Statistical modelling of the characterization of low-strength mortars adopting the drilling resistance test

André Filipe Rosário Marcelino Silva
andre.f.rosario.marcelino.silva@ist.utl.pt

Instituto Superior Técnico

October 2018

Abstract

The historical and cultural relevance that old renders possess, lead to an increasing importance in the correct diagnosis of the state of conservation of these old renders. Thus, the diagnostic and intervention actions, carried out through in situ characterization tests, should be the least intrusive possible. This particularity leads to the drilling resistance method due to its advantage over the remaining characterization tests, since it allows to obtain information, such as the mechanical resistance and thickness of the layer, with a very small element intrusion.

In this work was develop a classification model of mortars present in old renders trough a statistical analysis of the drilling resistance method results. The main objective of this work was to be able to define the constitution of a mortar using this method. Based on a sample of 18 mortars developed by Nogueira (2016), the statistical analysis performed was based in the transposition of the information in the drilling profiles to a histogram, and the consequent adjustment of a probability distribution to it. This allowed to characterize the mortar present in the sample with the parameters that define the probability distribution adjusted and determine the constitution of the test mortar by comparing with a mortar from the data base.

The results obtained by this work allowed to complement the information obtained by the drilling resistance method and define a minimal number of measurements. Thus, providing a greater diversity of characteristics provided by this method for the mortars tested.

***Keywords**:* Drilling resistance, low-strength mortars, heterogeneity, log-normal distribution function, mechanical behavior, classification model.

1. Introduction

1.1. Historical background

From the earliest civilizations to the Industrial Revolution, the technology used in the formulation and application of mortars had remained approximately constant. The industrialization of the construction sector, the increase of scientific knowledge and the arising of new materials, led to an evolution of the technology regarding renders execution. Therefore, the old practices of execution of these mortars, based on empirical knowledge, practically disappeared [1] [2].

1.2. Requirements of old renders

Generally, renders' main functions encompass the protection and decoration of walls and are thus considered as sacrificial layers. To ensure protection, renders should constitute a barrier to prevent or delay contact between environmental aggressive agents and the substrate. When a rendering is applied as external coating directly on the substrate that is part of the structure of the building, the protection function becomes more important, since the degradation of the render will affect the integrity of the building. Renders have also a relevant role in the aesthetic appearance of buildings, and therefore, the finishing layer and the renders' surface should comply with some performance requirements. Accordingly, the mortar compositions must consider the required aspects like flatness, texture, colour and brightness [1] [3].

1.3. *In situ* characterization tests

The cultural and historical importance of old renders led to a growing concern related with their state of conservation. For the development of protection and conservation policies applied to old renders, the adoption of *in situ* characterization methods, which provide important information about the render's composition, is mandatory [4] [5].

Therefore, the drilling resistance method appears to be a useful characterization method, capable of providing important information, e.g. the strength and the layer thickness, with a small intrusion on renders, usually only a 5 mm hole. When compared to other characterization methods, especially destructive methods, such as the pull-off test, the drilling test presents some advantages, namely little intrusion, simplicity of execution. On the other and, when compared with non- or minor-destructive methods such as ultrasonic pulse velocity or superficial hardness, the drilling resistance has the advantage of providing the in-depth characterization of the material [3] [6].

1.4. Drilling resistance method in low strength mortars

The drilling resistance method provides an in-depth drilling force measurement. This drilling force corresponds to the thrust to be exerted on the drill to drive the bit at a constant penetration rate and revolution speed, which leads to an irregular strength profile when applied to low strength mortars [7] [8]. Even though the strength profile gives valuable information regarding the characteristics of the material, the high value of the standard deviation, when compared with the average of the drilling resistance, introduces a high uncertainty in the analysis of the output. Therefore, Nogueira [6] proposed a statistical analysis of the measurements, by converting the information of the strength profile into a histogram, using the statistical parameters to obtain more information with the same data [6].

1.5. Dissertation objectives

This dissertation intends to develop a classification model to characterise low-strength mortars, by using the drilling resistance method. The proposed model allows to identify the main characteristics of a mortar under analysis (test mortar), concerning the type of binder (lime, hydraulic or a mixture of both), the size and nature of the aggregates (siliceous or carbonate aggregates), the binder–aggregate and the water-binder ratios. The identification of the characteristics of the test mortar is possible through the comparison between its drilling resistance profile and the drilling resistance profiles of different mortars that characterise ancient mortars, with low-strength (compressive strength comprised between 1 and 11 MPa). Additionally, the classification model allows obtaining other characterisation parameters, namely the compressive strength and the heterogeneity degree.

