Financial Engineering
A Structured Financial Product applied to Renewable Energies

Manuel Soveral Padeira Hopffer Navarro

Thesis to obtain the Master of Science Degree in

Energy Engineering and Management

Supervisor: Prof. Maria Margarida Martelo Catalão Lopes de Oliveira Pires Pina

Examination Committee

Chairperson: Prof. Duarte de Mesquita e Sousa

Supervisor: Prof. Maria Margarida Martelo Catalão Lopes de Oliveira Pires Pina

Member of the Committee: Prof. Maria Isabel Craveiro Pedro

June 2018

"A nation that can't control its energy sources can't control its future."

BARACK OBAMA, *THE AUDACITY OF HOPE*

Acknowledgements

To my Supervisor, Professora Doutora Margarida Catalão Lopes who not only accepted promptly to coordinate this project while giving me the freedom to develop my critical thinking and problem solving by figuring out which way was best to address this self proposed investigation, but also for always being available to provide with much required help and guidance.

A word of appreciation to my dear friend, Tiago Baptista, for introducing me to his uncle Vasco Nuno who has proved to be essential for this investigation. Hence, to Vasco I owe my gratitude for making this project possible by providing the initially required guidance, although he had such a busy schedule.

To my cousin Francisca I also dedicate a word of appreciation for helping me with the technicalities required for successfully carrying this investigation, by providing with studying material from her master's degree courses and also by being such a rich source of conversation.

To my cousin Maria Flor who was the first to help with what was initially an idea that took its final shape, and is now presented here, as a consequence of a suggestion made by her.

To my dear girlfriend Madalena, whose love and support during this project have given me the motivation to accomplish this achievement.

Finally, to my parents and my brother André, for providing me with unfailing support and encouragement throughout my years of study.

Abstract

Faced with the globally spread increase in electricity consumption, renewable energies are rushing to set themselves as leaders on the already ongoing "next energy transition". As a consequence, it becomes evident the need for investigating a new strategy that allows retail investors, producers and financial institutions to benefit from this transition, without jeopardizing consumers, through the creation of a socially responsible structured financial product applied to electricity generation from renewable sources. To cope with this investigation we first decided to go back in history and learn from the mistakes that have left their mark on financial engineering. We then continued by establishing the legal framework that currently drives this industry as a whole in Portugal, as well as the incentives that were created to foster this transition, proving this way the potential interest of introducing the product we propose. Lastly we have defined a strategy that enables one to create such financial product with special emphasis on the variable component, which was made possible by using the models mostly recommended by the scientific community. Finally we were able to conclude that there is in fact a market from which everyone could benefit, but its success is subject to transparency and openness.

Keywords: Derivatives, jump-diffusion model, electricity, structured product, renewable energy

Resumo

Dado o aumento do consumo energético registado globalmente, cada vez mais as energias renováveis se apressam em definir-se como líderes naquela que é já "a próxima transição energética". Torna-se, pois, pertinente investigar uma estratégia que permita a investidores do retalho, juntamente com produtores e instituições financeiras, tirar partido desta transição sem que os consumidores sejam afetados, através da criação de um produto financeiro estruturado, socialmente responsável, aplicado à eletricidade proveniente de fontes renováveis. Para isso optamos por começar com um regresso ao passado no sentido de aprender com os erros que marcaram a engenharia financeira. Posteriormente definimos a contextualização legal atual que o sector da electricidade, como um todo, vive em Portugal, bem como a evolução dos incentivos à produção renovável, transmitindo assim o interesse inerente à criação de um produto como o que aqui sugerimos. Por fim definimos uma estratégia que possibilite a criação, com sucesso, de um produto desta natureza, com especial enfoque na componente variável, usando para isso os modelos mais recomendados pela comunidade científica. É possível concluir que existe de facto um mercado do qual todos podem beneficiar, mas é necessário transparência e abertura.

Palavras-chave: Derivados, modelo de salto-difusão, electricidade, produto estruturado, energias renováveis

Table of Contents

Acknowledgements ii

Abstract iii

Resumo iv

List of Abbreviations vii

List of Tables ix

List of Figures x

1. Introduction 1

 1.1. Objectives and Methodology 1

 1.2. Structure of this investigation 2

2. Structured Financial Products 3

 2.1. Growth and importance 3

 2.2. Structured Financial Products and the U.S credit crisis 3

  2.2.1. The rise of CDOs 5

  2.2.2. Introducing CDSs to the market 9

3. Renewable Energy industry and its main financing tools 11

 3.1. Portuguese context – policies and their impact 12

 3.2. The Renewable Energy Sector today 18

 3.3. Electricity Market – Key features 21

4. Making personal finance a part of renewable energy funding 25

5. Electricity Derivatives on Foreign Markets 27

 5.1. Financial Products' Development – Acceptance and Growth 28

  5.1.1. Nord Pool 28

  5.1.2. EEX – a member of the EEX Group 29

 5.2. A Working Example – Wind Power Futures 30

6. Creating Financial Products 33

 6.1. Addressing Complexity 34

 6.2. How to create a structured financial product 35

  6.2.1. Main Features 36

 6.3. Structured Product – a standard contract 41

  6.3.1. Highly volatile market – Barrier option as a solution 49

7. Pricing Derivatives 51
7.1. The Black-Scholes-Merton model 52
7.2. Monte Carlo 58
8. Focusing on the Portuguese Electricity Market 61
8.1. Market Modeling 63
8.1.1. Review of existing models 63
8.1.2. A jump-diffusion mean-reverting approach 66
8.2. Calibration 68
8.3. Results & Discussion 72
9. Option Pricing 75
10. Conclusion 77
Bibliography 80
Appendix A – "German Austria Power week future" 84
Appendix B – example of required information on a structured financial contract 86
Appendix C – strategies with options 87
Appendix D – Barrier options terminal payoff 89
Appendix E – risk neutral measure 90
Appendix F – Black Scholes Calculator 91

List of Abbreviations

APREN	Associação Portuguesa das Energias Renováveis (Portuguese Renewable Energy Association)
ATM	At-the-Money
BSM	Black-Scholes-Merton formula
CCGTs	Combined Cycle Gas Turbines
CCP	Central Counterparty
CDO	Collaterized Debt Obligation
CDS	Credit Default Swap
CFTC	Commodity Futures Trading Commission
CMVM	Comissão de Mercado de Valores Mobiliários (Portuguese Securities Market Commission)
CNE	Comissión Nacional de Energia (National Energy Comission – Spain)
DL	Decree Law
DS	Deferred Settlement
EEF	Energy Efficiency Fund
EEX	European Energy Exchange
ERSE	Entidade Regulardora do Sector Energético (Regulatory Entity of the Energy Sector)
FAI (FSI)	Fundo de Apoio à Inovação (Fund to Support Innovation)
FIT	Feed-in Tariff
FTSE	Financial Times Stock Exchange
GCV	Gross Calorific Value
GDP	Gross Domestic Product
GHG	Green House Gases
ITM	In-the-Money
MBS	Mortgage Backed Security
MDGs	Millennium Development Goals
MIBEL	Iberian Electricity Market
NES2020	National Energy Strategy
NIIKEI	Tokyo Stock Exchange
NREAP	National Renewable Energy Action Plan
OMIClear	Iberian Energy Derivatives Exchange Clearing house

OMIP Operador do Mercado Ibérico de Energia – Pólo Portuguê (Iberian Energy Market Operator – Portugues Division)

OTC Over the Counter

OTM Out-the-Money

P&L Profits and Losses

PRE Special Regime Production (SRP)

QREN (NSRF) Quadro de Referência Estratégico Nacional (National Strategic Reference Framework)

RES Renewable Energy Sources

SDE Stochastic Differential Equation

SDGs Sustainable Development Goals

SMI Swiss Market Index

SPE Special Purpose Entity

SPV Special Purpose Vehicle

TSO Transmission System Operator

VR Variable Return

wph Wind Production Hour

List of Tables

Table 1– Action plan measure | Source: Plano Nacional de Acção para as Energias Renováveis (2009) 14
Table 2 – Prices in $/Unit | Adapted from: International Energy Agency (2016)........ 20
Table 3 – Contract specifications for illustrative purposes only | Adapted from the communicating tools from EEX 30
Table 4 – Option position | Adapted from Nunes (2015)........ 38
Table 5– Strike price and Current price | Adapted from Nunes (2015)........ 39
Table 6 – Call and Put prices calculated under the Black-Scholes-Merton assumptions........ 44
Table 7 – Variable Return subject to Growth. 46
Table 8 – Index growth cap 47
Table 9 – Barrier Options 50
Table 10 – Parameters provided by data fitting via least squares. 72
Table 11 – Jump intensity shape s(t) parameters. 73
Table 12 – Model parameters. 73
Table 13 – Statistical moments matching for empirical data and simulated. 74
Table 14 – Pricing options using two different methods. Values calculated for three maturities (in months) and three different strike prices to cover ITM, ATM, OTM. 76
Table 15 – Automated calculator for pricing options with the BSM model. The chart in blue provides “historical data” and the red the path simulated. The boxplot on the right represents the confidence level from registered spot prices from the market. 91

List of Figures

Figure 1 – CDO exemplification 3 tranches 6
Figure 2 – Leverage ratio across banks | Adapted from Kalemli-Ozcan *et al.* (2011) 7
Figure 3 - Maintaining AAA ratings through financial engineering | Source: International Monetary Fund (2008) 8
Figure 4 - New Paradigm | Source Griggs *et al.* (2013) 11
Figure 5 - Evolution of Renewable Energy penetration in Portugal (%) | Source: (APREN) 19
Figure 6 - RES share (%) in Electricity Production | Source: Direção Geral de Energia e Geologia (2016) 21
Figure 7 – Electricity spot price history. 24
Figure 8 - Energy mix by share in Portugal | Source: Direção Geral de Energia e Geologia (2016) 25
Figure 9 – Simplified model of the power stack function | Adapted from (EDP Power trade)..... 27
Figure 10 – The impact of introducing Deferred Settlement Futures | Source Bjønnes & Saakvitne (2015) 29
Figure 11 – Contract Fulfillment | Adapted from (EEX) - Phelix Power Futures Presentation Data © 32
Figure 12 - Top-Down approach | Adapted from Lee & Zhong (2014) 33
Figure 13 - Long Call vs Short Call 39
Figure 14 - Capital Protection 40
Figure 16 – distribution of electricity injection by sources | Source (APREN) 43
Figure 17 – Simplified version of a possible structured product contract | Adapted (Banco BPI) 43
Figure 15 – Renewable energy penetration as a percentage of total production | Adapted from PORDATA (2018) 43
Figure 18 – Structured product's payoff yearly distributed. 44
Figure 19 – Profit and Loss chart for a Long Call 47
Figure 20 – Comparison between first and new scenario P&L chart. 48
Figure 21 – Implied volatility' smile effect 57
Figure 22 – Option worth by segment and respective Greek 58
Figure 23 – Electricity spot price in Portugal | Source (OMIP) 64
Figure 24 & Figure 25 – Logarithmic returns (above) and Spot prices (below) show deviating levels from the normal curve due to the occurrence of extreme events. 67
Figure 26 – Seasonality function ploted against the (log)spot prices (above) and the *Deseasonalized* function (below) 69
Figure 27 – Overlapping of the intensity shape function with the deseasonalized data series. . 70
Figure 29 – Jump-Diffusion model path 74
Figure 30 – "German-Austria power week future" trading on the 13th December 2017 | As seen on EEX 84

Figure 31 – Settlement price path example as an indicator on what to expect. | As seen on EEX 84
Figure 32 – Fundamental Information to the Investor | As seen on the original document (Banco BPI)........ 86
Figure 33 - Bear Vertical Spread........ 87
Figure 34 – Butterfly Spread 87

1. Introduction

Considering the observed deceleration of renewable energy growth, this dissertation proposes the creation of a financial structured product applied to this sector. With the liberalization of the electricity market, the conditions for implementing a financial product to apply on renewable energies are established. The product analyzed in this project is at the centre of very ambiguous opinions worldwide – a structured financial product. For that reason, the following section is dedicated to the history of this type of products, and their impact on economies on a global scale from a financial engineering perspective. When financial products are created, one does not usually foresee what can go wrong, so it is in our interest to develop such product taking into consideration what history tells us.

The current Portuguese economic situation should not stop us from investing in a sector in which we already are at the forefront among large economies in Europe. It is in Portugal's interest to remain competitive and take advantage of endogenous resources provided by the country's geography (insulation, wind, currents, etc). As government incentives towards these technologies are now becoming unsustainable for the financial system, the solution we propose aims to pull out some weight from tax payers at the same time that enables producers to hedge their activities. Policy makers should keep an active role in this subject; however, instead of financing activities, we propose a solution based on encouraging retail investors who want to be a part of renewable energy business via, e.g., tax benefits.

1.1. Objectives and Methodology

Although forward contracts are the most natural vehicles to trade electricity, due to the non-storable nature of this utility as argued in Geman (2005), in this project we explore with further detail the use of option contracts as the derivative component of our structured financial product, since they are most common for hedging purposes. That way, producers are able to securitize their investments whilst providing the opportunity for retail investors to profit from risk sharing.

Since our target is the retail market, the product should not be too complicated for common citizens to understand. To cope with that, we propose the integration of continuous models (such as the Black-Scholes) with some of the assumptions of discretized ones. We understand that such assumptions make the product more comprehensive for any user, although they might produce results that deviate from the reality. The reason for not using only a discretized model resides in the fact that these usually resort to Monte Carlo simulations, which as we shall find, can take too much time (sometimes several hours) and might require advanced computational equipment.

Also, in this way we are able to deal with one of the major problems that active trading companies face when attempting to choose a model that reaches for simplicity while capturing the most important moments of the markets in "real life" as posed by Hafner (2003). For that purpose we will try to meet two series of criteria when representing our market, as in Geman (2005):

Trajectorial – meaning that the simulations should generate paths fairly similar to those observed in the markets;

Statistical – we should make an effort to take into consideration the matching of the first four statistical moments between simulated and real market. In that sense, average, standard deviation, skewness and Kurtosis will be addressed as a measure of quality of our model.

As this investigation provides nothing but a first approach to study the possibility of integrating such products in this industry, we will take these errors to be acceptable but we shall propose future studies to deal with them without removing the simplicity of such financial product in the best way possible. The same reasoning applies to the fact that our study will focus on options written on spot prices, although we reckon that markets worldwide trade on futures and forward contracts, where higher liquidity can be found.

Given that, we shall use as underlying the baseload, i.e. the arithmetic average of hourly prices for the day-ahead market and we will assume that there are no limitations regarding the time horizon for trading in this market – instead of being the typical daily market that usually trades (at best) for three days ahead, we will allow it to trade for as long as we wish to prove some point.

A structured financial product can be decomposed in both a straight bond and an option portfolio. Since the first component only includes the guaranteed redemption of the face value and the minimum fixed coupon, we will assume the same structure for all products investigated. For that we will first present a simple structured product and carry on our investigation solely on the option portfolio using different strategies and different fictitious products.

1.2. Structure of this dissertation

Before we initiate our investigation on whether a structured financial product could be applied to the market under investigation, we agree that it would be wise to first conduct an investigation on what history tells us about such products. The next chapter will hence focus on the U.S credit crisis, not because the product we are trying to achieve resembles some of the products used during this time of financial history, but to enhance the fact that financial engineering could do worse than good to an economy. Our point being, acknowledging that the success of any financial product is implied on the intentions of all market participants. We wish to avoid such damages at all costs!

After this review on financial history we will make a review in chapter 3 on how is the market currently going in Portugal while addressing the major drawbacks due to financial crisis. Also in this chapter we wish to address the most important features of this market in order to make the following chapters easier to understand. Chapter 5 takes us on a journey through international markets, for we believe there is much to learn from them. Chapters 6 and 7 are meant to properly explain what is there to know when creating structured products with special focus on derivatives. Chapter 8 will address the issue of modeling the Portuguese electricity market whilst on 9 we provide an analysis on option pricing as a tool for investing in renewable energy.

2. Structured Financial Products

2.1. Growth and importance

One of the core tasks of investment banking is the constant search for opportunities to create new financial instruments. Typically, this task is fulfilled by innovatively combining already existing components to form new financial instruments through financial engineering.

Since the end of the 1990's something like 2 trillion euros of highly complex financial products were designed and traded between European financial institutions and households – the so called retail structured products, as presented in Célérier & Vallée (2015).

These investment products can be seen as the combination of two or more financial instruments being one of them, at the very least, a derivative – *derives* its value from an underlying asset or a combination of assets. They can be characterized for giving investors the access to a set of assets that would otherwise be unattainable (due to high investing requirements, physical space restrictions, etc), allowing them to benefit from different risk-return profiles that would not be possible with traditional products (bank loans, etc). These products tend to face demand increase, when traditional products are presented with low interest rates, as often literature suggests "…the great expansion took place at the beginning of the millennia, in a period when market interest rates were low" (Pinto, 2013, p. 1)

Ambiguous opinions arose since the creation of structured products, due to their complexity. Breuer & Perst (2007, p. 828) classified them as "one of the most prominent groups of newly introduced financial instruments resulting from such financial engineering (…) which exhibit structures with special risk/return profiles that may not be otherwise attainable on the capital market without significant transaction costs being incurred – at least for private investors". However, for other authors a more cautious approach was adopted as "financial complexity is one of the key developments of modern finance, and has been pointed out as a catalyst of the recent financial crisis" Célérier & Vallée (2017, p. 2). Buffet (2003, p. 13) went further, and classified these products as "time bombs for both parties that deal in them and the economic system" when specifically referring to those that were created during the housing bubble in the US, enabling investors to bet on virtually anything, which would later on lead the world to plunge into a crisis with a dimension never seen since the Great Depression (1930's) as we shall explore further on.

2.2. Structured Financial Products and the U.S credit crisis

As Boz and Mendoza (2013) suggest, the U.S. credit boom arose in a period where significant financial innovation – that "securitized" the payment streams generated by a wide variety of assets – met with important reforms that radically changed financial regulation. In reality "the gradual introduction of collaterized debt obligations[1] (CDO's) dates back to the early 1980's", however it was in the 1990's that the most significant financial reforms since the Great Depression were introduced,

[1] CDO's are a structured financial product that pools together cash-flow generating assets and repackages this asset pool into discrete tranches that can be sold to investors.

aiming to remove the "barriers separating bank and non-bank financial intermediaries set in the 1933 Glass-Steagall Banking Act[2]". For this essay, two major acts are of significant importance, for they reveal how relevant financial regulation is when dealing with the creation of financial structured products. The first is the 1999 Gramm-Leach-Bliley Act, which removed the prohibition on banking holding companies from owning other financial companies, enabling commercial banks to invest depositors' money. The second is the 2000 Commodity Futures Modernization Act which, quoting Boz & Mendoza (2013, p. 2), "left over-the-counter[3] financial derivatives beyond the reach of regulators". Only few people actually tried to go against this "de-regulation" of the financial sector, such as Brooksley Born[4] who became best known as the sole regulator in the Clinton administration for attempting to regulate derivatives and hence becoming the target of bullying by the Treasury Secretary Robert Rubin, his enforcer, Larry Summers, and Fed Chair Alan Greenspan, who defended that "financial markets are self regulating and that financial firms are capable of policing themselves." In the end, Born lost the battle to Wall Street and ended up resigning her post, after Congress passed legislation prohibiting her agency from regulating derivatives.

With the de-regulation along with financial innovation, banks were now able to bet depositors' money, on almost everything. Traditionally banks and common investors use the Federal Reserve for investing their money, through treasury bills, and believed this to be the safest investment (specially on stable economies, such as the US) and with rates that are usually appealing. However, in the wake of the ".com bubble"[5] Federal Reserve chairman Alan Greenspan lowered the rates to only 1% to keep the economy strong. Since return on investment was at a very low rate (1%), investors had to turn to more attractive applications. However, even though a 1% rate may not be good for investment, it represents a great opportunity for banks to borrow money. Adding to that, countries like China and Japan at the time were generating surpluses that they needed to invest, so they turned to the US, meaning that there was an abundance of "cheap credit" for banks to take and leverage their operations.

On the other hand, there were home owners who needed to borrow money to pay for their properties. In the beginning, they would contact a local mortgage broker to apply for a line of credit, who would then contact a mortgage lender (e.g. Countrywide). The mortgage lender would consider the household income, credit history and demand for a down payment in exchange for the mortgage. Since the housing market was on the rise, the first group of derivatives emerged and became the largest at the time of the crash – mortgage backed securities (MBS). As Danny Myers (2016) suggests, these bonds were formed by mortgage loans that were pooled together into a parcel, and sold together as a collection of mortgage loans in blocks of 1000 to 5000 to investors, spread all over

---

[2] The Glass-Steagall Banking Act prohibited commercial banks from participating in the investment banking business. It was repealed in 1999.

[3] Over-the-counter Markets (OTC) are decentralized markets, without a central physical location, where market participants trade with one another. The dealers act as market makers by quoting prices at which they will buy/sell a security or currency. Trades can occur between two parties without others being aware. Hence, OTC markets are therefore less transparent than exchanges and are also subject to fewer regulations.

[4] American attorney, who was chairperson of the Commodity Futures Trading Commission (CFTC) from August 26, 1996, to June 1, 1999.

[5] In the late 1990's the Dotcom Bubble grew out of a combination of speculation, abundance of venture capital funding for startups and the failure of dotcoms.

the world. These derivatives were created by investment banks that would buy the mortgages from the Mortgage Lenders and pool them together to sell to investors through a scheme that enabled them to remove the mortgages from their balance sheet – the so called Special Purpose Vehicle (SPV) or Special Purpose Entity (SPE) - i.e. the monthly payments of the mortgages would be theirs by right after buying their "share" of the mortgage pool. To this group it was a good deal, since the interest rates that they got from buying MBSs were higher than those of Treasury Bills, and also this investment seemed to be safe, for the home prices were going up, which meant that, worst case scenario, if the borrowers went default, lenders could just take back the house and place it once again on the market for more money than initially sold.

2.2.1. The rise of CDOs

At the same time, banks that were creating these products were also paying credit rating agencies[6], such as Moody's, S&P or Fitch to evaluate the securities they were selling. Initially those bonds were mainly AAA-rated guaranteed by the US government, but later with the separation of the bonds into tranches these agencies would rate them accordingly, ranging from AAA's – top layers - to BBs and lower due to an increase in subprime mortgages. This separation into tranches and different ratings would mean that the lower layers would become more difficult to sell due to the inherent risk, since investment coming from e.g. pension funds would be automatically repelled, for these public backed funds can only bet on safer investments. Now, in order to cope with market requirements, banks turned to financial engineering to create a product where everyone could invest, i.e., in order to meet with different investor's risk profiles banks created derivatives from those MBS tranches that were not selling and bundled them together along with other forms of debt. The tranches that were once considered BBs and lower were now being given AAA ratings because they had been merged with other types of debts, hence being considered diversified and safe. That's when the CDOs boom started. Using the definition by Danny Myers (2016), Collaterized Debt Obligations are structured asset-backed securities that combine a number of other securities (mostly Residential MBS and Commercial MBS, but also Asset-backed Securities such as car loans, student loans, etc.) into tranches with different levels of risk. "A typical CDO can contain as many as 150 other packages of securities, structured into seven or eight tranches ranging from «super senior», which has the least risk of default, to «toxic waste», which carries far greater risks and has higher chance of default."

Figure 1 illustrates in a simplified way the concept behind the Collaterized Debt Obligations[7].

In this example, for the sake of simplicity, we use only three degrees of separation and consider that the debt being generated is only for the housing market. Suppose a given bank lent $1Billion to a given number of clients that were seeking to buy a house, at a simple interest rate of 10%.

[6] Rating is an assessment of the "creditworthiness" of a borrower in general terms or in respect to a particular debt or financial obligation. Different agencies use different alphabetical combinations, hence to simplify we will use AAA as the benchmark for best grading.
[7] A more detailed investigation was carried out by Barnett-Hart (2009).

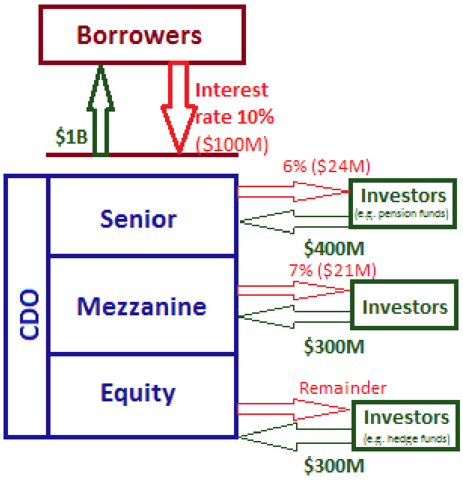


Figure 1 – CDO exemplification 3 tranches

Technically, since not every customer has the same credit history, nor income, the risk of default is different for each one. Given that, the banks should pool the borrowers into three tranches regarding their risk of default: "less-risky" end up in Senior Tranche, "more-risky" in Equity Tranche. Or, at least, that is what investors thought banks were doing when, in reality they were not – they just separated the whole group of borrowers into tranches, regardless of their risk of default and if eventually the entire product starts to lose money, the "less-risky" tranche gets paid first (at a fixed rate of return), all the way down until there is no more money. Hence the Equity Tranche does not have a fixed rate of income, but the remainder of what the borrowers paid to the other two tranches, as we will explain later. By doing so, the bank can now sell the CDOs to investors depending on the risk they are willing to take. At the same time, the bank pays rating agencies to rate its products – typically Senior Tranches were given AA's and higher, whilst Mezzanine Tranches were given BB's. They would not even bother to rate the Equity Tranche, for their target was risk takers, whom would already be aware of the type of product they were investing in.

In this small example, we assumed that a group of investors worth $400M decided to buy CDOs from the Senior Tranche, for this seemed to be a safe investment, with a good rating and a payoff of 6% ($24M) which is reasonable since, as mentioned before, Treasury Bills were offering only 1%. We represented this group using the pension funds as an example, to show how everyone assumed this was a safe product, as these funds are only allowed to invest at minimal rates of AA's due to the nature of the money it represents. At the mid-level of risk takers, we have a group investing their $300M worth in the Mezzanine Tranche that offers also a relatively safe investment opportunity with a rate of 7% ($21M). Finally, the Equity Tranche is sold to a group of investors worth $300M, represented by the example of Hedge Funds. For these funds risk management may vary with the investors' profiles and the policies of the company itself (note that not all hedge funds engage in such risky deals). We assume they are a group of investors willing to take on high risk investment since, as mentioned before, these tranches often were not even classified by rating agencies and, when they were, the classification would be "Junk". Also, as the figure shows, there is no fixed rate of income because they will be paid with the remainder of the inflow of the CDO, which means that, with a zero default rate from the borrowers, they will get $55M (Remainder = Borrowers – Senior (6%) + Mezzanine (7%) = $100M – ($24 + $21)). This represents a rate of return of 18,3%, which is extremely attractive. However, this simulation is based on the assumption that there are no defaults among the borrowers. Let us now assume that there is a 20% default with a recovery of 50%, i.e. 20% of borrowers default and the value of the assets can only be recovered by 50%. This means that there is an overall loss of 10% that translates into a decrease of $10M for the income of the CDO. Since the first and second Tranches have fixed rate of return, those investors will still receive 6% and 7%

respectively. However, considering that the Equity Tranche gets the remainder, those investors will now receive $45M, which corresponds to a rate of return of 15%. This is still very attractive but it represents how exposed this tranche is to default, as we will prove further on by getting back to this example.

For as long as banks were selling AAA rated CDOs to investors, provided that the borrowers pool consisted essentially of mortgages from borrowers with good credit history, this would be in fact a good investment strategy, for the risk of massive default was contained and with low probability of occurrence. And even if some of the borrowers went default, since the housing market was on the rise, banks could just take the houses and place them back on the market for higher prices. Given that, investors spread worldwide wanted to take part in this investment opportunity – by late 1990s these derivatives were already worth $50 Trillion in unregulated markets - which meant that banks needed to create more and more of these products to meet the demand. This is the reason why between 2000 and 2003 the number of mortgage loans in the US nearly quadrupled. Banks were borrowing heavily during the bubble to leverage their operations, in order to keep buying more and more loans, as shown in Figure 2. As the asset prices were going up, banks needed to keep borrowing money – procyclical leverage[8] as in Kalemli-Ozcan *et al.* (2011), which amplifies the business cycle but potentially leads to systemic risk if asset prices do not properly reflect fundamental values (the so called “bubbles”). By 2007, with banks reaching leverage ratios of 33:1, that would mean that a small decrease in value of their asset-based would leave them insolvent, as Daniel Alpert stated in his interview for the documentary Inside Job (2010).

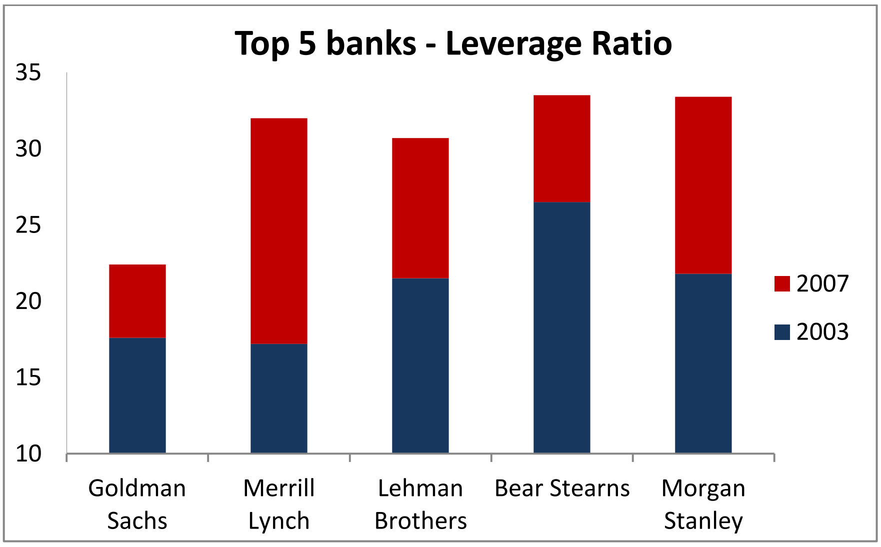


Figure 2 – Leverage ratio across banks | Adapted from Kalemli-Ozcan *et al.* (2011)

However, banks can only create “safe” products – from *prime* loans – for as long as there are borrowers with good credit history and stable jobs and income to take them. Faced with the massive demand for the CDOs, banks needed to generate more mortgages in order to sell them to investors. With the securitization chain in motion, lenders were now becoming less demanding regarding the borrowers’ ability to repay their debts, as long as there were investors willing to buy the products. This

[8] In an asset price boom, banks mark their asset values up as they increase in value, which makes their balance sheet look favorable.

is why banks were only focusing on the profits they would get by selling CDOs, regardless of the inherent risk of default. Loans were now being massively assigned to virtually anyone who wanted a home through the so-called *subprime* loans[9], which besides increasing the number of mortgages, would also increase the rates, translating into higher returns for the investors. Through financial engineering, banks managed to attract more and more mortgages by making use of predatory lending practices, i.e. they would design complex contracts to home owners, enabling them to pay small fixed interest rates in the first two years of their mortgage (making it attractive) but by the third year would jump to much higher and variable rates, alongside with the rise of the housing market. For some people, the increase on the rate due to the rapid increase on the market housing value became so strong that they would have to apply for a second mortgage, just to be able to pay the first! Subprime loans was now "the word of the day", leading borrowers to become needlessly placed in these expensive loans with adjustable rates, along with people that could hardly repay them. This is why, in ten years this industry rose from $30 Billion to $600 Billion and, in 2006, nearly 40% of all profit at S&P500 was coming from the financial institutions trading with these products, as investigated by Charles Ferguson (Inside Job, 2010). Note that the ratings agencies were still rating most of these derivatives as AAA's despite many of them were mainly constituted by subprime loans. Financial engineers were constantly "engineering" the worst tranches of the products into bigger groupings by what was known as squaring CDO, as shown below (figure 3).

