Integrating Inventory Management and Distribution Scheduling for Clinical Supplies in a Hospital Context

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Abstract

Healthcare costs have been increasing over the past years, leading to an interest in improving logistic activities in hospitals. Hospitals usually have a central warehouse supporting the services and replenishing their warehouses. They are responsible for multiple items with different particularities, often with expiration dates. Each service is responsible for placing inventory orders to the central warehouse according to its needs. The complexity lies on deciding when and how much to order of each supply and scheduling the deliveries, while dealing with multiple constraints. Inventory management is therefore a challenging task where ordering, distribution and consumption must be coordinated between the different services and the central warehouse, maintaining wastage levels to a minimum and preventing stock-outs. With the aim of reducing costs and decreasing the variability in the workload of deliveries, a model that allows the optimization of inventory control and delivery scheduling is proposed. This work is motivated by the case of a Portuguese hospital, focusing on clinical supplies. Inventory policies for the items in each service and a schedule of deliveries are determined, considering capacity and human-resource constraints and the services' location considering routing. A robust optimization model is developed, complementary to the deterministic model, to address demand uncertainty. This work proposes an innovative approach to hospital inventory management, integrating inventory control with delivery scheduling. Both models are tested with hospital data and the obtained solutions show heavier schedules, with more deliveries, and less inventory at the services when compared with the hospital's current situation.

Keywords: Health Care Services Management, Hospital Inventory Management, Inventory Control, Distribution Scheduling, Clinical Products

1. Introduction

Healthcare costs have been gradually increasing in developed countries over the past years. This leads to an emerging interest in improving logistic activities in hospital, that constitute a large portion of hospital expenses, i.e. generally more than 30%, only surpassed by staff related costs [1]. Furthermore, there is a growing pressure in improving performance and delivering healthcare more efficiently [2], and, as a matter of fact, cost reductions from optimizing logistics activities do not affect patient care and safety [3]. Hospitals are complex settings and logistic activities tend to be performed by clinical staff without a background on logistics or inventory management, leading to experienced-based decisions as opposed to optimized solutions. Appropriate inventory policies must be identified and applied in order to increase efficiency, decrease wastage and avoid stock-outs, allowing not only proper operation in each service but also coordination between each service and the central warehouse (CW), improving the functioning and performance of the hospital as a whole. In addition, it is important to have practical schedules of deliveries from the CW to each service, adapted to the necessities of each service, but focusing as well on a balanced workload throughout the workdays.

The aim of this work is to propose an optimization model for a new approach in healthcare inventory management, the Inventory Control and Distribution problem, minimizing inventory holding costs and the variability in the workload of deliveries, to improve the clinical inventory management and distribution activities in hospitals. Capacity and human-resource constraints, uncertainty in the demand and routing considerations are also taken into account. This problem originates from a private hospital in Portugal (PPH), composed by multiple services that receive clinical supplies from the CW.

To clarify the scope of this particular problem, this work is classified as a multi-item multi-location

Inventory Control and Distribution problem. First, the work is focused on multiple items and on multiple services within the hospital. Regarding hospital inventory management decisions, these can be driven by two approaches: (i) inventory oriented, regarding inventory policies and control, and (ii) schedule oriented, that are concerned with planning and scheduling of distribution activities. This work integrates both. By aiming to determine appropriate inventory policies for the clinical items consumed by each service, falls within inventory control (also commonly referred to as inventory management or policy). The distribution and scheduling stream approaches are commonly classified as inventory distribution problems, and by determining a schedule of deliveries to the services this work considers inventory distribution. It is worth nothing that although this work addresses routing, it is not classified as an inventory routing problem since routing is not the focus. It is included to better estimate the time spent in deliveries and better perform the distribution, guaranteeing that the time available for deliveries per day is not exceeded. It is a part of the constraints and not a part of the objective function, and thus is not a crucial part of the problem. Accordingly, this work falls within a new approach in healthcare inventory management, the Inventory Control and Distribution problem.

The contributions of this study are fourth-fold. First, a new problem is addressed and modelled integrating both inventory control decisions and distribution decisions, considering multiple services within a hospital. It is an innovative strategy aiming to fill a gap in the hospital inventory management literature. Second, this work proposes a robust optimization model, to address the uncertainty in demand. This is the first work in the hospital inventory management field to consider a robust optimization approach. Third, routing is considered to better estimate the time spent in deliveries and take into account the distance between the services, which was not considered so far in this field. Finally, this work is applied to a real-case scenario of PPH, to address the current Inventory Control and Distribution problem, being tested with hospital data. A large number of items and services are considered, creating a complex setting and providing insights to hospital inventory management. Authentic recommendations and findings can be provided to the hospital and real improvements achieved.

The remainder of this paper is organized as follows. Section 2 describes the inventory control and distribution problem and gives insights on the hospital. Section 3 introduces an overview of hospital inventory management literature. Section 4 presents the deterministic and the robust optimization models. Section 5 concerns the application of the model to the hospital case and discusses the results of the computational experiments. Section 6 provides conclusions and future work suggestions.