2. Classification model

2.1. Definition of the sample

Table 1: Mortar mixtures and main testing features

Mortar/ Category	Binder	Aggregate		B/A[1]	W/B[2]	Flow[3] (mm)	*fc*[4] (MPa)	Number of drilling tests	Depth of the drilling tests (mm)	N° measurements
		Nature	Size							
USL1	CL90	Siliceous	Coarse	1:3	1,6	220	0,9	11	25	2750
USL2					1,5	207	1,1	6		1500
USL3					1,4	192	1,3	5		1250
USL4					1,3	171	1,6	4		1000
USL5					1,28	165	1,7	9		2250
USL6					1,2	142	2,1	6		1500
TSL1	CL90	Siliceous	Fine	1:1	1	222	2,4	8	33	2640
TSL2					0,9	194	2,9	4		1320
TSL3					0,85	172	3,4	8		2640
TSL4					0,8	159	3,9	4		1320
UCL1	CL90	Carbonate	Coarse	1.3	1,6	220	1,8	6	25	1500
UCL6					1,2	124	3,4	6		1500
TCL1	CL90	Carbonate	Fine	1:1	1	211	3,6	6	25	1500
TCL3					0,85	151	4,2	7		1750
USH1	50%CL90 50%NHL5	Siliceous	Coarse	1:3	0,9	144	4,2	4	25	1000
USH2	NHL5				0,9	169	5,9	4		1000
USH3	90%NHL5 10%CEM				0,81	164	8,6	3		750
USH4	50%CL90 50%CEM				0,9	154	10,6	4		1000

1-Water/binder ratio
2-Binder/aggregate volumetric ratio
4-Compressive strength test according with EN1015-11:199
3-Flow table test according with EN1015-3:1999

In order to develop a classification model, a sample of mortars that comprise the main low-strength mortars present in ancient renders was used in this work. The sample is composed of 18 mortar mixtures (Table 1), produced on the thesis of Nogueira [6]. These mortar mixtures were design with the aim of comprising a large diversity of characteristics and of being representative of the mortars present in ancient renders.

The mortars included in the database are grouped into five categories. Each category is distinguished by 3 letters. The first letter of the mortar's identification regards a topcoat or undercoat mortar composition ("T" and "U", respectively). The topcoat mortars are lime-rich mortars composed of fine aggregates and the undercoat mortars have a coarse aggregate and a weaker binder-aggregate ratio. The second letter indicates the nature of the aggregate - siliceous ("S") or carbonate ("C"). The final letter of the mortar's identification is related with the type of binder: non-hydraulic binder lime ("L") and hydraulic binder ("H").

The drilling data was collected using a penetration rate of 40 mm/min and a revolution speed of 100 rpm. Table 1 shows the depth and the number of drilling tests made for each mortar, which gives the information of the number of measurements obtain (around 1500 measurements per mortar). Each category has a depth of 25 mm, except the TSL. The drilling tests were performed intending to gather the maximum number of measurements and keeping the same depth in each category, allowing a more reliable comparison between the results obtained for the different mortars.

2.2. Adjustment of a probability distribution function to the sample

The drilling profile of each mortar was translated into a histogram. After this, 65 probability distributions functions were adjusted to each histogram, using the software *EasyFit 5.3* (Figure 1). However, only six of the 65 probability distributions were considered: i) Normal; ii) Lognormal; iii) Gamma; iv) Weibull; v) Gumbel; and vi) Logistic. These six distributions are the most commonly used in the civil engineering field. The selection of the best distribution is made based on three goodness-of-fit tests: i) Anderson-Darling test; ii) Kolmogorov-Smirnov test; and iii) Chi-square test [9].