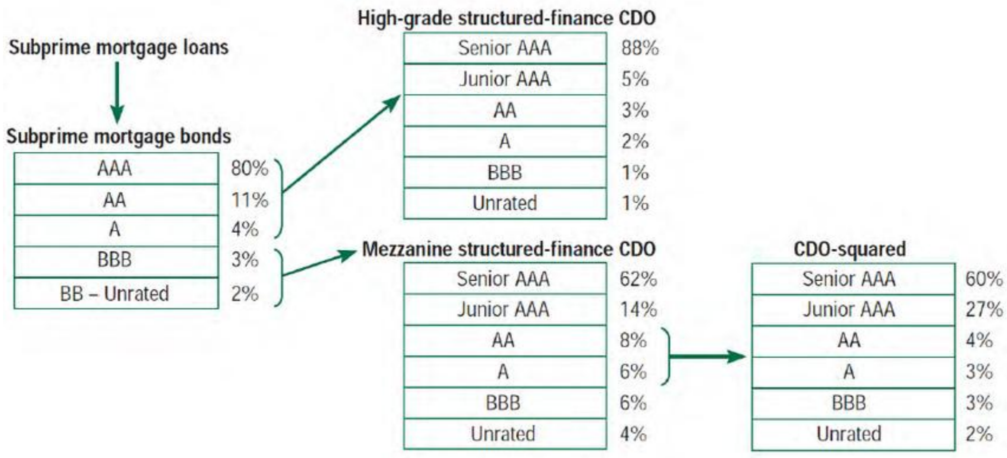


Figure 3 - Maintaining AAA ratings through financial engineering | Source: International Monetary Fund (2008)

Some people actually started to question themselves for how long would the market be able to endure such complexity, since it was based essentially on subprime loans that would most probably go into default. As Michael Lewis (2010) investigated, Mike Burry was one of the first to actually look at what banks were doing and decided to invest on his suspicion: the housing market was going to crash. It was not a matter of "if" but rather "when". Moreover, data shows that some of these bonds were only 65% AAA-rated out of which 95% were based on subprime[10] with a FICO score[11] below 550, which

[9] A type of loan offered at a rate above prime, to individuals who do not qualify for prime rate loans.
[10] A more cautious and detailed research was carried out by Barnett-Hart (2009).
[11] A type of credit score created by Fair Isaac Corporation that allows lenders to assess credit risk on borrowers. It ranges between 300 and 850 and, generally, a figure above 650 indicates a good credit history in contrast to those with scores below 620, whom might find it difficult to obtain financing.

means that the majority of the players in this market were either unaware of what they were doing, or looking to the other side.

2.2.2. Introducing CDSs to the market

It was in this environment that a new derivative entered the market: The Credit Default Swap (CDS) – name given to an insurance look-alike contract between one investor/counterparty that seeks protection from the declining value of another product (such as CDOs, MBSs, etc). On the other side the other counterparty is willing to provide such protection in exchange for insurance premiums. The buyer will get the protection from the insurance if the loss recovery due to credit events, or defaults, exceeds the amount paid in premiums (depending on the contract agreed between the parties). The investor that buys the protection and gains from the loss of the insured product is known for taking a Short Position, since he/she is betting on the decreasing value of a bond/asset. However, there are two major differences that separate the parallel between an insurance contract and a credit default swap. First, when buying an insurance contract, one must be directly exposed economically, whereas on CDSs you are not required to hold the bond or asset at stake. Secondly, as mentioned before, unlike insurance contracts CDSs are traded over the counter basis (OTC), which means that their trades can be untraceable enabling the creation of multiple contracts for the same product, and sold to different investors.

Theoretically, credit default swaps should enable the efficiency of financial markets by improving the allocation of capital as suggested by Stulz (2010), since the separation of cost of funding and credit risk would introduce greater transparency in the pricing of credit, which would eventually reduce the cost of capital for firms. A CDS market "would turn out to be a better place to assess a company's credit risk than the market for that company's bonds". However, as we will discuss further on, the same way credit default swaps provide theoretical benefits to financial markets, they can also create problems regarding the incentives for monitoring and for working out situations of financial distress – by spreading the risk of credit, a given loaning company might lower their standards regarding the monitoring of their loans that are protected via CDS and, "of course, the seller of protection cannot monitor the firm in the same way as the bank [that loaned the money in the first place] would, because it has no contractual relationship with the firm." Also on a CDS market, there can be a modification on the incentives in the investors' perspective if we consider the gains they might have if they hold the bonds (protected via a credit default swap) of a given company in financial distress seeking to restructure its debt. Eventually, an investor might choose to drive the firm into bankruptcy, hence triggering payments under the protection contract he/she holds, rather than working out a refinancing plan. However, even though a CDS market on firms' debts can lead to some shady situations and financial inefficiencies, credit events are still very clear and well defined, with the company often being led to bankruptcy or to the restructuring of its debt. But what happens when you try to insure a given tranche of a pool consisting of thousands of mortgage debts – CDOs, MBSs, etc? What is at stake is that a rising level of defaults on the underlying mortgages will lead to a reduction in debt payments, not implying that the whole product will go "bankrupt" since there is still a positive cash flow. As Michael Lewis (2010) stated in his later work - *The Big Short:* "Buying insurance on a pool of U.S.

home mortgages was more complicated, [than insurance on corporate bonds] because the pool didn't default all at once; rather, one homeowner at a time defaulted." So, once again through financial engineering the dealers – led by Deutsche Bank and Goldman Sachs – created a solution that would adapt to this market: a form of *pay-as-you-go* CDS in which the "buyer of the swap would be paid off not all at once, if and when the entire pool of mortgages went bust, but incrementally, as homeowners went into default.", i.e. they created a multi-name CDS system (CDSs referenced to various entities), specifically a Index CDS type[12], that was designed to trigger a compensation (via a CDS settlement[13]) for any reduction in payments to the tranche's holder. However, that CDS would still exist until the maturity of the contract, but with a reduction of its notional amount by the defaulting part of the pool share.

As Stulz suggested (2010), by 2006, the introduction of the ABX indices by the Markit Group Ltd on subprime securitizations came to, theoretically, introduce "greater transparency in the market for subprime debt as their trading facilitated price discovery for that debt". It would be working as a synthetic tradable index referencing a basket of 20 subprime mortgage-backed securities designed to provide feedback on the market, allowing investors to take views/positions on it without actually owning subprime mortgages directly or indirectly. According to Fender and Scheicher, (2008), each index relies on five individual sub indices, each referencing to the same 20 underlying subprime mortgage securitizations, though at different levels of the liability structure. Using their example, "the ABX 06-1 AAA index represents tranches with an original rating of AAA from a pool of MBS originated in the latter half of 2005. The other sub indices, in turn, are backed by tranches of the same securitizations at the AA, A, BBB and BBB- levels of credit quality." Once they are created, the indices remain static, despite the fact that underlying credits can reach lower ratings. However, since the prices on ABX contracts are based on the willingness of investors to invest in each subindex, this becomes more transparent than the typical previous OTC market. This made it possible for investors to take more exposure to subprime mortgages than there were actual mortgages, as now any investor could assume a position on those mortgages. Even banks started to invest in these products as they came to the conclusion that by the time it was no longer possible for them to sell their CDOs, they were stuck with all those bad loans. As banks bought the swaps on their own MBSs, they created yet another product, called synthetic CDO, which consists of a CDS embedded on the typical CDO. Since it was securitized, rating agencies would call them safe bets. However, in case of default of the underlying assets, investors would not be compensated, but the bank would. A more detailed analysis in this product was carried out by Gibson, (2004).

By the time the housing market collapsed, billions of dollars in pension funds were lost in the CDO markets, which obviously pushes us to the ongoing debate on the motives for developing these complex instruments, as there is a lack of protection towards the common investors who are often led to products whose risk and functioning they do not understand.

[12] According to Deutsche Bank Research by Christian Weistroffer (2009), Index CDSs are the key product among multi-names. They use an index of debtors as reference entity, incorporating up to 125 corporate entities. If a firm in the index defaults, the protection buyer is compensated for the loss and the CDS notional amount is reduced by the defaulting firm's pro rata share.

3. Renewable Energy industry and its main financing tools

As the economies worldwide are evolving, energy requirements tend to grow continuously. For this reason words like “efficiency”, “sustainability”, “green energy” are becoming more and more present on our vocabulary due to the urgent need to avoid catastrophic damage to our planet and human race itself. There is in fact a growing awareness in order to cope with human kind´s energetic rising requirements, since the world population is set to climb to 9 billion by 2050. Hence, when one speaks about sustainable development it should (and must!) be taken into account that it is not only Earth’s welfare that is at stake, but also to securitize a safe growth for human population. As Griggs *et al.* (2013) argues “defining a unified set of SDGs [Sustainable Development Goals] is challenging, especially when there can be conflict between individual goals, such as energy provision and climate-change prevention.” This means that, in order to tackle world’s problems it is important to address the pillars that constitute “sustainable development” as one. Those foundations are based on what the author defined as the New Paradigm, as presented in Figure 4, i.e. we should focus our efforts regarding the new definition of sustainable development as the “development that meets the needs of the present while safeguarding Earth’s life-support system, on which the welfare of current and future generations depends.” Griggs, *et al* (2013, p. 306)

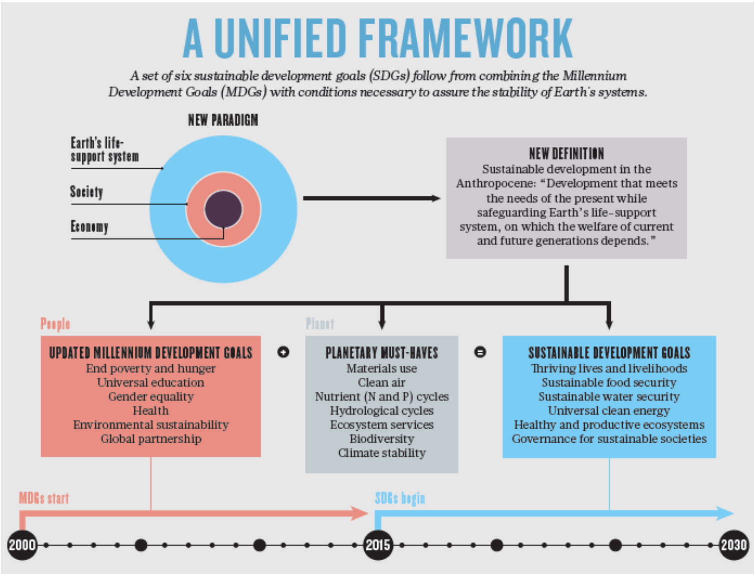


Figure 4 – New Paradigm | Source Griggs *et al.* (2013)

The authors propose a set of goals (six) that could allow us to move in the right direction. Since this essay is based on making personal finance a part of renewable energy industry, we shall focus only on three:

- Thriving lives and livelihoods. End poverty and improve well-being through access to education, employment and information, better health and housing, and reduced inequality while moving towards sustainable consumption and production;
- Universal clean energy. Improve universal, affordable access to clean energy that minimizes local pollution and health impacts and mitigates global warming;

- Governance for sustainable societies. Transform governance and institutions at all levels to address all sustainable development goals.

Creating a socially responsible financial product would be a step towards contributing to the United Nations' commitment to sustainable energy for all, whilst providing equal opportunities for business aiming to reduce inequalities and reducing carbon emissions as a consequence. Governing acts towards sustainable societies would prove to be essential to build on Millennium Development Goals (MDGs)' partnerships and incorporate environmental and social targets into global trade, investment and finance, as in Griggs *et al.* (2013).

However, one must first understand and tackle how the economy in this utility works. The history of industrial development tells us that growth has been supported by energy transitions "as supplies of firewood and other biomass energy proved insufficient to support growing economies in Europe and the United States, people turned to hydropower (…) then to coal during the nineteenth century, and then to oil and natural gas during the twentieth century. In the 1950s nuclear power was introduced into the energy mix." (Timmons, Harris, & Roach, 2014, p. 3) It is easily recognizable that each stage of economic development has been accompanied by a characteristic energy transition from one. Now, on the 21st century, we are already witnessing the next great transition, as renewable sources are conquering market shares of the energy mix on a global scale. Timmons *et al.* (2014) believe that "Society will eventually adopt renewable energy, since fossil fuels are limited in supply and only created over geologic time. Thus the question is not whether society will shift to renewable energy, but when". Some authors, however, might discredit this statement, as they do not believe that technologies can evolve in such a way that fossil fuels can be fully replaced, instead of just being reduced in the energy mix. Either way, it is directly observable that renewable energies are gaining more and more space in day-to-day market trading and do not seem to be slowing down worldwide.

Despite the existence of conflicting theories in this matter, there is a common ground where both meet since investment opportunities that arise from this transition are recognizable for all, as Timmons *et al.* (2014) suggests. Due to the dependence on fossil-fuel energy use that constitute the base for the capital stock and infrastructure of modern economic systems, any transition from these sources dependence will require massive restructuring and new investment. This means that major government policy changes will have to take place to foster the transition, clearing the way for private markets and small communities (small-scale projects) to play their critical role.

3.1. Portuguese context – policies and their impact

The present essay relies on the same principle – achieving sustainability by increasing the renewable energy share – but focusing on the financing tools that need to be adapted to the market in order to allow common citizens to become a part of this technological boom. Since (some) government budgets are limited, they must be the ones to set the pace - as policy makers - for public, private and institutional investors to step in and play an important role for future sustainable development. Portugal being a Member State of the European Union and in compliance with the Kyoto protocol, had legislation approved through Cabinet Resolution No. 29/2010, of April 15th - National Energy Strategy 2020 (henceforth also called NES 2020) – to pursue its commitments.

Concerning the “Directive 2009/28/EC of the European Parliament and of the Council, of 23 April 2009, on the promotion of the consumption of energy from renewable sources…”, it was defined that all Member States were to develop a National Renewable Energy Action Plan (NREAP) focusing on each individual country’s targets concerning the share increase of renewable energies on the energy mix, reducing fossil fuels dependence by 2020, which would then be approved by the EU. In order to achieve this approval, the Portuguese government delivered its action plan with all measures to be applied in the years to come.

Concerning the Portuguese commitment, all the objectives for the energy policy were incorporated on the later on approved action plan, planned for a time frame until 2020, seeking to “maintain Portugal at the forefront of the energy revolution”. The main objectives the country self proposed includes:

- To guarantee compliance with Portugal’s commitments in the context of European energy policies and policies to combat climate change, ensuring that a 31%, 60% and 10% quota from renewable sources is reached in, respectively, gross final consumption, electricity produced and transportation sector;
- Reduce the dependence on external sources, regarding consumption and imports of fossil fuels to around 74% in 2020, by means of increasing use of endogenous energy sources (estimated reduction using a Brent reference of 80 USD/bbl[14]);
- To reduce the balance of energy imports by 25% with the energy produced from endogenous sources, making it possible to reduce imports by an estimated 60 million barrels of oil;
- To consolidate the industrial cluster associated with wind energy and to create new clusters associated with new technologies in the renewable sector, ensuring a Gross Added Value of 3.8 billion Euros and creating 100,000 new jobs in addition to the existing 35,000 jobs associated with the production of electricity from RES[15] by 2020;
- To promote sustainable development, creating the necessary conditions to meet the commitments that Portugal has made with regard to reducing GHG[16], by means of a greater use of RES and energy efficiency.

In fact, by the time this document was written “more than 40% of the electricity produced in Portugal…” was already based on the use of RES and the potential for these numbers to continuously grow was recognized by policy makers, especially if one considers that from the security of supply’s point of view, Portugal did not have any known fossil fuel reserve available.

Indeed the Portuguese efforts towards sustainability (regarding the use of RES) started prior to the development of the action plan, as by 2010 the country already had a scheme to access the electricity network which prioritized the endogenous resources, “both with regard to planning and developing the network as well as in relation to everyday management, by giving priority to such dispatches.”

[14] Reference value used on the document – NREAP - in 2010. bbl = an oil barrel.
[15] Renewable Energy Sources
[16] Green House Gasses

Moreover the creation of differentiated tariffs for electricity produced in renewable plants was already in motion, with the so called feed-in tariffs (FIT) which, according to the definition used by Couture and Gagnon, (2010), has as main principle "to offer guaranteed prices for fixed periods of time for electricity produced from Renewable Energy Sources. These prices are generally offered in a non-discriminatory manner for every kWh of electricity produced, and can be differentiated according to the type of technology, the size of the installation, the quality of resource, the location of the project, as well as a number of other project-specific variables." and its implications will be addressed later on.

The following table shows the measures[17] approved in the action plan:

Table 1– Action plan measure | Source: Plano Nacional de Acção para as Energias Renováveis (2009)

Reference	Name of Measure	Type of Measure	Expected Results	Targeted group and/or activity
DL 225/2007	Special tariffs for electricity produced from renewable energy sources	Regulatory	Increase electricity produced from RES	PRE
RCM 169/2005	Intensification and diversification of the use of all sources of renewable energy to produce electricity, especially hydro and wind energy	Voluntary	By 2010, 39% of final electricity will be derived from renewable energy	Producers
DL 50/2010	Energy Efficiency Fund (EEF)	Financial	To ensure compliance with the national energy efficiency targets established in the action plan	State/ Companies/ End user
Law No. 3-B/2010: State Budget for 2010	30% tax reduction of amounts spent on acquiring the following goods, as long as they are for personal use, up to a limit of 803€: a) New equipment to use renewable energy; (…)	Regulatory/ Financial	Increase the use of renewable energy	End user
Order 32276-A/2008	Creation of the Fund to Support Innovation (FAI)	Regulatory/ Financial	Promote research and development in Renewable energy's field	Renewable energy sector
RCM 86/2007	QREN - Funding for innovative pilot initiatives for the rational use and production of energy (from renewable sources)	Financial	Increase the use of renewable energy	Producers/ Companies

In addition to those presented, the energy policy, by means of the NES 2020 (which establishes the global framework to implement the action plan), established a series of specific measures[18] in order to promote compliance with those targets:

- By 2012 a tariff equilibrium should be created, contributing towards minimizing the variations in electricity tariffs aiming to benefit consumers and creating a framework for economic sustainability that will support a long-term growth in the use of renewable energy;
- To develop, during 2010, lines of support for investments in the field of renewable energy, within the scope of the implementation of the national strategic reference framework (QREN[19]) and other instruments to support economic development, aiming to increase exporting capacity.

[17] We present only the ones that are relevant for our discussion
[18] We will discuss only those that are relevant for the current discussion.
[19] Establishes the framework for the implementation of the community policies on economic and social cohesion in Portugal.

In order to accomplish the proposed targets, the NES 2020 had to be divided into 5 different axes, each dedicated to a set of priorities in different domains and its corresponding measures to tackle them. Again, for objectivity purposes we will only focus on axes 1 and 2, as follows:

- Axis 1 – NES 2020 is an agenda for competitiveness, growth and both financial and energetic independence invigorating different economic sectors, creating value and employment through the investment in innovative projects in the fields of energetic efficiency, renewable energies – including decentralized production – and mobility. The agenda promotes market competitiveness through the consolidation of the Iberian Electricity Market (MIBEL).
- Axis 2 – NES 2020 focuses on renewable energies, promoting the development of industrial lines that induce economic growth and employment, enabling the achievement of the national renewable energy production targets that intensify the diversification of renewable sources on the country's energy mix. This promotion will allow Portugal to become less dependent on imports, as well as the enhancement of security of supply.

Axis 1 aims to reinforce the country's objective of belonging at the forefront of the renewable sector, along with major economies through the rest of Europe by betting in mature technologies, as well as encouraging new and innovative projects in energetic efficiency through the investment in smart grids, etc. Moreover, one of the key pillars of this axis is to allow the creation of a decentralized production system in a way that promotes the opportunity of development in regions that are still to be seized.

The materialization of an internal market in Europe has granted an increase in competition, making the liberalization of these markets a key factor to improve the economy's competitiveness along with a cost reduction in energy. This liberalization has in fact benefited from structuring initiatives throughout the value chain of the electrical sector that contributed to the viability and stimulation of the required market dynamics. However, at this stage it is relevant to state that axis 1 aims to foster the energy markets competitiveness by progressively phasing out the regulated tariffs, i.e. gradually end the regulated market transitioning to a dynamic one where costumers are allowed to chose their suppliers and negotiate the price to pay. However, phasing out regulated tariffs does not imply that costumers will be fully free of all tariffs, since it is mandatory to expand our production capacity and reinforce the interconnection between the Iberian Peninsula and France in order to gain access to the rest of the European markets, hence there will always be a remaining tariff component.

Axis 2 on the other hand comes to tackle the priority of increasing the share of renewable energies in the Portuguese energy mix by setting individual objectives for each endogenous source, as well as measures to implement them. This can be interpreted as the national vision for this sector, which is to broaden and diversify the portfolio using mature technologies that are able to actively and more immediately make a significant contribution to the electrical production system, and also to encourage research and development of technologies and projects that are undergoing the testing phases but prove to a certain degree the potential to contribute with value to the national economy.

In what concerns the feed-in tariffs as an incentive, as Couture and Gangon (2010) defined they can be seen as the most effective measure at stimulating the rapid development of renewable energy sources as long as they are well adapted. In fact, the way this tariff is designed is a great contributor for decentralizing electricity production, for it enables multiple "investors to participate, including homeowners, landowners, farmers, municipalities and small business owners, while helping to stimulate rapid renewable energy deployment in a wide variety of different technology classes." This rapid market growth is also explained with the risk reduction when investing in renewable energy technologies due to the structure of the FIT that bases the payment levels on the cost required to develop the projects and ensures those payments for the life time of the technology, hence generating a securitization of future cash flows that can be particularly valuable when financing "capital-intensive projects with high upfront costs…"

However, to guarantee the efficiency of the payments under the FIT policy it is essential to have an adequate and specific design for the remuneration in order to achieve a stability that maintains investors' confidence. In Portugal, the remuneration system was based on setting the FIT above the market price, accompanied by a purchase guarantee, i.e. all power produced is dispatched to the grid ((Decreto-Lei n.º 189/88, 1988) updated in (Decreto-Lei n.º 225/2007, 2007)). This pricing model would vary according to the technologies being used, and considered avoided costs (regarding traditional power plants) along with differentiated costs, such as operating costs, reduction of CO_2 emissions, etc.

The Iberian Electricity Market (MIBEL) also came as an incentive and arose from the idea of creating an internal energy market throughout Europe. Both the Portuguese and Spanish administrations agreed – via a protocol, signed in 14th November of 2001 – to create this market in order to take a step in that direction. Due on January 1st, the starting of the MIBEL should guarantee all established agents, in both countries, access to the Iberian Market Operator and to the interconnections with third parties (other countries), based on equality and freedom of bilateral contracting conditions (Modelo de Organização do Mercado Ibérico de Electricidade, 2002). Moreover, this principle guided market aims to allow all consumers to benefit from an efficient competition between active agents, along with the required transparency that would foster the credibility of this system while objectively promoting the use of renewable energies in electricity production.

When both countries agreed to develop the market, a study was carried forecasting a continuously high growth in the demand for electricity, meaning that high investments were required in order to respond positively with an increase in production capacity. At the same time, agents involved in designing the market were concerned with the high degree of subsidized investment in production capacity from renewable sources along with the need to consider and adjust the investment recovery for those operating in an ordinary regime (all non-renewable sources) in order to guarantee that the effective recovery on the transitioning costs to a free market would not distort the competition between producers nor the price of electricity.

In order to deal with those issues, the entities responsible for the development of the market (CNE[20] and ERSE[21]) designed a list of priorities that are now presented:

[20] Comissión Nacional de Energia (National Energy Commission – SP)
[21] Entidade Reguladora do Sector Energético (Regulatory Entity of the Electricity Sector – PT)

- Equal opportunity for organized market and bilateral bargaining: the system cannot indulge one form of negotiating over the other, hence the need to create an intermediary system between the *spot*[22] and bilateral[23] markets – an organized forward[24] market.
- To guarantee MIBEL's transparency and the organized market liquidity: due to the high degree of centralization and vertical integration, it is very important that the MIBEL behaves transparently and to assure the required liquidity for the organized market, in order to maintain prices stability. Since a gradual transition by costumers from the regulated market to the free market is expected, the liquidity will be assured by those who remain in the regulated tariff regime.
- All consumers must be granted with equal opportunities.
- Idle costs should not distort the market: the amount and the payment mechanism to producers should be transparent in order to guarantee market's correct operation.

With the priorities defined, the market was developed and fully activated (on all its dimensions) on July 1st of 2007. However, at this stage it is important to deal with the way the market treats the production in special regime, especially because this segment of the market has undergone several changes (adaptations) since it was created.

In the past, until 2012 the last resort supplier had the obligation to purchase all the energy produced by the PRE (special regime production) and its corresponding prices were established with the previously explained feed-in tariff. I.e. the last resort supplier would place its bids on the MIBEL taking into account all the energy already acquired to the special regime production. This way, this type of production would not appear explicitly in the market, but had an influence on the price as it would weight on the volume of the purchase offer.

However, according to a study carried out by PLMJ Sociedade de Advogados (2014) (a major law firm in Portugal) such incentives towards renewable energy were put a limit despite all progress achieved by then regarding the 2020 targets, due to the global financial crisis that shook the world in 2009 leaving Portugal on the edge of bankruptcy. With the arrival of the Troika[25] in 2011, the energy sector suffered a major drawback when several restrictive measures were imposed, especially on renewable technologies. Their subsidies dependence, were considered "most expensive" hence leading to its growth being revised down, result of the impact caused by the rising deficit[26] on the sector's tariff system.

One of the touchstones on the agreement between the government and the Troika was in fact the efficiency in promoting renewable energies. This so called *memorandum of understanding* provided

[22] The spot market establishes schemes of sale (production) and purchase of electricity for the day after the trading, using both daily contracts and intraday adjustments. See daily market (Mercado Diário).
[23] In the bilateral market, agents negotiate the purchase and sale of electricity for various time horizons. See (Contratação)
[24] A futures market, which establishes future commitments for the production and purchase of electricity. This market may carry out physical settlement (energy delivery) or financial settlement (clearing of the monetary values underlying the trading). See the futures' market (Mercado a Prazo).
[25] To prevent an insolvency situation Portugal applied for a bail-out program from the International Monetary Fund, the European Commission and the European Central Bank – also known as Troika.
[26] The consistent investment in state-of-the-art technology over the years, and not charging consumers with its costs, generated a cumulative tariff debt.

for a reevaluation of the support schemes for electricity produced from renewable sources, to be achieved by reducing feed-in tariffs on on-going and future contracts. Readjusting tariff parameters to balance incentives in the context of markets' technicalities would promote its underlying efficiency by ensuring that producers would limit their project costs towards cost-benefit equilibrium.

3.2. The Renewable Energy Sector today

Although the country had great gains by 2011 in meeting its 2020 targets, 2009 crisis drove the electricity market to a scenario of excess of supply, as a result of a reduction in demand. In fact, many companies went bankrupt and had to end their activities leaving factories and other facilities abandoned. Adding to that we had an increase in unemployment and consequent significant decrease in household's budgets which also contributed to the apparent excess of supply.

Accordingly the strategic reconfiguration needed to be "based on reaching an appropriate level of national generation capacity by applying a logic of economic rationality and freedom of initiative of the promoters, without depending on subsidies, guaranteed remuneration and risk mitigation", i.e. that of a free market. The corresponding amendments on the previously discussed document – NREAP – could be simply translated into: any future requirement of additional power for any given installation (which would most certainly occur) would have its subsidies and incentives limited, prioritizing technologies at entry to the system.

For that reason, the Decree Law 215-B/2012 came to define a remuneration system for electricity produced from renewable sources capable of working under market logic, through either organized markets, or bilateral contracts, unlike what had happened until then, i.e. special regimes would cease to distinguish itself from the other in a way that "special regime production will be contemplated under the market remuneration regime." However, in order to assist on the materialization of this system, a "market facilitator" figure had to be created to account for the full acquisition of the electricity produced through renewable sources, in order to help mitigating the inherent risk. At the same time, the guaranteed remuneration regime was to remain for those that had been previously approved until the end of the contract, which according to Decree Law 33-A/2005 would be enough to assure full investment return as well as a small profit. This latter DL also stated that, at the end of the guaranteed remuneration, all entities (except hydro power plants) were to be paid based on the electricity sold on the market, plus the incentives of green certificates – tradable commodities proving that electricity is being generated using endogenous resources. Moreover it also declared that if at the end of the contract any installation was not able to generate green certificates, then it could apply for an extension of 5 years of the guaranteed remunerating regime (unspecified at the time). All those remunerating systems regarding renewable sources were defined and approved by Decree Law 35/2013, 28th February.

Given that feed-in tariffs represented the safest system of income for producers (as means of a guarantee), as they were faced with its removal many questioned about their business' resilience to withstand such a change. In fact, later this year (2017) at the annual conference on renewable energy, EDP *Renováveis* joined Professor Sá da Costa[27] in alerting to the fact that recent changes in policies regarding remunerating systems would slow Portugal in the race to the forefront of renewable energies in Europe. The reason for this statement lies behind the deadline imposed on feed-in tariffs across the sector. In fact, it is estimated that by 2023 no more than half of installed wind power plants will continue to benefit from this incentive. The issue at stake is that, at the same time the tariff goes null, most of the plants will reach the end of its useful life, diminishing its production capacity and increasing costs of operation and maintenance. Producers believe that the solution for this problem relies in repowering the stations through equipment update to more efficient and powerful technologies. However, this operation is extremely costly in terms of investment and also because there is no legal framework to contemplate this issue and, to mitigate risk investors might have to think twice before putting up their money. This has come to prove that policy makers are failing to see the benefits of increasing renewable energy' share in the energy mix as means to decrease electricity prices to consumers as Professor Sá da Costa stated in an interview later this year by showing that by means of marginal costs solar energy (e.g.) is accounted for more or less 40€/MWh while conventional technologies such as Natural Gas is 60-65€/MWh.

