis on the necessary modifications made in the original methods.

Chapter 5: Method development

The inherent complexity in developing a method with high robustness in terms of its mathematical foundations led to the creation of a programming code that respects all the requirements and the procedure presented in the previous chapter. An *Excel* file was created with a default format to receive the input data from a DM. This data is then manipulated through a programming code written in *python* to execute the procedure of the ELECTRE TRI-nC method with MCHP. The output of the model, and all the data required to achieve that output are computed in the code and presented in the same *Excel* file previously mentioned. A specific package of the *python* library was also used to allow the presentation of several outputs in a browser page that can be access by different devices at the same time. This chapter is organized into five subchapters. The first one addresses all the requirements that the model must respect. The second one describes the method's procedure. The third presents the model's architecture and the interactions between the *Excel* file and the computational program. The fourth subchapter consists of describing the computational program and the data generated during the several procedures present in that program. Lastly presented are the limitations and capabilities of the computational program.

5.1 Concerns and requirements

The major concern in the development of the ELECTRE TRI-nC method with MCHP was creating it with a general format. This enables the possibility of receiving inputs that represent any specific problem of a certain DM, and presenting outputs in conformity with the theoretical requirements of this outranking method. The ELECTRE TRI-nC method is based in a co-construction process between a DM and an analyst, who is the actor that interacts with the DM to collect the data required to create a specific model representing the problem at hand (Almeida-Dias et al., 2012). The focus of the analyst is to explain to the DM what is required from him/her and understand his/her preferences (Figueira et al., 2010). This assumption enhances the importance of having a general format model, allowing the analyst not to have to change any feature to adapt it to the problem that is being addressed. Logically, developing a computational program emerges as the only solution to create a model with this type of format. The framework that compiles this program with a platform to insert the input data and present the outputs of the model must also consider several characteristics of the underlying procedure of the ELECTRE TRI-nC method with MCHP. This also imposes restrictions on the tool selected to insert the data and visualize, not only the results, but also the parameters defined during the model's execution.

As explained in Chapter **4**, the initial input data that is inserted in the model is obtained through the interaction between the analyst and the DM. This data consists of the following elements:

- A set of potential actions A;

- A set of completely ordered categories C_h, with $h = 1, \dots, q$;

- A set of criteria organized in an hierarchical structure $\check{G}$, which reveals the type of criterion of each specific criterion existent in that set;

- The direction of preference of each criterion EL;

- The discriminating thresholds associated with each criterion EL, which are the indifference threshold q_j, the preference threshold p_j, and, if the DM wishes to, the veto threshold v_j (with $j = 1, \ldots, n$);

- A subset of reference actions for each category B_h, with $h = 1, \ldots, q$;

- The performances of the potential actions, $g_j(a)$, and reference actions, $g_j(b_h^r)$, with $a \in A$, $j \in [1, n]$, $h = 1, \ldots, q$, $r = 1, \ldots, m_h$ and $m_h \geq 1$.

Since the criteria are organized in a hierarchical structure it is possible to obtain three new variables that allow to distinguish their location in the hierarchy:

- The type of criterion;

- The index of the criterion;

- The hierarchy level of the criterion;

The developed tool must have the capability of receiving these inputs and store them in the file memory to be used in the computational program (*CP*). However, the interaction between the analyst and the DM is not finished after collecting this initial input data. The ELECTRE TRI-nC method with MCHP allows for the DM to redefine some model data during the construction of the model. However, even if he/she does not wish to redefine any of the input data, his/her intervention is always required during the SRF weighing procedure and when defining the cutting level associated with each criterion $\mathrm{G_r}$, $r \in L_{\check{G}} \setminus EL$. It is only possible for the DM to define the value of the cutting levels after the estimation of the intrinsic weights associated with each criterion $\mathrm{G_r}$, $r \in L_{\check{G}}$. This implies the need for having a tool that can store the initial data and present it in a visual format that aid the analyst when constructing the model. Furthermore, it is also mandatory that the *CP* accesses the information in this tool to use it when applying the calculations required in the method, and then insert new data resulting from those calculations in it. This data is then used to aid the DM in defining new parameters in the model, which means that there is once again the need to re-access the tool to define them. The manner in which the method is conceived, and the continuous interaction between the two already mentioned actors, enhance the need for developing a tool that is easily accessed during the method's execution

5.2 Method implementation

In order to understand how the method was developed, it is crucial to clarify the entire procedure from the insertion of the input data to the assignment of potential actions to the pre-defined categories at the comprehensive level, but also considering specific criteria of the problem analyzed. Figure **5.1** is illustrative of that procedure. Each step existent in the flowchart contains several operations that are performed, not only to obtain the intended results of the method, but also to verify all the theoretical requirements that validate the model.

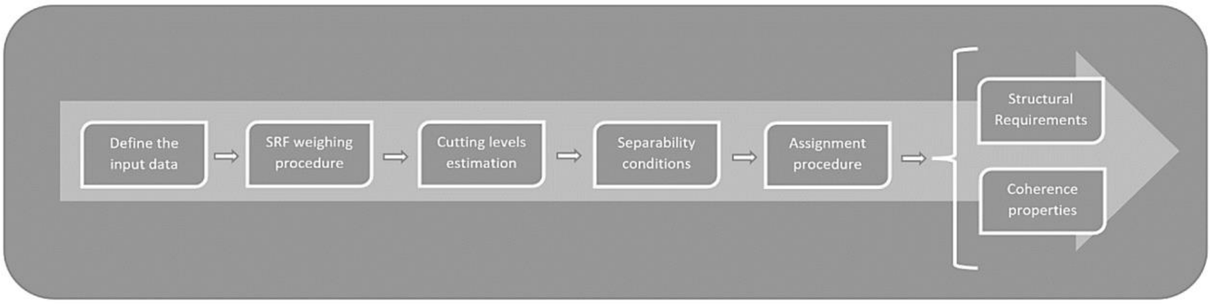

Figure 5.1 - Flowchart of the method procedure

In the first step ("*Define the input data*") the DM specifies all the initial input variables that were mentioned in Section **5.1.1**. This data then has to be handled by the *CP* to organize the criteria in the specific format required to apply the SRF weighing procedure. As mentioned in Section **4.3.2**, there is the need to follow a top-to-bottom approach to use this specific weighing procedure in a problem that has a hierarchy structure of criteria. This approach leads to a specific organization of the criteria that is the basis of the interactive process between the analyst and the DM. The result of this interaction is the insertion of new variables defined by the DM, that are used to estimate the values of the intrinsic weight of all criteria in the model. This procedure is made during the second step ("*SRF weighing procedure*"), and it requires a continuous flow of information between the *CP* and the tool used to visualize the data organized in the intended format. Defining the value of the weights is imperative for all the steps forward.

The third step consists of estimating the cutting level's value associated with each criterion $\mathrm{G_r}$, $r \in L_{\check{G}} \backslash\, EL$. There are specific rules for defining those values. As explained in Theorem **1.1**, it is possible to estimate the value of the cutting level of a specific criterion $\mathrm{G_r}$, with $r \in L_{\check{G}} \backslash\, \{EL \cap LBO\}$, if all the subcriteria descending from it, $G_{(r,j)}$, are already defined. This enables the possibility of the DM defining only the cutting levels' values of the criteria in the last but one level of the hierarchy (LBO). The value of the cutting level associated with any criterion G_r must be in the interval $\left[\frac{w_r}{2}, w_r\right]$ for all $r \in \mathrm{L}_{\check{G}} \backslash\, EL$. This condition is always respected when the DM defines the cutting levels' values for all criteria LBO between that interval. The value of all criteria ascending from that level, that is, λ_r for all $r \in L_{\check{G}} \backslash\, \{EL \cup LBO\}$, are computed by adding the cutting levels' values of the subcriteria that descend from them in the subsequent level, which implies the need to follow a bottom-to-top approach when defining them. It is also important to notice that this procedure of estimating the values of the cutting levels have a direct impact in the two coherence properties of the model, which is described in Section **5.3**.

If the DM does not wish to change any of the parameters already defined in the model, then, from the moment the third step is concluded, there is no further intervention from him/her in the model. All the variables already defined are then used to verify all conditions and properties that validate the model, and to execute the assignment procedure.

The fourth step consists of verifying the separability conditions between the different categories defined by the DM. As mentioned in **Appendix C**, the purpose of the separability conditions is to verify if the comparison of the subsets of reference actions that define adjacent categories allows to conclude that those categories are in fact distinct (Almeida-Dias et al, 2010). The procedure used to make this conclusion is based on the comparison of each reference action of a certain category C_h, with each reference action of the next higher category, that is C_{h+1}. The objective of this method is to assign each potential action to one or more consecutive categories, not only at the comprehensive level, but also considering only specific subcriteria in the hierarchical tree (Corrente et al, 2016). This implies the need to verify the separability condition for all the categories in all subcriteria of the hierarchical tree. However, by imposing those conditions in all subcriteria located in the LBO level, it is verified that they are respected in all criteria in the higher levels (including the comprehensive level). There are three different separability conditions: the weak separability, the strict separability and the hyper-strict separability (for more details, see **Appendix A**).

The index used to conclude about the separability conditions between adjacent categories is the partial credibility index between two characteristic reference actions that belong to different categories. To compute the value of this index it is necessary to first compute the value of three others: the elementary concordance index, the elementary discordance index and the partial concordance index. It is easy to conclude that this elaborate procedure leads to a high number of mathematical calculations to define all necessary indexes, which has impact in the program's computational effort.

It is essential to have a tool that allows, not only to observe if the example used verifies the separability conditions, but also, if not, which are the subsets of reference actions, and in which subcriterion, that not respect those conditions. This feature will help the analyst to rapidly visualize what are the reference actions that must be changed.

The fifth step of the method is the assignment procedure, which is completely described in Section **4.3.4**. It is also important to acknowledge, that in order not to waste time, the separability conditions should be verified before the assignment procedure is made. This procedure will lead to the sorting of all potential actions to one, or more, categories, considering each criterion $\mathrm{G_r}$, with $r \in L_{\check{\mathrm{G}}} \setminus EL$. Since the rest of the natural requirements of the method use information provided by the assignment procedure, it is mandatory that it be executed first.

The final step includes the rest of the natural requirements of the ELECTRE TRI-nC method, extended to the case of having a hierarchical structure of criteria:

Structural Requirements

There are four structural requirements that were presented in Section **4.1.3**. Regarding the conformity one, since the insertion of the characteristic reference actions in the model is made by the analyst, and considering that he/she understands their role in the method, there is no need to verify this condition because it will always be respected. The second structural requirement, homogeneity, imposes the need to compare all the possible pairs of potential actions (one being different from the other). This comparison is made by analyzing the values of the outranking credibility indexes of each action in the pair with respect to each reference action in the problem, and observing if those indexes are exactly the same for both actions. In that case, it is mandatory that both actions are assigned to the same category. Similarly, to the separability conditions, this comparison is not only made at the comprehensive level, but also considering only specific criteria.

The third structural requirement, monotonicity, evaluates the strictly dominance condition between two actions, to conclude if their assignment respects what is intended. The elementary concordance index between two actions allows to observe if one of them is strictly preferred to the other. In that case, that action should at least be assigned to the same category as the other.

The last requirement is used to analyze the repercussions in the assignment when applying a merging or a splitting operation. It is also possible to define a model in which the merging and splitting operations are not made in any step existent in Figure **5.1,** excluding the first one. This means that, every time the DM wishes to make one of these operations, it is mandatory to go back to the first step and execute the rest of the procedure with the new input data. Logically, if the model is built with this assumption, there is no need to ascertain whether the stability condition is respected, because all the natural requirements of the method will be analyzed considering the new input data.

Coherence properties

As mentioned in **Appendix B**, there are two coherence properties that must be respected for all criteria, with exception to the last two levels of the hierarchy tree of criteria. These properties are based on the comparison of the outranking relation between each potential action and each characteristic reference action in a certain criterion, with the outranking relations in the subcriteria directly descending from the one previously mentioned. The incorporation of the MCHP in the ELECTRE TRI-nC method led to the definition of three possible outranking relations for each non-elementary criterion. To compute those relations, it is necessary to define the four existent indexes of the method for each pair of actions under study.

As mentioned in the above natural requirements, the procedure that allows to make conclusions regarding the coherence properties, should be visually presented in the tool used. This allows the analyst to rapidly understand where the error is, when the two properties are not respected.

5.3 Architecture of the tool

The framework is composed of an *Excel* file and a *CP* written in *python* that executes all the method's procedure. The *Excel* software program was selected because of its ability to organize and store data and present it in a functional and visual format. On the other hand, *python* is a high-level programming language used worldwide in several applications. It has several properties, including the possibility of using multiple packages of its library that are particularly useful when using the data from an *Excel* file, or even to visualize the results in different formats, which is very advantageous in this method. Due to the high number of operations and analyses that must be made in this method, it is convenient to organize the data in different spreadsheets of the *Excel* file. These spreadsheets have a default format consisting of several headers that define the position of each input data set, thus facilitating the process of entering the data. Logically, if the format of the *Excel* file is modified, there is the need to also adapt the *python* program.

The *Excel* file is composed of twelve spreadsheets that are presented in Figure **5.2**.

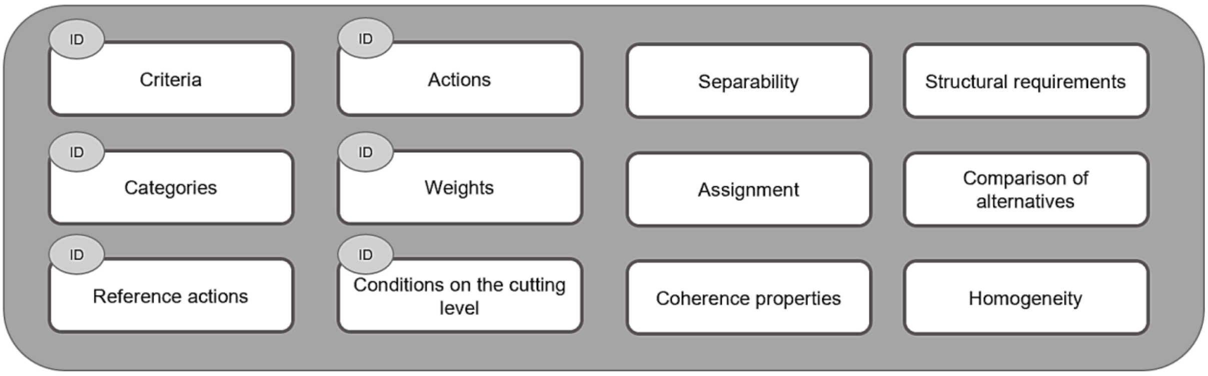

Figure 5.2 - *Excel*' spreadsheets defined in the *Excel* file

*ID – Insertion of Data

In order to fully understand the framework, an adaption of Figure **5.1** is displayed in Figure **5.3**, presenting the interactions between the *Excel* file and the *CP*.

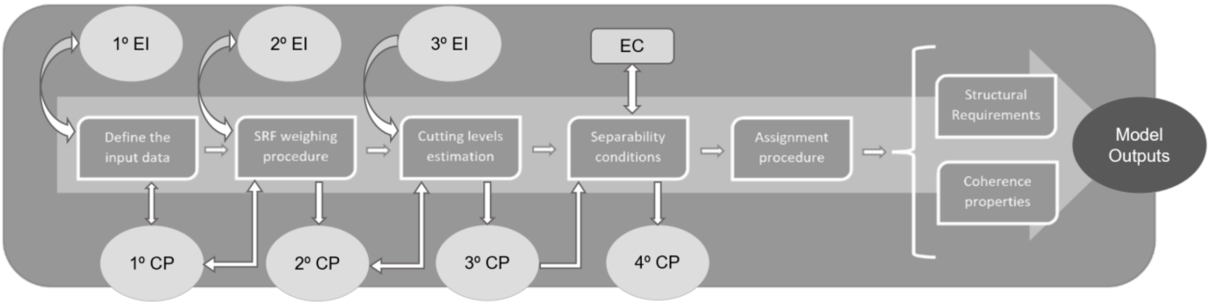

Figure 5.3 - Framework' architecture

*EI – *Excel* Intervention

*CP – Computational Program

*EC – *Excel* Confirmation

Define the input data

Initially the *Excel* file only has the spreadsheets mentioned and the headers in those spreadsheets. The rest of the file is completely blanked and ready to receive the data provided by the DM. The first *EI* is divided into two parts. In the first part the data is inserted into four different spreadsheets: *Criteria*, *Categories*, *Reference actions* and *Actions*. It is then executed the *1º CP* intervention, which transposes the elementary criteria to the *Actions* and *Reference actions* sheets. The second part of the *EI* consists of entering the performances of all actions in each elementary criteria in the *Actions* and *Reference actions* sheets. The criteria data are then used in the second part of the *1º CP* intervention to organize it in the necessary way to apply the SRF weighing procedure.

SRF weighing procedure

The organized data set obtained in the end of the *1º CP* is inserted into the *Weights* sheet. Then occurs the *2º EI*, in which the analyst observes the organized data in the *Weights* sheet and interacts with the DM to apply the SRF weighing procedure. Applying this procedure in a case study that has a hierarchical structure of criteria requires the use of a top-to-bottom approach. The *CP* organizes the data of the first level criteria to define the intrinsic weights of each criterion on that level. The entering of the data required to compute the weights is made through the interaction between the two actors. The variables' values generated during this interaction (empty cards, levels of importance, ratio) are inserted in the correspondent area in the *Excel* file. The values of the weights associated with each criterion on the first level are then computed by the *CP*, that also organizes the data for executing the same procedure in the second level. It is not possible to apply the SRF weighing procedure to every level in the hierarchy at the same time, because the values of the criteria's weights on the higher level are used to compute the values in the next lower level. This justifies why it is necessary to perform the *1º CP* intervention several times in conjunction with the *2º EI* until defining the weight of all criteria.

Cutting levels estimation

After this process, and once the intervals of possible cutting levels' values for each criterion LBO have already been defined, the *2º CP* intervention is executed. This intervention defines those intervals in the *Conditions on the cutting level* spreadsheet. The next step is the *3º EI*, when the DM defines the cutting level's value for each criterion LBO, within the interval previously established. The cutting levels of the remaining criteria are computed based on the values defined by the DM for the LBO level. This process is also performed in the *2º CP* intervention.

Separability Conditions

The separability conditions of the categories previously defined by the DM are verified in the *3º CP* intervention. It is advisable for the analyst to open the *Excel* file after performing this intervention to conclude if the separability conditions are respected, particularly the weak separability one.

Assignment procedure & Structural requirements & Coherence properties

The final step is the *4º CP* intervention. It consists of executing the method's assignment procedure and verify if all structural requirements and coherence properties are respected. If so, the process is concluded.

5.4 Computational program

The *python* program is composed of 24 specific functions. As it is possible to observe in Figure **5.3**, there are four *CP* interventions until the model is completed. In each intervention, one or more functions are executed. Table **5.1** highlights the functions that are executed in each intervention. One of the focuses when developing the program was its modularity. A modular program allows to have different modules, in which only a specific functionality is executed (Brogi et al., 1994). The ELECTRE TRI-nC method with MCHP benefits from this approach since it is required to insert new data in different steps of the model and to verify certain conditions before proceeding for the foregoing steps.

Table 5.1 - Computation program' functions per block

<table>
<tr><th>1º CP</th><th>2º CP</th><th>3º CP</th><th>4º CP</th></tr>
<tr><td>TransposeToReferenceAndActionsSheet</td><td rowspan="5">TransferDataToCuttingLevel</td><td rowspan="4">TransferCuttingToCriteria</td><td>TransferDataToAssignment</td></tr>
<tr><td>TransferDataToWeight</td><td>ComputePartialIndexesAssignment</td></tr>
<tr><td>RootCriterionTable</td><td>ComputeTheFinalTableOfTheAssignment</td></tr>
<tr><td>TransferDataToCriteriaOf2level</td><td>ComputeDescendingRule</td></tr>
<tr><td>ProcessDataToCriteriaOf2level</td><td rowspan="3">TransferDataToSeparabilityTable</td><td>ComputeAscendingRule</td></tr>
<tr><td>TransferDataToCriteriaOf3level</td><td rowspan="5">ComputeCuttingLevel</td><td>CoherenceProperties</td></tr>
<tr><td>ProcessDataToCriteriaOf3level</td><td>Homogeneity</td></tr>
<tr><td rowspan="3">TransferWeightsToCriteria</td><td rowspan="3">ComputePartialIndexes</td><td>Monotonicity</td></tr>
<tr><td>IndividualStictability</td></tr>
<tr><td>MonotonicityFinal</td></tr>
</table>

Notice that in order to execute a function that requires accessing the information in an *Excel* file, it is imperative that this file is not open. Since all the functions mentioned in Table **5.1** must access the data in the *Excel* file and then store some new data in it, it is mandatory for each function to open that file in the computation program and save the modifications in the end. Each individual function in each of the four blocks is explained below.

Block *1º CP*:

TransposeToReferenceAndActionsSheet

The objective of this function is to analyze the data entered in the *Criteria*' sheet to check which are the elementary criteria and transpose them for a specific row in the *Actions* and *Reference actions* sheets. Firstly, the last row filled with data in the *Criteria* sheet is defined, and then, a *cycle* that examines each row with data to observe the type of criterion, is executed. There are four possibilities: *root criterion*, *subcriterion*, LBO and EL. If the criterion examined is characterized as EL, it is transposed into a specific cell in the *Actions* and *Reference actions* sheets.

TransferDataToWeight

This basic function transfers the criteria data into a specific table located in the *Weights* sheet.

RootCriterionTable

This function organizes the criteria that belong to the first level of the hierarchy, in order to apply the SRF weighing procedure. The objective is to develop a table, in which each row is composed of a first level criterion and its index. To define this table, the *CP* verifies each criterion in the *Criteria* sheet and examines if it is on the first level of the hierarchy. If so, it is transferred to the desired table.

ComputeDataRootCriterionTable

After inserting the level of importance of each criterion, the number of empty cards between those levels and the ratio Z in the *Excel* file*,* it is possible to determine the criteria' weights in that table.

The expression to compute the non-normalized weight of a criterion considers several elements:

- The sum of empty cards used in the lower levels of the one under analysis, $\sum_{s=1}^{l(r)-1} e_{(r,s)}$;

- The sum of empty cards considering all levels of importance, $\sum_{r=1}^{v(r)-1} e_r$;

- The level of importance of the criterion, $l(r)$;

- The ratio, Z_r.

In order to make the explanation of the *CP* process more explicit, it is accompanied with the example used in Table **5.2**, that is composed of the following structure of criteria:

- Five criteria in the first level (g_1, g_2, g_3, g_4, g_5);

- Seventeen elementary criteria:

  - Directly descending from g_1: $g_{(1,1)}$, $g_{(1,2)}$, $g_{(1,3)}$, $g_{(1,4)}$;
  - Directly descending from g_2: $g_{(2,1)}$, $g_{(2,2)}$, $g_{(2,3)}$;
  - Directly descending from g_3: $g_{(3,1)}$, $g_{(3,2)}$, $g_{(3,3)}$, $g_{(3,4)}$, $g_{(3,5)}$;
  - Directly descending from g_4: $g_{(4,1)}$, $g_{(4,2)}$;
  - Directly descending from g_5: $g_{(5,1)}$, $g_{(5,2)}$, $g_{(5,3)}$.

Table 5.2 - Example of the root criterion table after executing the *ComputeDataRootCriterionTable*' function

Root Criterion						
Criterion	Index	Level of importance	Empty cards	Ratio	Non-Normalized Weight	Normalized Weight
g_1	1	3	1	10	7.545	0.236
g_2	2	4	0	10	9.182	0.287
g_3	3	5	0	10	10	0.313
g_4	4	1	3	10	1	0.031
g_5	5	2	3	10	4.273	0.134

– Data from *RootCriterionTable'* function

– Data from the interaction between the analyst and the DM

– Data from the *ComputeDataRootCriterionTable'* function

The *CP* process is composed of several operations:

1) Defining the last row in the *root criterion*'s table that has data;
2) Executing a *cycle* that starts in the first criterion in the table and finishes in the last row already defined. This *cycle* analyzes the level of importance of the criterion, l_r, and considers the number of empty cards associated with it, e_{l_r+1}. It also defines the maximum l_r existent.
   - $l_1 = 3$, $e_4 = 1$, $l_2 = 4$, $e_5 = 0$, $l_3 = 5$, $e_6 = 0$, $l_4 = 1$, $e_2 = 3$, $l_5 = 2$, $e_3 = 3$, $\max l_r = 5$.
3) Applying the following equation for e_r (with $r = 2, \ldots, \max l_r + 1$): $e_r = e_r + e_{r-1}$.
   - $e_2 = 3$, $e_3 = 6$, $e_4 = 7$, $e_5 = 7$, $e_6 = 7$.
4) Defining the total of empty cards by observing which e_n (with $n = 2, \ldots, 11$) has the higher value. This condition is required because the *CP* was defined to handle up to ten levels of importance, so the variables not used in this example, l_r and e_{r+1} (with $r = 0, 6, 7, \ldots, 10$) have the value 0.
   - $\sum_{s=1}^{v(r)-1} e_r = 7$.