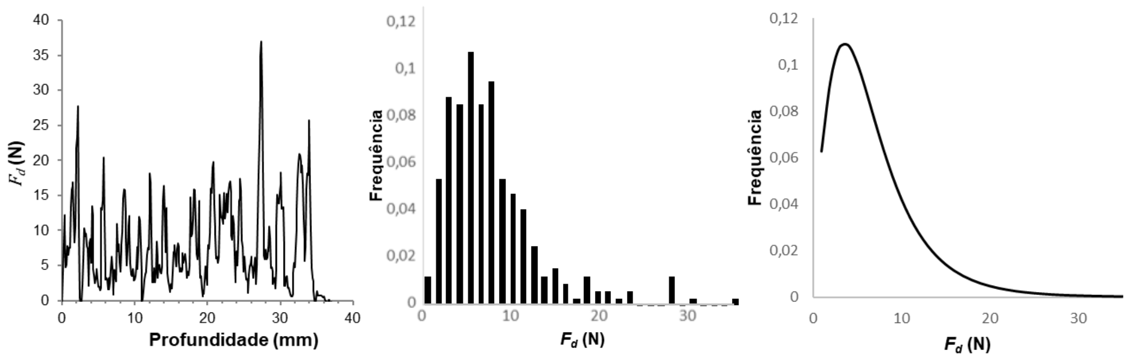


Figure 1: Transposition of the drilling resistance profile to a probability distribution, for the USL4 mortar

The results of the statistical tests revealed that the log-normal probability distribution (with three parameters - 3P) was the one with the best goodness-of-fit to all the histograms of the sample.

Therefore, the parameters that define the log-normal probability distribution of each mortar were calculated in order to define the database of de classification model (Table 2).

2.3. Validation of the classification model

The classification model proposed in this study intends to classify a new mortar of unknown composition (test mortar) by comparing the log-normal distribution that characterises the drilling resistance profile of this new mortar (test mortar) and the probability distribution functions obtained for the eighteen mortars included in the database. This comparison is performed using the software Wolfram Mathematica, obtaining the absolute value of the difference between the integrals of the two distributions, which provides an indicator ($P_{overlapping}$) of the overlapping area between those two distributions [4]. In this study, this indicator was divided into five intervals of overlapping percentage [5] [6]: i) a $P_{overlapping}$ equal to 100% corresponds to distributions that are coincident (i.e. the test mortar perfectly corresponds to a mortar included in the database); ii) a $P_{overlapping}$ in the [0.9; 1.0[range presents a very good correspondence between the test mortar and the database; iii) a $P_{overlapping}$ in the [0.8; 0.9[range presents a good correspondence between the test mortar and the database; iv) a $P_{overlapping}$ in the [0.7; 0.8[range presents an acceptable correspondence between the test mortar and the database; and v) a $P_{overlapping}$ below 0.7 shows an inaccurate result and it is not possible to establish an unequivocal correspondence between the test mortar and the database. The validation of the classification model was performed in two different ways: i) internal validation, where some values of the database were used to test the model; and ii) external validation, where other mortars, not present in the database, were used to test the model.

Table 2: Parameters of the log-normal distribution for each mortar

Mortar	σ	μ	γ
USL1	0.618	1.744	-1.896
USL2	0.573	1.982	-2.777
USL3	0.592	1.851	-2.239
USL4	0.519	2.085	-2.557
USL5	0.548	2.088	-2.34
USL6	0.628	1.902	-1.316
TSL1	0.220	2.299	-4.18
TSL2	0.191	2.593	-5.762
TSL3	0.249	2.779	-6.057
TSL4	0.224	2.757	-5.996
UCL1	0.431	2.236	-1.636
UCL6	0.464	2.891	-2.622
TCL1	0.088	3.248	-17.44
TCL3	0.260	2.457	-2.593
USH1	0.394	2.654	-3.468
USH2	0.205	3.464	-17.08
USH3	0.089	4.984	-105.9
USH4	0.118	4.589	-69.84

The results from the internal validation, shown in the Figure 2, show a symmetric matrix with 100% in the main diagonal, as expected. This matrix also allows establishing some conclusions regarding the characteristics of the mortars included in the database:

1. In the USL group, a high percentage of overlapping area for every mortar in the group was observed, which reveals the similarity of characteristics among the mortars included in this group;
2. In the UCL group, the low overlapping percentage between the mortars UCL1 and UCL6

was detected, which indicates that these two mortars have very different drilling resistance profiles, even though presenting similar composition, which seems to indicate that, in the case of carbonate aggregate mortars, the differences in water/binder ratio strongly influence the results obtained in the drilling resistance test;

3. The mortars from the groups TCL and TSL have a high overlapping percentage between them, which indicates that, for mortars with fine aggregates, the nature of the aggregate takes a lower influence on the characteristics of the mortar;
4. Finally, the USH group shows a low overlapping percentage between the mortars included in this group, revealing that the use of different binders has a significant effect on the properties of the mortar.