2. Background

Hospitals are responsible for a significant amount of items, of different kinds and often with expiration dates, such as clinical supplies, pharmaceutical products, administrative and laundry items. Clinical supplies (e.g. syringes and needles) are the focus of the current work, since it is the PPH department with a more evident need. Hospitals are organized in multiple services distributed among different floors and separate locations and usually have a CW supporting their activity. Each service is responsible for placing inventory orders to the CW according to its needs. The complexity lies on (i) deciding when and how much to order each clinical supply, (ii) organizing and scheduling transportation and deliveries from the CW to the services, as well as on (iii) dealing with capacity and human resource constraints, in an environment such as healthcare where the clinical inventory demand is uncertain and stock-outs are usually not allowed. Therefore, in order to improve the clinical inventory management, preventing stock-outs and maintaining the wastage levels to a minimum, it is important to have proper inventory policies (i.e., inventory rules and guidelines) adapted to the activity, needs and particularities of eachservice but subject to the existing limitations (such as limited storage space). These should be supported by practical schedules with, for example, a balanced workload throughout the week.

Typically, the material distribution in hospitals follows a multi-echelon network, with a CW receiving items from suppliers and delivering them to each service. Inventory control decisions focus on determining the optimal policy, which includes i) defining the inventory cycle and ii) defining the parameter setting for the s, R, Q and/or S levels. There are usually two possible review cycles to be adopted: periodic and continuous. In continuous replenishment, an order is placed every time an item falls bellow a reorder point s. In periodic models an order can be placed in every review period R, or following a schedule of deliveries where each service has prespecified days for inventory deliveries. An order can be made only if the stock level is bellow a reorder point s, or in every review period regardless of the stock level. A fixed reorder quantity Q can be ordered, or the inventory level can be increased to an order-up-to level S. It is worth noting that there are variations to these policies and other parameters may be defined.

Inventory logistics and management problems in healthcare can be divided in three hierarchical lev-

Item	Stock Level at 31/12	Avg. Daily Demand
6.1	400	89
6.2	998	48
6.3	0	39
6.4	400	17
6.5	633	15
6.6	907	14
6.7	168	7

Table 1: Inventory levels at 31/12/2018 and average daily consumption in 2018.

els: strategic, tactical and operational, and the latter is further distinguished between offline and online [4]. The strategic level addresses structural long-term decisions (e.g. warehouse location). Tactical decisions (e.g. supplier selection) concern the implementation of the strategy and planning of processes, and have a shorter planning horizon than strategic decisions. Operational planning is characterized by short-term decisions concerning the execution of processes. Decisions in advance (proactive) are offline decisions (e.g. placing inventory orders), while online operational planning are real-time decisions (e.g. emergency replenishment). In this work, two main decisions are addressed in a hospital context: to determine a schedule for clinical inventory deliveries to the services, and to settle inventory policies for the items in each service, for short planning periods. These decisions fall into the offline operational planning category.

The work is motivated by the case of PPH. The goal is to determine inventory policies for the clinical items in each service and a new schedule of deliveries from the hospital's CW to the services. A schedule of clinical supply deliveries is already in place, defined by the department responsible for the distribution activities together with the services. The current schedule of deliveries shows a considerable difference in the time spent in deliveries and in the number of services visited per day (between 142 and 100 minutes, and between 5 to 9 services). The inventory quantities in many services are not aligned with the average consumption, and excessive stock levels are commonly found. To quantify this, an example is shown in Table 1 for the 7 most used items in the Emergency Room. Furthermore, due to obsolesce and expiration, a high wastage level is registered at PPH. Therefore, appropriate inventory policies and schedule of deliveries must be identified and put into practice.

3. Literature Review

Although significantly studied in industrial environments, material logistics is less explored in the healthcare setting. Nevertheless, the literature on healthcare and hospital logistics, including inventory management and distribution activities, has been increasing in the past years [1]. The attention given to this area continues to enlarge, given the further understanding of the importance of logistics activities and appropriate inventory management at healthcare providers. [1] presents a state-of-the-art of the literature on material logistics management in hospitals, categorizing the studies in four research streams: (i) Supply and procurement, (ii) Inventory management, (iii) Scheduling and distribution, and (iv) Holistic supply chain management. Inventory management (ii) concernslocation planning, inventory policies and control and inventory classification. Scheduling and distribution (iii) focus on the transportation of the inventory itself, divided in internal and external distribution. These areas are the focus of the current work, in particular inventory policies and internal delivery scheduling.

Within inventory management, and particularly within inventory policies and control, multiple studies focus on a single warehouse or service. [5] determine s, c, and S levels for the (R, s, c, S) policy, an extension of the (R, s, S) policy, for multiple items in a single location, and demand is considered normally distributed. [6] determine the service level, review period R and order-up-to level S for sterile and bulk items within an intensive care unit, modeling the demand as stochastic with a normal distribution and considering space constraints. [7] propose two models for the disposable items in a service, considering capacity limitations and service level. Here, the demand is assumed to follow a Poisson distribution. Two periodic-review policies are compared: the (R, s, Q) policy, and the (R, s, S) policy, concluding that the latter uses capacity more efficiently (the fill rate - the fraction of demand satisfied directly from stock on hand - is higher). Similarly, [8] determine the reorder point s and order-up-to level S for multiple items within a single service, with stochastic demand. [9] combine a periodic-replenishment with a (s, S) policy, with lower costs, with a continuous replenishment, to avoid stock-outs, under both deterministic and normally distributed demand.