Despite divergences in domestic grounds, external entities have recognized Portuguese efforts over the last years. As it can be read in the latest publication by the International Energy Agency (2016), "despite the difficult economic climate, Portugal has continued to develop and reform its energy policy with undoubted benefits (…) [which] include greater economic activity in the energy sector, rapid increases in renewable energy deployment, further market liberalization in the electricity sector…" Moreover, the benefits of renewable energy penetration in the electric system include less dependence on importing fossil fuels, as well as declining carbon emissions. In fact, if we analyze data provided by APREN, as presented in the Figure 5 [28] one can understand this importance.

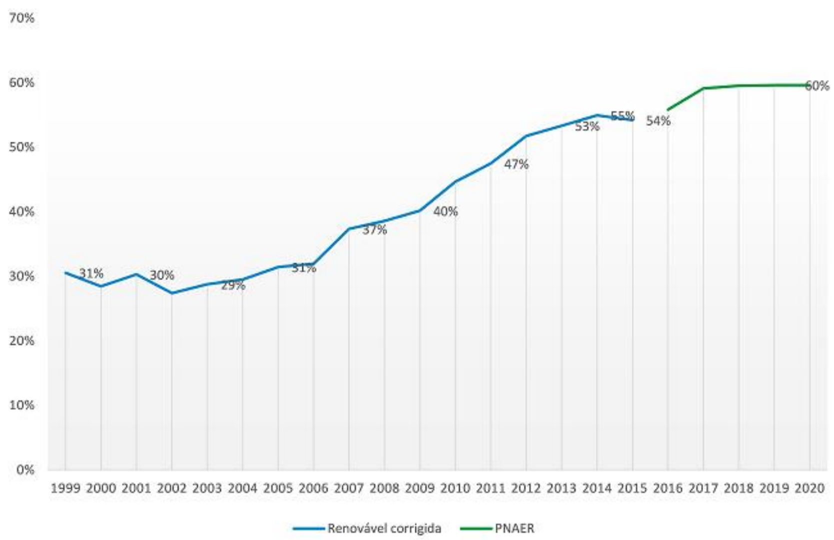


Figure 5 - Evolution of Renewable Energy penetration in Portugal (%) | Source: (APREN)

[27] Chairman of APREN (*Associação Portuguesa das Energias Renováveis*)

[28] Takes into account the cumulative contribution of renewable energies in the sectors: Heating and Cooling, Transport and Electricity.

In fact, since 2013 more than half of electricity production became full dependent on renewable sources, that translates in terms of avoided costs in 1.479M€ for both natural gas and coal (APREN).

Over the last ten years, the weight of renewable energies in electricity production has been rising to levels that are closing in on the targets for 2020. There is still clearly a long way to go, and a lot of room to evolve as well as space for new players to step in. However, the contributions that this sector has already given to this country are noticeable and it is in our interest to keep it as a priority in our economy, especially if we are to consider one of the main benefits for the promoting of renewable energy investment, i.e. costs avoided with fossil fuels, which according to the International Energy Agency, are among the highest in Portugal (**Erro! A origem da referência não foi encontrada.**).

Table 2 – Prices in $/Unit | Adapted from: International Energy Agency (2016)

Country	Fuel Oil (tone)	Automotive Diesel (litre)	Unleaded Premium (litre)	Natural Gas (MWh GCV[29])	Coal (tonne)
Portugal	513,72	1,064	1,459	44,1	109,2
Spain	249,71	0,861	1,215	33,23	...
Belgium	197,66	0,999	1,404	28,29	...
Denmark	414,56	0,962	1,481	...[30]	...
France	300,68	0,948	1,388	40,5	...
Germany	180,15	0,912	1,36	34,12	...
Greece	287,05	0,887	1,463	36,92	...
Poland	320,29	0,783	1,017	31,84	70,09

The countries presented were chosen somehow randomly[31] from a list of OECD members with the purpose of proving that fossil fuel prices in Portugal are in fact leading as the most expensive. Although we have objectively achieved satisfactory levels in becoming less dependent on fossil fuels, electricity prices are still very high due to imports of expensive raw materials – mainly Natural Gas and Coal. In fact, concerning the electricity prices among those countries Portugal has one of the most expensive with the average costs[32] for both electricity in industry and for households being 138,4€/MWh and 235€/MWh, respectively. Note that this occurs in a country where average income *per capita* is among the worst in European countries; Germany where electricity prices are also among the highest, has a minimum wage of 1.440€ (averaged monthly value) while Portugal's is 618,33€.

This is why we need to be creative in finding solutions in order to increase the share of renewable energies, without depending on a rising tariff deficit, and move closer to meet our targets for 2020. It is important to maintain the country at the forefront of this target (see Figure 6) in order to increase its presence in the European internal energetic market and use it as a means to recover from financial distress.

[29] Gross Calorific Value
[30] … data unavailable
[31] Note that the list of countries consists of Spain, which is geographically meaningful; Belgium, Denmark, France and Germany which are typically chosen as benchmarks; Poland and Greece which for being similar economies to ours are frequently used as terms of comparison.
[32] Data from (PORDATA)refereeing to the year 2016.

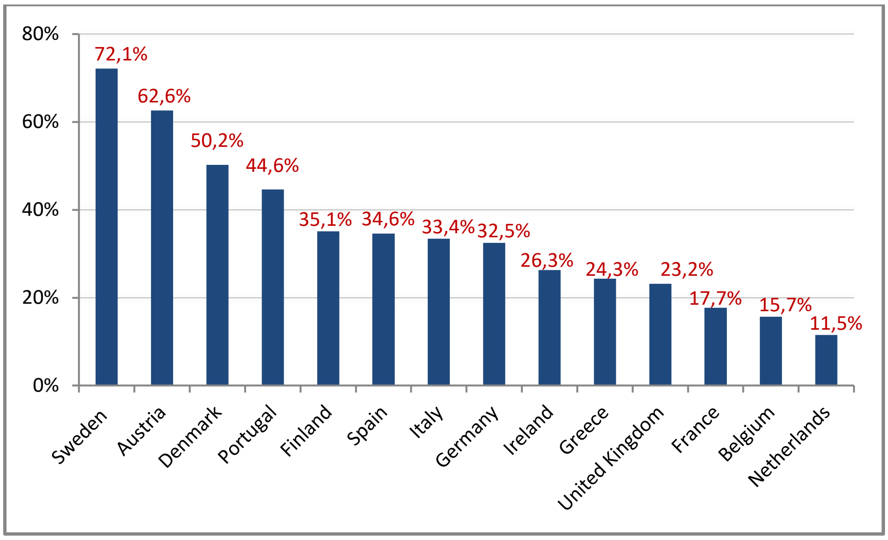


Figure 6 - RES share (%) in Electricity Production[33] | Source: Direção Geral de Energia e Geologia (2016)

Obviously there is still a long way to go before Portugal gets to accomplish its goals until 2020, which in our opinion represent a good standoff point for business opportunities, especially if we consider the contributions this sector has had towards development in our country. In fact, considering the study carried out by Deloitte alongside with APREN, in 2013 renewable energies recorded an impact on GDP of 2.730M€ whereas they contributed with 37.916 indirect jobs and 2.811 direct jobs.

Portuguese development could in fact be based on a sustainable society as explained in the beginning, and renewable energies can offer a good opportunity for this.

3.3. Electricity Market – Key features

According to Castro (2012) the initiation of the market liberalization process dates back to 1995 when the first important steps were taken aiming to reshape the (monopolized) market by establishing the coexistence of both a public service and an independent electrical system. It was in this latter one that the first principles and orientation of market-place arguments were implemented. However, it is only through the Cabinet Resolution no. 169/2005 that the basic principles of organization and operation based on the promotion of the competition in the energetic market were established. This was later on implemented through the Decree Law (DL) no. 29/2006 and further developed via DL 172/2006 and DL 264/2007.

Under this new regime, all agents involved trade electricity with a monetary value defined by the point where supply meets demand. With market deregulation and under fair market principles, both supply and commercialization become activities that are open to competition amongst all players. However due to technical motives all activities related to the network were to remain as a regulated (natural) monopoly designed especially to achieve maximum efficiency. Given that, in order to the market properly work under this liberalization strategy a power exchange had to be created (as seen previously); a place where all trading takes place. Hence, the formation of the electricity price under

[33] Takes into account Import/Export balance and hydro power whilst in pumping.

these trading activities is subject to the well functioning of the organized market where the marginal price of energy is established, as Gil (2010) argues.

Since it is in this briefly described market that all trading takes place, it becomes impossible for us to adequately separate the electricity being generated from renewable sources from the others. We will have to focus our efforts on the market as a whole, but always bearing in mind that the product we are trying to develop focuses on the energy generated and sold by those sources. In fact, this is not a wrongful line of thinking since these technologies are always dispatched with priority on the same market, as we explained in the previous chapter, and the price is always defined by the marginal cost of production taking into consideration all technologies in the energetic mix. The challenge remains in agreeing on the quantity of electricity that could be made available for our product as we will see further on.

Since our beliefs are similar to those on Geman & Roncoroni (2006), making use of the authors' words we do find it important that in the wake of deregulation of power markets "a proper representation of the dynamics of spot prices becomes a necessary tool for trading purposes…" which is what we are trying to accomplish, i.e. an adequate model for the spot price of electricity. To cope with that one needs to bear in mind that some fundamental properties of this commodity need to be well captured and represented. However, before digging into those properties there is an important observation that needs to be made regarding the capacity to store electricity. From common knowledge we say that unlike any other commodity, electricity cannot be stored, i.e. apart from pumping technologies[34] used in (and on that technology itself) hydropower plants, there is no other form of industrial storage that can be used to address the concept of convenience yield[35]. This concept, as we will see further on, is highly important to price derivatives on commodities when using standard methods that were initially designed for instruments on financial equity. Moreover, as Thornton (2010)suggests "non-storability can cause temporary over- and under-supply…" and consequently influence downward or upward price movements and for this reason it is important to correctly incorporate this phenomenon in pricing models.

Regarding the key features that one needs to address when modeling electricity spot prices, we can first point out the mean reverting trend that most markets exhibit. This means that there exists a mean (probably deterministic) level to which prices fall after an extreme event, and this level can be described by the average marginal cost of production as Burger *et al.* (2006) and Geman & Roncoroni (2006) propose. As in Geman & Roncoroni (2006) we will present this feature by mean *reverting* the process to a deterministic periodical trend driven by seasonal effects. In fact, prices do exhibit variations under a more or less predictable pattern and to understand that, one just needs to think of the difference in demand during summer days and winter.

The second property (and from our understanding dependent on the first) that needs to be well captured is the occurrence of small random movements of the price process around the trend which

[34] Note that, as Burger, *et al* (2006) suggests pumping technologies can result in a loss of approximately 30% energy

[35] The concept was introduced for agricultural commodities to represent the benefit from holding the commodity as opposed to a forward contract.

come as a consequence of temporary supply/demand imbalances, as Geman & Roncoroni (2006) proposes. These are “locally unpredictable and may be represented by a white-noise term affecting daily price variations” as we will prove further on.

Lastly (but not less important) there is a feature that is, in our opinion, intimately related to the (lack of) capacity of generating inventories when referring to electricity production and also to the inelastic nature of demand for this commodity. We are evidently referring to the presence of the so called price spikes, which are price movements that can either be up- or downward directed.

If in some other commodities a shortage can be smoothed out by resorting to the buffering effect of inventories, in electricity as we stated before one is not able to do so. However, from our understanding shortages are inadmissible in our market which is why the free market created between Portugal and Spain is so important (among many other reasons), i.e. if there is a fall in production in either countries, they can resort to importation from the other. Imagine the scenario where Portugal per example has a weather event causing the production from the cheapest technologies not to be enough to satisfy demand internally; assume also that “peaking[36]” technologies have also already been deployed but are still not enough. Before deploying (e.g.) a coal power station – which is very costly – if Spain can complement production with a least expensive technology, then the electricity will be imported hence lowering prices a little, i.e. the price will most likely be higher than the average trend, but at least will not be as high as it would if the Portuguese most expensive technology was deployed. We believe that due to this fact, positive spikes will not occur as often as a downward ones which are more likely to be found during high precipitation months, as we will see further on.

However, before moving on it matters to state that many authors agree that whichever event occurs, it will be followed by another on the opposite direction to restore market equilibrium. Let us admit that during the weekend in Portugal high precipitation and windy events take place which, due to the high installed capacity in these technologies, forces electricity prices to go down. These elements combined with lower demand (remember that working days have higher demand than weekends) can bring prices close to zero. After these events occur, there is a natural market force that brings the prices back to “normal” values. To correctly capture this effect can pose as a delicate task as we will see. To hold that previous statement and in order to present what we are aiming to capture in Chapter 8 Figure 7shows the evolution of electricity spot price since 2009 (baseload).

[36] Some technologies are known as “peakers” due to their remarkable flexibility that allows plants being switched on without increasing prices too much. Technologies such as Combined Cycle Gas Turbines (CCGTs) are smaller and cheaper.

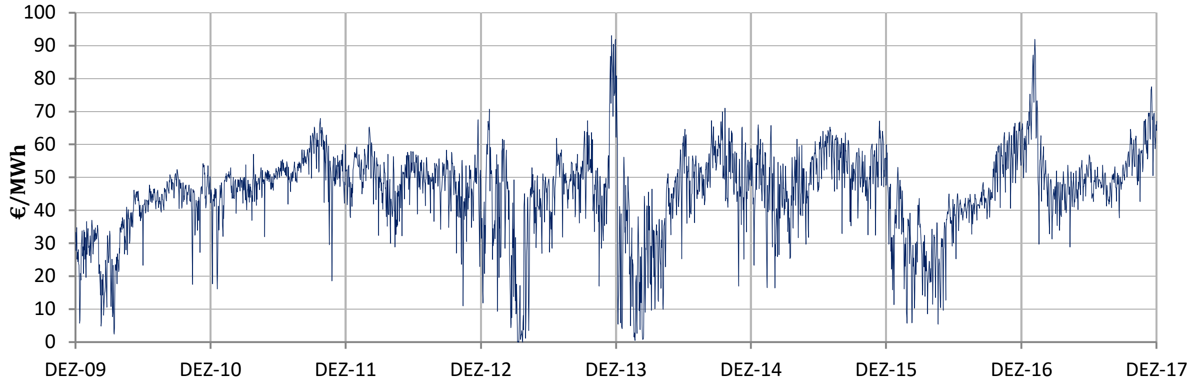


Figure 7 – Electricity spot price history.

At a first glance we can clearly observe that downward directed spikes occur more often than upward, which as we stated before was somehow predictable.

According to *Boletim Climatológico Anual* (2014) 2014 was an exceptional year in what concerns weather variables. In fact it was registered a deviation of nearly +25% (1971-2000) on precipitation which, due to high dependence of hydroelectricity, translated into a price decrease of different magnitudes, i.e. February per example registered precipitation levels of 100mm higher than homolog periods in other years which translated into price decreases on the gross market. Conversely the prices for January 2017 peaked to levels only seen nearly 4 years before. There were several factors behind this event with demand being the most obvious one. Although in what concerns temperatures (Instituto Português do Mar e da Atmosfera, 2014) considered this to be a normal month, average temperatures reached 8,26ºC (0.55ºC below average) which, as usual leads the population to seek for heating equipment (air conditioning, etc). However, this year was not a normal year in what refers to precipitation, i.e. only 53% of regular values were reached, hence considering this as a *very dry* year. Again, due to the increase in demand – according to REN it was registered an increase of 4.6% – and the lack of hydro capacity electricity prices were expected to climb. However, these two factors could not alone be responsible for price increases as large as those observed. What lies behind this is that at the same time that these events took place in Portugal, France had also turned to the Iberian market to purchase electricity due to the fact that 20 out of their 58 nuclear power plants were stopped for either maintenance works or inspections. Alongside with this the prices for coal and natural gas were also peaking due to high demand across all Europe.

Despite the differences between both examples given, it is obvious that the mean reverting feature of electricity still holds in our market. In fact, as Hafner (2003) argues the higher the deviation from the deterministic trend the stronger will be the force that drives prices back to this trend. Nevertheless, if we consider the possibility of spike occurrence to be somehow also deterministic in the sense that these events tend to cluster around some defined period, this statement can be subject to some form of questioning. This is what is going to pose as a major task when we present our modeling results in chapter 8.

4. Making personal finance a part of renewable energy funding

The uncertainty regarding the future of renewable energy's incentives has created an urge to design new innovative mechanisms to develop these technologies without depending on public funds that will certainly increase the tariff deficit. With constant changes in policies that are destabilizing the sector, due to economic uncertainty, regarding ongoing guaranteed remunerations to producers it is imperative to create new solutions in order to avoid falling back in the race towards our international commitments, as well as reinforcing our position in this competitive market. Countries like Germany also saw their governments reduce renewable energy incentives, thus slowing down the markets. Even though they had achieved significant development in the sector, with great impact nationwide, as Yildiz (2014) suggests. Despite this drawback common knowledge realized that it was important to proceed with investment if they were to tackle both "energy transition and achieve the postulated goals".

The situation in Portugal is similar if we want to achieve a RES share of 15.800 MW in installed capacity by 2020 (International Energy Agency, 2016). Currently the status of our energy mix is as follows:

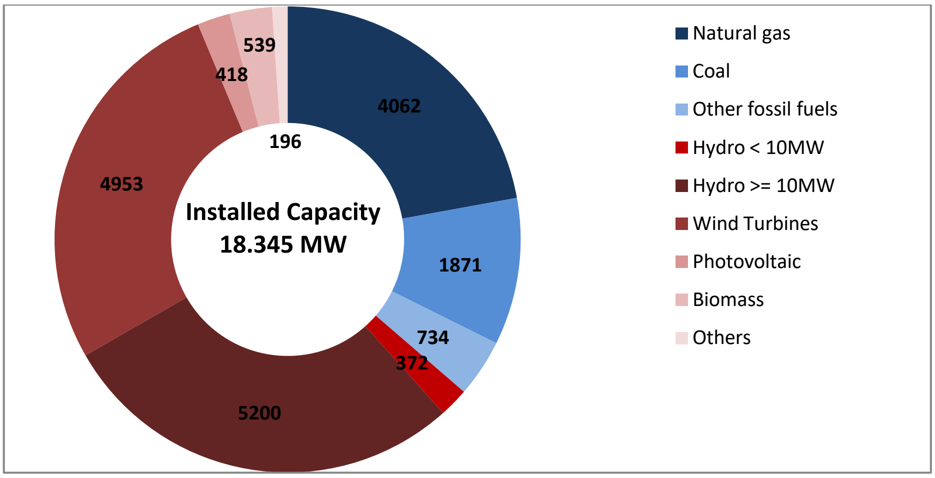


Figure 8 - Energy mix by share in Portugal | Source: Direção Geral de Energia e Geologia (2016)

By July 2017, the same report suggested an installed capacity of 13.657 MW for renewable energies. Hydro power showed the highest growth rate, because it includes an increase in reversible capacity (ability to pump the water from down to upstream, and re-use it when needed the most) which enables the creation of synergies between hydro and wind, in order to optimize local resources and ensure more effective management of the electricity generation system.

Despite indicators proving an expected growth, they also prove that the rate that it does is decreasing due to lack of new investment. Innumerous reasons for this investment hampering (besides those that we already described) are behind the deceleration of this sector, which by itself carries a great obstacle with the requirements for high investments. Moreover, as stated by Yildiz (2014), one must also consider that other financial restraints take place. High transaction costs across the entire developing cycle (for instance, it costs more to a financial institution to evaluate the credit-worthiness of several wind farms, than one fossil fueled power plant with equivalent installed capacity) and under investment problems. These latter restraints arise from institutional constraints like asset specificity[37] and lock-in effects, typical from investments in long term capital assets particularly in scenarios where changing the system carries high switching costs.

Incorporating citizens (as retail investors) in the business that this sector represents is the main focus of this project as means to recover investment levels in renewable sources. However it must also be in our agenda to address the issue proposed (that of creating a structured financial product) taking into account that financial innovation and overconfidence about the risk of new financial products may pose as a liability to market efficiency, hence the introduction on the U.S. credit crisis which we considered interesting from a financial engineering point of view.

Yet for simplicity we will carry on our analysis based on the assumption that there are no restrictions in trading on the spot price market, i.e. we take it as a possibility to disregard the time frame on which this market typically operates. We agreed to accept this assumption based on the fact that this hypothetical market is easier to understand while enabling us to avoid words like “futures” or “forwards” which sometimes can be ambiguous and susceptible to misinterpretations and consequently confusing to any “uneducated” investor.

[37] The more specific an asset, the lower its potential resale value or redeploy ability.

5. Electricity Derivatives on Foreign Markets

According to the definition of a spot market provided earlier under the deregulation basis, they can be organized in two forms. First we can point out the *Pool and Single buyer* where the system operator (acting as the single buyer) is responsible for collecting all the bids placed from the producers. The way the operator than organizes those bids is known as the power stack function in which technologies are ranked according to a merit order from the least expensive one to the most. For illustrative purposes consider Figure 9.

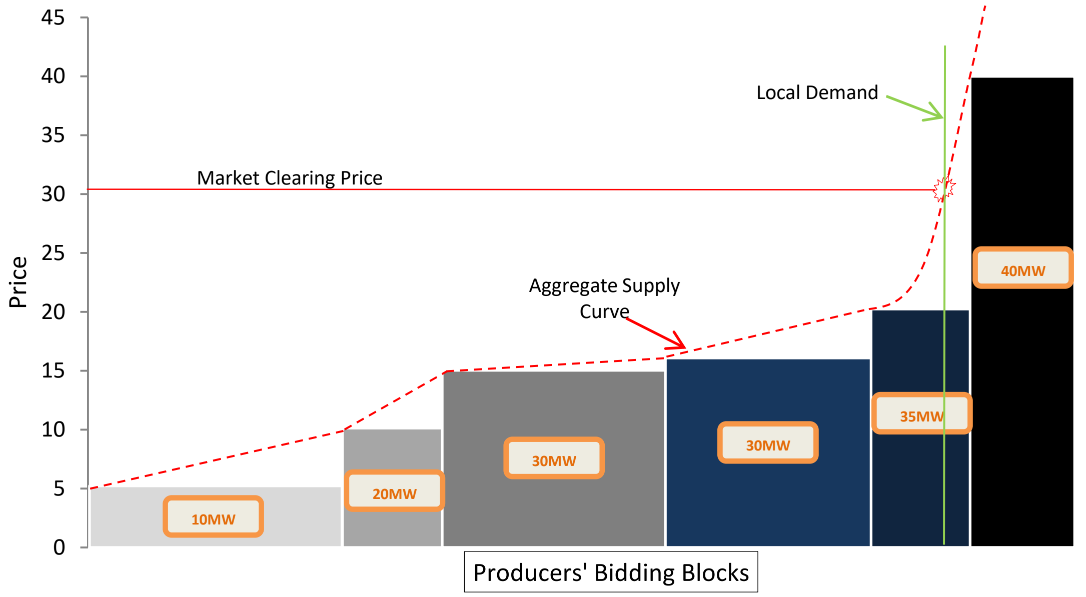


Figure 9 – Simplified model of the power stack function | Adapted from (EDP Power trade)

Although the figure provided is merely illustrative we can point out that at some point prices start to rise exponentially as Geman (2005) noted. That is due to the fact that the last technologies to be deployed are the least efficient ones. Given that, the market operator has two choices when considering the market design. Either the suppliers are the only actors making their bids to the market – and in which case the operator is responsible for computing the expected demand for every hour of the following day – or both buyers and suppliers place their bids and, in this case, the operator builds the demand and supply functions, although the demand function is quasi-vertical due to the inelasticity to price changes. Either way, both designs have the system marginal price defined where demand meets supply.

The second structure for the spot power market is known as a *power exchange* as Geman (2005) states, where the wholesale trading takes place by market participants. As the author suggests trading takes place through bilateral contracts, although it is not mandatory. However when they occur, bidders must place their offers and quantities the day before delivery in order to provide the remaining participants the necessary time to handle the physical aspects of delivery. Note that this always has to include the Transmission System Operator (TSO), but this does not mean that the contracts are made public. In fact, the main purpose of such contracts is profit optimization by “identifying counterparties to directly trade power within the best conditions”. Once the spot price trading on the exchange reaches a satisfactory liquidity, matching supply and demand leads to a public market-clearing price. These hourly prices are the ones that lead to an exchange index, on which derivatives are written.

5.1. Financial Products' Development – Acceptance and Growth

No matter how the market is organized, the number one priority in all of them is to always satisfy demand. That is why we want to focus our studies on markets that are known for their efficiency as we do believe that there is still a long way for our (Portuguese) market to evolve and a lot to learn from those markets.

5.1.1. Nord Pool

Given that, we evidently have to write about the Nord Pool, as for some authors, (e.g. Hafner (2003) and Villaplana (2003)) the exemplary way this market operates since its creation around 1991[38] is worthy of focusing efforts to study as it can be seen as a "role model" for many other markets, otherwise it would not have 380 companies operating and trading until today and trading 505TWh in one year[39]. Note that traders operating in this market are able to rely on a certain degree of transparency which can be one of the reasons why the market has proven to be worthy of its success – as an example of such transparency in October 2010 management made a public announcement reporting on suspicious activities by one of its tradders[40]. Moreover the Nord Pool covers several markets, particularly a financial market where no physical delivery takes place and all traded products serve for hedging, speculating and risk management purposes, namely what was formerly known as Eloption and Eltermin as Geman (2005) presents. The financial market is nowadays known as NASDAQ OMX Commodities Europe.

However, it matters to state that one of the reasons why this market is so successful, and in our opinion a vital one, is that the largest producer[41] operating "only" has a 22% share of the electricity supplied (according to the company's website) which facilitates and grants the entrance to new participants and financial traders as price manipulation is not easy – note that this is not the case in Portugal as the largest operating company detains an astonishing share of 45,3% of total production in deregulated market.

Regarding the financial market (the one we are interested in) one can find several products across the platform. Such products include base and peak load Futures, Monthly Average Rate Futures, Deferred Settlement (DS) Futures, Monthly DS Futures, Options and Electricity Price Area Differentials. Although options do exist, we could point out that we were not able to find any data regarding liquidity worth mentioning on those contracts. On the other hand an incredible amount of 3217.96 GWh (for

[38] Under a different name at first and operating solely on Norwegian soil, it was only in 1993 that the Nord Pool was given a cross-border trading license. The Nordic market became fully integrated in the year 2000 when Denmark joined in.

[39] 2016

[40] "No. 54/2010 NPS - Nord Pool Spot AS issues written warning for insider trading and breach of disclosure requirements" (Nord Pool Group, 2010)

[41] Vattenfall

different maturities) was traded on futures on March the 1st 2018 alone[42]. Such trading volume could be explained by the existence of financial products that were particularly designed to fit the cash flow profile of electricity producers and consumers, as the Deferred Settlement Futures (see Bjønnes & Saakvitne (2015, pp. 12-13)) where the "key difference between [these] and ordinary futures contract is that market participants can post bank guarantees[43] as variation margin for their DS futures in the trade period." This is only made possible due to a particular feature of the Nord pool market that is, the existence of a clearing entity – Nasdaq Clearing. In other words, after a bilateral contract is agreed between two parties the clearing house takes over the deal and provides "clearing" for whichever contract, i.e. all derivatives on commodities. This agent acting as a contractual counterparty offers the netting positions, enabling members to use their capital in the most efficient manner.

The following picture (as extracted from the original report) depicts the growth of futures' contracts after the introduction of those – through the words of the authors – "tailor-made" instruments.

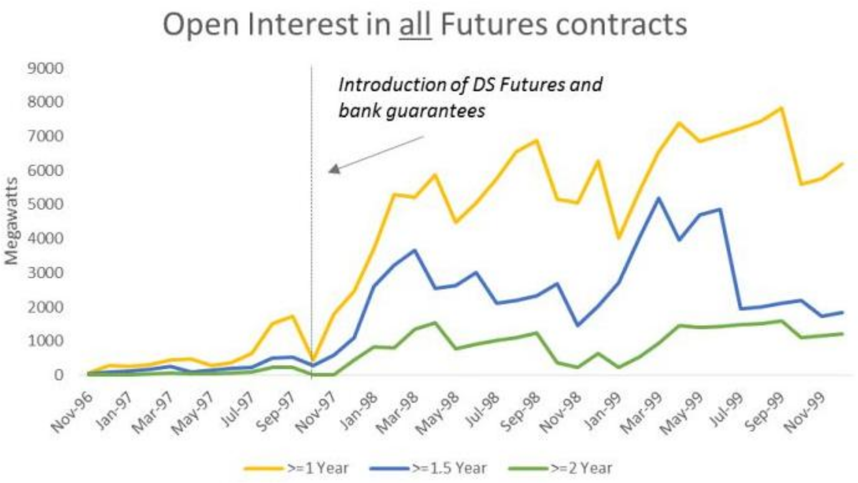


Figure 10 – The impact of introducing Deferred Settlement Futures | Source Bjønnes & Saakvitne (2015)

Note that this report focuses on investigating if the use of bank guarantees as collateral poses any systemic risk for the market as we can recall from previous chapters they might resemble the financial products used during the financial crisis in 2008 – the particular case of the CDS market. This is, in our point of view, another feature provided by the market that makes it trustworthy and justifies the liquidity i.e. not only it possesses interesting investment products but also enforces transparency by keeping track of all trading activities and assuming the responsibility of clearance.

5.1.2. EEX – a member of the EEX Group

Now at this point it seems only fair to us to mention the European Energy Exchange, for this market also provides several tools for our domestic market to learn and evolve. The way it operates resembles the previous case, as it operates in 30 countries worldwide and offers several investment solutions and easy market access. Focusing on the Power Derivatives Market segment, the company provides financial products that range from futures, options and spreads. Moreover entering through

[42] Data only for the Nordic market.
[43] Collateral is mandatory for all trades.

this market allows the investor to have access to the most liquid financial power future contract in Europe – the Phelix-DE Future[44].

However what caught up or attention in this market is the recent addition of a financial product that is somehow relatable to what we propose on our essay – EEX Wind Power Futures. As the Managing Director Exchange, Tim Greenwood said "the Energiewende [German for energy transition] poses new risks, and requires new instruments to hedge these. EEX is shaping the power markets of the future with new and innovative products like the Wind Power Future". Introducing this derivative to the market allows for a growing number of companies too exposed to risk, from their wind park portfolios, to hedge their positions while at the same time addressing the steadily growing price impact for conventional power revenues due to "increasing power generation from renewable, in combination with the uncertainty of weather…"

One of the features that make this product so interesting is the fact that in order to cope with transparency it is traded in a regulated fashion, i.e. it is an "Exchange Traded Derivative" (ETD) which unlike traditional OTC products it offers standardization and elimination of default risk. Standardization in the sense that the terms and specifications are standard for each derivative contract making it easy to understand for any investor – note that this makes it well suited for retail investors. The elimination of counterparty risk or default risk, on the other hand comes from the fact that all contracts are cleared via CCP (Central Clearing Counterparty House). Moreover the ETD (in this case EEX) itself acts as counterparty for every derivative trade, i.e. it acts as a buyer for every seller and the seller for every buyer eliminating this way the risk that either side of the transaction will default on its obligations.

