# Fatigue Strength Assessment of Welded Joints Employing Peak Stress Method

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Abstract: The objective of the present work is to perform a finite element analysis (FEA) in order to apply the Peak Stress Method (PSM) in a fatigue strength assessment. The PSM is based on the analysis of a singular linear elastic peak stress at the singularity point, requiring a less computational and user effort in a comparison with other methods, mainly due to the use of coarser meshes and a single stress value. The PSM has been developed and calibrated by Meneghetti and Lazzarin using the FEA software *Ansys* with two-dimensional four nodes linear quadrilateral elements. The present study uses the software *Adina* with two-dimensional four nodes linear quadrilateral elements. The final objective is to prove that the results obtained by different softwares and consequently different element formulation and interpolation functions provide similar results therefore proving the applicability and usefulness of the PSM. The analysis focuses on two-dimensional cruciform welded joints geometries subjected to the Mode I linear elastic loading.

**Key words**: fatigue, ADINA, Peak Stress Method, finite element analysis, Meneghetti, Lazzarin.

## 1 Introduction

The fatigue strength assessment has a major importance in structural design. The first standards in the area were introduced in the 1970's while the first international recommendations were created in 1982 by the International Institute of Welding (IIW) [1]. The more commonly used local approaches in fatigue strength assessments in the last 20 to 30 years are categorized as: structural stress approaches, notch stress approaches, notch strain approaches and crack propagation approaches. There is a new "category" of fatigue strength assessments based on the concept of the Notch Stress Intensity Factor (NSIF), first introduced by Williams in 1952 [2] that takes into consideration the effect of the weld notch. This NSIF allows for the stress to be analysed at the notch tip, and is the base for different types of fatigue assessments in which the PSM is included. The PSM is based in the cyclic nature of the NSIF and its relation with the applied nominal load. The main advantages reside on the coarse mesh required and the need of only a single value of peak stress to assess the fatigue strength as opposed to the other approaches that require a whole stress field [3]. Since the objective of any finite element analysis is to provide accurate results at the lowest cost possible, both from the numerical and from the operator point of view, the applicability of the PSM is obvious and therefore the main objective is to study it. A general overview on the current fatigue strength evaluation methods in use is provided. The objective is to situate the reader as to where does the

peak stress method stand in the "spectrum" of fatigue strength analysis methods available. A short description of each method is provided, as well as its applications, advantages and disadvantages.

In this work all FE analyses are linear, static and are performed with the software ADINA [4].

## 2 Overview of current fatigue strength assessment methods

A brief overview of the current fatigue strength analysis methods is provided in order to situate the method in study in the "spectrum" of current methodologies. The current methods utilised to perform fatigue strength analysis can be divided in two main categories: nominal stress and local concepts. The latter is further divided in structural stress/strain, notch stress/strain and fracture mechanics concepts [5].

**Nominal Stress Approach**

The fatigue strength analysis based on the nominal stress approach utilises the nominal stress applied to the structure and a corresponding S-N curve to estimate the fatigue life. This approach disregards local stress increases due to discontinuities or defects in the material or weld. The nominal stress approach uses predetermined S-N curves that correspond to the structure in study, limiting the usage of this approach to already known and common geometries [6].

**Local Concepts – Structural Stress**

The structural stress concept was firstly introduced in the analysis of steel tubular joints and was then adapted to a wider range of geometries, which lead to its codification as a fatigue strength assessment. Contrarily to the nominal stress approach, the stress increases due to misalignments or defects in the material or welds are taken into account. The structural stress is primarily based on the measured surface stress in predetermined points whose location depends on the thickness of the plating. The value of the stress in the weld notch is then estimated from extrapolating the surface stresses applied to the desired location following one of various methods. The analysis of the fatigue life is made in terms of number of cycles using S-N curves [5]. Following IIW recommendations [1] this type of approach is to be used only for evaluating the fatigue strength of weld notches in plate type structure analysis. [5].

**Local Concepts – Notch Stress**

The notch stress approach, contrarily to the structural stress approach, takes into consideration the increase in the local stress due to the notch formed by the weld chord. Similarly to the structural stress approach, the analysis of the fatigue life is made in terms of number of cycles using S-N curves [5]. The effective notch stress approach utilises a fictitious notch radius together with a predetermined S-N curve to assess the fatigue strength of a structure. The notch radius is usually assumed to be 1 mm except in thin-sheet joints where the radius assumed is significantly smaller. This approach takes into consideration the geometry properties of the weld chord such as thickness and fillet angle, as well as the thickness of the plating. However, the geometry of the weld is considered to be "perfect" when performing finite element analysis so any misalignments or imperfections on the weld itself are not taken into consideration [6].

