Propagation of density currents in a porous medium

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Abstract

Residual wastewater can be treated in constructed wetlands (CW), consisting in basins with porous filler through which wastewater flows, denser than the water present in the bed. In these, the wastewater flows in the form of a density current. This thesis aims to clarify the dynamics of density currents over and within granular beds, with special emphasis on the description of kinematic aspects. Laboratory tests of lock-exchange type were performed (fluid release in a channel containing a fluid with lower density), and the data was acquired through the image analysis technique. The porous medium consists in glass spheres, and between experiments there was a variation in the height of the porous medium and the height of fluid above it. Three types of flows were observed. For each type the most significant results are presented.

– Density currents at the surface medium: occur when there is fluid above the porous medium presenting longitudinal propagation. The upstream load and the density difference determine the evolution of the current front. The fundamental geometric scale is the position of the gate and the kinematic scale is obtained from the acceleration of the reduced gravity and initial load. The obtained results are compatible with previous studies in currents over smooth beds (self-similar) and rough beds, which present an attenuation in the propagation speed. The novelty introduced is the characterization of density currents over porous mediums, for which lower propagation velocities occur due to the loss of mass into the medium.

– Infiltration of the superficial current body into the medium: occurs when the height of the medium is sufficiently high for its development. It occurs in the form of plumes with descending vertical development. The infiltration driver mechanism is similar to that of Rayleigh-Taylor instability, the inverse stratification of the fluids. The growth rate and dominant wavelength are independent of the fluid load above the medium. The growth rate will depend only on the bed material. The depletion of fluid above the bed causes a stop in the development of the plume in relation to the propagation of the infiltration front. These results are an absolute novelty in the context of the study of density currents on porous beds.

– Density currents inside the medium: horizontal development with a much slower velocity than the superficial current and is proportional to the Reynolds number. The current is neither self-similar nor dimensionless with the total height of the fluid, which will result in the viscous effects being very relevant. Infiltration leads to increasing velocity of the current within the medium. The current inside the medium causes the deceleration of the plumes development.

Keywords: Gravity Currents; Porous Media; Rayleigh-Taylor Instabilities; Image Analysis Technique; Lock-exchange

1. Introduction

Not treated wastewater are prejudicial to the environment and to human health. Treatment processes need to be applied before wastewaters are returned to the receiving aquatic environment.

One treatment system frequently used are constructed wetlands (CW). Mainly applied for small populations, CW consists in the reproduction of natural wetlands. CW are basins with a porous filler, colonized by macrophytes, through which flows the wastewater. The affluent wastewater is denser than the water present inside the medium. The developed flow inside the CW takes the form of a density current, with longitudinal direction. In CW there can be situations when superficial flow is present (i.e. affluent flow is much higher than the flow that the CW was dimensioned). This water present at the surface will infiltrate vertically to the inside of the porous medium.

The main objective of the present work is to clarify the mechanisms associated with the propagation of a density current in a porous medium, particularly when coexists superficial flow and flow inside the medium. The scientific questions leading this investigation are the following:

1) Which are the regimes and temporal scales relative to the superficial flow?
2) Which are the mechanisms that originate and drive the infiltration of the superficial current at initial times?
3) How is the interaction between the propagation inside the media (horizontal) and the infiltration flow (vertical)?

The specific objectives aim to answer the scientific questions displayed above. Laboratory tests of lock-exchange type were performed (fluid release in a channel containing a fluid with lower density), and the data was acquired through the image analysis technique. The porous medium consists in glass spheres, and between experiments there was a variation in the height of the porous medium and the height of fluid above it.

2. Literature review

2.1. Density currents

A density current, also known as gravity current, occurs when a denser fluid (contaminant fluid) flows into a less denser fluid (ambient fluid), motioning mostly in the horizontal direction (Simpson, 1982). The main impulse force of the motion is the density difference between fluids. This can be due to dissolved material present in the fluid, suspended matter, or temperature differences.

The experimental study of density currents started in the middle of the 20th century with von Kárman (1940). Most studies have been trying to clarify factors that affect the dynamic and kinematic of the currents, with particular emphasis on the effects of density difference and bottom slope (Rottman and Simpson, 1983; Lombardi et al. 2015). However, detailed studies of the behaviour of a density current propagating over a granular porous medium are not found.

