Improving patient flow and delivery of care: a bed assignment optimization model

Francisca Botelho Silva Gomes

Thesis to obtain the Master of Science Degree in

Biomedical Engineering

Supervisors: Prof. Dr. Tânia Rute Xavier de Matos Pinto Varela

Dr. Maria Inês dos Santos Mendonça Padre Eterno

Examination Committee

Chairperson: Prof. Dr. Mónica Duarte Correia de Oliveira

Supervisor: Prof. Dr. Tânia Rute Xavier de Matos Pinto Varela

Members of the Committee: Prof. Dr. Ana Isabel Cerqueira de Sousa Gouveia Carvalho

May 2018

Acknowledgments

First of all, I would like to thank Prof. Tânia Varela for her guidance and support throughout this thesis.

I would like to thank Dr. Inês Mendonça who gave me the opportunity to perform this thesis in Hospital Beatriz Ângelo. To the nurse chief, Teresa Simões, and the bed managers, Catarina Rodrigues, Vitor Vaz Pinto and João Celestino who transmitted me the technical knowledge regarding bed management that was crucial in the model development.

To all my friends and family, and particularly to my parents I would like to thank for the strength, encouragement, support and opportunities they gave me through all these years.

Abstract

The world is ageing, healthcare professionals are a limited resource and, as a result, hospitals are facing constant bottlenecks. This increase in patient flow makes hospital bed management key to keep the system in sync. Choosing a bed for a patient is not a trivial task, given the different criteria and constraints that should be considered when assigning a patient. Although we are living in the era of innovation and technology, this matter has not been deeply explored yet. This thesis lies in a case study of a Portuguese public hospital, where this process is performed daily on a paper base by a nurse. Given the amount of information to be considered and the different stakeholders involved in the process, the information can end up being mistreated, leading to a deficient bed assignment. This thesis proposes a multi-objective mathematical model as a decision support tool. Different scenarios were designed to assess the model behavior with the variation of the hospital occupancy rate. The optimization of the formulation was performed using two different techniques, goal programming (GP) and lexicographic goal programming (LexGP). GP performed better across all scenarios in a smaller period, however, LexGP ended up mimicking more accurately the BA task. A sensitivity analysis was also performed to study how the model behaved when varying the batch dimension and the maximum number of transfers. It is important for the hospital to have a model developed in line with its decision criteria and goals that may positively impact its performance.

Keywords: Bed management, Bed Assignment, Multi-Objective model, Optimization, Decision support model

Resumo

O envelhecimento da população aliado ao insuficiente número de profissionais de saúde resulta num constante congestionamento dos hospitais. Este aumento ao nível do fluxo de doentes torna o processo de gestão de camas fundamental. Determinar qual a melhor cama a atribuir a um doente é um processo complexo, tendo em conta os diversos critérios e restrições que devem ser tidos em conta na tomada de decisão. Esta tese assenta no caso de estudo de um hospital público português no qual este processo é realizado diariamente em suporte de papel por um enfermeiro. Dada a quantidade de informação a ser considerada e o número de profissionais envolvidos no processo a informação pode acabar por ser descurada, conduzindo a uma deficiente atribuição de camas. Esta tese propõe um modelo matemático multiobjetivo como ferramenta de apoio à decisão. Foram elaborados diferentes cenários para avaliar o comportamento do modelo perante a variação da taxa de ocupação hospitalar. A otimização foi efectuada utilizando duas técnicas, *goal programming* (GP) e lexi*cographic goal programming* (LexGP). GP apresentou um melhor desempenho em todos os cenários num mais curto espaço de tempo, no entanto, LexGP acabou por representar com maior precisão a tarefa de atribuição de camas. Foi também realizada uma análise de sensibilidade para estudar o comportamento do modelo perante a variação do *batch* e do número máximo de transferências. Para o hospital, é relevante ter um modelo alinhado com os seus critérios de decisão que possa vir a apresentar um impacto positivo no seu desempenho.

Palavras-Chave: Gestão de camas, Atribuição de camas hospitalares, Modelo multiobjetivo, Otimização, Modelo de apoio à decisão

Table of Contents

List of Tables

List of Figures

List of Acronyms

BA	Bed assignment
BT	Boarding time
BM	Bed management
EOR	Expected occupancy rate
GP	Goal programming
HBA	Hospital Beatriz Ângelo
ILP	Integer linear programming
LexGP	Lexicographic goal programming
LOS	Length of Stay
MOP	Multi-objective optimization problem
OR	Operations research
OF	Objective function

1. Introduction

1.1. Motivation

In the last couple decades, the world has been facing the ageing of its population. Child mortality has been reduced, employment and education have become more accessible, gender equality is closer, and the access to family planning has contributed to a reduction in birth rates. Besides, improvements in public health, technology and living conditions have contributed to an increase in life expectancy, people not only live longer but healthier. Portugal is not an exception. Due to an increase in longevity and decline in fertility, the country is currently ranked as the fifth country with the highest percentage of people over 60 years old worldwide and this position is expected to be maintained until 2050. (ONU 2015)

The situation mentioned above and the lack of general practitioners in outpatient clinics results in a constant hospital overcrowding. Patients are facing long waiting lists, long times in queue, long length of stay (LOS) and at times leave without being seen (LWBS). This increase in patient flow combined with limited resources makes hospital bed management (BM) key to keep the system in sync. An optimal BA is central to avoid waste, reduce costs, increase revenues, improve efficiency and more important to guarantee a positive experience for the patient. (Maia 2016)

Data collection and analysis are necessary in this process, allowing the recognition of patterns and identification of opportunities for improvement. Defining the right metrics to assess patient flow across several hospital departments is a relevant step to obtain results that can be set in motion and used on a daily basis. The uncertainty of many parameters, such as, the number of emergency admissions, discharges, movements and transfers between departments, LOS and variability in patients, makes hospital BM a complex and challenging task. (Adamski 2014)

It is in this context that Hospital Beatriz Ângelo (HBA) aims to address the overcrowding of its units, patients' boarding time (BT) and LOS, which not only have an impact on the patient satisfaction but also may cause clinical complications. As a reference unit in the country, HBA wants to approach this issue through the improvement of its bed managing practices, more specifically through the optimization of its bed assignment (BA) process. Currently, this process relies on the bed manager's judgement, who should consider all assignment criteria and restrictions among thousands of potential alternatives. The reduction of HBA's inpatients BT and LOS is relevant for the hospital, being parameters assessed by the Join Commission International (JCI), from which HBA is accredited since 2013. However, despite this achievement, there are still opportunities for further improvement in the hospital BM field. (Bash 2015)

This thesis addresses the HBA's BA process, proposing a mathematical model as a future decision support tool for the bed manager. It is expected that this model will add value to the hospital through the improvement of its efficiency and assignment precision. This study contributes to the literature proposing a model that mimics the bed manager's practice, considering parameters, constraints and objectives that weren't previously explored in the literature. Additionally, it implements two different optimization techniques, allowing the comparison of its performance in this context. Particularly for HBA, it is important to have a model supported by its decision criteria and in line with its goals that may positively impact its performance.

1.2. Thesis Outline

This dissertation is structured in eight chapters. In the first chapter, a brief introduction is presented, in which the problem addressed is contextualized, the motivation is clarified, and the objective is described. The second chapter, consists on the presentation and description of Luz Saúde Group, HBA and the its current approach to the BA process. The third chapter comprehends firstly the state of the art literature review on operations research (OR) in healthcare settings and the methods applied in this regard, namely, simulation and optimization. This chapter ends with a section focused on hospital BA and the solutions that have been developed to address it. In the fourth chapter the problem is characterized and the solution designed is presented. The fifth chapter includes an overview of the methodology applied in this dissertation, with a section dedicated to the theoretical clarification of the implemented techniques. In the sixth chapter, the model formulation developed to mimic the bed manager's decision making is presented, followed by a detailed interpretation of the formulation. A section in this chapter is dedicated to a description of the scenario optimization performed. The chapter ends with the presentation of the considered assumptions. In the seventh chapter the results of the multi-objective optimization and sensitivity analysis are presented. The eighth chapter consists of the discussion of the results presented in chapter 7. In the final chapter, the main conclusions of this dissertation are presented, followed by a suggestion of future work.

In Figure 1.1, the main stages of this dissertation are presented.

Figure 1.1 Main stages of the dissertation

2. Context

In this chapter, a brief introduction of HBA and Luz Saúde Group is presented in sections 2.1 and 2.1.1, respectively, as well as of HBA's current approach to BA in section 2.1.2.

2.1. Hospital Beatriz Ângelo

HBA is a public hospital integrated in the National Health Service whose activity started in 19th January 2012. The hospital is operated under a public-private partnership program involving three parties: the Portuguese state, SGHL (Sociedade Gestora do Hospital de Loures, SA) and HL (Sociedade Gestora do Edifício, SA). HL is responsible for the design, financing, construction, maintenance and management of the hospital, while SGHL ensures the healthcare delivery. HBA was founded with the purpose of giving response to the need for healthcare services in Lisbon and Tagus River Valley region. The hospital is located in Loures and serves a population of 300.000 residents form the municipalities of Loures, Mafra, Odivelas and Sobral de Monte Agraço. It holds 424 inpatient beds, 44 offices for outpatient consultations, 8 operating rooms, 5 delivery rooms, 3 c-section rooms, a day hospital with a capacity for 64 patient and an ED. (Luz Saúde 2016)

HBA's vision of being a reference healthcare provider recognized by its medical practice and various level of care strengthens its mission of providing healthcare to the population of its area of influence, respecting patient's individuality and needs, and supported by principles of efficacy, efficiency, quality and training.

Since 2013, HBA is accredited by JCI after a process of evaluation and audit. This accreditation endorses the high quality and safety standards of the hospital in different areas such as hospital infection control, medicine safety, medical practice and hospital management. By the end, JCI evaluated 1300 parameters, being HBA in the first audit 98% in conformity with the international standards established by JCI. (Luz Saúde (2016))

2.1.1. Luz Saúde Group

Luz Saúde was founded in 2000 as a private healthcare group. Since then, it became one of the largest healthcare delivery groups in terms of income in the country, being a reference of innovation and medical excellence. The group aims the establishment of an integrated healthcare delivery network composed by hospital units, outpatient clinics and senior residences. Luz Saúde comprises 29 units (twelve private hospitals, a National Health Service hospital operated under a public-private partnership (PPP), fourteen private outpatient clinics, two senior residences) spread across the North, Center and South-Center regions of the country. (Bash (2015))

On January 2012, Luz Saúde established its first PPP, HBA, making its entrance on the public healthcare sector.

On February 2014, Luz Saúde became the first private company in the health care sector publicly listed. Since October of the same year, the insurance company Fidelidade is the largest shareholder having currently 98.7% of the company's shares. (Sa (2017))

In 2016, Luz Saúde invested 35 million euros in a geographic growth of its private care network with the acquisition of Hospital da Luz Guimarães, Hospital do Mar Gaia and the construction of a new hospital unit in Vila Real. It also started the implementation of an ambitious program to increase the capacity of Hospital da Luz Lisboa, Hospital da Luz Oeiras and Hospital da Luz Arrábida. In this year, the Group increased its operating income in 6.4% compared to the previous year reaching 450.7 million euros, powered by the 8.3% growth of the private segment. (ANUAIS (2016))

2.2. Chapter Conclusions

In this chapter Luz Saúde Group and HBA are characterized and the BM process is described.

Luz Saúde Group is growing and is committed to achieve high standards of quality, innovation, and service excellence. Its PPP, HBA, obtained in 2003 the JCI accreditation, having since then the ambition of keeping its high standards and continuously improve its processes. Therefore, a demanding and effective management approach is a must. (Caldas,2012)

This thesis focus the BM process, more specifically the BA task, since this is a source of hospital overcrowding, high boarding time, poor resources usage and high costs.

Currently the BA is performed manually which presents several limitations such as:

- The assignment relies on the bed manager judgment
- There is a significant amount of possible combinations to be evaluated when assigning patients, making the decision process very complex to be manually performed
- The information is transmitted to the bed manager over the phone
- Human errors due to lack of complete and prompt information

In the next chapter, a literature review containing the state of the art on hospital OR and BA is presented, as well as, the different techniques and methods that have been applied to approach these topics.

3. Literature Review

This chapter presents a literature review on hospital OR and BA. Section 3.1 consists in an overview of healthcare OR with a special emphasis on patient flow, resources allocation and staff scheduling, along with the different methods used in this field. Section 3.2 refers specifically to BA, presenting different models and techniques that can be applied to improve effectiveness and efficiency. In the final section, 3.3, conclusions regarding the state of the art literature are presented as well as its applicability on this thesis.

3.1. Hospital Operations Research

Facing the pressure of bottlenecks, backlog and a more limited budget, hospitals are forced to rationalize its resources, seek improvement and guarantee customer satisfaction. Operations research (OR) applies problem solving techniques such as mathematical optimization, simulation, decision analysis, among others, to improve decision making. OR is an important tool in healthcare to assess patient flow, allocate resources, schedule staff and operating rooms, manage supply chains, finances and quality of care (Abe et al. (2016)). Different methods have been used to approach healthcare operations, such as, deterministic modeling, heuristic methods, stochastic modeling, discrete event simulation, system dynamics simulation and agent based modeling simulation (Gunal (2012)).

3.1.1. Mathematical Optimization

Mathematical optimization is a branch of applied mathematics, consisting on the selection of the best solutions amongst a set of alternatives given a variety of objective functions (OFs) and different domains. Different algorithms may be used to solve these problems such as optimization algorithms, iterative methods, global convergence and heuristics.

Ma & Demeulemeester (2013) studied the impact of a given patient group in profitability, developing an integer linear programming (ILP) model. The number of elective patients of a certain patient group that should be admitted to a hospital to maximize the overall financial contribution was determined, given the expected financial reward, operating time and LOS of each patient group. Several constraints were considered, namely, bed capacity, daily occupancy of each ward, time-phased allocation of operating rooms, total surgery time for each surgeon group and admission volume of each patient group. The main limitation of this study was the lack of variability introduced, resulting from the lack of definition of the resource requirements for each type of patient.

Mathematical optimization also presents a broad application on scheduling staff and surgeries. Güler (2013) developed a hierarchical goal programming model (HPG) to schedule residents and senior academic staff to outpatient clinics. This process was not efficient when performed manually due to the significant number of constraints and preferences of the residents. The formulation’s objective function

(OF) minimizes the deviations from the soft constraints, penalizing with a fictitious cost the deviation variables when the soft constraints are violated. The costs were defined by the decision maker through an analytical hierarchical process (AHP) which led to the determination of the constraints' relative importance. The same methodology was applied by Idi (2013) for scheduling the residents' shifts in the Anesthesia and Reanimation Department of the same university. In contrast with the previous authors, Marques et al. (2014), Latorre-Núñez et al. (2016) and Belciug & Gorunescu (2016) explored heuristics. Marques et al. (2014) adopted a population based approach to increase hospital efficiency and reduce the elective surgery's waiting list. After applying a genetic algorithm (GA), the researchers obtained a surgical plan of a significant number of elective surgeries in a short amount of time. In order to study and compare the performance of a mathematical approach and an heuristic approach, Latorre-Núñez et al. (2016) applied two optimization methods to the scheduling of operating rooms, a mixed integer linear programming (MILP) model combined with a constraint programming (CP) model and a metaheuristic based on a GA and a constructive heuristic (CH). The required human resources, equipment, materials, post-anesthesia recovery beds and possible emergency surgeries were included. The OF aimed to minimize the closing time of the last operating room in use. The authors concluded that the CP model delivered good quality solutions in run times that were shorter than those of MILP. On the other hand, the metaheuristic model deliverd higher quality solutions than CP in short run times.