	USL1	USL2	USL3	USL4	USL5	USL6	UCL1	UCL6	TSL1	TSL2	TSL3	TSL4	TCL1	TCL3	USH1	USH2	USH3	USH4
USL1	100%	91,8%	96,3%	82,5%	80,9%	83,6%	59,1%	31,3%	58,9%	47,8%	42,1%	40,8%	39,6%	40,8%	45,2%	30,4%	5,2%	13,3%
USL2	91,8%	100%	95,4%	89,1%	87,6%	89,6%	66,0%	37,0%	61,0%	52,7%	48,7%	47,2%	44,2%	46,8%	51,8%	35,8%	6,4%	15,9%
USL3	96,3%	95,4%	100%	86,1%	84,3%	87,0%	62,5%	33,9%	60,6%	50,5%	45,0%	43,6%	42,0%	43,7%	48,1%	32,7%	5,6%	14,2%
USL4	82,5%	89,1%	86,1%	100%	97,4%	96,5%	75,9%	41,5%	68,2%	61,2%	56,3%	55,0%	51,9%	55,2%	58,9%	40,2%	6,6%	16,9%
USL5	80,9%	87,6%	84,3%	97,4%	100%	95,7%	77,4%	42,4%	67,2%	61,4%	58,4%	56,6%	52,3%	56,3%	61,4%	42,7%	7,6%	18,5%
USL6	83,6%	89,6%	87,0%	96,5%	95,7%	100%	74,2%	41,9%	67,0%	59,0%	55,1%	53,4%	65,4%	53,2%	58,4%	40,6%	7,8%	18,1%
UCL1	59,1%	66,0%	62,5%	75,9%	77,4%	74,2%	100%	51,6%	70,7%	79,0%	76,2%	75,5%	69,6%	76,2%	75,2%	50,5%	7,3%	19,3%
UCL6	31,3%	37,0%	33,9%	41,5%	42,4%	41,9%	51,6%	100%	27,1%	39,0%	62,0%	57,5%	38,6%	50,8%	73,2%	89,2%	28,5%	52,6%
TSL1	58,9%	61,0%	60,6%	68,2%	67,2%	67,0%	70,7%	27,1%	100%	69,9%	47,2%	47,9%	57,8%	52,2%	46,7%	25,7%	1,6%	7,1%
TSL2	47,8%	52,7%	50,5%	61,2%	61,4%	59,0%	79,0%	39,0%	69,9%	100%	69,0%	71,6%	86,4%	79,2%	62,7%	37,9%	2,7%	10,6%
TSL3	42,1%	48,7%	45,0%	56,3%	58,4%	55,1%	76,2%	62,0%	47,2%	69,0%	100%	94,1%	70,6%	85,2%	84,9%	62,9%	7,4%	21,7%
TSL4	40,8%	47,2%	43,6%	55,0%	56,6%	53,4%	75,5%	57,5%	47,9%	71,6%	94,1%	100%	75,3%	90,3%	80,2%	57,9%	6,0%	18,8%
TCL1	39,6%	44,2%	42,0%	51,9%	52,3%	65,4%	69,6%	38,6%	57,8%	86,4%	70,6%	75,3%	100%	84,4%	60,6%	37,5%	2,0%	9,4%
TCL3	40,8%	46,8%	43,7%	55,2%	56,3%	53,2%	76,2%	50,8%	52,2%	79,2%	85,2%	90,3%	84,4%	100%	73,3%	50,5%	4,5%	15,7%
USH1	45,2%	51,8%	48,1%	58,9%	61,4%	58,4%	75,2%	73,2%	46,7%	62,7%	84,9%	80,2%	60,6%	73,3%	100%	73,6%	13,7%	31,4%
USH2	30,4%	35,8%	32,7%	40,2%	42,7%	40,6%	50,5%	89,2%	25,7%	37,9%	62,9%	57,9%	37,5%	50,5%	73,6%	100%	19,9%	44,1%
USH3	5,2%	6,4%	5,6%	6,6%	7,6%	7,8%	7,3%	28,5%	1,6%	2,7%	7,4%	6,0%	2,0%	4,5%	13,7%	19,9%	100%	64,1%
USH4	13,3%	15,9%	14,2%	16,9%	18,5%	18,1%	19,3%	52,6%	7,1%	10,6%	21,7%	18,8%	9,4%	15,7%	31,4%	44,1%	64,1%	100%

Figure 2: Results obtained in the internal validation

For the external validation, six test mortars were used, and a comparison between each test mortar and each one of the 18 mortars within the database were performed. Table 3 presents the characteristics of each test mortar and the identification of the mortar in the database with a higher overlapping percentage, using a different number of measurements to the test mortar.

