Differential GPS Positioning

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***Abstract*—The Global Positioning System (GPS) is used as a sensor capable of providing users navigation and positioning services anywhere on Earth's surface. However, there are some errors that can end up affecting the accuracy of the positioning calculations performed by the GPS. Among them, we highlight satellite and receiver clock errors, ephemeris errors, multipath errors and signal propapgation errors, such as ionosphere delay and troposphere delay. Therefore, in order to correct these errors and increase the GPS accuracy, we use a technique called DGPS. This technique consists of using a base station which position is known, and from this station differential corrections in pseudoranges are calculated. These differential corrections are then broadcasted to a GPS user, thus allowing the user's position to be more accurately determined. For this project, the data collected by Telecommunications Institute (IT) during a survey work were used. The GPS receiver used as base station, in this work, is installed on the top of the Instituto Superior Técnico's (IST) North Tower, in Alameda.**

***Index Terms*—GPS, DGPS, Base station, Differential corrections, Pseudoranges**

I. Introduction

For many year, humans have been devising many ways to go to faraway locations... in the past, procedures that allowed man to navigate through the stars were used. The space age, however, began in 1957 with the launch of the first artificial satellite (Sputnik). During this period, scientists realized that by observing changes in the transmission of a satellite's radio signal, it was able to compute a satellite's position, allowing them to determine the position of a receiver on Earth [1]. However, the GPS project was established by the Department of Defense (DoD) in 1973. Other satellite navigation systems, such as Galileo, Global Navigation Satellite System (GLONASS), BeiDou Navigation Satellite System (BDS), Navigation Indian Constellation (NavIC), and the Quasi-Zenith Satellite System (QZSS), were being developed and launched [2].

Galileo is a European Union-developed system. It has been in use since 2016. It now has 26 satellites in orbit, but the constellation is anticipated to expand to 30 satellites in the future. GLONASS is a Russian satellite navigation system that has been active since 2012 and has a total of 24 satellites in orbit. BDS, a Chinese government-developed satellite constellation, was launched in 2012 to cover primarily Chinese land, but it became active globally in 2020, with a total of 35 satellites in its constellation. NavIC- Created by Indian authorities, this system operates at a regional level and has a constellation of seven satellites. QZSS - created by the Japanese government, this system only functions at a regional level and has been active since 2018, with a constellation of four satellites, but an expansion to seven satellites is projected for 2023 [2].

Outside of navigation, GPS has grown in popularity and use, and is now widely utilized for medical applications, mobile gaming, and search and rescue operations, among other things. GPS receivers can receive satellite signals from any location on the planet, regardless of weather conditions, 24 hours a day. By default, GPS receivers are used to determine a specific location and, in conjunction with a map, for navigation. The GPS receiver receives a signal from each satellite on a regular basis, and after receiving data from four or more satellites, it uses trilateration to calculate its position [3]. GPS technology is limited in terms of precision due to flaws in satellite signal transmission, causing the GPS receiver to estimate its position with a horizontal position error of 22 meters and a vertical position error of 27.7 meters. Multipath, ionosphere delay, troposphere delay, satellite clock errors, ephemeris errors, and other factors all contribute to GPS errors [4].

However, GPS errors must be addressed when greater/better precision about the user's position is required. There are a variety of GPS techniques that can be used to improve the user's positioning, some of which could be implemented in real time, such as the DGPS technique, which will be examined in this research, and the Wide Area Augmentation System (WAAS) methodology. After obtaining GPS data, it is still possible to correct it (post-processing). DGPS is a system that corrects the position of the rover receiver by using one or more base stations whose positions are known, allowing differential corrections to be calculated and then sent to the rover receiver to obtain more accurate information about the rover receiver positioning. Thus, the goal of this dissertation is to look into how the differential GPS positioning technique is used [5].

In chapter II the GPS basic concepts are presented. The chapter III presents the place where the used antennas were installed and describes the results collected throughout the process of estimating the receiver's position. Chapter IV is dedicated to the work's conclusions.

II. GPS BASIC CONCEPTS

A. Reference Coordinate Systems

The GPS system is mostly used on the Earth's surface, with satellites orbiting the planet over time. A coordinate system is used to determine the position of a certain point on the earth's surface. Three of these systems will be discussed: Earth-Centered, Earth Fixed; World Geodetic System; Local Coordinate System [6].