The following authors focused the resource allocation and utilization in hospital operations. Belciug & Gorunescu (2016) applied a hybrid genetic algorithm-queuing multi-compartment model to optimize bed allocation and associated costs. Without using artificial intelligence technologies this task reveals to be challenging, being simplified by the utilization of an evolutionary-based approach. Moreover, this methodology with slight modifications in structure and parameterization, could be extended to different medical departments. Kamran (2016) developed a multi-objective MILP model, aiming to optimize four primary goals: minimize resources costs; maximize the number of admitted patients; minimize travelled distance by non-admitted patients to the nearest hospital; minimize the time of patients in the system. Some of the primary and secondary goals (added to make the model more realistic and logical) were conflicting, leading to the application of weighted goal programming (WGP) and consequent minimization of the gaps between the obtained values and determined goals. The weights were determined using multi-criteria decision making methods. The patients' condition was characterized in 8 categories, each with the following specificities: equipment required, average bed occupancy time interval and importance factor. The number of patients a doctor can treat, the percentage of patients with each condition and the usage costs of a resource were also considered. The results obtained revealed a 12.7% improvement compared with the current state. Burdett and Kozan (2016) developed a holistic multi-objective MILP model to determine the maximum number of patient treatments that can be performed over a specified period considering technical constraints and providing a plan of the resource assignments and utilizations. The solutions were selected using variants of the standard epsilon constraint method (ECM). Before applying the ECM, the most equitable patient case mix (EQPCM) is identified using a minimum Euclidean distance metric and a separable programming approach. Reenberg and Friis (2017) modeled the hospital ward occupancy density functions using a homogeneous continuous time Markov chain. The hospital operations are interpreted as a queueing system with N different patient types arriving at N parallel service stations, in which the number of servers depends on the number of beds available at each ward. The station is blocked when all the servers of the station are occupied, and if that occurs, the arrivals are distributed according to a probability to the other stations, or disappear from the system without creating a queue. The authors derived the ward blocking probability and the expected number of primary rejections. A local search algorithm was then used to optimize the system. When applied to a Danish hospital, a reduction of 11.8% on the number of primary rejections were verified.

3.1.2. Simulation techniques

As mentioned in the beginning of this chapter, different simulation methods have been used in hospital modelling, being discrete event simulation (DES), system dynamics (SD) and agent-based simulations (ABS) the most common.

DES is used for modelling systems that change states dynamically, stochastically and in discrete intervals. It is characterized by its flexibility to model different levels of detail, individual patient focus, easy modeling of stochastic factors (random emergency arrivals, LOS, clinic appointments), ease of use in reusable components, waiting time related performance and queues and visual representation of patient flows. DES is particularly designed for operational level problems (Gunal (2012)). The following authors used DES to approach hospital operations. Levin et al. (2008) focused on the study of ED's BT, particularly on the time interval between the cardiology admission request to the allocation of an inpatient bed. All patients who interacted with the cardiology macro system (telemetry, cardiovascular intensive care, post anesthesia care units, operating rooms and cardiac catheterization laboratory) were included in the stochastic DES model. Results showed that shifting 1 afternoon elective catheterization case to before noon on the weekends results in a 6.4% (20 min) reduction in the average BT, while adding one bed to the telemetry unit results in a 2.9% (9min) reduction in the average BT. Khare et al. (2009) assessed and compared the impact of two common bottlenecks, the number of beds and the rate of discharge from the ED to an inpatient bed. The study led to the conclusion that increasing the number of beds without increasing the rate of departure from the ED as no effect on patients' LOS. Ben-Tovim et al. (2016) focused the study on patients with LOS > 21 days. Although, these patients represented less than 3% of the admissions, their discharge on the 21st day resulted in a significant improvement in the mean occupancy. Additionally, earlier discharges not only altered boarding and ED buffer time, but also left time to perform more elective surgeries. Oh et al. (2016) established strategies to increase the volume of patient with LOS < 3 hours from less than 50% to more than 80%, reduce the LWBS in 2.5% and increase ED's patient volume in 7%. These values were achieved after targeting inefficiencies in the radiology turnaround, and as a result, optimizing the re-collection rate, CT oral drink time and radiologist self-editing dictation use. To stress the idea that not always an increase on the number of resources implies an improvement on the performance levels (service achievements for patients, LOS, time to consultation, time to triage and doctor utilization level), Wong (2016) studied the impact of increasing the number of ED physicians against the reduction on the patient journey. All processes inherent to the patient journey were considered in the model, namely, patient arrival patterns, patient flows by category, priority based on triage level, patient/staff interactions and rules, resource schedules and capacity, main ED processes and operation times, disposition routes and decisions on medical treatment and reassessment. According to the results, an increase in the physician level not always contributes to an increase in the ED performance. However, a reduction in the patient journey within the ED for a small number of non-emergent patients (source of bottlenecks) can result in improved times for a larger number.

SD contrasts with DES, approaching systems from a higher level and thus focusing on strategic level problems (Gunal (2012)). It is used for modeling continuous and mostly deterministic systems, tracking the instantaneous behavior of the system described by a set of differential equations. In this method, entities represent cohorts instead of individuals. Sabbah & Selamat (2015) proposed a BM SD model based on a statistical estimate of the average time of hospitalization for each DRG (diagnosis related group). Each patient was assigned to a DRG, in which information regarding the diagnosis codes and procedures, age and sex of the patient, duration of admission and discharge were registered. Each DRG has an upper LOS limit associated, which when exceeded, the patient is recognized as an outlier of the system. Making use of this information the bed manager is notified about the patient and can take action, improving the turnover rate if possible. Applying this model a reduction of about 8% in the number of patients on gurneys was achieved. Muller & Hart (2016) used SD to model an antenatal clinic, in which the relationships between patients and physicians' activities were represented using causal loop diagrams. The model was used to study the impact of physicians' availability on the waiting time, testing the impact of improving measures, such as the investment on the physician attendance rate and the technology factor associated with the clinic time saved. It allowed to define strategies to increase the physicians' attendance rate time with the patients and understand the impact of investing in advanced medical technologies in the treatment quality and the influence of the system optimization in the working time.

In line with Gunal (2012), ABS is a method used to model dynamic, adaptive, autonomous and mostly deterministic systems. These models have a high capacity range, being able not only to react in a simple if-then behavior, but also learn and adapt based on experience. Queues exist implicitly. Cabrera et al. (2011) developed an ABS model to minimize patients' waiting time through the optimization of the ED staff configuration (doctors, nurses and admission personnel, all distinguished by expertise, as junior or senior). The active agents represent individuals who act upon their own initiative and are described by Moore machines. The passive agents represent systems (loudspeaker, patient information system, pneumatic pipes, central diagnosis services). The distances between different areas were considered to model the environment (admissions, triage box, waiting room, diagnostic and treatment zone). A constant demand was assumed, as well as a certain arrival time step and four different arrival probabilities. The results met the expected (larger and more experienced staff led to shorter average patient waiting time) and were encouraging, however it implied exhaustive search techniques, becoming time consuming. Kaushal et al. (2015) developed an ABS model to study the impact of the ED Fast Track Treatment (FTT) in the reduction of patient's waiting time. The concept of FTT was introduced with the idea of reducing the ED process for non-urgent patients, resulting in an earlier discharge. The two performance measures considered in this research were waiting to be seen (WTBS) and LOS in the ED. Regarding the model's elements, nurses, doctors and humans with reasoning abilities were defined as entities, while patients and beds were defined as state objects. The state objects have state variables associated, such as arrival mode, age, triage scale, desired and actual treatment location. Changes in the bed state result updates the system state object and the original patient state. FTT methods are analyzed considering two configuration of ED operations, static (fixed duration) and dynamic processes (ED condition triggers the FTT, such as number of patients waiting and beds availability), in a way that WTBS for low acuity patients is reduced without any significant impact on other ED patients. The charge nurse can use the information presented on the monitors in the ED regarding the longest and average waiting time and the information available about the patient's status in each bed in the treatment rooms and trigger the FTT process if relevant.

3.2. Hospital Bed Assignment

Hospital BA is a branch of the hospital BM field which focus the process of assigning beds to patients. Several analytical methods of OR have been used to approach this task. Although hospital BA is not as deeply explored in the literature, it is not less important. Hospital BA has become an issue since the demand for hospital beds has increased, affecting directly the quality of care delivered. There is a permanent competition for beds within and between emergency and elective patients, and between medical and surgical clinical typologies. Additionally, there is a set of criteria to be considered and constraints to be respected when assigning a bed to a patient.

In line with Proudlove, Gordon, and Boaden (2003), hospital BM goes hand in hand with all phases of the inpatient's stay, being in practice characterized by bed finding, and firefighting once a crisis is occurring. Emergency demand, elective demand and discharge are frequently the source of problems, such as, trolley wait, high LOS and cancelled operations (due to bed shortage). According to Commission and Ld (2013), holding patients in the ED or in another temporary location after the decision to admit or transfer has been made – boarding - may not only decrease patient satisfaction but also potentially have adverse clinical outcomes. JCI presents in the Standard LD.04.03.11 a 4h BT as a guideline in the interest of patient safety and quality of care, recognizing that BT frames fluctuates between the organizations and that meeting such a short time frame is not at times within the control of the organization. Therefore, it is not a requirement for JCI accreditation.

Demeester et al. (2010) was one of the first to use computational integer programming to approach BA through a single OF that maximizes the number of allocated patients and minimizes the number of transfers, considering the admission and discharge date, gender, quarantine conditions and needed treatment. This solution revealed to be impractical due to the required computational time, being a Tabu search procedure applied instead. Although this approach was successful, the fact that the arrival and discharge times were required to be known in advance did not leverage its adoption. Ã and Schaerf (2011) reformulated Demeester et al. (2010)'s model to improve search times, proposing two different local search procedures, however not solving the limitation of having to know the arrival and discharge times in advance. In a different study, Ceschia and Schaerf (2012) revisited the model, integrating the information on unplanned admissions and uncertain LOS. The problem was solved daily based on the patients currently registered. A penalty was applied ever since a delay in the admission date occurred. Both an ILP and a local neighborhood search algorithm were developed to solve both dynamic and static scenarios. The first method, contrasting with the second, was unable to find an optimal solution for large "dynamic" instances due to complexity. However, both methods achieved similar results for small dynamic scenarios. According to the authors, finding a good balance between the components of the OF was extremely difficult and would be interesting to explore a multi-objective approach and a deep analysis on the relative influence and correlation between objectives. Similarly, Ben, Guinet, and Hajri-gabouj (2012) proposed a decision support tool based on ILP for hospital bed planning occupancy by elective and acute patients, considering among other criteria, incompatibilities between pathologies, patient gender, contagious disease and LOS. It focused the minimization of costs, namely, the cost generated by the lateness of the patient's hospitalization and the cost of a refused admission due to bed unavailability. The model was tested to schedule ahead, either elective and acute patients. Different solvers were used to optimize the model, being CPLEX and LINGO the most powerful solvers tested.

Schmidt, Geisler, and Spreckelsen (2013) studied the elective patient admission and assignment planning through the definition of a cost function for patient admission considering adaptable LOS estimations and aggregated resources. The affinity between clinics and wards, the ward occupancy, the change of ward occupancy and the assignment delay were considered in the model's OF. To address the problem, four algorithmic methodologies were evaluated according to the cost outcome, performance and dismissal ratio, namely, a mixed integer programming solver SCIP, the longest expected processing time, the shortest expected processing time and random choice (heuristic approaches). The use of this assignment model resulted in a reduction of the dismissal ration by more than 30%. The exact approach presented a marginal advantage over the heuristic approaches of 3%, however, its computational times are fifty times larger than the computational times of the heuristic. Turhan and Bilgen (2016) solved the Patient Admission Scheduling (PAS) problem using two different heuristic approaches based on Mixed Integer Programming (MIP), namely, Fix and Relax (F&R) and Fix and Optimized (F&O). Applying time and patient decomposition, the authors decomposed the problem into sub problems. Solutions generated by F&R heuristic were used as input to the F&O heuristic and were iteratively improved until there was no sub problem left to solve. The OF considered minimizes the total penalties associated with assigning patients to rooms for the duration of their stay, considering patient LOS, room preference, admission date, specialism choice, and age. Starting from the first night, the optimization window was decided, from which the room costs for all patients staying within the period were calculated. A CPLEX matrix was constructed and optimized, and the patients were assigned to the rooms. This process was repeated until all nights within the planning horizon were investigated. F&R provided feasible solutions in short calculation times and F&O improved the initial solution received by the F&R in an iterative nature. Hoff (2017) developed a multi-objective model that maximizes the overall criticality of patients admitted and minimizes movements of previously admitted patients. The integer program model was used in Monte Carlo simulations. The model was created to determine the ideal number of private and semi-private rooms and the ideal timeframe over which to batch patient admissions. To achieve this goal, a stream of patients randomly generated for each experiment was utilized, as well as scenarios in which the demand, the number of beds per rooms and the admission and discharge intervals varied. As a result, the number of movements, utilization and occupancy rates were studied for the several scenarios.

3.3. Chapter Conclusions

Hospital OR is a broad field. Simulation and optimization models have been developed for years with the aim of responding to bottlenecks, backlog and a limited budget. Assessing patient flow, operation room scheduling, resources usage and staff allocation are examples amongst the many OR applications. The process of assigning a patient into a bed can be supported by a structured model that may provide some direction on the decision-making. Given the several combinations of internal movements and assignment possibilities when a patient requires a bed, it is essential to quantify several parameters and thus find the most suitable solution for the patient and the hospital operations (patient flow, overcrowding of departments, wards occupancy, human resources usage, among others).

In the next chapter, the problem characterization is presented, followed by the solution proposal.

4. Problem Characterization

Prior to characterizing the problem, a meeting with the chief nurse took place in HBA, in which the BA process and the issues inherent to the current method were presented. The interest of HBA in developing an automated solution to overcome the limitations presently faced was clear. The BA process map (Figure 2.1 – Appendix C) and the paper based tools used to register the list of bed requests (Figure 2.2 – Appendix C) and the inpatients bed assignment map (Figure 2.3 – Appendix C) were provided, as well as, a description of the criteria that should be considered when assigning patients into beds. Job shadowing was crucial to observe how the theoretical information presented by the nurse chief is put into practice by the bed manager.

When assigning a patient into a room, the bed manager should try to achieve the following objectives:

- Maximization of the assignments
- Maximization of patients' priority according to:
    - Boarding time (a patient who is waiting for a long time for a bed should have priority over a patient who isn't waiting for such a long time);
    - Occupancy of the patients' origin ward (if the occupation level of the origin ward is higher, that patient should be prioritized to avoid ward overloading);
    - Time of the day the bed is required (the MDH and surgical day hospital (SDH) close in the afternoon, therefore MDH and SDH patients should have priority over patients who come from the ED after a certain time of day, otherwise they would have to go to the ED and wait there until a bed is free).
    - Patient's medical condition (patients in a more serious and critical clinical status should be prioritized);
- Minimization of the internal movements, considering patient's history (number of transfers a patient has already been subjected to and how long a patient has been accommodated in a room) and the type of transfer performed, if it is within or between a ward (a transfer within ward is preferred over a transfer between wards).
- Maximization of the affinity between clinical typologies and wards (there is a preference regarding which clinical typologies to allocate in a certain ward).
- Minimization of the differences in occupancy across wards (there is an interest in ensuring equity of occupancy across wards and avoid significant differences in terms of workload of the resources).

Moreover, the bed manager can only assign a bed to a patient when a set of constraints are respected:

- Two patients allocated in the same room should have the same gender;
- Two patients allocated in the same room should be categorized in the same clinical typology;

- Two infected patients allocated in the same room must have the same contagious disease;
- If a patient requires a certain equipment he/she must be allocated to a room with that equipment.

A diagram that reflects the decision making that is behind the BA process was created and is represented in Figure 4.1.