This study also intends to analyse the number of tests (and measurements to collect) that should be performed in order to accurately classify the test mortar. Based on the results obtained, 1000 measurements seem to provide enough information to accurately characterise the probability distribution of the mortar.

The results obtained revealed that the classification model was able to correctly identify the mortars tested (Figure 3), having an overlapping percentage higher than 85% for all the mortars, with the exception of A5, thus showing a strong discriminating power. The lower overlapping percentage of A5 mortar is also a good result, since it is composed of an aggregate with a nature different from those used in the database mortars (Table 3).

Table 3: Characteristics of the mortars used in the external validation

Test mortar	Substrate	Group of the database that fits better the test mortar	Comparation with the mortars on the data base
A1	Prism	UCL	Same binder and same nature of the aggregate.
A2	Render	TSL1	Same mortar but applied on a render, instead of a prism.
A3	Prism	USL5	Same mortar but 3 to 4 months older than the mortar on the database.
A4	Prism	USL	Same binder and same nature of the aggregate.
A5	Prism	USL	Same binder and aggregate with different nature
A6	Render	USL	Similar composition [6]

	USL1	USL2	USL3	USL4	USL5	USL6	UCL1	UCL6	TSL1	TSL2	TSL3	TSL4	TCL1	TCL3	USH1	USH2	USH3	USH4
A1	66,5%	73,2%	69,9%	84,1%	85,5%	82,7%	88,9%	51,9%	72,2%	69,8%	68,5%	66,3%	60,1%	65,9%	72,1%	50,4%	10,1%	22,4%
A2	68,7%	72,3%	71,0%	80,9%	80,1%	79,3%	79,0%	34,7%	85,8%	73,7%	55,7%	45,5%	61,9%	59,3%	55,3%	33,5%	3,1%	10,8%
A3	86,6%	92,1%	89,9%	94,9%	93,3%	96,8%	71,4%	39,5%	66,2%	57,2%	52,5%	50,9%	48,1%	50,9%	55,5%	38,3%	7,1%	16,8%
A4	91,6%	84,7%	88,2%	74,3%	72,6%	75,1%	50,9%	25,2%	54,2%	41,6%	34,8%	33,7%	34,0%	33,9%	37,8%	24,6%	6,2%	10,2%
A5	75,3%	67,5%	71,7%	58,8%	57,2%	59,9%	36,0%	15,1%	41,8%	27,8%	22,1%	21,0%	21,7%	20,6%	25,1%	15,3%	1,8%	6,0%
A6	74,1%	81,0%	77,5%	91,1%	92,8%	89,3%	84,6%	46,4%	69,7%	67,2%	64,4%	62,9%	57,9%	62,8%	66,4%	55,9%	7,3%	18,8%

Figure 3: Results of the external validation obtained for 1000 measurements for each mortar

2.4. Determination of compressive strength using the classification model

Figure 4 shows the statistical correlation between the μ parameter of the log-normal distribution of the mortars within the database (Table 2), and their compressive strength. A good correlation between these two indicators was obtained, which allows obtaining a good estimative of the compressive strength using the μ parameter.

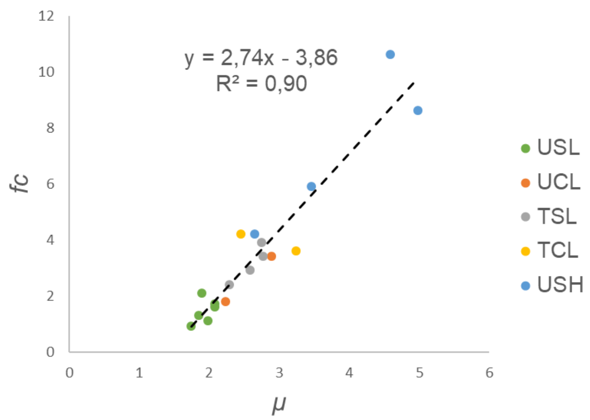


Figure 4: Correlation between compressive strength and the μ parameter.