**Fracture Mechanics Concepts – NSIF**

When applying the Notch Stress Intensity Factor (NSIF) based approach, the weld notch is modelled in the finite element analysis software as a sharp V-notch. However, the stress at a sharp V-notch is theoretically infinite. In order to describe the stress distribution, a mathematical factor, NSIF, was used by Lazzarin and Tovo [7] in the fatigue strength analysis of welded joints [6]. The NSIF describes the local stress field in the weld toe. It depends on the loading and on the geometrical parameters of the structure (plate and weld thickness, weld fillet angle). The NSIF approach is mostly suited for evaluating fatigue strength up to the crack initiation. The main disadvantage of this approach lays in the fact that for each weld geometry (thickness, fillet angle, etc.) a new set of reference data is necessary, meaning a new empirical scatter $K_1 - N$ curve. This limits the usage of this approach to very common

and already known weld geometries. It also must be taken into consideration that the finite element modellation implies a "perfect" weld geometry which is usually not the case, with every weld containing small defects and imperfections [5].

**Fracture Mechanics Concepts – PSM**

The peak stress method is an approach that obtains the NSIF through the calculation of a single value of linear elastic peak stress at the singularity point using finite element analysis. Unlike the NSIF approach mentioned earlier, the PSM requires much coarser meshes and a singular stress value to produce similar results, hence the interest in studying it. Meneghetti [8] used the correlation between the notch stress intensity factor and the peak stress calculated using finite element analysis along with defined mesh and element properties to obtain a stress value on the weld notch and weld root, which together with a standard S-N curve allows the assessment of the structure fatigue strength [8]. This particular approach is the approach in study in this work.

## 2 Peak Stress Method

When analysing plane stress problems, the linear elastic stress fields on the vicinities of the singularities, see Fig. 1, can be expressed as functions of the Notch Stress Intensity Factor (NSIF), a mathematical parameter related with the stress distributions [8].

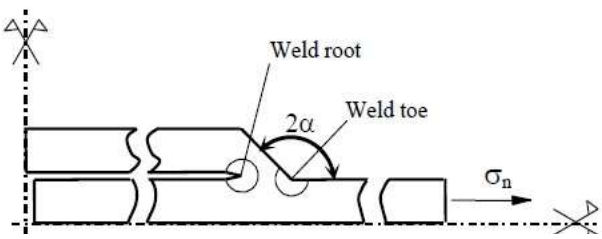

Figure 1 – Singularity points: weld toe and weld root [3]

The mode I NSIF of sharp and open V-notches loading causes the crack to open, and is defined in eq. (1). The exponent $\lambda_1$ [2] depends on the notch opening angle $2\alpha$, while the stress component (peak stress) $(\sigma_\theta)_{\theta=0}$ is calculated on the notch bisector line, where the stress value is maximum ($\theta = 0°$).

The PSM is a method capable of numerically evaluating the NSIF's by adopting finite elements (FE) models with a relatively coarse mesh. The method was theoretically proven [3] to be used in the analysis of the mode I NSIF of sharp and open V-notches as well as the mode II NSIF of open V-notches. The present work focuses on the analysis of the mode I notches. Prior to its utilization, the PSM requires calibration of the following factors:

- FE code or software used;
- Element type and formulation;
- FE model mesh pattern;
- Stress extrapolation at FE nodes.

It was proved in [3] that the $\frac{K_I^V}{\sigma_{peak}}$ ratio is constant with respect to the crack size *a* provided that the same mesh pattern is employed throughout the FE analysis. In order to take into account the effect of the FE size *d*, a new ratio was introduced, see eq. (2) [3]. The advantage of the PSM is in providing the results of good quality while using a coarser mesh. To calibrate the method a number of FE models were tested [3] with different geometries, crack size and notch opening angle, and it was proven that for $\frac{a}{d} \geq 3$ the ratio $K_{FE}^*$ is constant, thus defining the limit of the validity of the PSM.