5) Executing a new *cycle* that determines the value of $\sum_{s=1}^{l(r)-1} e_r$ for each l_r and computes the weights' values. That *cycle* re-analyzes each row of Table **5.2**, to fix the variable Z, and l_r. If l_r is equal to p, the value of $\sum_{s=1}^{l(r)-1} e_r$ is equal to e_p.
   - $\sum_{s=1}^{l(1)-1} e_1 = 0$, $\sum_{s=1}^{l(2)-1} e_2 = 3$, $\sum_{s=1}^{l(3)-1} e_3 = 6$, $\sum_{s=1}^{l(4)-1} e_4 = 7$, $\sum_{s=1}^{l(5)-1} e_5 = 7$.

   The second part of this *cycle* is to compute the non-normalized weights' values, by applying the equation described in Section **4.3.2**, and to define a variable with the sum of those weights.
6) Executing a new *cycle* that inserts in each row of Table **5.2**, the normalized weight's value. This value is computed by dividing the value of the non-normalized weight in that row by the value of the variable that represents the sum of the non-normalized weights.

TransferDataToCriteriaOf2level

The weight associated with each criterion on the second level is influenced by the weight of the criterion on the first level that ascends from it. This enhances the importance of organizing the second level criteria. The objective of this function is to build as many tables as the number of criteria in the first level. From now on, every time that the *CP* generates multiple small tables to be inserted inside one major one, they will be mentioned as *Mini Tables*. This function is based on two *cycles*, one inside the other, that are described below:

1) The *1º cycle* examines each row in the *Root Criterion*'s table previously established (Table **5.2** is an example) in order to fix the index of each criterion therein.
2) The *2º cycle* fixes the index of each criterion defined in the table obtained in the *TransferDataToWeight'* function, and compare that index with the one fixed in the *1º cycle*. That comparison allows to conclude if the criterion fixed in the *2º cycle* belongs to the second level of the hierarchical tree, and if it descends from the criterion fixed in the *1º cycle.* When this comparison is verified, the *CP* inserts the information of both criteria in the *Criteria of 2nd level*'s table in the *Excel* file.

Table **5.3** presents the *Mini Tables* generated by executing this function.

Table 5.3 - Example of the table defined in the *TransferDataToCriteriaOf2level*' function

Criteria of 2nd level

Weight of criterion of 1º level	Criterion of 1º level	Criterion of 2º level	Index	Level of importance	empty cards	Zr	Non-Normalized Weight	Locally Normalized Weight	Globally Normalized Weight
0.236	g_1	$g_{1,1}$	1,1						
0.236	g_1	$g_{1,2}$	1,2						
0.236	g_1	$g_{1,3}$	1,3						
0.236	g_1	$g_{1,4}$	1,4						
0.236	g_1	$g_{1,5}$	1,5						

Weight of criterion of 1º level	Criterion of 1º level	Criterion of 2º level	Index	Level of importance	Empty cards	Zr	Non-Normalized Weight	Locally Normalized Weight	Globally Normalized Weight
0.287	g_2	$g_{2,1}$	2,1						
0.287	g_2	$g_{2,2}$	2,2						
0.287	g_2	$g_{2,3}$	2,3						
0.287	g_2	$g_{2,4}$	2,4						
0.287	g_2	$g_{2,5}$	2,5						

Weight of criterion of 1º level	Criterion of 1º level	Criterion of 2º level	Index	Level of importance	Empty cards	Zr	Non-Normalized Weight	Locally Normalized Weight	Globally Normalized Weight
0.313	g_3	$g_{3,1}$	3,1						
0.313	g_3	$g_{3,2}$	3,2						
0.313	g_3	$g_{3,3}$	3,3						
0.313	g_3	$g_{3,4}$	3,4						
0.313	g_3	$g_{3,5}$	3,5						
0.313	g_3	$g_{3,6}$	3,6						

Weight of criterion of 1º level	Criterion of 1º level	Criterion of 2º level	Index	Level of importance	Empty cards	Zr	Non-Normalized Weight	Locally Normalized Weight	Globally Normalized Weight
0.031	g_4	$g_{4,1}$	4,1						
0.031	g_4	$g_{4,2}$	4,2						
0.031	g_4	$g_{4,3}$	4,3						
0.031	g_4	$g_{4,4}$	4,4						
0.031	g_4	$g_{4,5}$	4,5						

Weight of criterion of 1º level	Criterion of 1º level	Criterion of 2º level	Index	Level of importance	Empty cards	Zr	Non-Normalized Weight	Locally Normalized Weight	Globally Normalized Weight
0.134	g_5	$g_{5,1}$	5,1						
0.134	g_5	$g_{5,2}$	5,2						
0.134	g_5	$g_{5,3}$	5,3						

It should be noted that when the *1º cycle* advances for the next criterion in the table, the program adds a blank row and a new header in the *Criteria of 2nd level'*s table, in order to have all data organized in a way that aids the analyst in its job.

ProcessDataToCriteriaOf2level

The SRF weighing procedure must be executed as many times as the number of *Mini Tables* defined in the *TransferDataToCriteriaOf2level'* function. The process for computing those values is described in a sequence of operations:

1) The *CP* executes a *cycle* that examines each row in the *Criteria of 2nd level's* table (Table **5.3** is an example). If it finds a blank row it recognizes that the previous rows that were examined form a *Mini Table.*
2) The *CP* applies the exact same procedure described in the *ComputeDataRootCriterionTable'* function in each *Mini Table*, until computing the non-normalized weights' values.
3) The *CP* re-examines each row in the *Mini Table*, and it computes the locally normalized weight's value by multiplying the value of the non-normalized weight by the weight associated with the criteria on the first level. It also defines a variable whose value is equal to the sum of all locally normalized weight's values in that *Mini Table*.
4) The *CP* re-examines each row in the *Mini Table* to compute the globally normalized weight's value for each row. This value is computed by dividing the value of the locally normalized weight by the value of the variable created in the last operation.

TransferDataToCriteriaOf3level* and *ProcessDataToCriteriaOf3level

When the case study considered has more than two levels in its hierarchical tree of criteria it is indispensable to execute both these functions. They are almost equal to the two ones referring to the second level, with the only difference being the table used to define the first *cycle*. This table is the one built in the *ProcessDataToCriteriaOf2level'* function.

TransferWeightsToCriteria

The last function of the first *CP'* block transfers all the final weights associated with each criterion in the model to the correspondent location in the *Criteria* sheet.

Block *2º CP*:

TransferDataToCuttingLevel

The second block of the *CP* deals with the computation of the cutting level's values for all non-elementary criteria. The procedure of this particular function is as follows:

1) The *CP* executes a *cycle* that transfers all the criteria' data (including the weights) from the *Criteria* sheet to a specific location in the *Conditions on the cutting level* sheet.
2) The *CP* examines each row in the *Conditions on the cutting level* sheet to identify each criterion LBO there inserted. Once the *CP* identifies one, it computes a minimum and maximum value for the cutting level's value, which are, respectively, half of the weight and the total weight associated with that criterion.

ComputeCuttingLevel

After the DM sets the cutting level's values in the criteria identified in the previous function, the *ComputeCuttingLevel'* function is executed. Its objective is to define the cutting level's values in the other criteria ascending from the LBO level. The *CP* process is the following one:

1) Executing the *1º cycle*, whose function is to examine each row in the *Conditions on the cutting level* sheet to fix each criterion G_r, $r \in L_{\check{G}} \setminus \{EL \cup LBO\}$. When one is fixed (except G_0), along with the information regarding it, the *2º cycle* is executed. If the criterion fixed is G_0 the *CP* advances for the *3º cycle*.
2) Executing the *2º cycle*, which analyzes each criterion in the *Conditions on the cutting level* sheet to conclude if it directly descends from the criteria fixed in the *1º cycle.* That conclusion is made by comparing the number of characters of their index and the variable h (level in the hierarchy). If it is concluded that it directly descend from the other, and that it has a cutting level's value associated, this value is stored.
3) In the end of the *2º cycle*, all stored values are summed. The result is the cutting level's value of the criterion fixed in the *1º cycle*, which is inserted in the same row where that criterion is.
4) Executing the *3º cycle*, that examines each h in the *Conditions on the cutting level* sheet to conclude if the criterion associated belongs to the first level of the hierarchy, and if it has a cutting level's value defined. If so, that value is stored.
5) In the end of the *3º cycle*, all the values stored are summed. The result is the cutting level's value of the root criterion, which is inserted in the same row where that criterion is.

This function has to be executed as many times as necessary, until all the cutting levels' values of each non-elementary criterion are defined.

Block *3º CP*:

TransferCuttingToCriteria

This function transfers all the cutting levels' values present in the *Conditions on the cutting level'* table to the *Criteria'* table. This last table has now all the information essential to execute the assignment procedure and the natural requirements of the method.

TransferDataToSeparabilityTable

To verify the separability conditions between the categories C_h and C_{h+1} it is necessary to compare each reference action belonging to the subset of reference actions B_h with B_{h+1}, in each criterion LBO in the model. The purpose of this function is to combine the data present in different *Excel* sheets, and organize it in a specific format that allows to compute the values required to verify the separability conditions. To make this format, five *cycles* were created, one within the other, and several conditions to analyze the data provided in each cycle:

1) The *1º cycle examines* the data inserted in each row of the *Reference action'*s table and fixes one reference action (its index and the category that it defines).
2) The *2º cycle* does exactly the same as the first one. If the categories fixed in the two *cycles* are adjacent, the *CP* advances for the next *cycle*.

3) The *3º cycle* inspects each row of data inserted in the *Criteria* table, and analyzes the type of criterion. If the criterion is in the LBO level, the *CP* stores the data associated with it and moves on to the next *cycle*. The criterion fixed in this *cycle* will be mentioned as *1º criterion*.
4) The 4º *cycle* also fixes a criterion (and its information) from the *Criteria* sheet. The criterion fixed will be mentioned as *2º criterion*. An if condition is then imposed to verify if the *2º criterion* is an elementary one, and if it descends from the *1º criterion*. If so, the information collected in the four *cycles* is inserted in the *Separability conditions'* table. However, in order to compute the indexes required to compare two reference actions with respect to a criterion LBO (*1º criterion*), it is necessary to know their performances in the elementary criterion (*2º criterion).* This requires the use of a *5º cycle*.
5) The *5º cycle* verifies each row with data in the *Reference actions'* table. If the cell that indicates the criterion's name is equal to the name of *2º criterion*, it inserts the performance correspondent to the reference actions previously fixed in the intended location in the *Separability conditions'* table. It is then possible to compute the values of the elementary indexes, since all the information required to compute them is already available in the *Separability conditions'* table. The equations to compute them are defined in Section **4.3.3**.

Every time that one of the first three *cycles* advances to the next row with data, the *CP* inserts a blank row and a new header in the *Separability conditions'* table. In this way, several *Mini Tables* are defined, facilitating the analyst's job when the separability conditions are not verified for all categories.

ComputePartialIndexes

With all the data organized in the *Separability conditions'* table it is then possible to determine the values of the partial indexes that allows to verify if the separability conditions are respected. The *CP* process follows an order that is described below:

1) Executes a *cycle* that examines each row with data in the *Separability conditions'* table to identify each *Mini Table* there inserted.
2) Each row in the *Mini Table* is then re-examined to compute a variable that multiplies the value of the weight of the elementary criterion there inserted by the elementary concordance index. The sum of those values is the partial concordance index.
3) Each row in the *Mini Table* is once again analyzed, in order to insert the partial concordance index' value in it, and to verify if the elementary discordance index' value is greater than the partial concordance index' value. If so, the value of the following equation is stored: $\frac{1-\mathrm{d_j}(\mathrm{a},b)}{1-\mathrm{C_r}(\mathrm{a},b)}$. After analyzing each row, the *CP* multiplies all the values stored (if there are more than one), and it multiplies that result by the partial concordance index' value. The result is the value of the partial credibility index.
4) A new *cycle* is executed to re-examine the *Mini Table* and insert the value of the partial credibility index. That value is also compared with the weight of the criterion LBO there entered. This comparison allows to partially conclude if the categories of the reference actions referenced in that *Mini Table*, respect the separability conditions, considering the criterion LBO

under scope. The reason why this conclusion is only partial is because this condition has to be verified considering all the possible pairwise combinations of the reference actions belonging to the subsets of reference actions that define those categories.

5) After making the partial conclusions in all *Mini Tables*, the *CP* executes the two final *cycles*. The first examines each row in the *Criteria'* table to fix each criterion LBO. When it is fixed it executes the next *cycle*, which in turn verifies each row of the *Separability conditions'* table. This verification allows to conclude if each separability condition is respected when the criterion LBO is referred.
6) The results of the conclusions withdrawn are presented into another table in the *Separability conditions sheet.*

Appendix D presents three figures related with the verification of the separability conditions. The Figure **D.1** is a small portion of the *python* code of this function. Figure **D.2** presents the structure of the main table defined in the *TransferDataToSeparabilityTable' function,* and complemented by the procedure of the *ComputePartialIndexes' function.* Figure **D.3** is an example of the table defined to make the final conclusions. The colors inserted in each figure intend to highlight the linkages between the *python* code and the data entered in the tables.

Block *4º CP*:

TransferDataToAssignment

As the name indicates, the purpose of this function is to transfer the data required to execute the assignment procedure to the *Assignment* sheet. The structure of the data entered there is very similar to the one presented in the *TransferDataToSeparabilityTable'* function. The *CP* procedure to define the *Mini Tables* is also very similar. The major differences are the following ones:

- The comparison to compute all the indexes requested is between each potential action and each reference action in the model;
- The comparison is not only made considering the LBO criteria, but also, all non-elementary criteria.

ComputePartialIndexesAssignment

This function computes all the indexes used in the *ComputePartialIndexes'* function with the same procedure, however, it computes them for two different situations. If a certain *Mini Table* is comparing a potential action a with a reference action b, those indexes must be computed for the comparison (a, b) and (b, a). The different steps of this function are the following ones:

- Analyze the partial indexes' values of (a, b) and (b, a) to make conclusions regarding the three possible outranking relations (*OR*) existent in this method. The objective of those three relations is to decide if there is an *OR* between the actions compared in a given non-elementary criterion. However, they consider different arguments to make this conclusion, which makes them different.
- Each of these *OR* is then used to make conclusions regarding the binary relations between (a, b). Since it is possible to use any of the three *OR* to verify the binary relations between those actions, nine different scenarios were considered.

ComputeTheFinalTableOfTheAssignment

Although the main table is already fully filled with the necessary data to perform the assignment procedure, it is crucial to organize this data considering the max operator mentioned in Section **4.3.4**. The objective of this function is to analyze the data in the *Assignment* sheet' main table, in order to define the maximum partial credibility index in any given subcriterion G_r, with $r \in \mathrm{L}_{\check{G}} \setminus \{EL\}$, for a given action a and any category C_h. This requires the comparison of all partial credibility *indexes* defined to compare that action a, with all reference actions that define the category C_h. Figure **5.4** and Table **5.4** display, respectively, a portion of the python code of this function and a table in which each row represent a *Mini Table* from the *Assignment's* sheet main table.

Python Code

Compute the Final Table of The Assignment

```python
for k in range(start_row_criteria, last_row_criteria + 1):
    criteria_name = ws_criteria.cell(k, 2).value
    criteria_λr = ws_criteria.cell(k, 14).value
    criteria_type = ws_criteria.cell(k, 9).value
    if criteria_type != 'EL':
        for i in range(start_row_action, last_row_actions + 1):
            action_name = ws_actions.cell(i, 3).value
            for j in range(start_row_category, last_row_category + 1):
                highest_partial_credibility_index_a_b = 0.000000000
                highest_partial_credibility_index_b_a = 0.000000000
                category_reference_index = ws_categories.cell(j, 2).value
                category_reference_number = category_reference_index[1:]
                for l in range(start_assignment, last_row_main_assignment_fill + 1):
                    main_criteria_name = ws_assignment.cell(l, 1).value
                    assignment_action = ws_assignment.cell(l, 5).value
                    assignment_reference_action = ws_assignment.cell(l, 6).value
                    if assignment_reference_action != None and assignment_reference_action != 'Reference profile':
                        reference_action_last_character = assignment_reference_action[2:]
                        if reference_action_last_character == category_reference_number and assignment_action == action_name and main_criteria_name == criteria_name:
                            partial_credibility_index_a_b = ws_assignment.cell(l, 18).value
                            partial_credibility_index_b_a = ws_assignment.cell(l, 22).value
                            if partial_credibility_index_a_b > highest_partial_credibility_index_a_b:
                                highest_partial_credibility_index_a_b = partial_credibility_index_a_b
                            if partial_credibility_index_b_a > highest_partial_credibility_index_b_a:
                                highest_partial_credibility_index_b_a = partial_credibility_index_b_a
                            pr_a_b = highest_partial_credibility_index_a_b
                            pr_b_a = highest_partial_credibility_index_b_a
                            if pr_a_b < pr_b_a:
                                pr_total = pr_a_b
                            else:
                                pr_total = pr_b_a
                ws_assignment.cell(last_row_final_assignment + 1, 39).value = criteria_name
```

Figure 5.4 - Portion of the *python* code of the *ComputeTheFinalTableOfTheAssignment*' function

Table 5.4 - Example of the *Mini Tables* defined in the *ComputeTheFinalTableOfTheAssignment*' function

Assignment Procedure Main Table					
Criterion	Index	Action	Reference action	$\sigma_r(a, b_h^r)$	$\sigma_r(b_h^r, a)$
g_0	0	a_3	b_2^1	0.825	0
g_0	0	a_3	b_2^2	0.852	0
g_0	0	a_3	b_2^3	0	0

As it is possible to observe in Figure **5.4**, the *CP* is based on four *cycles*, that are described below:

1) The *1º cycle* fixes a non-elementary criterion G_r, with $r \in \mathrm{L}_{\check{G}} \setminus \{EL\}$, and its cutting level, from the table defined in the *Criteria* sheet. If this criteria is a non-elementary one, the *CP* advances to the next *cycle*.
   - In the example presented in Table **5.4**, the G_r fixed was g_0.
2) The *2º cycle* fixes an action from the *Actions sheet'* table.
   - In the example presented in Table **5.4**, the action fixed was a_3.
3) The *3º cycle* fixes a category, namely its index (C_h) and the final character of that index (h), from the table inserted in the *Categories* sheet. It also defines two new variables that help

defining the maximum partial credibility indexes resultant from the comparison between the previously fixed action and all reference actions that define C_h.

- In the example presented in Table **5.4**, the fixed category was C_2.

4) The *4º cycle* analyzes each row of the *Assignment's* main table, and it fixes the criterion, the action, and the reference action, of each *Mini Table* there inserted. The criterion and the action fixed are directly compared with the same variables fixed in the first two *cycles*. The last character of the reference action is also removed to compare it with the one removed in the *3º cycle*. If all variables are equal, it means that the *Mini Table* that has been fixed represents one of those that must be considered, for choosing the maximum partial credibility indexes with respect to the variables fixed in the first three *cycles*. The credibility indexes defined in that *Mini Table* are then compared with the two variables defined in the *3º cycle*. This process is repeated for all the *Mini* Tables that fulfill the necessary conditions. This process allows to compute the maximum partial credibility indexes (a, b) and (b, a) and the value of $\rho_r(a, b)$.

   - In the example presented in Table **5.4**, the reference actions that characterize C_2 are b_2^1, b_2^2 and b_2^3. The maximum partial credibility indexes, by considering all the *Mini Tables*, are $\sigma_0(a_3, B_2) = 0{,}852$ and $\sigma_0(B_2, a_3) = 0$.

All this information is then entered into a new table in the *Assignment* sheet. This table is composed of several *Mini Tables*, each of which will be used to make the assignment procedure of the action there inserted, regarding the criterion also referred there.

Table **5.5** presents an example of one of those *Mini Tables*.

Table 5.5 - Example of a *Mini Table* defined in the *Organized Data for the Final Assignment*' table

Organized Data for the Final Assignment						
Criteria	**Λ_r**	**Action**	**Category**	$\boldsymbol{\sigma_r(a, B_h)}$	$\boldsymbol{\sigma_r(B_h, a)}$	$\boldsymbol{\rho_r(a, B_h)}$
g_0	0.6	a_3	C_6	0	1	1
g_0	0.6	a_3	C_5	0	0.959	0.959
g_0	0.6	a_3	C_4	0	0.902	0.902
g_0	0.6	a_3	C_3	0	0.674	0.674
g_0	0.6	a_3	C_2	0.852	0	0
g_0	0.6	a_3	C_1	0.906	0	0
g_0	0.6	a_3	C_0	1	0	0

ComputeDescendingRule* & *ComputeAscendingRule

The next step of the *CP* is to analyze the information stored in the table defined when executing the *ComputeTheFinalDataOfTheAssignment'* function. The logic beyond the *python* code is presented below:

1) A *cycle* to identify each *Mini Table* available in the *Organized Data for the Final Assignment'* table. The criteria and action defined in that *Mini Table* are inserted into a new table.
2) Each *Mini Table* is examined to execute the procedure imposed by the descending and ascending rules, in order to select two specific categories where a certain action should be assigned to. This procedure is described in Section **4.3.4**. These categories can be equal or different, in this last case it is considered that one of them represents the minimum category

where the action should be allocated to, and the other the maximum. These results are inserted into the new table that already has defined the criteria and action to which the *Mini Table* refers to.

This entire procedure is made until executing the descending and ascending rule for each *Mini Table* defined in the *Organized Data for the Final Assignment'* table.

Coherence properties

Appendix E presents three figures that help to fathom the procedure of this function. The *OR* established in the *ComputePartialIndexesAssignment'* function are fundamental to check the coherence properties imposed by this method, that must be verified for all criteria G_r, with $r \in L_{\check{G}} \setminus \{EL \cup LBO\}$. The *CP* procedure is as follows:

1) The *1º cycle* fixes each action from the table defined in the *Actions* sheet. After fixing it the *CP* advances to the next *cycle*.
2) The *2º cycle* fixes each reference action from the table defined in the *Reference Actions* sheet. After fixing it the *CP* advances to the next *cycle*.
3) The *3º cycle* fixes a criterion, and the correspondent information regarding it, from the table defined in the *Criteria* sheet. It also defines 14 new variables that can be visualized in the portion of *python* code presented in Figure **E.1** of **Appendix E**. If the criterion's type is *subcriteria* (which means that it is criterion G_r, with $r \in L_{\check{G}} \setminus \{EL \cup LBO\}$), the *CP* advances to the next *cycle*.
4) The *4º cycle* examines each *Mini Table* defined in the main table of the *Assignments* sheet to check the criteria associated with it. This checking consists in verifying if the criteria associated is directly descending from the one fixed in the *3º cycle*. If so, the action and reference action of that *Mini Table* are compared with the ones fixed on the first two *cycles*. If all of them are equal, the *ORs* established in that *Mini Table* must be studied. Table **E.1** in **Appendix E** presents an example of one of those *Mini Tables*.

   The possible strings for each *OR* are either 'YES' or 'NO', which logically implies if a certain *OR* is verified or not. The 14 variables previously defined are then used to count the number of times that a certain *OR* has a certain string (12 possibilities), and the number of times that each row fulfills the necessary requisites. These variables are then processed to check if $aS_{(r,j)}b$, $not(aS_{(r,j)}b)$, $bS_{(r,j)}a$ or $not(bS_{(r,j)}a)$ for all $j = 1, \dots, n(r)$, with $r \in L_{\check{G}} \setminus \{EL \cup LBO\}$), for every possible *OR*. If any of them is verified, then it is fundamental to prove that the same *OR* is established for the ascending subcriteria firstly fixed. The information resultant from this strategy is inserted in the *Coherence properties* sheet.
5) The last part of the *Coherenceproperties'* function is to verify all the information available in the *Coherence properties* sheet, to analyze whether it allows to conclude that the two coherence properties are respected or not. The results of this conclusion are inserted into a specific table of the *Coherence properties'* sheet. This table is presented in Figure **E.3** of **Appendix E.**

To better understand on how the data is organized in the *Coherence properties'* sheet it is presented an example in **Appendix E**.