[10] study an one warehouse, one pharmacy, replenishment problem, considering inventory and transportation costs, and the vehicle capacity, following the Economic Order Quantity model, with deterministic demand. [11] follow an order-up-to level S policy for a hospital with one central pharmacy and multiple services with capacity limitations, modeling the demand as stochastic. The authors show that the optimal policy is obtained by setting the reorder point s one unit lower than the order-up-to level S. [2] compare two possible networks within hospital inventory management for a single item and a periodic order-up-to-level S policy: a three-echelon system, with a CW supporting multiple hospitals each serving numerous ser-

vices, and a two-echelon system where a distributor would operate the CW and deliver directly to the services. The latter results in lower costs and equivalent service levels when compared to the three-echelon system.[12] propose an optimization model that integrates production and distribution in a two-echelon network with one pharmaceutical company and one hospital, for a continuous review policy (s, Q) with normally distributed demand. In an extension of this work, [13] address other considerations such as uncertainty in the hospital's expiry rate and holding cost following a fuzzy-approach and uncertainty on the quantity received by the hospital following a normal distribution.

On what concerns distribution planning and scheduling decisions, [14] propose a scheduling approach for the inventory delivery from the CW to multiple services, with known safety stocks and considering capacity restrictions and human resource constraints (with a limited time for deliveries), while focusing on a balanced schedule. In [15] a weekly distribution plan to minimize service visits is determined considering prespecified safety stocks and service storage capacity. The vehicle capacity and number of services visited per day are limited. In an extension of this work, [16] three types of vehicles with different capacities and restrictions and two teams can perform the distribution, guaranteeing a balanced workload between the teams. The demand is assumed to be known (i.e. deterministic) in these three works ([14]-[16]).

To the best of our knowledge, determining both inventory policies and distribution scheduling for the services within hospital inventory management has not yet been done. Uncertainty is already considered through stochastic models, but other methods such as robust optimization are yet to be considered. In stochastic approaches the demand is considered to follow a probability distribution, but it may not be available or be difficult to accurately estimate [17]. On the other hand, robust optimization does not need a probability distribution. Instead, the uncertain parameter is assumed to belong to an uncertainty set, characterized only by a nominal value and a maximum deviation. Additionally, it has a high and relatively simple applicability to different problems, as well as computational tractability [18] [19]. Therefore, this approach is further explored in this work with the aim of addressing demand uncertainty. Routing has never been considered in a hospital setting for the inventory distribution to multiple services. Nevertheless, it enables a better estimate of the time spent in deliveries and allows to plan the distribution considering the distance among the services (e.g. making the deliveries in the same day to services that are close to one another). This paper aims to fulfill this gap. First, both the inventory policy and schedule of deliveries are determined in an integrate way, for multiple items in services of an hospital. Uncertainty in the demand is considered through a robust optimization model following the cardinality-constrained approach [20]. Routing constraints are modelled to accurately account for the traveling time between services, in order to not exceed the limited time available for deliveries while guaranteeing a balanced schedule.

4. Optimization models

Two models are developed: a deterministic integer programming model, where demand for the clinical items is assumed to be known in advance, and a robust optimization model where uncertainty in the demand is introduced to better capture the real conditions of inventory management.

The models determine a schedule of clinical inventory deliveries from the CW to the services as well as an inventory policy for the items in each service, minimizing the inventory holding costs and decreasing the variability in the workload of deliveries. A reorder point, order-up-to-level (s, S) policy is followed, with $S = s + 1$ (following the results of [11]). Thus, a service orders an item if the stock level (Q^0) is bellow or equal to the reorder point s. An order can be placed at the scheduled delivery days for that service. The quantity to order is such that the stock increases to an order-up-to level S.

In a real hospital setting, inventory management faces multiple requirements that need to be incorporated in the model. First, the demand must be satisfied, avoiding stock-outs. In addition, there are human-resource and capacity constraints. The delivery vehicle and the service warehouses have limited storage space and so the inventory transported and kept at the services cannot exceed the available capacity. The daily time available for distribution of inventory from the CW to the services is restricted. Therefore, the time needed to perform the deliveries in each service is taken into account, as well as the set-up-time in the CW to prepare the vehicle for the distribution. Moreover, it is important to consider the traveling time between the services, which requires the introduction of vehicle routing constraints in the model. Usually, services in hospitals are located in a central building where multiple services are located in the same floor. In these cases, the traveling time between services might be negligible. So, instead of considering each service as an individual location, services are grouped in clusters according to proximity. The traveling time among the services in the same cluster is negligible as they are located close to each other. The model defines a route for the clusters to be visited each day, making it possible to account for the travel time.

Some assumptions are made: (i) the time spent at each service is independent of the quantity to deliver, (ii) the set-up time does not depend of the quantity to deliver, (iii) the order is made and delivered in the same time period (when an order is placed, the order quantities are determined based on the sotck-level at the beginning of each time period), and (iv) the stock level at the CW is sufficient to guarantee the services' demand.