$$K_I^V = \sqrt{2\pi} * \lim_{r\to\infty}(\sigma_\theta)_{\theta=0} * r^{1-\lambda_1} \quad (1)$$

$$K_{FE}^* = \frac{K_I^V}{\sigma_{peak} * d^{1-\lambda_1}} \cong 1.38 \quad (2)$$

## 3 Geometry in Study

The model in study is a representation of three plates joined perpendicularly by fillet welds, see Fig. 2. A nominal load range is applied on the horizontal plating, $\Delta\sigma_n$. In order to further simplify the calculations, symmetry boundary conditions (SBC) were applied when modelling the 2D geometries in ADINA. This resulted into a model size of a quarter of the original one while requiring a much smaller computational effort. The final geometry to be modelled and studied in the present work is presented in Fig. 2.

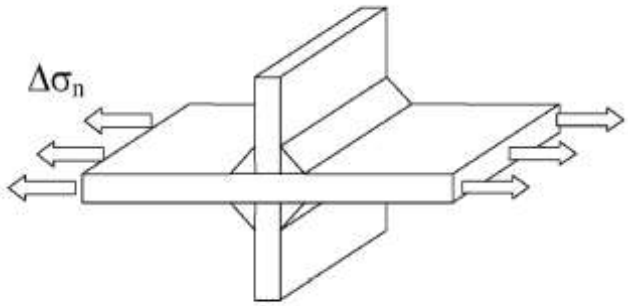


Figure 2 – 2D cruciform geometry [9]

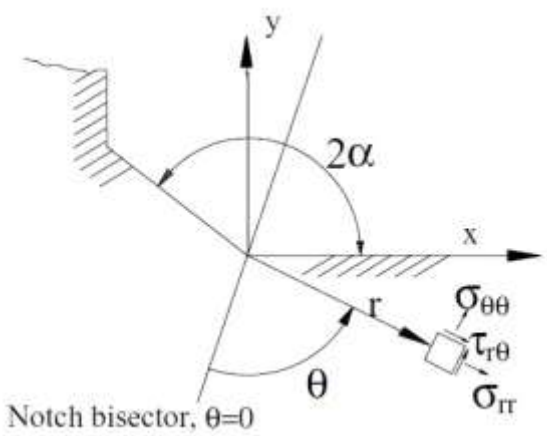


Figure 3 – Stress direction on weld toe [9]

The advantage of the PSM is in providing the results of good quality while using a coarser mesh. To calibrate the method a number of FE models were tested [3] with different geometries, crack size and notch opening angle, and it was proven that for $\frac{a}{d} \geq 3$ the ratio $K_{FE}^{*}$ is constant, thus defining the limit of the validity of the PSM. Two types of geometries are analysed: the load carrying (LC), see Fig. 4, and the non-load carrying (NLC), see Fig. 3, welded joints. In the LC case the model consists of a vertical plate with two welded horizontal plates. The type of failure analysed is the weld root crack, which occurs in the inside of the plating. In the NLC case the model consists of a horizontal plate with two vertical plates welded to it, with the type of failure analysed being the crack initiating at the weld toe.

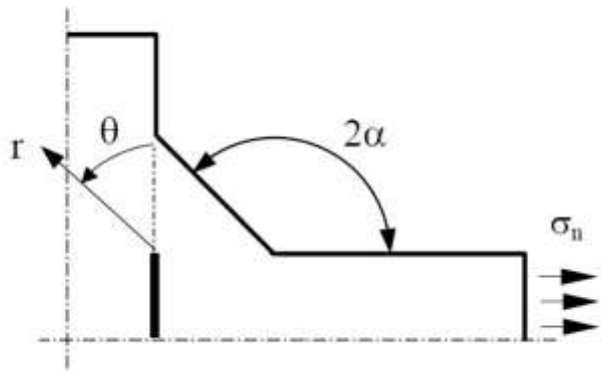


Figure 4 – Weld root analysis [9]

## 3 PSM - Calibration / Application

The main objective of this study is to use the PSM to obtain similar results of the ones obtained with other FEA methods while using much coarser meshes. However, a calibration must be performed in order to establish a limit of the $\frac{a}{d}$ ratio to which the PSM remains valid. To obtain this limit a series of tests were performed in which the $\frac{a}{d}$ ratio was altered [3]. The results, see Fig. 5, showed that the $K_{FE}^{*}$ ratio is constant with the notch depth, *a*, as long as the mesh pattern and element size remain unaltered, thus defining the limit of validity of the PSM.