Reproduction of density currents in a laboratory is commonly done through lock-exchange experiments (Rottman and Simpson, 1983, Hallworth et al., 1996, Nogueira et al, 2013). These experiments consist in the removal of a gate separating two fluids with different densities promoting the flow of the heavier fluid through the bottom of the channel and the lighter fluid in the opposite direction.

Anatomically, a density current has two distinct zones: the head and body (figure 1). The head is a strong mixture zone due to the unstable flow resulting of gravitational and shear instabilities. These instabilities are seen in the form of Kelvin-Helmoltz (KH) instabilities and the formation of lobes and clefts (see figure 2).

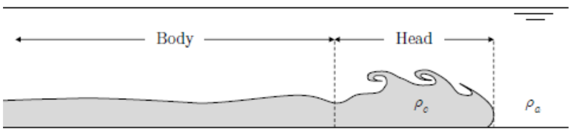


Figure 1 – Generalized shape of a density current (Silvestre, 2017)

a) b)

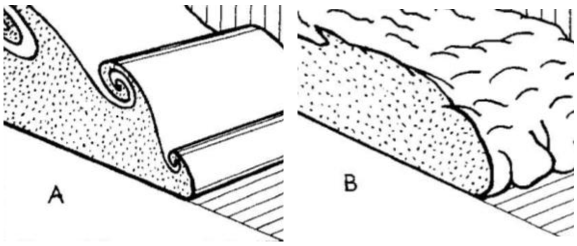


Figure 2 - Instabilities on the density currents head: a) KH instabilities; b) lobes and clefts (Simpson, 1997).

Three different stages are seen in the propagation of density currents (Simpson, 1997):

a) "slumping phase": initial adjustment phase, dependent on the initial conditions. It is formed the head of the current and it develops with constant velocity;
b) Self-similar phase: governed by balance of the buoyancy and inertial forces. The velocity of the current diminishes with $t^{-1/3}$.
c) Viscous phase: viscous forces are predominant in the propagation of the flow.

When a current is propagating over a rough bottom, the velocity of the current will also be smaller than the velocity of a density current in a smooth bottom due to the drag force applied by the rough bottom (Nogueira et al. 2013).

A density current propagating over a porous or a permeable medium will exchange fluid with the porous substrate (Simpson, 1997). The current is in everything similar to the current over smooth bed, but the loss of mass into to the medium leads to a slower velocity of the current.

The knowledge of density currents inside a porous medium is not as prolific as the knowledge of density currents in smooth beds. Its study has been made through Hele-Shaw cells (Huppert and Woods, 1995; Hesse et al, 2007; Szulczewski and Juanes, 2013). However, this technique despises the superficial tension between the fluids and the medium particles, the interfacial tension between the fluids inside the porous, drag effects present in density currents.

2.2. Infiltration

The flow of heavier fluid at the surface will infiltrate into the granular medium. Although phenomena associated with the infiltration of water in unsaturated granular media are known, detailed studies on the infiltration of a viscous fluid into a medium saturated by other viscous fluid are fewer. Likewise, studies on the propagation of density currents inside granular media are incomplete in terms of interaction with the surface current. In addition, these studies were performed on Hele-Shaw cells, which do not allow the three-dimensional development of the current, presumably leading to systematic errors in the time scales associated with the propagation of the density currents.

The infiltration into a porous medium is observable as the development of plumes of infiltrating fluid. In the present work, it is considered that the formation of plumes is the development of Rayleigh-Taylor (R-T) instabilities (considered in previous studies such as Kalisch et al., 2016).

R-T instabilities are present when a heavier fluid is present over a porous medium saturated with a lighter fluid. When the resulting force is directed to the lighter fluid, it will lead to a disturbance in the fluids interface, presented as waves. The hydrostatic pressure will promote the development of the amplitude of the waves as intrusion of the heavier fluid into the lighter one (Andrews and Dalziel, 2010), in the form of plumes, with wavelength λ.

In early stages, the evolution of the waves is exponential, with $e^{\sigma t}$, where the growth rate is real and provided by $\sigma = \sqrt{At\, g\, k}$, where k is the spatial wavenumber, At is the Atwood number and g is the acceleration of gravity (Drazin, 2002).

Through a linear stability analysis, the solutions are of the form:

$$\tilde{z}_p(t) = \Delta z_0 e^{ikz+\sigma t} \qquad (2.1)$$

being $\tilde{z}_p(t)$ the position of the plume on instant t, and Δz_0 a constant (Manickam and Homsy, 1995).