The ECEF coordinate system is a frequently used coordinate system for describing the location of a specific station on the earth's surface. It is a cartesian coordinate system that describes the position in space (x, y, z). The ECEF system's origin is aligned with the Earth's center of mass, and its Z axis is aligned with the Earth's rotation axis. The right hand rule can be used to identify the Y axis, which points in the direction of the reference meridian. It is feasible to represent the set of Latitude, ϕ), Longitude (λ) e Altitude (h), (LLH), coordinates of any location on Earth using ECEF coordinates (as shown in Fig. 1). The world geodetic system can also be used to express ECEF coordinates [6].

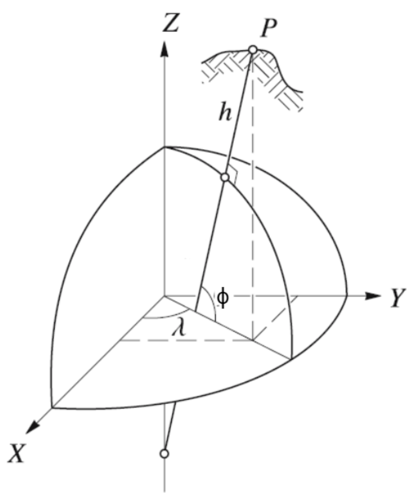

Fig. 1. LLH coordinates and Cartesian coordinates [7].

The World Geodetic System of 1984 is a global geodetic system developed by the Department of Defense and serves as the GPS reference system. The WGS 84 is a model that is used to collect data on the Earth's gravitational anomalies. This information is required in order to acquire correct ephemeris data. The essential goal, however, is to determine the GPS receiver's latitude, longitude, and altitude. WGS 84 specifies the Earth's shape as an ellipsoid for this purpose [4].

It is typically more practical to use a coordinate system that favors the GPS receiver's positioning when it's in motion. The East, North, Up (ENU) coordinates are presented in this context. The X and Y axes are in the horizontal plane in this coordinate system. The ENU system is a coordinate system with an axis pointing north, an axis pointing up, and an axis pointing east. It is also feasible to determine the elevation and azimuth using ENU coordinates [8].

B. GPS

GPS is a global navigation satellite system (GNSS) that can provide positioning and time to an endless number of appropriately equipped users 24 hours a day, seven days a week. GPS is a passive system, which means that only the satellite transmits signals to the receiver. The Department of Defense collaborated with the Department of Transportation to develop this system, which is overseen by the US government. Originally, it was created for military objectives. The GPS service is currently offered in two areas: the precise positioning service (PPS), which is for military services and some authorized users, and the standard positioning service (SPS), which is for anyone. Although there are 24 satellites in the GPS constellation, the GPS receiver only needs to connect with 4 of them to calculate its position. The GPS receiver estimates the signal's travel time and, using this information, determines the pseudorange to the satellite, as given in (16), and so determines its position using the trilateration technique [4].

C. Estimation of the receiver's position

If four or more satellites are available, the Least Squares approach can be used to estimate the receiver's position (x, y, z) and the clock's offset (δ^{clk}) [8].

Assuming that the receiver's initial coordinates r_0 = (x_0, y_0, z_0), the pseudorange, $\rho^{(n)}$, and the position of n satellites, $S^{(n)}$ = (x_s, y_s, z_s), are known, then the linearized model $X = (H^T H)^{-1} H^T \cdot z$ can be used to estimate the receiver's new position. Where $X^T = \begin{bmatrix} x & y & z & \delta^{clk} \end{bmatrix}$, H is the matrix of cosines directors of the unit vectors that point from the receiver to each of the n satellites, and z is the matrix of pseudoranges. H is denoted by [8]:

$$H = \begin{bmatrix} -e_0^{(1)^T} & 1 \\ -e_0^{(2)^T} & 1 \\ \vdots & \vdots \\ -e_0^{(n)^T} & 1 \end{bmatrix} \tag{1}$$

with

$$e_0^{(n)} = \frac{S^{(n)} - r_0}{||S^{(n)} - r_0||} \tag{2}$$

And z is given by:

$$z = \begin{bmatrix} \rho^{(1)} - e_0^{(1)^T} \cdot S^{(1)} \\ \rho^{(2)} - e_0^{(2)^T} \cdot S^{(2)} \\ \vdots \\ \rho^{(n)} - e_0^{(n)^T} \cdot S^{(n)} \end{bmatrix} \tag{3}$$

However, in order to improve the estimation of the receiver's positioning, the pseudoranges must be corrected.

D. GPS Time

All GPS satellites have atomic clocks that ensure the accuracy of GPS broadcasts. GPS users employ a variety of time systems for various purposes. Coordinated Universal Time

(UTC) and GPS Time (GPST) are the most essential among them. UTC is an atomic time scale based on International Atomic Time (TAI), which means that it has the same tempo as TAI. Following its introduction, UTC started to base all Legal hours around the world. The TAI is a uniform time scale calculated by the *Bureau International desPoids et Mesures* (BIPM) using the average time value of atomic clocks around the world [9].