It should be remarked that if all elementary criteria in the model are pseudo-criteria, then properties **(C1)** and **(C2)** are respected iff $\lambda_r = \sum_{j=1}^{n(r)} \lambda_{r,j} \ \ for\ all\ r \in L_{\check{G}} \setminus \{EL \cup LBO\}$ (for more details, see Appendix **B**). The way in which the cutting levels' values have been defined evidence that there is no need to verify if the coherence properties are respected in case all elementary criteria in the model are pseudo-criteria. However, since it is possible to have a model that does not respect this assumption, it was implemented in the *CP* the *Coherence properties'* function.

Homogeneity

The objective of this function is to define three tables in the *Homogeneity* sheet (**Appendix F** presents an example). The process to define those tables is presented in the following steps:

1) The *CP* gathers information from the main table in the *Assignment'* sheet, and inserts it into a new table in the *Coherence properties* sheet. Table **F.1** of **Appendix F** represents that table.
2) Compare the credibility indexes obtained by different actions with respect to the same reference profile and subcriterion. To make this analysis it were defined multiple *Mini Tables*. Each one compares the indexes of two different actions, with respect to a certain non-elementary criterion in all reference actions of the model. The *CP* then analyzes each row to examine if the credibility indexes obtained for each action have exactly the same value. The conclusion of this analysis is inserted into another table (*2º Table*) in the *Coherence properties* sheet, in conjunction with the information of the variables under study. Figure **F.2** of **Appendix F** represents that table.
3) The third and final table in the *Coherence properties* sheet (presented in Figure **F.3** of **Appendix F**) is defined by collecting information from the *2º Table* and also from the results obtained by the assignment procedure. The *CP* defines three *cycles* to fix a non-elementary criterion from the *Criteria* sheet and two different action from the *Actions* sheet. These variables are inserted into a row of the *3º Table*. Then, it examines each *Mini Table* generated in the *2º Table* to examine if the variables there inserted are equal to the ones defined in the three previous *cycles*. If so, the *CP* examines the last column of each row of the *Mini Table* to conclude if all the credibility indexes are equal or not (considering the two actions and the criterion defined). The conclusion withdrawn from that analysis is inserted in the same row of the *3º Table*. The last data inserted in that row concerns the results of the assignment procedure for each action, considering the criteria under analysis.
4) After inserting all the data in the *3º Table*, the *CP* verifies each row in it. When the string there inserted indicates that the credibility indexes of the specific case that each row represents, are not all equal, the homogeneity requirement is respected, and that conclusion is also inserted in the end of that row. In case the credibility indexes are all the same, the *CP* compares the categories assigned for each action to check if they match. If so, it is concluded that the homogeneity condition is respected between those actions considering the subcriterion under analysis.
5) The final step of the *CP* is to verify if the homogeneity condition is respected in each row of the third table, and, if so, to insert a string in a specific cell of the *Structural requirements* sheet indicating it. In case it is not, the string indicates the opposite.

Monotonicity & IndividualStrictability & MonotinicityFinal

The monotonicity condition imposes the need to compute the elementary indexes for each pair of different actions with respect to each non-elementary criterion in the model. The *Monotonicity* function's objective is exactly the computation of that indexes. The *CP* defines multiple *Mini Tables* in the *Comparison of alternatives* sheet, with the same structure of the ones defined in the *TransferDataToAssignment'* function. However, instead of comparing one action with a reference action, they compare two different actions. Table **5.6** presents three examples of those *Mini Tables*.

Table 5.6 - Example of the *Mini Tables* defined by executing the *Monotonicity*' function

Criteria				Index		Elementary Criteria						Performances		Elementary Indexes	
Criterion	Index	Wr	λ_r	Action 1	Action 2	Elementary	Weight	Index	q_t	p_t	v_t	Action 1	Action 2	Øt (a_1, a_2)	dt (a_2, a_1)
g_1	1	0.249	0.15	a_1	a_2	$g_{1,1}$	0.021	1,1	2	5	Ø	76.1	95.8	0	0.155
g_1	1	0.249	0.15	a_1	a_2	$g_{1,2}$	0.028	1,2	2	5	Ø	96	99.2	0.6	0
g_1	1	0.249	0.15	a_1	a_2	$g_{1,3}$	0.076	1,3	2	3	Ø	3.2	7.6	1	0
g_1	1	0.249	0.15	a_1	a_2	$g_{1,4}$	0.076	1,4	0.4	0.5	Ø	1.142	1.189	1	0
g_1	1	0.249	0.15	a_1	a_2	$g_{1,5}$	0.048	1,5	0.4	0.5	Ø	1.799	1.832	1	0
Criterion	Index	Wr	λ_r	Action 1	Action 2	Elementary	Weight	Index	q_t	p_t	v_t	Action 1	Action 2	Øt (a_1, a_2)	dt (a_2, a_1)
g_2	2	0.277	0.166	a_1	a_2	$g_{2,1}$	0.038	2,1	3	5	20	83.1	69.8	1	0
g_2	2	0.277	0.166	a_1	a_2	$g_{2,2}$	0.287	2,2	1	2	4	7.3	6.56	1	0
g_2	2	0.277	0.166	a_1	a_2	$g_{2,3}$	0.01	2,3	0.3	0.5	2	3.96	1.52	0	1
g_2	2	0.277	0.166	a_1	a_2	$g_{2,4}$	0.095	2,4	3	5	Ø	29.7	84.3	0	0.522
g_2	2	0.277	0.166	a_1	a_2	$g_{2,5}$	0.095	2,5	0.2	0.3	Ø	0.75	0.54	0.9	0
Criterion	Index	Wr	λ_r	Action 1	Action 2	Elementary	Weight	Index	q_t	p_t	v_t	Action 1	Action 2	Øt (a_1, a_2)	dt (a_2, a_1)
g_5	3	0.14	0.084	a_1	a_2	$g_{5,1}$	0.07	5,1	3	5	Ø	31.7	25.2	0	0.016
g_5	3	0.14	0.084	a_1	a_2	$g_{5,2}$	0.035	5,2	3	5	Ø	33	28	0	0
g_5	3	0.14	0.084	a_1	a_2	$g_{5,3}$	0.035	5,3	1	5	Ø	33	28	0	0.021

Pyhton Code

Monotinicity example

```
for i in range(rows, rows + num_rows):
    elementary_concordance_index_a_b = ws_comparasion.cell(rows, 15).value
    if elementary_concordance_index_a_b == 0:
        number_of_rows_strict_condition = number_of_rows_strict_condition + 1
    rows = rows + 1

criteria_name = ws_comparasion.cell(rows - 2, 1).value
action_1_name = ws_comparasion.cell(rows - 2, 5).value
action_2_name = ws_comparasion.cell(rows - 2, 6).value
ws_comparasion.cell(last_row_final_comparasion + 1, 18).value = criteria_name
ws_comparasion.cell(last_row_final_comparasion + 1, 18).alignment = Alignment(horizontal='center', vertical='center', wrap_text=True)
ws_comparasion.cell(last_row_final_comparasion + 1, 19).value = action_1_name
ws_comparasion.cell(last_row_final_comparasion + 1, 19).alignment = Alignment(horizontal='center', vertical='center', wrap_text=True)
ws_comparasion.cell(last_row_final_comparasion + 1, 20).value = action_2_name
ws_comparasion.cell(last_row_final_comparasion + 1, 20).alignment = Alignment(horizontal='center', vertical='center', wrap_text=True)
if num_rows == number_of_rows_strict_condition:
    ws_comparasion.cell(last_row_final_comparasion + 1, 21).value = 'TRUE'
```

Figure 5.5 - Portion of the *python* code of the *IndividualStrictability*' function

Figure **5.5** displays a portion of the *python* code of the *IndividualStrictability'* function. This function is focused on analyzing the data entered in the *Mini Tables* defined in the Monotonicity' function, and compare the categories in which each action was assigned to, if necessary. To do so, the *CP* identifies each *Mini Table* defined in the *Comparison of alternatives* sheet and verifies if the value of the elementary concordance index, $\phi_t(a, b)$, in each row is equal to zero. If so, it means that action b is strictly preferred to action a considering the subcriterion presented in that *Mini Table*. The *CP* also identifies in the *Assignment* sheet which are the categories that those actions were assigned to, with respect to the same non-elementary criterion. All this information is inserted into one row of the second table existent in the *Comparison of alternatives* sheet. Table **5.7** displays an example of this table.

Table 5.7 - Example of the rows defined by executing the *MonotonicityFinal*' function

Final assignment								
Strict Separability analysis				Action 1		Action 2		Monoticity
Criteria	Action 1	Action 2	Is action 2 stricly preferred to action 1?	Min Category	Max Category	Min Category	Max Category	
g_1	a_1	a_2	FALSE	C_2	C_2	C_1	C_2	RESPECTED
g_2	a_1	a_2	FALSE	C_3	C_3	C_4	C_4	RESPECTED
g_5	a_1	a_2	FALSE	C_3	C_5	C_4	C_5	RESPECTED

The *CP* then inspects each row of this table to verify if the monotonicity condition is individually respected. There are two scenarios consistent with this conclusion:

- The characters of the string in the fourth column of the table indicate that action b is not strictly preferred to action a. The first two rows of Table **5.7** corroborate this scenario;

- The characters of the string in the fourth column of the table indicate that action b is strictly preferred to action a, and the comparison between the categories in which each action were assigned to, allows to conclude that action b was assigned at least to the same categories that action a was assigned to. The third row of Table **5.7** confirms this scenario.

The final step is to verify if the monotonicity condition is respected, or not, in each row of this last table, and to insert a string consistent with that conclusion in the *Structural Requirement* sheet.

5.4.1 Presentation of results

The results of the model are presented in the same *Excel* file used to construct it. It was also defined a new feature to present all the relevant data of the model, including the final results into a web browser. This browser can be accessed at real time by any device, which is advantageous in a situation where there are multiple people that wants to observe this data. To establish this feature, it was used the *Streamlit* and *Pandas python* packages. The second one allows to manipulate the data of the model into a structure that may be presented. The first one allows to define a web browser page, which presents the manipulated data into a specific format that can be easily modified by the analyst. The computer where the model is being executed is the server of that web browser page. That browser is terminated when the computer terminates the *CP*. This capability was introduced with the objective of further involving the DM in the entire method development, as he/she main gain another insight by observing the multiple data inserted or obtained in the model. Its use may also be interesting when there are multiple DMs that are constructing the model conjointly.

5.4.2 Limitations and capabilities of the computational program

The major concern when developing the *CP* was its capability of executing all the procedures imposed by the method, with any input data defined by the DM. This capability is also indirectly linked with the objective of aiding the analyst in its role, since a flexible tool allows for him/her to be only focused on capturing the DM preferences. Nevertheless, it was also essential to seek the reduction of the computational effort of the program when it was possible. Specially, when the selected method has several different features that can be incorporated into it, as for instance the possibility of having interactions between the criteria considered. All of these concerns were considered when developing the program, and led to the definition of some assumptions:

- When applying the SRF weighing procedure, the level of importance assigned to each criterion must not be higher than ten. Otherwise, it is imperative to add a small portion of programming code in the respective function;

- The hierarchical tree of criteria must not have more than three levels of criteria. If this principle is not respected, it is imperative to define two new functions for each additional level, similar to the ones defined for the second and third level;

- All the criteria's scales are quantitative. If the DM wishes to have one or more criteria with a qualitative scale it is fundamental to transform this scale into a quantitative one, which can prove to be very useful to establish discrimination thresholds for that criteria. The analyst is responsible for the selection of the procedure to execute this transformation;

- If the DM wishes to change any parameter of the data entered in the model (including the splitting or merging operations of certain categories) it is essential to restart the method execution.

- There is no interaction between the criteria.

- The manner in which the characteristic reference actions are entered in the *Excel* file, and the need to identify the categories that they belong to, makes the structural requirement of *Conformity* to always be respected. Thus, it is not necessary to verify it.

5.5 Summary

This chapter presented firstly all the concerns and requirements that were considered when implementing the method. The method's framework was then defined, including the interface between the *CP* and the *Excel* file that highlights when it is necessary to insert specific data in the model resultant from the interaction between the analyst and the DM. The structure of the *Excel* file and the programing code were then described, with emphasis on the logic behind each function in the *python* code. Finally, it was defined the limitations of the method developed and the reasons that led to considered them.

Chapter 6: Case study

This chapter presents firstly a brief description of the Portuguese healthcare sector, followed by the definition of the model used in this dissertation, and all its input data. The results of the implementation of the model in the *CP* developed are then presented and interpreted. In last, it was made a sensitivity analysis of the model to evaluate its robustness.

6.1 Overview

As described in Section **2.1**, the Portuguese NHS is a universal coverage system, established in 1979, that should grant the health conditions defined in the Portuguese constitution of 1976. The Portuguese government has implemented this system with that purpose, allowing mainly to guarantee the access of all citizens to health care. The NHS has a predominantly public matrix, and it is mainly funded by the tax system imposed to the Portuguese population, which makes it a Beveridge model-based system. Nonetheless, it is also important to consider that there are some features of the Bismarck model in the NHS (Tavares & Marques, 2020). One of those features is the existence of some organized groups of professionals, for instance, the civil servants, that pay a contribution to a social health insurance. This social health insurance reenforces the coverage to health, allowing for a greater easiness in accessing the healthcare services. Logically, these features have an impact in the access to health care.

The Ministry of Health is the highest health entity in Portugal, and it is in charge of managing the NHS and evaluating the results of that management. NHS provides care for the Portuguese citizens in three specialized segments: Primary care, Secondary Care and Tertiary Care. The differentiated care (Secondary and Tertiary) are mainly provided in the Portuguese hospitals, while the Primary ones are provided by the Primary Healthcare Center groups. According to the information gathered by INE, there are 238 hospitals in Portugal, being 108 public, 127 private, and the remaining three a result of a public-private partnership[4]. The constant goal of improving the efficiency and quality of the NHS led to the implementation of some adjustments in the organizational structure of the healthcare providers. The emerging of the hospital center groups, responsible for integrating and coordinating several public hospitals of a certain region, and the creation of Local Health Units, that have the task of integrating healthcare units of both primary and differentiated care in a limited demographic region, are two of the major examples of such adjustments. This merging of health care units allowed, respectively, to improve the horizontal integration of the public hospitals, and the vertical integration of the hospitals and the HCC. Nonetheless, there are not still evidence reviling the effectiveness of the creation of the HCC (Fronteira et al, 2018). Another important modification of the system was the corporatization of the healthcare units, which had a substantial impact on the funding of those units, and the establishment of some public-private partnerships.

The Portuguese Government have tried to make some modifications in the NHS during the years since its implementation, with the purpose of improving efficiency. This idea is even more pronounced

[4] https://www.ine.pt/xportal/xmain?xpid=INE&xpgid=ine_indicadores&contecto=pi&indOcorrCod=0008101&selTab=tab0

since the financial crisis of 2009. This crisis, that had a huge impact in the Portuguese economy, led to the need of acquiring a loan from external entities, and consequently to the signing of a MoU between Portugal and those entities. In that MoU it was specified several measures for the Portuguese government to implement, in order to be able to reduce the debt created within the stipulated deadlines. The public health area was significantly affected by those measures.

The post-crisis period, that began in 2016, brought new reforms in the healthcare area focused on improving the indicators that have worsened during the austerity period. Nevertheless, these reforms also seek to continuously improving the efficiency of the NHS, particularly in the relation between the expenditures and the quality and access to that system. The expenditures in health have been growing in an inefficient way, which is easily proved by the constant negative deviations from the NHS balance. This balance measures the difference between the total expenditures in health in a specific year and the budget that were stipulated in the state budget for that area. The 2018 and 2019 years were particularly worrying considering that indicator, as the value of the negative deviations were, respectively, 732 M€ and 628 M€. It is also crucial to acknowledge the value of the debt to the NHS external suppliers, that in the end of 2020 was 1516 M€ (Portuguese Public Finance Council, Budgetary Evolution of the NHS, Nº06/2021), which is clear evidence of the unsustainability of the NHS. The budget stipulated for the healthcare is allocated to different healthcare units according to the guidance of the Ministry of Health. The corporatization has introduced some modifications on the healthcare units funding system, mainly because each individual hospital started to be more accountable for its expenditures (Andrews et al., 2019), which has shifted the funding paradigm. Nowadays, the global budget allocated for the public hospitals is based on contracts between them and the Ministry of Health. The value defined in those contracts is essentially based on the diagnosis-related group (DRG) hospital payment system information, that has been implemented by Medicare in the United States in 1983 (Fetter, 1991), and also the non-adjusted hospital outpatient volume. The DRG implementation has introduced an activity-based resource allocation model (Simões et al., 2017), that requires to constantly collect data from the individual patients that enter in any healthcare unit. The funding is then calculated by considering the activities and productivity of the healthcare unit analyzed, which is correlated with the expenditures of that same unit. It is also important to mention that there is another part of the budget that is complemented by the payment made by third parties to the public hospitals. This prospective funding model was implemented with the purpose of gaining effectiveness and containing costs at the same time, although, as previously specified, the real expenditures of the majority of the healthcare units have largely exceeded the budget allocated to them. There are several possible causes for those negative deviations, however they exist undeniably, and the focus must be on reducing, or even eliminating them. Nevertheless, it is crucial to seek this goal without jeopardizing the quality and the access of the Portuguese population to healthcare.

The latest two years, which coincide with the appearing and spreading of the COVID-19 pandemic, have, once again, enhanced the importance of having an organized and efficient NHS. Portugal was deeply affected by this virus and despite the fact that the vaccination process is being immensely efficient, it is expected that there will be a global recession due to the effects of this pandemic. The real GDP in 2020 has descended in 8,4% in comparison with the previous year, which makes this year the worst one, in terms of economic activity, since 1995[5]. Alongside with that increment, it also occurred an increase of 7.8% (nominal) of the national public expenditure as result of the measures imposed during the COVID-19 pandemic[6]. This contrast between the increase in expenses and a decrease in the GDP reveals the importance of reducing costs in the near future. If the healthcare sector was already, particularly since the 2009 crisis, an area in which there was a constant concern in improving efficiency and reducing costs, this objective could be even more important considering the financial and social repercussions on the country due to the COVID-19 pandemic.

All this contextualization reveals the importance of correctly measuring some crucial subjective indicators as the quality and access to the health care providers, because it is not advisable to seek an improvement on the efficiency without considering simultaneously those two dimensions. However, these dimensions are particularly hard to measure, especially because they are subjective and there are a huge variety of criteria that could be considered. This chronic problem motivated this dissertation. The use of a robust MCDA model in the evaluation of the quality and access to the Portuguese public hospitals intends to evaluate those two dimensions by accounting some inherent specifications of the problem in hands. Two of those specifications are the need to have a non-compensatory method, and its flexibility in capturing the preferences of the person who is building the model with the analyst. Another important aspect is the fact that the majority of the real-life problems are better described by a hierarchical tree of criteria. These requirements led to the choice of the ELECTRE TRI-nC method with MCHP.

Rocha et al. (2021) studied the quality assessment of the Portuguese public hospitals with a multiple criteria approach. The family of criteria was structured in a hierarchical way, however, a hierarchic process was not used to deal with this problem. The MCDA method used in that paper was the ELECTRE TRI-nC, and also the ELECTRE TRI-C to construct the scales of the elementary criteria based on the scales of the subcriteria. Since the used method was also within the ELECTRE family of methods, there is the possibility of using the model there presented to be applied in the method developed in this dissertation. Nevertheless, it will be necessary to make some adjustments in certain parameters, for it to be applied in a method that uses a hierarchical process.

6.2 Construction of the model

As mentioned in Subchapter **6.1**, the decision model used in this dissertation is almost equal to the one presented by Rocha et al. (2021). The main differences are related with the use of MCHP to deal with the hierarchical structure of criteria and the absence of a DM, which does not have an impact in the method, since the decision model constructed presented by Rocha et al. (2021) was already constructed with a DM.

The decision model using ELECTRE TRI-nC and MCHP and the modifications made are presented in the following subsections.

[5] https://www.ine.pt/xportal/xmain?xpid=INE&xpgid=ine_destaques&DESTAQUESdest_boui=473168285&DESTAQUESmodo=2
[6] https://www.ine.pt/xportal/xmain?xpid=INE&xpgid=ine_destaques&DESTAQUESdest_boui=507673594&DESTAQUESmodo=2

6.2.1 Performances assessment source

Assessing the performances of all potential and reference actions of the model in the elementary criteria selected by the DM is a critical step. This data should be as reliable as possible. Choosing an official source for collecting these data emerged as the best possible alternative to maximize that reliability. The ACSS benchmarking database, mentioned in Section **2.2**, was the official source of data selected to collect the performances data. The benchmarking is available online, and all the information there inserted is public and exportable. This initiative started in 2011, nonetheless, the first year considered in the benchmarking process was 2013. There are multiple healthcare facilities contemplated in that process, and each one of them has a specific performance value attributed to it according to the healthcare indicator considered. The ACSS benchmarking database organizes the data by month/year, providing it from January of 2013 until July of 2021[7].

It is important to remark, that there are several health care providers that have uncomplete information in some months or years, or even, do not have it at all in some indicators. This information has influenced the choice of the public hospitals that will be considered in the model, and also the time interval selected. The data collected by Rocha et al. (2021) concerned the years 2017 and 2018, however in this dissertation only the 2018 data is used.

6.2.2 Potential actions

The benchmarking database has defined five different groups, from Group B until F, of hospitals through a hierarchical clustering process. In total, 43 different hospitals were considered in that process. From these 43, 25 public secondary healthcare providers were selected from which five are hospitals, and 20 are hospital centers. These healthcare providers, that from now on will be mentioned as potential actions, are presented in Table **6.1**.

Table 6.1 - Potential Actions of the model

Name	Code	a_i
Centro Hospitalar do Médio Ave	CHMA	a_1
Centro Hospitalar Póvoa de Varzim/Vila do Conde	CHPV	a_2
Centro Hospitalar Barreiro/Montijo	CHBM	a_3
Centro Hospitalar de Leiria	CHL	a_4
Centro Hospitalar de Setúbal	CHS	a_5
Centro Hospitalar do Baixo Vouga	CHBV	a_6
Centro Hospitalar Entre Douro e Vouga	CHCHDV	a_7
Centro Hospitalar Médio Tejo	CHMT	a_8
Centro Hospitalar Tâmega e Sousa	CHTS	a_9
Centro Hospitalar Universitário Cova da Beira	CHUCB	a_{10}
Hospital da Senhora da Oliveira, Guimãres	HSO	a_{11}
Hospital Distrital de Santarém	HDS	a_{12}
Centro Hospitalar Tondela-Viseu	CHTV	a_{13}
Centro Hospitalar Trás-os-Montes e Alto Douro	CHTAD	a_{14}
Centro Hospitalar Universitário do Algarve	CHUA	a_{15}
Centro Hospitalar Vila Nova de Gaia/Espinho	CHVNG	a_{16}
Hospital Espírito Santo de Évora	HESE	a_{17}
Hospital Fernando da Fonseca	HFF	a_{18}
Hospital Garcia de Orta	HGO	a_{19}
Centro Hospitalar de Lisboa Ocidental	CHLO	a_{20}
Centro Hospitalar e Universitário de Coimbra	CHUCB	a_{21}
Centro Hospitalar Universitário de Lisboa Central	CHULC	a_{22}
Centro Hospitalar Universitário de São João	CHUSJ	a_{23}
Centro Hospitalar Universitário do Porto	CHUP	a_{24}
Centro Hospitalar Universitário Lisboa Norte	CHULN	a_{25}

[7] https://benchmarking-acss.min-saude.pt/BH_AcessoDashboard

6.2.3 Criteria

Table **6.2**, which was adapted from Rocha et al. (2021), identifies all the criteria defined in the model. The elementary criteria defined in the model must be operationalized, which means that they need to be associated with an indicator that describes how to measure the performances of the actions in each one of them. These indicators are also defined in Table **6.2**, as the direction of preference and the performance's scale of each elementary criteria. Defining a minimum and a maximum value for the performances is mandatory in the ELECTRE TRI-nC method with MCHP, because it allows to establish two subsets of reference actions, with one reference action each, that have the worst and the best possible performances in all elementary criteria of the model.

Appendix G displays a figure that defines the hierarchy tree of criteria of this model.