3. Experimental work

The laboratorial work was performed in the Environmental Laboratory of the Civil Department of Instituto Superior Técnico. Lock-exchange experiments were performed in a 300 cm length, 40 cm height and 5 cm width tank. The gate confining the contaminate fluid is positioned in position x = 20 cm and a PVC plate was placed in position x = 233 cm, dividing the channel in two, to have a calibration zone (figure 3).

The porous medium consists in glass spheres with a diameter, d, of 3 mm.

Between tests, there was variation of the height of the medium, H_b, the height of fluid above the medium, h_0, and consequently total height, $H_0 = H_b + h_0$. Each test was performed in two different positions: pos.1: 10 to 50 cm; pos.2: 70 to 110 cm. In table 1 are displayed the tests specifications.

Table 1 - Tests specifications (exp. - experiment; pos. - position)

Test	H_b (cm)	h_0 (cm)	H_0 (cm)	Exp.	Pos.
1	10	10	20	1.1.1	1
				1.2.1	2
2	10	2.5	12.5	2.1.1	1
				2.2.1	2
3	10	0	10	3.1.1	1
				3.2.1	2
4	5	0	5	4.1.1	1
				4.2.1	2
5	1	10	11	5.2.1	2
6	0.3	10	10.3	6.2.1	2
7	0	10	10	7.2.1	2

The heavier and lighter fluids were obtained through the mixture of water and salt and water and ethanol, respectively. The components added were chosen in order to have a similar refractive index between fluids and a difference of density of 15 g/l between fluids. The refractive index matching was performed according to Daviero et al. (2001).

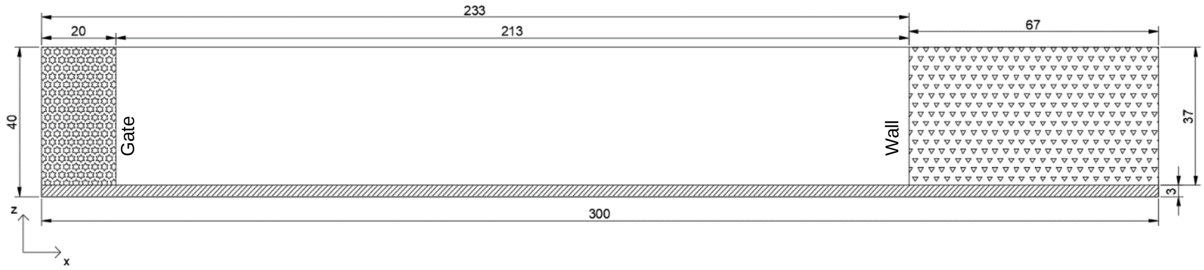


Figure 3 - Channel scheme. Stars – contaminant fluid contained before the experiment; triangles – calibration zone.

The data was acquired through an image analysis technique. This technique allows the study of a density current over and inside a porous medium. It is used to estimate the bidimensional velocity field of the current density and to investigate the dynamics of the same. It's used a dye in the heavier fluid, that will attenuate the LED light strategically positioned to illuminate the region in study, altering the gray levels of the images obtained. Through a calibration process it is possible to correspond a certain gray level to a density of contaminant fluid. While in the zone without spheres it is possible to make a calibration pixel by pixel, in the medium area it's not possible due to the variation in the gray levels of the porous medium area from experiment to experiment due to the rearrangement of the spheres. For that area, it is made an average of values of four different rearrangements of the spheres followed by the average of gray levels values correspondent to all pixels of the medium area. Even though it is not possible to correspond a correct value to a pixel, it allows to perceive zones with higher or lower density.

The raw images obtained were treated with MATLAB software. After orthorectifying the images, a clear view of the contaminant fluid was obtained through the subtraction of a reference image. Inside the medium was applied a gauss filter to smooth the medium. The outline of the current allows the tracking of the currents front position, both superficial and inside the medium.

4. Results and discussion

4.1. Initial considerations

Through the experiments performed, three distinct flows were observed:

– Superficial density currents (above the porous medium) formed only if $h_0 > 0$ cm, with longitudinal development;
– Infiltration of contaminant fluid, in the form of plumes, from the body of the superficial current. It is seen whenever $H_b \geq 5$ cm and $h_0 > 0$ cm, and it propagates with vertical direction.
– Density currents inside the porous medium, visible when $H_b \geq 5$ cm, with longitudinal development.