4.1. Deterministic Model

A deterministic approach is first proposed through an integer programming model. The notation is in Table 2.

Indices and Sets	
$i \in I$	clinical items
$w \in W$	services
$t \in T$	time periods
$f \in F$	clusters of services with cluster 1 corresponding to the CW
Subsets	
I_w	items used in service w
W_f	services in cluster f
Parameters	
Items	
d_{iwt}	demand of item i in service w at time period t
$\overline{d}_{iw}$	average time period demand of item i in service w
v_i	volume of item i
p_i	cost of item i
Services	
c_w	capacity of the warehouse of service w
q^0_{iw}	initial quantity of item i in the warehouse of service w
Delivery	
a_t	available time for deliveries in time period t (min)
h	vehicle capacity
m^0	set up time (min)
m_w	time spent in service w (min)
$m_{f_1 f_2}$	time between a service in cluster f_1 to a service in cluster f_2 (min)
α	constant to adjust the relevance of balancing the workload of deliveries
Variables	
Auxiliary	
Y_{wt}	1, if there is a delivery to service w in time period t 0, otherwise
N_w	maximum number of consecutive time periods without a delivery to service w
A_{wt}	number of consecutive time periods without a delivery to service w at time period t
Q_{iwt}	quantity of item i in the warehouse of service w at the end of time period t
DT_t	total delivery time in time period t (min)
DT_{max}	maximum delivery time in the planning period (min)
DT_{min}	minimum delivery time in the planning period (min)
Z_{ft}	1, if cluster f is visited in time period t 0, otherwise
$R_{f_1 f_2 t}$	1, if cluster f_2 is visited after cluster f_1 in a time period t 0, otherwise
Decision	
S_{iw}	order-up-to level of item i in service w
X_{iwt}	quantity of item i delivered to service w at time period t

Table 2: Notation

The model is organized by groups of constraints. Constraints (2) to (5) concern inventory management, with the definition of inventory levels. The inventory policy is introduced in constraints (6) to (9).The scheduling and distribution is defined in constraints (10) to (16). Constraints (17) and (18) concern capacity limitations, followed by routing constraints (19) to (25). The delivery time is addressed in constraints (26) to (29).

$$\min \sum_{i\in I_w}\sum_{w}\sum_{t} p_i Q_{iwt} + \alpha * (DT_{max} - DT_{min}) \quad (1)$$

$$\text{s.t. } Q_{iw1} = q^0_{iw} - d_{iw1} + X_{iw1}, \ \forall i \in I_w, w \in W \quad (2)$$

$$Q_{iwt} = Q_{i,w,t-1} - d_{iwt} + X_{iwt}, \ \forall i \in I_w, w \in W, t \in T \setminus \{1\} \quad (3)$$

$$Q_{iwt} \geq d_{i,w,t+1}, \ \forall i \in I_w, w \in W, t \in T \setminus \{|T|\} \quad (4)$$

$$Q_{iw|T|} \geq \overline{d}_{iw}, \ \forall i \in I_w, w \in W \quad (5)$$

$$X_{iw1} \geq S_{iw} - q^0_{iw} - M(1 - Y_{w1}) \ \forall i \in I_w, w \in W \quad (6)$$

$$X_{iwt} \geq S_{iw} - Q_{i,w,t-1} - M(1 - Y_{wt}) \ \forall i \in I_w, w \in W, t \in T \setminus \{1\} \quad (7)$$

$$X_{1wt} \leq \max(0, S_{iw} - q^0_{iw}) \ \forall i \in I_w, w \in W \quad (8)$$

$$X_{iwt} \leq \max(0, S_{iw} - Q_{iwt}) \ \forall i \in I_w, w \in W, t \in T \setminus \{1\} \quad (9)$$

$$A_{w,t-1} + 1 - Y_{wt}|T| \leq A_{wt} \leq A_{w,t-1} + 1 + Y_{wt}|T| \ \forall w \in W, t \in T \setminus \{1\} \quad (10)$$

$$A_{wt} \leq (Y_{wt} - 1)|T| \ \forall w \in W, t \in T \quad (11)$$

$$A_{w,1} = 1 - Y_{w1} \ \forall w \in W \quad (12)$$

$$N_w \geq A_{wt} \ \forall w \in W, t \in T \quad (13)$$

$$S_{iw} \geq (N_w + 1)\overline{d}_{iw} \ \forall i \in I_w, w \in W \quad (14)$$

$$\sum_{i\in I_w} X_{iwt} \geq Y_{wt}, \ \forall w \in W, t \in T \quad (15)$$

$$X_{iwt} \leq \frac{\min(h, c_w)}{\min_i v_i} Y_{wt}, \;\; \forall i \in I_w, w \in W, t \in T \quad (16)$$

$$\sum_i Q_{iwt} v_i \leq c_w, \;\; \forall w \in W, t \in T \quad (17)$$

$$\sum_w \sum_i X_{iwt} v_i \leq h, \;\; \forall t \in T \quad (18)$$

$$Z_{ft} \geq Y_{wt}, \;\; \forall f \in F \setminus \{1\}, w \in W_f, t \in T \quad (19)$$