Table 6.2 - Model's criteria and correspondent information

Subcriteria	Elementary Criterion	Indicator	Direction of Preference	Scale
Access (g_1)	First medical appointments timeliness ($g_{1,1}$)	Number of non-urgent first medical appointments performed in adequate time per 100 first medical appointments	Maximization	[0; 100]
	Enrolled patients for surgery ($g_{1,2}$)	Number of enrolled in the surgical waiting list within the mean guaranteed response time	Maximization	[0; 100]
	Availability of beds ($g_{1,3}$)	Difference between the real occupancy rate and the ideal occupancy rate	Minimization	[13; 0]
	Availability of doctors ($g_{1,4}$)	Doctors per 1000 inhabitants	Maximization	[0; 5.8]
	Availability of nurses ($g_{1,5}$)	Nurses per 1000 inhabitants	Maximization	[0; 8.2]
Care Appropriateness (g_2)	Minor surgeries appropriateness ($g_{2,1}$)	Number of outpatient surgeries per 100 potential outpatient procedure	Maximization	[0; 100]
	Avoidable re-admission prior 30 days after discharge ($g_{2,2}$)	Number of readmissions in 30 days after discharge per 100 inpatients	Minimization	[13.2; 0]
	Excessive staying delay ($g_{2,3}$)	Number of long-stay inpatients per 100 admissions	Minimization	[6.5; 0]
	Hip surgery timeliness ($g_{2,4}$)	Number of hip surgeries performed in the first 48 hours per 100 hip surgeries	Maximization	[0; 100]
	Delay before surgery ($g_{2,5}$)	Average waiting time before surgery	Minimization	[1.9; 0]

Patient Safety (g_3)	Bedsores ($g_{3,1}$)	Number of bedsores per 100 inpatients	Minimization	[0.2; 0]
	Bloodstream infections related to CVC ($g_{3,2}$)	Bloodstream infection rate related to CVC per 100 inpatients	Minimization	[0.1; 0]
	Postoperative pulmonary embolism or thrombosis ($g_{3,3}$)	Postoperative pulmonary embolism/deep venous thrombosis cases per 100 surgical procedures	Minimization	[0.4; 0]
Patient Safety (g_3)	Postoperative septicaemia ($g_{3,4}$)	Postoperative septicaemia cases per 100 inpatients	Minimization	[1.4; 0]
	Non-instrumental vaginal deliveries with severe laceration ($g_{3,5}$)	Cases of trauma on vaginal delivery (third and fourth degree lacerations), without instrumentation, per 100 assisted deliveries	Minimization	[1.6; 0]
	Assisted vaginal deliveries with severe laceration ($g_{3,6}$)	Cases of trauma on vaginal delivery (third and fourth degree lacerations), with instrumentation, per 100 assisted deliveries	Minimization	[6.9; 0]
Efficiency (g_4)	Expenses with staff ($g_{4,1}$)	Expenses with staff per severity-adjusted patient	Minimization	[3107.3; 0]
	Expenses with drugs, pharmaceutical products and clinical consumables ($g_{4,2}$)	Expenses with drugs, pharmaceutical products and clinical consumables per severity-adjusted patient	Minimization	[2924.7; 0]
	Expenses with supplies and external services ($g_{4,3}$)	Expenses with supplies and external services per severity-adjusted patient	Minimization	[1032.6; 0]
	Expenses with overtime ($g_{4,4}$)	Expenses with overtime per total expenses with staff	Minimization	[18.4; 0]
	Expenses with outsourcing ($g_{4,5}$)	Expenses with outsourcing per total expenses with staff	Minimization	[16; 0]
Caesarean Appropriateness (g_5)	Volume of caesarean sections ($g_{5,1}$)	Number of caesarean sections per 100 deliveries	Minimization	[43.4; 0]
	Caesarean sections in UCFTPs ($g_{5,2}$)	Number of caesarean sections in UCFTPs per 100 sections in UCFTPs	Minimization	[59.6; 0]
	First caesarean sections in UCFTPs ($g_{5,3}$)	Number of first caesarean sections in UCFTPs per 100 deliveries in UCFTPs without caesarean section before	Minimization	[100; 0]

The five criteria in the model represent five dimensions that the DM considered crucial to evaluate the quality of the Portuguese public hospitals. Thus, it is important to define each of these criteria:

Access (g_1):

- Most healthcare authorities consider this dimension particularly difficult to define, and consequently to operationalize. Nevertheless, its importance in the healthcare and social rights area, is indisputably meaningful. Different authors have presented multiple approaches for this theme, during the years. Penchansky et al. (1981), consider that this dimension summarizes five specific ones: availability, accessibility, accommodation, acceptability and affordability. The elementary criteria descending from the subcriteria access reveal that the DM was focused on the first four specific dimensions, especially in the availability one. This is completely justified, since the scarcity of resources in the healthcare providers could compromise the access to them (Aday et al., 1981).

Care Appropriateness (g_2):

- As in the case of access, there is no standardized operational definition for care appropriateness. However, it is directly linked with the maximization of the benefits and the minimization of the risks for the patient associated with the care provided (Sanmartin, et al., 2008). The level of delays and readmissions is also correlated with the appropriateness of care.

Patient Safety (g_3):

- In 1999, the Institute of Medicine (IOM) wrote a report addressing some actions to reduce the human errors that constantly occur in a healthcare provider, and an estimation of the number of deaths in hospitals per year because of those errors (Leape & Berwick, 2005). IOM has also defined patient safety as the *prevention of harms to patients* (Mitchell, 2008). Thus, this criterion considers subcriteria that are directly linked to human error in a secondary healthcare provider, and that have a negative impact in the safety of the patient.

Efficiency (g_4):

- Efficiency in healthcare is usually associated with an economical view, in which, the objective is to optimize the use of the existent resources (Williams, 1988). As in many other areas, to make conclusions regarding that use it is required to analyze the relation between the inputs and outputs. The first ones concern the expenditures, while the second ones are focused on indicators directly related with the patients (Palmer & Torgerson, 1999).

Caesarean Appropriateness (g_5):

- Logically, this criterion is related to the Care Appropriateness one, however, since it has a specific technology, it was considered that it should be evaluated independently from g_2. Caesarean section rates have been a matter of discussion for long time, nonetheless, the utmost important indicator concerning this should be if the caesarean is appropriated for each individual situation rather than their rate (Robson, 2001). However, at a macroscopic view, this indicator is almost impossible to consider, which enhances the importance of considering the section rates. There is no indication that the increase in rates of caesarean delivery, in cases that there is no medical justification to perform it, is associated with any clear overall benefit for the newborn or his/her mother. This becomes even more worrying, because there is an association between that increase and the maternal and perinatal morbidity (Villar et al., 2007). According to the World Health Organization, *there is no justification for any region to have caesarean section rates higher than 10-15%* (Betrán et al., 2007). The same organization has concluded that this rate in 2030, will be 29%, in global terms, which is very far from the previously mentioned interval (Betrán et al., 2021).

6.2.4 Indifference thresholds

In order to take into account, the imperfect character of the data regarding the computation of performances in the elementary criteria, $g_j(a)$ with $j = 1, \ldots, 24$ and $a \in A$ (Almeida-Dias, et al., 2012), it was established for each one of them, two indifference thresholds, q_j and p_j. The veto thresholds, v_j, that the DM has decided to implement in the model defined by Rocha et al. (2021) were adopted to fit into this method. **Appendix H** describes the transformation process made to make the necessary adaptations. Table **6.3** presents the thresholds associated with each criterion.

Table 6.3 - Thresholds' value for each elementary criterion

Elementary Criterion	Technical name	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$
	Indiference threshold, q_j	2	2	2	0.4	0.4	3	1	0.3	3	0.2	0.01	0.01	0.01	0.01	0.02	0.02	50	50	25	0.5	0.5	3	3	1
	Preference threshold, p_j	5	5	3	0.5	0.5	5	2	0.5	5	0.3	0.01	0.01	0.01	0.01	0.02	0.02	100	100	50	1	1	5	5	3
	Veto threshold, v_j	∅	∅	∅	∅	∅	∅	5.3	2.6	∅	∅	∅	0.04	0.16	0.56	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅

6.2.5 Categories and characteristic reference actions

The goal of the ELECTRE TRI-nC method with MCHP in this case study is the assigning of each potential action a_i, with $i = 1, \ldots, 25$, to one or more continuous categories that represent a certain degree of quality and access. The set of categories was established a priori, and it comprises five different categories, $C = \{C_1, C_2, C_3, C_4, C_5\}$. To be in line with what was presented in Section **4.1.1**, there is the need to add two additional categories, C_0 and C_6, that, respectively, represent the worst and best possible degree of quality and access. Hence, the set of pre-ordered categories of the model is $C = \{C_0, C_1, C_2, C_3, C_4, C_5, C_6\}$. The meaning of each category is described in Table **6.4**.

Table 6.4 - Model's categories description

Categories						
C_0	C_1	C_2	C_3	C_4	C_5	C_6
Worst	Very poor	Poor	Neutral	Good	Very good	Best

As mentioned in Section **4.3.1**, the ELECTRE TRI-nC method with MCHP requires the definition of a subset of reference actions, B_h, with $h = 0, \ldots, 6$, to characterize each category. Each subset is composed by one or more reference actions, $b_h^r, r = 1, \ldots, m_h$, with $h = 1, \ldots, 6$. The subset B_0 and B_6, are respectively composed by a single reference action that has the worst and best possible performances in all elementary criteria, g_j. The rest of the reference actions are defined by the DM. In Rocha et al. (2021), the DM has selected one reference action per subset. However, in order to test the possibility of defining an unlimited number of reference actions per subset in the developed method, and to consider some underlying variability in the process that led to their definition, four new reference actions were added. **Appendix I** describes the process to compute the performances in all elementary criteria for those four reference actions, and also a description of how the worst and best possible performances were computed.

Table **6.5** presents the subsets of reference actions and the categories that they characterize.

Table 6.5 - Definition of the subset of reference actions

Category	C_0	C_1	C_2	C_3	C_4	C_5	C_6
Subset of reference actions	$B_0 = \{b_0^1\}$	$B_1 = \{b_1^1\}$	$B_2 = \{b_2^1, b_2^2, b_2^3\}$	$B_3 = \{b_3^1, b_3^2\}$	$B_4 = \{b_4^1, b_4^2\}$	$B_5 = \{b_5^1\}$	$B_6 = \{b_6^1\}$

6.3 Model implementation

The implementation of the model followed the framework presented in Figure **5.3** of Section **5.3**. The first step was to enter the model's input data into the correct spreadsheets in the *Excel* file, which resulted in four tables with data. Table **6.6** presents the table defined in the *Criteria's* sheet.

Table 6.6 - Criteria's table after the *1º EI*

Criteria	Technical name	Index	h	Type of criterion	qt	pt	vt	Direction
Quality and Access	g_0	0	0	Root Criterion				
Access	g_1	1	1	LBO				
Care Appropriateness	g_2	2	1	LBO				
Patient Safety	g_3	3	1	LBO				
Efficiency	g_4	4	1	LBO				
Caesarean Appropriatenes	g_5	5	1	LBO				
First medical appointments timeliness	$g_{1,1}$	1.1	2	EL	2	5	∅	Max
Enrolled patients for surgery	$g_{1,2}$	1.2	2	EL	2	5	∅	Max
Availability of beds	$g_{1,3}$	1.3	2	EL	2	3	∅	Min
Availability of doctors	$g_{1,4}$	1.4	2	EL	0.4	0.5	∅	Max
Availability of nurses	$g_{1,5}$	1.5	2	EL	0.4	0.5	∅	Max
Minor surgeries appropriateness	$g_{2,1}$	2.1	2	EL	3	5	∅	Max
Avoid re-admission prior 30 days after discharge	$g_{2,2}$	2.2	2	EL	1	2	5.3	Min
Excessive staying delay	$g_{2,3}$	2.3	2	EL	0.3	0.5	2.6	Min
Hip surgery timeliness	$g_{2,4}$	2.4	2	EL	3	5	∅	Max
Delay before surgery	$g_{2,5}$	2.5	2	EL	0.2	0.3	∅	Min
Bedsores	$g_{3,1}$	3.1	2	EL	0.01	0.01	∅	Min
Bloodstream infections related to CVC	$g_{3,2}$	3.2	2	EL	0.01	0.01	0.04	Min
Postoperative pulmonary embolisms or thrombosis	$g_{3,3}$	3.3	2	EL	0.01	0.01	0.16	Min
Postoperative septicaemia	$g_{3,4}$	3.4	2	EL	0.01	0.01	0.56	Min
Non-instrumental vaginal deliveries with severe laceration	$g_{3,5}$	3.5	2	EL	0.02	0.02	∅	Min
Assisted vaginal deliveries with severe laceration	$g_{3,6}$	3.6	2	EL	0.02	0.02	∅	Min
Expenses with staff	$g_{4,1}$	4.1	2	EL	50	100	∅	Min
Expenses with drugs, pharmaceutical products and clinical consumables	$g_{4,2}$	4.2	2	EL	50	100	∅	Min
Expenses with supplies and external services	$g_{4,3}$	4.3	2	EL	25	50	∅	Min
Expenses with overtime	$g_{4,4}$	4.4	2	EL	0.5	1	∅	Min
Expenses with outsourcing	$g_{4,5}$	4.5	2	EL	0.5	1	∅	Min
Volume of caesarean sections	$g_{5,1}$	5.1	2	EL	3	5	∅	Min
Caesarean sections in UCFTPs	$g_{5,2}$	5.2	2	EL	3	5	∅	Min
First caesarean sections in UCFTPs	$g_{5,3}$	5.3	2	EL	1	3	∅	Min

The performances of the reference actions and potential actions in all elementary criteria were then inserted after the first *CP* intervention. **Appendix H** presents a table with the performances of all the potential actions. Table **6.7** presents the performances of all reference actions.

Table 6.7 - Reference actions' performances in all elementary criteria

Reference Actions		Performances																								
Category	Index	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$
C_6	b_6^1	100	100	0	5.8	8.2	100	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_5	b_5^1	95	95	0	4.3	6.4	90	5	2.7	90	0.5	0	0	0	0	0.05	0.15	1408	537.04	348.12	10.6	1.14	15	15	80
C_4	b_4^1	85	85	2	2.7	4.8	85	6.5	3.2	80	0.6	0.03	0.02	0.08	0.03	0.16	0.35	1536	689.56	407.24	11.6	2.62	20	20	85
	b_4^2	82.5	90	3.5	2.4	5.6	82.5	5.75	3.45	65	0.55	0.04	0.01	0.04	0.045	0.105	0.5	1683	613	459	11.84	3.92	24.2	17.5	82.5
C_3	b_3^1	80	80	5	2.1	3.5	80	7.4	3.7	50	0.9	0.05	0.04	0.15	0.06	0.33	0.64	1829	847.82	509.85	12.08	5.22	28.4	31.2	90
	b_3^2	75	77.5	6	1.65	4.14	77.5	6.95	4.1	40	1	0.075	0.025	0.18	0.045	0.245	0.5	1934	768.7	557.78	13.03	3.92	29.2	30.6	92.5
C_2	b_2^1	70	75	7	1.2	2.1	75	8.3	4.5	30	1.1	0.1	0.06	0.21	0.76	0.64	1	2039	1082.11	605.71	13.98	7.29	30	30	95
	b_2^2	75	72.5	8	1.1	2.8	77.5	7.85	4.85	40	1	0.075	0.075	0.26	0.98	0.485	1.525	1934	1184.92	632.895	14.4	6.26	32.5	30.6	92.5
	b_2^3	65	77.5	6	1.65	1.9	72.5	9.05	4.1	25	1.25	1.1	0.05	0.18	0.041	0.725	0.82	2210	964.97	557.78	13.03	7.9	29.2	32.5	97.5
C_1	b_1^1	60	70	9	1	1.7	70	9.8	5.2	20	1.4	0.12	0.09	0.31	1.2	0.81	2.05	2381	1287.73	660.08	14.82	8.51	35	35	100
C_0	b_0^1	0	0	13	0	0	0	13.2	6.5	0	1.9	0.2	0.1	0.4	1.4	1.6	6.9	3107.3	2924.7	1032.6	18.4	16	43.4	59.6	100

6.3.1 Criteria' weights

The next phase of the developed method was to compute the weights of each criterion in the model. The SRF weighing procedure, explained in Section **4.3.2**, was applied to determine those weights. The data resultant from the interaction between the analyst and the DM when applied the SRF weighing procedure by Rocha et al. (2021), were inserted into the *Excel* file, and were then handle by the CP to compute the intrinsic weight of each criterion. **Appendix J** presents the value of the weights of all elementary criteria, g_j, with $j = 1, \dots, 25$. The weights associated with each criteria in the first level of the hierarchy are presented in Table **6.8**.

Table 6.8 - Subcriteria' weights

Criteria	Name	Access	Care Appropriateness	Patient Safety	Efficiency	Caesarean appropriateness
	Technical name	g_1	g_2	g_3	g_4	g_5
	Weight	0.2365	0.287	0.313	0.031	0.134

6.3.2 Cutting levels

The next phase of the method's framework was to define the value of the cutting levels. The procedure to compute those values was already described in the *Block 2º CP* of Section **5.4**. However, it is important to mention, that the ELECTRE TRI-nC method with MCHP requires the computation of the cutting levels in all criteria $\mathrm{G_r}$, with $r \in L_{\check{\mathrm{G}}} \setminus EL$. Since that Rocha et al. (2021), the DM only has defined the cutting level' value at a comprehensive level, it is necessary to compute the value of the non-elementary criteria of the model in a congruous way with the value defined by him. The same proportion (weight/λ_r) was used to determine the cutting level' value of each of those criteria.

Table **6.9** illustrates the cutting levels' values defined in each criterion G_r, with $r \in L_{\check{G}} \setminus EL$, and also at the comprehensive level (root criterion).

Table 6.9 - Cutting level's value of each non-elementary criterion

Non-elementary criterion					
Technical name	**Type of criterion**	**Weight**	**min λr**	**max λr**	**λr**
g_0	Root Criterion	1	0.5	1	0.6
g_1	LBO	0.236	0.118	0.236	0.141
g_2	LBO	0.287	0.143	0.287	0.172
g_3	LBO	0.313	0.156	0.313	0.188
g_4	LBO	0.031	0.016	0.031	0.019
g_5	LBO	0.134	0.067	0.134	0.08

6.3.3 Coherence properties and natural requirements

With the cutting levels' values and the weights of each criterion already computed, it was possible to verify the Separability Conditions. The results of this analysis are displayed in Table **6.10**.

Table 6.10 - Separability Conditions in each criterion LBO

Subcriteria	Weak Separability	Strict Separability	Hyper-Strict Separability
g_1	TRUE	TRUE	FALSE
g_2	TRUE	FALSE	FALSE
g_3	TRUE	TRUE	FALSE
g_4	TRUE	TRUE	FALSE
g_5	TRUE	TRUE	FALSE

The ELECTRE TRI-nC method with MCHP requires that the weak separability condition is respected for each criterion LBO (for more details, see **Appendix C**). As it is possible to conclude in Table **6.10**, this condition is respected.

The next phase of the *CP* was the execution of the Assignment procedure and the verification of the Coherence properties and Structural requirements imposed by the method. These verifications allowed to conclude that the two Coherence properties, defined in **Appendix B**, and the four Structural requirements, explained in Section **4.3.1**, were all respected.

6.3.4 Assignment procedure

The assignment procedure of the ELECTRE TRI-nC method with MCHP is based on the conjoint application of the two rules defined in Section **4.3.4**. As described, those rules compare the value of the partial credibility indexes $\sigma_r(a, B_h)$ and $\sigma_r(B_h, a)$, with λ_r ($r = 0, 1, \ldots, 5$ and $h = 0,\ 2, \ldots, 6$), in order to determine which category/categories should action a be assigned to. This comparation is made considering each non-elementary criterion, which results in a vast amount of data. Table **6.11** exposes all the credibility indexes for each action considering the root criterion g_0 (comprehensive level).

By observing the data there inserted it is possible to conclude that each action outranks the reference action that has the worst possible performance in each elementary criterion (C_0), as intended. The value of this index decreases as this action is compared to reference actions of higher categories, until reaching the minimum possible value (0), when compared with the one that has the best possible performance in all elementary criteria. The same analysis can be done analogously for the other credibility index, $\sigma_r(B_h, a)$.

Table 6.11 - Credibility Indexes of each action considering the root criterion

Actions		Categories													
		$\sigma_0(a, B_h)$							$\sigma_0(B_h, a)$						
Name	a_i	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_0	C_1	C_2	C_3	C_4	C_5	C_6
Centro Hospitalar do Médio Ave	a_1	1	0.994	0.994	0.802	0.215	0.001	0	0	0	0	0.51	0.831	0.947	1
Centro Hospitalar Póvoa de Varzim/Vila do Conde	a_2	1	0.994	0.907	0.728	0.603	0.526	0	0	0	0	0	0.458	0.817	1
Centro Hospitalar Barreiro/Montijo	a_3	1	0.991	0.885	0.445	0.078	0	0	0	0	0	0.674	0.894	0.94	1
Centro Hospitalar de Leiria	a_4	1	0.9	0.874	0.691	0.5	0.104	0	0	0	0	0	0.628	0.903	1
Centro Hospitalar de Setúbal	a_5	1	0.869	0.778	0	0	0	0	0	0	0	0.619	0.94	0.947	1
Centro Hospitalar do Baixo Vouga	a_6	1	0.982	0.912	0	0	0	0	0	0	0	0.399	0.87	0.947	1
Centro Hospitalar Entre Douro e Vouga	a_7	1	1	0.893	0.64	0.218	0	0	0	0	0	0.568	0.823	0.947	1
Centro Hospitalar Médio Tejo	a_8	1	0.99	0.893	0	0	0	0	0	0	0	0.104	0.8992	0.938	1
Centro Hospitalar Tâmega e Sousa	a_9	1	1	0.887	0.756	0.517	0.002	0	0	0	0	0.18	0.93	0.938	1
Centro Hospitalar Universitário Cova da Beira	a_{10}	1	0.889	0.801	0.624	0.135	0.001	0	0	0	0	0.642	0.937	0.938	1
Hospital da Senhora da Oliveira, Guimães	a_{11}	1	0.972	0.87	0	0	0	0	0	0	0	0.63	0.894	0.947	1
Hospital Distrital de Santarém	a_{12}	1	0.916	0.797	0.216	0.015	0	0	0	0	0	0.581	0.893	0.947	1
Centro Hospitalar Tondela-Viseu	a_{13}	1	0.914	0.775	0	0	0	0	0	0	0	0.502	0.831	0.87	1
Centro Hospitalar Trás-os-Montes e Alto Douro	a_{14}	1	0.876	0.69	0.245	0.046	0	0	0	0	0	0.495	0.833	0.947	1
Centro Hospitalar Universitário do Algarve	a_{15}	1	0.907	0.824	0	0	0	0	0	0	0	0.818	0.947	0.947	1
Centro Hospitalar Vila Nova de Gaia/Espinho	a_{16}	1	0.951	0.927	0.051	0	0	0	0	0	0.015	0.737	0.938	0.947	1
Hospital Espírito Santo de Évora	a_{17}	1	0.906	0.748	0	0	0	0	0	0.002	0.292	0.883	0.915	0.947	1
Hospital Fernando da Fonseca	a_{18}	1	0.798	0.549	0	0	0	0	0	0	0	0.595	0.894	0.947	1
Hospital Garcia de Orta	a_{19}	1	0.922	0.901	0.361	0.006	0	0	0	0	0	0.557	0.795	0.947	1
Centro Hospitalar de Lisboa Ocidental	a_{20}	1	0.94	0.869	0	0	0	0	0	0	0.559	0.842	0.9	0.94	1
Centro Hospitalar e Universitário de Coimbra	a_{21}	1	0.954	0.878	0.669	0.157	0	0	0	0	0	0.599	0.855	0.938	1
Centro Hospitalar Universitário de Lisboa Central	a_{22}	1	0.931	0.906	0.127	0.008	0	0	0	0	0	0.705	0.859	0.867	1
Centro Hospitalar Universitário de São João	a_{23}	1	0.927	0.837	0.203	0	0	0	0	0	0.024	0.442	0.846	0.867	1
Centro Hospitalar Universitário do Porto	a_{24}	1	0.982	0.95	0.799	0.275	0.004	0	0	0	0	0	0.709	0.832	1
Centro Hospitalar Universitário Lisboa Norte	a_{25}	1	0.982	0.712	0	0	0	0	0	0	0	0.698	0.866	0.947	1

The data entered in this table was then used to execute both mentioned rules conjointly, thus allowing the assignment of each action to one or more continuous categories. Figure **6.1** displays the assignment of each action at the comprehensive level in a Chart.