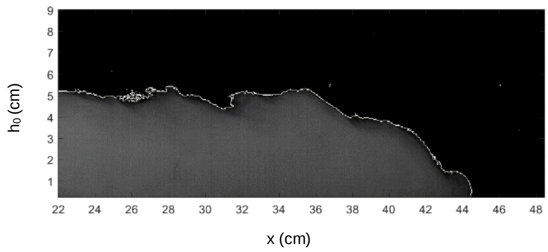
When infiltration and the propagation of density currents inside the medium coexist, their development is affected by each other.

4.2. Superficial density current

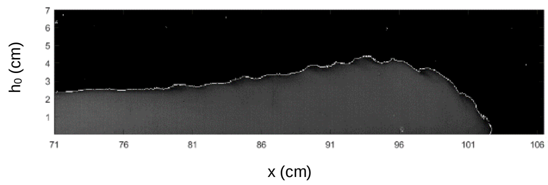
The presence of fluid above a porous medium leads to the propagation of a density current along the medium surface. In figure 1 is presented the delimitation of the superficial current and in figure 2 are presented the dimensionless evolutions of the current front. The non-dimensional position scales with x/X, where x is the front position of the current and $X = x_0 = 20$ cm (gate position). The non-dimensional time is defined by t/T, with $T = X/u$, where $u = \sqrt{g' \times h_0}$. According to Rottman and Simpson (1983), density currents are in the slumping phase till $t/T = 20$.

The upstream load is determinant to the current evolution. Lower load leads to currents with small velocity where viscous effects are predominant (experiment 2.1.1). This current has no instabilities on the head, having a smooth interface (figure 4c).

a) Experiment 1.1.1 (t = 5 s)



b) Experiment 1.2.1 (t = 12 s)



c) Experiment 2.1.1 (t = 12 s)

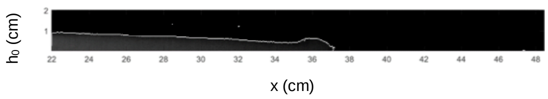


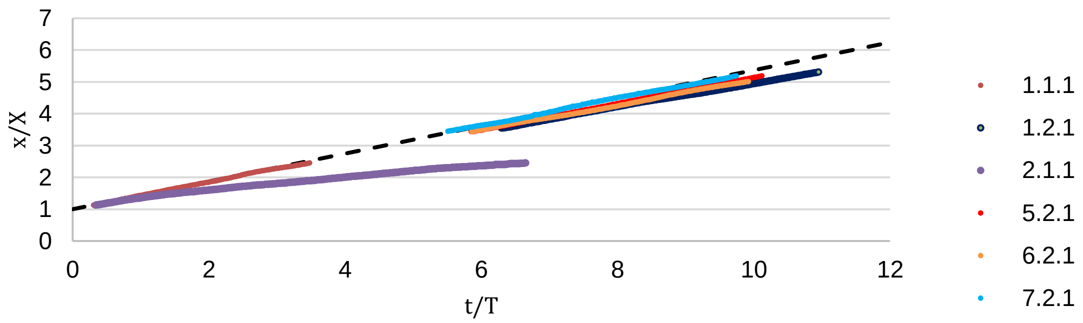
Figure 4 - Superficial density current delimitation

The currents with higher load have instabilities on the interface and are in the slumping phase (t/T < 20).

The evolution of the smooth bed density current (experiment 7.2.1) is over the Rottman and Simpson (1983) dimensionless evolution. As for the evolution of experiments 5.1.2 and 6.1.2 is smaller than experiment 7.1.2, due to the rough bottom that increases the drag force. These results are compatible with previous studies such as Rottman and Simpson (1983) and Nogueira et al. (2013).

The evolution of experiments with higher medium (1.1.1, 1.2.1 and 2.1.1) are slower because the current loses mass into the porous medium.

a)



b)