Although GPST is an atomic time scale, it is only used as a reference for GPS signals.It is determined using a time scale generated by satellite and control station atomic clocks. GPST does not require the introduction of a second scalar to adjust its time scale, indicating that it is a continuous time scale. After some adjustments, the GPST time scale was determined to be the same as UTC. The GPST timeframe is now 18 seconds ahead of UTC due to UTC timescale corrections [10].

E. GPS Signals

The fact that all components of the sent signal are strictly controlled by atomic clocks is crucial to the GPS system's accuracy. The fundamental frequency, f_0, of 10.23 MHz is generated by these extremely accurate clocks. Two carrier waves, L1 and L2, are derived from this fundamental frequency. The frequencies of L1 and L2 are calculated as follows [7]:

$$L1 = 154 \cdot f_0 = 1575.42MHz \tag{4}$$

$$L2 = 120 \cdot f_0 = 1227.60MHz \tag{5}$$

The navigation message is transmitted at a rate of 50 Hz and is modulated on both L1 and L2 frequencies at a rate of 50 bits/second. It is made up of 25 frames of 1500 bits. The frequencies L1 and L2 are essential for the correction of the Ionosphere layer, which is the largest source of inaccuracy. A method known as Code Division Multiple Access (CDMA) is used to broadcast multiple signals at the same frequency. The modulation of the carrier is done through two Pseudorandom Noise (PRN) codes, which are: the C/A code and the P(Y) code [7].

The C/A is a 1023-bit code with a period of 1 ms, created at a rate of 1,023 MHz (one tenth of the fundamental frequency). The C/A code is available to any user and is only broadcasted on the L1 frequency [4]. P(Y) is a code that is similar to C/A except that it has $2.3547 \cdot 10^4$ bits and is created at the fundamental frequency rate. The P(Y) code is used for military navigation and by some authorized users. It is transmitted in L1 (in quadrature phase with the C/A code, that is, 90º apart in phase) and L2 (see Fig. 2) [4].

F. GPS segments

The GPS system is divided into three segments: space segment, control segment, and user segment [4].

The space segment represents the constellation of satellites in orbit and the signals they transmit. The space segment consists of a constellation of 24 orbital satellites orbiting the Earth at a distance of around 20200 kilometers from the user. These satellites are arranged in a system with six orbital planes, each of which is made up of four satellites. A GPS satellite's nominal orbital period is 11 hours and 58 minutes. The orbits are approximately circular, with a 55º inclination to the equator in each plane. The orbital radius (the distance between the Earth's center of mass and the satellite) is about 26600 kilometers. This constellation of satellites enables the user to use a global navigation service 24 hours a day [4].

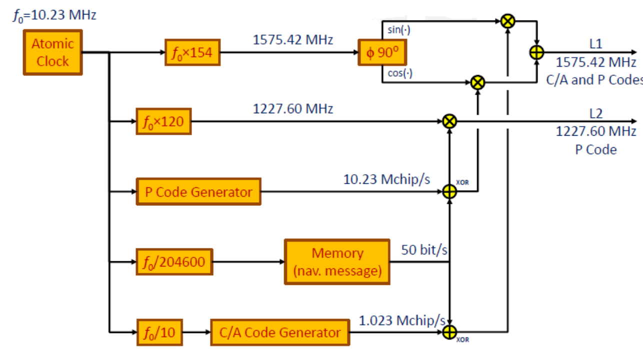

Fig. 2. GPS signal [8].

The control segment is in charge of monitoring the GPS satellite constellation and comprises of several control stations positioned throughout the world, with the main station in Colorado, USA. The main tasks of the control segment are as follows: controlling the position of the orbiting satellites and their clock parameters, and uploading the navigation message data to the satellites [7].

The user segment include both the type of GPS receiver and the people who use it. Users are divided into classes: military and civil users and authorized and unauthorized users. All GPS signals are not available to civilians and unauthorized users. GPS receivers have a processing unit that can decode the information sent by each satellite in real time and calculate their own position [4].

G. GPS error sources

Several forms of errors affect the signal transmitted by the satellite, ranging from systematic errors to atmospheric propagation delays. Due to these errors, the time of arrival of the signal transmitted by the satellite to the GPS receiver is longer than predicted, influencing the receiver's position results. The key errors that effect pseudo-distance and, as a result, GPS accuracy are then discussed and defined.