In order for the PSM to be applied correctly, some influencing parameters are calibrated by employing the commercial software Ansys [10] prior to any

simulations. The parameters for producing the FE models in both softwares are:

- 2D 4 nodes quadrilateral elements;

- Linear shape functions;

- Finite element size $d$ = 1 [mm];

- Automatic mesh generator employed;

- Weld toe node shared by 2 elements;

- Weld root node shared by 4 elements;

- Three-dimensional 8 nodes elements.

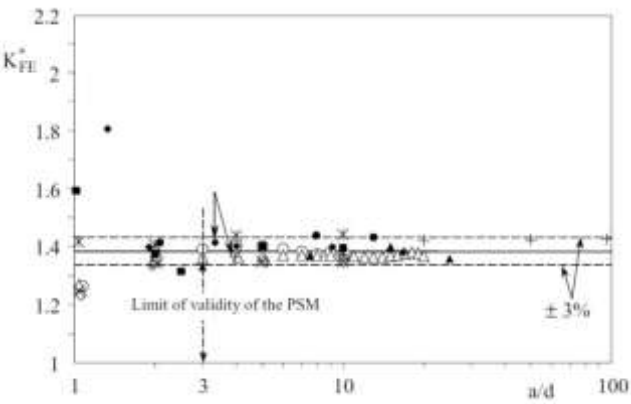


Figure 5 – Results obtained for the $\boldsymbol{K^*_{FE}}$ ratio [3]

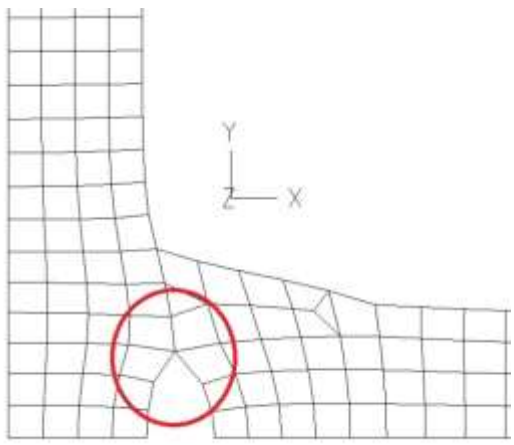


Figure 6 – Weld root detail in ADINA

The 2-D assessments in Ansys [10] were performed using PLANE 42 and PLANE 182 elements. The automatic mesh generator was employed throughout the analysis. The 2-D assessments in ADINA [4] were performed using 2D Solid Elements set up as a linear quadrilateral 4 node element. All geometries modelled [3] have two elements sharing the node on the singularity point where the peak stress is evaluated for the non-load carrying cases (singularity point on the weld toe), see Fig. 6 and four elements for the load carrying cases (singularity point on the weld root), see Fig. 7.

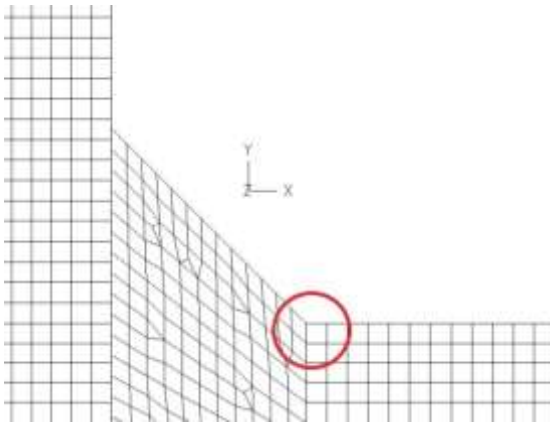


Figure 7 – Weld toe detail in ADINA

## 4 2D Peak Stress Analysis

20 cruciform steel and aluminium 2D NLC welded joint geometries were analysed in this section [3] with different weld and plate sizes, see Table 1 and 2. The geometrical parameters of the models produced in ADINA (plate thicknesses, weld thickness and weld fillet angle) are presented in Fig. 8. The focus of the analysis is in the weld toe in order to produce the $\frac{\sigma_\theta}{\sigma_n}$ ratio for each model and compare them with the values obtained in [3]. The PSM is applied by evaluating the $\frac{\sigma_\theta}{\sigma_n}$ ratio where $\sigma_\theta$ is the peak stress and $\sigma_n$ is the nominal stress applied. The nominal load value is useful only when compared to the peak stress. Therefore a single valued nominal load is used to immediately obtain the $\frac{\sigma_\theta}{\sigma_n}$ ratio. The results were analysed in terms of the difference (E1) between the $\frac{\sigma_\theta}{\sigma_n}$ ratios from [3] and the ones obtained with ADINA and in terms of the difference between the $K^*_{FE}$ ratio from [3] and the one calculated with the peak stress obtained in ADINA. An absolute precision error (APE) was also produced to assess the dispersion of the results.