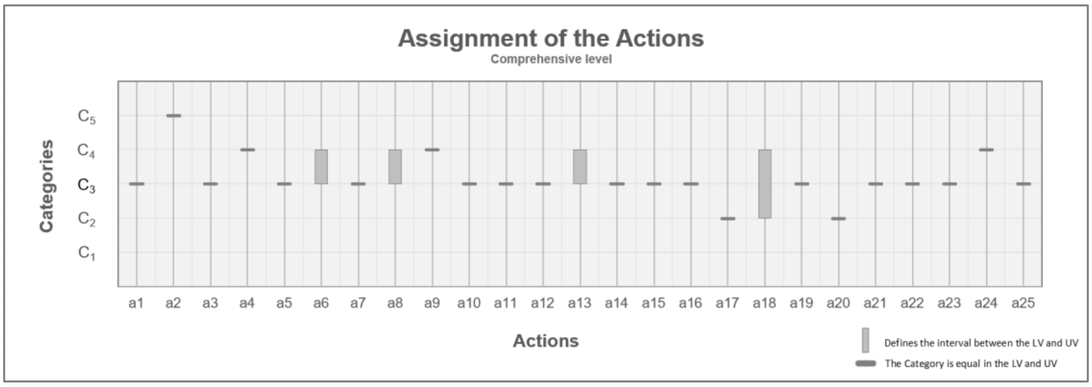

Figure 6.1 - Graph of the assignment of each action at the comprehensive level

Figure **6.2** presents five similar Charts to the one presented in Figure **6.1**. The only difference is that each one concerns a specific criterion G_r, with $r = 1, \dots, 5$, instead of the root criterion one. **Appendix L** presents a Table that has all the information required to conceive these Charts.

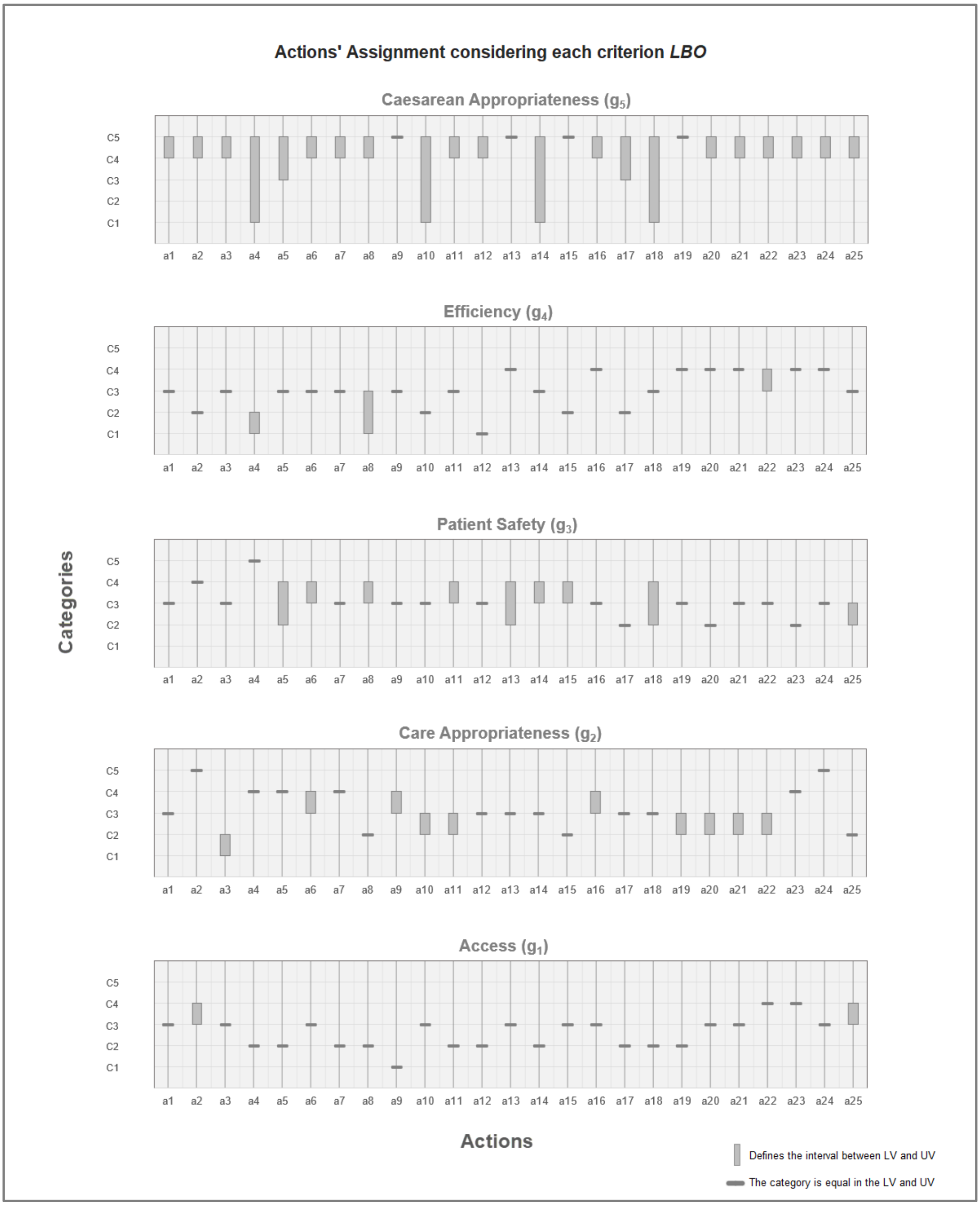

Figure 6.2 - Graphs of the assignment of each action considering each criterion LBO individually

Figure **6.2** illustrates one of the major features of this method, which is the possibility of assigning each action to one or more continuous categories in each non-elementary criterion. This feature is considerably helpful, because it allows for the DM to have information regarding specific subcriteria that he/she has defined in the hierarchical tree of the problem. A further analysis could also expose why a specific action at the comprehensive level was assign to a certain category, even when it has been assigned for a lower category in certain subcriteria. This reason is completely related with the intrinsic weight assigned in those subcriteria. If their weights have lower value regarding the others considered in the problem, it is logical that they will have a lower impact in the results at the comprehensive level. The same conclusion can be made analogously, which can help the DM in defining the area that is crucial to improve in the majority of the Portuguese public hospitals.

The graph of the *Caesarean Appropriateness* (g_5) in Figure **6.2** has revealed a substantial interval between the categories in which some actions were assigned to. However, this larger gap between categories is justified by the comparison between the actions' performances and the reference actions' performances in the elementary criteria that directly descend from the *Caesarean Appropriateness'* subcriterion. One can observe in Table **6.7**, that the performance of the reference action b_5^1 (that characterizes the category *Very Good*, C_5) in the elementary criterion $g_{5,3}$ is very high.

In a similar way, Table **H.1** in **Appendix H** reveals that the worst performance considering all potential actions, in the same elementary criterion, is still high. Which reveals that all of them have a smaller performance in this criterion than the performance that characterizes category C_5. Since the direction of preference in this elementary criterion is to minimize, and the weight of this elementary criterion is 40% of the weight' value associated with the subcriterion *Caesarean Appropriateness*, it is completely perceptible that this elementary criterion is distorting the assignment procedure of the actions in subcriterion g_5. If a certain action has bad performances in the other two criteria that descend from g_5, this creates a considerable interval between the categories in which that action was assigned, considering that subcriterion. This observation, that was easy to identify as a result of the implementation of the previous mentioned feature, could led to consider that there has been some ill-determination of the performances of the reference actions in elementary criteria g_5.

Table **6.12** displays the overall assignment of the Portuguese public hospitals in each non-elementary criterion and defines the percentage of actions that were assigned to a specific interval of categories considering each non-elementary criterion.

Table 6.12 - Percentage of hospitals assigned to a certain categories' range in each non-elementary criterion

Categories' Range		Non-elementary critera					
		Access (g_1)	Care Appropriateness (g_2)	Patient Safety (g_3)	Efficiency (g_4)	Caesarean Appropriateness (g_5)	Comprehensive level (g_0)
C_1	C_1	4%	0%	0%	4%	0%	0%
C_1	C_2	0%	4%	0%	4%	0%	0%
C_1	C_3	0%	0%	0%	4%	0%	0%
C_1	C_5	0%	0%	0%	0%	16%	0%
C_2	C_2	40%	12%	12%	16%	0%	8%
C_2	C_3	0%	24%	4%	0%	0%	0%
C_2	C_4	0%	0%	12%	0%	0%	4%
C_3	C_3	40%	24%	44%	40%	0%	60%
C_3	C_4	8%	12%	20%	4%	0%	12%
C_3	C_5	0%	0%	0%	0%	8%	0%
C_4	C_4	8%	16%	4%	28%	0%	12%
C_4	C_5	0%	0%	0%	0%	60%	0%
C_5	C_5	0%	8%	4%	0%	16%	4%

It was possible to withdraw several conclusions from the information displayed in Table **6.12:**

- The most represented range of categories considering the subcriterion g_j, with $j = 1, \dots, 5$, was the interval $[C_3; C_3]$, with a mean of $29{,}6\%$ per criterion;

- The *Caesarean Appropriateness'* subcriterion is the one that has the higher percentage of hospitals assigned to a Good (C_4) or better category, more precisely 76%;

- There is not a single hospital that was assigned to category C_5 considering the subcriteria Access and Efficiency;

- 84% of the hospitals were assigned to a category equal to or lower than C_3 considering the subcriterion Access, of which 4% were assigned to category C_1, 40% to category C_2;

- Considering the Efficiency's subcriteria, 32% and 28% of the hospitals were assigned to a higher and lower range of categories than $[C_3; C_3]$, respectively;

- The lowest diversity in terms of the number of ranges that have at least one hospital assigned to was found in the Caesarean Appropriateness' subcriteria, with only four ranges represented;

- The only subcriteria that has hospitals assigned to a range of categories composed by an interval of more than two categories is the Caesarean Appropriateness' one, as it was also possible to observe in Figure 6.2.

- In criterion g_1, there was 92% of hospitals that were assigned to a single category instead of an interval, while for the other remaining criteria g_j, with $j = 2, \ldots, 5$, the percentages were 60%, 64%, 88% and 16%, respectively.

Figure **6.3** display the results that concern the comprehensive level to a better visualization.

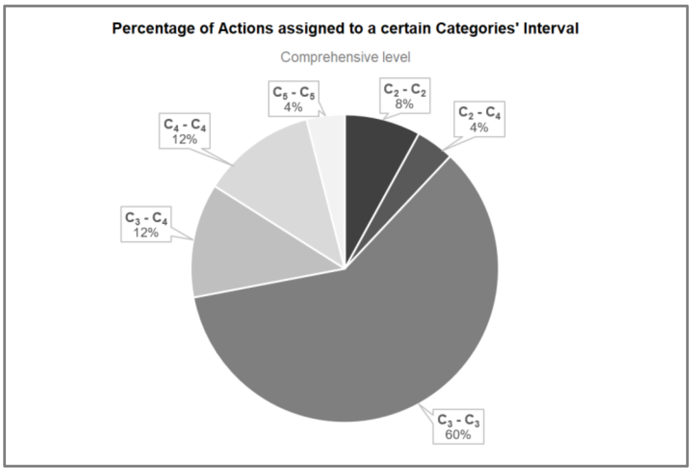

Figure 6.3 - Percentage of actions assigned to a specific categories' range at the comprehensive level

The comprehensive level measures, in global terms, the level of quality and access to the Portuguese public hospitals. Figure **6.3** reveals that 60% of the 25 analyzed hospitals were assigned to C_3 (*Neutral*), that is, the majority of them. Only 4% of them, which corresponds to action a_2, were assigned to C_5 (*Very Good)*. In negative terms, it is noteworthy that two hospitals, namely a_{17} and a_{20}, were assigned to C_2 (*Poor*). The remaining 28% of hospitals were assigned to a range of categories.

The fact that 72% of the hospitals were assigned to a unique category it is advantageous for the DM, since it allows to have a clearer idea of the level of quality and access of each hospital.

Nevertheless, it is important to mention that this conversion is influenced by the disparity of performances in the criteria of an action.

6.4 Robustness analysis

The method used in this dissertation, to the best of our knowledge, has never been used to model a complex real-world situation. As mentioned in Delgado et al., (2004), a model is a simplified version of a part of reality that offers a comprehensible description of a problem situation. Defining a perfect model, that considers all the variables in a specific situation and their influence in the outputs is a utopia, because that influence is very hard to gage, and it is closely related with the reliability of the model. Due to this fact, it was made a robustness analysis. This analysis allows to check the variation of the outputs, when certain input parameters are varied in a specific range. These parameters can be varied individually, or also combined (Qureshi et al., 1999). The DM's perception of the influence of certain parameters on the model output may even lead him/her to change the model itself.

Due to the high number of parameters contemplated in this model, it was chosen three specific ones to be modified individually and combined. Two of them concerned the SRF weighing procedure, which estimates the values of the criteria' weights. The variations were only applied in the process of determining the weights of the non-elementary criteria. The other parameter is the cutting levels' values associated with each non-elementary criteria in the model. The parameters and the range chosen to vary them were the same as those used by Rocha et al., (2021). The reason for this choice was to allow a comparison, with the obvious limitations, between the method used in this dissertation and the one used in that paper. The parameters and their variations are described below:

SRF weighing procedure

- Variation of the ratio Z. The original value of Z is 10, and this value was varied to 9 and 11;

- Variation of the number of empty cards between specific subcriteria:

  - **Scenario A**: The number of empty cards between the subcriteria g_5 and g_1 was modified to 2, instead of the original 3;

  - **Scenario B:** The number of empty cards between the subcriteria g_4 and g_5 was modified to 4, instead of the original 3;

  - **Scenario C:** Scenario A and Scenario B were combined together.

Cutting levels' values

- Variation of the cutting level' value of the root criterion to $0{,}54$ and $0{,}66$, that is, respectively, 10% less and more than the original value. The ELECTRE TRI-nC method with MCHP requires the setting of values for the cutting levels in each non-elementary criterion. In order to change the set for the root criterion, it was changed in the same proportion the values of each subcriteria g_j,with $j = 1, \dots, 5$.

Table **6.13** displays the results from the sensitivity analysis. The darkest row with data represents the assignment procedure results with the data defined by the DM. Every time that there was a variation in the percentage of hospitals assigned to a certain categories' range regarding the percentage in the darkest row, it was inserted in that cell that percentual variation.

Table 6. 13 - Assignment procedure at the comprehensive level for each possible parameters' variations

			Categories' Range						
Value of Z	Root Criterion Cutting Level's Value	Weight's Scenario	C_2 – C_2	C_2 – C_4	C_3 – C_3	C_3 – C_4	C_4 – C_4	C_4 – C_5	C_5 – C_5
Z = 10	CL = 0,6	Original	8%	4%	60%	12%	12%	0%	4%
		A	8%	4%	60%	12%	12%	0%	4%
		B	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
		C	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
	CL = 0,54	Original	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		A	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		B	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		C	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
	CL = 0,66	Original	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		A	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		B	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		C	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
Z = 9	CL = 0,6	Original	8%	4%	60%	12%	12%	0%	4%
		A	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
		B	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
		C	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
	CL = 0,54	Original	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		A	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		B	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		C	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
	CL = 0,66	Original	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		A	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		B	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		C	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
Z = 11	CL = 0,6	Original	8%	4%	60%	12%	12%	0%	4%
		A	8%	4%	60%	12%	12%	0%	4%
		B	8%	4%	60%	12%	12%	0%	4%
		C	8%	4%	60%	12%	12%	4% (+4%)	0% (-4%)
	CL = 0,54	Original	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		A	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		B	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
		C	8%	0% (-4%)	64% (+4%)	12%	12%	0%	4%
	CL = 0,66	Original	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		A	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		B	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)
		C	8%	4%	52% (-8%)	20% (+8%)	12%	4% (+4%)	0% (-4%)

To observe the impact that each varied parameter has on the results of the assignment procedure of the method, they will be analyzed individually and then combined:

Variation of the parameter Z:

- The variation of parameter Z did not imply any change in the assignment procedure. This confirms that even if the DM was in doubt when choosing one of the three values considered, this choice would have no impact on the result of the method.

Variations of the number of empty cards (cells filled in orange color):

- There were no alterations registered by implementing Scenario A;

- In Scenario B and C, the same change occurred, the only hospital (a_2) assigned to C_5 with the original parameters' value were assigned to the interval $[C_4; C_5]$. This reflects a variation of 4% in each of these scenarios;

- In total, there was a variation of $2{,}67\%$ in the results of the assignment procedure by applying the three scenarios defined individually.

Variation of the cutting levels' values (cells filled in blue color):

- Implementing a root criterion cutting level's value equal to 0.54, and the correspondent modifications in the descending subcriteria, resulted in a 4% variation of the assignment procedure results. The hospital a_{18} was assigned to a unique category, C_3, instead of the interval $[C_2; C_4]$ that was achieved with a root criterion cutting level's value equal to 0.6;

- Modifying the root criterion cutting level's value to 0.66 has resulted in a 12% variation in the assignment procedure's results. Two hospitals (8%), a_5 and a_{11} were assigned to the interval $[C_3; C_4]$, instead of the unique category, C_3, that resulted from the assignment procedure with a root criterion cutting level's value to 0.6. The other hospital that was assigned was a_2, and the modification was the same that occurred in Scenario B and C.

- In total, four hospitals were assigned to a different categories' range, which means that there was a variation of 8% in the output of the method by applying a modification of $\pm 10\%$ in the cutting levels' values of all non-elementary criteria in the model. It is also interesting to notice that a decrease in the cutting levels' values as resulted in a higher percentage of hospitals assigned to a unique category, while an increase in that value resulted in a 12% reduction of that percentage. One possible interpretation of this fact is that the cutting levels' values defined by the DM could be reduced to have a stronger assignment relation.

The cells filled in green represent a variation resulting from combining a variation in the cutting levels' parameter and the scenarios defined by the change of empty cards. The cells filled in yellow represent a variation resulting from combining all three parameters' variation established.

The number of assignment procedures performed by combining all the previously established parameters' variation and the individual variation was 36, which includes the original one. This number of tests resulted in the assignment of 900 hospitals to a specific categories' range. That number becomes 875 when excluding the assignment made with the original parameters' values. From 875 hospitals, only 54 were assigned to a different categories' range, which is equivalent to a 6.17% variation of the method's output. This number highlights the robustness of this model.

6.5 Comparison of results

As previously mentioned, the model defined in this dissertation was based on the one presented by Rocha et al. (2021). Despite the obvious limitations, arising from the adaptations that had to be made in the model for it to be applied in the method proposed in this dissertation, it is relevant to observe the differences between the results obtained with the two different methods. Those results concern the year of 2018, and they can only be compared by considering the assignment of hospitals into a categories' range at the comprehensive level. In Rocha et al. (2021) this assignment was made by considering two different perspectives, the descending view and the ascending view. Those views where considered due to the way the scales of the criteria belonging to the first level of the hierarchical tree were defined. The descending view can be judged as the most optimistic of the two.

The results of the assignment procedure were considerably different between the two different approaches. In fact, only 28% and 20% of the hospitals were assigned to the same categories' range, when comparing the results obtained in this dissertation and the ones obtained in the upper-level and lower-level view in Rocha et al. (2021). There is a greater similarity between the results obtained in the descending view with the ones obtained with the ELECTRE TRI-nC method with MCHP, which is justified, not only because there are more hospitals assigned to the same categories' range than in the descending view, but also because 20% of the ones that are not in that list were assigned to an adjacent categories' range. It is also interesting to observe that the two hospitals with the worst assignment results at the comprehensive level in the method proposed in this work, were also two of the worst three in the method used by Rocha et al. (2021). Furthermore, the only hospital assigned to category C_5 in that research paper, was also the *Centro Hospitalar da Póvoa do Varzim/Vila do Conde*. These informative output reveal a certain degree of similarity between the results obtained in both methods.

As the robustness analysis made in this dissertation is equal to the one presented by Rocha et al. (2021) it is meaningful to compare the results. Nevertheless, one can notice, that since that research paper considered two views in the years 2017 and 2018 in its model, the number of tests were considerably higher. Therefore, the percentages of variations is the only variable adjusted to compare the output of the robustness analysis. The one performed in this dissertation resulted in a 6.17% variation of the method's output, while the one performed by Rocha et al. (2021) resulted in a variation of 8%. These data reveal the high level of robustness in both cases.

6.6 Summary

This chapter presented firstly a brief contextualization of the Portuguese healthcare system, and more precisely the NHS. In Subchapter **6.2** it was made a full description of the model used in this dissertation, which was based in the one presented by Rocha et al. (2021). Subchapter **6.3** presents the results from implementing the model into the *CP* developed to apply the ELECTRE TRI-nC method with MCHP, and their interpretation. The robustness analysis of the model, and the conclusions resulting from it, were then described in Subchapter **6.4**. Lastly, it was presented a brief comparison between the results obtained in this dissertation, and the ones by Rocha et al. (2021).

Chapter 7: Conclusions

The Portuguese health sector is characterized by having an NHS that ensures health care to the entire population. The recent transformations that have occurred in this sector, particularly those carried out during the last financial crisis, led to a change in the health management policy. The focus on reducing costs and improving efficiency is the pinnacle of these changes, which led to cuts in the NHS funding. The measures taken had a negative impact on the NHS, which was evidential by several indicators linked to public health at the end of the external intervention period (2015). This decline in quality and access to healthcare, led to a paradigm shift adopted by the government, which introduced new measures to improve the efficiency and quality of the health care services provided to the population, without jeopardizing the sustainability of the system. Despite some good indicators, the short temporal analysis does not allow to withdraw major conclusions. Nevertheless, in order to do so, it is crucial to have adequate tools to accurately measure different indicators, or even groupings of several.

The public hospitals are the most important secondary care providers of the NHS, making their evaluation in terms of quality and access a central theme. However, making a general assessment considering all the levels underlying these major criteria is not enough. In order to obtain a detailed analysis, it is imperative to evaluate each hospital at the comprehensive level, but also considering specific segments, thus enabling identifying which areas need intervention, to increase their overall performance, and consequently, the performance of the Portuguese public hospitals.

The high number of criteria to be analyzed, and the complexity and subjectivity inherent to the concepts of quality and access in health care led to the choice of using an MCDA approach to analyze this situation, and within that approach, the selection of the ELECTRE TRI-nC method with MCHP. Following an interactive approach helps defining complex situations, and capturing the DM´s preferences, which leads to the creation of a specific model that represents his/her perception of the situation. Since this problem, as most real-life ones, is better represented with a hierarchical structure of criteria, it is fundamental to have a process that enables this type of structure. Thus, facilitating the DM's task in defining the model and allow the assessment of the quality level considering specific subsets of the criteria, or all, hence the use of the MCHP.

This research work intends to evaluate the quality and access to health care in the Portuguese public hospitals by using an MCDA approach. The conclusions withdrawn from this study have to be based on the current state of health in Portugal and the method considered. To implement the ELECTRE TRI-nC with a MCHP a computational program was developed with a general format. This program is based on an *Excel* file and a programming code written in *python* to execute the method's procedure based on the model inserted into that file. The developed program allows to apply the proposed method to any model with compatible input data, which includes a hierarchic structure of criteria. The method's framework and the *CP* procedure are characterized in Chapter **5**.

With the objective of evaluating the quality and access to health care in the Portuguese public hospitals, based on the study data from Rocha et al. (2021), it was applied the same model defined in that research paper into the computational program developed. That model was also applied into the ELECTRE TRI-nC method, however, and although the criteria are represented in a hierarchical structure, a hierarchical process was not applied by Rocha et al. (2021). Instead, another ELECTRE method (ELECTRE TRI-C) was used to determine the scales of criteria at the higher levels based on the scales of criteria at the lower levels. The differences between the approach proposed in this dissertation and the previous one, led to some adjustments of the model's input data.

The model suggested is composed of 25 public secondary healthcare providers, which were evaluated according to a hierarchical tree of criteria composed by five criteria in the first level, and 24 in the second level. These last criteria have preference and indifference thresholds associated with them, so as to consider the imperfect character of the data and the arbitrariness when defining the entire set of criteria (Almeida-Dias et al., 2010). Five criteria in the second level also contain a veto threshold associated with them, which is utmost useful in the majority of problems in the healthcare sector, since a considerable poor performance of an action in a specific "life threatening" criterion can jeopardize the possibility of that action being better than another. The weight of each criterion was computed through the use of the SRF weighing procedure, that was also adapted to be applied in a problem that has a hierarchic structure of criteria. As Rocha et al. (2021), the source of the performances of the public hospitals was the ACSS benchmarking database, which concerns the year of 2018. The model also included a set of five pre-ordered categories ($C = \{C_1, C_2, C_3, C_4, C_5\}$), that represent a certain level of quality and access. Each category is characterized by one or more reference actions, which ultimately represent a fictitious hospital that fits well into that category.

The execution of the computational program developed in this dissertation allowed to apply the ELECTRE TRI-nC method with MCHP in the model described. The output of the method was the assignment of each public hospital into one or more continuous categories by considering each criterion in the first level individually, or all of them combined. These results, which are presented in Section **6.3.3,** highlight the lack of public hospitals with high levels of quality and access at the comprehensive level. Only 4% of the set of hospitals considered were assigned to the maximum level of quality and access (C_5) at the comprehensive level. From that said set, 8% and 60% of hospitals were assigned to category C_2 (poor level of quality and access) and C_3 (neutral level of quality and access) at the comprehensive level, respectively, proving the previously mentioned conclusion.