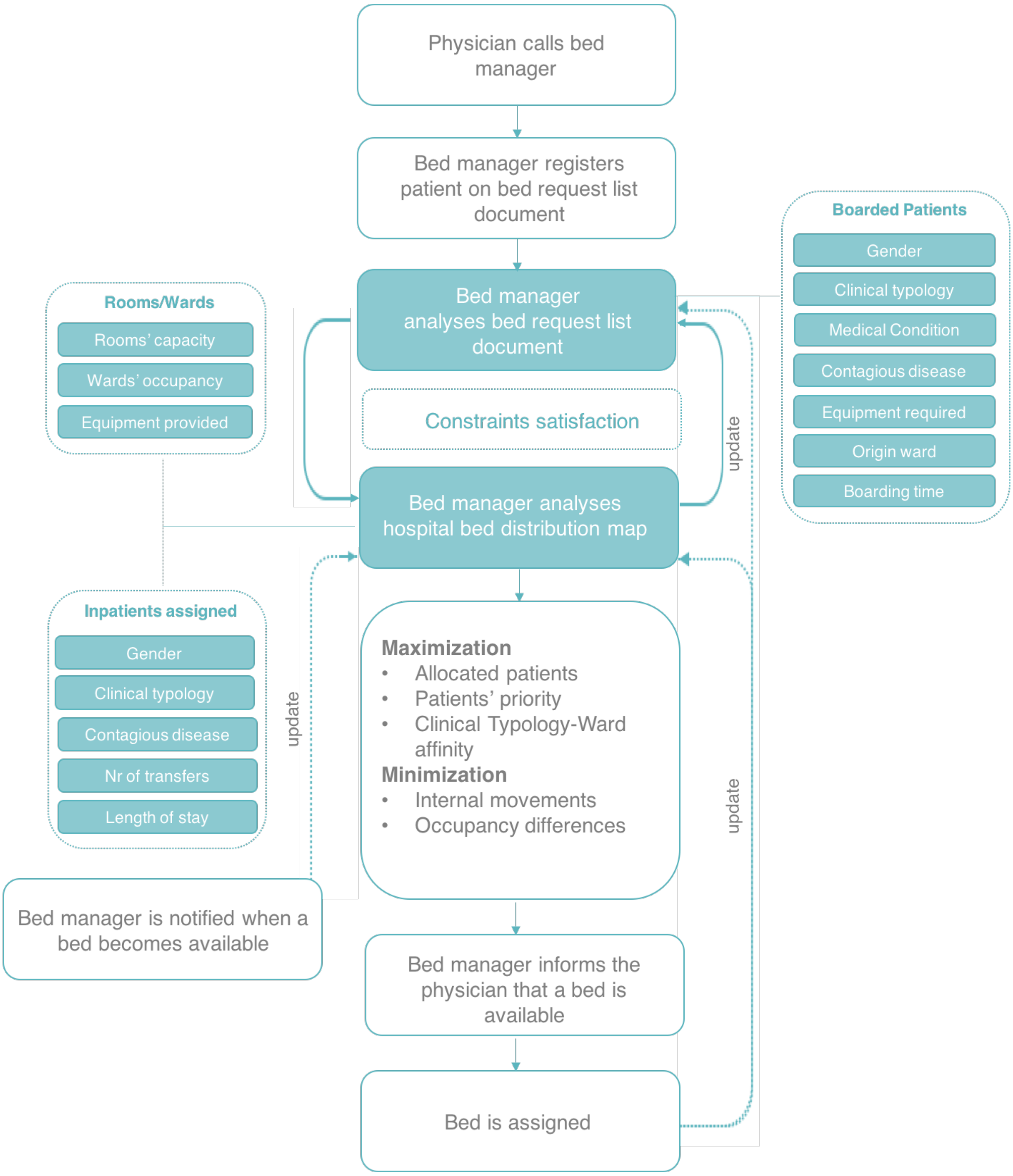

Figure 4.1 Bed assignment diagram

After having identified and characterized the problem, a solution was designed to automate it - a mathematical model that is expected to receive the list of patients requiring beds and its respective characteristics as input, and compute the optimal BA, providing the new patients assignment suggestion as output. Given the several objectives that should be attained when assigning a patient into a bed, a multi-objective model appeared to be the most adequate approach to mimic the bed manager decision making. Indexes, sets and parameters were defined, equations that reflected the different objectives were formulated and constraints were added to the model to make it as realistic as possible.

In the next chapter, the methodology applied to develop and solve the model is presented.

5. Methodology

In this chapter, the methodology framework applied to the HBA's problem is introduced followed by a more detailed and theoretical explanation of multi-objective optimization and multi-objective problem solving techniques applied in this thesis.

5.1. Methodology overview

The methodology framework is structured in four stages, which were followed sequentially as represented in Figure 5.1. The first stage consisted of collecting information, interviews and job shadowing. This stage was crucial to understand the root of the problem and design a solution for it. The next stage consisted of information processing in which the information gathered in the previous stage was deeply analyzed and used to formulate a multi-objective mathematical model. In the third stage, four different scenarios were designed and the two multi-objective problem solving techniques, goal programming (GP) and lexicographic goal programming (LexGP), used to solve the model explored. In the fourth stage, a sensitivity analysis was performed to study the impact of the batch dimension and the maximum number of transfers on each OF. Finally, the results obtained with the different optimization techniques for the different scenarios (stage 3) and the sensitivity analysis (stage 4) are compared and interpreted in stage five.

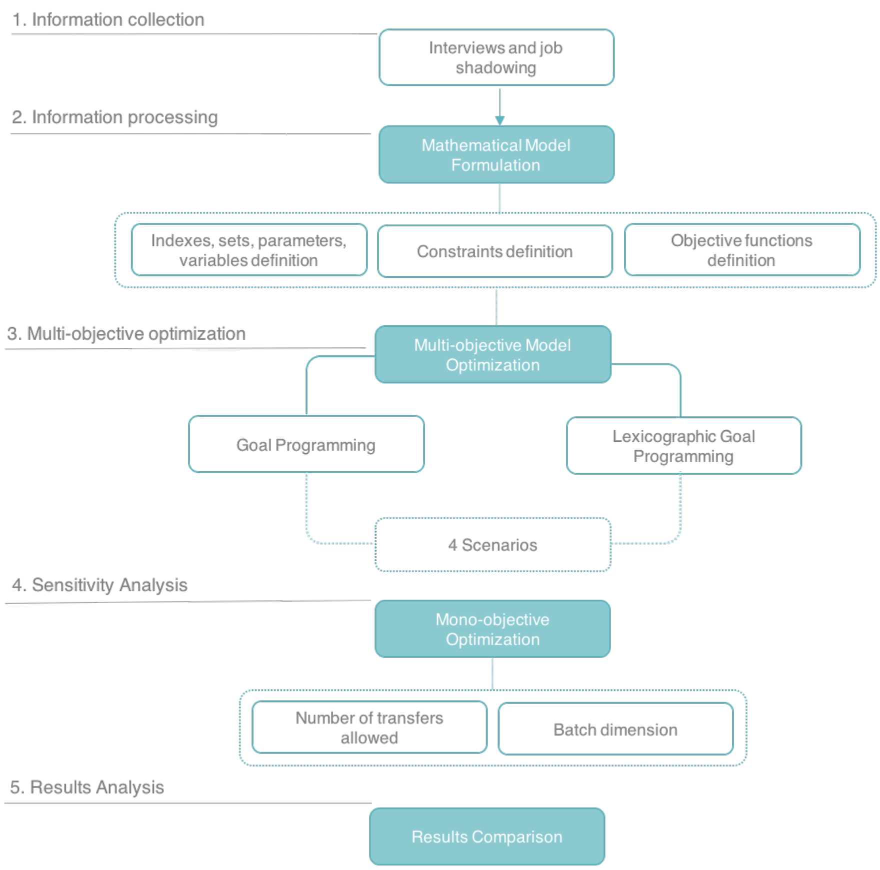

Figure 5.1 Methodology Framework

5.2. Multi-objective optimization

Multi-objective optimization techniques are used to solve multi-objective optimization problems (MOPs), in which the objectives are defined in incomparable units and present a degree of conflict between them. Contrasting with a single objective optimization problem where it is possible to obtain a single optimal solution between a given pair of solutions, in a multi-objective optimization there is no straightforward method to determine if one solution is better than the other. The *Pareto dominance relation* is the most frequently adopted method to compare solutions, presenting a set of alternatives with different trade-offs among the objectives (*Pareto optimal solutions or non-dominated solutions*), out of which the most preferred is chosen based on the subjective input of the decision maker. (Antonio, Coello, and Mart (2009))

5.2.1. Background concepts

In line with Antonio, Coello, and Mart (2009), a MOP is formally defined as:

$$\text{Minimize} \quad f(x) = [f_1(x), f_2(x), \dots, f_k(x)]^T \qquad 5.1$$

$$\text{subject to } x \in \chi$$

where the vector $x \in \mathbb{R}^n$ is formed by n decision variables. The feasible set $\chi \subseteq \mathbb{R}^n$ is determined by a set of equality and inequality constraints. The vector function $f: \mathbb{R}^n \to \mathbb{R}^k$ is composed by k scalar OFs $f_i: \mathbb{R}^n \to \mathbb{R}(i = 1, \dots, k; k \geq 2)$. The set $\mathbb{R}^n$ represents the decision variable space while the set $\mathbb{R}^k$ represents the OF space. The image of χ is a subset of the OF space $\mathcal{Z} = f(\chi)$, referred to as the feasible set in the OF space.

A graphical representation of the feasible MOP's set is presented in Figure 5.2.

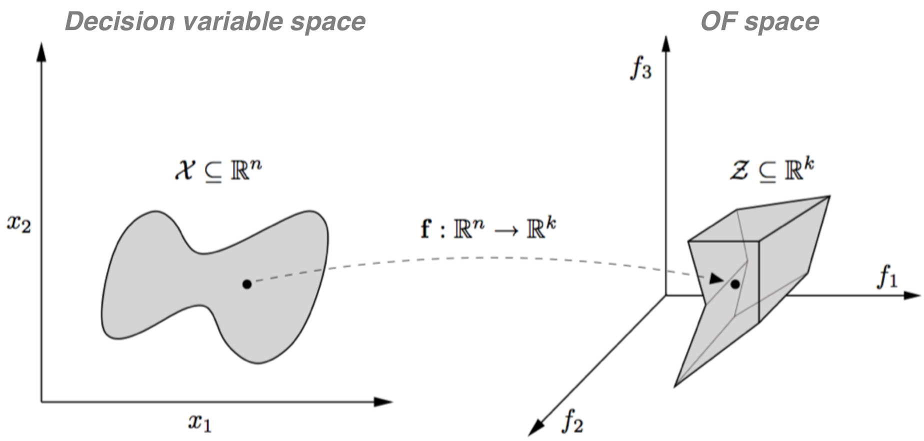

Figure 5.2 Feasible MOP's set. (Antonio, Coello, and Mart (2009))

On one hand, in a single objective optimization, the minimization in $\mathbb{R}^k$assumes the use of the relation "less than or equal to" to compare the scalar OFs. Using this relation there may be different solutions but only one optimal value $f_{min} = min\{f_i(x)|x \in \chi\}$, for each function f_i, since the relation ≤ induces a total order in $\mathbb{R}$. On the other hand, in MOPs, there is no canonical order on $\mathbb{R}^k$ and thus the Pareto dominance relation is usually adopted.

Considering a minimization problem, a vector z^1Pareto-dominates vector z^2, denoted by $z^1 \prec_{pareto} z^2$ if and only if:

$$\forall i \in \{1, \dots, k\}: z_i^1 \leq z_i^2$$

and

$$\exists i \in \{1, \dots, k\}: z_i^1 \leq z_i^2$$

This Pareto dominance relation is graphically represented in Figure 5.3.

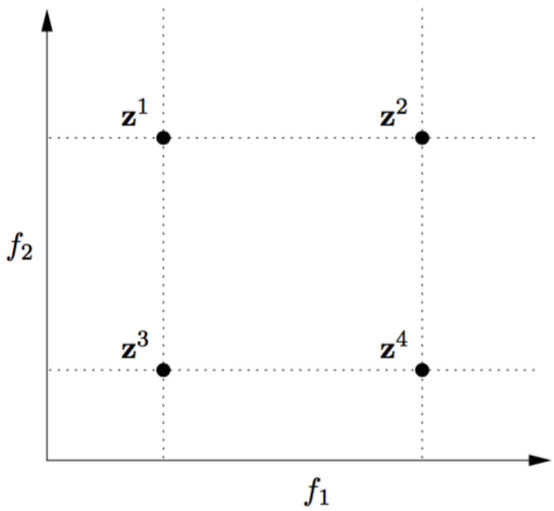

Figure 5.3 Pareto dominance relation. (Antonio, Coello, and Mart (2009))

To solve a MOP the solutions $x \in \chi$ whose images $z = f(x)$ are not Pareto-dominated by any other vector in the feasible space must be find. Based on Figure 5.3 there is no vector dominating z^3, consequently z^3 can be defined as non-dominated or Pareto optimal. While $z^3 \prec_{pareto} z^4, z^1 \prec_{pareto} z^2$ and $z^4 \prec_{pareto} z^2$, some elements are incomparable such as z^1 and z^4, $z^1 \nprec_{pareto} z^4$ and $z^4 \nprec_{pareto} z^1$.

For a Pareto optimal set $\mathcal{P}^*$ defined as:

$$\mathcal{P}^* = \{x \in \chi \mid \nexists\, y \in \chi :\ f(y) \preccurlyeq f(x)\}$$

the Pareto front, $\mathcal{PF}^*$ is defined as:

$$\mathcal{PF}^* = \{f(x) = (f_1(x), \dots, f_k(x)) \mid x \in \mathcal{P}^*\}$$

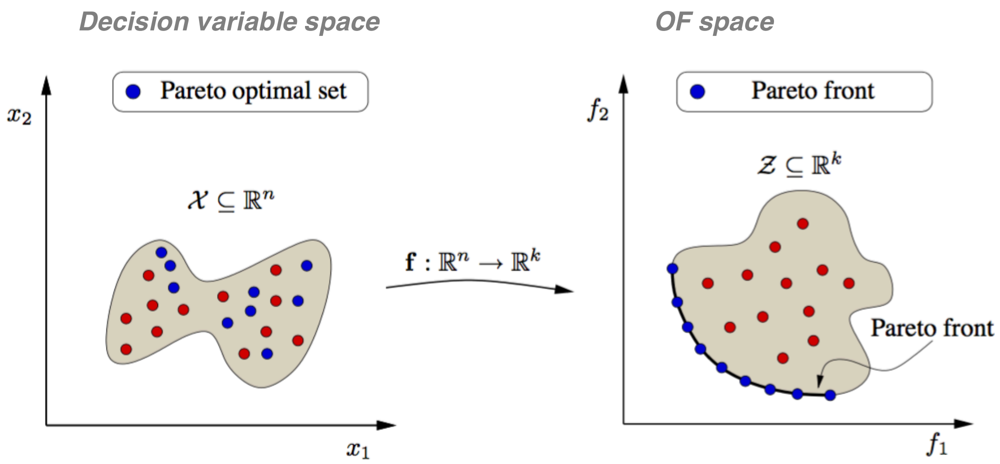

Figure 5.4 Pareto optimal set and respective Pareto front. (Antonio, Coello, and Mart (2009))

In the decision variable space, Figure 5.4, the Pareto optimal decision vectors are represented in blue, i.e. the non-dominated vectors which in the OF space represent the Pareto optimal objective vectors, comprising the Pareto front. Lower and upper bounds are defined respectively by the ideal point, $z_i^* = min_{z \in Z} z_i$ for all $i = 1, \dots, k$, and the nadir point, $z_i^{nad} = max_{z \in Z} z_i$ for all $i = 1, \dots, k$. (Antonio, Coello, and Mart (2009))

5.2.2. Multi-objective problem solving techniques

The multi-objective problem solving techniques are classified as *a priori*, *interactive* or *a posteriori*, depending on the moment the decision maker provides his/her input. If *a priori,* the preference information is given prior to the search; if *interactive* it is given during the search; if *a posteriori* it is provided after the search. In the following sections, a detailed description of the *a priori* techniques explored in this thesis is presented.