- Satellite and receiver clock error: as previously stated, the satellite's system includes an atomic clock, which is more accurate than GPS receiver clocks. As a result, the satellite inaccuracy is caused by a difference in satellite time. Clock errors can affect GPS accuracy with an error of about 0.8 m [4]. To fix this type of inaccuracy, the relativity error must be taken into consideration, which is caused by the fact that the receiver's clock and the satellites are in separate gravitational fields and move at different speeds. The receiver's clock, on the other hand, is not as accurate as the satellite's atomic clock, resulting in slight mistakes in signal arrival time estimation. Receiver's clock errors are typically measured in millimeters and are not taken into account due to their minor impact on GPS accuracy [1]. The receiver clock offset, δ^{clk}, is estimated using the receiver coordinates, and the satellite clock offset in seconds is calculated using the following equation [1]:

$$\delta t_{sv} = a_{f0} + a_{f2}(t_s - t_{oc}) + a_{f2}(t_s - t_{oc})^2 + \Delta t_r \quad (6)$$

Where:
a_{f0}, a_{f1} e a_{f2} = polynomial coefficients
t_{oc} = clock data reference instant.
t_s = GPS system time.
Δt_r = relativistic correction caused by the orbital eccentricity.
The relativistic correction caused by the orbital eccentricity is given by [1]:

$$\Delta t_r = -4.442807633 \cdot 10^{-10} \cdot e \cdot \sqrt{A} \cdot sin(E_k) \quad (7)$$

However, the polynomial coefficients, the GPS system time, the clock data reference instant, the square root of the semi-major axis ($\sqrt{A}$) and the orbit eccentricity (e) are provided by the navigation message, but the eccentric anomaly (E_k) is determined during the satellite position estimation procedure.

- Multipath error: multipath is a phenomenon in which the GPS signal travels over many paths to reach the receiving antenna, i.e., the signal is reflected on one or more surfaces near the receiving antenna. Multipath error affects pseudorange with an error approximately equal to 0.2 m [4].
- Ephemeris error: the navigation message delivered by the satellites is used by any GPS user to calculate their position. In the position estimate procedure, the receiver has to know the position of the satellites, hence an ephemeris error would affect the receiver's accuracy. Despite daily updates, it is extremely difficult to prevent ephemeris errors. The difference between the satellite's true position and its computed position via navigation messages causes these inaccuracies. Due to this difference, ephemeris introduce an error in pseudoranges of up to 2.5 m [11].
- Atmosphere delay: the speed of the transmitted signal is affected by the atmosphere around the planet, resulting in measurement errors in pseudo-distances. The atmosphere is separated into two layers for GPS purposes: the ionosphere and the troposphere [12]. The troposphere is the layer closest to the earth's surface, and the ionosphere is the layer above it. The ionosphere, on the other hand, is a dispersive medium, which means that its effect is frequency dependant. As a result, at frequencies L1 and L2, it works differently, allowing the magnitude of the error to be estimated. The inaccuracy from the ionosphere is roughly 7 meters on average [4]. The Klobuchar model described in [13] is used to correct the ionosphere delay. Unlike the ionosphere, the troposphere is a non-dispersive medium, so satellites do not transmit any data to correct the delay in the troposphere. For a satellite in the zenith, the delay is around 2 m, and it increases as the satellite's line-of-sight angle decreases. In order to corect the troposphere delay, the Saastamoinen model can be used to estimate the delay towards the Zenith, and the mapping functions proposed by Chao can be used to alter the delay while taking the elevation angles of the satellites into consideration. This two model are described in [14]:

H. Dilution of Precision

The accuracy in GPS positioning depends, on the one hand, on the accuracy of the pseudorange measurement, and on the other hand, on the geometric configuration of the constellation of utilized satellites, expressed by a scalar value designated Dilution of Precision (DOP) or Geometric Dilution of Precision (GDOP). Fig. 3 shows the optimal geometric configuration when the satellites are not too close together, that is, when one satellite is directly above the user and three others are evenly separated and have an elevation of 25 to 30 degrees (in order to reduce atmospheric refraction). This would result in a DOP with a low/good value. When the contrary occurs, the geometry is considered poor and the DOP value is high. The DOP value, on the other hand, varies as satellites move through their orbits [15].

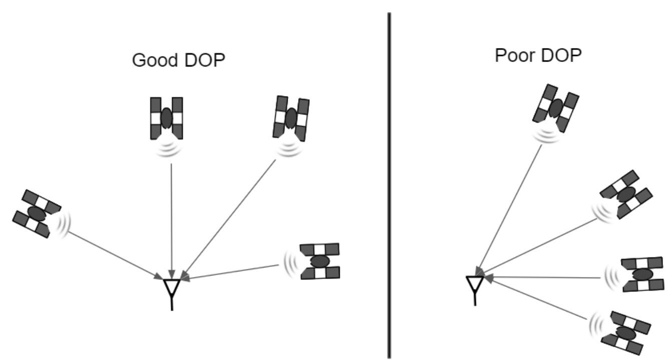

Fig. 3. Satellite geometric configuration and DOP.