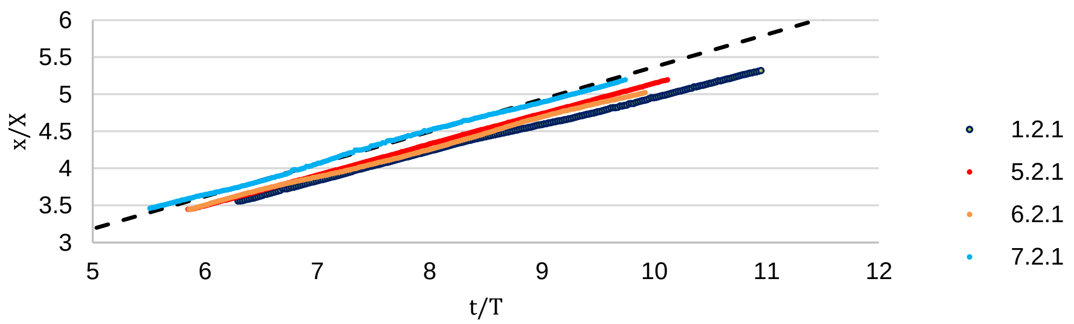


Figure 5 – Dimensionless evolution of the superficial density current front. a) both positions; b) pos.2. (- - -) Rottman and Simpson (1983) dimensionless evolution.

4.3. Infiltration

4.3.1. Plumes characterization

The infiltration process occurs as plumes, with vertical downwards orientation. Their development begins with the passage of the superficial current. Density maps of the infiltration process are presented in figure 6.

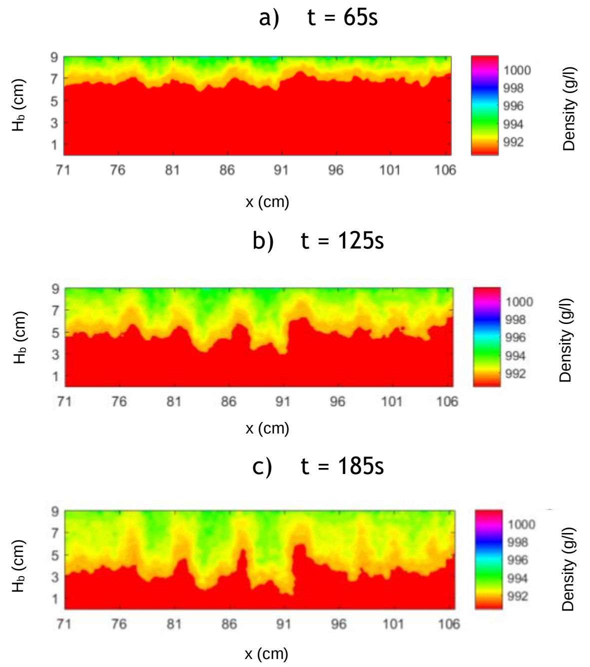


Figure 6 – Density maps of the infiltration process (referent to experiment 1.2.1).

The plumes are denser in its core, becoming lighter towards the interface. Between plumes, the density is much lower due to the escape of ambient fluid. As the time progresses, the front of the plume will also diminish its density, due to effects of diffusion.

4.3.2. Dominant wavelenght

The dominant wavelength of the plumes was obtained via visual inspection, identifying the centre of the plumes, calculating the distance between them and applying the average to these values. The dominant wavelength for the different experiments are similar, indicating that the dominant wavelength is not affected by the initial conditions (table 2), particularly the height of fluid above the medium.

Table 2 - Dominant wavelength

Experiment	Dominant Wavelength (cm)
1.1.1	4.18
2.1.1	3.87
1.2.1	3.99
2.2.1	3.62

4.3.3. Growth rate

For the determination of the growth rate were chosen 5 plumes from the experiments 1.1.1, 1.2.1, 2.1.1 and 2.2.1 (figure 7).