$$Z_{ft} \leq \sum_{w \in W_f} Y_{wt}, \;\; \forall f \in F \setminus \{1\}, t \in T \quad (20)$$

$$Z_{1t} \geq Y_{wt}, \;\; \forall w \in W, t \in T \quad (21)$$

$$Z_{1t} \leq \sum_w Y_{wt}, \;\; \forall w \in W, t \in T \quad (22)$$

$$\sum_{f_1 \neq f_2} R_{f_1,f_2,t} = Z_{f_2,t}, \;\; \forall f_2 \in F, t \in T \quad (23)$$

$$\sum_{f_1 \neq f_2} R_{f_1,f_2,t} = \sum_{f_1 \neq f_2} R_{f_2,f_1,t}, \quad (24)$$
$$\forall f_2 \in F, t \in T$$

$$\sum_{f_1} \sum_{f_2} R_{f_1,f_2,t} \leq |C| - 1, \quad (25)$$
$$\forall t \in T, C \subseteq F, |S| \geq 2$$

$$DT_t = \sum_w (m^0 + m_w) * Y_{wt} + \quad (26)$$
$$\sum_{f_1} \sum_{f_2} m_{f_1,f_2} R_{f_1,f_2,t}, \;\; \forall t \in T$$

$$DT_t \leq a_t, \;\; \forall t \in T \quad (27)$$

$$DT_{max} \geq DT_t, \;\; \forall t \in T \quad (28)$$

$$DT_{min} \leq DT_t, \;\; \forall t \in T \quad (29)$$

$$S_{iw}, Q_{iwt}, X_{iwt}, DT_t, DT_{max}, DT_{min}, A_{wt} \geq 0 \quad (30)$$
$$\forall i \in I_w, w \in W, t \in T$$

$$Y_{wt}, Z_{ft}, R_{f_1,f_2,t} \in \{0, 1\}, \quad (31)$$
$$\forall f, f_1, f_2 \in F, w \in W, t \in T$$

The objective function (1) minimizes the total inventory holding costs along the planning period, in order to minimize inventory costs across all services and time periods (first term). The difference between the maximum and minimum time spent in deliveries in the planning period is also minimized, to obtain a delivery schedule with a balanced workload over the working days (second term). The parameter α allows to give a relative importance to the second term and to countervail it regarding the difference in order of magnitude to the first term. Constraints (2) and (3) balance the inventory level over time, for each clinical item, service and time period. Constraints (4) and (5) ensure that the inventory level at each service is enough to satisfy the demand in all time periods. For this deterministic model, demand is assumed to be known in advance. So, the inventory level at the end of a time period is equal to the inventory level at the beginning of the next period, and must be enough to satisfy the demand of this next time period. This is guaranteed by constraints (4). Following this reasoning, constraints (5) ensure that the inventory level at the end of the last time period is equal or higher than the average daily demand for each item and service, to keep quantity available for upcoming periods. Constraints (6)-(9) model the periodic (s, S) inventory policy: if there is a delivery to a service, all the clinical items with a quantity bellow the corresponding reorder point s are replenished with a quantity that increases the inventory level to the order-up-to level S. So, the quantity to be delivered of each item, to a given service and in a given time period is equal to the order-up-to level of the item and service minus the inventory quantity. Constraints (8) and (9) are presented as non-linear for simplification purposes, to facilitate the reading and understanding of the model. However, in order to develop an integer linear programming model, the two sets of constraints can be linearized. Constraints (10)-(13) determine the maximum number of time periods a service is without a clinical inventory delivery. Constraints (14) ensure that the order-up-to level is enough to cover all the periods without delivery (given by N_w). Because the order-up-to-level of an item in a service is the inventory level of that item after a replenishment, it needs to be able to satisfy the demand everyday until the following delivery. Constraints (15) imply that if there is a delivery there is at least one unit to be delivered. Constraints (16) guarantee that products at the services can only be replenished if the service is visited and link variables X and Y. Each service warehouse has a limited capacity, and constraints (17) state that the total volume of items in each service does not exceed it. Constraints (18) guarantee that the transportation vehicle capacity is not exceeded. Constraints (19) and (20) concern the clusters of services, ensuring that if a service within a cluster is replenished, then the cluster is visited. Constraints (21) and (22) ensure that if there are deliveries in a time period the CW cluster is visited. Constraints (23) and (24) are routing constraints, implying that if a cluster of services is visited at a given time period then the route must include that cluster. These are linking constraints between variables Z and R. Constraints (25) are sub-tour elimination constraints. Constraints (26) define the delivery time in each time period as the sum of the set up time, plus the time spent in each visited service, and the traveling time between clusters of services. Constraints (27) guarantee that the time spent in deliveries does not exceed the available time, and constraints (28) and (29) calculate its the maximum and minimum values. Constraints (30) and (31) are domain constraints.

4.2. Robust Optimization Model

In order to better capture the real conditions of inventory management, a stochastic approach is considered to handle demand uncertainty. As of now, the demand was assumed to be known. However, in the real setting of clinical inventory management, demand has an uncertain behavior. While some items can maintain a relatively stable demand, this is not the case for all clinical materials, which can show volatility. To address this uncertainty, complementary to a deterministic model, a robust optimization approach is developed.