When DOP is less than 2, it is regarded great, although this is not always the case; it usually necessitates a clear line of sight to the horizon. It's regarded favorable if the value is between 2 and 4. Only in low-precision scenarios is a DOP value of 6 regarded acceptable. The DOP consists of four parameters [15]:

- Position DOP (PDOP): is a three-dimensional dilution of precision.
- Horizontal DOP (HDOP): is the precision dilution in two horizontal dimensions. Because it ignores the vertical dimension, its value is frequently lower than PDOP.
- Vertical (VDOP): is the dilution of precision in one dimension, the vertical.
- Time DOP (TDOP): is the dilution of precision relative to time.

The DOP parameters are calculated as follows [15]: we begin by stating that the DOP value is related to the estimate error's covariance matrix, $c_{\tilde{x}}$, and we assume that the system is in an optimal state and

$$c_{\tilde{x}} = (H^T C_m^{-1} H)^{-1} \tag{8}$$

therefore

$$C_m = \begin{bmatrix} \sigma_{m1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{m2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{mn}^2 \end{bmatrix} \tag{9}$$

All pseudoranges are also assumed to have the same variance, σ_ρ^2, which means that the error covariance is stated as follows: $c_{\tilde{x}} = \sigma_\rho^2 (H^T H)^{-1} = \sigma_\rho^2 M$.

The covariance of the mistake is given matrically by:

$$c_{\tilde{x}} = \sigma_\rho^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \tag{10}$$

As a result, the parameters GDOP, PDOP, and TDOP can be calculated as follows:

$$GDOP = \sqrt{M_{11} + M_{22} + M_{33} + M_{44}} \tag{11}$$

$$PDOP = \sqrt{M_{11} + M_{22} + M_{33}} \tag{12}$$

$$TDOP = \sqrt{M_{44}} \tag{13}$$

The matrix H is written in ENU coordinates for the calculation of HDOP and VDOP parameters in particular. The first two values, E and N, are estimates of the horizontal plane position, while the last position, U, is an estimate of the vertical plane position. The equations are as follows:

$$HDOP = \sqrt{M_{11} + M_{22}} \tag{14}$$

$$VDOP = \sqrt{M_{33}} \tag{15}$$

I. Differential GPS

The goal of the DGPS is to improve the GPS system's accuracy. Two receivers are used in the DGPS system, one of which is fixed in a known position and the other of which is a common user who obtains nearly the same positioning data as the permanent receiver. Because the error components of the receivers are practically identical and because some errors that impact the user's positioning vary continuously, it is critical that both receivers capture the same data for the same satellites and at the same time [16].

A minimum of four satellites and two GPS receivers are required to determine the position of a particular receiver using a DGPS system, as shown in Fig. 4. The base station or reference station is the receiver whose position is known with high precision, whereas the rover receiver is the other receiver. The rover receiver must be within range of the base station to ensure a good connection for differential corrections transmission from the base station to the rover receiver [16].

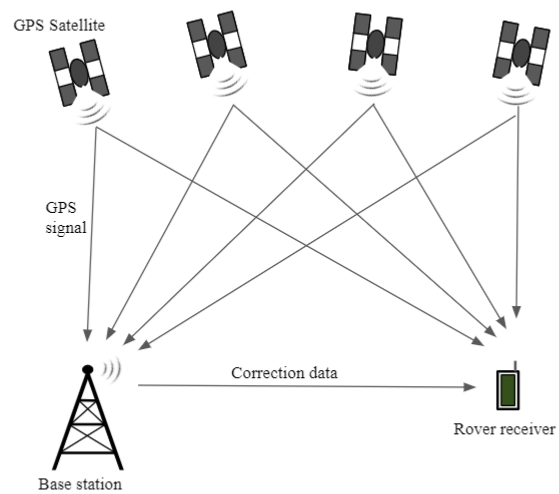

Fig. 4. Satellite geometric configuration and DOP.

DGPS techniques are divided into two categories: techniques based on pseudoranges and techniques based on carrier phase or Real-Time Kinematik (RTK). These techniques can be applied in real time or in post-processing. RTK is the technique that produces the best results (when used in post-processing), with accuracy in the decimeter range. Despite its high precision, this technique is limited to a region inside a 20 kilometers radius circle. The distance between the base station and the rover receiver, on the other hand, should be roughly 300 kilometers for pseudoranges based techniquescite [15]. The technique based on pseudo-distances will be used in this project, as well as the post-processing method.