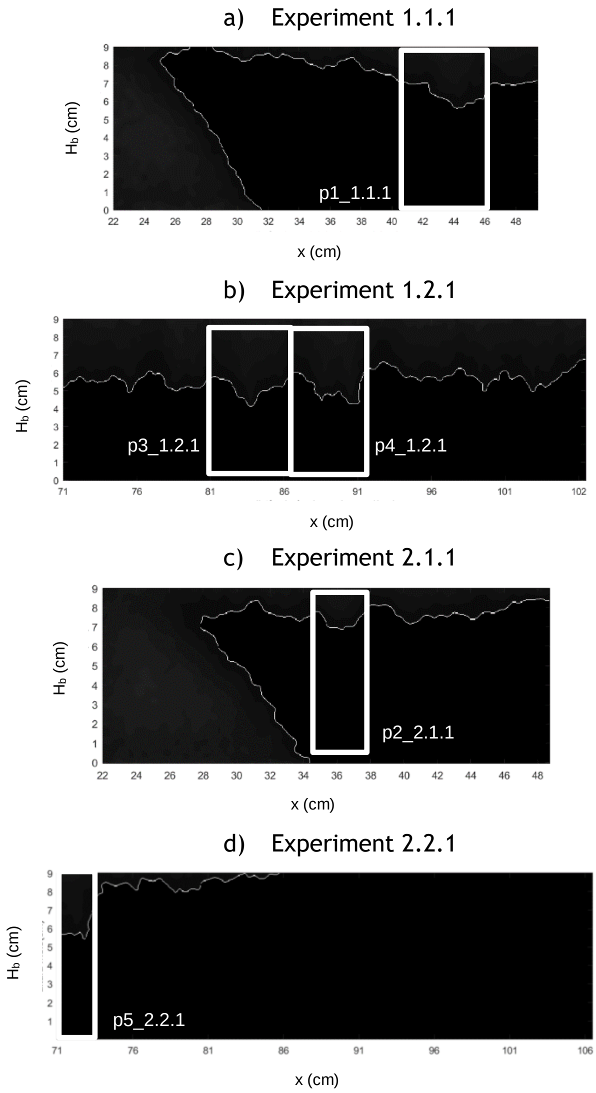


Figure 7 – Plumes identification.

The evolution of the average of the infiltration front, Z, is presented in figure 8. The evolution of the front in the experiments of test 1 are similar among them.

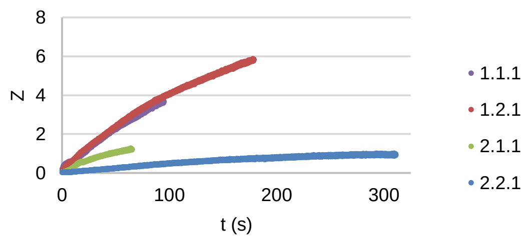


Figure 8 – Evolution of the average infiltration front.

The small development of Z from experiment 2.1.1 is due to the small velocity of the superficial current, along with the development of the density current inside the porous medium (see chapter 4.5). In experiment 2.2.1, there is only the development of one plume because of the low velocity and mass of the superficial current (superficial current stops propagating in the middle of position 2).

The growth rate of the dominant wavelength is obtained through the modification of equation (2.1), that corresponds to the amplitude growth associated to this wavelength relatively to the average infiltration front, described by:

$$\tilde{z}_p(t) = \Delta z_0 e^{\sigma t} \quad (4.1)$$

where $\tilde{z}_p(t)$ is the position of the plume relatively to the average infiltration front (Z). Applying the logarithm to the equation (4.2), it is obtained:

$$ln\left(\tilde{z}_p(t)\right) = ln(\Delta z_0) + \sigma t \ . \quad (4.2)$$

The growth rate is obtained through a linear regression of equation (4.2). In figure 9 is presented this evolution for experiments 1.1.1, 1.2.1 and 2.2.1. It is not shown the progression of plume p2_2.1.1, since it is affected by the interior current being referred in section 4.5.

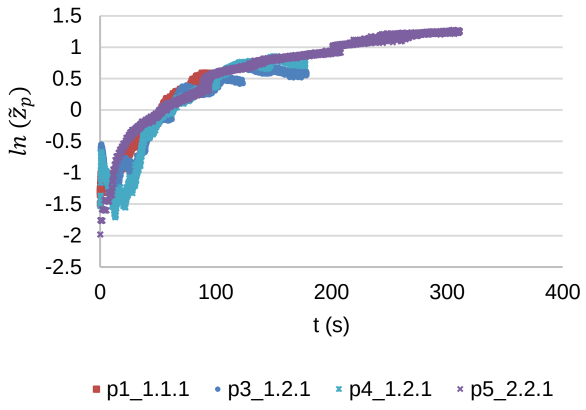


Figure 9 – Logaritmic evolution of $\tilde{z}_p$

The plumes of test 1 (p1_1.1.1, p3_1.2.1 and p4_1.2.1), present an initial phase where the growth of the plume is equal or smaller than the infiltration front (t < 40 s) and final stage (t > 100 s) where the growth stagnates in relation to Z. This situation is not present in plume p5_2.2.1, because of the smaller development of Z, promoting the growth of the plume.

For the interval 50 < t (s) < 100, there is a similar growth of the different plumes, being valid a normalization of the results. Not following a known scale, the non-dimensional evolution of $\tilde{z}_p$ is a function of the medium material (d_{50}), mainly $\tilde{z}_p/d_{50}$. The time is normalized through $t\sqrt{g/d_{50}}$. The dimensionless evolution is presented in figure 10.