5.2.2.1. Lexicographic method

The lexicographic optimization is a technique used to perform multi-objective optimization considering only one objective at a time. The objectives are ranked by the decision maker (form best to worst) according to its relevance. The optimal value $f_i^*(i = 1, \dots, k)$ is obtained by a sequential optimization of the OFs, starting with the most important one and ending with the least important. The optimal value of each objective is added as a constraint for the subsequent optimizations. Optimizations performed in following stages must all be alternative optima of the highest priority OF. When the alternative optima are rare, the lexicographic approach essentially optimizes the priority objective while ignoring the others. (Orumie and Ebong(2014))

The first objective optimization is formulated as:

$$Minimize\ f_1(x) \tag{5.2}$$

subject to $x \in \chi$

Although only one optimal value $f_1^* = min\{f_1(x)|\ x \in \chi\}$ is generated, it is possible to obtain different optimal solutions $x^* \in \chi$. Nonetheless, regarding the original multi-objective problem, only one of these solutions is Pareto optimal. If a unique optimal solution is obtained, this solution is the optimal solution of the original multi-objective problem, and, therefore, the optimization process is interrupted, if not, the next objective must be optimized:

$$Minimize\ f_i(x), i = (2, \dots, k) \tag{5.3}$$

subject to $x \in \chi$

$$f_l(x) = \ f_l^*, l = 1, \dots, i-1 \tag{5.4}$$

5.2.2.2. Goal Programming (GP)

In line with Orumie and Ebong(2014), in a GP approach, the decision maker assigns targets or goals to achieve with each OF, which are added to the formulation as additional constraints. The OF aims to minimize the sum of the deviations, positive (δ_i^+) and negative (δ_i^-), of the obtained solution to the goal for each OF.

$$Minimize \sum_{i=1}^{k} (\delta_i^+ + \delta_i^-) \tag{5.5}$$

subject to

$$f_i(x) - \delta_i^+ + \delta_i^- = G_i,\ i = 1, \dots, k \tag{5.6}$$

$$\delta_i^+, \delta_i^- \geq 0, i = 1, \dots, k \qquad 5.7$$

$$\delta_i^+ . \delta_i^- = 0, i = 1, \dots, k \qquad 5.8$$

$$x \in \chi$$

Sometimes there is an interest in expressing a preference for over or under achievement of a goal (weighted goal programming), and thus the decision maker may add weights to the positive (w_i^+) and negative (w_i^-) deviations from each target G_i.

$$Minimize \sum_{i=1}^{k} (w_i^+ . \delta_i^+ + w_i^- . \delta_i^-) \qquad 5.9$$

subject to

$$f_i(x) - \delta_i^+ + \delta_i^- = G_i, i = 1, \dots, k \qquad 5.10$$

$$\delta_i^+, \delta_i^- \geq 0, i = 1, \dots, k \qquad 5.11$$

$$\delta_i^+ . \delta_i^- = 0, i = 1, \dots, k \qquad 5.12$$

$$x \in \chi$$

To sum up, GP contrast with lexicographic in the following aspects: objectives are conceptualized as goals; there is only one OF which consists in minimizing the sum of the deviational variables of the different objectives; deviational variables, δ_i^+ and δ_i^-, are used to measure overachievement and underachievement from the target; weights may be assigned to the deviational variables.

5.2.2.3. Lexicographic Goal Programming (LexGP)

To overcome the difficulty of quantifying the importance of each objective and to determine a weight for it, it is possible to rank the goals according to its importance degree ("lexicographic ordering") and considering it while optimizing the formulation. This method, first tries to meet the most important goal before proceeding to the next, and assures that the lower order goals do not degrade the solution attained with the higher relevance goals. The lexicographic GP is a dynamic proces, since the optimization stages cannot be solved simultaneously, but sequentially. The LGP's approach formulation is given by the following equations in which P_k represents the priority factor, where $P_k \gg P_{k+1}$, with "$\gg$" meaning infinitely more important:

$$Min \left\{ \sum_k P_k(\delta_k^+ + \delta_k^-) \right\} \quad 5.13$$

subject to

$$f_k - \delta_k^+ + \delta_k^- = t_k \quad 5.14$$

$$\delta_k^+, \delta_k^- \geq 0 \quad 5.15$$

$$\delta_k^+ . \delta_k^- = 0 \quad 5.16$$

$$x \in \chi$$

The LGP can became more flexible and reach a feasible solution using relaxation quantities. An example of a second-stage LGP problem is presented below:

$$Minimize\ (\delta_2^+ + \delta_2^-), \quad 5.17$$

subject to

$$f_2(x) - \delta_2^+ + \delta_2^- = G_2 \quad 5.18$$

$$f_1(x) - \delta_1^{+^*} + \delta_1^{-^*} + \Delta_1 = G_1 \quad 5.19$$

$$\delta_2^+, \delta_2^- \geq 0 \quad 5.20$$

$$\delta_2^+ . \delta_2^- = 0 \quad 5.21$$

$$x \in \chi$$

in which the relaxation quantity is presented by Δ_1. This approach may or may not work succefully due to the ad hoc nature of selecting the relaxation quantity. ((Radin 1998))

6. Mathematical Formulation

This chapter describes the mathematical formulation used to solve the multi-objective problem of BA. The formulation captures the process that the bed manager goes through when executing this task. In section 6.1, the ILP is characterized, followed by sections 6.2 and 6.3, in which the mathematical formulation behind the multi-objective problem solving techniques, goal programming (GP) and lexicographic goal programming (LexGP), respectively, is presented.

6.1. Model Formulation

To characterize the process described in chapter 4, the model was developed as a multi-objective ILP model composed by five OFs.

The formulation assumes that the hospital configuration (number of single and double rooms), patient's clinical typology and medical condition do not change during the decision period. It is also assumed that a patient cannot be discharged from the hospital without the bed manager's permission.

Table 6.1 contains the indexes, set, parameters, constants, decision variable and objective functions considered in the formulation.

Table 6.1 Model description

INDEXES	
i	Patient
j	Room
e	Equipment
t	Clinical typology
s	Medical condition
w	Ward
g	Gender
c	Contagious disease
o	Patient's origin ward
SETS	
P	Set of all patients
J	Set of rooms
E	Set of equipment
T	Set of clinical typologies
S	Set of medical conditions
W	Set of wards
G	Set of genders

C	Set of contagious diseases
O	Set of origin wards
PARAMETERS	
$gen_{i,g}$	Gender g of patient i, $i \in P$, $g \in G$
$ct_{i,t}$	Clinical typology t of patient i, $i \in P$, $t \in T$
$cd_{i,c}$	Contagious disease c of patient i, $i \in P$,$c \in C$
$ori_{i,o}$	Origin o of patient i, $i \in P$, $o \in O$
$eqr_{i,e}$	Equipment e required by patient i, $i \in P$, $e \in E$
$wa_{j,w}$	Ward w of room j, $j \in J$, $w \in W$
cap_j	Capacity of room j, $j \in J$
$eqp_{e,w}$	Equipment e provided in ward w, $w \in W$, $e \in E$
ueq_e	Total units of equipment e available, $e \in E$
$ctwa_{t,w}$	Affinity between clinical typology t and ward w, $w \in W, t \in T$
$mc_{i,s}$	Medical condition s of patient i, $i \in P$, , $s \in S$
td_o	Time of day impact on the priority of patients coming from origin o, $o \in O$
ocl_o	Occupation level of origin ward o, $o \in O$
bt_i	Boarding time of patient i, $i \in P$
sc_s	Severity of medical condition s, $s \in S$
nt_i	Number of times patient i has been transferred, $i \in P$
ti_i	Period of time patient i has been in room j, $i \in P$, $j \in J$
$y_{i,j}$	Patient i is currently assigned to room j, $i \in P$, $j \in J$
$nrequests$	Total number of requests
h	Time of the day the model is run (h=1 if run after 5pm or h=0 of before 5pm)
$maxbt$	Maximum boarding time of the patients waiting for admission
$maxti$	Maximum period of time a patient has been in a room
$maxtr$	Maximum number of transfers a patients has been subjected to
CONSTANTS	
w_{td}	Weight of the time of day on patients' priority
w_{ol}	Weight of the occupation level of patients' origin ward on patients' priority
w_{mc}	Weight of the patients' medical condition on patients' priority
w_{bt}	Weight of the BT on patients' priority
w_{bw}	Weight of transfers between wards
w_{iw}	Weight of transfers within ward
w_{nt}	Weight of the number of transfers patients have been submitted to
w_{ti}	Weight of the time interval a patient has been accommodated in a room
DECISION VARIABLE	

$x_{i,j}$	$= \begin{cases} 1 \text{ if patient } i \text{ is assigned to a bed in room } j, i \in P, j \in J \\ 0 \text{ otherwise} \end{cases}$
OFS	
f_1	Assignment capacity
f_2	Patient priority
f_3	Internal movements
f_4	Clinical typology-ward affinity
f_5	Occupancy difference across wards

The OFs are represented bellow by equations 6.1-6.6.

$$Max \sum_{i,j} x_{i,j}\, , i \in P, \mathrm{j} \in \mathrm{J} \tag{6.1}$$

$$\begin{aligned} Max \sum_{i,j} x_{i,j} \times (1 - y_{i,j}) \Bigg[& \left[\frac{w_{mc}}{w_{ol} + w_{td}h + w_{bt} + w_{mc}} \sum_{s} sc_s\, mc_{i,s} \right] \\ & + \left[\frac{w_{td}h}{w_{ol} + w_{td}h + w_{bt} + w_{mc}} \sum_{o} td_o ori_{i,o} \right] \\ & + \left[\frac{w_{ol}}{w_{ol} + w_{td}h + w_{bt} + w_{mc}} \sum_{o} ocl_o ori_{i,o} \right] \\ & + \frac{w_{bt}}{w_{ol} + w_{td}h + w_{bt} + w_{mc}} \frac{bt_i}{max(bt_i) + 1^*} \Bigg], i \in P, j \in J, o \in O, s \in S \end{aligned} \tag{6.2}$$

*Guarantees that the denominator is never zero

$$\begin{aligned} Min \sum_{i,j,jj,w,ww:j \neq jj, w \neq ww} & x_{i,j} y_{i,jj} wa_{j,w} \left(w_{iw} wa_{jj,w} + w_{bw} wa_{jj,ww} \right) \left(w_{nt} \frac{nt_i}{maxtr + 1^*} \right. \\ & \left. + w_{ti} \frac{ti_i}{maxti + 1^*} \right), i \in P, jj \in J, j \in J, w \in W, ww \in W \end{aligned} \tag{6.3}$$

*Guarantees that the denominator is never zero

$$Max \sum_{i,j,t,w} ctwa_{t,w} x_{i,j} ct_{i,t} wa_{j,w}, i \in P, j \in J, w \in W, t \in T \quad (6.4)$$

$$Min \sum_{w,ww} \frac{|oc_w - oc_{ww}|}{2}, w \in W, ww \in W \quad (6.5)$$

Since the formulation is linear and equation 6.5 is nonlinear, it was submitted to linearization using Bertsimas and John N. Tsitsiklis (1997) (Appendix A). Being rewritten as:

$$Min \sum_{w,ww} \frac{ocdif_{w,ww}}{2}, w \in W \, , ww \in W \quad (6.6)$$

$$s.t$$

$$oc_w - oc_{ww} \leq ocdif_{w,ww}, w \in W, ww \in W \quad (6.7)$$

$$-(oc_w - oc_{ww}) \leq ocdif_{w,ww}, w \in W, ww \in W \quad (6.8)$$

Apart the two constraints represented by equations 6.7 and 6.8, several constraints were applied to ensure an appropriate BA considering the patients' and rooms' specificities.

$$\sum_{j} x_{i,j} \leq 1, \; \forall \; i \in P, \mathrm{j} \in \mathrm{J} \quad (6.9)$$

$$\sum_{i} x_{i,j} \leq cap_j, \; \forall \; \mathrm{i} \in \mathrm{P}, j \in J \quad (6.10)$$

$$eqr_{i,e} wa_{j,w} x_{i,j} - eqp_{e,w} \leq 0, \forall \; i \in P, j \in J, w \in W, e \in E \quad (6.11)$$

$$\sum_{i,j,w} x_{i,j} wa_{j,w} eqp_{e,w} \; eqr_{i,e} \leq ueq_e, \forall \; i \in P, j \in J, w \in W, e \in E \quad (6.12)$$

$$\sum_{j} x_{i,j} = 1, \forall \sum_{j} y_{i,j} = 1, i \in P, j \in J \quad 6.13$$

$$x_{i,j} + x_{ii,j} \leq 1, \forall\, i \neq ii, i \in P, ii \in P, j \in J, cap_j = 2, ct_{i,t} \neq ct_{ii,t}, t \in \mathrm{T} \quad 6.14$$

$$x_{i,j} + x_{ii,j} \leq 1, \forall\, i \neq ii, i \in P, ii \in P, j \in J, cap_j = 2, cd_{i,c} \neq cd_{ii,c} \quad 6.15$$

$$x_{i,j} + x_{ii,j} \leq 1, \forall\, i \neq ii, i \in P, ii \in P, j \in J, cap_j = 2, gen_{i,g} \neq gen_{ii,g} \quad 6.16$$

$$oc_w = \sum_{i,j} x_{i,j} wa_{j,w}, \forall\, i \in P, j \in J, w \in W \quad 6.17$$

$$x_{i,j} wa_{j,w} ct_{i,t} = 0, \forall\, ctwa_{t,w} = 0, i \in P, j \in J, \mathrm{w} \in W, t \in \mathrm{T} \quad 6.18$$

$$nr_{transf} = \sum_{i,j} -\left(x_{i,j} - y_{i,j}\right) y_{i,j}, \forall\, i \in P, j \in J \quad 6.19$$

$$nr_{transf} \leq 0.2\; nrequests \quad 6.20$$

Equation 6.1 represents the main concern of hospitals in general which is to maximize the number of patients assigned, and therefore minimize the BT and the number of patients who are LWBS. Equation 6.2 prioritizes patients, distributing weights according to the importance degree of each criteria. It considers only patients who require a bed, defined by the term $x(i,j)\times(1 - y(i,j))$. If a patient is already in a room ($y(i,j) = 1$), that patient is not in the waiting list and thus, the equation is 0 for those patients. The first term of the equation is related with the patient's medical condition, the second with its BT, the third with the occupancy of the patient's origin ward, and the forth with the time of the day the request is done. Equation 6.3 minimizes the total number of internal movements, within and between wards, represented by the first and second terms of the equation, respectively. It also considers the number of times a patient has already been transferred, represented by the third term of the equation in which the sum of the absolute pairwise deviation of the number of transfers patients have been subjected to is minimized. Finally, how long the patient has been accommodated in a room is represented by the fourth term. To each of this equation segments, a penalty/weight is associated according to its relevance. It is important to note that the denominator of the fraction presented in the third term of the equation is summed by 1, which is used to guarantee that the denominator is never 0. Equation 6.4 describes the maximization of the affinity between clinical typologies and wards. This preference is described by a matrix of scores, in which all clinical typologies and wards are combined and characterized by a value. This value is as high as the affinity between those clinical typologies and wards. Equation 6.5 minimizes the occupancy difference across wards to avoid overloading of the health professionals. This equity is represented by the sum of the pairwise absolute deviation (the hospital is interested in ensuring equity of occupancy across wards, and thus, avoid overloading some of its resources). Since the proposed formulation is linear and the equation is nonlinear, it was linearized using Bertsimas and John N. Tsitsiklis (1997) (Appendix A). The linearization is represented by equations 6.6-6.8. Equation 6.9 ensures that each patient is allocated to only one bed. Equation 6.10 guarantees that the number of patients allocated to a room is less or equal to the capacity of that room. Equation 6.11 assigns a patient who needs a certain equipment to a ward that provides that equipment. Equation 6.12 ensures that the number of patients who need a certain equipment does not exceed the total number of units available of that equipment in the hospital. Equation 6.13 prevents patients already allocated to beds to be automatically discharged from the hospital without the bed manager consent, thus, a patient with $\sum_{\mathrm{j}} y_{i,j} = 1, i \in P, j \in J$, can't become $x(i,j) = 0, i \in P, j \in J$. Equation 6.14 ensures that patients from different clinical typologies are not allocated into the same room. Equation 6.15 ensures that patients with different contagious diseases are not allocated into the same room. Equation 6.16 ensures that patients with different genders are not allocated into the same room. Equation 6.17 describes the occupation of a ward as the sum of the total number of patients in that ward. Equation 6.18 represents a constraint used to prevent that a patient with a zero score of affinity between clinical typology and ward would be allocated to that ward. Equations 6.19 and 6.20 were added to limit the total number of transfers performed each time the model is ran. It was assumed that the number of transfers suggested cannot be higher than 20% of the total number of hospitalization requests.