J. Post-processing data

Post-processing is a method of differential correction that does not require any form of link between the base station and the rover receiver during the process of gathering data from both receivers, that is, the method of processing entails downloading the entire survey data and processing it in the office. It is, nevertheless, usual for the base station to begin collecting data before the rover receiver and/or to continue collecting data after the mobile receiver has completed collecting data [15].

K. Real time data

The idea of the real-time DGPS system is similar to that of the post-processing mode corrections. The primary difference is that the data from the receivers is not saved because corrections are calculated and sent to the rover receiver at the time of data collection. Unfortunately, this correction method has as its main disadvantages the fact that it is more expensive to execute and that it is less precise in position calculation (since accurate real-time orbits and satellite clock correction are required), with an error of around 3 meters [15].

L. Differential data sources

Differential correction data can be acquired from a variety of sources for use by the DGPS system in post-processing or real-time. The most straightforward method is to set up your own base station. Differential correction data for the DGPS system can also be obtained through companies that specialize in this field. Some of these businesses have their own satellites and receivers, among other things, in order to apply methods that allow them to obtain differential correction data. Finally, there are data sources owned by governments of specific countries, which supply correcting data at free charge to the customer [7].

M. Pseudoranges correction

Pseudorange is a measure of the distance between the satellite and the GPS receiver. This distance is obtained based on the measurement of the GPS signal transmitted by the satellite to the GPS receiver. The pseudorange obtained by the base station and rover receiver for each satellite in line of sight, however, must be corrected in order to increase the accuracy of the receiver positioning [6]. Before the correction, the pseudo-distance in meters to the i-th satellite is determined using the following expression [4]:

$$\rho^i = (T_R - T_{Ti}) \cdot c \tag{16}$$

where:
c = speed of light = 299792458 (m/s)
T_r = receive time of the GPS receiver's clock (seconds)
T_{Ti} = transmit time based on the Satellite clock (seconds)

To correct the errors, the general equation for computing the pseudo-distance in meters is as follows [13]:

$$\rho^i_{cor} = \rho^i + (\delta t_{sv} - T_{tropo} - T_{iono}) \cdot c \tag{17}$$

where: δt_{sv} = satellite clock offset (seconds)
T_{tropo} = troposphere delay (seconds)
T_{iono} = ionosphere delay (seconds)

The base station estimates the errors (which will later be transmitted to the rover receiver) of the pseudoranges for each satellite in line-of-sight after correcting them. It is vital to take into account the correct knowledge of the base station's ECEF coordinates in order for the rover receiver to accurately estimate its position on the earth's surface. In the scenario where $(x_i,\ y_i,\ z_i)$ represents the i-th satellite's position in meters and (x_m, y_m, z_m) represents the base station's position in meters, the geometric distance, R^i_m, in meters between the satellite and the base station is determined using the following equation [4]:

$$R^i_m = \sqrt{(x_i - x_m)^2 + (y_i - y_m)^2 + (z_i - z_m)^2} \tag{18}$$

The pseudorange value for the base station or rover receiver is calculated using the equation mentioned in (16). However, the pseudo-distance of the base station in meters can still be written as follows:

$$\rho^i_m = R^i_m + \delta^{clk}_m + \varepsilon_m \tag{19}$$

Where, δ^{clk}_m is base station clock offset, and ε_m represents the errors in the base station-to-satellite distance. The base station clock offset is determined using the Least Square approach during the base station position estimation procedure. In (19), we may isolate ε_m and get:

$$\varepsilon_m = \rho^i_m - R^i_m - \delta^{clk}_m \tag{20}$$

This error, which can be positive or negative and is measured in meters, is transmitted to the rover receiver, correcting the pseudorange error for the same satellite. It's also well knowledge that the general equation for the rover receiver's pseudorange is:

$$\rho^i_{u,cor} = \rho^i_u + (\delta t_{sv} - T_{tropo} - T_{iono}) \cdot c \tag{21}$$

By subtracting the error determined in (20) in (21), we can apply differential corrections to the rover receiver's pseudoranges in order to get a more accurate result about its position. Then we get:

$$\rho^i_{u,dif.} = \rho^i_u + (\delta t_{sv} - T_{tropo} - T_{iono}) \cdot c - \varepsilon_m \tag{22}$$

After correcting the pseudorange, the Least Square approach can be used to calculate the rover's position $(x_u,\ y_u,\ z_u)$.