The assignment of the hospitals by considering only a specific criterion in the first level of the hierarchy has also revealed that there is not a single hospital assigned to C_5 in the *Access* or *Efficiency* criteria. In fact, 84% of the hospitals were assigned to a category equal to or lower than C_3 considering the criterion *Access*, which is congruent with the worsening of access to health care services observed due to structural reforms adopted during the external intervention period (Nunes et al., 2019). The *Efficiency* criterion is based on the hospitals' expenses. The fact that 24% of the hospitals were assigned to a category equal to or lower than C_2 in this criterion reveals that the strategies adopted in the next upcoming years (to improve the quality and access to the public hospitals), must be implemented without compromising the economic sustainability of the system.

The possibility of assigning hospitals not only at the comprehensive level, but also considering specific criteria in the hierarchy, has proven to be very useful to observe disparities between the assignment results of different hospitals. These disparities are considerable in this case study, as they reveal the inefficient benchmarking process that is being made in this sector. However, the exact feature that allowed to observe these disparities can also be one of the solutions to the problem. The possibility of knowing the assignment of each hospital in a specific criterion can be used to position the best hospital in that criterion as a benchmarking for the others. A complementary study can be made to compartmentalize the expenses of a certain hospital per criterion. Despite the obvious difficulties of this process, it would allow to compare the level of expenses in a certain criterion with the assignment results obtained in that same criterion, for any hospital. The results would allow to conclude which is the most cost-efficient hospital considering a certain criterion, thus becoming meaningful to study the behaviors and procedures carried out there, to try to replicate them in the most inefficient ones.

The changes made to revitalize the NHS after the external intervention period are still too recent to withdraw major conclusions. Nevertheless, it is noteworthy that the expenditures in the healthcare area are increasing at a higher pace than the GDP, which may be even more noticeable in the years to come, due to the impact of the COVID-19 pandemic. This major concern highlights the importance of having different “measuring” tools, as the one presented in this dissertation, complemented by financing tools, to allow to control the level of quality of different components of the NHS. Thus, making it possible to seek to raise this level of quality in a sustainable and financially efficient way.

To evaluate the robustness of the model, and the conclusions withdrawn, an analysis of different scenarios were made. The input parameters modified were the cutting level’s values and two others, related to the SRF weighing procedure, that influence the criteria’ weights. In total, 35 new models were tested and 875 hospitals were assigned to one or more categories at the comprehensive level. From those, only 54 (6.3%) were assigned to a different category, when comparing to the original results. This percentage reenforces the robustness of the model to its inputs’ imperfect knowledge.

Despite that robustness there is always limitations associated with developing a model and applying it into a specific method. In this case, one of them is the inability of the computational program to consider the effects os possible interactions between the criteria. Another limitation is the need to reformulate the *python* code whenever the hierarchical tree of criteria has more than three levels, or there are more than ten levels of importance when applying the SRF weighing procedure.

In terms of future research, since the use of the ELECTRE TRI-nC method with MCHP to assess the public hospitals level of quality and access has proven to be an adequate tool, its use in this sector could be explored in different contexts to take advantage of its flexibility. An interesting possibility would be its application in a context focused only on the economic aspect considering the structure of costs in the public hospitals, since the unsustainable increase in NHS expenses is one of the major problems in this sector. Nevertheless, it will also be interesting to study the insertion of a new feature in the method that allows to consider the economic view in a problem without interpreting it as a criterion, when there is the opportunity of compartmentalizing costs for each criterion in the case study. In this way, it will be possible to consider those costs directly in the assignment procedure, allowing for two actions with the same performances in all elementary criteria descending from a specific subcriterion, but with completely different level of costs associated, to be assigned to different categories considering that said subcriterion.

Bibliography

Aday, L. A., & Andersen, R. M. (1981). Equity of Access to Medical Care: A Conceptual and Empirical Overview. *Medical Care*, *19*(12), 4–27. http://www.jstor.org/stable/3763937

Allen, R. G. D. (1934). The Nature of Indifference Curves. *The Review of Economic Studies, 1(2)*, 110. https://doi.org/10.2307/2967617

Almeida-Dias, J., Figueira, J. R., & Roy, B. (2010). ELECTRE TRI-C: A multiple criteria sorting method based on characteristic reference actions. *European Journal of Operational Research*, *204*(3), 565–580. https://doi.org/10.1016/j.ejor.2009.10.018

Almeida-Dias, J., Figueira, J. R., & Roy, B. (2012). A multiple criteria sorting method where each category is characterized by several reference actions: The ELECTRE TRI-nC method. *European Journal of Operational Research, 217(3)*, 567–579. https://doi.org/10.1016/j.ejor.2011.09.047

Andrews, R., Beynon, M. J. & McDermott, A. (2019). Configurations of New Public Management reforms and the efficiency, effectiveness and equity of public healthcare systems: A fuzzy-set Qualitative Comparative Analysis. *Public Management Review,* 1–25. https://doi.org/10.1080/14719037.2018.1561927

Article 64.º of the Portuguese Constitution, of 10th April 1976. Republic Journal: I series, nº 86/1976. Lastly modified by the article 37.º of the constitutional law nº1/97 (Republic Journal: I-A series, nº 218/1997 of 20th September 1997). Accessed on 4th March 2021. Available via: https://dre.pt/web/guest/legislacao-consolidada/-/lc/337/201903120346/73938588/diploma/indice

Barros, P. P. (2012). Health policy reform in tough times : The case of Portugal. *Health Policy*, *106*(1), 17–22. https://doi.org/10.1016/j.healthpol.2012.04.008

Belton, V., & Stewart, T. J. (2002). Outranking Methods. *Multiple Criteria Decision Analysis*, 233–259. Springer, Boston, MA. https://doi.org/10.1007/978-1-4615-1495-4_8

Betrán, A. P., Merialdi, M., Lauer, J. A., Bing-Shun, W., Thomas, J., Van Look, P., & Wagner, M. (2007). Rates of caesarean section: Analysis of global, regional and national estimates. *Paediatric and Perinatal Epidemiology, 21(2)*, 98–113. doi:10.1111/j.1365-3016.2007.00786.x

Betrán A. P., Ye J., Moller A.-B., Souza J.P., Zhang J. (2021). Trends and projections of caesarean section rates: Global and regional estimates. *British Medical Journal, Global Health, 6(6), e005671.* doi:10.1136/bmjgh-2021-005671

Bezdek, J. C., Spillman, B., & Spillman, R. (1978). A fuzzy relation space for group decision theory. *Fuzzy Sets and Systems, 1(4)*, 255–268. https://doi.org/10.1016/0165-0114(78)90017-9

Bouyssou, D., & Marchant, T. (2015). On the relations between ELECTRE TRI-B and ELECTRE TRI-C and on a new variant of ELECTRE TRI-B. *European Journal of Operational Research, 242(1)*, 201–211. https://doi.org/10.1016/j.ejor.2014.09.057

Brogi, A., Mancarella, P., Pedreschi, D., & Turini, F. (1994). Modular logic programming. *Associating for Computing Machinery, Transactions on Programming Languages and Systems, 16(4)*, 1361–1398. https://doi.org/10.1145/183432.183528Carlon, G. C., & Combs, A. H. (2005). If you cannot measure it, you cannot improve it. *Critical Care Medicine, 33(5)*, 1146–1147. https://doi.org/10.1097/01.CCM.0000162564.77999.F8

Choo, E. U., Schoner, B., & Wedley, W. C. (1999). Interpretation of criteria weights in multicriteria decision making. *Computers & Industrial Engineering, 37(3)*, 527–541. https://doi.org/10.1016/s0360-8352(00)00019-x

Corrente, S., Doumpos, M., Greco, S., Słowiński, R., & Zopounidis, C. (2015). Multiple criteria hierarchy process for sorting problems based on ordinal regression with additive value functions. *Annals of Operations Research, 251(1-2)*, 117–139. https://doi.org/10.1007/s10479-015-1898-1

Corrente, S., Figueira, J. R., & Greco, S. (2014). The SMAA-PROMETHEE method. *European Journal of Operational Research, 239(2)*, 514–522. https://doi.org/10.1016/j.ejor.2014.05.026

Corrente, S., Figueira, J. R., Greco, S., & Słowiński, R. (2017). A robust ranking method extending ELECTRE III to hierarchy of interacting criteria, imprecise weights and stochastic analysis. *The International Journal of Management Science 73*, 1–17. https://doi.org/10.1016/j.omega.2016.11.008

Corrente, S., Greco, S., & Słowiński, R. (2012). Multiple Criteria Hierarchy Process in Robust Ordinal Regression. *Decision Support Systems, 53(3)*, 660–674. https://doi.org/10.1016/j.dss.2012.03.004

Corrente, S., Greco, S., & Słowiński, R. (2013). Multiple Criteria Hierarchy Process with ELECTRE and PROMETHEE. *Omega - The International Journal of Management Science*, *41(5)*, 820–846. https://doi.org/10.1016/j.omega.2012.10.009

Corrente, S., Greco, S., & Słowiński, R. (2016). Multiple Criteria Hierarchy Process for ELECTRE TRI methods. *European Journal of Operational Research , 252,* 191-203. https://doi.org/10.1016/j.ejor.2015.12.053

Dawes, R. M., & Corrigan, B. (1974). Linear models in decision making. *Psychological Bulletin, 81*(2), 95–106. https://doi.org/10.1037/h0037613

Delgado, M. G., & Sendra, J. B. (2004). Sensitivity Analysis in Multicriteria Spatial Decision-Making: A Review. *Human and Ecological Risk Assessment: An International Journal, 10(6)*, 1173–1187. doi:10.1080/10807030490887221

Escoval, A., Santos, A. T. L. & Barbosa, P. (2016). Contributo para a compreensão do financiamento de saúde em Portugal: O caso dos cuidados de saúde primários, dos cuidados hospitalares e dos cuidados continuados. *Tempus, 10(1)*, 17-27. https://doi.org/10.18569/tempus.v10i1.1858

Fernandes, A. C., & Nunes, A. M. (2016). Os Hospitais e a Combinação Público-Privado no Sistema de Saúde Português. *Acta Med Port, 29(3)*, 217-223. https://doi.org/10.20344/amp.6712

Ferreira, D. C., & Marques, R. C. (2020). Public-private partnerships in health care services - Do they outperform public hospitals regarding quality and access - Evidence from Portugal. *Socio-Economic Planning Sciences, 100798*. https://doi.org/10.1016/j.seps.2020.100798

Fetter, R. B. (1991). Diagnosis Related Groups: Understanding Hospital Performance. *INFORMS Journal on Applied Analytics 21(1)*, 6-26. https://doi.org/10.1287/inte.21.1.6

Figueira, J. R., Greco, S., & Roy, B. (2009a). ELECTRE methods with interaction between criteria: An extension of the concordance index. *European Journal of Operational Research, 199(2)*, 478–495. https://doi.org/10.1016/j.ejor.2008.11.025

Figueira, J. R., Greco, S., & Słowiński, R. (2009b) Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. *European Journal of Operational Research 195 (2)*, 460–486. https://doi.org/10.1016/j.ejor.2008.02.006

Figueira, J. R., Mousseau, V., & Roy, B. (2016). ELECTRE methods. *International Series in Operations Research and Management Science (Vol. 233)*. Springer, New York. https://doi.org/10.1007/978-1-4939-3094-4_5

Figueira, J. R., & Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research, 139(2)*, 317–326. https://doi.org/10.1016/s0377-2217(01)00370-8

Figueira, J. R., Greco, S., Roy, B., Słowiński, R., (2010) ELECTRE methods: main features and recent developments. *Handbook of multicriteria analysis*, 51–89. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-92828-7_3

Fronteira, I., Simões, J. & Augusto, G. F. (2018). Looking back at the Portuguese crisis: what legacy for the Portuguese NHS. https://doi.org/10.25761/anaisihmt.250

Oliveira, M.D. and Pinto, C.G. (2005), Health care reform in Portugal: an evaluation of the NHS experience. *Health Economics, 14(S1*), 203–220*.* https://doi.org/10.1002/hec.1037

Govindan, K., & Jepsen, M. B. (2016). ELECTRE: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research, 250(1)*, 1–29. https://doi.org/10.1016/j.ejor.2015.07.019

Greco, S., Kadziński, M., Mousseau, V., & Słowiński, R. (2011). ELECTRE^{GKMS}: Robust ordinal regression for outranking methods. *European Journal of Operational Research, 214(1)*, 118–135. https://doi.org/10.1016/j.ejor.2011.03.045

Greco, S., Mousseau, V., & Słowiński, R. (2008). Ordinal regression revisited: Multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research, 191(2)*, 416–436. https://doi.org/10.1016/j.ejor.2007.08.013

Guitouni, A., & Martel, J.-M. (1998). Tentative guidelines to help choosing an appropriate MCDA method. *European Journal of Operational Research, 109(2)*, 501–521. https://doi.org/10.1016/s0377-2217(98)00073-3

Hands, D. W. (1983). Paul Samuelson and Revealed Preference Theory. *History of Political Economy, 46*, 85-116. https://doi.org/10.1215/00182702-2398939

Köksalan, M., Wallenius, J., & Zionts, S. (2016). An early history of multiple criteria decision making. *International Series in Operations Research and Management Science*, *233*, 3–17. https://doi.org/10.1007/978-1-4939-3094-4_1

Laffy, R. (1966): La méthode MARSAN pour la recherche de produits nouveaux. Presented at the ESOMAR congress, Copenhagen (1966).

Lahdelma, R., Hokkanen, J., & Salminen, P. (1998). SMAA - Stochastic multiobjective acceptability analysis. *European Journal of Operational Research, 106(1)*, 137–143. https://doi.org/ 10.1016/s0377-2217(97)00163-x

Law-Decree (Decreto-lei) nº 413/71, of 27th September from Health Ministry. Republic Journal: I series, nº 228/1971. Accessed on 4th March 2021. Available at https://data.dre.pt/eli/dec-lei/413/1971/09/27/p/dre/pt/html

Law-Decree (Decreto-lei) nº 124/2011, of 29th December from Health Ministry. Republic Journal: I series, nº 249/2011. Accessed on 4th March 2021. Available at https://data.dre.pt/eli/dec-lei/124/2011/p/cons/20170109/pt/html

Leape, L. L., & Berwick, D. M. (2005). Five Years After To Err Is Human. *The Journal of the American Medical Association, 293(19)*, *2384*. doi:10.1001/jama.293.19.2384

Mitchell, P., H. (2008). Defining Patient Safety and Quality Care. *Patient Safety and Quality: An Evidence-Based Handbook for Nurses.* Rockville (MD): Agency for Healthcare Research and Quality (US); 2008 Apr. Chapter 1. Available from: https://www.ncbi.nlm.nih.gov/books/NBK2681/

Nunes, A. M. (2018a). Reforms in the Portuguese health care sector : Challenges and proposals. *The International Journal of Health Planning and Management 34(1),* 1–13. https://doi.org/10.1002/hpm.2695

Nunes, A. M. (2018b). The health care reform in Portugal : Outcomes from both the New Public Management and the economic crisis. *The International Journal of Health Planning and Management 34(1),* 196-215. https://doi.org/10.1002/hpm.2613

Nunes, A. M. (2019). Financial Crisis in Portugal : Effects in the Health Care Sector. The International Journal of Health Planning and Management *49(2),* 237–259. https://doi.org/10.1177/0020731418822227

Orlovsky, S. A. (1978). Decision-making with a fuzzy preference relation. *Fuzzy Sets and Systems, 1(3)*, 155–167. https://doi.org/10.1016/0165-0114(78)90001-5

Palmer, S. & Torgerson, D. J. (1999). Economics notes: Definitions of efficiency. *British Medical Journal, 318(7191)*, 1136–1136. doi:10.1136/bmj.318.7191.1136

Penchansky, R. & Thomas, J. W. (1981). The Concept of Access: Definition and Relationship to Consumer Satisfaction. *Medical Care*, *19*(2), 127–140. http://www.jstor.org/stable/3764310

Portuguese Public Finance Council (2020). *Evolução Orçamental do Serviço Nacional de Saúde 2013-2019*. Report N.º 10/2020.

Portuguese Public Finance Council (2021). *Evolução Orçamental do Serviço Nacional de Saúde 2014-2020*. Report N.º 06/2021.

Qureshi, M. E., Harrison, S. R., & Wegener, M. K. (1999). Validation of multicriteria analysis models. *Agricultural Systems, 62(2),* 105–116. doi:10.1016/s0308-521x(99)00059-1

Robson, M. S. (2001). Can we reduce the caesarean section rate? *Best Practice & Research Clinical Obstetrics & Gynaecology, 15(1)*, 179–194. doi:10.1053/beog.2000.0156

Rocha, A., Costa, A. S., Figueira, J. R., Ferreira, D. C., & Marques, R. C. (2021). Quality assessment of the Portuguese public hospitals: A multiple criteria approach. *Omega - The International Journal of Management Science, 105, 102505.* doi:10.1016/j.omega.2021.102505

Rogers, M., Bruen, M., & Maystre, L.-Y. (2000). The ELECTRE Methodology. *ELECTRE and Decision Support*, 45–85. https://doi.org/10.1007/978-1-4757-5057-7_3

Roy, B. (1968) Classement et choix en présence de points de vue multiples (La méthode ELECTRE). *Revue française d'Informatique et de Recherche Opérationnelle, 6(8)*. https://doi.org/10.1051/ro/196802v100571

Roy, B. (1978) ELECTRE III; Algorithme de classement base sur une representation floue des preferences en presence de criteres multiples. *Cahiers de CERO, 20(1)*, 3-24.

Roy, B. (1990). The Outranking Approach and the Foundations of ELECTRE Methods. *Readings in Multiple Criteria Decision Aid*, 155–183. https://doi.org/10.1007/978-3-642-75935-2_8

Roy, B., & Bertier, P. (1971). La méthode ELECTRE II: Une méthode de classesment en présence de critères multiples. *Direction scientifique, Note de travail, 142*. *SEMA (Metra International),* Paris.

Roy, B., Bouyssou, D., 1986. Comparison of two decision-aid models applied to a nuclear power plant siting example. *European Journal of Operational Research 25*, 200-215. https://doi.org/10.1016/0377-2217(86)90086-x

Roy, B., Bouyssou, D. (1993). *Aide Multicritère à la Décision: Méthodes et Cas.* Economica, Paris

Roy, B., Figueira, J. R., & Almeida-Dias, J. (2014). Discriminating thresholds as a tool to cope with imperfect knowledge in multiple criteria decision aiding: Theoretical results and practical issues. *Omega - The International Journal of Management Science*, *43*, 9–20. https://doi.org/10.1016/j.omega.2013.05.003

Roy, B., & Hugonnard, J. C. (1982). Ranking of suburban line extension projects on the Paris metro system by a multicriteria method. *Transportation Research Part A: General, 16(4)*, 301–312. https://doi.org/10.1016/0191-2607(82)90057-7

Roy, B., & Skalka, J. (1987). ELECTRE IS: Aspects méthodologiques et guide d'utilisation. Document du LAMSADE 30, Université Paris Dauphine (1984).

Samuelson, P. A. (1938). A Note on the Pure Theory of Consumer's Behaviour. *Economica, 5(17)*, 61. https://doi.org/10.2307/2548836

Sanmartin, C., Murphy, K., Choptain, N., et al. (2008). Appropriateness of healthcare interventions: Concepts and scoping of the published literature. *International Journal of Technology Assessment in Health Care, 24(3)*, 342–349. doi:10.1017/S0266462308080458

Schoner, B., & Wedley, W. C. (1989). Ambiguous Criteria Weights in AHP: Consequences and Solutions. *Decision Sciences, 20(3),* 462–475. https://doi.org/10.1111/j.1540-5915.1989.tb01561.x

Shaaban, A. N., Peleteiro, B. & Martins, M. R. O. (2020). COVID-19: What is next for Portugal? *Fronties in Public Health*. https://doi.org/10.3389/fpubh.2020.00392

Simões, J. A., Augusto, G. F., Fronteira I., Hernández-Quevedo, C. (2017). Portugal: Health system review. *Health systems in transition*, *19(2)*, 1–184. https://apps.who.int/iris/handle/10665/330211

Simos, J., 1990a. L'évaluation environnementale: Un processus cognitif négocié. Thèse de doctorat, DGF-EPFL, Lausanne.

Simos, J., 1990b. Evaluer l'impact sur l'environnement: Une approche originale par l'analyse multicritère et la négociation. Presses Polytechniques et Universitaires Romandes, Lausanne.

Stiglitz, J. E. (1981). Pareto Optimality and Competition. *The Journal of Finance, 36(2)*, 235–251. https://doi.org/10.1111/j.1540-6261.1981.tb00437.x

Tavares, A. I., & Marques, I. (2020). Multi-layer health insurance coverage, medical services use and health in a Universal National Health System, the case of Portugal. *The European Journal of Health Economics*, *22*, 141–153. doi:10.1007/s10198-020-01242-4

Villar, J., Carroli, G., Zavaleta, N., Donner, A., Wojdyla, D., Faundes, A., Acosta, A. (2007). Maternal and neonatal individual risks and benefits associated with caesarean delivery: Multicenter prospective study. *British Medical Journal*, *335(7628),* 1025. doi:10.1136/bmj.39363.706956.55

Weil, D. N. (2014). Health and Economic Growth. *Handbook of Economic Growth (Vol. 2). Elsevier B.V*. https://doi.org/10.1016/B978-0-444-53540-5.00003-3

Williams, A. (1988). Priority setting in public and private health care. *Journal of Health Economics, 7(2),* 173–183. *doi:10.1016/0167-6296(88)90015-x*

Williams, A. (1998) Quality and care: patients' perceptions. *Journal of Nursing Care Quality. 12(6)*, 18–25. https://doi.org/10.1097/00001786-199808000-00006

Yu, W., 1992. Aide Multicritere a la Decision dans le Cadre de la Problematique du Tri: Concepts Methodes et Applications. These de Doctorat, LAMSADE, Universite Paris-Dauphine, Paris, France.

Zanakis, S. H., Solomon, A., Wishart, N., & Dublish, S. (1998). Multi-attribute decision making: A simulation comparison of select methods. *European Journal of Operational Research, 107(3)*, 507–529. https://doi.org/10.1016/s0377-2217(97)00147-1

Appendix A

Separability conditions of the ELECTRE TRI-nC

This appendix defines the necessary conditions of the ELECTRE TRI-nC method to ensure that two consecutive categories (B_{h+1} and B_h) are distinct, the dominance condition and the weak separability condition.

Dominance condition: The set of reference actions, B, fulfills the dominance condition if, and only if, $\forall j,\ g_j(b_{h+1}^s) - g_j(b_h^r) \geq 0, s = 1, \dots, m_{h+1};\ r = 1, \dots, m_h; h = 1, \dots, (q-1)$.

Condition 1 (weak separability): The set of reference actions, B, fulfills the weak separability condition if, and only if: it respects the dominance condition and, at least in one criterion, each reference action belonging to B_{h+1} is weakly preferred to each reference action belonging to B_h, $\sigma(b_h^r, b_{h+1}^s) < 1,\ r = 1, \dots, m_h; s = 1, \dots, m_{h+1}; h = 1, \dots, (q-1)$.

It is possible to replace the weak separability condition for one of the two additional stronger conditions.

Condition 2 (strict separability): The set of reference actions, B, fulfills the weak separability condition if, and only if: it respects the dominance condition and, at least in one criterion, each reference action belonging to B_{h+1} is weakly preferred to each reference action belonging to B_h, $\sigma(b_h^r, b_{h+1}^s) < \frac{1}{2},\ r = 1, \dots, m_h; s = 1, \dots, m_{h+1}; h = 1, \dots, (q-1)$.

Condition 3 (hyper-strict separability): The set of reference actions, B, fulfills the weak separability condition if, and only if: it respects the dominance condition and, at least in one criterion, each reference action belonging to B_{h+1} is weakly preferred to each reference action belonging to B_h, $\sigma(b_h^r, b_{h+1}^s) = 0,\ r = 1, \dots, m_h; s = 1, \dots, m_{h+1}; h = 1, \dots, (q-1)$.

Appendix B

Coherence properties and Theorem 1.1

Appendix B presents two coherence properties required to impose a logical relation between the criteria, and a theorem that must be respected to ensure that those properties hold.

Considering a subcriterion G_r, $r \in L_{\check{G}} \setminus \{EL, LBO\}$, and the subcriteria descending from it:

(C1) if $aS_{(r,j)}b, j = 1, \ldots, n(r)$, then $aS_r b$,

(C2) if $not(aS_{(r,j)}b), j = 1, \ldots, n(r)$, then $not(aS_r b)$.