6.1.1. Goal programming formulation

To perform a GP optimization, goals were defined for each OF (G_1, G_2, G_3, G_4, G_5) with the support of the decision maker. The OFs were then converted into constraints, characterized by equations 6.22-6.23, and replaced by a single OF that minimizes the sum of the deviations of each constraint to its respective goal, characterized by equation 6.21. Each deviation was multiplied by a normalization constant (target goal) to allow a direct comparison between them.

$$Min\left\{\frac{\delta_1^-}{G1} + \frac{\delta_2^-}{G2} + \frac{\delta_3^+}{G3} + \frac{\delta_4^-}{G4} + \frac{\delta_5^+}{G5}\right\} \qquad 6.21$$

$$s.t$$

6.1-6.20

$$f_k + \delta_k^- \geq G_k, k = 1,2,4 \quad 6.22$$

$$f_k - \delta_k^+ \leq G_k, k = 3,5 \quad 6.23$$

$$\delta_k^+ \geq 0, k = 3,5 \quad 6.24$$

$$\delta_k^- \geq 0, \mathrm{k} = 1,2,4 \quad 6.25$$

6.1.2. Lexicographic goal programming formulation

To apply LexGP, besides defining the goals for each OF (which were the same as the applied in GP), the decision maker ranked the OFs based on their importance level. The OF prioritization defined is represented in Table 6.2.

Table 6.2 Objective functions prioritization

Ranking	OF	Goal	Context
1st	f_1	G_1	*Assignment capacity*
2nd	f_2	G_2	*Patient priority*
3rd	f_3	G_3	*Internal movements*
4th	f_4	G_4	*Clinical typology-ward affinity*
5th	f_5	G_5	*Occupancy difference*

The aim is to minimize the deviations from the targets, represented by $\delta_k^+$and δ_k^-. The LexGP model proposed is represented by equations 6.1-6.20 and 6.26-6.32:

$$Minimize\ \{P_1(\delta_1^-) + P_2(\delta_2^-) + P_3(\delta_3^+) + P_4(\delta_4^-) + P_5(\delta_5^+)\} \quad 6.26$$

s.t

6.1-6.20

$$f_1 + \delta_1^- \geq G_1 \tag{6.27}$$

$$f_2 + \delta_2^- \geq G_2 \tag{6.28}$$

$$f_3 - \delta_3^+ \leq G_3 \tag{6.29}$$

$$f_4 + \delta_4^- \geq G_4 \tag{6.30}$$

$$f_5 - \delta_5^+ \leq G_5 \tag{6.31}$$

$$\delta_k^+ \geq 0,\ \delta_k^- \geq 0, (\mathrm{k} = 1,2,3,4,5) \tag{6.32}$$

The LexGP's optimization method steps are represented in Figure 6.1.

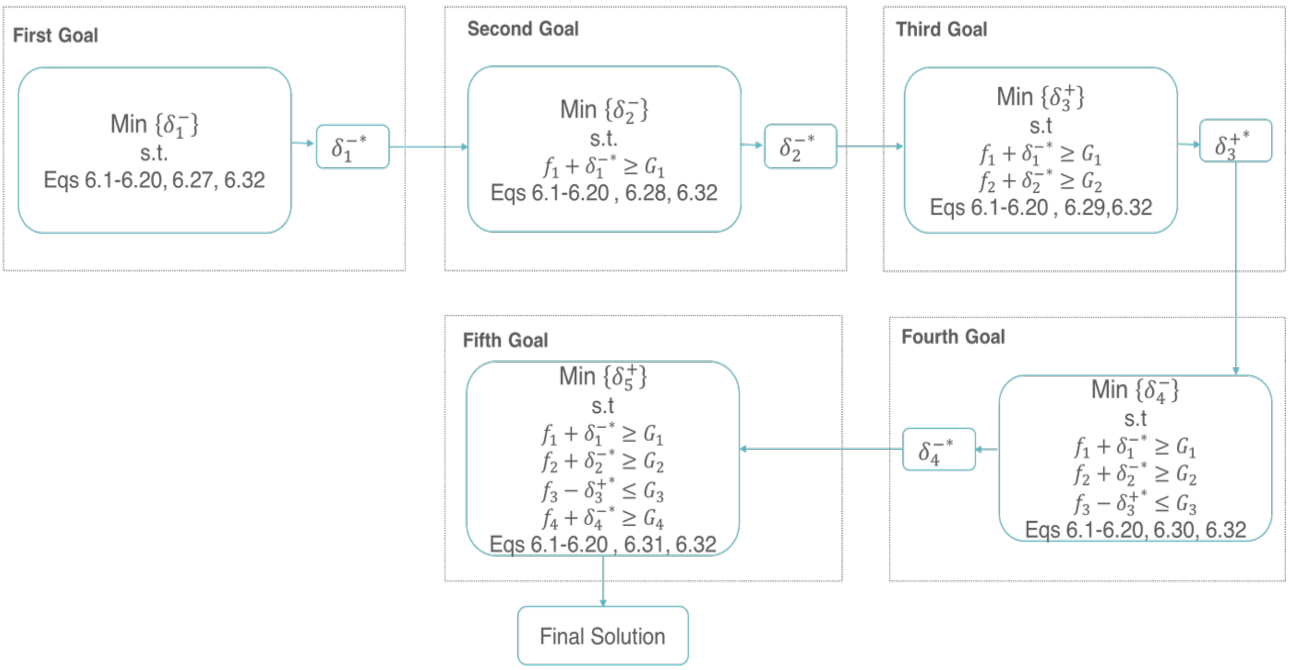

Figure 6.1 LexGP Optimization steps

The optimization was sequential, starting with the optimization of the function with highest importance and ending with the least important function. The LGP optimization can be described by the following steps:

- The first goal of the LexGP approach aims to minimize the deviation, δ_1^-, between the actual number of allocated patients f_1 and its target G_1. The value of δ_1^{-*} is fixed in the following objective function minimization.
- The second goal aims to minimize the deviation, δ_2^-, between the patient's priority actual value f_2 and its target G_2. The value of δ_2^{-*} is fixed in the following objective function minimization.
- The third goal represents the internal transfers and aims to minimize the deviation, δ_3^+, between the actual value f_3 and the target value G_3. The value of δ_3^{+*} is fixed in the following objective function minimization.
- The fourth goal is associated with the affinity score between clinical typologies and wards and aims to minimize the deviation, δ_4^-, between the actual value f_4 and the target value G_4. The value of δ_4^{-*} is fixed in the following objective function minimization.
- The fifth goal is associated with the difference of occupancy between wards and aims to minimize the deviation, δ_5^+, between the actual value f_5 and the target value G_5.

6.2. Scenario optimization

As mentioned in the beginning of this chapter a deterministic approach was the one chosen to optimize this mathematical formulation. Both GP and LexGP methods were applied and the results obtained were compared.

To assess how the formulation would respond to an increase in occupancy and a constant demand for beds, four scenarios were tested. In the first scenario, the hospital was modeled as empty, having an occupancy of 0% and a request of 45 beds. In the second scenario, 20 more beds were requested. The resultant inpatients distribution, which corresponds to an expected bed occupancy (EOR) of 45%, was then used as input for the third scenario, followed by a demand for 20 more beds. Finally, and due to the motivating model's capacity of 99 beds, 14 beds were still requested. These scenarios were sequentially solved, such that the hospital's inpatients distribution resulting from the previous scenario optimization would be used as input for the following scenario optimization. The solution approach used is described in Figure 6.2.

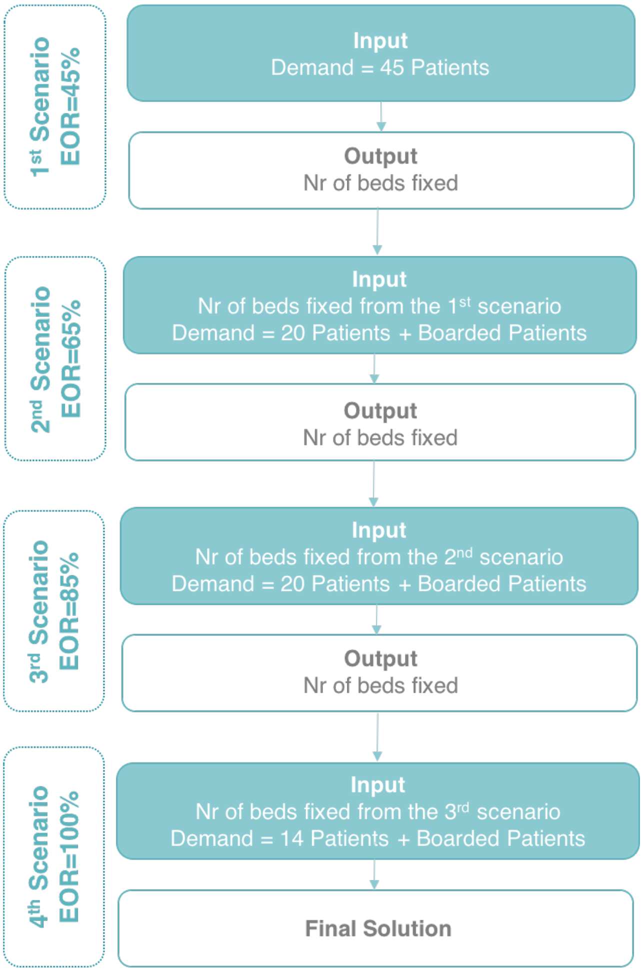

Figure 6.2 Solution approach

Both GP and LexGP methodologies were explored over the scenarios. The GP method minimizes the sum of the relative deviations of each objective to its respective goal, while the LexGP method solves prioritized objectives. The CPLEX solver from GAMS 24.8.5 was used as an optimization tool. The stopping criteria considered was the optimization time length of 500 seconds, which was the interval considered adequate to the demanding response time of bed requests HBA is under.

A sensitivity analysis over the parameters batch dimension and the maximum number of transfers was performed and will be further explored.

6.3. Assumptions

The formulation was validated over a motivating model. In this chapter, the data and assumptions that were used as input to the model are characterized. In Table 6.3 are represented the main characteristics of the motiving model.

Table 6.3 Motivating model characterization

	Motivating model
Wards	11
Rooms per ward	5
Double rooms per ward	4
Single rooms per ward	1
Beds	99
Clinical Typologies	12
Types of equipment	3
Origin wards	8

The formulation assumes that:

- The hospital configuration (number of single and double rooms), patient's clinical typology and medical condition do not change during the current decision period.
- The patient cannot be discharged from the hospital without the permission of the bed manager.
- The number of transfers suggested by the formulation shouldn't be higher that 20% of the total number of beds requested.

The parameters and weights considered were assumptions made based on the characterization of the relative importance and compatibilities of the different parameters by the decision maker. The values presented below are only representative of HBA, and therefore, in case the model is to be implemented a multi-criteria decision analysis should be performed to add more precision to the model.

In Table 6.4 the nomenclature applied to the indexes associated with the equipment, origin wards, assignment wards, contagious diseases, clinical typologies and clinical conditions is presented. In Table 6.5 the impact of the time of the day on the priority of patients coming from a certain origin is presented, followed by the distribution of the equipment across wards in Table 6.6. Table 6.7 presents the affinities between patient's clinical typologies and hospital wards. The severity of the medical condition is presented in Table 6.8. And finally, Table 6.9 presents the weights applied on the formulation are presented.

Table 6.4 Nomenclature

Definition	Index
Equipment	
Telemetry	$e1$
Non invasive ventilation	$e2$
Chest drain	$e3$
Origin	

URG-SO	$o1$
URG	$o2$
UCI	$o3$
UCInt	$o4$
UCPA	$o5$
SDH	$o6$
MDH	$o7$
Consultations / Electives	$o8$
Ward	
2.1	$w1$
2.2	$w2$
2.3	$w3$
3.1	$w4$
3.2	$w5$
3.3	$w6$
3.4	$w7$
4.1	$w8$
4.2	$w9$
4.3	$w10$
4.4	$w11$
Contagious Diseases	
MRSA	$c1$
DESC	$c2$
Clostridium	$c3$
Outro	$c4$
Clinical Typology	
General surgery	$t1$
Other surgical clinical typologies	$t2$
Orthopaedics	$t3$
Medicine	$t4$
Pulmonology	$t5$
Neurology	$t6$
Cardiothoracic	$t7$
KRC	$t8$
ERC	$t9$
Discharge	$t10$
Cardiology	$t11$
Infectious diseases	$t12$

Clinical Condition	
Good	$s1$
Fair	$s2$
Serious	$s3$
Critical	$s4$

Table 6.5 Impact of time of day on the priority of patients from a certain origin ward

Index	Time of the day (td)
$o1$	0
$o2$	0
$o3$	0
$o4$	0
$o5$	0
$o6$	1
$o7$	1
$o8$	1

Table 6.6 Hospital equipment distribution

Index	Location
$e1$	$w5$
$e2$	$w5$
$e3$	$w9$

Table 6.7 Clinical typologies – ward affinity

	Affinities between clinical typologies and wards											
	Clinical typologies											
Hospital wards	$t1$	$t2$	$t3$	$t4$	$t5$	$t6$	$t7$	$t8$	$t9$	$t10$	$t11$	$t12$
$w1$	1	0.5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0.1	0.1
$w2$	0.5	1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0.1	0.1
$w3$	0.1	0.1	1	0.3	0.3	0.3	0.3	0.1	0.1	0	0.3	0.1
$w4$	0.1	0.1	0.3	1	0.5	0.5	0.5	0.1	0.1	0	0.5	0.1
$w5$	0.1	0.1	0.3	0.5	1	1	1	0.1	0.1	0	0.5	0.1
$w6$	0.5	0.5	0.1	0.1	0.1	0.1	0.1	1	1	0.8	0.1	0.1
$w7$	0	0	0	0	0	0	0	0	0	1	0	0
$w8$	0.1	0.1	0.3	1	0.5	0.5	0.5	0.1	0.1	0	0.5	0.1
$w9$	0.1	0.1	0.3	0.5	0.5	0.5	0.5	0.1	0.1	0	1	
$w10$	0.1	0.1	0.3	1	0.5	0.5	0.5	0.1	0.1	0	0.5	0.1
$w11$	0	0	0	0	0	0	0	0.5	0.5	0	0	1

Table 6.8 Severity of the medical condition

Index	Severity (sc)
s_1	0
s_2	0.3
s_3	0.6
s_4	1

Table 6.9 Weights

Weights	
w_{td}	0.5
w_{ol}	0.5
w_{bt}	0.2
w_{cs}	0.3
w_{bw}	0.7
w_{iw}	0.3
w_{nt}	0.4
w_{ti}	0.6

7. Results

7.1. Multi-Objective Optimization

In this section, the results of the optimization of the different scenarios using two different multi-objective problem solving techniques, GP and LexGP, are presented.

Figure 7.1 - Figure 7.4 present the assignment results of the 99 patients into the 55 rooms (x_{ij}) after the 4th scenario optimization when applying GP and LexGP. It consists on the decision variable (x_{ij}) directly exported from GAMS after the optimization, which reflects the operational application of the model. If $x_{ij} = 1$ it means that a bed was assigned to a patient i in room j, and $x_{ij} = 0$ otherwise. The patients identified with a grey color represent the patients for whom a bed was not find. The results obtained after the 1st-3rd scenarios optimization are presented in appendix B. After optimizing the formulation and obtaining x_{ij} for each scenario using GP and LexGP, the dependent variables' values of each objective function were then used to calculate the proportion of the deviation between the solution obtained for each OF and its respective goal, which is presented for the different scenarios after applying GP and LexGP in Figure 7.5-Figure 7.9. Figure 7.10 presents the average of the proportion of the deviation between the solution and its respective goals across all OFs for each scenario. The proportion of the deviation was calculated such that for all objective functions, a 0% proportion the achievement of the goal, a positive proportion represents the overachievement of the target goal and a negative proportion represents its underachievement. Figure 7.7 presents the resource (time) usage associated with the optimization of each scenario using both techniques.