The robust approach is based on the cardinality-constrained approach proposed by [20]. Uncertainty is considered on the demand of each item in each service at each time period, d_{iwt}. The constraints regarding the inventory level over time (2) and (3) and the constraints ensuring that the inventory level is enough to satisfy the demand (4) are therefore subject to uncertainty, specifically including uncertainty on parameters d_{iwt}.

Each parameter d_{iwt} is now modelled as an independent, symmetric and bounded random variable $\tilde{d}_{iwt}$, with values in $[\bar{d}_{iwt}$-$\hat{d}_{iwt}$;$\bar{d}_{iwt}$+$\hat{d}_{iwt}]$, where $\bar{d}_{iwt}$ is the expected demand and $\hat{d}_{iwt}$ is the maximum variation of the demand of item i in service w at time period t (it is assumed that $\hat{d}_{iwt} < \bar{d}_{iwt}$). No specific distribution is assumed. These random parameters are now included in constraints (2)-(4), and therefore any possible value of $\tilde{d}_{iwt}$ needs to be taken into consideration.

In the present approach, the total scaled deviation of variable $\tilde{d}_{iwt}$ from $\bar{d}_{iwt}$ cannot exceed a prespecified value Γ_{iwt}, i.e., parameters $\tilde{d}_{iwt}$ can vary from their nominal value $\bar{d}_{iwt}$ a maximum of Γ_{iwt} %, with Γ_{iwt} varying from 0 to 1 (or to 100%).

The cardinality-constrained uncertainty set ($w \in W, i \in I_w, t \in T$) is then given by:

$$U_{iwt} = \{ \quad (\tilde{d}_{iwt}) : \frac{|\tilde{d}_{iwt} - \bar{d}_{iwt}|}{\hat{d}_{iwt}} \leq \Gamma_{iwt}, \quad \tilde{d}_{iwt} \in [\bar{d}_{iwt} - \hat{d}_{iwt}; \bar{d}_{iwt} + \hat{d}_{iwt}]\}$$

Robust optimization is only logic for inequality constraints. The constraints (2) - (4 can be transformed from equality constraints to inequality constraints since, as the goal is minimizing inventory holding costs, having = or ≥ is the same. The robust equivalent of constraints (2) - (4) can be formulated as:

$$Q_{iw1} \geq q^0_{iw} - \tilde{d}_{iw1} + X_{iw1}, \quad \forall \tilde{d}_{iw1} \in U_{iw1}, w \in W, i \in I_w \tag{2R}$$

$$Q_{iwt} \geq Q_{i,w,t-1} - \tilde{d}_{iwt} + X_{iwt}, \quad \forall \tilde{d}_{iwt} \in U_{iwt}, w \in W, i \in I_w, t \in T \setminus \{1\} \tag{3R}$$

$$Q_{iwt} \geq \tilde{d}_{i,w,t+1}, \quad \forall \tilde{d}_{iwt} \in U_{iwt}, w \in W, i \in I_w, t \in T \setminus \{|T|\} \tag{4R}$$

Since constraints (2R) - (4R) need to be satisfied for every value in the uncertainty set U_{iwt}, then they must also hold for the worst case, which in this approach is the maximum of the right-hand side. In other words, we can reformulated them as:

$$Q_{iw1} \geq q^0_{iw} + X_{iw1} - \min_{\tilde{d}_{iw1}} \quad \forall w \in W, i \in I_w \tag{2R'}$$

$$Q_{iwt} \geq Q_{i,w,t-1} + X_{iwt} - \min_{\tilde{d}_{iwt}} \quad \forall w \in W, i \in I_w, t \in T \setminus \{1\} \tag{3R'}$$

$$Q_{iwt} \geq \max_{\tilde{d}_{iwt}}, \quad \forall w \in W, i \in I_w, t \in T \setminus \{|T|\} \tag{4R'}$$

Observe that in this case, the maximum and the minimum values in the above expressions can be easily determined. In fact, max $= \bar{d}_{iwt} + \Gamma_{iwt} * \hat{d}_{iwt} and \min = \bar{d}_{iwt} - \Gamma_{iwt}\hat{d}_{iwt}$.

Now, the non-linear constraints (2R'), (3R') and (4R') can be linearized as follows:

$$Q_{iw1} \geq q^0_{iw} - (\bar{d}_{iw1} - \Gamma_{iw1}\hat{d}_{iw1}) + X_{iw1}, \quad \forall w \in W, i \in I_w$$
$$Q_{iwt} \geq Q_{i,w,t-1} - (\bar{d}_{iwt} - \Gamma_{iwt}\hat{d}_{iwt}) + X_{iwt}, \quad \forall w \in W, i \in I_w, t \in T \setminus \{1\}$$
$$Q_{iwt} \geq \bar{d}_{i,w,t+1} + \Gamma_{i,w,t+1} * \hat{d}_{i,w,t+1}, \quad \forall w \in W, i \in I_w, t \in T \setminus \{|T|\}$$