They impose that if in all the criteria descending from G_r in its subsequent level, "a outranks b", or "a does not outrank b", then, respectively, "a outranks b on criterion G_r" **(C1)**, or "a does not outranks b on criterion G_r" **(C2)**. It is crucial that the outranking relationships previously defined respect these two properties. Since the way to establish them is through a comparison with the cutting level, it seems logical to define some conditions regarding this particular variable to ensure that the two properties hold. Theorem 1.1 presents those conditions.

Before presenting the theorem, it is relevant to recap an important notion about the criteria and the elementary concordant index. As in the original ELECTRE TRI-nC, this method contemplates the possibility of having some elementary quasi-criteria, which means the indifference threshold associated with the criterion, q_j, is equal to the preference threshold, p_j. Through the observation of the elementary concordance index equation, ϕ_j, it is possible to verify that if a certain elementary criterion g_j is a quasi-criterion, then the value of the concordance elementary index is $\{0,1\}$. If the elementary criterion is a pseudo-criterion, then that value could be any value in the range $[0, 1]$.

Theorem 1.1

Considering a criterion G_r, with G_r, $r \in L_{\check{G}} \setminus \{EL, LBO\}$, for each $j = 1, \ldots, n(r)$:

- Set $YS_{(r,j)} = \{ \sum_{t \in A \subseteq E(G_{(r,j)})} w_t \phi_t : \sum_{t \in A \subseteq E(G_{(r,j)})} w_t \phi_t \geq \lambda_{(r,j)} \}$,

- $m_{(r,j)} = \min YS_{(r,j)}$.

The objective of this part of the theorem is to compute the minimum of the set of the reachable concordance index $C_{(r,j)}$ for each criterion $G_{(r,j)}$, value that must not be smaller than the cutting level $\lambda_{(r,j)}$ associated. Variable $m_{(r,j)}$ represents that value. The elementary criteria descending from subcriterion $G_{(r,j)}$, can be quasi-criterion, pseudo-criterion, or a mix of the two. This will influence the range of values of the elementary concordance index for each of them. Considering that the value of this index will be multiplied by the intrinsic weight of that elementary criterion, it is required a simple process of summing the range of values resulting from that operation, to find the reachable range of values of the concordance index $C_{(r,j)}$ of the subcriterion $G_{(r,j)}$. If the value of the cutting level, $\lambda_{(r,j)}$, can be reached in any of possible ranges of values of $C_{(r,j)}$, then $m_{(r,j)}$ will be equal to this value, otherwise it will be equal to the minimum value that can be reached, with this value being higher than the cutting level. The second part of the theorem states that, considering the same criterion G_r, $r \in \mathrm{L}_{\check{G}} \setminus \{EL \cup LBO\}$, for each $j = 1, \ldots, n(r)$:

- Set $NS_{(r,j)} = \{ \sum_{t \in B \subseteq E(G_{(r,j)})} w_t \, \Phi_t : \sum_{t \in A \subseteq E(G_{(r,j)})} w_t \, \Phi_t < \lambda_{(r,j)} \}$.

- $M_{(r,j)} = \sup MS_{(r,j)}$.

The objective of this part of the theorem is to compute the supremum of the set of the reachable concordance index $C_{(r,j)}$ for each criterion $G_{(r,j)}$, value that must be smaller than the cutting level $\lambda_{(r,j)}$ associated. Variable $M_{(r,j)}$ represents that value. The explanation for the value of $M_{(r,j)}$ is analogously to the one presented for the $m_{(r,j)}$. The last part of this theorem is stated as follows:

- $m_r = \sum_{j=1}^{n(r)} m_{(r,j)} \wedge M_r = \sum_{j=1}^{n(r)} M_{(r,j)}$,

Then properties **(C1)** and **(C2)** hold iff:

$$\begin{cases} M_r < \lambda_r < m_r & if \;\; M_{(r,j)} = \max NS_{(r,j)}, \;\; for\ all\ j = 1, \ldots, \quad n(r), \\ M_r \leq \lambda_r \leq m_r & if \; M_{(r,j)} = \sup NS_{(r,j)} = \lambda_{(r,j)} \;\; for\ at\ least\ one\ j \in \{1, \ldots, n(r)\} \end{cases}$$

The last part states that if the value of the cutting level of the criterion G_r is between the interval of the sum of all $m_{(r,j)}$, and $M_{(r,j)}$ descending from that criterion, then properties **(C1)** and **(C2)** hold. It is very important to notice that this theorem also clarifies the possibility of estimating the value of the cutting level of a certain subcriterion G_r, when the values of the cutting levels of all subcriterion $G_{(r,j)}$ descending from it are defined.

There is a particular case that simplifies the verification of the coherence properties when considering the cutting levels' values, that occurs when all elementary criteria in the model are pseudo-criteria. In this case it is possible to conclude that properties **(C1)** and **(C2)** hold iff:

$$\lambda_r = \sum_{j=1}^{n(r)} \lambda_{r,j} \;\; for\ all\ r \in \mathrm{L}_{\check{G}} \setminus \{EL \cup LBO\}.$$

The veto threshold has a high importance in ELECTRE TRI-nC method, and in its way of defining an outranking relation. Even in this new method, two of the three outranking relations previously established, **(OR2)** and **(OR3)**, are defined considering the effect of the veto threshold. Nevertheless, this theorem do not mention it in any stage, creating only constraints in the values of the cutting levels. This happens because the way in which the veto threshold conditions the outranking relation of two actions in any of the levels of the hierarchy, is restrictive enough to ensure that its own form respect the two coherence properties presented. The following fully explain its role regarding the two properties:

- If $aS_{(r,j)}b$, for all $j = 1, \ldots, n(r)$, then $g_t(b) - g_t(a) < v_t$ for all $t \in \mathrm{E}(\mathrm{G}_{(r,j)})$ and, consequently, $g_t(b) - g_t(a) < v_t$ for all $t \in \mathrm{E}(\mathrm{G}_r)$, implying that $aS_r b$;

- If $not(aS_{(r,j)}b)$ for at least one $j \in \{1, \ldots, n(r)\}$, then $g_t(b) - g_t(a) \geq v_t$ for at least one $t \in \mathrm{E}(\mathrm{G}_{(r,j)})$ and, consequently, $g_t(b) - g_t(a) \geq v_t$ for at least one $t \in \mathrm{E}(\mathrm{G}_r)$, implying that $not(aS_r b)$.

This also clarifies that, in a hierarchy structure, the veto threshold restricts the outranking relations at the comprehensive level, but firstly, in the subsequent levels. This creates the possibility of having an action a not outranking another action b, at the comprehensive level, and, at the same time, having a outranking b in multiple subcriteria $\mathrm{G_r}$. This possibility could be of great interest for the DM. Imagine that a specific action a was not sorted for the next higher category (B_{h+1}) because, in a certain elementary criterion, the differences in performances between that action and one or more characteristic reference actions of the higher category (b_{h+1}^{r} with $r = 1, \dots, m_{h+1}$) were higher than the veto threshold associated with that criterion. In this new method, the DM will have the possibility of analyzing which categories were recommended to receive that action in different areas of the problem (different subcriteria). This possibility will aid the DM to make a deeper study, on the specific characteristics of the different actions, in the different areas of analysis that he/she created, and their impact at the comprehensive level.

Appendix C

Separability conditions of the ELECTRE TRI-nC with MCHP

This appendix defines the existent conditions of the ELECTRE TRI-nC method with MCHP to ensure that two consecutive categories (B_{h+1} and B_h) are distinct. The weak separability condition must always be fulfilled to grant the minimum distinction level accepted. The remaining two conditions are stronger, and can replace the other if that is the wish of the DM.

- **Weak separability condition:** $\sigma_{\mathrm{s}}(b_h^{\mathrm{k}}, b_{\mathrm{h+1}}^{\mathrm{l}}) < W_s$, $\forall\, s \in LBO, k = 1, \dots, m_h, l = 1, \dots, m_{h+1}$, and $h = 1, \dots, p-1$;
- **Strict separability condition:** $\sigma_{\mathrm{s}}(b_h^{\mathrm{k}}, b_{\mathrm{h+1}}^{\mathrm{l}}) < \frac{W_s}{2}$, $\forall\, s \in LBO, k = 1, \dots, m_h, l = 1, \dots, m_{h+1}$, and $h = 1, \dots, p-1$;
- **Hyper-strict separability condition:** $\sigma_{\mathrm{s}}(b_h^{\mathrm{k}}, b_{\mathrm{h+1}}^{\mathrm{l}}) = 0$, $\forall\, s \in LBO, k = 1, \dots, m_h, l = 1, \dots, m_{h+1}$, and $h = 1, \dots, p-1$

Notice that the weight used for the comparison, W_s, is the weight of criterion G_s, that is, the sum of the weight of all criteria descending from G_s. Since $s \in LBO$, and LBO contains all the subcriteria of the last but one level of the hierarchy, which in is hand, ascend from all the elementary criteria, it is logical to conclude that the indices used are enough to ensure that there are separability conditions for the whole hierarchy of criteria.

Appendix D

Separability example

Pyhton Code

Separability example

```
# Define the separability main table
for i in range(start_row_reference, last_row_reference + 1):
    reference_category_1 = ws_reference.cell(i, 1).value
    reference_index_1 = ws_reference.cell(i, 4).value
    for j in range(start_row_reference, last_row_reference + 1):
        reference_category_2 = ws_reference.cell(j, 1).value
        reference_index_2 = ws_reference.cell(j, 4).value
        if reference_category_2 - reference_category_1 == 1:
            for k in range(start_row_criteria, last_row_criteria + 1):
                criteria_main_type = ws_criteria.cell(k, 9).value
                if criteria_main_type == 'LBO':
                    criteria_lbo = ws_criteria.cell(k, 2).value
                    criteria_lbo_weight = ws_criteria.cell(k, 13).value
                    criteria_lbo_index = ws_criteria.cell(k, 7).value
                    criteria_lbo_index_plus_one = criteria_lbo_index + 1
                    for l in range(start_row_criteria, last_row_criteria + 1):
                        criteria_geral_name = ws_criteria.cell(l, 2).value
                        criteria_geral_objective = ws_criteria.cell(l, 16).value
                        criteria_geral_index = ws_criteria.cell(l, 7).value
                        criteria_geral_type = ws_criteria.cell(l, 9).value
                        criteria_geral_qt = ws_criteria.cell(l, 10).value
                        criteria_geral_pt = ws_criteria.cell(l, 11).value
                        criteria_geral_vt = ws_criteria.cell(l, 12).value
                        criteria_geral_wt = ws_criteria.cell(l, 13).value
                        criteria_geral_highest_performance = ws_criteria.cell(l, 15).value
                        if criteria_geral_type == 'EL' and criteria_geral_index > criteria_lbo_index and criteria_geral_index < criteria_lbo_index_plus_one:
                            ws_separability.cell(last_row_separability + 1, 1).value = criteria_lbo_weight
```

Figure D.1 - Portion of the *python* code of the *TransferDataToSeparabilityTable'* function

Table D.1 - Example of the main table defined in the *Separability conditions* sheet

Criteria		Elementary		RP 1		RP 2		Elementary Index		Partial Index		Separability Conditions		
W_s	Criterion	Criterion	w_t	Index	Performance	Index	Performance	$Ø_t$ (rp1, rp2)	d_t (rp1, rp2)	C_s (rp1, rp2)	$σ_s$ (rp1, rp2)	Weak	Strict	Hyper-Strict
0.249	g_1	$g_{1,1}$	0.021	b_5^1	95	b_6^1	100	0	0	0.076	0.059	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,2}$	0.028	b_5^1	95	b_6^1	100	0	0	0.076	0.059	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,3}$	0.076	b_5^1	0	b_6^1	0	1	0	0.076	0.059	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,4}$	0.076	b_5^1	4.3	b_6^1	6	0	0.218	0.076	0.059	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,5}$	0.048	b_5^1	6.4	b_6^1	8	0	0.147	0.076	0.059	TRUE	TRUE	FALSE

W_s	Criterion	Criterion	w_t	Index	Performance	Index	Performance	$Ø_t$ (rp1, rp2)	d_t (rp1, rp2)	C_s (rp1, rp2)	$σ_s$ (rp1, rp2)	Weak	Strict	Hyper-Strict
0.277	g_2	$g_{2,1}$	0.038	b_5^1	90	b_6^1	100	0	0.333	0	0	TRUE	TRUE	TRUE
0.277	g_2	$g_{2,2}$	0.01	b_5^1	5	b_6^1	0	0	1	0	0	TRUE	TRUE	TRUE
0.277	g_2	$g_{2,3}$	0.038	b_5^1	2.7	b_6^1	0	0	1	0	0	TRUE	TRUE	TRUE
0.277	g_2	$g_{2,4}$	0.095	b_5^1	90	b_6^1	100	0	0.053	0	0	TRUE	TRUE	TRUE
0.277	g_2	$g_{2,5}$	0.095	b_5^1	0.5	b_6^1	0	0	0.118	0	0	TRUE	TRUE	TRUE

W_s	Criterion	Criterion	w_t	Index	Performance	Index	Performance	$Ø_t$ (rp1, rp2)	d_t (rp1, rp2)	C_s (rp1, rp2)	$σ_s$ (rp1, rp2)	Weak	Strict	Hyper-Strict
0.249	g_1	$g_{1,1}$	0.021	b_4^1	85	b_5^1	95	0	0.053	0.076	0.061	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,2}$	0.028	b_4^1	85	b_5^1	95	0	0.053	0.076	0.061	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,3}$	0.076	b_4^1	2	b_5^1	0	1	0	0.076	0.061	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,4}$	0.076	b_4^1	2.7	b_5^1	4.3	0	0.2	0.076	0.061	TRUE	TRUE	FALSE
0.249	g_1	$g_{1,5}$	0.048	b_4^1	4.8	b_5^1	6.4	0	0.147	0.076	0.061	TRUE	TRUE	FALSE

Table D.2 - Example of the final table defined in the *Separability conditions* sheet

Subcriteria	Weak Separability	Strict Separability	Hyper-Strict Separability
g_1	TRUE	FALSE	FALSE
g_2	TRUE	TRUE	FALSE
g_3	TRUE	TRUE	FALSE
g_4	TRUE	TRUE	FALSE
g_5	TRUE	TRUE	FALSE

Appendix E

Coherence properties example

In order to better understand the example presented, it will be complemented with four figures. The first figure represents a portion of the *python* code that defines the *Coherence properties'* function.

Python Code

Coherence properties

```
for i in range(start_row_action, last_row_actions + 1):
    action_1_index = ws_actions.cell(i, 3).value
    for j in range(start_row_reference, last_row_reference + 1):
        reference_1_index = ws_reference.cell(j, 4).value
        for k in range(start_row_criteria, last_row_criteria + 1):
            # there is the need to differentitate the conditions' variables since they will be used for the the differents S (3 of them), and then 2 for each one to check if
            number_of_lines_verifying_1_negative = 0
            number_of_lines_verifying_1_positive = 0
            variable_for_condition_1_negative_a_s_b = 0
            variable_for_condition_2_negative_a_s_b = 0
            variable_for_condition_3_negative_a_s_b = 0
            variable_for_condition_1_positive_a_s_b = 0
            variable_for_condition_2_positive_a_s_b = 0
            variable_for_condition_3_positive_a_s_b = 0
            variable_for_condition_1_negative_b_s_a = 0
            variable_for_condition_2_negative_b_s_a = 0
            variable_for_condition_3_negative_b_s_a = 0
            variable_for_condition_1_positive_b_s_a = 0
            variable_for_condition_2_positive_b_s_a = 0
            variable_for_condition_3_positive_b_s_a = 0
            criteria_1_name = ws_criteria.cell(k, 2).value
            criteria_1_type = ws_criteria.cell(k, 9).value
            size_of_criteria_1 = len(criteria_1_name)
            # all the subcriteria without the root one
            # The first cycle is to search and analyze all the relations of the criteria descending from the subcriteria in a certain action and a certain reference profile
            if criteria_1_type == 'Subcriterion':
                for l in range(start_row_assignment, last_row_assignment_fill + 1):
                    criteria_2_name = ws_assignment.cell(l, 1).value
                    if criteria_2_name != None and criteria_2_name != 'Subcriteria':
                        action_2_index = ws_assignment.cell(l, 5).value
                        reference_2_index = ws_assignment.cell(l, 6).value
                        size_of_criteria_2 = len(criteria_2_name)
                        criteria_2_name_with_size_1 = criteria_2_name[0:size_of_criteria_1]
                        if criteria_1_name == criteria_2_name_with_size_1 and size_of_criteria_2 - 1 == size_of_criteria_1 and action_2_index == action_1_index and reference
                            a_outranking_b_1 = ws_assignment.cell(l, 23).value
                            a_outranking_b_2 = ws_assignment.cell(l, 24).value
                            a_outranking_b_3 = ws_assignment.cell(l, 25).value
                            b_outranking_a_1 = ws_assignment.cell(l, 26).value
                            b_outranking_a_2 = ws_assignment.cell(l, 27).value
```

Figure E.1 – Portion of the *python* code of the *Coherence properties*' function

The first three cycles of the *CP* have fixed the action a_1, the reference profile B_{16}, and the subcriterion g_0. This imposes the need to verify in the Assignment's sheet, all the *Mini Tables* associated with the subcriteria that directly descend from the root one and the one that concerns the criterion g_0.

Table E.1 - *Mini tables'* example

Criteria	Index	Action	Reference action	aS'b	aS''b	aS'''b	bS'a	bS''a	bS'''a
g_1	1	a_1	b_6^1	NO	NO	NO	YES	YES	YES
g_2	2	a_1	b_6^1	NO	NO	NO	YES	YES	YES
g_3	3	a_1	b_6^1	NO	NO	NO	YES	YES	YES
g_4	4	a_1	b_6^1	NO	NO	NO	YES	NO	YES
g_0	0	a_1	b_6^1	NO	NO	NO	YES	YES	YES

Each row in Table **E.1** represents one of those *Mini Tables*. In this example, the subcriteria that directly descend from the root one are g_1 ($G_{0,1} = g_1$), g_2, g_3 and g_4, which means that there are five *Mini Tables* that must be analyzed, the ones referring those subcriteria, and the one referring g_0. By observing the *OR* in the first four rows of Table **E.1** it is possible to conclude, for instance, that

$not(aS'_{(r,j)}b)$ for all $j = 1, \ldots, 4$ and $r = 0$. This conclusion imposes the need to analyze if $not(aS'_0 b)$ to verify if *Coherence property 2* is respected. By observing the same Figure is possible to observe that this condition is respected. This information is then inserted in Table **E.2**.

Table E.2 - *Coherence properties* main table's example

Coherence properties						
Criterion	Action	Reference Action	OR	(C1)	(C2)	Information
g_0	a_1	b_6^1	aS'b		VERIFIED	NOT(aS'b) for all criteria descending from g_0
g_0	a_1	b_6^1	aS''b		VERIFIED	NOT(aS''b) for all criteria descending from g_0
g_0	a_1	b_6^1	aS'''b		VERIFIED	NOT(aS'''b) for all criteria descending from g_0
g_0	a_1	b_6^1	bS'a	VERIFIED		bS'a for all criteria descending from g_0
g_0	a_1	b_6^1	bS''a			All the criteria descending from g_0 are either NOT(bS''a) or bS''a
g_0	a_1	b_6^1	bS'''a	VERIFIED		bS'''a for all criteria descending from g_0

Notice, that neither $not(bS''_{(r,j)}a)$ or $bS''_{(r,j)}a$, for all $j = 1, \ldots, 4$ and $r = 0$. These information is also inserted in Table **E.2**, however, in this particular cases there is no coherence properties to verify.

Each row in the *Coherence properties main table* is then checked to conclude if the Coherence properties (C1) and (C2) are always respected. The conclusions are then presented in Table **E.3**.

Table E.3 - *Coherence properties* final table's example

Conclusions regarding the Coherence properties	
(C1)	**(C2)**
Respected	Respected

Appendix F

Homogeneity example

Table F.1 - *Homogeneity* first table's example

Homogeneity analysis				
Criteria	Action	Reference action	$\sigma_r(a, B_h)$	$\sigma_r(B_h, a)$
g_0	a_1	b_6^1	0	1
g_0	a_1	b_5^1	0	0.965
g_0	a_1	b_4^1	0	0.886
g_0	a_1	b_4^2	0	0.874
g_0	a_2	b_6^1	0	1
g_0	a_2	b_5^1	0	0.905
g_0	a_2	b_4^1	0.167	0.514
g_0	a_2	b_4^2	0.3	0.119

Table F.2 - *Homogeneity* second table's example

Homogeneity comparation								
Criteria	Action 1	Action 2	Reference action	Action 1		Action 2		Equal credibility indexes?
				$\sigma_r(a, B_h)$	$\sigma_r(B_h, a)$	$\sigma_r(a, B_h)$	$\sigma_r(B_h, a)$	
g_0	a_1	a_2	b_6^1	0	1	0	1	YES
g_0	a_1	a_2	b_5^1	0	0.965	0	0.905	NO
g_0	a_1	a_2	b_4^1	0	0.886	0.167	0.514	NO
g_0	a_1	a_2	b_4^2	0	0.874	0.3	0.119	NO

Table F. 3 - *Homogeneity* third table's example

Homogeneity comparation								
Criteria	Action 1	Action 2	All credibility indexes are equal?	Action 1		Action 2		Homogeneity respected?
				Min category	Max category	Min category	Max category	
g_0	a_1	a_2	NO	C_3	C_3	C_5	C_5	YES