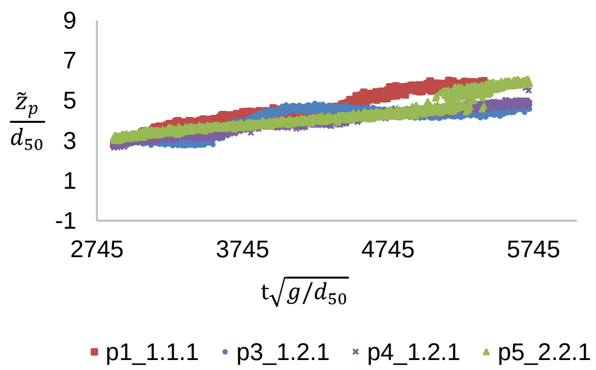


Figure 10 – Dimensionless evolution of $\tilde{z}_p$.

In table 3 are presented the growth rates, from applying the linear regression to the values of figure 9 for the interval 50 < t (s) < 100. These results show that the growth rate is independent of the height of fluid above the medium.

Table 3 – Plumes growth rate for the interval $50 < t\ (s) < 100$.

Plume	Growth rate, σ
p1_1.1.1	0.014
p3_1.2.1	0.010
p4_1.2.1	0.011
p5_2.2.1	0.012

4.4. Density currents inside the porous medium

The delimitation of the density current inside the medium is presented in figure 11. For the early stages, the current presents an S shape that will evolve to a straight line, consistent with Szulczewski and Juanes (2013).

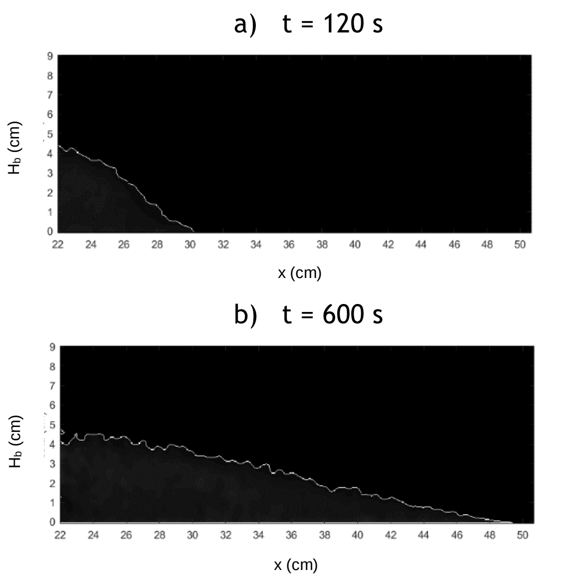


Figure 11 – Delimitation of the density current inside the medium (experiment 3.1.1).

The dimensionless and absolute evolution of the currents front are presented in figure 12. The dimensionless evolution of the current front scales with x/H_0 and the time scales with $t\sqrt{g\ /\ H_0}$.

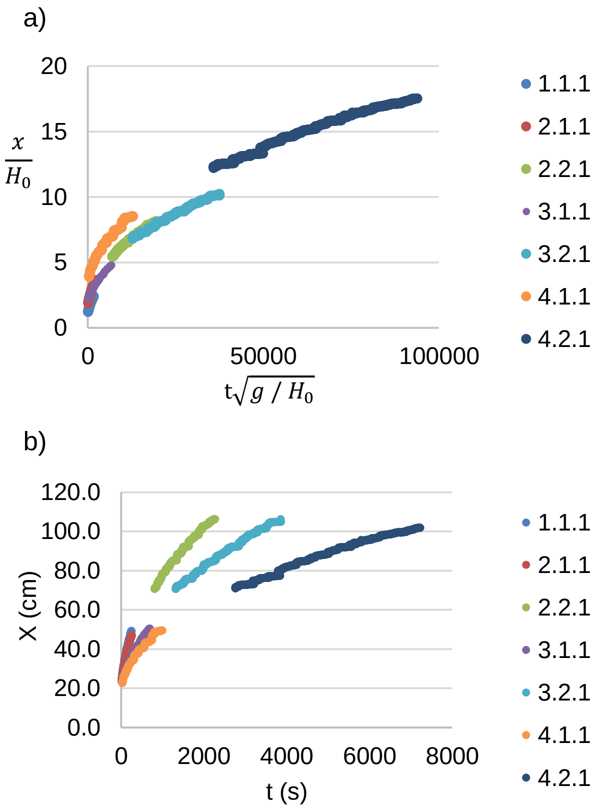


Figure 12 – Evolution of the current front inside the medium: a) dimensionless, b) dimensional

The currents inside the medium are much slower than the currents in the surface of the medium and the viscous effects are predominant.