Table 1 – Results obtained for the peak stress

| Model | E1( $\frac{\sigma_\theta}{\sigma_n}$ ) | E1 ( $K^*_{FE}$) |
|---|---|---|
| **Steel #1** | 0.068 | 0.098 |
| **Steel #2** | 0.073 | 0.101 |
| **Steel #3** | 0.081 | 0.075 |
| **Steel #4** | 0.032 | 0.061 |
| **Steel #5** | 0.005 | 0.029 |
| **Steel #6** | 0.049 | 0.071 |
| **Steel #7** | 0.040 | 0.071 |
| **Steel #8** | 0.001 | 0.033 |
| **Steel #9** | 0.062 | 0.073 |
| **Steel #10** | 0.039 | 0.069 |
| **Steel #11** | 0.061 | 0.088 |
| **Steel #12** | 0.060 | 0.089 |
| **Steel #13** | 0.050 | 0.078 |
| **Steel #14** | 0.073 | 0.100 |
| **Mean** | 0.050 | 0.074 |
| **APE** | 0.030 | 0.034 |

Table 2 – Results obtained for the peak stress

| Model | E1( $\frac{\sigma_\theta}{\sigma_n}$ ) | E1 ( $K^*_{FE}$) |
|---|---|---|
| **Aluminium #1** | 0.086 | 0.118 |
| **Aluminium #2** | 0.051 | 0.088 |
| **Aluminium #3** | 0.042 | 0.084 |
| **Aluminium #4** | 0.074 | 0.112 |
| **Aluminium #5** | 0.071 | 0.106 |
| **Aluminium #6** | 0.064 | 0.099 |
| **Mean** | 0.065 | 0.101 |
| **APE** | 0.016 | 0.013 |

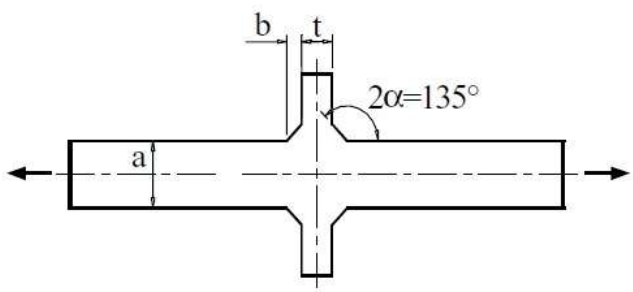


Figure 8 - Geometrical parameters [3]

# 5 Weighted Peak Stress

The Strain Energy Density (SED) method is a method to perform a fatigue strength assessment by analysing the strain energy mean value in a defined volume enclosing the point where the fatigue crack initiates (weld toe or weld root) [11]. By relating the SED method with the PSM [12] a new parameter is included, $f_w$, which is a correction factor that allows to obtain peak stress data for a multitude of geometries which in turns allows to convert multiple scatter bands referring to multiple geometries into a single scatter band, see Fig. 9. The parameter $f_w.\Delta\sigma_{peak}$ is referred to as weighted peak stress [11] and is applicable to fatigue strength assessments of welded joints failure from both the weld toe and on the weld root. The correction factor is calculated according to eq. (3):

$$f_w.\Delta\sigma_{peak} = 1.38.\sqrt{\frac{2e_1}{1-\nu^2}}.\Delta\sigma_{peak}.\left(\frac{d}{R_0}\right)^{1-\lambda_1} \quad (3)$$

- $d$: mesh size [mm];
- $R_0$: radius of control volume [mm] [11];
- $e_1$: parameter dependent on the notch opening angle and Poisson ratio;
- $\nu$: Poisson ratio;
- $\lambda_1$: mode I Eigenvalue in William's equation [2].