Appendix G

Hierarchical tree of criteria of the case study

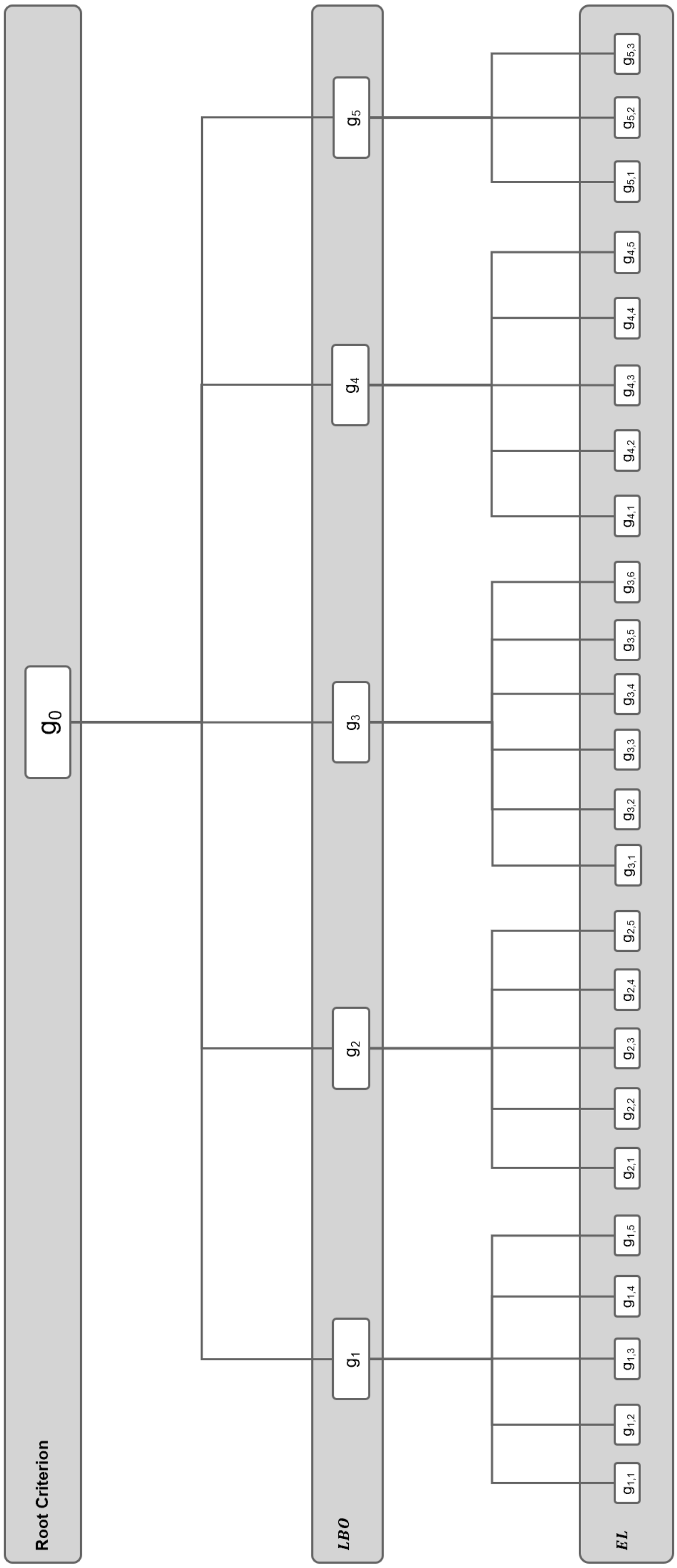

Figure G.1 - Hierarchical tree of criteria

Appendix H

Potential Actions’ performances

Table H.1 - Potential actions' performances

Potential Actions			Elementary criteria																								
Name	Code	a_i	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$
Centro Hospitalar do Médio Ave	CHMA	a_1	76.1	96,00	3.2	1.143	1.8	83.1	7.3	3.96	29.7	0.75	0.025	0	0.143	0.463	0.17	0.93	2558	542	567	10.574	6.8	31.7	33	33
Centro Hospitalar Póvoa de Varzim/Vila do Conde	CHPV	a_2	95.8	99.2	7.6	1.189	1.832	69.8	6.56	1.52	84.3	0.54	0	0	0	0.216	0.34	1.52	2566	341	599	14.434	7.296	29	26.1	26.1
Centro Hospitalar Barreiro/Montijo	CHBM	a_3	83	69.4	0.1	1.627	3.092	70.2	8.38	4.9	45.4	0.99	0.017	0.013	0.08	0.484	0.28	2.97	2063	845	526	9.275	6.04	29.1	29.9	29.9
Centro Hospitalar de Leiria	CHL	a_4	56.2	82.7	0.6	1.062	2.149	88.5	8.18	2.31	32.4	0.74	0.013	0	0	0.09	0	0.27	1759	2543	538	11.921	6.719	28.7	48.3	53.4
Centro Hospitalar de Setúbal	CHS	a_5	74.1	62.8	2.8	1.808	3.088	81.8	5.36	3.85	65	0.87	0.1	0	0.079	1.018	0.99	4.35	2022	691	535	10.47	8.425	31.9	40.6	41.8
Centro Hospitalar do Baixo Vouga	CHBV	a_6	66.4	82.2	1.1	1.48	2.458	83.1	6.93	2.96	66.5	0.48	0.052	0	0.227	0.655	1.35	2.26	2135	525	509	12.059	5.25	23.5	29	28.2
Centro Hospitalar Entre Douro e Vouga	CHCHDV	a_7	59	80.9	4.1	1.153	1.652	84.6	6.9	2.65	23.8	0.63	0	0	0.204	0.212	0.37	2.04	2091	633	491	13.179	5.863	25.2	28	28
Centro Hospitalar Médio Tejo	CHMT	a_8	83.9	72.7	10.4	1.412	3.157	85.2	9.78	4.13	31.3	0.64	0	0	0.033	0.648	0.46	0.57	2373	140	898	11.756	13.907	27.4	28.2	28.2
Centro Hospitalar Tâmega e Sousa	CHTS	a_9	60.4	85.9	6.2	0.867	1.349	85.7	6.69	3.5	59	0.85	0.042	0	0.137	0.363	0.33	0	1739	617	624	11.999	8.48	22.7	22.8	22.8
Centro Hospitalar Universitário Cova da Beira	CHUCB	a_{10}	76.3	68,00	5.1	2.45	4.837	72.9	7.2	3.71	53.1	0.7	0.135	0	0.106	0.388	0.45	0	2472	862	629	15.994	2.743	36.7	48	48
Hospital da Senhora da Oliveira, Guimãres	HSO	a_{11}	57.1	74.1	1.5	1.085	1.573	79.6	7.98	4.57	43.5	0.6	0	0	0.066	0.635	0.64	3.22	1474	1200	499	11.512	5.038	28.5	29.5	29.5
Hospital Distrital de Santarém	HDS	a_{12}	67.6	57.2	8.6	1.877	3.511	81.8	10.5	3.13	33.2	0.54	0.014	0	0.146	0.538	0.35	3.21	2702	945	828	13.694	6.58	28.2	29.6	29.5
Centro Hospitalar Tondela-Viseu	CHTV	a_{13}	80.6	49.5	3.4	2.923	3.515	96.8	5.4	4.05	36.7	1.67	0.116	0	0.193	1.09	0.98	3.86	1736	841	419	11.332	2.427	22.4	28.3	28.3
Centro Hospitalar Trás-os-Montes e Alto Douro	CHTAD	a_{14}	64.1	70.1	2.6	1.246	2.21	86.2	11.44	2.91	73.5	1.11	0.019	0.007	0.085	0.443	0.29	0.62	1932	696	611	12.001	4.329	37.7	48.7	48.7
Centro Hospitalar Universitário do Algarve	CHUA	a_{15}	73.2	66.6	4.1	2.032	3.749	79.8	7.09	5.64	12.5	1.38	0.052	0.005	0.261	0.639	0.54	0.9	2318	1019	493	12.807	7.635	26.6	25.8	25.5
Centro Hospitalar Vila Nova de Gaia/Espinho	CHVNG	a_{16}	55.7	80.8	1.9	2.777	3.513	84.6	7.44	3.76	71.8	0.96	0.084	0.023	0.184	0.607	0.32	1.99	1617	1109	330	13.16	2.581	29.4	30.6	30.6
Hospital Espírito Santo de Évora	HESE	a_{17}	61.5	65.4	3.4	3.555	3.475	76	6.07	3.94	16.5	0.44	0.113	0.058	0.182	0.746	1.01	2.43	1872	988	625	14.395	4.227	34	38.8	31
Hospital Fernando da Fonseca	HFF	a_{18}	76.7	66.1	3	1.382	1.73	82.7	4.84	5.53	34.9	0.49	0.028	0	0.321	1.2	0.6	2.48	1412	822	573	15.084	7.264	31.6	51.8	38
Hospital Garcia de Orta	HGO	a_{19}	86.5	51.8	4.4	1.565	2.598	90.2	7.09	4.5	26.8	1.29	0.023	0.007	0.235	0.485	1.17	3.62	1378	874	404	11.399	5.134	26.3	25.8	25.4
Centro Hospitalar de Lisboa Ocidental	CHLO	a_{20}	70.2	63.9	6.7	3.984	5.759	80.2	7.05	4.5	51.2	1.35	0.075	0.047	0.242	0.852	0.09	0.7	1878	1210	406	9.553	3.564	27.5	31.7	31.7
Centro Hospitalar e Universitário de Coimbra	CHUCB	a_{21}	66	65.7	8	3.454	5.381	81.1	8.83	4.06	39.2	1.35	0.026	0.003	0.143	0.39	0.14	1.03	1612	152	431	11.613	0.634	29.9	33.2	33.2
Centro Hospitalar Universitário de Lisboa Central	CHULC	a_{22}	75.1	60.9	5.7	5.075	7.035	83.8	6.54	4.82	35.4	1.3	0.073	0.005	0.211	0.564	0.82	3.27	2019	1217	436	10.235	1.139	30.9	30.8	30.7
Centro Hospitalar Universitário de São João	CHUSJ	a_{23}	50	74.6	2.3	4.823	7.11	81.4	5.11	3.25	62.7	0.95	0.043	0.03	0.307	0.55	0.18	3.19	1594	1150	348	10.68	1.959	28	27.2	26.7
Centro Hospitalar Universitário do Porto	CHUP	a_{24}	72.7	79.7	11.3	3.5	5.2	78.4	1.4	2.73	58.3	0.37	0.095	0	0.155	0.089	0.41	1.65	1433	1410	310	11.866	1.445	27.6	29.2	26.3
Centro Hospitalar Universitário Lisboa Norte	CHULN	a_{25}	63.5	68.4	0.5	4.516	5.463	79.8	9.44	4.53	43.8	1.05	0.025	0.011	0.252	1.1	0.69	5.97	1491	1539	403	14.693	0.967	24.9	28.8	28.8

Appendix I

Veto threshold

The veto thresholds in the ELECTRE TRI-nC method with MCHP must be associated with the elementary criteria. In Rocha et al. (2021) the DM has defined a veto threshold for the criteria g_2 and g_3. The values defined by the DM for the veto thresholds were related with the scale defined for those criteria. They corresponded to 40% of the max value in that scale. Since in this new method these two criteria are interpreted as subcriteria due to the incorporation of the MCHP, it becomes necessary to adapt those values to be implemented in the elementary criteria descending from g_2 and g_3.

It were selected two elementary criteria $(g_{2,2}, g_{2,3})$ that directly descend from the subcriteria g_2, and three elementary criteria $(g_{3,2}, g_{3,3}$ and $g_{3,4})$ that directly descend from the subcriteria g_3, to be associated with a veto threshold. The process to define the values of those thresholds was the following equation:

$$v_t = 0{,}4 * Max\ performance_t$$

The value obtained was then rounded to only have two decimal places.

The results of this procedure are presented in Table **I.1**:

Table I.1 - A portion of the *Criteria* sheet's table with the values of the veto thresholds

Criteria	Technical name	Type of criterion	q_t	p_t	v_t	Max performance
Avoid re-admission prior 30 days after discharge	$g_{2,2}$	EL	1	2	5.3	13.2
Excessive staying delay	$g_{2,3}$	EL	0.3	0.5	2.6	6.5
Bloodstream infections related to CVC	$g_{3,2}$	EL	0.01	0.01	0.04	0.1
Postoperative pulmonary embolisms or thrombosis	$g_{3,3}$	EL	0.01	0.01	0.16	0.4
Postoperative septicaemia	$g_{3,4}$	EL	0.01	0.01	0.56	1.4

With this procedure, it was kept the same proportion of the criterion's scale, used by Rocha et al. (2021), to compute the veto thresholds' value for criteria g_2 and g_3, to define the values of the veto thresholds in the selected elementary criterion.

Although the DM decided in Rocha et al. (2021) that a veto threshold should be associated with g_2 and g_3, there is a strong possibility that he/she did not consider that all elementary criteria descending from them should have one associated with them. There is why only two and three elementary criteria, which respectively descend from subcriteria g_2 and g_3, were chosen to be associated with a veto threshold, instead of all the elementary criteria that descend from those subcriteria. This selection was made randomly.

Appendix J

Defining the Criteria’ scale and the four new Reference Actions

The following figure identifies all reference actions in the model and their performance on the elementary criteria. The cells filled with green and yellow color represent, respectively, the performances of the four reference actions created and the worst and best possible reference actions.

Table J.1 - Performances of all reference actions

Reference Actions		Performances																						
Category	Index	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$
C_6	b_6^1	100	100	0	5.8	8.2	100	0	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_5	b_5^1	95	95	0	4.3	6.4	90	5	2.7	90	0.5	0	0	0	0	0.05	0.15	1408	537.04	348.12	10.6	1.14	15	15	80
C_4	b_4^1	85	85	2	2.7	4.8	85	6.5	3.2	80	0.6	0.03	0.02	0.08	0.03	0.16	0.35	1536	689.56	407.24	11.6	2.62	20	20	85
	b_4^2	82.5	90	3.5	2.4	5.6	82.5	5.75	3.45	65	0.55	0.04	0.01	0.04	0.045	0.105	0.5	1683	613	459	11.84	3.92	24.2	17.5	82.5
C_3	b_3^1	80	80	5	2.1	3.5	80	7.4	3.7	50	0.9	0.05	0.04	0.15	0.06	0.33	0.64	1829	847.82	509.85	12.08	5.22	28.4	31.2	90
	b_3^2	75	77.5	6	1.65	4.14	77.5	6.95	4.1	40	1	0.075	0.025	0.18	0.045	0.245	0.5	1934	768.7	557.78	13.03	3.92	29.2	30.6	92.5
C_2	b_2^1	70	75	7	1.2	2.1	75	8.3	4.5	30	1.1	0.1	0.06	0.21	0.76	0.64	1	2039	1082.11	605.71	13.98	7.29	30	30	95
	b_2^2	75	72.5	8	1.1	2.8	77.5	7.85	4.85	40	1	0.075	0.075	0.26	0.98	0.485	1.525	1934	1184.92	632.895	14.4	6.26	32.5	30.6	92.5
	b_2^3	65	77.5	6	1.65	1.9	72.5	9.05	4.1	25	1.25	1.1	0.05	0.18	0.041	0.725	0.82	2210	964.97	557.78	13.03	7.9	29.2	32.5	97.5
C_1	b_1^1	60	70	9	1	1.7	70	9.8	5.2	20	1.4	0.12	0.09	0.31	1.2	0.81	2.05	2381	1287.73	660.08	14.82	8.51	35	35	100
C_0	b_0^1	0	0	13	0	0	0	13.2	6.5	0	1.9	0.2	0.1	0.4	1.4	1.6	6.9	3107.3	2924.7	1032.6	18.4	16	43.4	59.6	100

Criteria’ scale

The application of the ELECTRE TRI-nC method with MCHP in this model requires the definition of two subsets of reference actions (B_0 and B_6), composed by a single reference action (b_0^1 and b_6^1), that have the worst and best possible performances in all elementary criteria, respectively. Since in Rocha, A., et al. (2021) the DM has not defined a worst and best possible performances in each elementary criteria, it was necessary to develop a procedure to estimate those values. The procedure was the following one:

1) Since a great portion of the elementary criteria are evaluated by considering the number of occurrences in 100 cases, it was defined that the criteria that have reference actions’ performances close to this value should have a criteria’ scale equal to $[0, 100]$, accordingly with the direction of preference in that criteria;
2) When all the values of the reference actions’ performances (b_h^r, with $h = 1, \dots 5$ and $r = 1$) were to far apart from 100, it was defined that the maximum or minimum performance, accordingly with the direction of preference in the criterion analyzed, should be established by also considering the potential actions performances’ values. The process was based in the verification of the maximum performances’ values in all reference actions (b_h^r, with $h = 1, \dots 5$ and $r = 1$), and potential actions (a_i, with $i = 1, \dots, 25$). The value obtained was then multiplied

by a coefficient of 1.15. The resultant value was then associated with the worst or best possible performance, accordingly with the direction of preference in that criterion. If that value was interpreted as being the best possible performance (the direction of preference was maximization), the value associated with the worst possible performance was 0. The same logic was applied to the reverse case.

New Reference Actions

In order to test the capability of the *CP* to deal with subsets of reference actions that have more than one reference action associated, it were defined four new reference actions in the model. However, it is important that these reference actions do not have performances' values that are two dissimilar from the ones defined by Rocha et al. (2021). The procedure developed to create those actions aimed to define performances' values similar to the ones that define the reference action of the subset of reference actions under analysis. The procedure will be explained based on the practical case presented in Table **J.2**.

Table J.2 - Reference action b_4^2 elaboration

Reference Actions		Performances																							
Category	Index	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$	
C_5	b_6^1	95	95	0	4.3	6.4	90	5	2.7	90	0.5	0	0	0	0	0.05	0.15	1408	537	348	10.6	1.14	15	15	80	
		90	90	1	3.5	5.6	87.5	5.75	2.95	85	0.55	0.015	0.01	0.04	0.015	0.105	0.25	1472	613	378	11.1	1.88	17.5	17.5	82.5	$=\frac{(b_5^1 + b_4^1)}{2}$
C_4	b_4^1	85	85	2	2.7	4.8	85	6.5	3.2	80	0.6	0.03	0.02	0.08	0.03	0.16	0.35	1536	690	407	11.6	2.62	20	20	85	
	b_4^2	82.5	90	3.5	2.4	5.6	82.5	5.75	3.45	65	0.55	0.04	0.01	0.04	0.045	0.105	0.5	1683	613	459	11.84	3.92	24.2	17.5	82.5	
		82.5	82.5	3.5	2.4	4.15	82.5	6.95	3.45	65	0.75	0.04	0.03	0.115	0.045	0.245	0.495	1682.5	769	459	11.84	3.92	24.2	25.6	87.5	$=\frac{(b_3^1 + b_4^1)}{2}$
C_3	b_3^1	80	80	5	2.1	3.5	80	7.4	3.7	50	0.9	0.05	0.04	0.15	0.06	0.33	0.64	1829	848	510	12.08	5.22	28.4	31.2	90	
		43	76	7	22	92	3	53	36	39	70	14	88	75	47	66	33	17	59	41	25	9	27	96	60	Random number [0;100]

Notice that in the following explanation the goal is to add a new reference action (b_4^2) to the subset B_4, and to define its performances values in each elementary criteria. The process is based on the comparison of the reference action (b_4^1) that defines a certain subset of reference actions (B_4), with the reference actions of the adjacent categories (b_5^1 and b_3^1). The comparison is made by computing the mean between the performances of the reference action (b_4^1) and each one of the other two mentioned (b_5^1 and b_3^1). The results of this operation are highlighted in the blue cells. The second operation is to define a random number between 0 and 100 for each elementary criteria. If the value of the random number computed in a given criteria is greater than or equal to 50, the performance's value of b_4^2 is equal to $\frac{b_5^1+b_4^1}{2}$, which means that the performance's value is between the reference action that defines the selected category to add a reference action, and the reference action that defines the next higher category. If the value of the random number computed in a given criteria is smaller than 50, the process is made in the reverse way.

This process was made to develop all reference actions highlighted with a green background in Table **J.1**.

Appendix K

Elementary criteria' weights

The following fives Figure presents the estimated values for the *Locally Normalized Weight* and *Globally Normalized Weight* of each elementary criterion.

Table K.1 - Weights associated with each elementary criteria descending from g_1

Criteria	Name	First medical appointments timeliness	Enrolled patients for surgery	Availability of beds	Availability of doctors	Availability of nurses
	Technical name	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$
	Locally Normalized Weight	0.271	0.254	0.136	0.136	0.203
	Globally Normalized Weight	0.064	0.06	0.032	0.032	0.048

Table K.2 - Weights associated with each elementary criteria descending from g_2

Criteria	Name	Minor surgeries appropriateness	Avoid re-admission prior 30 days after discharge	Excessive staying delay	Hip surgery timeliness	Delay before surgery
	Technical name	$g_{2,1}$	$g_{2,2}$	$g_{2,3}$	$g_{2,4}$	$g_{2,5}$
	Locally Normalized Weight	0.269	0.385	0.269	0.038	0.038
	Globally Normalized Weight	0.077	0.11	0.077	0.011	0.011

Table K.3 - Weights associated with each elementary criteria descending from g_3

Criteria	Name	Bedsores	Bloodstream infections related to CVC	Postoperative pulmonary embolisms or thrombosis	Postoperative septicaemia	Non-instrumental vaginal deliveries with severe laceration	Assisted vaginal deliveries with severe laceration
	Technical name	$g_{3,1}$	$g_{3,2}$	$g_{3,3}$	$g_{3,4}$	$g_{3,5}$	$g_{3,6}$
	Locally Normalized Weight	0.147	0.218	0.289	0.289	0.0289	0.029
	Globally Normalized Weight	0.046	0.068	0.09	0.09	0.009	0.009

Table K.4 - Weights associated with each elementary criteria descending from g_4

Criteria	Name	Expenses with staff	Expenses with drugs, pharmaceutical products and clinical consumables	Expenses with supplies and external services	Expenses with overtime	Expenses with outsourcing
	Technical name	$g_{4,1}$	$g_{4,2}$	$g_{4,3}$	$g_{4,4}$	$g_{4,5}$
	Locally Normalized Weight	0.207	0.279	0.279	0.207	0.028
	Globally Normalized Weight	0.006	0.009	0.009	0.006	0.001

Table K.5 - Weights associated with each elementary criteria descending from g_5

Criteria	Name	Volume of caesarean sections	Caesarean sections in UCFTPs	First caesarean sections in UCFTPs
	Technical name	$g_{5,1}$	$g_{5,2}$	$g_{5,3}$
	Locally Normalized Weight	0.2	0.4	0.4
	Globally Normalized Weight	0.0267	0.053	0.053

As mentioned in Section **4.3.2**, the *Locally Normalized Weight* is the weight associated with each elementary criterion without considering their position in the hierarchical tree. The *Locally Normalized Weight* of the criteria in each table are computed as if these criteria were the only ones defined in the problem. To adjust them to the hierarchical process it is necessary to multiply their *Locally Normalized Weight,* with the weight associated with the criteria from which they are descending. Thus, obtaining the value of the *Globally Normalized Weight.*

Appendix L

Assignment procedure of the method

Table L.1 - Assignment procedure of each action

Actions		Assignment Procedure: Access g_1		Care Appropriateness g_2		Patient Safety g_3		Efficiency g_4		Caesarean Appropriateness g_5		Comprehensive level g_0	
Name	a_i	Lower Category	Upper Category	Lower Category	Upper Category	Lower Category	Upper Category	Lower Category	Upper Category	Lower Category	Upper Category	Lower Category	Upper Category
Centro Hospitalar do Médio Ave	a_1	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_4	C_5	C_3	C_3
Centro Hospitalar Póvoa de Varzim/Vila do Conde	a_2	C_3	C_4	C_5	C_5	C_4	C_4	C_2	C_2	C_4	C_5	C_5	C_5
Centro Hospitalar Barreiro/Montijo	a_3	C_3	C_3	C_1	C_2	C_3	C_3	C_3	C_3	C_4	C_5	C_3	C_3
Centro Hospitalar de Leiria	a_4	C_2	C_2	C_4	C_4	C_5	C_5	C_1	C_2	C_1	C_5	C_4	C_4
Centro Hospitalar de Setúbal	a_5	C_2	C_2	C_4	C_4	C_2	C_4	C_3	C_3	C_3	C_5	C_3	C_3
Centro Hospitalar do Baixo Vouga	a_6	C_3	C_3	C_3	C_4	C_3	C_4	C_3	C_3	C_4	C_5	C_3	C_4
Centro Hospitalar Entre Douro e Vouga	a_7	C_2	C_2	C_4	C_4	C_3	C_3	C_3	C_3	C_4	C_5	C_3	C_3
Centro Hospitalar Médio Tejo	a_8	C_2	C_2	C_2	C_2	C_3	C_4	C_1	C_3	C_4	C_5	C_3	C_4
Centro Hospitalar Tâmega e Sousa	a_9	C_1	C_1	C_3	C_4	C_3	C_3	C_3	C_3	C_5	C_5	C_4	C_4
Centro Hospitalar Universite1-rio Cova da Beira	a_{10}	C_3	C_3	C_2	C_3	C_3	C_3	C_2	C_2	C_1	C_5	C_3	C_3
Hospital da Senhora da Oliveira, Guimãres	a_{11}	C_2	C_2	C_2	C_3	C_3	C_4	C_3	C_3	C_4	C_5	C_3	C_3
Hospital Distrital de Santarém	a_{12}	C_2	C_2	C_3	C_3	C_3	C_3	C_1	C_1	C_4	C_5	C_3	C_3
Centro Hospitalar Tondela-Viseu	a_{13}	C_3	C_3	C_3	C_3	C_2	C_4	C_4	C_4	C_5	C_5	C_3	C_4
Centro Hospitalar Te1-rs-os-Montes e Alto Douro	a_{14}	C_2	C_2	C_3	C_3	C_3	C_4	C_3	C_3	C_1	C_5	C_3	C_3
Centro Hospitalar Universite1-rio do Algarve	a_{15}	C_3	C_3	C_2	C_2	C_3	C_4	C_2	C_2	C_5	C_5	C_3	C_3
Centro Hospitalar Vila Nova de Gaia/Espinho	a_{16}	C_3	C_3	C_3	C_4	C_3	C_3	C_4	C_4	C_4	C_5	C_3	C_3
Hospital Espírito Santo de Évora	a_{17}	C_2	C_2	C_3	C_3	C_2	C_2	C_2	C_2	C_3	C_5	C_2	C_2
Hospital Fernando da Fonseca	a_{18}	C_2	C_2	C_3	C_3	C_2	C_4	C_3	C_3	C_1	C_5	C_2	C_4
Hospital Garcia de Orta	a_{19}	C_2	C_2	C_2	C_3	C_3	C_3	C_4	C_4	C_5	C_5	C_3	C_3
Centro Hospitalar de Lisboa Ocidental	a_{20}	C_3	C_3	C_2	C_3	C_2	C_2	C_4	C_4	C_4	C_5	C_2	C_2
Centro Hospitalar e Universite-1-rio de Coimbra	a_{21}	C_3	C_3	C_2	C_3	C_3	C_3	C_4	C_4	C_4	C_5	C_3	C_3
Centro Hospitalar Universite1-rio de Lisboa Central	a_{22}	C_4	C_4	C_2	C_3	C_3	C_3	C_3	C_4	C_4	C_5	C_3	C_3
Centro Hospitalar Universite1-rio de São João	a_{23}	C_4	C_4	C_4	C_4	C_2	C_2	C_4	C_4	C_4	C_5	C_3	C_3
Centro Hospitalar Universite1-rio do Porto	a_{24}	C_3	C_3	C_5	C_5	C_3	C_3	C_4	C_4	C_4	C_5	C_4	C_4
Centro Hospitalar Universite1-rio Lisboa Norte	a_{25}	C_3	C_4	C_2	C_2	C_2	C_3	C_3	C_3	C_4	C_5	C_3	C_3