2.5. Determination of heterogeneity using the classification model

According to Nogueira [6], the coefficient of variation (ratio between the standard deviation and the average) have a good correlation with the heterogeneity of the mortars and can even be used to classify the heterogeneity in 3 classes: i) H1 corresponds to the more heterogeneous mortars; H2 corresponds to an intermediate situation; and iii) H3 corresponds to the less heterogeneous. The analysis of the database allowed to conclude that both σ/μ and σ parameters have a good correlation (R^2 = 0.93 and R^2 = 0.90) with the coefficient of variation. Hence, using the classes defined by Nogueira [6], and the correlation with the coefficient of variation, the matrices in Figure

5 were developed, where the database mortars are shown divided into three heterogeneity degrees, allowing to obtain a estimative of the heterogeneity of the mortar tested.

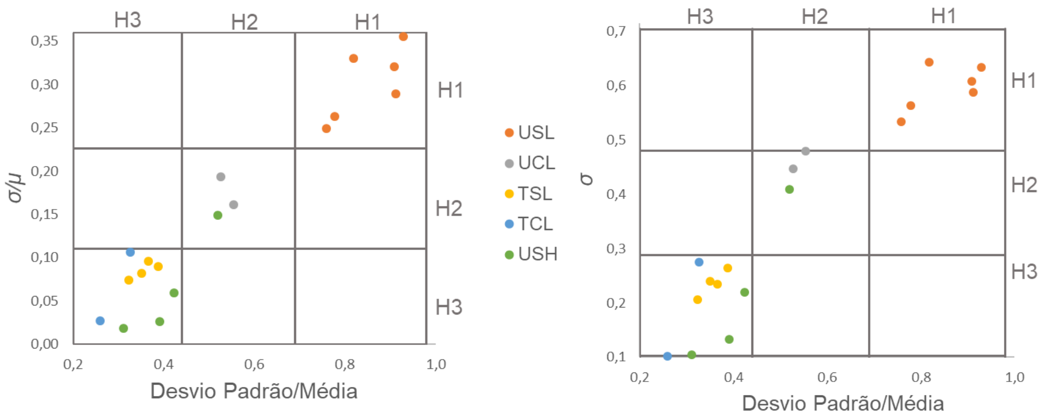
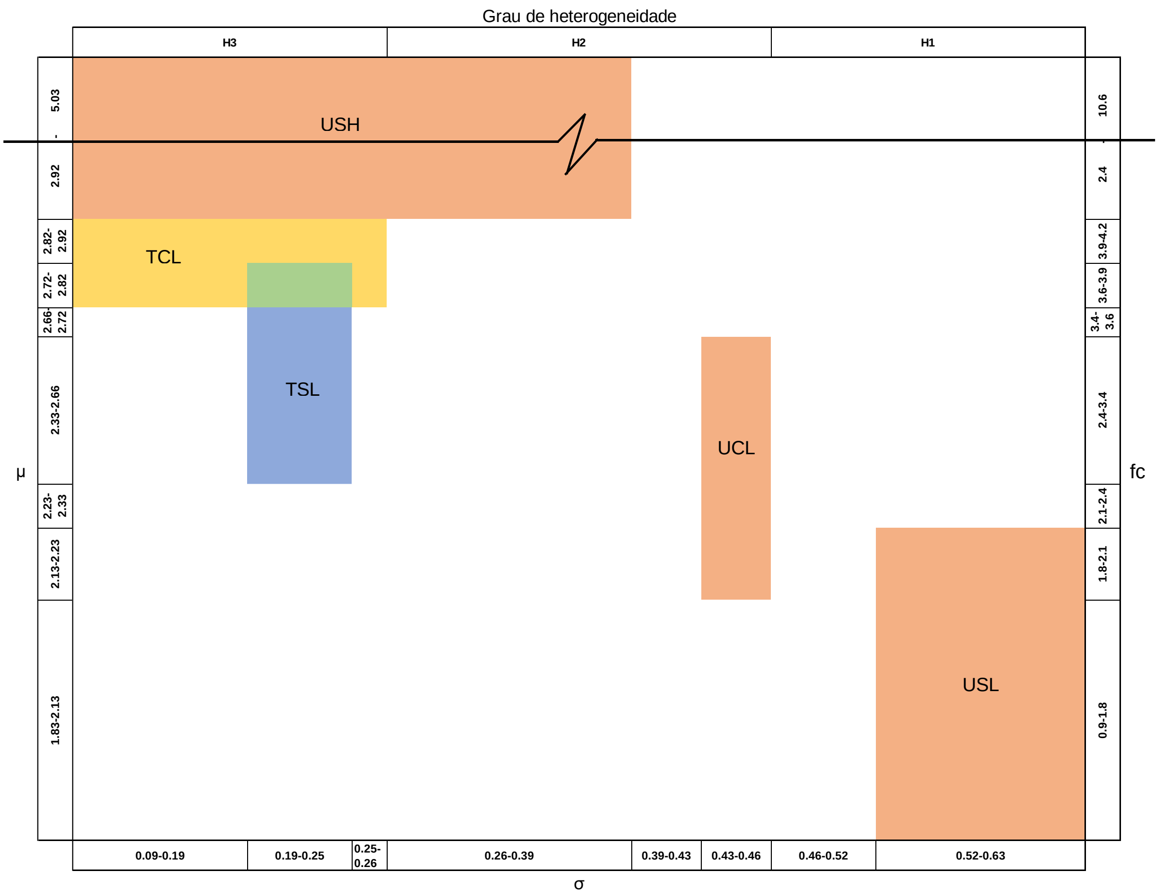