5.2. A Working Example – Wind Power Futures

Following the brief introduction on a derivative that has been recently introduced on the EEX and given the resemblances to what we propose, we have decided to investigate it a little further in order to understand which features we can adapt to that of our own. At the same time we wish to present in detail each of the quotes that appear in contracts of this nature in order to make it easily understandable for a potential retail investor.

Table 3 – Contract specifications for illustrative purposes only | Adapted from the communicating tools from EEX

Contract Specifications	
Underlying	Average Wind Load Factor per contract period
Load Profile	Baseload
Regions	Germany/Austria
Maturities	Weeks, Months, Quarters, Years
Quotation	1% = 1€/wph
Tick Size	0.01
Contract Volume	-
Trading	Exchange trading and trade registration
Fulfillment	Cascading of quarters, seasons and years; Cash Settlement against the index

[44] On March the 1st as an example an option contracts ("Apr/18") written on "PhelixDE Baseload Month Futures" registered a volume of exchange of 7.200 contracts; Phelix stands for Physical Electricity Index.

Starting from the top we can first point out that the underlying in this case is based on the Load Factor representing wind power generation normalized do MWh per available MW over the delivery period, i.e. the average power produced divided by the rated power. This leads us directly to the Quotation as it is presented in a percentage of injected electricity by means of wind turbines. Data is continuously monitored and provided by (EuroWind) relying on a model-based calculation of wind power generation in 15 minute resolution supported by meteorological data by official institutions. Each percentage point of the daily expected wind generation normalized to available capacity represents the price differential of one euro per wind production hour (wph) of injected power, hence producing a reference price for wind power production.

Right after "quotation" we see the "Tick Size" which is the smallest movement the future prices (or any other financial instrument for that matter) are allowed to make in either direction. Typical for European markets, tick sizes can change according to the price level, which is our case as the tick is represented as a two decimal points variation of the quotation. In other words, assume that a contract is issued for a power injection of 30%, hence making its size worth 30€. A tick size of .01 corresponds to a minimum fluctuation of 0.3€ which means that for every 0.01 movement in the quantity injected, the price moves 0.3€ in the corresponding direction (since it is in percentage it means that in order to change by 0.3€ there needs to be an injection variation of 1%). Note that if the asset is trading at (e.g.) 30.38€, the broker only quotes the corresponding minimum allowed 30.3€. As we said, each future contract has different variables such as size, quantity, valuation etc., which is why the tick size varies along with them.

Regarding the contract volume (which we could not find) we can state that it denotes the delivery rate of electricity. Consider this rate per contract is 1 MW during each delivery hour of the delivery period. Using Geman (2005) example for a baseload monthly future for September, this amounts to: $24h \times 30days \times 1MW = 720MWh$

This finally leads us to the Fulfillment as it is directly related to the previous example when considering the delivery period. That is the market allows us to write contracts for as long as a year, however the way they are settled is through a process that is known as Cascading. This process means that the long-term contract is replaced by the next shortest period of validity on the last trading day. For better understanding consider the following Figure 11:

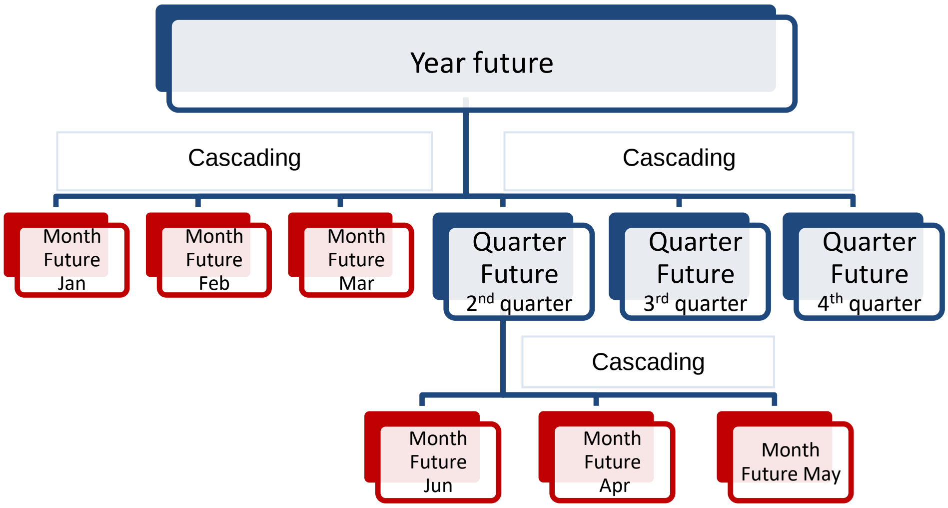


Figure 11 – Contract Fulfillment | Adapted from (EEX) - Phelix Power Futures Presentation Data ©

From here we can see that (under this example) on the last trading day before the delivery of a year contract, this contract is divided into the month contracts for January, February and March, and three other Quarter futures for the remaining second, third and fourth quarters – Note that all the contracts must sum together in volume equal to the year contract. Two days before or on the last day prior to the beginning of the second quarter contract it is once again divided into the equivalent months it represents and so forth.

When the trader finally enters a contract of this nature he/she is faced with a table that might look something like what we present in Appendix A (where the reader can also find an example of the Settlement prices' path over a 1 month period).

6. Creating Financial Products

Investors consistently and rationally choose to invest in any sector (finance, industry, energy, manufacturing, etc) by weighing the levels of risk and possible opportunities of return, which will then be picked if they are adequate to the investor's profile. Renewable energies, in the energy sector, tend to be at a disadvantage when compared to traditional energy due to environmental externalities, as shown in Wüstenhagen & Menichetti (2011). Those externalities include visual, acoustic and odor nuisance (considering biofuels) which tend to be more perceptible for the general population as in Möllendorff & Welsh (2015). This does not mean that externalities on fossil fuels can be disregarded, however when we are considering the well being of citizens, renewable energies can have greater impact (e.g. consider the noise a wind farm can generate to surrounding populations). Externalities on fossil fuels are not so perceptible due to the fact that most power plants are allocated to industrial areas, hence the notion of e.g. pollution can be sometimes forgotten.

Once a given sector is identified and its performance is understood, one can determine an investment vehicle that benefits investors without violating his/hers constraints, (such as liquidity, time horizon, tax concerns etc) Lee & Zhong (2014) suggest a top down approach (Figure 12).

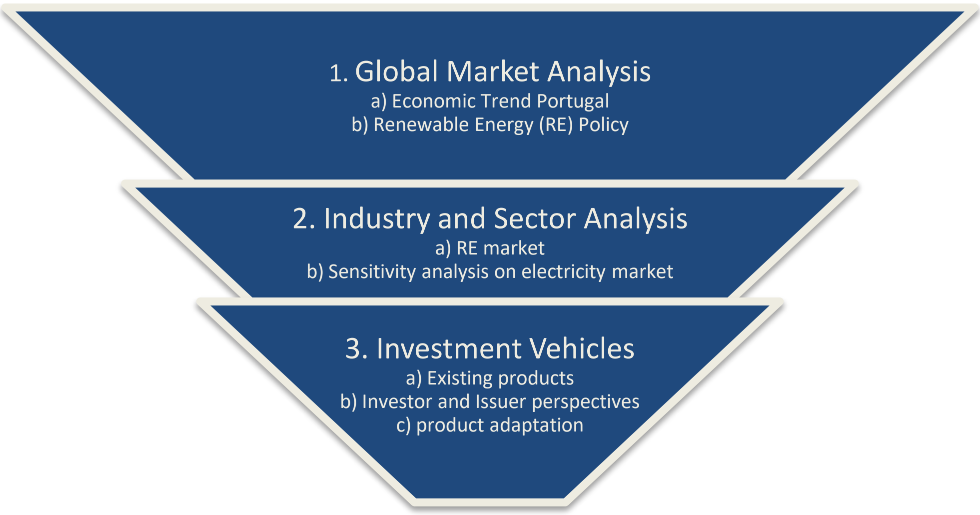


Figure 12 - Top-Down approach | Adapted from Lee & Zhong (2014)

Investment vehicles are of numerous types, including:

- Cash and cash equivalents
- Fixed interest securities
- Stocks
- Commodities
- Commercial or residential real estate
- Insurance products
- Derivatives
- Foreign currency
- Private equity

In order to be able to chose among such variety one must dedicate his efforts to an analysis both quantitative and qualitative of all variables of the sector, aiming to pinpoint the investment strategy that best represents the interest of investors. Qualitative analysis is important in a way that allows the gathering of "understanding of human behavior and the reasons that govern such behavior", while a quantitative approach provides the necessary assessment of certain phenomena "via statistical, mathematical or computational techniques." as stated in Lee and Zhong (2014, p. 763).

6.1. Addressing Complexity

The subject at stake here is no different from what has been made in other sectors, which is why in order to create a certain financial product a top down approach was partially conducted, as Lee and Zhong (2014) suggested. First, a global market analysis is to take place – which we already addressed when briefly describing the current Portuguese economic status – followed by another one specifically directed to the renewable energy sector, which we have also detailed previously. To complete this analysis the following subsection (6.2) is dedicated to the financial contract and its legal aspects.

Global market analysis should first consider economic indicators that can be used to assess the economy's health, which reflects the various sectors. Such indicators can include GDP (growth, expenditure, etc) – it provides a general picture of how active the private market and government are - average consumer prices, population, etc. The study should then focus on renewable energy policies as "governments enact fiscal and monetary policies that have direct or indirect, and positive or negative effects on the profitability" Lee and Zhong (2014, p. 765). Although this latter suggestion has already been studied in this paper, further detailed **quantitative** analysis is suggested as means of current renewable energy conditions - intuitively, the larger the gap between current renewable energy share and the target is, the larger the room for investment opportunities - and capital flow trends as they provide with the ability to trace investment flows and to gauge trends in attitudes and perceptions of investors towards renewable energies.

The renewable energy sector should then be analyzed in terms of predicting the evolution of the free market, specifically the prices for electricity from these sources as well as its injection on the grid. The impossibility of tracking injected power should not stop us from conducting an analysis on the underlying, which is why we try to capture the first four statistical moments of the electricity spot price on Chapter 8.

As mentioned previously the final analysis on the top down approach concerns the existing financial tools. Valuating known investment vehicles allows a comparison among them to make it possible to choose the most valuable.

For the purpose of this paper, we will focus our efforts on what has to be done in order to create a product that will be eligible among such investment vehicles providing the tools for the reader to compare solutions. However, due to the very nature of the products at stake here, we must find a meeting point where both costumers (notice that we are focusing on retail investors, i.e. homeowners) and issuers can be satisfied. Such nature of structured products, often times leads to a degree of complexity from issuers that alienates common investors due to lack of knowledge. Those that remain willing to invest in such complex products usually are unaware of the true payoff formula that they are going to be subjected. Good marketing teams behind those products allure costumers using sentences like “super yield” based on promises of yield rates of e.g. 8%. Moreover, as Célérier and Vallée, (2015) argued the complexity and multidimensionality involved in these contracts is to make it more difficult for retail investors to understand and compare with other products. Consider the following example taken from Célérier and Vallée, (2015) :

> “This is a growth product linked to a basket composed of the FTSE Euro First 80, the FTSE 100, the SMI and the NIKKEI 225. The Annual Performance is set at 5% for the first three years. In the following years, if the performance since the start date of the worst-performing index is positive or null, then the Annual Performance for that year is registered at 5%, otherwise 0%. The Basket Performance since the start date is registered every six months. The Final Basket Performance is calculated as the average of all these six-monthly readings, capped at a maximum basket performance of 100%. After 8 years, the product offers a guaranteed capital return of 100%, plus the greater of either the sum of the Annual Performances, or 100% of the Final Basket performance.”

Such complex products are created because their flexibility in terms of pay-off formulas allows banks to engineer patterns that are potentially attractive to unsophisticated investors. Although this example has less information than the original contract probably has, it is not uncommon to find out there on the market products with pay-off formulas that can lead to situations like the one described. Note that we are not saying it is entirely wrong, however the issue here is that these products are typically sold to unsophisticated investors who do not realize how the markets react to certain adversities and what will it imply to their invested money. Note also that we are not implying that our (proposed) product will not be prone to such losses however it is important for us to remain transparent while exposing all possible outcomes.

Given that, it is important, when creating a product that focuses on renewable energies and in order to make it eligible to any investor, to make it easy to understand its risks and the way it works in a way that any outcome will not come as a surprise. This means that to create such product, the market needs to be well detailed and understood to forecast the best way possible its behavior, making it somehow predictable. It is particularly important to do so in renewable energies sources because its reputation cannot be jeopardized.

6.2. How to create a structured financial product

Now, in order to be able to create that product, one must understand the dynamics involved, which is why we will now focus on describing the four dimensions that define a structured product: the underlying asset, payoff formula, maturity and format as in Célérier & Vallée, (2015). Briefly explaining, a structured product is actually a type of risk-return profile that derives its value through a composition of two or more financial instruments with at least one of which being a derivative, Pinto (2013), which is why it is fair to say that they indeed have a very personalized structure varying from institutions, targets, assets, etc. The expected return will depend directly on the performance of the underlying asset, which in turn may vary with different situations, depending on its type. Equity is the most common underlying asset class, which means that the products will rely on the performance of a single stock, single index, basket of shares or even a basket of indices. Other asset classes can be found, such as interest rates or commodities. Hence, the design of a structured product is seen taking into account an analysis of a set of factors, resulting such design in the combination of the requirements and objectives for both investors and issuers and also the expected future evolution of the market. Investors' objectives may vary from protecting their capital, obtain better returns – otherwise unachievable from traditional products – gain access/exposure to a particular market, enjoy a periodic income, hedge and/or diversify investment portfolios, "noting that risk can be reduced by combining different assets", Wüstenhagen & Menichetti (2011, p. 4). This can provide an advantage for the issue at stake regarding renewable energies, as there is a systematic difference between the risk-return ratio of one single investment and that of a portfolio of investments, i.e. taking advantage of investing in multiple sources for the same utility.

Whichever form of contract the issuer puts up to the market, it will be subject to very strict regulation that aims to protect uniformed investors. As one can read in the regulatory entity's website, financial innovation came with misinformation to clients who unknowingly were investing in products they did not understand and were caught by surprise when they saw negative values in their positions. In fact, in 2009, 2.417 complaints were addressed to the Portuguese Securities Market Commission (in Portuguese known as CMVM), from which 98% targeted financial intermediaries that were neither properly explaining the nature of the financial products nor pinpointing potential losses.

To address this issue the regulatory entity issued the Regulation "CMVM nº1/2009" regarding the information and marketing of such products. This Regulation, along with the Decree Law 211-A/2008 from November 3rd, came to obligate the issuers to communicate in a very detailed way the products at stake, including the creation of the worst case scenario and present it to the investor/client. One example of such mandatory form of communication is presented in Appendix B – example of required information on a structured financial contract. Although it comes in Portuguese, the summary of the risk level of such product is easily identifiable which for that case represents the risk of total investment losses.

Concerning the issuers' interests, an assessment on commissions, transaction costs and ease to cover positions needs to be made and thus choose the best financial instruments to incorporate in the structures. Also the volatility and market trends need to be assessed as they will also provide essential tools for designing the product and for future effective profitability. Only then can the product be established in all its dimensions, as previously described.

6.2.1. Main Features

Typically a structured product consists of two main components that may vary with a lot of other considerations. However they can be distinguished as a part that establishes a capital guarantee or minimum profitability and the other that works with the variable payment. The first component (that of the capital guarantee) can be seen as an "insurance" on the money being invested, depending on the client's risk profile, or as a means to minimize eventual losses with the variable part. It can guarantee the investor's full or partially recover of the initial capital being invested. Traditionally this is achieved by applying a part of the initial capital in a fixed rent asset, as a zero coupon bond, or treasury bonds that at maturity date of the product will recover the pre established amount. The second component is trickier because there are a lot of different strategies that can be used. However, most of them rely on the use of **options** as argued in Pinto (2013). In other words what generates the variable return component is an option portfolio that is enhanced in periods of high volatility, as we will see later on. This is what is going to be the main feature in the payoff formula, allowing the investor to participate in a market (regarding the chosen underlying asset) that would otherwise be nearly impossible due to several constraints. We will focus on those that are financially settled, as there are options in which the buyer may wish to take physical delivery of the underlying, as shown in Geman (2005).

Among the option type contracts we can distinguish those known as "plain vanilla", that are contracts under the form of either European or American styles. When the underlying asset or the strike price (among many other features) is considered to be non-standard, Exotic options takes in and there are several instruments that belong to this group, namely Digital options, Barrier options or Asian options, among many others. We will try to further detail some of them later on, as they can prove to be somehow interesting for what we are trying to achieve. But for this moment let us focus on the simpler versions of these contracts, particularly on the example provided in the following chapter that considers the valuation of standard options. To accomplish this allow us to first make a generalization which, from our understanding, is valid for each and every form of contract of this nature. Using Matos (2008), one can argue that the payoff of an option is given by the value of holding that contract until exercise. This exercise will be subject to different conditions, depending on whether we are considering a call option or a put option. If we are to consider a call option, we must state that to exercise such contract at maturity it needs to hold that the difference between the value of the underlying asset (S_t) at maturity ($t = T$) and the strike price (k) is higher than zero, otherwise the payoff will be zero with the reasoning being that no one will exercise an option out of the money which would lead to certain financial losses. In other words, at the moment of a potential exercise the payoff of a call can be written as:

$$max(0, S_t - k) \quad (1)$$

Inversely, considering a put contract at maturity it needs to hold that $S < k$ in order to exercise it, hence the payoff can be written as:

$$max(0, k - S_t) \quad (2)$$

Both these definitions are what is known as the intrinsic value of an option contract, which is nothing more than the cash flow obtained through the immediate exercise. Obviously in the case of a European contract this value is purely theoretical.

As we will see these generalizations will prove to be useful on the following chapter when we start presenting a working example of a structured product that makes use of option contracts to derive part of its value, and further detailed in upcoming chapters when describing extensively the complex nature of these instruments.

The other value we can distinguish on these financial instruments is the time value that is given by the difference between the option premium and its intrinsic value. Since it results from the probability of a favorable move on the underlying spot price, as we reach maturity this value approaches zero – as the probability of the value changing is a decreasing function of time.

Now, options belong to the family of derivatives (derive their value from an underlying asset) in the form of a contract conferring the right (and not obligation, unlike forward and future contracts) to buy/sell a given position on an asset at a fixed price and date. Here is what distinguishes what we previously referred to as European options and American. Unlike what the name suggests the reason is not geographical but related to the exercising style of each of the contracts. Whilst European options can only be exercised **at** maturity, the American equivalent can be exercised **until** maturity during any trading day. This latter feature poses a challenge when trying to produce a closed form solution for the valuation of such option, as we will see.

The basic positions in financial options are presented below focusing on the right underlying the contract:

Table 4 – Option position | Adapted from Nunes (2015)

	Call Option	Put Option
Long Position	Long Call (right to buy)	Long Put (right to sell)
Short Position	Short Call (obligated to sell)	Short Put (obligated to buy)

One can pinpoint that the most common form of contract is the *call* option, in which the holder is entitled to the right of buying an asset (or position) at a given date, through the payment of a premium. I.e. if one believes that a given asset is going to see its value increase, but is not certain of it, by paying a premium to the issuer, when the contract reaches its maturity and the asset's value has in fact increased, then he can buy it through the payment of the *strike* price. This is also the price that an asset price must go above (for calls) or below (for puts) before a position can be exercised for profit. Hence, the breakeven price is given by the sum of both premium and strike. (Figure 13 - Long Call vs Short Call[45])

[45] Note that P&L stands for Profits and Losses;

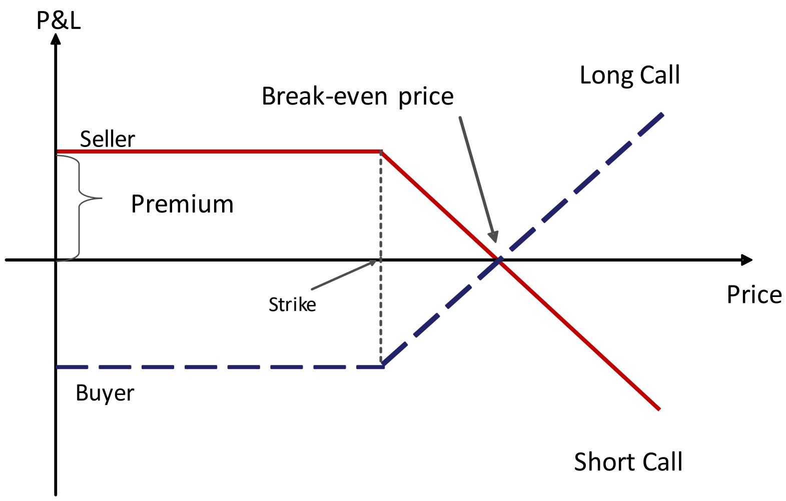


Figure 13 - Long Call vs Short Call

What we stated previously is enforced by the figure provided. Here the party who wishes to acquire the right to buy an asset in the future enters a long call by paying a premium to the short call (seller), hence the initial "profit" for the buyer is negative in an equal amount as the premium paid. The strike price is the price both parties agreed upon to proceed with the transaction at the expiry date, should the long call position decide so. Evidently if the price at maturity is below the strike (out-the-money) he/she will not exercise and his/her losses will stick to the premium paid initially. For this reason between buyer and seller is said to exist a zero sum game or, in other words buyers' gains are sellers' losses. Note that the buyer pays a premium and has unlimited gain potential.

This leads us to presenting the following table with the different classifications one can attribute an option when going through a comparison between the spot price of the underlying asset (S_t) and the strike price (k) of the option contract.

Table 5– Strike price and Current price | Adapted from Nunes (2015)

	In the Money ITM	At the Money ATM	Out the Money OTM
Call	$S_t > k$	$S_t = k$	$S_t < k$
Put	$S_t < k$	$S_t = k$	$S_t > k$

Profitability can take place at one time, at maturity, or as regular coupons over the life of the product, on predetermined dates, as in Pinto (2013) depending on the structure chosen. This way, a typical retail investor is faced with the opportunity to enter a market that would otherwise be unattainable, as he is now entitled to the right of buying/selling a given position on an asset that could be otherwise impossible (consider the price you would have to pay for the installation of a small power plant in order to be able to sell electricity to the grid, e.g. solar panels).

To offer such a structured product, banks or other entities need to define the term of the contract – maturity – as Célérier and Vallée (2015, p. 7) argued when investigating approximately 55.000 products. "The maturity of a structured product is 4,2 years, on average, and ranges from less than one year up to more than 10 years." I.e. it is important to define an adequate time span for the product until it reaches its maturity date, as structured products can be short, medium or long-term types depending on their maturity. However as Pinto (2013) suggests, even though the products have a predetermined maturity date, some can include the ability to end before that date, either by issuer's decision (*callable*) or investor's (*putable*) or even automatically if a given and pre-established situation occurs (*autocallable*).

Concerning the format, structured products can be issued in different ways, such as deposits, bonds, insurance contracts, funds or the combination of one or more of these. At this stage, it is important to define if there is going to be any form of guaranteed capital (as mentioned, in the form of e.g. zero coupon bonds) – the so called capital protection. Some products can be structured in a way to guarantee the total amount of invested capital, while others only a part or even none of it. However, this capital protection (when there is any) can only be assured if the product is kept until its expiring date. Only in this way will the issuer be able to define how much of the total capital will be available for investing in derivatives, increasing both return (potential) and risk. Also we should note that when there is some form of capital guarantee, what it means is that the bank is taking some of the risk away from the investor and retaining it for itself. This obviously has to mean that the payoff of the variable component of the product will be subject to some form of cap or, in other words, only if the underlying asset exceeds a given performance threshold should the investor be paid, otherwise the bank will retain the profits fairly for itself. This is also true to protect the financial institution from losing the premium paid for a given position, but this we will discuss later on.

The following figure provides a simplification of what was explained:

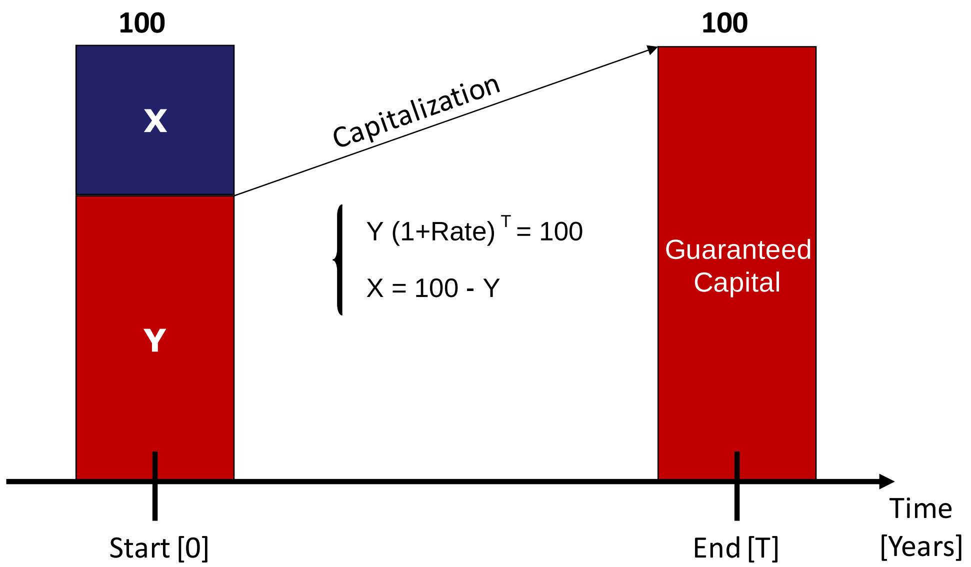


Figure 14 - Capital Protection

What is depicted in this figure can be easily explained. Let us assume that the investor initially puts up 1000€ for investing, but the minimum that he accepts at maturity is to fully recover this capital. Luckily for him/her the bank offers a product with a three year maturity at a 3% rate that guarantees redemption of the face value. This means that at time "Start [0]" the banks sets aside Y = 915,14€ in (e.g.) treasury bonds that offers a 3% interest rate, leaving a remainder of X = 84,86€ that will be available for investing during those three years using the variable component of the product. Further on we will extend this example to a more detailed scenario for better understanding.

On a side note, it is important to state here that whichever form of contract in our case it has to have access to a secondary market that we are targeting, for we must not forget that being electricity a commodity it is virtually impossible to trade small amounts and not all structured products' formats allow doing so. In other words, this is important since the electricity market most likely does not deal "small amounts" of electricity, i.e. one would only be able to buy e.g. a future if the quantity was enough for the producer to consider it. This is where the markets we described in previous chapters (case of Nord Pool and EEX) take place and where all transactions occur and with a price. This statement along with what we previously explained regarding the risk taking from the banking institution as a whole is what we will use to justify the commissions paid, as logically some form of compensation has to take place for the risk taker.

Concerning the payoff mechanism, there is no specific formula to address this issue, as there is no structured product equal to another. Also, because these products depend directly on the performance of a given underlying asset, the payoff will most certainly be of a variable nature. However, given the chosen structure for the product one can predict, based on incorporated instruments and concerning both investors and issuers' requirements, as well as market tendencies, the potential gains and losses. And this is what the regulatory entity demands financial institutions to present to their clients. Through the presentation of admissible scenarios for the performance of the underlying clients will know what to expect.

Thus the payoff formula will depend on the price of the underlying asset at the beginning and at maturity, and in some cases it can even consider its performance at specific pre-defined stages during the life of the product. However, despite the fact that anticipating the payoff of a given structured product (the variable part) can be difficult, it is possible to estimate/evaluate option positions that are to be used as we shall see.

6.3. Structured Product – a standard contract

At this stage, before going any further we should walk through a simple form of contract of a financial structured product, following the reasoning stated previously. For that we will use a working contract but try and make it simpler just for the sake of pinpointing the important aspects of such products. We will not yet go into detail on how the variable return of the product works, since for that we will dedicate an entire chapter to investigate our underlying and consequently how would a financial product perform under such market.

First let us consider a synthetic index that accurately tracks the amount of electricity being injected in the grid from renewable sources. We shall name it EL-index (randomly named after the short form EL for electricity) and this will be our base for the underlying. For the sake of simplicity let us also assume that each percentage point injected represents 1€, hence if 50% of power in a given hour came from renewable sources, that represents a valuation of 50€ per production unit hour – say r-kwh as in "renewable-kilowatt hour".

The reason why we decided to "create" such an index lies behind the fact that since 2010 the governments have been trying to create a system that differentiates renewable sources from conventional technologies. This is precisely what our product needs in order to make sense. Although there is still no working mechanism to address this issue we believe that sooner than later a system involving “certificates” will be fully operational, allowing the market to differentiate between those two sources. As the State Secretary for Energy assured in an interview to a known newspaper (see Silva (2018)) 2019 was the target defined to the creation of “origin certificates” that copes with this differentiation.

To support what we have been explaining so far, consider the following example which was adapted from a working contract (from a very well known Portuguese bank, Banco BPI) and merged together with an example provided in Nunes (2015). Notice however that this example was created to represent an extremely simple version of a product in order to ease understanding, for from what we know much more information is usually contained in these contracts; also their remunerating systems tend to be more complex than the one we are presenting. Moreover it is usually mandatory that these contracts contain a worst case scenario explaining clients what can happen if the market does not perform the way they expect. For example, in the original contract in this case, it was presented to customers that if all stocks from the basket did not grow altogether, then they would only gain the minimum guaranteed.

We will then proceed with some simple calculations to explain how the product will perform under a manipulated market.

Product description	Bonds with guaranteed capital recovery with variable return annually distributed and indexed to the performance of the EL-index that tracks the amount of electricity injected from renewable sources. Coupon rate (minimum guaranteed return) = 1% Variable return to be paid at the maturity date = x% of the growth rate of the El-index until maturity
Remunerating system	Concerning the annual interest payment date: **If** the **index grows** during the time until maturity: **X**% of the growth rate to be paid at maturity. Otherwise, Gross annual income = 1% x face value Index growth: seen from the difference between start value and annual value. **Start Value**: the spot value on the closing market for the index on the start of the contract. **Annual Value**: the closing value of the market for the pre-defined observation date. **Observation Date**: every 8th of November. Should any date miss a trading day, than it shall be replaced by the immediately subsequent trading day. **Payment Date:** every 20th of November. Should any date miss a trading day, then it shall be replaced by the immediately subsequent trading day.