The linear robust optimization model (RLM) can therefore be written as:

$$\begin{aligned}
\min \quad & (1) \\
\text{s.t.} \quad & (5) - (31) \\
& Q_{iw1} \geq q^0_{iw} + X_{iw1} - \bar{d}_{iw1} + \Gamma_{iw1}\hat{d}_{iw1}, \quad \forall i \in I_w, w \in W && (32) \\
& Q_{iwt} \geq Q_{i,w,t-1} + X_{iwt} - \bar{d}_{iwt} + \Gamma_{iwt}\hat{d}_{iwt}, \quad \forall i \in I_w, w \in W, t \in T \setminus \{1\} && (33) \\
& Q_{iwt} \geq \bar{d}_{i,w,t+1} + \Gamma_{i,w,t+1} v_{i,w,t+1} \hat{d}_{i,w,t+1}, \quad \forall i \in I_w, w \in W, t \in T \setminus \{|T|\} && (34) \\
& && (35)
\end{aligned}$$

The cardinality-constrained robust optimization approach allows to include uncertainty in the problem without much knowledge on the uncertain parameters. Only the nominal value and maximum deviation are needed for each uncertain parameter. In addition, the level of robustness can be controlled (Γ_{iwt}), adjusting the level of conservatism of the solution. One of the advantages of this approach is its computational tractability, which enables a high applicability to real-life cases. The robust counterpart of the model is also a linear programming model, which is particularly of interest for large problems. Nevertheless, this approach can still be computationally expensive for large instances and might depend of the value of the Γ_{iwt} parameters. Also, the fact that only two values (nominal value and maximum deviation) are used to characterize the uncertain parameters might be insufficient to describe the behavior of particular parameters [21].

5. Results

Both models are tested with PPH data using different instances. The deterministic model is first addressed (tested for two α values), followed by the robust approach (tested for two α and six Γ values). The models are coded and solved using IBM ILOG CPLEX 12.5.1, and tests are performed on a computer with an Intel(R) Xeon(R) processor of 3.33 GigaHertz and 6 cores, and 24 GigaByte RAM.

The models are applied to the PPH real setting. The hospital is open 24h a day, every day. It is composed by a total of 36 services and is organized in a main building but with some services at close individual locations. Of the 36 services, 19 receive clinical items from the CW and are the ones considered in this work. There is a significant amount of clinical items at use in the hospital but not all services consume the same items.

The demand data of PPH used in this work corresponds to the consumption of clinical inventory registered in each service in 2018, consisting of two thousand different items. Since not all of these items have a meaningful consumption an ABC analysis is performed to reduce the items to the most significant ones. This analysis was performed to each service selecting the items in class A that correspond to 80% of the service's annual consumption. The total number of different items is reduced to 104. In addition to a decrease on the complexity, this strategy allows to focus the inventory policy on the commonly used clinical items. Considering all the items consumed by the hospital in 2018 would mean that an inventory policy would also be determined for items with sporadic consumption, which is not only unnecessary but may generate inaccurate results due to the lack of consumption data.

5.1. Deterministic Model

The deterministic model is tested for 10 real-word instances, representing 10 weeks in PPH in 2018. This represents a situation where the demand is perfectly known in advance. Two α values (1 and 5) are employed to vary the relevance of the balanced workload term in the objective function. The running time is 600 seconds. For simplification purposes the results are only presented for one instance (Table 2). The remaining ones have similar results.

The instances generate feasible results with a small gap (from 0,46% to 19,24%) within a reasonable running time in 18 runs. In two runs, no integer solution is found. The deterministic results show appropriate decisions, adequate to the needs of each service, on the inventory policy and schedule of deliveries. Compared to the PPH case, the model proposes more regular deliveries and lower inventory quantities at the services, with the possibility of balanced schedules.

5.2. Robust Optimization Model

The demand for most of the items in a hospital environment is unknown in advance. While some items might maintain a high and relatively stable demand, this is not the case for all clinical materials, that can show volatility and demand uncertainty. Therefore, in order to better capture the setting of hospital inventory management, demand uncertain must be included in the model. So, the developed robust optimization model is applied to the PPH case. Twelve different instances are employed, each corresponding to the average demand per time period and demand range of each month of 2018, for each item in each service, registered in PPH. A running time of 1800 seconds is employed. The instances are tested for two different values of parameter α (1 and 5), and for 6 different equally spaced values of Γ_{iwt}, from the deterministic case (Γ_{iwt}=0) to the worst case scenario in terms of deviation from the expected value (Γ_{iwt}=1).

The tests generate feasible results with a considerable gap (from 2,75% to 23,82%), in 100 of 144 runs. In the remaining ones, no integer solution is found. Similarly to the deterministic case, the results are only presented for one instance (Table 3). The ”—” indicates that no solution is found.