Figure 5: Heterogeneity degree

2.6. Development of a classification matrix

Using the conclusions obtained from the previous sections, the matrices shown in Figure 6 were developed. These matrices show the database mortars categories classified in terms of compressive strength (columns) and heterogeneity degree (lines). The *σ*/*μ* per *μ* matrix shall be more accurate since *σ*/*μ* has a higher correlation with the heterogeneity of the mortars. Conversely, the *σ* per *μ* matrix is prompter since it does not require the extra calculation to obtain the ratio *σ*/*μ*; the loss of accuracy is small (3% difference in the R^2 value).



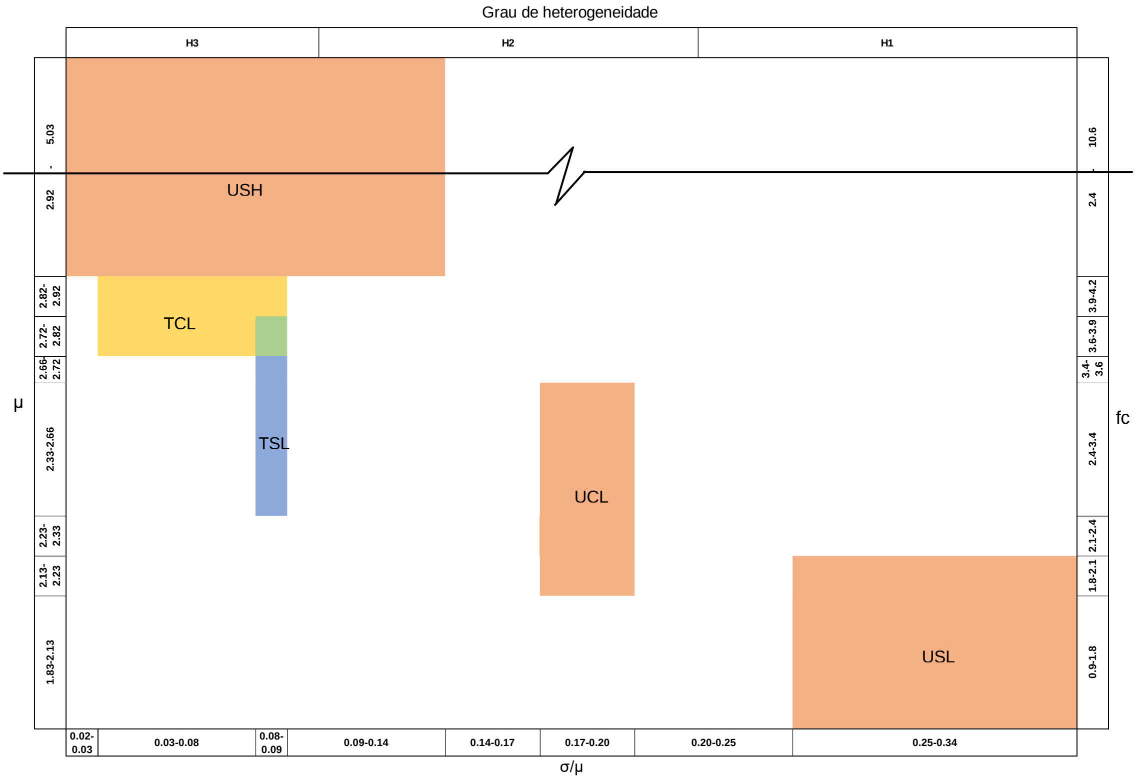


Figure 6: Classification matrices: *σ* per *μ* (above) and *σ*/*μ* per *μ* (below).