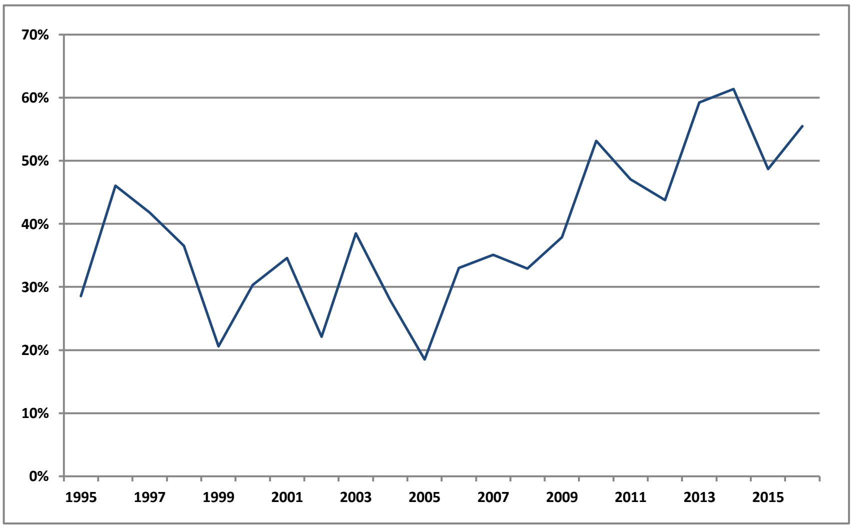
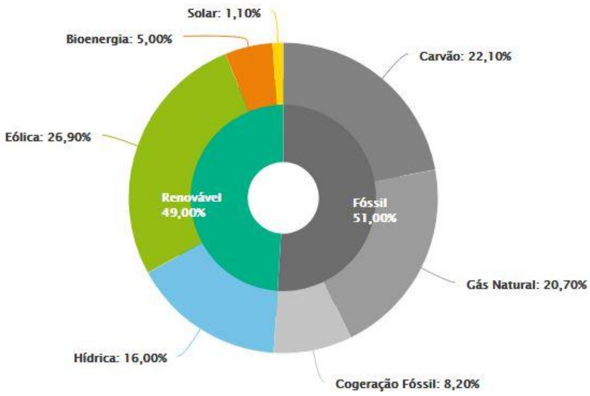
Historical Data (results in the past are in no way future outcome guarantees)	Evolution of renewable injection compared to total production throughout the years.  **Figure 15 – Renewable energy penetration as a percentage of total production | Adapted from PORDATA (2018)** Injection by source for the month of February 2018.  **Figure 16 – distribution of electricity injection by sources | Source (APREN)**
Issue Price	100% of the face value
Costs	Here should follow the information regarding the commissions paid to the bank for access to the secondary market. Note that all transactions have different costs throughout different banking institutions.
Face Value	1.000,00€
Maturity	3 years
Contract Multiplier (size)	1
Tick	$Size = 0{,}5$ $Value = 0.5\% = 0.5€$

Figure 17 – Simplified version of a possible structured product contract | Adapted (Banco BPI)

So what we are now interested in is to understand what kind of static hedging strategy the bank would use with our money, as a potential client. However we should first mention that we left the variable X% blank on purpose due to the fact that banks also have to deal with the dimensioning of that variable in order to build the hedging strategy they have to present to the client to fulfill what they promise. Evidently in such contracts this would already be calculated but to show both the perspectives we opted to do it this way.

On a side note, we have chosen to keep the tick size at 0,5 assuring that the minimum asset movement allowed for our product is equivalent to 0,5%. We did so to keep the example as simple as possible.

Let us start with the easier part (using the same reasoning as the previous example) but this time taking into consideration that the product not only assures a bullet guaranteed redemption of the face value but also a 1% coupon. Note that bullet means that all potential returns will all be paid at maturity.

Also, consider the following valuation of different positions in European style options presented below. The values were obtained using the Black-Scholes formula with an estimated volatility of 30% (reasonable for electricity prices). This process will be further described in 8.1.

Table 6 – Call and Put prices calculated under the Black-Scholes-Merton assumptions.

Strikes (%)	29,5	30,0	30,5	39,0
Call Premium	7,07	6,86	6,65	3,96
Put Premium	4,9	5,16	5,43	10,75

The interest rate we used for both the valuation of the structured product and for pricing the options is 1,939% which corresponds to the latest known treasury bonds issued by the government; moreover, due to the nature of our simulated contract it made sense for us to use such rate.

Finally we will also assume that on the initial trading day of the contract the spot value of the index is set at 30 points (30% renewable power injected).

We will then investigate how the product performs under different scenarios for the market at maturity.

- **Setting the Hedging Strategy**

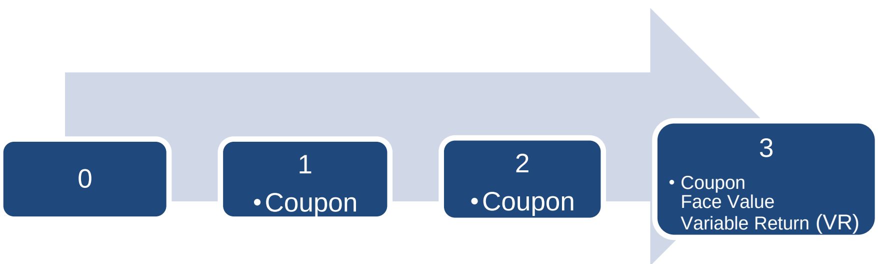


Figure 18 – Structured product's payoff yearly distributed.

$$\frac{1\%}{(1+1{,}939\%)}+\frac{1\%}{(1+1{,}939\%)^2}+\frac{100\%+1\%}{(1+1{,}939\%)^3}$$

$$= \mathbf{97{,}289}\%$$

At the beginning of the contract 97,289% of the amount invested is immediately deposited, which for us represents 972,89€.

Hence for the variable return we are left (at $t = 0$) with **2,711%** (or 27,11€) to invest. This is the basis for calculating the percentage of the index growth (if there is any) that the bank will pay at maturity. Translating this into mathematical reasoning it reads as:

$$VR_3 = \max\left(0\% \,; \frac{S_3 - S_0}{S_0}\right) \times X\% = \frac{X\%}{S_0} \max\left(0 \,; S_3 - S_0\right)$$

where $\max(0\,; S_3 - S_0)$ component is nothing more than the terminal value of a European style option[46]. Through some manipulation (that we will leave aside for now) we are faced with the following result:

$$\max(0\,; S_3 - S_0) = c_0(S_0; k = S_0; T = 3)$$

which is the value of an At-the-money call option at date t=0 that expires in 3 years. This means that the present value of the variable return is given by:

$$VR_0 = \frac{X\%}{S_0} \times c_0(S_0; k = S_0; T = 3)$$

$$\Leftrightarrow 2{,}711\% = \frac{X\%}{30{,}0} \times 6{,}86€ \Leftrightarrow \boldsymbol{X}\% = \mathbf{11{,}856}\%$$

That is, **if the index grows** this is the percentage of that growth that the issuer will offer in the contract to the bondholders. Note that the inverse calculation is the proof that the issuance of this product is worth 100% (97,289%+2,711%) of its face value, which is the limiting value for any investor to consider the contract. In other words, the issue price is equal to the face value of the bond.

Finally this leads us to:

$$\#Calls = \frac{1000€ \times 2{,}711\%}{6{,}86€}$$

That makes the **hedging strategy**:

1. Deposit 972,89€
2. Buy 3,95 (given by $\#Calls$)[47] standard European ATM Calls with maturity in 3 years.

[46] If we remember $max(0, S_t - k)$ as the payoff of a call option from the previous chapter, making S_t the value at maturity t=3 and k equal to the spot price, S_0 at t=0 (which represents an ATM call option) it holds that we are facing the terminal value of such call option.

[47] The financial institution operates in the secondary market and compensates investors proportionally to their equivalent position.

- **Performance under different scenarios**

Table 7 – Variable Return subject to Growth.

Index	20,00	31,5	35,00
Growth	-33,3%	+5,00%	16,67%
VR_3(%)	0	0,5928%	1,9764%
VR_3(€)	0	5,928€	19,76€
Payoff From the long ATM calls	-27,097€	5,925€	19,75€

In the first scenario the EL-index decreased due to some unforeseen climatic event which translated into a production decrease from renewable sources. The bank that initially paid the premium lost that money.

The second and third scenario, unlike the first recorded a significant growth probably due to good weather conditions and an increase in installed capacity throughout the years. In the second case a 5% increase represented an extra profit for the client of 0,5928% of the initially invested money, which corresponds to 5,928€ that is approximately the same amount as the profit from the derivatives' market in which the issuer invested. By exercising the position at maturity the bank will retain the remaining profits.

Notice that although the payoff in the first scenario is negative the client will not take any losses, since the contract guarantees the full redemption of the face value at maturity. However the bank will incur in that loss, since the institution paid the premium for the equivalent of 3,95 call options at the beginning of the contract. Obviously this is also the main reason that lies behind the fact that the issuer needs to create a compensation scheme – the hedging strategy – to protect itself from potential losses. This compensation comes in the gains from the options market when the index grows, by retaining to itself the difference between the gross income and what it has to pay to the client.

However, we need to remember from Figure 13 - Long Call vs Short Call that despite the fact that the index grew there might still be losses. Notice that:

1. Premium paid = 6,86€ (per option contract), hence the balance at t=0 is negative.
2. The premium paid gives us the right to buy the asset at 30,00€ at maturity.
3. At maturity we buy the asset for 30,00 and sell it for 31,5€ generating a profit of 1,5€.
4. However we still have a negative balance from the premium paid of 5,36€ per contract.

In our case this translates into Figure 19 – Profit and Loss chart for a Long Call:

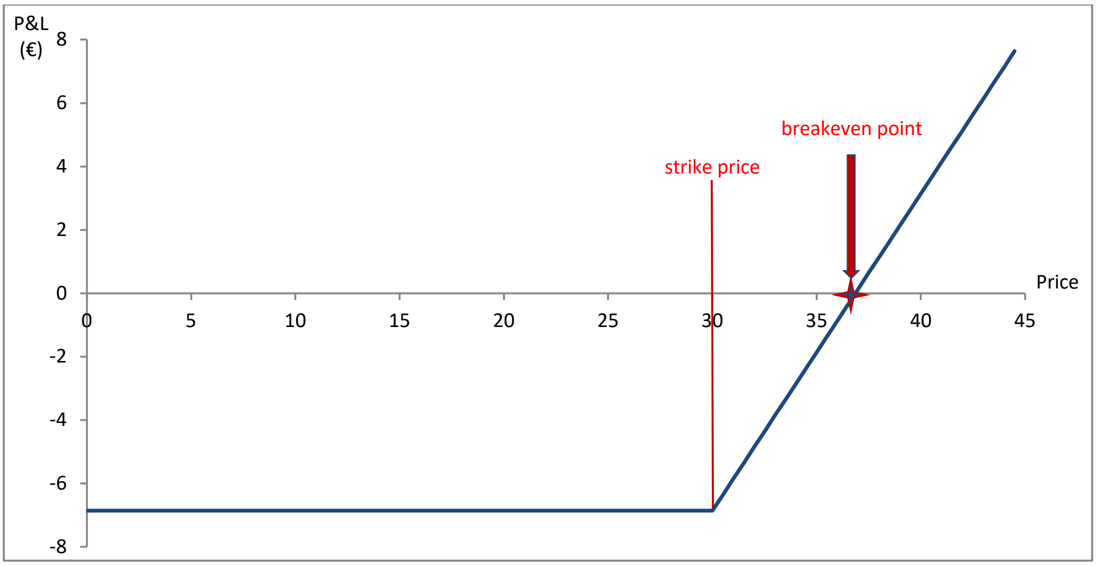


Figure 19 – Profit and Loss chart for a Long Call

This brings us to another strategy that financial institutions usually employ to address this issue. That is the application of a growth lower bound below which the client must not be paid.

From our example we notice that only above the breakeven point will the product start to profit. This means that unless 36,86% of injected power comes from renewable sources, the variable return of the product will not be profitable. However, as stated in the contract the minimum movement for the asset was placed in 0,5% which means that it has to reach maturity above 37% of injected renewable power which translates into:

Table 8 – Index growth cap

Index	**37,00**
Growth	**23,33%**

Since losses cannot be predicted what the financial institution does is setting an upper limit on the index growth. It might sound strange but in fact what it does is limiting the payments to the client if the index grows above a certain level and retaining those profits. Remember that we argued in past chapters that the risk takers should be compensated.

Let us admit a cap on growth of 30% (equivalent to a power injection of 39%), meaning that it is the maximum admissible growth rate on which the client is paid. In that case, we have to investigate if the product is still fairly priced and worthy of investment. Maintaining all other things equal it reads as:

$$VR_3 = 11{,}856\% \begin{cases} 30\% & if\ S_3 > 1{,}3S_0 \\ \dfrac{S_3 - S_0}{S_0} & if\ S_0 < S_3 < 1{,}3S_0 \\ 0 & if\ S_0 > S_3 \end{cases}$$

$$\Leftrightarrow VR_3 = \frac{11{,}856\%}{S_0} \begin{cases} 0{,}3S_0 & if\ S_3 > 1{,}3S_0 \\ S_3 - S_0 & if\ S_0 < S_3 < 1{,}3S_0 \\ 0 & if\ S_0 > S_3 \end{cases}$$

The previous result can be translated into a discounted position in two call options (one long ATM and one short OTM):

$$VR_0 = \frac{11,856\%}{S_0} \times [c_0(S_0\,; k = S_0\,; T = 3y) - c_0(S_0\,; k = 1,3S_0\,; T = 3y)]$$

$\Leftrightarrow VR_0 = \frac{11,856\%}{30,00} \times$ (6,86-3,96)

$\Leftrightarrow VR_0 = 1,146\%$

This implies that the issue price of such product

$B = 97,289\% + 1,146\% = 98,435\% < 100\% \rightarrow$ The client should **not** buy this structured product. The only way for it to be profitable for the client would imply that the bank could offer at least 28,045% of the index growth.

What is interesting from the last part of the example is that through financial engineering it is possible to reduce losses by creating different positions based on what we know from the market. The solution provided previously makes sense due to the fact that electricity markets tend to have high levels of volatility implied by the non storability characteristics of this sector. These levels tend to reflect on the prices of option contracts, which is why in our case buying an At-the-Money call option with a three year maturity is so expensive. However, let us leave the pricing of options issue for later on and focus now on the strategy that was applied, disregarding the fact that it was not interesting for the client.

Notice that the portfolio built consists in the following instruments:

1. 1 Long ATM call, T=3y
2. 1 Short OTM call, T=3y

What this means is that the financial institution buys a Call option with a strike price equal to the current spot price of the underlying asset, at the same time that it sells a call option with a strike price higher than the current underlying asset spot price. From the first transaction the issuer is asked to pay a premium of 6,86€ per contract, whilst in the second it receives an amount equal to 3,96€. This strategy is known as a “Bullish Vertical Spread”.

Going back to our P&L plot this can be explained as follows:

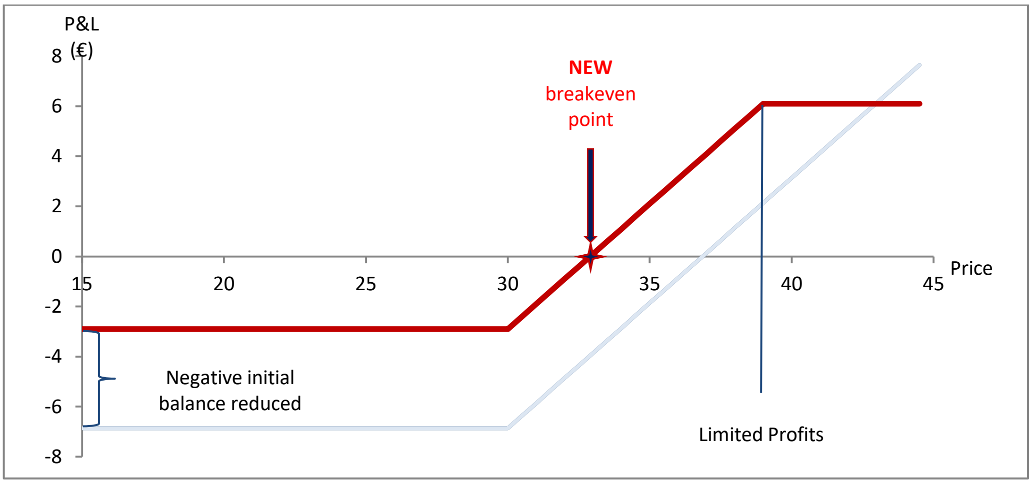


Figure 20 – Comparison between first and new scenario P&L chart.

By applying this strategy the bank is reducing its losses from 6,86€ per contract to 2,9€. At first it might seem that the difference is little but one must not forget that traditionally banks deal with positions much bigger than these and can (and some do) incur in losses proportionally bigger. The disadvantage of this strategy is that not only the losses are contained, but also the potential profits are limited to:

$$MaxProfit = k_2 - k_1 - Net\ Premium \quad (3)$$

In our example this maximum profit is limited to $(39 - 30 - 2{,}9 = 6{,}1€/contract)$.

In Appendix C – strategies with options we provide with different strategies for dealing with different perspectives regarding the market.

Note that our example deals with an extremely simplified version of a structured product as its variable return depends solely on different strategies applied to standard options. There are contracts that were designed in a much more complex manner to deal with different perspectives towards the market. These are based on what we previously referred to as Exotic options. Each and every one of them is worthy of intensive study as they can come in such structures that is quite easy for one to misinterpret and consequently incur in major financial losses.

One of the financial instruments used in such products captured our interest as its basic form presents as a clever instrument to deal with highly volatile markets, which is our case. In fact, such instruments could provide with a solution to cope with the example we gave previously. The option contract we are referring to is what is known as Barrier options.

6.3.1. Highly volatile market – Barrier option as a solution

These contracts belong to the class of *path-dependent* options, for its terminal payoff depends not only on the final price of the underlying but also on the path followed by such price from the valuation date until maturity, as in Nunes (2015). It is that path that determines the inception (*knock-in*) or extension (*knock-out*) of the contract or, in other words, the occurrence of each states only takes place if (and only if) the spot price touches some pre-specified barrier (H).

Two categories can be distinguished among these contracts: *knock-in* and *knock-out* options. The first category starts its life (at the valuation date) worthless and only becomes active if (and only if) a certain barrier is touched; if this event never occurs a *rebate* (possibly zero) is paid. If the option is "knocked-in" its terminal payoff is equal to that of European options. Conversely *knock-out* options also possess a terminal payoff equal to standard European options but only if the barrier is never touched, i.e. it starts its life active and expires worthless if the threshold is breached. It is very important to understand that in either case once the underlying touches the pre-specified barrier the option becomes permanently active (*knock-in*) or inactive (knock-out), which means that it does not depend on the path it will eventually follow to change its outcome.

Regarding the type of barriers used in the contracts these options can also be distinguished from the direction of the path's perspective. In other words as Nunes (2015) explains, if the barrier can only be touched from above – that is the spot price is above the threshold $H < S_t$– the barrier option is known as *down-and-in* or *down-and-out*. Similarly if the barrier can only be hit from below – current spot price below the barrier $H > S_t$– the contract is called *up-and-in* or *up-and-out*.

Given what was stated and knowing that each form can be applied to either calls or puts, 8 different types of barrier options can be distinguished:

Table 9 – Barrier Options

	-In	-Out
Down-and	Call or Put	Call or Put
Up-and	Call or Put	Call or Put

On a side note concerning the rebate that these types of contract pay (if the option expires inactive) it is to be seen as a fraction of the premium that was previously paid to the writer of the option in the first place. This rebate can be zero depending on the conditions agreed upon between both parties. This is one of the arguments used for the fact that these contracts are less expensive than those standard equivalent formats. Another opinion regarding the attractiveness of these products also concerning the cost when compared to plain vanillas is, from a different perspective, the fact that for any pair of down or up and put or call barrier option (and as long as the rebate is zero) the following result holds that:

$$premium\ (knock - in) + premium\ (knock - out) = premium\ (regular\ option) \qquad (4)$$

Although the rebates make these contracts very attractive under the OTC markets, it also poses as a disadvantage from our perspective, as they tend to make the valuation process much more complicated.

Since from our understanding (also based on the previous example) we find that these instruments can have some impact on investment decisions especially in markets with high volatility we believe that it would be interesting to address them for the case we are dealing with. However in order to prevent an extremely dense reading, we will leave it for the reader to draw his/her own conclusions. (In Appendix D – Barrier options terminal payoff you will find the terminal payoffs of these option contracts.)

7. Pricing Derivatives

Before we present the models that we are going to investigate for pricing our derivatives, a result first needs to be established not based on the underlying asset's dynamics but one that will allow us to reduce the problem of pricing calls and puts to simply pricing calls. That principle is called **Put-Call parity** and it needs to be taken into account in order to establish a correlation that must exist between put and call options with the same maturity date and strike price. This relationship states that the value of a call option, at a given strike price, implies a fair value for the corresponding put option and vice versa. This is required before pricing an option in order to counteract an arbitrage[48] opportunity that would arise if there was no parity.

The equation that this statement must obey to is as follows, Geman (2005)[49]:

$$P(t) + S(t) = C(t) + ke^{-r(T-t)} \tag{5}$$

P(t): European Put price

C(t): European Call price

r: continuously compounded risk free interest rate

k: strike

T: maturity

t: at date

S: underlying asset

Summing up, the put-call parity "states that a portfolio consisting of a stock and a European put can be *statically* replicated by a call and a position in money market account (...) This money market account is the riskless asset in the situation of constant interest rates and grows at continuously compounded rate r.", Geman (2005, p. 82)

To put it simple consider the example provided in Matos (2008) whilst verifying the validity of the previous result we established. If we borrow ke^{-rT} from the bank at rate r while at the same time selling a call option (both the loan and the option have the same maturity) and using that money (loan plus premium) to buy a put and the underlying asset, the value for that portfolio is given by:

$$\frac{k}{(1+r)^T} + C - P - S \tag{6}$$

If at maturity $S_T \leq k$, the put is exercised, we lose the asset but get the exercise price. The call is not exercised but there is still a loan to be paid back to the bank. Hence, we are left with the net value of $k(premium) - k(loan) = 0$, i.e. a worthless portfolio. The same would apply if at maturity $S_T > k$.

---

48 Arbitrage opportunities refer to the possibility of buying and selling the same security at the same time and taking advantage of a price difference between the two. In our case it refers to put and call positions.

49 An example is provided by the author.

7.1. The Black-Scholes-Merton model

Given what was presented previously we are now able to introduce the mostly used model for valuing a European call/put option, the so called Black-Scholes (1973) model or yet, more precisely, with the extensions provided by Merton (1973), the **Black-Scholes-Merton** model. Despite the fact that it is the mostly used process, the Black-Scholes formula depends on at least "10 unrealistic assumptions". Making the assumptions more realistic has not produced a formula that works better across a wide range of circumstances." (Black, 1988) Some of those assumptions are presented next:

- The asset's price volatility is known and does not change over the lifetime of the option;
- The underlying stock price varies smoothly without major movements in a short period of time;
- Interest rates are constant over the lifetime of the option and will denote the continuously compounded rate;
- Trading in the underlying stock takes place continuously;
- There are no-arbitrage opportunities in the market;

However, despite the assumptions followed by this model, we decided to investigate its applicability in our market due to its simplicity. Moreover, the Black-Scholes-Merton model has proved to be the chosen method to price options due to the fact that the users do not need to develop much elaborate mathematical models in order to produce accurate results as it only depends on the adequate choice of the inputs to use. Further on we will compare the results obtained with another model (Monte Carlo) in order to explore the influence of the introduction of multiple possible scenarios (in our case 10.000 simulations) rather than a continuous process with high volatility levels without admitting jumps as important features of the data.

Now, some institutions have been able to adjust the model in situations where some of the assumptions could not be valid. If we are to use this model, we must understand if the assumptions are acceptable in our market. Given that, let us now focus in presenting the model itself:

$$C(t) = S(t)e^{-g(T-t)}N(d_1) - ke^{-r(T-t)}N(d_2) \qquad (7)$$

where:

$$d_1 = \frac{\ln\left(\frac{S(t)e^{-g(T-t)}}{ke^{-r(T-t)}}\right) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{(T-t)}} \qquad (8)$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \qquad (9)$$

S(t): current price of the underlying stock (asset)

C(t): call price at date t

k: exercise price (strike)

r: continuously compounded risk-free interest rate

g:continuously compounded dividend yield of the underlying asset

T: maturity

t: at date

σ: asset price volatility

N(.): standard normal cumulative distribution function

Breaking down this model, it values a given option by taking into account that one has the choice of investing in an asset earning the risk-free market during the interval (t, T), i.e. the referred riskless money market account (the second term of the equation). In other words, that term discounts the price at which, upon expiration date, you will have the right to buy it (exercise price) back to its present value using the risk-free market rate. That result is multiplied by the normal distribution which is given by the standard deviation of stock's daily volatility adjusted for time, t. The first term corresponds to the variation of the value of the underlying stock price.

Despite the formula containing certain assumptions that are considered "unorthodox" or inadequate the fact is that the model has demonstrated to yield prices very close to the observed market prices, which is probably why it granted the Nobel Prize (1997) in Economics to two of its creators[50]. Moreover, adding to its main purpose, the formula can also provide with the hedging strategy for the holder of a short position in a call option, as Geman (2005) proved. As we will see further on, the Greeks that are derived from the model prove to be vital for this issue attesting this way the importance of such model.

Yet another reason for us to decide to investigate if this model is adequate resides in the fact that under the framework provided in Geman (2005), the Merton extension could be used to price options in commodity spot prices as long as we consider its behavior as that of a stock paying continuous dividend equal to the convenience yield. However, since this concept is directly related with the ability to maintain a storage level, we should question its applicability in electricity markets as (apart from hydroelectricity) no efficient form of electricity is known to be storable. We leave it to the reader to look up the construction of such statement (the author provides with very intuitive demonstrations, in our view) and draw his/her own conclusions.

For now, let us understand the Black-Scholes-Merton model (henceforth sometimes designated BSM model) in order to allow us to properly investigate and comment on future results. Here, we first need to point out that the main underlying assumption of the model is that the underlying asset price follows a *geometric Brownian motion*:

$$\frac{dS_t}{S_t} = (r - g)dt + \sigma W_t^{\mathbb{Q}} \qquad (10)$$

where $W_t^{\mathbb{Q}}$ is a $\mathbb{Q}$-measured[51] standard Brownian motion[52]. The equivalent martingale measure $\mathbb{Q}$ is used due to the fact that if we were to value a derivative contract based on the expected rate of return of a given asset, it would be subjective as it may vary from one investor to another. Hence this measure is said to be "preference free", Nunes (2015).

In other words, under the martingale pricing measure any stochastic process $(X_t)_{t\geq 0}$ that is adapted to the filtration $(\mathcal{F}_t)_{t\geq 0}$ is said to be a martingale under measure $\mathbb{P}$ if:

---

50 Fisher Black had passed away before the prize was awarded.

51 Measure $\mathbb{Q}$ or Risk neutral measure assumes that any financial asset generates a rate of return equal to the risk-free interest rate, i.e. it is a measure obtained when the *numéraire* of the economy is taken to be a money-market account.

52 A key property for this process is $W(t) \sim N^1(0, t)$, i.e. Possesses normal distribution with 0 mean and t variance.

$$E_{\mathbb{P}}(X_t|\mathcal{F}_s) = X_s, \forall s \leq t \quad (11)$$

(i.e. conditioned to the known previous value), meaning that it does not possess any upward or downward trend.

Consider a money-market account (cash account; savings) that earns the risk-free rate "r" with initial value β_0. Assuming that interest rates are constant so that 1 unit of invested money will be worth:

$$\beta_t = \beta_0 e^{rt} \quad (12)$$

Also, consider that the measure $\mathbb{Q}$ is equivalent to the original $\mathbb{P}$ such that the discounted price of any asset is a $\mathbb{Q}$-martingale. As in (11) and making $T \geq t$ any financial asset X:

$$\frac{X_t}{\beta_t} = E_{\mathbb{Q}}\left(\frac{X_T}{\beta_T}|\mathcal{F}_t\right) \Rightarrow E_{\mathbb{Q}}(X_T|\mathcal{F}_t) = X_t e^{r(T-t)}$$

$$\Leftrightarrow X_t = e^{-r(T-t)} E_{\mathbb{Q}}(X_T|\mathcal{F}_t) \quad (13)$$

Since $\mathbb{P}$ is irrelevant (due to the subjectivity we previously explained) for derivatives' valuation one can state, based on what was exposed, that anticipating or forecasting the (real) rate of return on the underlying asset is not necessary. That is, the $\mathbb{Q}$ measure considers that all assets generate a rate of return equal to the risk free interest rate, i.e. as if we were living in a risk free world leaving aside all investor's expectations. To put it simply, valuating these derivatives under this framework is nothing more than valuating the present value of the replicating portfolio under such conditions.

We understand that as a consequence of using $W_t^{\mathbb{Q}}$ to replicate the asset price, this Brownian motion is by itself responsible for the application of the filtration needed to replicate the uncertainty under this "risk-free world", i.e. the probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. However a standard Brownian motion would not suffice to correctly represent the spot price of a financial asset since it allows for the occurrence of negative values with zero mean, as Nunes (2015) shown.

To correct this effect a more generalized form of stochastic process needs to be considered. For that let us investigate the application of Ito's lemma – that proves by application to be a key figure when used in stochastic calculus– as it is used to find the **stochastic differential equation** (SDE) followed by an Ito's Process.

Let us start by defining an Ito's process, i.e. a stochastic process that includes no jumps, which means that it is a diffusion process defined by:

$$dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t) \quad (14)$$

This is the stochastic differential equation that represents the infinitesimal dynamic of the process $X(t)$ where $W(t)$ represents the Brownian motion generating the filtration to the process. $\mu(t)$ is the component responsible for capturing the trend of such process, hence known as drift. The previously mentioned lemma applied to this equation represents one of the cornerstones of finance since the Brownian motion is non differentiable and, consequently traditional techniques for (ordinary) differential equations cannot be applied. However a modification to the equation still needs to take place as (among other reasons that we leave aside) still allows for negative results to take place, which is unrealistic when considering an asset's price movement. However, the same is not valid if we consider the returns on an asset to represent its spot price, i.e.:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t) \tag{15}$$

which is the form used on the Black-Scholes model and where negative values are admissible. Moreover it maintains one of the main properties that we wanted to preserve, which is that the rate of return of any financial asset possesses a normal distribution: $\frac{dS(t)}{S(t)} \sim N^1(\mu dt, \sigma dt)$.

Also we must remember that the return over the period $(t-1, t)$ is given by:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}} \approx ln \frac{S_t}{S_{t-1}} \tag{16}$$

This form is very important from the financial point of view, since returns are commonly given in terms of their logarithmic form, particularly in this case where the normality of the distribution is a property that one wishes to preserve while modeling our underlying. Remember that we are dealing with the continuously compounded rate of return on a given asset, which means that it is important to preserve its characteristics throughout time. In fact if we consider the rate of return on a given asset (r_i) through a time span of n and calculate the arithmetic rate it reads as:

$$(1 + r_1)(1 + r_2) \dots (1 + r_n) = \prod_i^n (1 + r_i) \tag{17}$$

But the problem with this method (besides the unpleasantly of the process itself) is that the product of normally distributed variables is not normally distributed. Instead one rather uses the logarithmic equivalent:

$$\sum_i^n \ln(1 + r_i) = \ln(1 + r_1) + \ln(1 + r_2) + \cdots + \ln(1 + r_n) = \ln(p_n) - ln\,(p_i) \tag{18}$$

where p is the asset price at any given moment. This way by forcing returns to be log-normally distributed one gets to accomplish the normally distribution intended for the prices.