III. Experimental Results

The results of the DGPS implementation will be presented in this section. In this research, data obtained by Telecommunications Institute (IT) was used to examine the performance of DGPS. Two antennas (RF1 and RF2) were installed on the top of the Instituto Superior Técnico's (IST) North Tower, in Alameda, to collect the GPS data. The ECEF coordinates of these antennas can be found in Table I, and using Google Earth, it is possible to show in Fig. 5 the antennas' location.

Since just two antennas were installed, the DGPS system was used in its most basic configuration, with only one base station and one rover receiver. The base station is specified as the antenna with code RF2, while the mobile receiver is defined as the antenna with code RF6 (**case A**). Meanwhile, a second scenario is considered, in which RF6 is now characterized as a base station and RF2 is described as a rover receiver (**case B**). A distance of 10.27 meters separates the antennas.

TABLE I
Antennas' ECEF coordinates

Antenna	ECEF $(x,\ y,\ z)$		
RF2	4918525.180 m	-791212.210 m	3969762.190 m
RF6	4918532.100 m	-791212.610 m	3969754.61 m

The analysis for cases A and B is the focus of the results reported below. The algorithm was put to the test for three

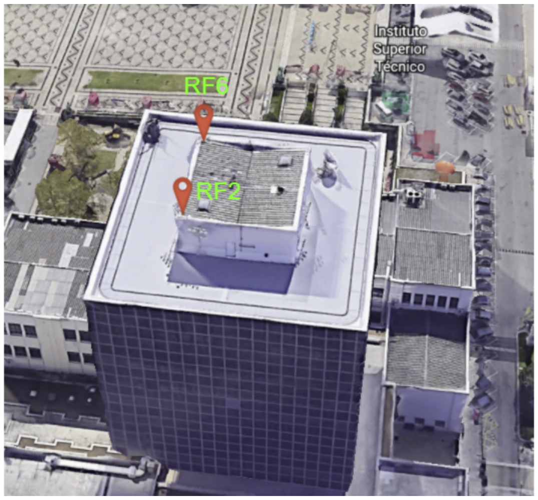

Fig. 5. Antennas' location.

different epochs, each with its own Time Of Week(TOW), this is the amount of time since the beginning of the week (from midnight Saturday to Sunday). And, because the results are nearly identical, just the results for TOW = 2800 will be shown.

A. Case A

In this case, there were a total of 10 satellites in common between the base station and the rover receiver. However, it was decided that the satellites' mask angles must be equal to or more than 5 degrees, and satellites with mask angles less than this number will be excluded. The 10 satellites in line-of-sight had an angle greater than 5 degrees at this period.

In addition, the position of the rover receiver was estimated using a conventional GPS system. Then, using the algorithm developed for the DGPS system, the position of the same receiver was computed. The results of the tests carried out are shown in Table II for a better comparative analysis and observation of the influence of the developed algorithm in determining the position of the rover receiver.

TABLE II
RESULTS FOR CASE A

Algorithm	ECEF (x, y, z)		
GPS	4918530.990 m	-791212.118 m	3969753.561 m
DGPS	4918533.103 m	-791211.743 m	3969754.113 m

Based on the data presented in Table II, we get a positioning error of 1.61 meters when using GPS and 1.42 meters when using DGPS. As we can see, when the DGPS system was used, a lower error was produced when compared to the GPS positioning result. This means that with the implementation of the DGPS system, the rover receiver's position may be calculated more accurately. Positioning errors were determined based on the difference of the coordinates obtained with the original position of the rover receiver (RF6) which is found in Table I.

B. Case B

The position of the rover receiver was estimated using the algorithm of a GPS and DGPS system, as in the previous case. Table III shows the test results.

TABLE III
RESULTS FOR CASE B

Algorithm	ECEF (x, y, z)		
GPS	4918523.067 m	-791212.584 m	3969761.638 m
DGPS	4918524.178 m	-791213.077 m	3969762.687 m

Based on the data presented in Table III, we get a positioning error of 2.22 meters when using GPS and 1.42 meters when using DGPS. This confirms that the DGPS has a positive impact on the estimation of the rover receiver's position. For this case, the positioning errors were computed based on the difference of the coordinates obtained with the original position of the rover receiver (RF2) which is found in Table I.

C. Pseudorange errors

Fig. 6 and 7 show the evolution of the pseudorange error as a function of time from each base station to satellite number 1, when TOW = 352800.