The dimensionless graph shows that currents form tests with the same H_b and different H_0 (tests 1, 2 and 3) will collapse similarly, despite the height of fluid above the medium. As for the current of the test with H_b = 5 cm (test 4) it presents a higher apparent velocity (in absolute terms it is the slowest). This is situation leads to believe that the propagation of density currents inside a porous medium is dependent of the Reynolds number. In the present case, there are two distinct Re families, one for H_b = 5 cm and another for H_b = 10 cm.

4.5. Interaction between infiltration and density current inside the medium

The presence of fluid above the medium has a direct effect in the propagation of the current inside the medium (figure 12b), increasing its velocity. This is consequence of the added pressure that the infiltration lays on the density current.

The evolution of plumes will also be affected by the current inside the medium. As seen in figure 13, the evolution of plume2_2.1.1 is much smaller than the plumes of the same tests or the same position

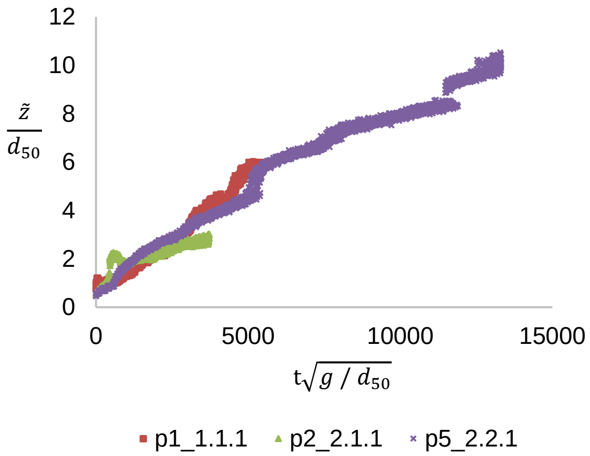


Figure 13 – Dimensionless evolution of plumes

This difference is due to the conjugation of two factors: the slow propagation of the superficial current (leading to a late development of the plume) and the propagation with normal speed of the current of the medium. This current will provide an additional factor of resistance to the plumes development, because it will push the ambient fluid upwards.

The current will also push the ambient fluid downstream, deforming the already developed plumes (figure 14).

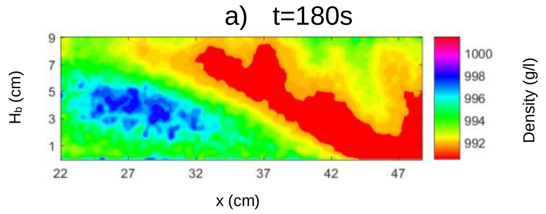


Figure 14 – Density map of experiment 1.1.1.

5. Conclusions

Through the experiments done, three different types of flows were studied:

– Superficial density currents: upstream load and the density difference determine the evolution of the current front. The fundamental geometric scale is the dimensions position of the gate and the kinematic scale is obtained from the acceleration of the reduced gravity and initial load. Currents over a porous medium have lower velocity velocities due to mass loss to the medium;
– Infiltration: in the form of plumes with downward vertical development. The infiltration driver mechanism is similar to that of Rayleigh-Taylor instability, the inverse stratification of the fluids. Growth rate and dominant wavelength are independent of the fluid load above the medium. The growth rate depends only on the bed material. The depletion of fluid above the medium causes a halt in the development of the plume in relation to the propagation of the infiltration front;
– Density currents inside the medium: horizontal development with a much slower velocity than the superficial current and proportional to the Reynolds number. The current is neither self-similar nor dimensionless with the total height of the fluid, which will result in the viscous effects being very relevant. Infiltration leads to increasing velocity of the current within the medium. The current inside the medium causes the deceleration of the plumes development.

These results are a novelty in density currents in porous media and can have many applications in different areas, such as provide new and more adequate time scales to apply in CW modelling. This work is also an incentive to the development of new analysis methods, such as evaluate the velocity inside the contaminating fluid inside the porous medium as well as what the preferential path of the contaminating fluid in the initial instants of the infiltration

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