Given that, returning to equation (14) and applying Ito's lemma to $f(S(t) = lnS(t))$ yields:

$$dlnS(t) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW(t) \tag{19}$$

The process applied to our equation (10) is the same, but the drift is replaced by the joint action of the interest rate and the convenience yield.

$$dlnS(t) = \left(r - q - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t^{\mathbb{Q}} \tag{20}$$

which by integration between t and T,

$$S_T = S_t \exp\left[\left(r - q - \frac{\sigma^2}{2}\right)\right] \tau + \sigma \int_t^T dW_u^{\mathbb{Q}} \tag{21}$$

where $\tau = T - t$ and

$$\int_t^T dW_u^{\mathbb{Q}} = \left(W_T^{\mathbb{Q}} - W_t^{\mathbb{Q}}\right) | \mathcal{F}_t \sim N^1\left(0, \sqrt{\tau}\right) \tag{22}$$

And because of that S_T is log-normally distributed.

Finally, from what we know (from previous chapters) to hold for European calls:

$$c(S,k,T) = [S_T - k]^+ \quad (23)$$

And knowing that the $\mathbb{Q}$-martingale is valid and also that interest rates are non-stochastic, then

$$c(S,k,T) = e^{-(T-t)r} E_{\mathbb{Q}}[(S_T - k)|\mathcal{F}_t] \quad (24)$$

which is the final step to start combining all elements that lead to the Black-Scholes model achieved through mathematical manipulation that we have decided not to include here. The reason for such decision lies behind the fact that many authors presented this task in such detail (see for instance Bärholm (2017), Nunes (2015) and Geman (2005)) that any attempt to recreate it would be useless and redundant. Moreover we wish to preserve some degree of simplicity throughout this paper in order to guarantee an easy reading for as long as it is possible, since our main goal is to provide with easily understandable tools to explain what we initially proposed, which by its very own nature is already too complicated.

Applying the same reasoning to put options we would be able to derive the same result. However since we have already established a result that saves us this task we can, from the put-call parity, derive the put pricing formula:

$$S(t)e^{-g(T-t)} + P(t) = ke^{-r(T-t)} + C(t) \quad (25)$$

which due to the symmetry of the normal distribution yields:

$$P(t) = -S(t)e^{-g(T-t)}[-N(d_1)] - ke^{-r(T-t)}[-N(d_2)] \quad (26)$$

However, this result is not needed since the parity established is enough to price both instruments, as we will see from our model later on.

This is the very foundation of our work in the upcoming chapters, particularly when we present our automated BSM calculator and the principles behind the simulations of our underlying asset, despite knowing that the model is questioned when used in electricity markets.

One of the benefits of this formula is that it provides the tools for a diversified analysis on the correlation between its multiple variables. Consider as an example the correlation between the asset price (S) and the value of a Call option, all the rest fixed. In fact this is one of the most important characteristics of the model under the risk free world, i.e. the value of the call increases with the value of the underlying but converging asymptotically to the present value of the payoff of that asset. Notice that:

$$\lim_{S\to\infty}(d_1) = \lim_{S\to\infty}(d_2) \Rightarrow \lim_{S\to\infty} N(d_1) = \lim_{S\to\infty} N(d_2) = 1 \quad (27)$$

Once this result is applied to the original formula it yields what we would expect, which is that the value of a call option is nothing but the present value of the payoff (value of the underlying minus the exercise price).

There is however a limiting behavior for the model that is derived from one of its main assumptions, that of constant volatility. In fact, the model depends on five (six if we consider the convenience yield) parameters (underlying asset's price, strike price, interest rate, time to maturity and volatility). All of those are directly observable from the market, except for the case of volatility. However, if we know the price of an option we can reverse the BSM model to calculate the volatility; or yet that volatility that is implied in that situation, hence the name implied volatility. Here an interesting experience emerges and produces results that deviate from what we would expect. By fixing all parameters except the exercise price and investigating how the implied volatility varies with k allows for an effect to take place that is known as the *smile effect.* (Figure 21)

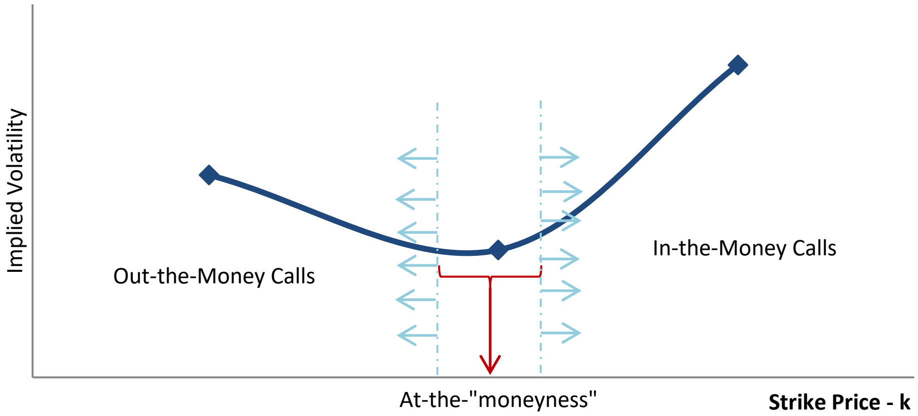


Figure 21 – Implied volatility' smile effect

If all the assumptions under the Black-Scholes were correct, this test would create a flat graphic, which means that we would reasonably expect that regardless of the strike price, options would have the same implied volatility. However, this does not occur and that makes this subject of intensive works from academics and investors to try and understand the consequences for the markets. We do believe that this effect takes place due to the basic principles of supply and demand. That is, the model replicates the valuation for all positions in the same option or, in other words, it does not take into account if the contract is In-, At- or Out- the money. However investors do take it into account and, traditionally, they tend to go for In-the-money and Out-the-money in detriment to At-the-money contracts. This preference over those bids seem to force the prices of such options to go up. Later on by making use of a working example we will show how important this indicator can be to traders in the sense that it can, to a certain extent, be a measure of potential profits and losses.

What was stated previously along with other key features of the Black-Scholes model defined what we will present next. From our point of view this is one of the aspects that makes the model so interesting since, even though its valuations sometimes do deviate from what we see in the market, there is still the possibility of investigating the influence of each variable in the final valuation and how exposed an investor is. Hence, in order to complement the analysis provided with Black-Scholes formula, many option traders rely on "the Greeks". These are partial derivatives of the call price used to evaluate option positions, using a collection of statistical values that measure the risk involved in an options contract in relation to the different parameters involved in the formula. Here we highlight the most important:

- Delta (Δ): provides a measure of the sensitivity to the underlying price. It is the partial derivative of the call price, in relation to the underlying asset price, playing a key role in "computing the exposures and Value at Risk [measure and quantify the level of financial risk] of a position": $\Delta = \frac{\partial C}{\partial S}(t, S_t)$

The delta is positive and the call price is an increasing function of S_t. "If we anticipate a rise in the underlying S, the return generated by any amount of money invested in the call is strictly higher than the return on the stock during the same time period". Geman (2005)

- Gamma (Γ): measures the sensitivity of Delta to price changes. Hence, it corresponds to the second derivative of Delta with respect to S: $\Gamma = \frac{\partial \Delta}{\partial S}(t, S_t)$

  Since delta is so important and since its values change at different rates, gamma is used to determine how stable an option's delta is. Thus, a higher gamma could implicate that delta would change dramatically in response to small movements in the underlying asset's price.
- Theta (Θ): a measure of sensitivity to time decay. I.e. it corresponds to the variation of the call price as time goes by: $\Theta = \frac{\partial C}{\partial t}(t, S_t)$

  Since its result is a negative quantity, it means that this "Greek" represents the time decay of an option as it gets closer to maturity. "One way of looking at it is to keep in mind that more time to maturity means more time for random moves of the underlying." Geman (2005) Moreover, Theta value increases when options are At-the-money and decrease when options are In- and Out-the-money, due to the decomposition of the result into the sum of intrinsic value (the pay-off of the option if it was exercised immediately) and time value of the option.
- Vega (not a Greek letter): measures the option's sensitivity to changes in the volatility of the asset: $Vega_{call} = \frac{\partial C}{\partial \sigma}$

  Increased volatility implies that the underlying asset is more likely to experience extreme values, hence an increase in the value of an option.

All that was previously described can be summarized in the following figure:

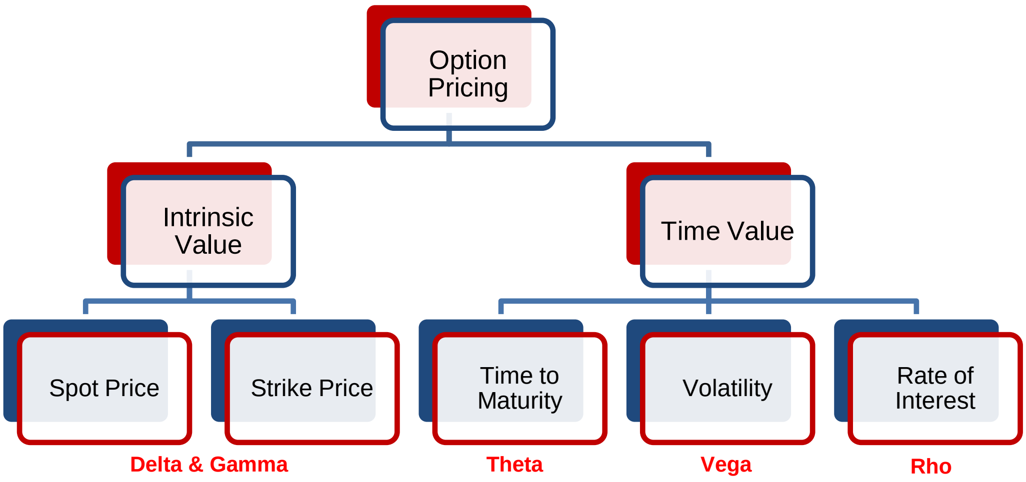


Figure 22 – Option worth by segment and respective Greek

7.2. Monte Carlo

Due to the fact that the previous method is highly questioned when used for electricity derivatives, we were compelled to investigate and compare the results to other methods that are accepted by the scientific community. In fact it is argued that this framework can accommodate more realistic stochastic processes, as is the case of the one we will present. However we are not going to run one method against the other as our main purpose is to rather present all the tools for a potential investor to understand the market. From our understanding one method can complement the other in the sense that the information provided in each one can be quite useful. See for instance the last property we discussed in the previous chapter when arguing about the importance of indicators such as the Greeks that enable an investor to gather some information on how exposed he/she is to any movement in any indicator of the underlying.

The Monte Carlo, on the other hand has the advantage of including the exposure of the investor to any movements in the pricing itself due to the fact that the nature of this process includes running several admissible scenarios that lead to different outcomes. Let us go through this process to make this statement clearer for the reader.

In what concerns the risk world we are in, the Monte Carlo is traditionally supported by the same assumptions used in the previous method; that is under the risk-neutral measure. The key difference lies in the discretization of the whole process itself with the time step varying from whichever size the simulation requires. Note that if one wanted, an hourly simulation could take place. However, such simulation would make an otherwise already time consuming process even longer.

Given this introductory note, as Nunes (2015) puts it, it is clear that the method under scrutiny here provides an estimate for the value of some function by simulating a sufficiently large number of times the path of its underlying independent variable. Given the time step and the number of simulations, by running the objective function it will provide with n paths that will have its own value, i.e. say you have 100 simulations for the possibility of rain during the next month, will provide you with 100 different (or not) possible outcomes depending on the function you fed to the process. At the end of the process by computing the average for each value one has the final estimate and the quality of this estimation is highly dependent on the number of simulations one uses.

However, and since we believe that enforcing this notion is required we should mention once again that achieving the risk free measure has proven to be a difficult task for us, since the methods suggested by many authors depend on the ability to acquire some information that the representatives in Portugal do not authorize us to get. As an example, the historical data of prices at which future contracts were traded and in which we were relying to compute such risk free measure. We will discuss this later on with further detail.

Applying this method can be summarized as in Nunes (2015), on whom we rely to present the following steps:

1. The stochastic process that represents the underlying asset needs to be discretized. To do so, the differential equation needs to be transformed to an equivalent time-discrete differential equation. In our case, and since electricity prices can come in different forms, we need to note that the discretization should be in agreement with the time step we use for the price registration (i.e. whether we are working with hourly prices, daily prices etc).

Given that, equation (15) will hence read as:

$$S_T = S_t \exp\left[\left(r - q - \frac{\sigma^2}{2}\right)\tau + \sigma\left(W_T^{\mathbb{Q}} - W_t^{\mathbb{Q}}\right)\right] \quad (28)$$

where $\left(W_T^{\mathbb{Q}} - W_t^{\mathbb{Q}}\right)|\mathcal{F}_t \sim N^1(0, \sqrt{\tau})$.

To include the time-step one simply applies any recursive technique, as in Euler's approximation to evolve the previous formula to:

$$S_{t+(i+1)\Delta t} = S_{t+i\Delta t} \exp\left[\left(r - q - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t \varepsilon_{i+1}}\right] \quad (29)$$

where ε is now the variable responsible for the randomness drawn from a standard normal univariate random variable $N^1(0,1)$.

2. Equation (21) is the one that is meant to recreate the spot price movement of the underlying and needs to be simulated a large number of times, as Monte Carlo simulations produce samples with decreasing standard errors inversely to the number of simulations.
3. At the end of each simulation, the terminal payoff should be computed. As we mentioned before, the terminal payoff depends on whether we are referring to a put or a call contract. In the first case it needs to hold that $S_T < k$. Conversely for a call option $S_T > k$
4. After all simulations (m) are ran and each with its terminal payoff, Monte Carlo estimates the option value ($\hat{V}_t$) by averaging all terminal payoffs:

$$\hat{\mathrm{V}}_{\mathrm{t}} = \mathrm{e}^{-\mathrm{r(T-t)}} \frac{\sum_{\mathrm{j=1}}^{\mathrm{m}} \mathrm{V}_{\mathrm{n,j}}}{\mathrm{m}} \quad (30)$$

5. Finally, and as mentioned the precision of the valuation is obtained by the standard deviation $\sigma(\hat{V}_t)$:

$$\sigma\left(\hat{V}_t\right) = \frac{\sigma(\mathrm{V}_{0,j})}{\sqrt{m}}, \quad (31)$$

with

$$\sigma(\mathrm{V}_{0,j}) = e^{-r(T-t)} \sqrt{\frac{\sum_{j=1}^{m} \mathrm{V}_{\mathrm{n,j}}^2 - \frac{1}{\mathrm{m}}\left(\sum_{j=1}^{m} \mathrm{V}_{\mathrm{n,j}}\right)^2}{m-1}} \quad (32)$$

Importantly enough to mention, the number of simulations can be pre established not to exceed a certain threshold as Nunes (2015) suggests by giving the example of such a limit being one such that $\frac{\sigma(\hat{V}_t)}{(\hat{V}_t)}$ does not exceed the bid-ask spread, i.e. the standard deviation does not "deviate" from the valuation obtained more than the interval between what the market is willing to give (bet) and to take (ask) for the same instrument. However, since we could not have access to such information, we used 10.000 simulations as it is standard for market practitioners as we will see later on, due to the simple fact that the standard error is inversely proportional to the square root of the number of simulations, hence making this number large enough (as it tends to infinity) the error tends to zero.

When you consider path dependent contracts one needs to have as many simulations as possible in order to produce meaningful results, but always under the risk neutrality assumptions. See, as an example the case of the barrier options where information on whether the asset reaches a certain threshold or not is too important to underestimate the number of simulations required.

8. Focusing on the Portuguese Electricity Market

Let us now take the final steps towards enabling us to set up all the information required to produce meaningful results. I.e. so far we have been presenting and producing results from a financial point of view with little consideration for the market that we are addressing, i.e. not specifying the applicability of these structures in commodities. It is very important to know and understand that some assumptions carried out through this study do not hold when we are referring to assets of this nature. Our efforts were dedicated to try the best way possible to overcome the difficulty that is inherent to adapting the financial world to that of commodities.

However, to begin with dealing with the self proposed task of adapting what was previously exposed we focused on a detailed analysis in the market that targets our subject (that of electricity). Such an analysis is required, in order to accordingly choose the form of derivative to include in such structured product. The reason for that is because electricity (from whichever source it comes from) is to be seen as a commodity, i.e. unlike a stock market or bond market where prices are established based on net present value of receivable cash flows (which we have been addressing extensively so far), commodity spot prices are defined by the intersections of supply and demand curves (and demand for commodities is generally inelastic to prices). According to Geman (2005) commodities represent a vast volume in financial transactions (forwards, futures, and options) and their prices are "closely related to spot prices in particular because physical delivery is a choice that is left to the buyer." Moreover, even though all actors involved in our sector engage mainly in spot trading with physical delivery (remember that, as mentioned in previous chapters, our spot market works on a daily basis, i.e. prices are hourly negotiated for the hours of the following day), "liquid derivative markets (…) paved the way for risk management and optimal design of supply and demand contracts" Geman (2005, p. 1).

This latter statement is what makes this investigation so interesting for us. As international markets have shown that under strict regulatory rules derivatives' markets have proven to a certain extent to improve the way those markets operate allowing them to become more efficient and hence reducing prices to final consumers. Making use of futures markets in our electric system could prove to become an important tool as well (even though they are already traded, the liquidity achieved is still very far from similar markets across Europe). However it will only be possible to become more efficient if the clearing house (OMIClear – The Iberian Energy Derivatives Exchange) takes an active role behind those exchanges in a similar manner as those markets we described in previous chapters. Although it already is extremely active we do believe that recent restructurings in the way the market operates and presents its results to general public became less transparent. Note that we struggled to get some information that would prove to be extremely helpful and help us achieve higher accuracy on our results. Moreover, investors tend to move away from markets that seem to be less transparent and less investors lead to less liquidity.

Despite all that, the most important role of such entity is (and in our opinion being carried out with great effectiveness) the responsibility for taking away any credit risk from the positions of the two participants engaged in the transaction, i.e. guaranteeing that both parties are good for the positions they are assuming. In fact, if we take the producer’s point of view, futures contracts (which unlike options are mandatory) can provide hedging against price risk by securing his future production, which in our case, as feed-in tariffs (guaranteed remuneration) are becoming obsolete, might present itself as a good solution. By securing his revenues the producer can allocate the proceeds to be received to the renovation of the power plant if needed for repowering, as we explained in section (3.2) regarding EDP *renováveis* statement. Note that the regulator is responsible for overseeing such transaction in order to guarantee that the producer is in fact able to commit to the transaction in the future. Moreover, this form of contract also benefits any class of investors when compared to spot market trading, as this one provides the flexibility for short or long positions, enabling the choice of positive or negative exposure to a rise in prices. However, as these contracts (unlike forwards) are marked-to-market[53] daily and the participants have to adjust their positions (e.g. if one has a long position in a futures contract acquired in a given day at a given price and on the following day the price is lower, then he will have to adjust his position by compensating his loss with an equal cash amount). Again it is the market operator that oversees that both parties are operating under law requirements, assuring that the investor keeps a credit for the future contract.

In order to draw some conclusions regarding this derivative it is important to proceed with an analysis on Futures prices and expectations of future spot prices. As Geman, (2005) argued it is important to understand and answer the question on whether the Futures prices are a good representation of a given quantity, and hence can be used as means for price discovery (or at least as a risk measure as we will see). For instance, if market participants could predict that in the near future, under-capacity would occur and accordingly invest in Futures market provoking a biased reaction on future spot prices, emphasizing its increase and eventually end up burdening consumers. It would in fact go against our idea – that a financial product does not necessarily have to prejudice any elements along the chain. In fact, an equilibrium between future spot and futures prices for the same time horizon is required in order to allow producers to hedge their positions, at the same time that investors who are willing to take on the risk, can profit without jeopardizing consumers position, i.e. “the spot price must exceed the forward price by the amount which the producer is ready to sacrifice in order to hedge himself…” Geman (2005) This principle is, as argued by many authors, the basis for establishing a risk neutrality principle when valuing options; i.e. using today’s price of a future contract with delivery at date T to find the risk premium for an option contract with the same maturity. This principle is, by itself open to a lot of debate as in Weron (2008) but we are going to assume as a reasonable assumption but pinpointing the importance of deriving such result as the authors did.

It is on these assumptions that our model will be based and we expect to get satisfactory results, which we will be able to determine if they are so by investigating the first four statistical moments, as we have stated before. The trajectorial path is obviously also under scrutiny here as we wish to produce simulations that are fairly similar to what is observed with real data (as shown in Figure 7).

However, we should point out that, as the literature suggests, there are models available known to produce better results than the ones we are about to present. To understand the difference, allow us

[53] Mark-to-market is a system for valuing assets by the most recent market price.

to go through some of them (picked based on the statistical quality achieved in many papers throughout the internet's universe) so that we can later on suggest different strategies to adapt, as well as explain why we did not opt for one that expectedly would produce more accurate results.

8.1. Market Modeling

As we are aware, correctly representing the dynamics of spot prices in electricity markets presents as a key element for trading purposes. For that, we have dedicated an extended amount of time in studying the models available and developing one of our own, i.e. fully adapted from what literature has taught us. Among innumerous authors that have been working on this issue, some stand out from the others due to the quality of their papers and the amount of times they are used to corroborate other empirical evidences in other readings. Such authors can be named after their excellence as Geman & Roncoroni (2006), Escribano *et al.* (2002) or Benth *et al.* (2007). Moreover, an extremely useful article from Benth *et al.* (2012) really helped us carrying out the analysis on the models we are about to present and helped us deciding which to apply for our market.

To produce an adequate model is our main target and, as we mentioned in previous chapters we are unable to rely on the spot-forward relationship to do so due to the simple fact that this relation does not hold as a consequence of the non storability of electricity. To investigate forward curves from the market and use them to try to produce results for our spot representation is a mistake as Geman (2005) suggests. However, we do believe that we can to some extent use this relationship to understand the market price of risk as we will see further on.

8.1.1. Review of existing models

The general characteristics of an electricity market can be taken to be somehow similar in different countries. Of course demand and supply will differ depending on climate, building quality, industry etc. However, we feel that the main assumptions that drive the prices up or down can be extended throughout different markets. Those features have been presented in section 3.3 and unless we are addressing some in particular we will not go into details here. Nevertheless those are the technicalities that different methods used in replicating spot prices try to capture.

There is however a common ground for nearly all authors; i.e. it is commonly accepted that electricity spot prices exhibit some form of (strong) seasonality on different levels (daily, weekly, yearly, etc) as well as mean reversion. The occurrence of small events around the trend due to locally unpredictable events is also included in this pattern and, as Geman & Roncoroni (2006) suggest this can be represented by the introduction of a white-noise term affecting daily prices, caused by imbalances in supply and demand as discussed in previous chapters.

It is also point of agreement among the scientific community that spot prices are characterized by extremely high volatility, as we have discussed, and also the occurrence of abrupt and short-lived extreme events, which we have referred to as jumps and/or spikes.

The first component we are addressing is the seasonality, which can be extrapolated for nearly all electricity markets depending of course on the periodicity to be considered. A country in sub Saharan

Africa will most likely not show the same pattern as one in Western Europe (e.g.). Nevertheless we can all agree that some form of cyclical path is followed on a yearly basis and that is what every model tries to capture when setting the seasonality.

Some authors simply combine the use of periodical formulas to reproduce these movements, while others include a moving weighted average, as in Janczuna *et al.* (2013) to maintain a certain degree of continuity on the process for they believe that high deviations from the trend can also be cyclical but not as steady as a pure periodic function. We believe however that this method is not suitable for pricing derivatives on electricity as they put to question a required feature that is guaranteeing it preserves the Markov[54] property, in the sense that the future value of a function is dependent on the present's level but not on prior knowledge. I.e. the process should preserve a degree of independence from the past since as weather itself as proven us it can differ a lot from one year to another. Below we show the seasonality obtained when using a weighted moving average for our situation:

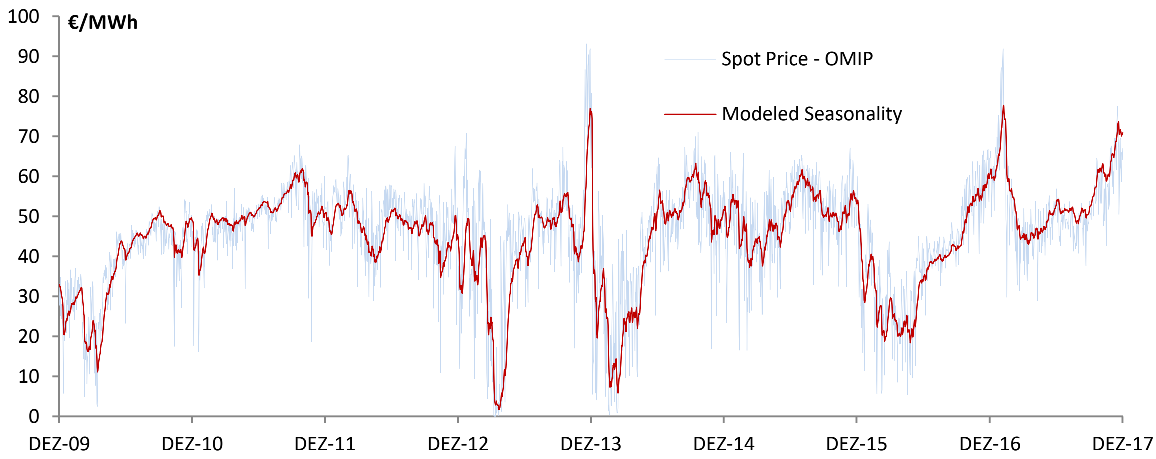


Figure 23 – Electricity spot price in Portugal | Source (OMIP)

Note that this is not how we computed our seasonality as we will see further on. We present it here for comparative purposes only and to support our statement regarding the Markov property. Any weighted moving average is obtained by attributing a degree of "importance" from previous values to the next, i.e. by averaging subsets of data points from historical data. In our case we centered the process on the last known data point and used a decay factor close to 1, in order to reduce the importance of further down in history points. Although this method could eventually provide us with more accurate results, we believe that its application should be not encouraged for what we are attempting to achieve, i.e. the valuation of derivatives under the assumptions of risk neutrality.

Apart from this technique nearly all authors agree with the method to apply to describe seasonality, and it has proven to efficiently reproduce the market (when combined with the stochastic process). In fact, Benth *et al.* (2012) in their comparative investigation regarding three models to later apply to pricing derivatives, agree on using the same deterministic function for the seasonal pattern and the only component that differs is that of the stochastic modeling. That is, the component that accounts for

[54] The Markov property is consistent with the efficient market hypothesis. It is important to preserve this feature in price modeling, even though on technical analysis one uses historical data to decide on whether to buy or sell a given asset.

the occurrence of extreme events, which is where most authors focus their efforts on trying to develop an adequate representation.

Some authors prefer to keep the whole process continuous by using the method initially proposed by Merton as a diffusion process, in the same manner as equation (14) yielding:

$$dS(t) = \mu(S_t, t)dt + \sigma S(t)dW_t \qquad (33)$$

where μ represents the seasonality function around which prices fluctuate. Introducing a high enough volatility (σ) attached to the Brownian motion (W_t) ensures the possibility of occurrence of high moments known as spikes, which will mean revert to the season. The volatility can be constant (as long as it is high enough to allow for jumps to take place) or deterministically stochastic, in which case a second function to account for this effect has to appear.

Other authors prefer to separate a third component to account for the jumps by adding it to the original equation. In doing such the volatility component attached to the Brownian motion is hence responsible solely for the local imbalances registered throughout normal days, as in:

$$dS(t) = \mu(S_t, t)dt + \sigma S_t dW_t + S_t U_t dN_t \qquad (34)$$

In this model the arrival of the events that carry with it the occurrence of spikes is determined by a Poisson process (N_t) with constant intensity λ. The magnitude of such jumps is brought by the real constant U_t. This model, or yet those that are built from these assumptions, as argued in Geman & Roncoroni (2006) pose the question on how to effectively bring the prices back to normal after an extreme event, because it has been proven that if an inversely (equivalent) magnitude is used it will make the typical local fluctuations disappear. It is based in this simple model that the authors on that paper start describing their improved method by introducing a *jump-reversion* model and representing the spot price of electricity in its logarithmic scale by:

$$\begin{cases} E(t) = lnS(t) \\ dE(t) = D\mu(t)dt + \theta_1[\mu(t) + E(t^-)]dt + \sigma dW(t) + h(t^-)dJ(t) \end{cases} \qquad (35)$$

Here, D stands for the first order derivative and the power (-) represents the left limit. The seasonal function around which prices fluctuate is represented in the same manner as other models by $\mu(t)$. The second term is the responsible for a smooth mean reversion of any fluctuation to its normal trend. $\theta_1$stands for the average variation of the price per unit of shift away from the trend. It is important to note that the speed at which the spot price reverts to the local average is dependent on the current price, since this force is multiplied by a difference between two variables and which size depends on where the price is located when compared to the trend. The volatility attached to the Brownian motion is the same as previously explained, whilst the last component is by itself one that improves to a large scale the quality of the results obtained by the authors. As they argue in their paper, jumps should be accounted for through an appropriate representation in order to improve the statistical quality of the model and also to "explain the significant deviations from normality in terms of high order moments observed in logarithmic prices." Geman & Roncoroni (2006, p. 1230) An analysis which, for that reason we will also include in the next chapter.

Returning to the model and to the last component, the authors consider that a spike is a cluster of large sized sharp upward directed movements followed by an equivalent shock in the opposite direction towards the normal price. To represent this discontinuous process, i.e. the periodic effect of spikes, the approach taken used a level-dependent sign for the jump component in the sense that "If the current price is below some threshold, prices are in the normal regime and any forthcoming jump is upward directed." (p. 1231). Conversely if the price is above, the market is likely to be experiencing an imbalance which has as a consequence high prices. Should this occur, then expectedly any jump would be downward directed. The jumps themselves are characterized by the direction, size and time of occurrence by the following reasoning:

$$J(t) = \sum_{i=1}^{N(t)} J_i \qquad (36)$$

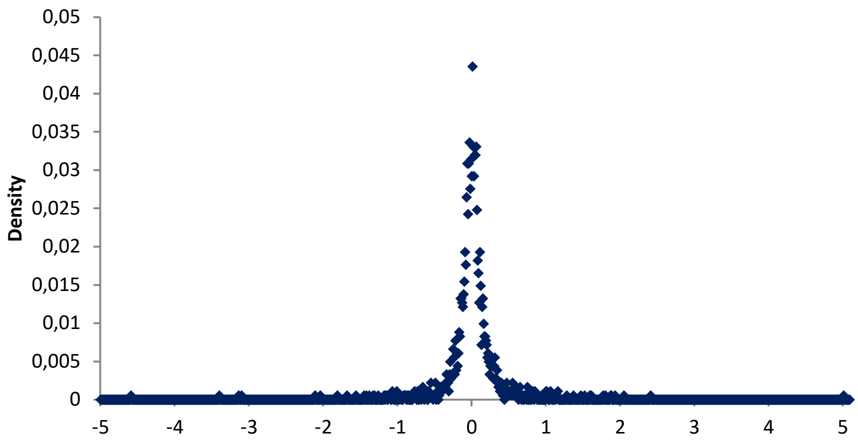
To put into words, where the arrival of the events follows a Poisson process and the jumps J_i are identically distributed random variables with exponential distribution. To investigate on how the author calibrates and computes such model we suggest either Benth *et al.* (2012) or Geman & Roncoroni (2006). However one should note that this is an extremely demanding mathematical process and this is the main reason why we abandoned it. Despite that to understand this model in order to improve further academic research regarding the Portuguese electricity market would prove to be rewarding.