The objective values for different Γ_{iwt} show that the robust approach for small values of Γ_{iwt} has lower objective values compared to the deterministic case. Since there is a possible higher demand, the model follows a conservative approach and assumes a higher consumption. So, even with changes to the inventory policy (higher (s, S) levels), the overall inventory levels are lower for Γ_{iwt}=0,2 and Γ_{iwt}=0,4 than in the deterministic case. Lower in-

[0] The Gap formula is given by: $\frac{|BestBound-BestInteger|}{(1^{-10}+|BestInteger|)}$

Nb of Constraints	Total Nb of Variables	Nb of Binary Variables	Nb of Integer Variables	α	RL Time (s)	Objective Value RL	Objective Value	Holding Cost (€)	Difference in Delivery Time (min)	Gap (%)	Best Bound
5578	3839	305	1501	1	0,03	6304,11	9514,29	9505,29	9	6,09	8935,05
				5	0,06	6342,74	9539,70	9474,7	13	5,06	9005,14

Table 3: Results of one instance with the deterministic model.

Nb of Constraints	Total Nb of Variables	Nb of Binary Variables	Nb of Integer Variables	Γ	α	RL Time (s)	Objective Value RL	Objective Value	Holding Cost (€)	Difference in Delivery Time (min)	Gap (%)	Best Bound
8950	6541	305	2238	0	1	0,11	4865,47	7432,25	7390,25	42	4,08	7129,37
					5	—	—	—	—	—	—	—
				0,2	1	0,23	4362,99	6523.48	6433,48	18	17,81	5361,27
					5	0,20	4362,99	6722,50	6707,50	3	21,41	5283,14
				0,4	1	0,08	4785,63	7899,01	7839,01	12	22,83	6095,76
					5	0,11	4785,62	7331,54	7301,54	6	18,41	5982,12
				0,6	1	0,14	5239,33	7935,20	7884,20	51	14,84	6757,34
					5	0,19	5239,33	8127,91	8182,91	9	18,15	6652,56
				0,8	1	0,06	5661,97	8530,79	8461,79	69	12,76	7442,68
					5	0,13	5661,97	8664,26	8604,26	12	12,84	7551,79
				1	1	0,12	6022,84	9543,41	9450,41	93	18,13	7813,54
					5	0,16	6022,84	9257,06	9197,06	12	12,69	8082,12

Table 4: Results of one instance with the robust optimization model.

ventory levels lead to lower holding costs, translating into a lower objective value. For higher values of Γ_{iwt} , the objective value increases. Now, even with possible higher demand, the changes to the inventory policy are more pronounced with usually much higher (s, S), leading to increased inventory levels. This translates into higher holding costs and consequently a higher objective value. In addition, for higher Γ_{iwt} values, the variation the delivery time is bigger, also increasing the objective value.

The robust approach has a conservative behavior: to account for a possible higher demand, the model proposes an overall increase on the regularity of deliveries and on the inventory policy parameters. This can be explained since most items have a very volatile demand and thus the possible variation range is very large. In order to account for these situations, ensuring that the demand is at all times satisfied, the higher the percentage deviation of the nominal value of demand (Γ_{iwt}), the more pronounced are the changes to the schedule and inventory policy. In addition, given that one of the model's objectives is to minimize the holding cost, the model chooses to perform more regular deliveries instead of delivering larger inventory quantities less often. Since there is no minimization on the number of deliveries, and transportation costs are considered, this behavior is logical. This leads to solutions that might not be so adequate and applicable to a real setting, and so setting a maximum on the number of deliveries per day or including transportation costs can be considered.

The complexity of the model is high, optimality is not achieved and not all tests obtain solutions. The robust instances are larger than the deterministic instances, and so larger optimality gaps and worse solutions are obtained in the former.

The insights gained with the deterministic and robust cases are shared with the PPH and it is concluded that, although there are aspects still to be considered, the obtained outcomes could be put into practice. Several advantages are identified, including the decrease of the services' stock levels and holding costs, less space occupied with inventory, more balanced schedules and a wastage decrease.

6. Conclusions

The proposed models determine inventory policies and scheduling of deliveries from the CW to each service, taking into account the limitations inherent to clinical inventory management. By integrating inventory control with scheduling, this work fills a gap in the hospital inventory management literature. Routing is taken into account and a robust optimization approach is employed to deal with uncertainty, two novel considerations within the topic. The model is applied to the PPH setting. The deterministic model obtains feasible solutions in most tests. The robust approach, which has a considerable larger size than the deterministic case, only 100 out of 144 tests succeed. Nevertheless, the model takes into account the uncertainty in the demand. Conservative solutions are obtained, since the model assumes a possible higher demand and, to satisfy it, increases significantly the regularity of the deliveries and the (s, S) levels. Appropriate insights and suggestions are identified with the aim of improving clinical inventory management at the hospital.

As for future work, an integration of the CW and the suppliers can be implemented, considering not only the internal supply chain (the focus of the current work), but also the external supply chain, to model the complete setting. Further investigation on the introduction of uncertainty in the demand might be considered. Since different items can have distinct degrees of demand uncertainty Γ parameters adapted to each item can be employed. To increase the applicability of the current findings in the PPH setting, an automatic replenishment with the inventory policy might be explored. One difficulty encountered in this work is the complexity of the hospital inventory management setting, leading to a complex model difficult to solve when applied to real-sized problems. Therefore, simplifications of the model might improve its results and ease the testing process. Nevertheless, several advantages are identified, opening the path for future work further exploring the hospital setting.

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