The peak stress is analysed for a set of 2D cruciform welded joint geometries in the weld toe and weld root area. An average of the results obtained for the $\frac{\sigma_\theta}{\sigma_n}$ ratio, similarly to the results in Table 1 and 2, is presented in Table 3 and 4.

Table 3 – Results obtained for the weighted peak stress – weld toe

| Model | E1 ($\frac{\sigma_\theta}{\sigma_n}$) |
|---|---|
| **Weld toe #1** | 0.100 |
| **Weld toe #2** | 0.059 |
| **Weld toe #3** | 0.062 |
| **Weld toe #4** | 0.120 |
| **Mean** | 0.085 |
| **APE** | 0.030 |

Table 4 – Results obtained for the weighted peak stress – weld root

| Model | E1 ($\frac{\sigma_\theta}{\sigma_n}$) |
|---|---|
| **Weld root #1** | 0.031 |
| **Weld root #2** | -0.096 |
| **Weld root #3** | -0.034 |
| **Weld root #4** | -0.090 |
| **Mean** | -0.056 |

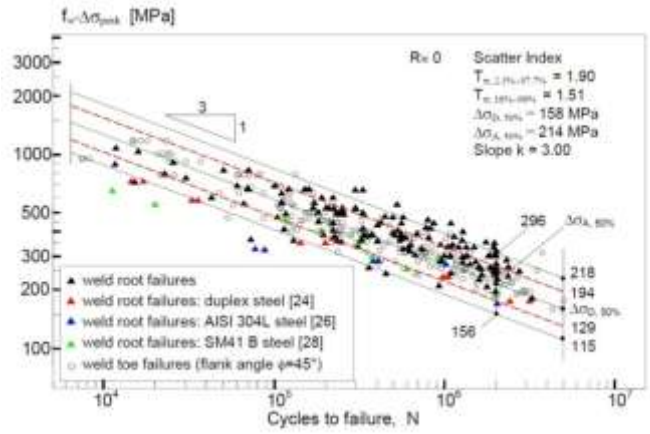


Figure 9 – S-N scatter diagram[12]

# 6 Discussion of the results

The analysis of the results is divided in two parts, the analysis of the $\frac{\sigma_\theta}{\sigma_n}$ ratio and the $K_{FE}^*$ ratio. When analysing the results corresponding to the $\frac{\sigma_\theta}{\sigma_n}$ ratio the objective is to compare the results obtained using 2D linear quadrilateral 4 node elements from ADINA with the results obtained using PLANE 42 and PLANE 182 elements from Ansys. In the analysis of the results regarding the $K_{FE}^*$ ratio the objective was to obtain the $K_{FE}^*$ value using eq. (2) for a multitude of geometries and analyse the span of the results obtained while comparing it to the span of results shown in Fig. 5.

**Analysis of the results for the $\frac{\sigma_\theta}{\sigma_n}$ ratio**

The results regarding the $\frac{\sigma_\theta}{\sigma_n}$ ratio are presented in Table 1, 2, 3 and 4. Observing the data referring to the NLC geometries in Tables 1, 2 and 3 it is possible to observe a relative mean difference between 5% and 9% along with an APE between 2% and 3%. These values are low and are considered satisfactory, thus proving the quality and robustness of the data obtained in ADINA when compared to the peak stress values obtained in the references using Ansys. It is also important to notice that the less quality results correspond to the ones obtained in Table 3 which are for fillet angles $\phi \neq 45°$. This may indicate that the PSM may not be suitable to perform a fatigue strength analysis on cruciform geometries with fillet angles $\phi \neq 45°$. The analysis of the LC geometries was performed, not based on a peak stress/nominal stress ratio such as the NLC geometries, but based on a specific value of applied nominal load. This value of nominal load is then applied to obtain a peak stress value. The exact nominal load values used to obtain the peak stress against which the values of peak stress obtained in this work are to be compared against, were not provided and had to be estimated by either a linear interpolation or a graphical extrapolation. This necessary procedure adds an uncertainty to the analysis which will decrease the quality of the results obtained. This uncertainty is present in the results from Table 4.