However, knowing from Benth *et al.* (2012) that it proves to produce such statistical quality results, we decided to adapt some of its assumptions in order to improve our model. We will hence combine some of the knowledge acquired from Villaplana (2003) and the previous authors and try to recreate our model.

8.1.2. A jump-diffusion mean-reverting approach

As we mentioned previously when we started modeling our market we realized that most models available out there were developed for markets that are consistently different from ours. Most probably because such models were developed in countries where winter season is much harsher than the Portuguese – known worldwide for its great weather conditions – making as a requirement for those countries to correctly model their spot prices in order to correctly develop their electric system to respond to abrupt demand increases during those periods. Countries in Scandinavia or Germany (those for which we found most of the data) go through extreme weather events throughout the year, which is why they need to properly dimension their electric grid. The financial markets take advantage of such models and use them to provide investors with financial tools that allow them to gain access to such markets.

Although different, our market also contains abrupt price changes that need to be captured. Using the same reasoning as in Geman & Roncoroni (2006) our model should be able to reflect those extreme events empirically registered and responsible for the significant deviations from normality in both logarithmic or "regular prices", as shown below.



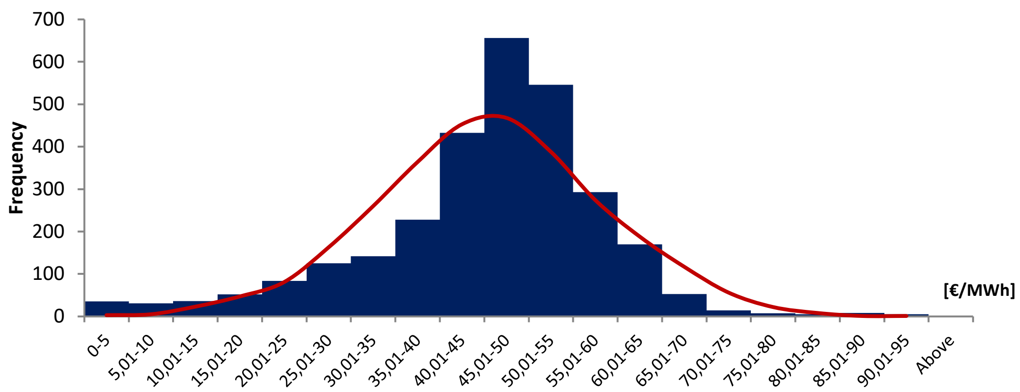


Figure 24 & Figure 25 – Logarithmic returns (above) and Spot prices (below) show deviating levels from the normal curve due to the occurrence of extreme events.

Given that, we have decided to conduct our experiment starting with the development of an adequate model to represent our spot price. The model translates the behavior of 1 megawatt-hour as traded in our market. To correctly accomplish this task, and given the reasons we presented previously, we will use the logarithmic scale of the price in order to guarantee the strict *positiveness* of the prices, enhancing the robustness of the calibration procedure as originally suggested by Cartea & Figueroa (2005). For that reason we will henceforth designate logarithmic prices simply as prices; although the graphics we present will be in its original form unless otherwise told.

Under that reasoning combined with the initial proposal by Lucia & Schwartz (2002), the model we employed is described as follows:

$$\ln(S_t) = \mu(t) + X(t) \qquad (37)$$

where (S_t) represents the spot price and $\mu(t)$ the trend it follows continuously throughout time and to which the process mean reverts. The last term $X(t)$ is the stochastic process responsible for modeling the random fluctuations around the trend. To capture such pattern we adapted the method used in Geman & Roncoroni (2006) without any prejudice to the model itself. The equation reads as:

$$\mu(t) = \theta_1 + \theta_2 t + \theta_3 \cos(\theta_4 + 2\pi t) + \theta_5 \cos(\theta_6 + 4\pi t) \qquad (38)$$

This deterministic function contains 6 variables $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$ that will be determined later on by direct application of a Least squares error, i.e. fitting the trend line to market observations. The first variable on the function can be seen as the fixed cost associated to power production (assumed to be a constant fixed cost) while the second term is responsible for the growth rate of the total production cost. The remaining components will attribute the required periodicity to the function translating into two maxima per year that can be interpreted as extreme seasons, i.e. both summer and winter. It makes sense if one thinks of energy requirements during those periods just for heating or cooling purposes.

In order to try to produce meaningful results we adopted a strategy for the stochastic component that assembles some assumptions of each of three models into one. Initially we model the process as a stationary mean reverting function, as described in Lucia & Schwartz (2002), i.e. as an Ornstein-Uhlenbeck process.

$$dX_t = -\kappa X_t dt + \sigma dW_t \qquad (39)$$

where κ is the force that drives the prices back to normal, and σdW_t is as previously explained, the volatility attached to the increments in a standard Brownian motion. Villaplana (2003) and Cartea & Figueroa (2005) adapt this model to account for the occurrence of jumps yielding in both cases models that are widely used for this purpose. However Cartea & Figueroa (2005) include also a deterministic component to express volatility; however, as Geman & Roncoroni (2006, p. 1240) argues that the addition of "a time-dependent volatility does not produce a significant improvement in the estimated process", we have also decided to keep the volatility constant.

The combination of the previously described models for the stochastic component yields the one we present as follows and based on which we elaborated our process:

$$dX_t = (\alpha - \kappa X_t)dt + \sigma dW_t + J(\mu_j, \sigma_j)d\prod(\lambda) \qquad (40)$$

where $(\alpha - \kappa)$ is again responsible for bringing prices back to normal, but this time it takes into account (α) with the purpose of including a component to consider the risk-neutral assumption as in Lucia & Schwartz (2002). The jump size, as in Cartea & Figueroa (2005) and Villaplana (2003) is modeled through a normal distribution with mean μ_j and standard deviation σ_j. Finally $\prod(\lambda)$ accounts for the arrival of the extreme events, that is, it is a Poisson process with a time-dependent constant rate of arrival (λ), i.e. jump intensity. However, we felt that we could introduce a minor change here in a similar way as in Geman & Roncoroni (2006) and transform it into an inhomogenous Poisson process. The rationale being that we have reasons to believe that the arrival of these events can be deterministic, as we will see.

To correctly calibrate our model we have decided to exclude all data previous to 01/01/2013 since there were years in between that could be considered an anomaly or, in other words, to fully incorporate them within this model would not be possible while aiming to produce good statistical results. For those years, an improved model would have to be applied to account for weather variables in a much more detailed way.

8.2. Calibration

To initiate the calibration process we uploaded the data of electricity spot price ranging from 01/01/2013 to 17/12/2017. We went further with using our model and tried our best to calibrate it in a similar manner as Geman & Roncoroni (2006) since we agree with some of the assumptions contained on that model, particularly the separation of the calibration procedure in two-steps. First we need to address all parameters that are considered “structural”, that is, those that translate into path properties of the process; and then the model parameters that are responsible for statistically fitting the model (since they are meant to be fitted to the filtered prices).

The prices used were, as already mentioned, the baseload prices. Then the logarithm of those prices was obtained in order to apply the least squares method to find the constants of the deterministic component, i.e. the first structural parameters. Below we present the calculated seasonality plotted against the spot prices as well as the *deseasonalized* data obtained and required to proceed:

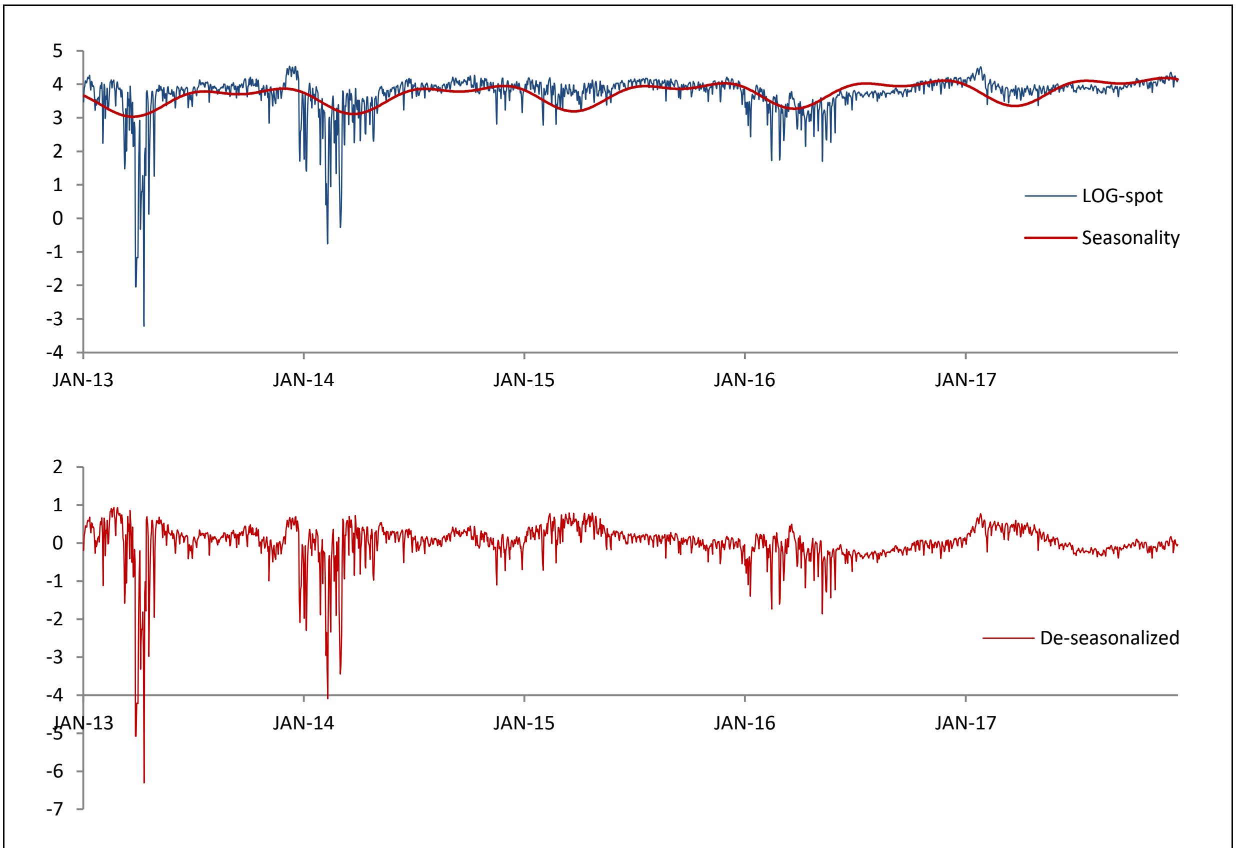


Figure 26 – Seasonality function ploted against the (log)spot prices (above) and the *Deseasonalized* function (below)

Once the deterministic function that describes the seasonality is removed one can focus on the issue of calibrating the stochastic component. But first we will obtain the structural parameters of this component, which is where we introduced some modifications to the model we are using.

First and as we stated previously we believe that spikes tend to cluster around (and peak) at April with the possibility of spreading to months before and after depending on those years’ weather conditions. However it is important to notice that most of our clusters are downward directed – and that feature is extremely important to be accounted for – unlike most electric systems that are currently found to be accurately modeled (as those we have been focusing on). To achieve that behavior we have decided to follow Geman & Roncoroni (2006) and model the intensity of the arrival of spike events as a deterministic function, but allowing for the occurrence of jumps during that period to be random (although the number of occurrences θ will also be calibrated to fit the data). This periodic jump intensity can be described as:

$$s(t) = \left[\left(\frac{2}{1 + \left|\cos\left[\frac{\pi(t-\tau)}{k}\right]\right|}\right) - 1\right]^{d} \qquad (41)$$

However, unlike that later model where spikes are mostly upward directed, we had to focus our efforts on those that are downward directed since those tend to occur rather more often than positive jumps. This can be explained by the fact that during those months rain events typically take place much more often, which leads to an increase in hydroelectric power production. This model takes into account that peaking levels are spread by k multiple years beginning at time τ. The exponent d is responsible for the events spread in time. And if one takes into account the weather events that traditionally occur in our country as explained in chapter 3.3, we can understand why such claim makes sense. More so if we take into account the graph presented below where the clustering of spikes around specific periods of the year is clear and evident.

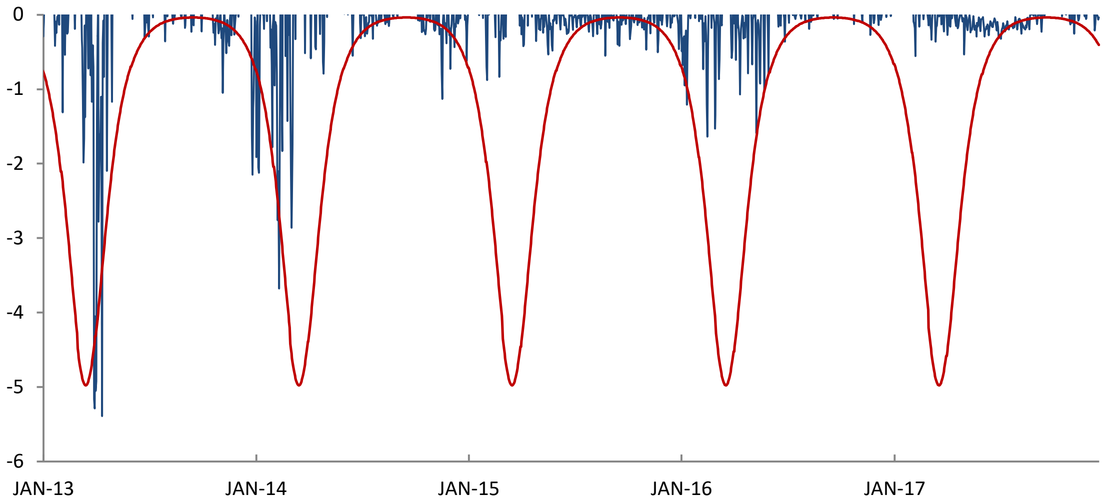


Figure 27 – Overlapping of the intensity shape function with the deseasonalized data series.

Evidently there is no such thing as negative distribution of occurrence of an event. We kept the graphic upside down for illustrative purposes only. Also, it is important to notice that on the original model where this deterministic function is used, the function is multiplied by a coefficient that determines the number of expected jumps per time unit at a clustering time. However, due to our calibration procedure (discretized) and given our historical data (note that in some years these events are less representative), we used a random number generator (0 or 1) with equal probability that determines whether or not a spike takes place, allowing this way for a certain degree of randomness of the process. Yet, since this model only works for one jump direction (either upward or downward) the nature of our market forced us to include a -1 parameter (θ – as seen in **Erro! A origem da referência não foi encontrada.**) to set the downward signal of the event. Any movement on the opposite direction should be captured by the mean reversion force and the Brownian volatility.

We have calibrated the curve so that we could match the (downward) peaking levels of electricity prices with around April but with a spreading level that allows for those events to take place from December until June, which was accomplished by filtering out seasonality and upward jumps. For those we believed that they could be addressed by a strong enough mean reverting force altogether with the Brownian volatility.

Finally, in order to calculate the parameters of the stochastic process we applied the same reasoning as in Villaplana (2003), i.e. the model is presented in its continuous form, but worked on under the discrete time version. Since these are very complex methods, trying to solve them in their continuous form has proven to be a demanding task (and computationally intensive) and for that reason we have also decided to discretize the model under the assumption that although the discretization of continuous-time stochastic equations has proven to introduce biased estimations, such bias can be minimized by shortening the sampling interval. For that reason, and since we are investigating baseload prices (daily average) we have chosen to discretize our model into a daily time step $dt = \frac{1}{365}$.

The discretization process we used can be found in Financial Instruments Toolbox™ (2018). The process applied to (40) is that of a Bernoulli process for the time step defined, i.e. there is at most one jump per day. For our model these events will tend to concentrate on those months that are determined by the previous function. Given that, the model reads as:

$$X_t = \alpha\Delta t + \emptyset X_{t-1} + \sigma\xi \tag{42}$$

Translating the equation – where $\emptyset = 1 - \kappa\Delta t$ to assure that if the force driving the price back to normal is too low $\emptyset = 1$ – we understand that the probability of a jump taking place is close to zero which forces the stochastic process to be fully dependent on the random movements attributed by the Brownian motion. When the function $s(t)$ starts to "kick in" the process becomes:

$$X_t = \alpha\Delta t + \emptyset X_{t-1} + \sigma\xi + \mu_j + \sigma_j\xi_j \tag{43}$$

where ξ and ξ_j are independent standard normal random variables. From here we can draw the density function (given by the equations (44) to (46))that drives X_t conditional to X_{t-1} in order to withdraw the parameter vector that we will want to calibrate:

$$v = \{\alpha, \emptyset, \mu_j, \sigma^2, \sigma_j^2, \theta\}$$

$$f(X_t|X_{t-1}) = (\theta s_t\Delta t)N_1(X_t|X_{t-1}) + (1 - (\theta s_t\Delta t))N_2(X_t|X_{t-1}) \tag{44}$$

$$N_1(X_t|X_{t-1}) = \left(2\pi\left(\sigma^2 + \sigma_j^2\right)\right)^{-\frac{1}{2}} \exp\left[\frac{-\left(X_t - \alpha\Delta t - \emptyset X_{t-1} - \mu_j\right)^2}{2\left(\sigma^2 + \sigma_j^2\right)}\right] \tag{45}$$

$$N_2(X_t|X_{t-1}) = \left(2\pi(\sigma^2)\right)^{-\frac{1}{2}} exp\left[\frac{-(X_t - \alpha\Delta t - \emptyset X_{t-1})^2}{2\sigma^2}\right] \tag{46}$$

For the calibration of the parameters, instead of using the traditional maximum likelihood we will use an equivalent approach since we will aim to minimize the negative logarithmic likelihood. That is because MATLAB® is incorporated with a tool that uses machine learning techniques. So instead of maximizing the product of the probabilities at each point (likelihood) the logarithmic scale is used due to similar reasons as presented before (subsection 7.1) and due to the fact that this measure produces tangible results for machine learning (remember the difference of adding and multiplying). This way through the parameters the algorithm is trained to produce the smallest number mispricing at each point.

8.3. Results & Discussion

As in Geman & Roncoroni (2006) we will start this section by presenting the results of our model and their implications, and later on we will discuss the statistical quality of such representation. We will hence start by presenting the values obtained for the structural parameters, as seen in **Erro! A origem da referência não foi encontrada.**.

Table 10 – Parameters provided by data fitting via least squares.

$R^2 = 0{,}23$ $p - value \ll 0{,}05$		
Parameter	**Interpretation**	**Estimate**
$\boldsymbol{\theta_1}$	Average (log) fixed cost/price level	3,5165
$\boldsymbol{\theta_2}$	Average (log) cost slope	0,0798
$\boldsymbol{\theta_3}$	Yearly trend	0,3177
$\boldsymbol{\theta_4}$	Trend displacement	1,6581
$\boldsymbol{\theta_5}$	6-month trend	0,1855
$\boldsymbol{\theta_6}$	Trend displacement	0,3700

The low figure obtained for R^2, might induce us into thinking that a seasonality with such a low correlation with empirical data cannot be used to explain the process. However, the model obtains a p-value far below the rejection point, which means that there is a high degree of confidence in adding this model to our simulation as from the predictor's perspective its changes are related to changes in actual data.

Observing the table (and calculating its exponential) we can argue that according to our seasonal pattern the electricity fixed price cost per unit averages around $33€$, which by comparison to empirical data that shows an average electricity cost for the same period differs in $12€$ ($\pm 45€$). This would be expected as our model is based on data provided by a gross market, where one still has to add to the final price tariffs and other costs that are not accounted for at this stage. Although the question about the price making in this industry is subject to extensive debate, based on our model we believe that the slope associated with the average tendency for the price is small due to (possibly) the presence of renewable sources. However, the market at the present day (2018) has not been responding in the same manner. Remember when we said previously that our market, unlike others in Europe, has a player capable of manipulating the gross market due to its share.

Finally, although θ_3 prevails over $\theta_5$we were expecting a bigger difference since from visual analysis of empirical price data we would suggest that a yearly trend would be clear whilst a 6-month would only be residual. However a downward tendency during summer months could be explained by a consumption decrease and an all growing photovoltaic industry. Even more so if we consider that during the last few years solar technologies have been the ones growing the most, which could also help explain why the seasonal trend deviates from the actual data during those summer months in the last two years (Figure 26).

To end the structural parameters one only has to address the deterministic function created to express the spike clustering.

Table 11 – Jump intensity shape s(t) parameters.

Parameter	Interpretation	Estimate
τ	Jump intensity displacement	0,722
κ	Jump periodicity	1
d	Clustering	1,5

Since $\cos\left[\frac{\pi(t-\tau)}{k}\right]$ is from our understanding aiming to capture a spike clustering moment, we had to set $\tau = 0{,}722$ which makes $12 \times (1 - 0.722) = 3{,}336$. Hence, these events will peak around March-April as expected. However it is important to note that there is a slight deviation due to the fact that our data spans from 1/01/2013 to 17/12/2017 that makes the period uneven for the computation[55]. In summary we have created such a function that determines one clustering moment per year ($k = 1$) but able to spread from December to June.

Finally by running the algorithm[56] to compute the statistical estimation of the model parameters we get:

Table 12 – Model parameters.

Parameters	Interpretation	Estimate
α	Mean reverting speed parameters	4,3468
κ		287,3376
σ	Volatility	4,8003
θ	Number of spikes per unit time[57]	-1
μ_j	Jump size mean	0.6689
σ_j	Jump size deviation	0.05

Since this model was created to account for positive jumps only, as Geman & Roncoroni (2006) suggest, it seems only logical that adapting it to capture downward movements is possible but probably at the expenses of some misguiding results. Usually mean reverting effects take place during times of price pressure in the electric system, which naturally tries to bring them back to normal. The problem is that when the same "treatment" is applied to downward jumps it might become an unrealistic assumption, as it does not account for the weather factor that determines the mean reversion force. Despite that, and despite the fact that the reverting forces are very high, the results we obtained are very satisfactory from the statistical perspective. Path wise however we feel somehow reluctant to comment on its significance since we were not able to investigate all 10.000 simulations. However, due to the statistical quality of the data obtained, one might think that among those simulations there can be some that better fit the path followed by empirical data and, consequently, we are able to suggest that perhaps strong reverting forces can also be used to explain how weather variables also tend to mean revert.

It is on that premises that we are finally able to address the quality of such model. However it is important to notice that after the maximum likelihood process we have ran several simulations (10.000) for small alterations on those parameters and then computed the average of such results.

[55] The reason for that lies behind the fact that we used t as fraction of a year. Function #yearfrac in MatLab®.

[56] By running on MatLab an algorithm to optimize a function by varying the set of constrained variables. ('optimfun','fmincon')

[57] We have modeled the number of spikes as a random number (0,1) with equal probability. Hence this parameter from the original formula stands for us as the direction of the event.

That has allowed us to state that the model we now present is one that adequately fits both requirements, as seen in Figure 28 and **Erro! A origem da referência não foi encontrada.**:

Table 13 – Statistical moments matching for empirical data and simulated.

	Empirical	Simulated
Average	0,000331	0.000375
Standard Deviation	0,379371	0,254794
Skewness	-0.34398	-1,98287
Kurtosis	42,8778	42,78973

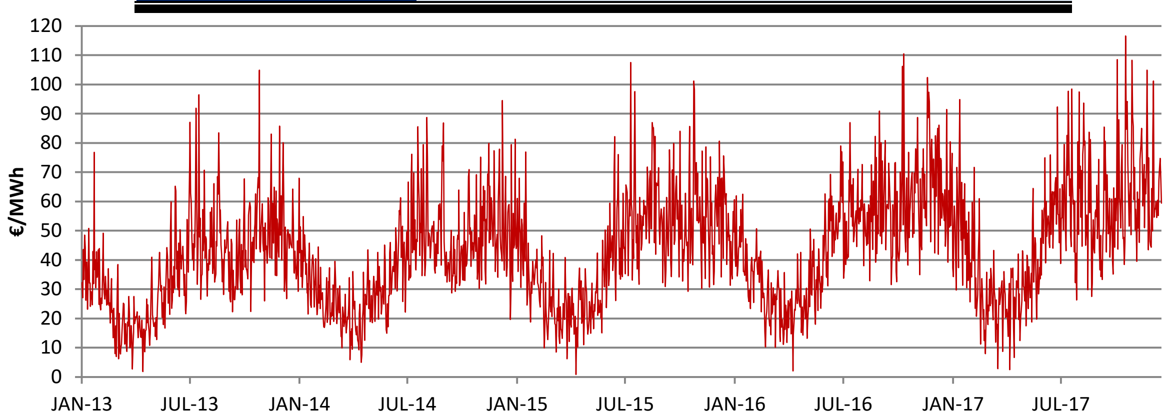


Figure 28 – Jump-Diffusion model path

The latter two indicators are the ones we find more relevant financially speaking. i.e. starting with the skewness one can observe that both empirical and simulated data are negatively skewed which means that they both have their “tails” pointing left. The implications of this from the financial perspective are that returns below (to the left of) the average occur less frequently but have higher magnitudes meaning that there can be major negative price returns movements, which can be rather interesting when considering short positions. On the other hand kurtosis is also an important indicator, and one we were able to perfectly match, in the sense that it reveals the degree to which exceptional values occur. In our case a high kurtosis number reveals us that these extreme events can be very frequent and this information not only can be useful in finance, but also in decision making for electricity markets' designing.

Although the model produced such satisfactory results in statistical terms we must point out that, as mentioned earlier in this section, this was only possible at the cost of some assumptions that did indeed influence the results in what concerns the path followed. These assumptions created the possibility for the market to reach maximums that were never observed from actual data history. The main cause for this effect is related to the mean reverting force that the algorithm had to apply in order to bring the prices back to normal after a downward spike clustering moment. Since the mean reversion remains constant over time, it means that it is still “working” during normal periods where only the Brownian motion and the seasonality are in place. I.e. every time there is a random movement (caused by the Brownian volatility) the mean reverting force will induce a movement in the opposite direction. Moreover, as many authors have suggested, this model removes the smoothness of the mean reverting process since it is calibrated to counteract the force of a spike event, i.e. the reverting force is so high that after a jump event any standard move following the shock disappear altogether. Despite this drawback, we have decided that the model was still an adequate representation for what we intended with this investigation.

9. Option Pricing

Ideally at this stage we would like to perform an analysis based on the same assumption as the one used when presenting the model for our structured product. I.e. we would like to be able to use a market based on an index that relies on the share of renewable energy being injected on the grid. Since such information is still not available in our country we will use the model created previously whilst trying our best to correlate it with what we are aiming to present – a financial instrument to attract more investment on renewable energies.

To correctly proceed with our investigation we should now estimate the risk premium for the market in order to properly price options in this asset. However, due to the reasons we have posed previously, regarding the difficulties we were faced when gathering information from the market, we were not able to do so since we would need to have access to the futures pricing curve and then implement it to our model. Such task could be done by:

$$risk\ premium = E(S_T) - F_{t,T} \qquad (47)$$

where, $E(S_T)$ is the expected spot price from our calibrated model and $F_{t,T}$ is the price that traders are willing to pay today for delivery in the same date as the spot. After this we would suggest following the steps clearly provided in Seifert & Uhrig-Homburg (2007) and presented in Appendix E – risk neutral measure.

On that note we will carry on with our investigation under the real probability world and take it as assumption not too far from reality since for both types of measures the set of all future admissible scenarios are the same. Note that we are aware that this assumption is far from the correct approach one should use, however due to the fact that information is inaccessible for us we will have to accept it. Moreover our only goal here is to provide an explanation on how our structured product's variable component would work and for that we only need the simulated market we have whichever risk measure is used.

Given that, the analysis that we provide hereafter is based on the spot price dynamics obtained with our model under the real world measure, since the model was calibrated to historical data. Moreover we start this analysis using the Black Scholes Merton formula, despite the fact that many authors argue its applicability in electricity markets, as Geman (2005) when arguing about the collapse of the main assumption that is the possibility of continuous trading making possible to dynamically adjust the hedging strategy.

To start with the BSM model, we randomly chose one of the paths obtained from the (10.000) simulations we ran for our model and will assume this to be a good representation to investigate the pricing formula. As we mentioned previously we wanted to present the BSM model despite the fact that we are aware that it fails in electricity markets, due to the fact that the Greeks provide with very useful information, as seen on the example provided in (Appendix F – Black Scholes Calculator).

On a side note, although this simulation is not dependent on the performance of an index regarding injected (renewable power) it would still be possible for the investor who believes in renewable energy to enter the market likewise. Hence a position to take on that trend would be to short a call option or going long on a put. I.e. believing the price will fall could be a synonym of believing that in the near future endogenous sources would be bursting. However, shorting a call option carries the risk that if the market rises, the investor will be obligated to buy the asset for the agreed price. For that reason we invite the reader to investigate the different strategies provided in Appendix C – strategies with optionsas well as in

Given that, what we are interested here however is to investigate the difference between the pricing methods especially because we our price simulations follow a model that is not suited for the BSM formula since it accounts for the occurrence of jumps and is not a pure geometric Brownian motion. And that is why we have decided to use it against another model that is very commonly used in electricity derivatives, i.e. the Monte Carlo. Given that we have 10.000 simulations that are statistically fit to empirical data, we will assume this to be a good measure for deciding on whether the BSM is a good indicator or not. Remember that this later model is much easier to use! To accomplish this task we created the scenario presented below for pricing options starting on 28/12/2012 and with an initial price of 42,56€ (gathered from historical data).

Table 14 – Pricing options using two different methods. Values calculated for three maturities (in months) and three different strike prices to cover ITM, ATM, OTM.

months	BSM σ=25,5%						Monte Carlo					
	k=20		k=35,5		k=50		k=20		k=35,5		k=50	
	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
T=3	15,59	0,00	1,56	1,42	0,00	14,32	3,55	1,19	0,10	13,20	0,00	27,55
T=6	15,70	0,00	2,64	2,30	0,08	14,11	22,11	0,00	8,14	1,39	1,70	9,32
T=12	15,92	0,02	3,88	3,20	0,50	14,04	26,45	0,00	12,00	0,76	3,57	6,56

It is clear from the table that the model that estimates the value under the Monte Carlo simulations is completely biased. That is, due to the fact that we were not able to compute the risk neutral measure, the market being used is the one we simulated and that was obtained with likelihood functions. I.e. it takes into account (and strongly relies on it) the real probability of occurrence of the events. As a consequence the more likely a given outcome is the more overpriced an option written on that outcome will be.