Equation 7.1 and 7.2 represent the formula applied to calculate the proportion of the deviation of functions aimed to be minimized (OF2 and OF5) and maximized (OF1, OF2, OF4), respectively.

$$\%\Delta_{optimal\ value-goal} = -\frac{Optimal\ Value - Goal}{Goal} \times 100 \quad 7.1$$

$$\%\Delta_{optimal\ value-goal} = \frac{Optimal\ Value - Goal}{Goal} \times 100 \quad 7.2$$

Figure 7.1 Scenario 4 bed assignment results using LexGP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																						1																																	
p2																																							1																
p3				1																																																			
p4																													1																										
p5					1																																																		
p6																							1																																
p7																																								1															
p8																			1																																				
p9							1																																																
p10																																																		1					
p11																												1																											
p12												1																																											
p13																																	1																						
p14																																										1													
p15																							1																																
p16	1																																																						
p17																																																				1			
p18																											1																												
p19																																																							
p20								1																																															
p21															1																																								
p22																						1																																	
p23			1																																																				
p24													1																																										
p25																									1																														
p26																	1																																						
p27																										1																													
p28																													1																										
p29														1																																									
p30																																									1														
p31																																					1																		
p32																																		1																					
p33																																												1											
p34																											1																												
p35																																												1											
p36																														1																									
p37																												1																											
p38																	1																																						
p39																																																						1	
p40											1																																												
p41		1																																																					
p42		1																																																					
p43																																																1							
p44																										1																													
p45																																																					1		

Figure 7.2 Scenario 4 bed assignment results using LexGP (p46-p99)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p46																																														1									
p47																																													1										
p48																					1																																		
p49																1																																							
p50																																						1																	
p51						1																																																	
p52																																		1																					
p53									1																																														
p54																								1																															
p55			1																																																				
p56																																																	1						
p57																																				1																			
p58																		1																																					
p59						1																																																	
p60									1																																														
p61																																	1																						
p62																																				1																			
p63																																																							1
p64																																											1												
p65								1																																															
p66																			1																																				
p67																																																	1						
p68											1																																												
p69																																1																							
p70																																															1								
p71																															1																								
p72																																									1														
p73																																																				1			
p74																																																							
p75																				1																																			
p76										1																																													
p77																																					1																		
p78																																										1													
p79																																																						1	
p80																																																			1				
p81																																																							
p82																																																							
p83																																																							
p84																					1																																		
p85																																							1																
p86																																																							
p87							1																																																
p88																																			1																				
p89																																															1								
p90																																																			1				
p91																																																							
p92																																																							
p93																																																							
p94																																											1												
p95																																																							
p96																																																					1		
p97																																																1							
p98																																																							
p99													1																																										

Figure 7.3 Scenario 4 bed assignment results using GP (p1-p45)

p1																																														1							
p2																			1																																		
p3		1																																																			
p4																													1																								
p5				1																																																	
p6																					1																																
p7																																							1														
p8																																					1																
p9														1																																							
p10																																																	1				
p11																																				1																	
p12											1																																										
p13																															1																						
p14					1																																																
p15																					1																																
p16	1																																																					
p17																																																			1		
p18																									1																												
p19									1																																												
p20								1																																													
p21																																																				1	
p22																																														1							
p23			1																																																		
p24													1																																								
p25																							1																														
p26																				1																																	
p27																											1																										
p28																																								1													
p29												1																																									
p30																																												1									
p31																																													1								
p32																																1																					
p33																																									1												
p34																																																		1			
p35																																											1										
p36																																			1																		
p37																																																		1			
p38																1																																					
p39																																																					1
p40										1																																											
p41		1																																																				
p42			1																																																		
p43																		1																																			
p44																											1																										
p45																																															1						

Figure 7.4 Scenario 4 bed assignment results using GP (p46-p99)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p46																											1																												
p47																																												1											
p48																							1																																
p49																		1																																					
p50																													1																										
p51						1																																																	
p52																																	1																						
p53		1																																																					
p54																									1																														
p55																																							1																
p56																1																																							
p57																	1																																						
p58																																																	1						
p59								1																																															
p60																																							1																
p61																															1																								
p62																					1																																		
p63																																																							1
p64																																											1												
p65							1																																																
p66																																						1																	
p67																																																							
p68											1																																												
p69																															1																								
p70									1																																														
p71																																		1																					
p72																																											1												
p73																																																				1			
p74																																																							
p75																1																																							
p76																																																							
p77																																														1									
p78																																																							
p79																																																							
p80																																																							
p81																																																							
p82																																																							
p83							1																																																
p84																																																							
p85																																																							
p86																																																							
p87														1																																							
p88																																			1																				
p89																																																							
p90																										1																													
p91																																																							
p92																																																							
p93																																																							
p94																																																							
p95																																																							
p96																																																							
p97																																																							
p98																																																							
p99																																																							

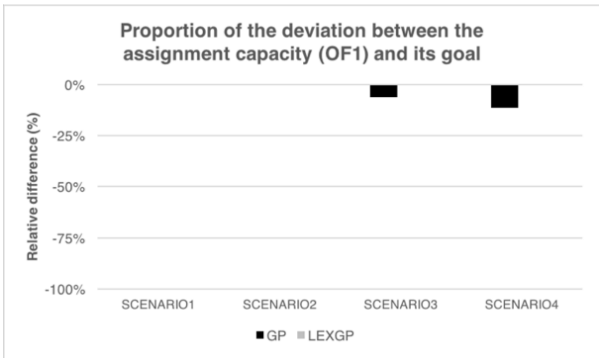

Figure 7.5 Proportion of the deviation between OF1 and its goal for each scenario using GP and LexGP

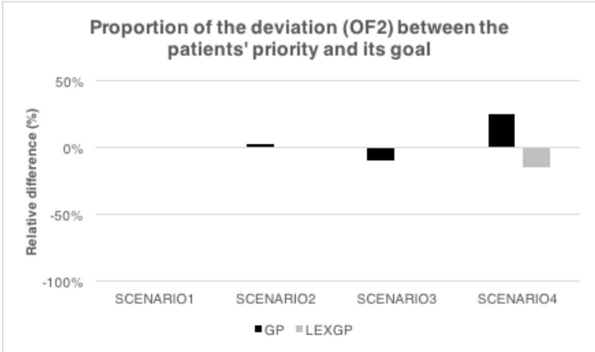

Figure 7.6 Proportion of the deviation between OF2 and its goal for each scenario using GP and LexGP

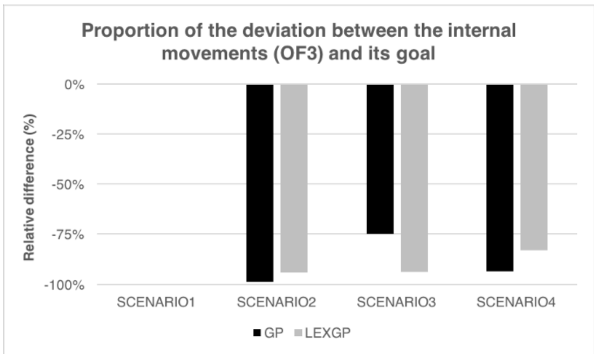

Figure 7.7 Proportion of the deviation between OF3 and its goal for each scenario using GP and LexGP

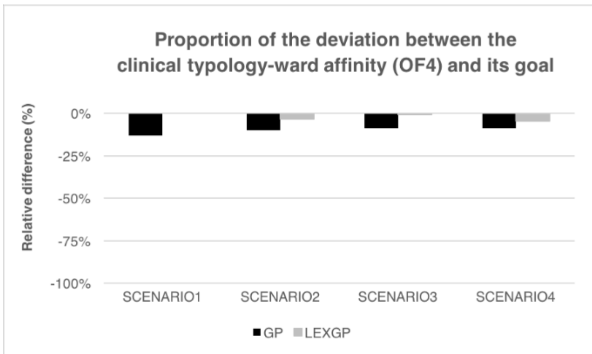

Figure 7.8 Proportion of the deviation between OF4 and its goal for each scenario using GP and LexGP

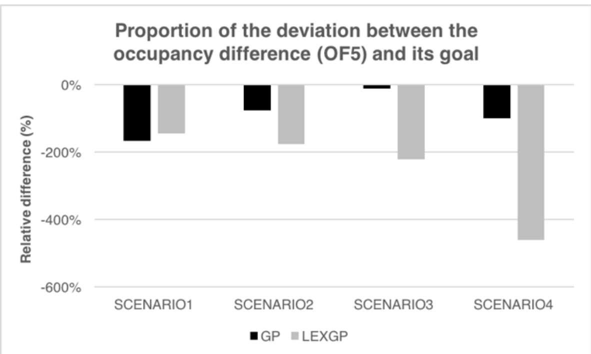

Figure 7.9 Proportion of the deviation between OF5 and its goal for each scenario using GP and LexGP

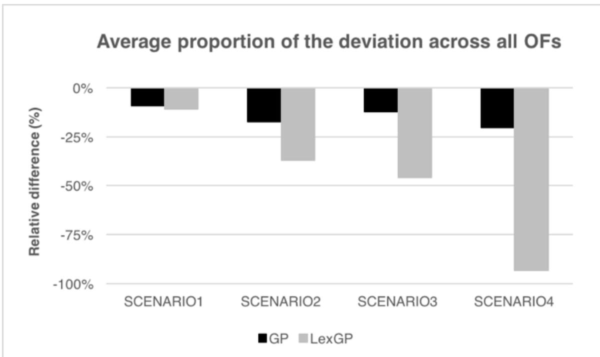

Figure 7.10 Average proportion of the deviation between the optimal solution and its goal across all OFs for each scenario using GP and LexGP

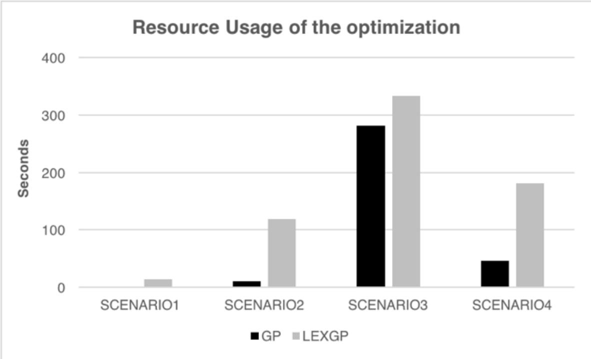

Figure 7.11 Resource usage of the optimization of each scenario using GP and LexGP

- Regarding the methods' performance on the assignment capacity (Figure 7.5), GP and LexGP performed equally in the 1st and 2nd scenarios, reaching the target goal. However, LexGP's results were 6% and 11% better than GP's in the 3rd and 4th scenarios, respectively.
- Concerning the patients' priority OF (Figure 7.6), GP and LexGP demonstrated similar results for the 1st scenario. LexGP performed 10% better than GP in the 3rd scenario and that situation was reversed for the 2nd and 4th scenarios, where GP performed 2% and 40% better and overachieved the target goal.
- Regarding the internal movements OF (Figure 7.7), GP and LexGP performed equally in the 1st scenario, reaching the target goal, as opposed to the other scenarios for which the target was not achieved. GP performed 19% better than LexGP in the 3rd scenario, while Lex GP performed 5% and 11% better in the 2nd and 4th scenarios.
- Concerning the clinical typology-ward affinity OF (Figure 7.8), LexGP outperformed GP across all scenarios, presenting a 13%, 6%, 8% and 4% better performance than GP in the 1st, 2nd , 3rd and 4th scenarios respectively.
- Concerning the occupancy difference across wards (Figure 7.9), neither of the methods achieved the target goal. LexGP presented 22% better results than GP for the 1st scenario, however GP performed 100% , 211% and 360% better than LexGP for the 2nd, 3rd and 4th scenarios, respectively.
- Respecting the overall performance of GP and LexGP across all OFs for each scenario (Figure 7.10), GP outperformed LexGP by 2%, 20%, 33% and 73% for the 1st, 2nd, 3rd and 4th scenarios, respectively.
- Regarding the resource usage (Figure 7.11), GP was always less time consuming than LexGP's, being this difference more perceptible in the 2nd and 4th scenario optimization, where there was difference of 110 and 135 seconds, respectively.

In chapter 8, the results presented above will be deeply explored and conclusions regarding the implementation of each method are presented.

7.2. Sensitivity Analysis

After comparing both GP and LexGP methods, a sensitivity analysis was performed with the aim of studying how some variables, namely the batch dimension and the maximum number of transfers, impacted the dependent variables under study (f_1, f_2, f_3, f_4, f_5). As opposed to the previous section, a mono-objective optimization was performed, meaning the OFs were optimized independently.

Batch Dimension

The batch dimension corresponds to the number of requests to be considered when the model is run. Although it is expected the model to be run each time a new patient is registered in the bed request list or an inpatient is discharged, it was interesting to analyze how batching would impact the optimal solution.

Batches of two different dimensions were created, 10 and 20 patients, and the formulation was run starting with an initial bed occupancy rate of 45%. Given the hospital dimension of 99 beds, 4 runs of 10 patients and 2 runs of 20 patients were executed. The intersection points between these two types of runs occur when EOR is of 66% (65 beds) and of 86% (85 beds), which are the ones used as reference to compare the results obtained.

Figure 7.12 – Figure 7.15 reflect how sensitive the OFs are to the variation of the batch's dimension in the 2nd and 3rd scenarios optimization.

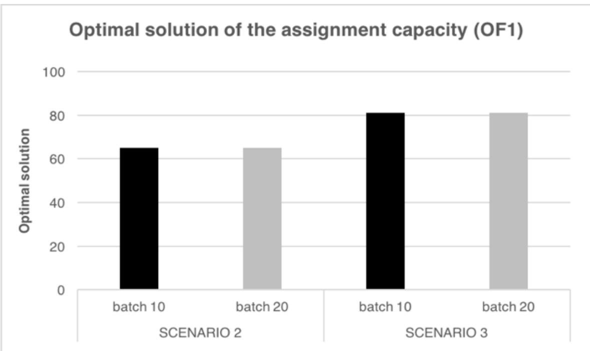

Figure 7.12 Optimization of OF1 for the 2nd and 3rd scenarios when varying the batch dimension

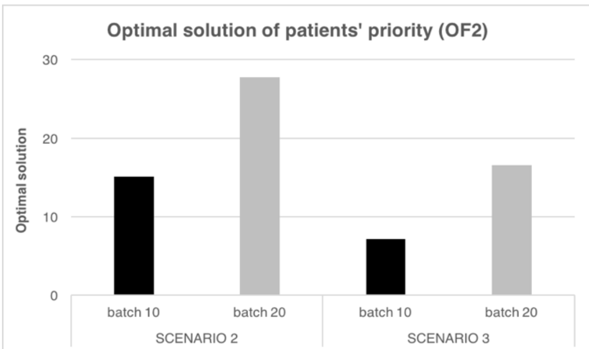

Figure 7.13 Optimization of OF2 for the 2nd and 3rd scenarios when varying the batch dimension

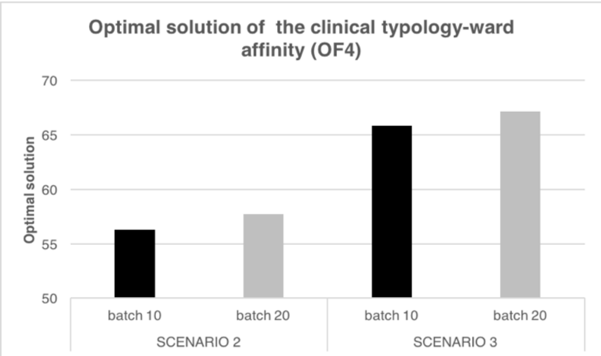

Figure 7.14 Optimization of OF4 for the 2nd and 3rd scenarios when varying the batch dimension

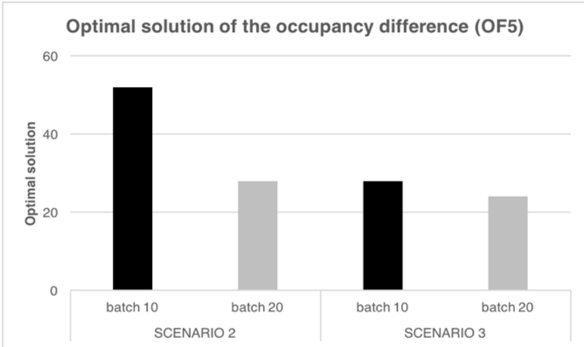

Figure 7.15 Optimization of OF5 for the 2nd and 3rd scenarios when varying the batch dimension

After optimizing each OF while varying the batch dimension, the relative difference between the optimal solution obtained with a batch of size 20 and a batch of size 10 for each OF was calculated, being presented in Table 7.1. Equation 7.3 was used to calculate the proportion of the deviation:

$$\%\Delta_{batch\ 10-batch\ 20}=\frac{f_{batch\ 20}-f_{batch\ 10}}{f_{batch\ 10}}\times 100 \qquad 7.3$$

Table 7.1 Proportion of the deviation between the optimal solution obtained with a batch of 10 and a batch of 20 patients

OF	Scenario	$\Delta_{batch10-batch20}$
OF1	**2^{nd}**	0%
	3^{rd}	0%
OF2	**2^{nd}**	84%
	3^{rd}	132%
OF3	**2^{nd}**	0%
	3^{rd}	0%
OF4	**2^{nd}**	2%
	3^{rd}	2%
OF5	**2^{nd}**	-46%
	3^{rd}	-14%

Concerning the influence of the batch dimension, the results show that this parameter does not impact the OFs related with the BA capacity, OF1 (Figure 7.12) and the internal movements, OF3. There is a 2% proportion of the deviation for OF4, related with the clinical typology-ward affinity (Figure 7.14). The OF related with the patient's priority (Figure 7.13) and the patient's distribution across wards (Figure

7.15) were the ones that revealed to be the most sensitive to the batch dimension. The proportion of the deviation of the optimal value of OF2 (Figure 7.13) was positive, 84% and 132%, for the 2nd and 3rd scenarios, respectively, while for OF5 (Figure 7.15), it was negative, -46% and -14%, for the 2nd and 3rd scenarios. It is important to highlight that for the OFs for which the interest is the maximization (OF1, OF2, OF4), a positive proportion of the deviation represents a better performance of batch 20, while for the OFs for which the interest is minimization (OF2, OF5), a positive proportion of the deviation represents a better performance of batch 10.