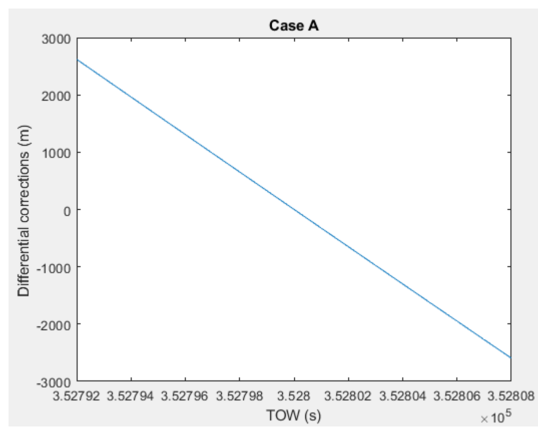

Fig. 6. Differential correction evolution as fucntion of time for case A.

Fig. 6 and 7 show that the errors have a similar evolution, as expected. However, the error correlation can be validated by subtracting the differential corrections of figures 2 and 3 from each other, instant by instant of time. Fig. 8 shows the result of this difference, and this error correlation is also shown in the improvement achieved when the rover receiver's position was estimated.

D. Geometry of the satellites

The geometric configuration of the satellite constellation was also studied through DOP at different times. Due to the similarity of the values in most situations, only the results for PDOP and HDOP will be shown. Fig. 9 shows a visual representation of these values based on the number of satellites available.

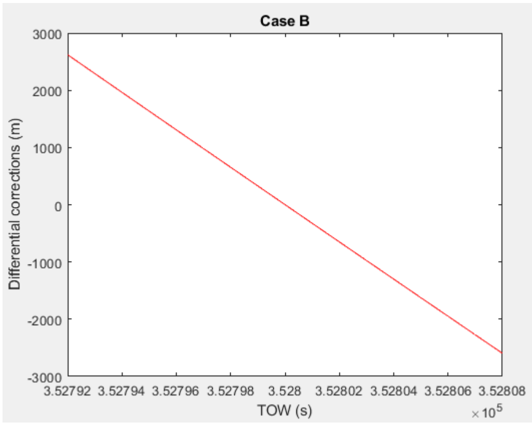

Fig. 7. Differential correction evolution as fucntion of time for case A.

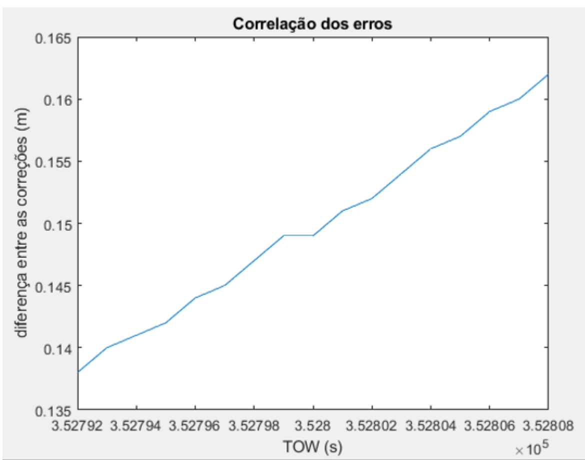

Fig. 8. Error correlation.

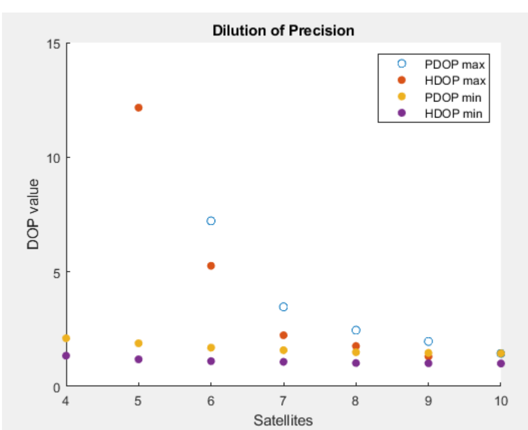

Fig. 9. PDOP and HDOP value.

Through a visual analysis on the graphics it is feasible to prove through visual inspection that the more satellites obtained in the measurement, the lower the PDOP and HDOP. And, because the calculated DOP values for the maximum satellites are around 2-3, the satellite constellation can be deemed to have good geometry at this epoch. Another point to consider based on the data is the HDOP numbers, which, as one would expect, are always lower than the PDOP.

IV. Conclusion

The objective of this project is to investigate how the DGPS system can increase the positioning accuracy of a specific GPS receiver. As illustrated in the research, GPS accuracy is limited by some errors, as a result, the DGPS system is used to improve receiver positioning. In this paper, we describe a method that takes advantage of pseudo-range data to reduce errors during the receiver positioning estimation using the Least Square method. We also looked into the geometry of the satellites to evaluate how crucial it is for receiver positioning estimation. In summary, the research presented in this paper demonstrated that the DGPS technique is an asset to correct a user's position.

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