**Analysis of the results for the $K_{FE}^*$ ratio**

Fig. 10 and 11 present the results obtained for the $K_{FE}^*$ ratio from Table 1 and 2. The values obtained are not important, as the ratio is merely a calibration factor. The convergence of the results however, is vital to prove the applicability of the PSM. The convergence observed in Fig. 10 and 11 for the values of $K_{FE}^*$ is a good indicator of the applicability of the PSM while using 2D linear quadrilateral 4 node elements from ADINA. It has to be taken into account that the PSM only applies for $\frac{a}{d} \geq 2$. The obtained value for $K_{FE}^*$, see eq. (4), sets a different calibration parameter than the one used for the fatigue strength analysis performed using Ansys, $K_{FE}^* \cong 1.38$.

$$K_{FE}^* \cong 1.275, \pm 3\% \tag{4}$$

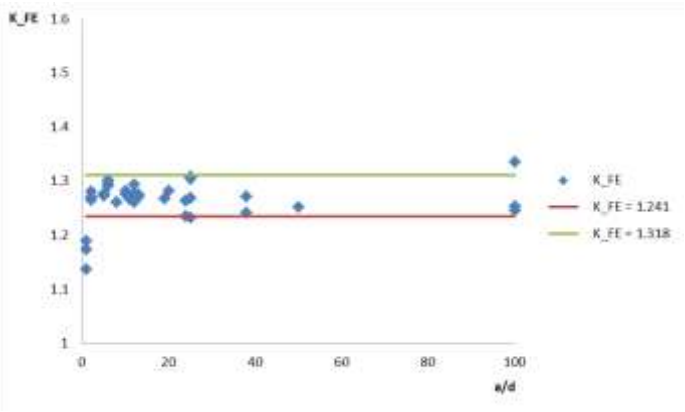


Figure 10 – Results obtained for the $K_{FE}^*$ ratio

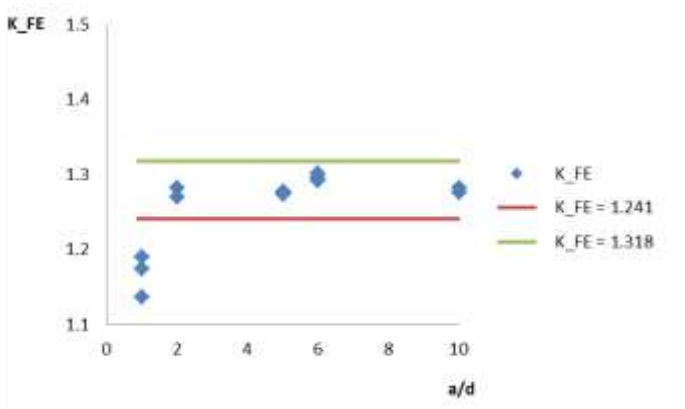


Figure 11 - Results obtained for the $K_{FE}^*$ ratio

# 7 Conclusions

This work performed a large amount of analyses where the PSM was applied for a fatigue strength analysis using various models with different geometrical parameters in order to validate the applicability of the PSM. The objective of this work in specific was to perform a fatigue strength analysis in a cruciform geometry, both load carrying and non-load carrying, in order to assess the quality of the results provided by the 2D linear quadrilateral 4 node elements from ADINA, which are the elements with the closest characteristics of the PLANE 42 and PLANE 182 elements from Ansys used in [3] when the PSM was first employed. In order to carry on the proposed assessment the analysis was focused on two parameters, the $\frac{\sigma_\theta}{\sigma_n}$ ratio and the $K_{FE}^*$ ratio.

Regarding the $\frac{\sigma_\theta}{\sigma_n}$ ratio, the quality of the results obtained is demonstrated by its proximity to the values from [3] and [12]. Regarding the $K_{FE}^*$ ratio, the quality of the results obtained is assessed by its convergence. The average value for $K_{FE}^*$ does not have to be the same as the one obtained in [3] as its value is merely a calibration parameter. It has, however, to show a convergence.

The results obtained are a good indicator of the applicability of the PSM in future fatigue strength assessments. Unlike the approach in which is based, the NSIF approach, the fatigue strength analysis using the PSM only requires a single linear stress value instead of data from a whole stress field. It also requires much coarser meshes to do so. This means that the effort to obtain results is much smaller, both from the numerical point of view as from the operator as the finite element models are simpler to produce and analyse. Adding to this, tests performed in [12] with the software *Ansys*, along with the results from this work, obtained with ADINA, are a good indicator that a single S-N scatter diagram is valid for analysing multiple geometries for both weld root and weld toe cases.

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