3. Application protocol

After the development of the classification model a protocol was defined, in which a sequence of the application of the different methodologies developed throughout this work is suggested. The application protocol is divided in five steps:

1. Performing the drilling resistance test in the test mortar, gathering at least 1000 measurements. Ideally, this number of measurements should be made in, at least, 3 different tests;
2. With the data gathered from the drilling resistance test, calculation of the parameters that define the adjusted log-normal distribution;
3. Using the classification matrix and the log-normal parameters, estimate the group of test mortar, the heterogeneity degree and the compressive strength;
4. With the equation developed in Figure 4, definition of an estimation of the compressive strength, using the *μ* parameter;
5. Using the database shown in the Table 2, calculation of the percentage of the overlapping area between the log-normal distributions of the test mortar and of each mortar of the database.

Although this protocol is divided in five steps, only the first two must be made consecutively, the other three steps can be made in any order. The order presented in this protocol is only a suggestion, aiding the practical use by a practitioner of the methodology proposed.

4. Conclusion and future developments

The classification model developed in this work allows obtaining the probable composition of an old mortar, as long as the mortar under analysis presents a similar composition to the mortars of the database. Since the database was defined by analysing the most frequent compositions present in ancient mortars, the application of this classification model has a broad scope.

This study also reveals that, even though the in-depth characterization of low-strength mortars shows a high complexity, the modelling of these characteristics is possible. The use of the data from the drilling resistance method to develop a probability distribution function, proved to be a great way to gather more information from the same amount of data.

This study also provided an estimation of the minimal number of measurements needed to gather enough data to obtain results. This minimal number of 1000 measurements, e.g. 4 drilling tests of 25 mm deep, leads to a lower intrusion on the analysed render, which is very important in the case of ancient ones, due to its historical and cultural value.

This work also enhances the knowledge of the drilling resistance method. In previous studies and applications, the drilling resistance method would only allow determining the thickness of the render and rough estimate of the material mechanical strength. In this work, with a lower amount of data, the composition, the heterogeneity degree and the compressive strength of the mortar can be estimated. Furthermore, the application protocol shows the flexibility and diversity of results produced by the method.

Regarding future developments, the database could be enriched with other mortars compositions, applied in other substrates or with older ages, in order to widen the application of this method.

5. References

[1] R. Nogueira, A. P. Ferreira Pinto and A. Gomes, “Rebocos à base de cal:passado e presente. Conhecimento empírico versus conhecimento cientifico,” 2007.

[2] Nogueira, Ferreira Pinto and Gomes, “Design and behavior of traditional lime-based plasters and renders. Review and critical appraisal of strengths and weaknesses,” *Cement and concrete composites,* vol. 89, pp. 192-204, 2018.

[3] R. Nogueira, A. P. Ferreira Pinto and G. Augusto, “The drilling test in the characterization of lime mortar renders in multilayer system,” Guimarâes, 2014b.

[4] J. Galvão, I. Flores-Colen, J. de Brito and M. Veiga, “Variability of in-situ testing on rendered walls in natural ageing conditions – Rebound hammer and ultrasound techniques,” *Construction and Building Materials,* vol. 170, pp. 167-181, 2018.

[5] A. Menezes, M. Glória Gomes and I. Flores-Colen, “In-situ assessment of physical performance and degradation analysis of renderig walls,” *Construction and building*

materials, vol. 75, pp. 283-292, 2015.

[6] R. M. V. Nogueira, “Lime plasters and renders. Characterization and assessment of consolidation treatments.,” Universidade de Lisboa. Instituto Superior Técnico, Lisboa, Portugal, 2016.

[7] R. Nogueira, A. P. Ferreira Pinto and A. Gomes, “Assessing mechanical behavior and heterogeneity of low-strength mortars by the drilling resistance method,” *Construction and building materials,* vol. 68, pp. 757-768, 2014a.

[8] R. Nogueira, A. P. Ferreira Pinto and A. Gomes, “The drilling resistance measurement on the characterisation of low-strength mortars,” Lisboa, Portugal, 2017b.

[9] A. Silva, “Modelação estatística da previsão da vida útil do revestimento de fachada,” Instituto Superior Técnico, 2015.

[10] C. D. Lewis, Demand forecasting and inventory control. A computer aided learning approach., 1st Edition ed., Cambridge: Woodhead Publishing Ltd. In association with the Institute of Operations Management, 1997.

[11] C. Piaw, Basic Resarch Statistics (Book 2) (In Malay), Kuala Lumpur: McGraw-Hill, 2006.