Maximum number of transfers

It was also performed a sensitivity analysis on the parameter associated with the maximum number of transfers constraint (6.20), proportional to the number of patients waiting for a bed. This constraint was necessary to include in the model since when using LexGP to optimize the model, the maximization of the assignment capacity (OF1) is the most prioritized function, while the minimization of the transfers only comes after (OF3). By default, the model would make internal movements to maximize the first equation without being constrained in any way, before even optimizing OF3 which would already be highly constrained by the optimization of OF1 and OF2.

The following results reflect how sensitive the OFs are to this parameter. An initial bed occupancy rate of 45% was assumed. Figure 7.16 – Figure 7.20 show the optimal values of each OF when varying the ratio between the maximum number of transfers and the number of requests across the different scenarios.

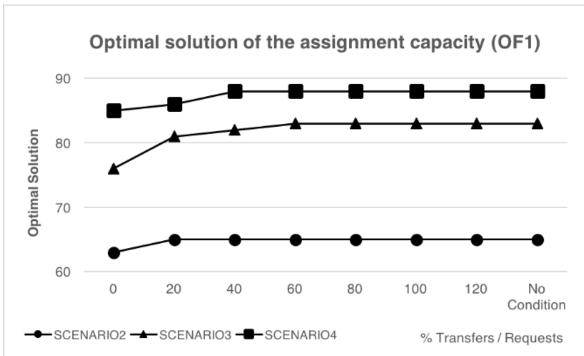

Figure 7.16 Optimization of OF1 for the 2nd, 3rd and 4th scenarios when varying the number of transfers

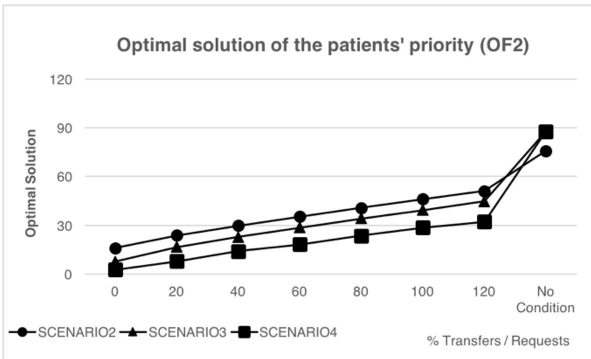

Figure 7.17 Optimization of OF2 for the 2nd, 3rd and 4th scenarios when varying the number of transfers

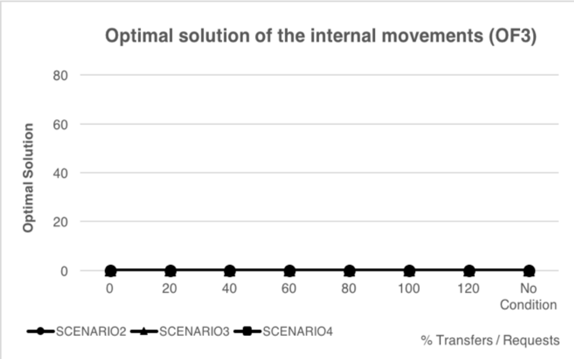

Figure 7.18 Optimization of OF3 for the 2nd, 3rd and 4th scenarios when varying the number of transfers

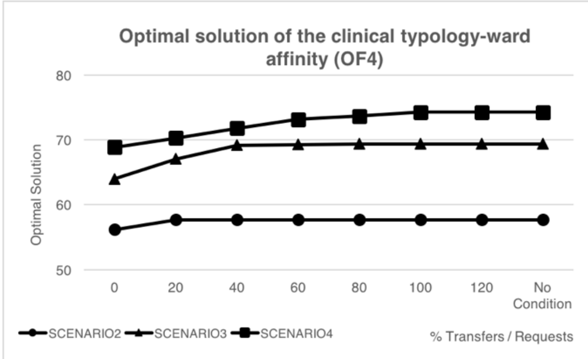

Figure 7.19 Optimization of OF4 for the 2nd, 3rd and 4th scenarios when varying the number of transfers

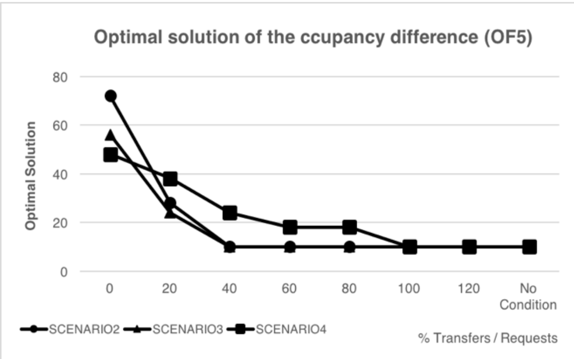

Figure 7.20 Optimization of OF5 for the 2nd, 3rd and 4th scenarios when varying the number of transfers

After optimizing each OF, the proportion of the deviation between the optimal solution obtained and its goal when varying the ratio between the number of transfers and the number of requests for the 2nd, 3rd and 4th scenarios is presented in Table 7.2. Equation 7.3 was used to calculate the proportion of the deviation between the optimal solutions for the different ratios.

Table 7.2 Proportion of the deviation between the optimal solution obtained when varying the maximum number of transfers

OF	Scenario	Δ_{0-20}	Δ_{20-40}	Δ_{40-60}	Δ_{60-80}	Δ_{80-100}	$\Delta_{100-120}$	$\Delta_{120-no\ con}$
OF1	**2nd**	3%	0%	0%	0%	0%	0%	0%
	3rd	7%	1%	1%	0%	0%	0%	0%
	4th	1%	2%	0%	0%	0%	0%	0%
OF2	**2nd**	49%	25%	19%	15%	13%	11%	48%
	3rd	121%	37%	26%	19%	15%	13%	97%
	4th	209%	78%	30%	30%	21%	13%	173%
OF3	**2nd**	0%	0%	0%	0%	0%	0%	0%
	3rd	0%	0%	0%	0%	0%	0%	0%
	4th	0%	0%	0%	0%	0%	0%	0%
OF4	**2nd**	3%	0%	0%	0%	0%	0%	0%
	3rd	5%	3%	0%	0%	0%	0%	0%
	4th	2%	2%	2%	1%	1%	0%	0%
OF5	**2nd**	-61%	-64%	0%	0%	0%	0%	0%
	3rd	-57%	-58%	0%	0%	0%	0%	0%
	4th	-21%	-37%	-25%	0%	-44%	0%	0%

The sensitivity of the different OFs to the maximum number of transfers in terms of variation follows a pattern across the different scenarios. OF2 (Figure 7.17) and OF5 (Figure 7.20) are the most sensitive. OF2 refers to the patients' priority, which tends to increase with the increase of the ratio. This increase is superior when the ratio varies between 0-20%. OF5 refers to the difference in the patients' distribution across wards, which tends to decrease with the increase of the ratio. This decrease is higher when the ratio varies between 0-20% and 20-40% of the numbers of bed requests.

Respecting equations OF1(Figure 7.16) and OF4 (Figure 7.19), the impact is observed when the number of transfers is very constrained (up to 40%), given that when it is upper to 40%, the number of assignments and affinity between clinical typologies and wards becomes constant. OF3 (Figure 7.18) is constantly null since the goal is to minimize the number of transfers, not being affected by the constraint.

8. Discussion

After applying GP and LexGP to optimize this multi-objective problem, it was possible to verify how the principals of each method affected the results obtained.

LexGP outperformed GP's assignment capacity, OF1, in the 3rd and 4th scenarios, which was a phenomenon to be expected since LexGP prioritizes this equation above all, while GP assumes that no function has priority over another.

Respecting the priority function, OF2, LexGP reached the target goal for all scenarios except for the 4th scenario, which may be due to the constrained nature associated with the model dimension and the fact that OF1 limits the performance on OF2. On the other hand, GP overcame the target goal in the 2nd and 4th scenarios, which may be due to the freedom of GP to obtain the best final results at any "cost", meaning it may favor the results of this functions and compromise the others.

OF5 was ranked last by LexGP, and thus it was expected the results to be highly constrained by the deviations obtained in the previous optimizations, which cannot be negatively affected by this function optimization. On the other hand, GP consistently outperforms LexGP.

In the overall GP performed better than LexGP, across all scenarios. The better performance of GP when compared with LexGP was more evident with the increase of the expected bed occupancy. The difference on the average performance of LexGP across all objective functions and its poor resource usage (in average 76 seconds longer than GP) was expected given the highly-constrained nature of LexGP as the optimization was occurring. Concerning the resource usage, it is important to remember the LexGP's sequential optimization process is more time consuming than GP that minimizes all deviations at once. Furthermore, for both methods the computational burden is as heavier as the model's dimension, i.e, the hospital's occupancy rate.

To sum up, GP ended up providing better results in the overall compared with LexGP, however it is important to highlight that the difference between GP's and LexGP's performance on OF5 highly impacts the average of the proportion of the deviation across all OFs, since the difference between GP and LexGP performance is substantial particularly as the expected bed occupancy increases. On the other hand, GP did not attain the mimicking effect of LexGP since there was no preference factor added to the OFs and all equations ended up being equally weighted, which is reflected on the great results obtained in OF5 (least relevant function) and poor results in OF1 (most important function).

Concerning the sensitivity analysis related with the batch dimension, the results showed that the patient's priority function (OF2) and the difference in occupancy function (OF5) were the most affected/sensible to the variation of the batch size, performing better in a 20-patient batch. After increasing the batch of 10 to a batch of 20 patients, OF2 went through an increase in both scenarios, more significantly in the 3rd scenario. On the other hand, OF5 went through a decrease, which was positive since that equation was aimed to be minimized, for both scenarios as well. This increase in performance may be due to different motives. The maximum number of transfers, which is as high as the number of bed requests, and the EOR before the optimization, which is lower in a batch of size 20 than in a batch of size 10 (leaving more beds available in the moment of assigning patients), end up providing more flexibility to the model, and therefore, the BA. It is important to highlight that OF2 naturally tends to increase with the batch dimension since the OF tends to increase with the number of requests. Therefore, a batch of more patients may lead to a better assignment performance.

Regarding the effect of the ratio between the number of transfers and the number of requests, the results showed an impact of this parameter on the different OFs, particularly on OF2, meaning that a higher ratio between the number of transfers and the number of requests gives more flexibility to the model allowing the assignment of beds to patients with a higher priority associated. OF5 is also evidently impacted by this parameter particularly when the ratio varies between 0 and 40%, allowing a more equal distribution of patients across wards. A variation of the ratio from 0 to 20% represents a meaningful impact on the improvement of the OFs optimal solutions, providing more flexibility to the model. A tradeoff between the number of transfers and the assignment capacity should be explored by the hospital when defining this parameter.

9. Conclusions and future work

The aim of this thesis was to provide HBA with a tool to support the bed manager's decision making when assigning patients into beds. The multi-objective mathematical model meets this goal, considering all criteria inherent to the BA decision making process. Moreover, some of the elements that were highlighted by the bed manager as important in the process and absorbed by the model are not easily accessible by the bed manager in the moment of deciding since it not only requires access to different information systems but also it is not easy to balance manually so many parameters.

Additionally, it was also of interest to compare the performance of two different multi-objective techniques that haven't previously been compared in this context. Hence, to solve the formulation two different multi-objective problem solving techniques were applied, GP and LexGP. GP presented in average better results in terms of meeting the target goals for most scenarios in a smaller amount of time when compared to LexGP. However, LexGP ended up mimicking more accurately the BA process, due to the OFs prioritization according to its relevance. Contrasting with the LexGP method, the GP approach applied assigned no weights or ranked the five equations, mitigating the prioritization that is verified in the previous case. This reasoning was verified when the multi-objective formulation was optimized.

Concerning the sensitivity analysis performed, with the aim of studying how different parameters impacted each OF, two parameters were chosen, the batch dimension and the maximum number of transfers. It led to the conclusion that in fact a larger batch may generate better results offering more flexibility to the model, however it may be unrealistic to wait until there is a waiting list of a certain number of patients to perform the optimization in a real context, where there is no planning ahead and the hospital is looking for an immediate response to avoid boarding. Regarding the increase of the maximum number of transfers, similarly to the increase of the batch dimension, it led to an increase in the model's flexibility, allowing better results in some OFs. The decision whether to fix a batch dimension or decide on the maximum number of transfer comes with a trade-off, and the hospital should balance the pros and cons when choosing these parameters.

It is expected that this model will add value to the hospital not only through the improvement of its efficiency, assignment precision and service quality, but also reducing patient's BT, LOS and consequently, the overcrowding of its wards. This study contributes to the literature proposing a multi-objective model that mimics the bed manager's practice, considering parameters, constraints and objectives that haven't been previously explored. Particularly for HBA, it is important to have a model supported by its decision criteria and in line with its goals that may positively impact its performance. Although this model was specifically designed according to the HBA's requirements it is generic enough to be applied to different hospitals and enriched with additional constraints, for example, patient's mobility or additional isolation needs.

In a future perspective, there is still room to explore other multi-objective problem solving techniques and study how they may impact the model's performance in terms of the quality of results and resource usage. In this thesis only LexGP and GP were applied, and thus it is interesting to explore the weighted goal programming method which may overcome the absence of preference between the OFs when the GP was applied. To achieve this, as well as to add more precision to the model formulation weights, a multiple criteria decision analysis should be performed. Heuristic methodologies should be explored as well, since they may find good enough solutions in a smaller period. The clinical typologies considered in this model were related with the specialty unit's affinity to each ward and used as a constraint of compatibility between patients sharing a room. Regarding the latter, it is important to develop a more granular approach to classify patients based on their diagnosis. An eventual implementation approach was not deeply explored in this thesis; however, it would require the integration of HBA's information systems, such that it automatically updates the bed occupation status as well as receives as input the characteristics of the patients waiting for a bed.

10. Bibliography

Ã, Sara Ceschia, and Andrea Schaerf. 2011. “Computers & Operations Research Local Search and Lower Bounds for the Patient Admission Scheduling Problem.” *Computers and Operation Research* 38(10): 1452–63.

Abe, Tolu K., Benita M. Beamon, Richard L. Storch, and Justin Agus. 2016. “Operations Research Applications in Hospital Operations: Part I.” *IIE Transactions on Healthcare Systems Engineering* 6(1): 42–54.

Adamski, Presenter Patricia. 2014. “Navigating the Challenges of Patient Flow and Boarding in Hospitals.” 32(7): 2013.

Antonio, L, Carlos A Coello Coello, and Zapotecas Mart. 2009. “An Introduction to Multi-Objective Optimization Techniques.” : 1–26.

“ANUAIS.” 2016.

Bash, Eleanor. 2015. “Manual de Acolhimento Do Colaborador.” *PhD Proposal* 1.

Belciug, Smaranda, and Florin Gorunescu. 2016. “A Hybrid Genetic Algorithm-Queuing Multi-Compartment Model for Optimizing Inpatient Bed Occupancy and Associated Costs.” *Artificial Intelligence in Medicine* 68: 59–69.

Ben-Tovim, D. et al. 2016. “Hospital Event Simulation Model: Arrivals to Discharge? Design, Development and Application.” *Simulation Modelling Practice and Theory* 68: 80–94.

Ben, Rym, Alain Guinet, and Sonia Hajri-gabouj. 2012. “Int . J . Production Economics An Integer Linear Model for Hospital Bed Planning.” *Intern. Journal of Production Economics* 140(2): 833–43.

Bertsimas, Demitris, and John N. Tsitsiklis. 1997. *Introduction to Linear Optimization*. ilustrada. ed. 1997 Athena Scientific.

Burdett, Robert, and Erhan Kozan. 2016. “A Multi-Criteria Approach for Hospital Capacity Analysis.” *European Journal of Operational Research* 255(2): 505–21.

Cabrera, Eduardo et al. 2011. “Optimization of Healthcare Emergency Departments by Agent-Based Simulation.” *Procedia Computer Science* 4: 1880–89.

Ceschia, Sara, and Andrea Schaerf. 2012. “Artificial Intelligence in Medicine Modeling and Solving the Dynamic Patient Admission Scheduling Problem under Uncertainty.” *Artificial Intelligence In Medicine* 56(3): 199–205.

Commission, The Joint, and Standard Ld. 2013. “The ‘patient Flow Standard’ and the 4-Hour Recommendation.” *Joint Commission perspectives. Joint Commission on Accreditation of Healthcare Organizations* 33(6): 1, 3–4.

Demeester, Peter, Wouter Souffriau, Patrick De Causmaecker, and Greet Vanden. 2010. “Artificial Intelligence in Medicine A Hybrid Tabu Search Algorithm for Automatically Assigning Patients to Beds.” 48: 61–70.

Güler, M Güray. 2013. “Expert Systems with Applic Ations A Hierarchical Goal Programming Model for

Scheduling the Outpatient Clinics.” 40: 4906–14.

Gunal, Murat M. 2012. “A Guide for Building Hospital Simulation Models.” *Health Systems* 1(1): 17–25.

Hoff, Brenden. 2017. “Multi-Objective Optimization of Hospital Inpatient Bed Assignment Multi-Objective Optimization of Hospital Inpatient Bed Assignment by.”

Idi, Kadir. 2013. “Expert Systems with Applications A Goal Programming Model for Scheduling Residents in an Anesthesia and Reanimation Department.” 40: 2117–26.

Kamran, Mehdi. 2016. “A Resource Allocation Model in a Healthcare Emergency Center Using Goal Programming.” 4(December): 81–97.

Kaushal, Arjun et al. 2015. “Evaluation of Fast Track Strategies Using Agent-Based Simulation Modeling to Reduce Waiting Time in a Hospital Emergency Department.” *Socio-Economic Planning Sciences* 50: 18–31.

Khare, Rahul K., Emilie S. Powell, Gilles Reinhardt, and Martin Lucenti. 2009. “Adding More Beds to the Emergency Department or Reducing Admitted Patient Boarding Times: Which Has a More Significant Influence on Emergency Department Congestion?” *Annals of Emergency Medicine* 53(5): 575–585.e2.

Latorre- Núñez, Guillermo et al. 2016. “Scheduling Operating Rooms with Consideration of All Resources, Post Anesthesia Beds and Emergency Surgeries.” *Computers and Industrial Engineering* 97: 248–57.

Levin, Scott R. et al. 2008. “Optimizing Cardiology Capacity to Reduce Emergency Department Boarding: A Systems Engineering Approach.” *American Heart Journal* 156(6): 1202–9.

Luz Saúde, S.A.; Sociedade aberta. 2016. “Hospital Beatriz Ângelo.”

Ma, G, and E Demeulemeester. 2013. “A Multilevel Integrative Approach to Hospital Case Mix and Capacity Planning.” *Computers and Operations Research* 40(9): 2198–2207.

Maia, Ana. 2016. “Mais de 44 Mil Doentes Foram Mais de Quatro Vezes Às Urgências Num Ano.” *Diário de Notícias*.

Marques, Inês, M. Eugénia Captivo, and Margarida Vaz Pato. 2014. “Scheduling Elective Surgeries in a Portuguese Hospital Using a Genetic Heuristic.” *Operations Research for Health Care* 3(2): 59–72.

Muller, Louis, and Mike Hart. 2016. : 2nd International Conference, ICDSST 2016, Plymouth, UK *Decision Support Systems VI - Addressing Sustainability and Societal Challenges*.

Oh, Chongsun et al. 2016. “Use of a Simulation-Based Decision Support Tool to Improve Emergency Department Throughput.” *Operations Research for Health Care* 9: 29–39.

ONU. 2015. “World Population, Ageing.” *Suggested citation: United Nations, Department of Economic and Social Affairs, Population Division (2015). World Population Ageing* United Nat((ST/ESA/SER.A/390): 164.

Orumie, Ukamaka Cynthia, and Daniel Ebong. 2014. “A Glorious Literature on Linear Goal Programming Algorithms.” (March): 59–71.

Proudlove, N C, K Gordon, and R Boaden. 2003. “Can Good Bed Management Solve the Overcrowding in Accident and Emergency Departments ?” : 149–55.

Radin, Ronald. 1998. *Optimization in Operations Research*.

Reenberg, Andersen Anders, and Nielsen Bo Friis. 2017. "PT." *European Journal of Operational Research*.

Sa, Luz. 2017. "Fidelidade Já Controla 98 , 7 % Da Luz Saúde." : 1–12.

Sabbah, Thabit, and Ali Selamat. 2015. 532 Communications in Computer and Information Science *Intelligent Software Methodologies, Tools and Techniques*.

Schmidt, Robert, Sandra Geisler, and Cord Spreckelsen. 2013. "Decision Support for Hospital Bed Management Using Adaptable Individual Length of Stay Estimations and Shared Resources." : 1–19.

Turhan, Aykut Melih, and Bilge Bilgen. 2016. "PT US CR." *Computers and Operations Research*.

Wong, Zoie. 2016. "A Discrete-Event Simulation Study for Emergency Room Capacity Management in a Hong Kong Hospital." : 1970–81.

11. Appendix

Appendix A – Linearization Methodology

According to Bertsimas and John N. Tsitsiklis (1997), when minimizing

$$Min\left\{\sum_{i=1}^{n} c_i|x_i|\right\} \tag{11.1}$$

s.t

$$\mathbf{Ax} \geq \mathbf{b} \tag{11.2}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$, and where the cost coefficients c_i are assumed to be nonnegative, we observe that $|x_i|$ is the smallest number z_i that satisfies $x_i \leq z_i$ and $-x_i \leq z_i$, and we obtain the linear programming formulation:

$$Min\left\{\sum_{i=1}^{n} c_i z_i\right\} \tag{11.3}$$

s.t

$$\mathbf{Ax} \geq \mathbf{b} \tag{11.4}$$

$$x_i \leq z_i,\ i = 1, \ldots, n \tag{11.5}$$

$$-x_i \leq z_i,\ i = 1, \ldots, n \tag{11.6}$$

Appendix B – Optimization bed assignment results

Figure 11.1 Scenario 3 bed assignment results using LexGP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																																																1							
p2																																																		1					
p3		1																																																					
p4																											1																												
p5					1																																																		
p6																							1																																
p7																																								1															
p8																			1																																				
p9															1																																								
p10																				1																																			
p11																												1																											
p12														1																																									
p13																																		1																					
p14							1																																																
p15																							1																																
p16				1																																																			
p17																																																				1			
p18																																					1																		
p19										1																																													
p20						1																																																	
p21																																															1								
p22																																																1							
p23	1																																																						
p24													1																																										
p25																						1																																	
p26																								1																															
p27																													1																										
p28																													1																										
p29											1																																												
p30																																												1											
p31																																																	1						
p32																																			1																				
p33																																									1														
p34																																					1																		
p35																																									1														
p36																																																						1	
p37																												1																											
p38																								1																															
p39																																																							1
p40												1																																											
p41			1																																																				
p42			1																																																				
p43																																						1																	
p44																											1																												
p45																														1																									

Figure 11.2 Scenario 3 bed assignment results using LexGP (p46-p85)

p46																																													1										
p47																																										1													
p48																					1																																		
p49																1																																							
p50																																																				1			
p51							1																																																
p52																																1																							
p53									1																																														
p54																									1																														
p55	1																																																						
p56																	1																																						
p57																		1																																					
p58																																			1																				
p59								1																																															
p60									1																																														
p61																															1																								
p62																																						1																	
p63																																																		1					1
p64																																									1														
p65																																												1											
p66																			1																																				
p67																	1																																						
p68												1																																											
p69																															1																								
p70																																																							
p71																														1																									
p72																																									1														
p73																																																			1				
p74																																																							
p75																																																							
p76																																																							
p77																																																1							
p78																																																							
p79																																																							
p80																																																							
p81																																																							
p82																																																							
p83																																																							
p84																																																				1			
p85																																																							

Figure 11.3 Scenario 2 bed assignment results using LexGP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																																						1																	
p2																																																		1					
p3								1																																															
p4																											1																												
p5																1																																							
p6																									1																														
p7																																								1															
p8																			1																																				
p9															1																																								
p10																				1																																			
p11																												1																											
p12														1																																									
p13																																		1																					
p14							1																																																
p15																							1																																
p16				1																																																			
p17																																																				1			
p18																										1																													
p19										1																																													
p20						1																																																	
p21																																																				1			
p22																																					1																		
p23	1																																																					
p24													1																																										
p25																						1																																	
p26																																															1								
p27																													1																										
p28																																																						1	
p29											1																																												
p30																																												1											
p31																																																	1						
p32																																			1																				
p33																																									1														
p34																										1																													
p35																																									1														
p36																														1																									
p37																												1																											
p38																																							1																
p39																																																							1
p40												1																																											
p41			1																																																				
p42			1																																																				
p43																																						1																	
p44																											1																												
p45																																				1																			

Figure 11.4 Scenario 2 bed assignment results using LexGP (p46-p65)

p46																																																					1
p47																																											1										
p48																					1																																
p49																	1																																				
p50																													1																								
p51									1																																												
p52																															1																						
p53	1																																																				
p54																								1																													
p55		1																																																			
p56																		1																																			
p57																																														1							
p58					1																																																
p59							1																																														
p60		1																																																			
p61																																	1																				
p62																																																	1				
p63																																																			1		
p64																																										1											
p65						1																																															

Figure 11.5 Scenario 1 bed assignment results using LexGP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																				1																																	
p2																																																	1				
p3		1																																																				
p4																										1																											
p5				1																																																	
p6																								1																													
p7																																							1														
p8																		1																																			
p9														1																																							
p10																			1																																		
p11																											1																										
p12													1																																								
p13																																	1																				
p14						1																																															
p15																						1																															
p16			1																																																		
p17																																																			1		
p18																									1																												
p19									1																																												
p20					1																																																
p21																																																			1		
p22																																				1																	
p23	1																																																					
p24												1																																									
p25																					1																																
p26																							1																														
p27																												1																									
p28																												1																									
p29										1																																											
p30																																											1										
p31																																																1					
p32																																		1																			
p33																																								1													
p34																									1																												
p35																																								1													
p36																																																					1
p37																											1																										
p38																							1																														
p39																																																						1
p40											1																																										
p41			1																																																			
p42			1																																																			
p43																				1																																	
p44																										1																											
p45																													1																								

Figure 11.6 Scenario 3 bed assignment results using GP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																																														1								
p2																				1																																		
p3			1																																																			
p4																														1																								
p5					1																																																	
p6																						1																																
p7																																							1															
p8																																					1																	
p9															1																																							
p10																																																		1					
p11																																				1																		
p12												1																																										
p13																																1																						
p14						1																																																
p15																						1																																
p16									1																																													
p17																																																			1			
p18																																				1																		
p19										1																																												
p20	1																																																					
p21																																																				1		
p22																																														1								
p23				1																																																		
p24																																																1						
p25																								1																														
p26																					1																																	
p27																												1																										
p28																																								1														
p29													1																																									
p30																																												1										
p31																																													1									
p32																																	1																					
p33																																									1													
p34																																																			1				
p35																																											1											
p36																										1																												
p37																																																			1				
p38																	1																																					
p39																																																					1	
p40											1																																											
p41														1																																								
p42				1																																																		
p43																			1																																			
p44																												1																										
p45																																															1							

Figure 11.7 Scenario 3 bed assignment results using GP (p46-p85)

p46																											1																											
p47																																											1											
p48																							1																															
p49																		1																																				
p50																													1																									
p51						1																																																
p52																																	1																					
p53														1																																								
p54																									1																													
p55																																						1																
p56																1																																						
p57																	1																																					
p58		1																																																				
p59								1																																														
p60																																						1																
p61																															1																							
p62																					1																																	
p63																																																						1
p64																																										1												
p65							1																																															
p66																																																						
p67																1																																						
p68											1																																											
p69																																1																						
p70																																																						
p71																																		1																				
p72																																										1												
p73																																																						
p74																																																					1	
p75																																																						
p76																																																						
p77																																													1									
p78								1																																													
p79																																																						
p80																																																						
p81																																																						
p82																																																						
p83	1																																																					
p84																													1																									
p85																																			1																			

Figure 11.8 Scenario 2 bed assignment results using GP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																																				1																	
p2																			1																																		
p3		1																																																			
p4																													1																								
p5				1																																																	
p6																					1																																
p7																																							1														
p8																	1																																				
p9													1																																								
p10																																																	1				
p11																												1																									
p12											1																																										
p13																															1																						
p14					1																																																
p15																					1																																
p16	1																																																					
p17																																																			1		
p18																																																		1			
p19									1																																												
p20								1																																													
p21																																																				1	
p22																		1																																			
p23		1																																																				
p24													1																																								
p25																							1																														
p26																																																1					
p27																																														1							
p28																																								1													
p29												1																																									
p30																																												1									
p31																						1																															
p32																																1																					
p33																																									1												
p34																												1																									
p35																																											1										
p36																																			1																		
p37																																																		1			
p38																																						1															
p39																																																					1
p40										1																																											
p41																1																																					
p42			1																																																		
p43																		1																																			
p44																										1																											
p45																																															1						

Figure 11.9 Scenario 2 bed assignment results using GP (p46-p65)

p46																											1																												
p47																																											1											
p48																								1																															
p49																																													1									
p50																										1																												
p51						1																																																	
p52																																		1																				
p53			1																																																				
p54																				1																																			
p55		1																																																					
p56															1																																								
p57																																																1						
p58																																					1																	
p59						1																																																	
p60														1																																									
p61																															1																							
p62																																						1																
p63																																																							1
p64																																										1												
p65							1																																																

Figure 11.10 Scenario 1 bed assignment results using GP (p1-p45)

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	r35	r36	r37	r38	r39	r40	r41	r42	r43	r44	r45	r46	r47	r48	r49	r50	r51	r52	r53	r54	r55
p1																																				1																	
p2																			1																																		
p3		1																																																			
p4																													1																								
p5				1																																																	
p6																				1																																	
p7																																							1														
p8																	1																																				
p9														1																																							
p10																																																	1				
p11																												1																									
p12											1																																										
p13																															1																						
p14					1																																																
p15																					1																																
p16	1																																																					
p17																																																			1		
p18																									1																												
p19									1																																												
p20								1																																													
p21																																																				1	
p22																		1																																			
p23		1																																																				
p24													1																																								
p25																							1																														
p26																																																1					
p27																																														1							
p28																																								1													
p29												1																																									
p30																																												1									
p31																						1																															
p32																																1																					
p33																																									1												
p34																												1																									
p35																																											1										
p36																																			1																		
p37																																																		1			
p38																																						1															
p39																																																					1
p40										1																																											
p41							1																																														
p42			1																																																		
p43																		1																																			
p44																										1																											
p45																																															1						