This will become even more evident when the volatility used in the BSM is the one provided from the standard deviation of the returns on our simulations (remember from **Erro! A origem da referência não foi encontrada.**) as it is our case. I.e. when considering short maturities (as in the first row) combined with a low volatility (as compared to the actual historical volatility) turns out it under prices the put option and completely overprices the call counterpart. The same contract on the Monte Carlo simulation is priced under the probability of in three months the price being trading at 20€/MWh, which is very likely. The BSM on the other hand takes as a principle the risk neutrality (although the input we provided is not) and hence overprices the possibility of buying the asset for less than 50% of its value today.

10. Conclusion

Although policy making concerning Renewable Energy has a great influence in Portugal, it cannot be considered the essential driver, or at least not alone. With or without policies, private capital has been growing in seeking opportunities for investment, since there is a sense of awareness that renewable energies do provide with a diversification effect on investment portfolios as “the return-risk ratio of a wind park and a set of solar power generation facilities together are more likely to be more favorable than the individual ones” Wüstenhagen & Menichetti (2011). Failing to see this implies being left behind on the race to the forefront of renewable energy development. Policy making is required no longer to subsidize but rather to allow and promote new environmentally and socially responsible solutions that target the increase in installed capacity in renewable sources, whilst avoiding harmful economic practices.

The liberalization of a regulated energy market came as an opportunity to outside investors to take part in the sector. As the former monopoly status of the electric sector did not prove to contribute to its efficient operation, the deregulation of the market had to take place. Removing control over the prices and leaving it subject to market competition provided benefits for consumers and private entities. However there is still a lack of financial assets/instruments that allow any investor to take part in this still apparently monopolized (by some) industry. As a consequence those instruments are still scarce and exclusive to a minor fraction of the population.

Under these premises we instated on our agenda to present the framework required to financially engineer a product that allows any investor to become active. We have focused on a socially responsible structured product that would protect companies issuing those products, companies willing to abandon traditional feed in tariffs by tax reduction and also and most importantly protecting the small investors who have been so harmed over the years due to irresponsible practices. This is where policy makers now become major players, i.e. in promoting the creation of new investment vehicles, while bearing in mind that constant monitoring is important to ensure that neither the financial market and electricity market, nor retail investors and consumers can be harmed for introducing such strategy.

By reducing taxes on financial products, such as the one we propose, and that help address the issue of gathering new renewable energy investment could be the first and most immediate solution towards sustainability, in the sense that it could help encourage retail investors to save a part of their income and use it to grow capital on renewable energy companies and consequently increase production. The market could benefit from it, especially if we consider major European players that are struggling to find solutions to replace nuclear power stations (such as Germany that aims to shut down 17 nuclear power stations by 2022) and other conventional technologies. Portugal has the resources to conquer a big market share in supplying the rest of Europe with clean energy. But that requires investment; i.e. private investment!

Financial institutions should also be encouraged to create these products through tax benefits, i.e. profits that are derived from these products should be taxed differently than the others. However those products should at all times be kept transparent and open to constant monitoring from public entities.

The reason why we had to assume that a structured financial product would be the best instrument is its versatile nature. On one side, we are allowed to choose a structure that can partially guarantee a full capital recovery, which can be good if one assumes that most of the products that banks use for this could be directly involved in financing companies in the green energy business in the form of debt securities, e.g. bonds. The benefits of this for investors are obvious, in the sense of risk mitigation. On the other side of the product – the part that deals with derivatives – we have a certain degree of freedom to design it in the most suitable way to target the market of interest. From a variety of instruments available and with some form of financial engineering, we can choose one and adapt it in order to take advantage of the market, in a sustainable manner.

However, we should note that we firmly believe that the success of such product would be dependent on the capacity of separating (or at least tracking) the percentage of energy injected in the grid from renewable sources. This way it would be possible to create a product specifically designed to incentivize new investment in installed capacity by allowing investors to take a share of the risk and profits.

Although such task was not possible, we were still able to draw some conclusions about the market. I.e. it has come to our attention that this is a very sensitive market that cannot be addressed with traditional techniques due to the nature of the underlying asset itself. We have proven that there are models already available to tackle this issue and they only depend on the transparency of the markets they are integrated in. Shamelessly we were not able to provide the flawless analysis we wanted due to several difficulties we encountered throughout the project, namely inaccessible data, mathematical processes that have proven to be too sophisticated for our comprehension and also computer requirement limitations.

However, solely by conducting this investigation we have proven that a financial structured product does not have to be by nature an incomprehensible instrument. We even dare to go further and argue that the more transparent one can make such product the bigger the success. Also the electricity market could adapt to this statement, as we do believe that transparency promotes liquidity!

It is important to know that this was only an attempt to produce a different (and yet interesting) point of view in what deals with renewable energy investments. We recognize that the "adaptations" we were able to produce could have been enriched with other tools that we have failed to apply due to the fact that it would require a much more extensive work to do so, however we feel compelled to briefly discuss them as follows as means to suggest future works on this matter.

To understand the real possibility of including such enriched products in both our financial markets and electricity markets would require a much more elaborate model in order to improve the quality of the results. Models that are adapted to the reality of our electrical system, taking into consideration variables that address real demand, weather dependence (in demand and production), gas and coal prices, etc. Evidently the model we employed does not cover all this, since its simplicity is only adequate for preliminary investigations, as it is our own. Moreover if one was to issue such products, by law it would be required to conduct a risk analysis, not only to protect potential investors but also the electrical system. This risk analysis should even cover the possibility of default on the capital guarantee component, since recent events from financial institutions in our country have taught us likewise.

It would also be extremely useful to understand how the actual price making in Portugal takes place. That is, a detailed investigation would be required to draw some conclusion in what concerns the influence of major players in Portugal's electricity market. What this means is that the fool extent to which any given player could manipulate the price in its favor should be understood in order to fully uncover the real risk behind our product. For all we know from economy basics, any player with a large share of both production and consumers could easily guide the prices in a favorable manner to its operation and consequently a financial product could be used wrongfully. It is relevant to understand this impact on our proposal.

We firmly believe that we have taken the first steps in an investigation that should by no means be simplified. Starting by correctly modeling the markets (under the assumption that tracking injected power by sources is possible) in order to correctly price financial derivatives are the major challenges. The rest comes from education, i.e. educating financial institutions and companies that socially responsible products can be created and, in our opinion should be compensated.

Bibliography

[1] APREN. (n.d.). *Peso da Electricidade de Origem Renovável*. Retrieved from APREN - Associação das Energias Renováveis: http://www.apren.pt/pt/dados-tecnicos-3/dados-nacionais-2/producao-2/a-producao-de-eletricidade-em-portugal-3/peso-da-eletricidade-de-origem-renovavel/

[2] Banco BPI. (n.d.). *Particulares*. Retrieved from BPI: http://www.bancobpi.pt/particulares/poupar-investir/produtos-estruturados

[3] Bärholm, P. (2017). Master of Science Thesis. *Implied volatility smile dynamics in the presence of jumps*. Tampere University of Technology.

[4] Barnett-Hart, A. K. (2009). *The Story of the CDO Market Meltdown: An Empirical Analysis.* Harvard College.

[5] Benth, F. E., Kallsen, J., & Brandis, T. M. (2007). A Non-Gaussian Ornstein-Uhlenbeck Process for Electricity Spot Price Modeling and Derivatives Pricing. *Applied Mathematical Finance 14*, 153-169.

[6] Benth, F. E., Kiesel, R., & Nazarova, A. (2012). A critical empirical study of three electricity spot price models. *Energy Economics 34*, 1589-1616.

[7] Bjønnes, G. H., & Saakvitne, J. (2015). *Electricity Derivatives and Bank Guarantees.* Working Party on Bank Guarantees in European Energy Markets.

[8] Black, F. (1988, March). How to Use the Holes in Black-Scholes. *Risk Management 1*, 455-461.

[9] Boz, E., & Mendoza, E. G. (2013). Financial Innovation, the Discovery of Risk, and the U.S. credit crisis. *Journal of Monetary Economics 62*, 1-22.

[10] Breuer, W., & Perst, A. (2007). Retail Banking and Behavioral Financial Engineering: The case of Structured Products. *Journal of Banking & Finance 31*, 817-844.

[11] Buffett, W. E. (2003). *Berkshire Hathaway INC. 2002 Annual Report.*

[12] Burger, M., Klar, B., Müller, A., & Schindlmayrt, G. (2006). A spot market model for pricing derivatives in electricity markets. *Journal of Quantitative Finance 4*, 109-122.

[13] Cartea, Á., & Figueroa, M. G. (2005). Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality. *Applied Mathematical Finance 12*, 313-335.

[14] Castro, R. (2012). *Uma introdução às Energias Renováveis: Eólica, Fotovoltaica e Mini-hidrica.* IST Press.

[15] Célérier, C., & Vallée, B. (2015). Catering to Investors Through Product Complexity. *ESRB Working Paper Series 14*.

[16] Célérier, C., & Vallée, B. (2017). Catering to Investors through Security Design: Headline Rate and Complexity. *Quarterly Journal of Economics, Forthcoming*.

[17] *Contratação*. (s.d.). Obtido de Entidade Reguladora dos Serviços Energéticos: http://www.erse.pt/pt/supervisaodemercados/mercadodeelectricidade/contratacao/Paginas/default.aspx

[18] Couture, T., & Gagnon, Y. (2010, February). An Analysis of the Feed-in Tariff Remuneration Models: Implications for Renewable Energy Investment. *Energy Policy 38*, 955-965.

[19] Decreto-Lei n.º 189/88, N.º 123 (Diário da República - I Série 27 de Maio de 1988).

[20] Decreto-Lei n.º 225/2007, N.º 105 (Diário da República - 1.ª Série 31 de Maio de 2007).

[21] Direção Geral de Energia e Geologia. (2016). *Renováveis - Estatísticas Rápidas.*

[22] EDP distribuição. (2017, December). *Quotas em Regime de Mercado*. Retrieved from EDP distribuição: https://www.edpdistribuicao.pt/pt/profissionais/EDP%20Documents/quotas_mercado.pdf

[23] EEX. (n.d.). *Phelix Power Futures Presentation.* Retrieved from European Energy Exchange: https://www.eex.com/blob/9160/363d1af22e2c87d23c58dec4e4e89db6/phelix-futures-presentation-data.pdf

[24] EEX. (n.d.). *Phelix-DE Options | EEX Power Derivatives*. Retrieved from EEX: https://www.eex.com/en/market-data/power/options/phelix-de-options#!/2018/04/13

[25] Entidade Reguladora do Sector Eléctrico. (2002). *Modelo de Organização do Mercado Ibérico de Electricidade.*

[26] Escribano, A., Peña, J. I., & Villaplana, P. (2002). Modeling Electricity Prices: International Evidence. *Economics Series 8, Working Paper*, 2-27.

[27] EuroWind. (s.d.). Obtido de www.eurowind.info

[28] Fender, I., & Scheiche, M. (2008, September). The ABX: How do the Markets Price Subprime Mortgage Risk? *BIS Quarterly Review*, pp. 67-81.

[29] Ferguson, C. (Director). (2010). *Inside Job* [Motion Picture].

[30] Fundação Francisco Manuel dos Santos. (2018, 01 31). *PORDATA*. Retrieved from Energia: https://www.pordata.pt

[31] Geman, H. (2005). *Commodities and Commodity Derivatives - Modeling and Pricing for Agriculturals, Metals and Energy.* John Wiley & Sons, Ltd.

[32] Geman, H., & Roncoroni, A. (2006). Understanding the Fine Structure of Electricity Prices. *The Journal of Business 79*, 1225-1261.

[33] Gibson, M. S. (2004, July). Understanding the Risk of Synthetic CDOs. *Working Paper, Board of Governors of the Federal Reserve System*.

[34] Gil, J. T. (2010, October). Análise e Previsão da Evolução do Custo da Electricidade. *Master Thesis*.

[35] Griggs, D. (2013). Sustainable Development Goals for People and Planet. *Nature's International Weekly Journal of Science 495*, 305-307.

[36] Hafner, C. M. (2003). Simple approximations for option pricing under mean reversion and stochastic volatility. *Computational Statistics 18*, 339-353.

[37] Instituto Português do Mar e da Atmosfera. (2014). *Boletim Climatológico Anual.*

[38] International Energy Agency. (2016). *Energy Policies of IEA Countries - Portugal 2016 review.*

[39] International Energy Agency. (2016). *Key World Energy Statistics.*

[40] International Monetary Fund. (2008). *Global Financial Stability Report: Containing Systemic Risks and Restoring Financial Soundness.*

[41] Janczuna, J., Trück, S., Weron, R., & Wolf, R. C. (2013). Identifying spikes and seasonal components in electricity spot price data: a guide to robust modeling. *Energy Economics 38*, 96-110.

[42] Kalemli-Ozcan, S., Bent, S., & Sevcan, Y. (2011, August). Leverage Across Firms, Banks, and Countries. *NBER Working Paper Series*.

[43] Lee, C. W., & Zhong, J. (2014). Top Down Strategy for Renewable Energy Investment: Conceptual Framework and Implementation. *Renewable Energy 68*, 761-773.

[44] Lewis, M. (2010). *The Big Short: Inside the Doomsday Machine.* W. W. Norton & Company.

[45] Li, L. W., Birmele, J., Schaich, H., & Konold, W. (2012). Transitioning to Community-owned Renewable Energy: Lessons from Germany. *Procedia Environmental Sciences 17*, 719-728.

[46] Lucia, J. J., & Schwartz, E. S. (2002). Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research 5*, 5-50.

[47] MATLAB. (2018). Financial Instruments Toolbox™. *User's Guide*.

[48] Matos, J. A. (2008, October). Lecture Notes. *Derivatives*. Nova SBE.

[49] *Mercado a Prazo*. (n.d.). Retrieved from Entidade Reguladora dos Serviços Energéticos: http://www.erse.pt/pt/supervisaodemercados/mercadodeelectricidade/mercadoaprazo/Paginas/default.aspx

[50] *Mercado Diário*. (n.d.). Retrieved from Entidade Reguladora dos Serviços Energéticos: http://www.erse.pt/pt/supervisaodemercados/mercadodeelectricidade/mercadodiario/Paginas/default.aspx

[51] Myers, D. (2016). *New Economic Thinking and Real Estate.* Wiley Blackwell.

[52] Nord Pool Group. (2010, October 13). *Newsroom*. Retrieved from Nord Pool: https://www.nordpoolgroup.com/message-center-container/newsroom/exchange-message-list/2010/10/No-542010-NPS---Nord-Pool-Spot-AS-issues-written-warning-for-insider-trading-and-breach-of-disclosure-requirements/

[53] Nunes, J. P. (2015, April). Lecture Notes. *Applied Derivatives*. Nova SBE.

[54] OMIP. (n.d.). *Derivatives Market - Products.* Retrieved from OMIP (The Iberian Energy Derivatives Exchange): http://www.omip.pt/MarketInfo/Produtos/tabid/76/language/en-GB/Default.aspx

[55] Pinto, A. S. (2013). *Master Thesis: "Os produtos Estruturados Colocados no Mercado Português: Evolução das Características e da Complexidade".* Faculdade de Economia da Universidade do Porto.

[56] Plano Nacional de Acção para as Energias Renováveis, Directiva 2009/28/CE (2009).

[57] PLMJ Sociedade de Advogados, RL. (2014, January). Informative Note - Energy and Natural Resources. *Renewable Energies*.

[58] Seifert, J., & Uhrig-Homburg, M. (2007, January). Modeling jumps in electricity prices: theory and empirical evidence. *Review of Derivatives Research 10*, 59-85.

[59] Silva, B. (2018, April 4). Governo avança com certificação de eletricidade renovável até 2019. *Diário de Notícias*.

[60] Stulz, R. M. (2010). Credit Default Swaps and the Credit Crisis. *Journal of Economic Perspectives 24*, 73-92.

[61] Thornton, B. (2010, February 5). Electricity Spot Price Modelling and Derivatives Pricing. *Master of Advanced Studies in Finance*. Zürich.

[62] Timmons, D., Harris, J. M., & Roach, B. (2014). A GDAE Teaching Module on Social and Environmental Issues in Economics. *The Economics of Renewable Energy*. Global Development And Environment Institute, Tufts University.

[63] Tymoigne, É. (2009, August). Securitization, Deregulation, Economic Stability, and Financial Crisis, Part I - The Evolution of Securitization. *Levy Economics Institute, Working Paper 573.1*.

[64] Villaplana, P. (2003). Pricing power derivatives: A two-factor Jump-Diffusion approach. *Business Economics Series 5, Working Paper*, 3-18.

[65] von Möllendorff, C., & Welsh, H. (2015). Measuring Renewable Energy Externalities: Evidence from Subjective Well-Being Data. *SOEPpapers on Multidisciplinary Panel Data Research*, 779-2015.

[66] Weistroffer, C. (2009, December 21). Deutsche Bank Research. *Credit Default Swaps: Heading Towards a More Stable System*.

[67] Weron, R. (2008). Market price of risk implied by Asian style electricity options and futures. *Energy Economics 30*, 1098-1115.

[68] Wüstenhagen, R., & Menichetti, E. (2011). Strategic Choices for Renewable Energy Investment: Conceptual Framework and Opportunities for Further Research. *Energy Policy 40*, 1-10.

[69] Yildiz, Ö. (2014). Financing Renewable Energy Infrastructures via Financial Citizen Participation - The case of Germany. *Renewable Energy 68*, 677-685.

Appendix A – "German Austria Power week future"

German-Austrian Wind Power Week Future

Name	Best Bid	Best Ask	No. of Contracts	Last Price	Abs. Change	Last Time	Last Vol.	Settl. Price	Vol. Exchange	Vol. Trade Registration	Open Interest Prev. Day
Week 50/17	-	-	-	-	1.67	-	-	38.90	-	-	25
Week 51/17	-	-	10	30.80	-3.40	12:12	1,680	30.38	1,680	-	10
Week 52/17	-	-	-	-	-1.30	-	-	43.20	-	-	-
Week 1/18	-	-	-	-	-	-	-	34.00	-	-	-
Week 2/18	-	-	-	-	-	-	-	35.10	-	-	-

Figure 29 – "German-Austria power week future" trading on the 13th December 2017 | As seen on EEX

Figure 30 – Settlement price path example as an indicator on what to expect. | As seen on EEX

Although we were not able to access some of the information (possibly due to the fact that we are not a registered trader at the platform) we can proceed with a brief explanation on each of the parameters that one might encounter in such trading platforms.

Starting with the "Best Bid" (and "Best Ask") we can easily understand that it refers to the highest priced buy order placed by a trader, i.e. it is the highest amount of money a trader is willing to pay to go long at that moment. It is useful information for the trader in the sense that it allows for some degree of perception on how he/she will have to place his/her offer, due to the obvious fact that in a trade there is a buyer and a seller, hence what we offer might not have a seller willing to accept. Analogously the "Best Ask" refers to the lowest registered sell order.

The joint action of both a seller and a buyer make one contract, which leads us another indicator in the table: "No. of Contracts". For every pair buyer/seller on whom there is an agreement there is said to exist a contract. Hence, from the time a buyer (or a seller) opens a new bid (or ask) until there is a counter party to agree, it remains open. However we should note that the same contract can change hands several times during the day, for as long as there is a buyer and a seller, but the number of contracts will stay the same, unless a new position is opened. For this reason we will jump to the final column of the table, since it represents the number of "open positions" left from the previous trading day. In other words, "Open Interest" refers to the total number of open or outstanding derivative contracts (that have not been settled nor delivered) and it is an indicator on what market participants are anticipating. By making use of expressions such as Bearish or Bullish we will go over this indicator further on.

The "Last Price" is like the name suggests the last known price the market traded before it officially closed, which in this case was at 12:12 as in "Last Time". This price leads us to the next column – "Abs. Change" – which shows us the difference between the "Last Price" and the settlement price for the previous trading session (in our case on the 12th December was 34.20€) hence the -3.40€. The "Settl. Price" on the other hand characterizes derivatives markets as they are standardized contracts that derive their value from the average price of the contract over the trading day (unlike equity markets). This price is an indicator on margin requirements for each trader hence this is the price that is used to mark-to-market on the trading day, which means that it is this one to look at when investigating profit and losses.

Finally we have the "Vol. Exchange" as in volume exchanged. This column is sometimes confused with the open interest one. Unlike the latter one, every time an already opened position changes hands the volume exchange figure changes, i.e. suppose you hold 5 of these future contracts and you decide to sell them. Since you are not opening a new position, after you sell them the number of contracts will not change, unlike the volume exchanged. Even if this means that a new trader entered the market. From our table we can interpret that on that day between open positions and contracts an amount equal to 1,680 exchanges took place.

Appendix B – example of required information on a structured financial contract

PRODUTO FINANCEIRO COMPLEXO

Um investimento responsável exige que conheça as suas implicações e que esteja disposto a aceitá-las

SG Europa e EUA EUR 2017-2022

ISIN: XS1619502203

Emitente: SG Issuer, sociedade anónima de direito luxemburguês (*société anonyme*), com sede social em 33, boulevard Prince Henri, L-1724 Luxemburgo, Luxemburgo.

1. Advertências Específicas aos Investidores

Este produto financeiro complexo:

- **Pode implicar a perda da totalidade do capital investido;**
- **Pode proporcionar rendimento nulo ou negativo;**
- **Proporciona uma taxa de rentabilidade inferior à exigida pelos investidores institucionais para níveis de risco idênticos;**
- **Pode ser reembolsado antecipadamente por verificação de evento de reembolso antecipado automático;**
- **Pode ser cancelado antecipadamente por opção do Emitente (SG Issuer) ou do Agente de Cálculo (Société Générale), caso em que os Investidores receberão um montante que poderá no limite ser nulo, perdendo os Investidores a totalidade do capital investido;**
- **Está sujeito ao risco de crédito do Emitente (SG Issuer) e do Garante (Société Générale);**
- **Implica ou pode vir a implicar que os Investidores suportem custos, comissões ou encargos;**
- **Implica ou pode vir a implicar que os Investidores suportem custos de cobertura de risco do Emitente ou outros;**
- **Está sujeito a potenciais conflitos de interesses na atuação do Emitente (SG Issuer) e do Agente de Cálculo (Société Générale);**
- **Não é equivalente à aquisição ou transação inicial dos ativos subjacentes;**
- **Não proporciona uma rentabilidade idêntica à taxa de variação dos Indexantes;**
- **Este produto financeiro é especialmente complexo e pode ser de difícil entendimento por investidores não qualificados.**

Texto manuscrito pelo Cliente "Tomei conhecimento das advertências":

Figure 31 – Fundamental Information to the Investor | As seen on the original document (Banco BPI)

At first sight the investor is faced with the summary of the risk inherent to the product – on the top right corner. From the figure one can state that this product carries the maximum risk, which is why it has a detailed message that reads "all capital invested is at risk to be fully lost". By itself, this warning could be enough, however the law states that the intrinsic risks should be specified, which is why there is a list called "specific warnings to investors" where all the implications involving the structured product are presented. At the bottom, there is a space for the client's signature which is worth mentioning because it is presented before all the details of the contract, meaning that before going into the specifications of the product (namely possible returns, etc) he/she is expected to read and agree to those warnings.

Appendix C – strategies with options

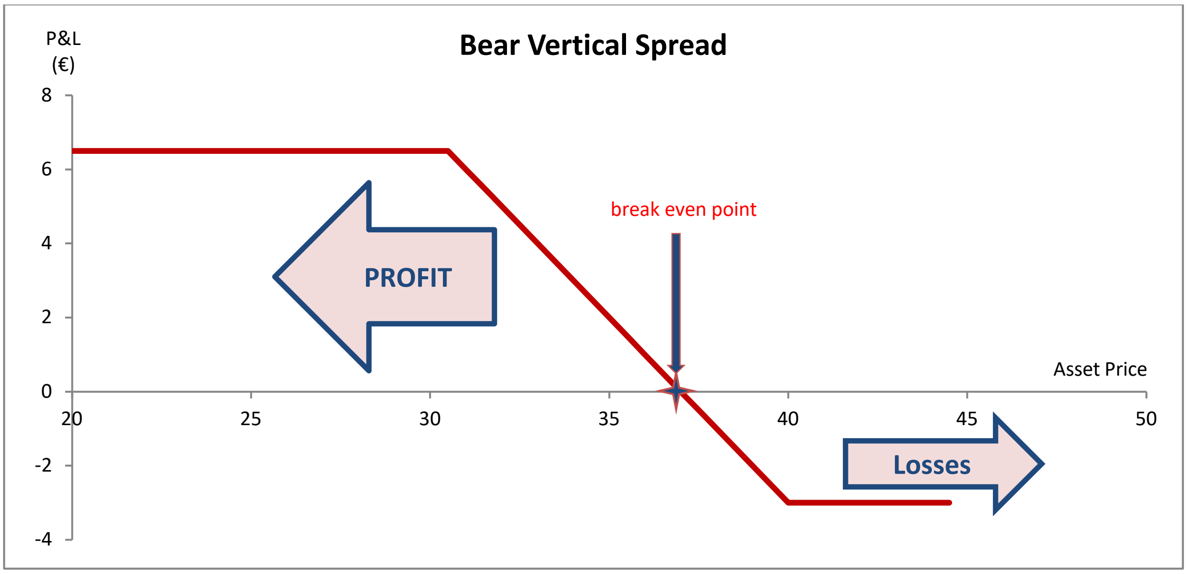


Figure 32 - Bear Vertical Spread

At time (t=0) an asset is trading at a price equal to the breakeven point in the figure and the **expectancy** is for the value to **decrease**. Buy an out-the-money call (long position) and Sell an In-the-money call (short position) – both with the same maturity date. The balance between the two generates a positive credit, which corresponds to the maximum profit possible. The risk is also contained.

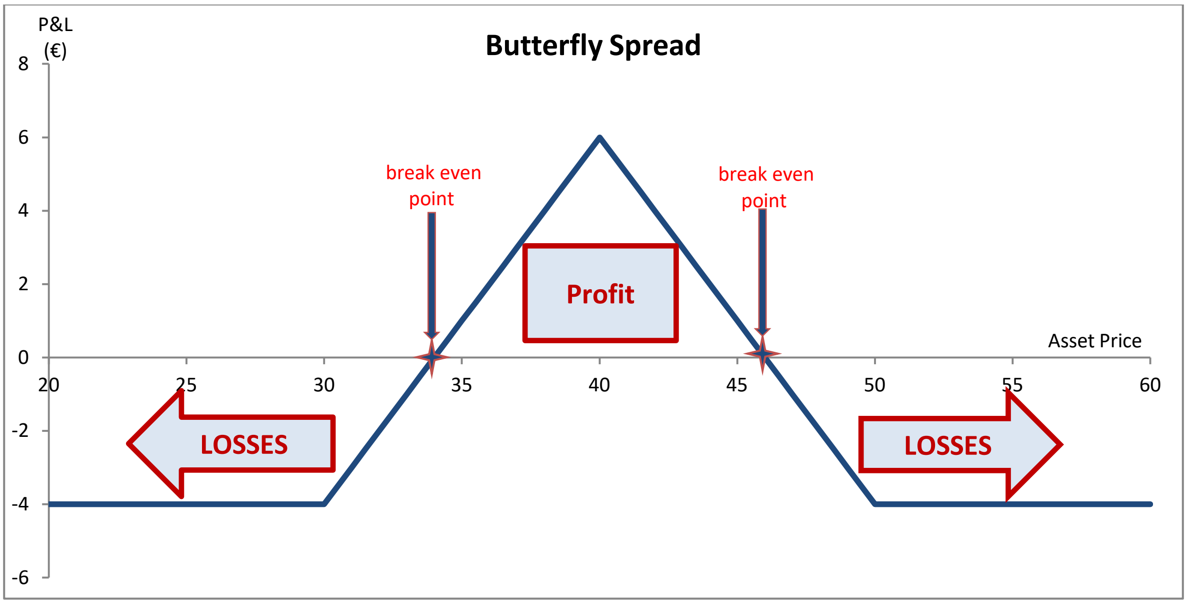


Figure 33 – Butterfly Spread

At time (t=0) an asset is trading at a price equal to the top vertex of the “triangle” and it is **not expected to rise or fall** by much until maturity. Buy one out-the-money call (long), sell two at-the-money calls (short) and buy one in-the-money call (long). Maximum profit is attained if at maturity if the asset expires at-the-money. As long as it remains above and below the lower and upper breakeven points there will be no losses. Losses are limited to the initial incurred debit to enter the position.

Among these two examples one can find many others like:

- Collar;
- Backspread;
- Straddle;
- Synthetic;
- Condor;
- Etc.

The choice of the strategy to apply depends on the perception the investor has concerning future market's developments based on expected tendencies. In our case, around april one could expect a downfall in electricity prices due to (expected) high precipitation. Depending on the risk profile a given investors makes the decision on what strategy best suits his/her interests and takes action accordingly.

Appendix D – Barrier options terminal payoff

Down-and-in	$DI(\theta)_T(S_T; k; L; R; T) = \begin{cases} [\theta S_T - \theta k]^+ & \Leftarrow \exists u \in]t, T]: S_u \leq L \\ R & \Leftarrow S_u > L, \forall u \in]t, T] \end{cases}$
Down-and-out	$DO(\theta)_T(S_T; k; L; R; T) = \begin{cases} [\theta S_T - \theta k]^+ & \Leftarrow S_u > L, \forall u \in]t, T] \\ R & \Leftarrow \exists u \in]t, T]: S_u \leq L \end{cases}$
Up-and-in	$UI(\theta)_T(S_T; k; U; R; T) = \begin{cases} [\theta S_T - \theta k]^+ & \Leftarrow \exists u \in]t, T]: S_u \geq U \\ R & \Leftarrow S_u < U, \forall u \in]t, T] \end{cases}$
Up-and-out	$UI(\theta)_T(S_T; k; U; R; T) = \begin{cases} [\theta S_T - \theta k]^+ & \Leftarrow S_u < U, \forall u \in]t, T] \\ R & \Leftarrow \exists u \in]t, T]: S_u \geq U \end{cases}$

Please note that θ=1 for barrier calls and θ=-1 for barrier puts. Also note that L stands for lower barrier and U for upper barrier.

Appendix E – risk neutral measure

Under the framework provided in (Seifert & Uhrig-Homburg, 2007) and (Burger, Klar, Müller, & Schindlmayrt, 2006) and adapting it to our model to price European options would be as follows:

First one needs to assume that it is possible to gather information from the market regarding futures prices. From the information on a monthly future contract we would assume that it was daily settled aiming to create the following relation:

$$F_{t,T} = E^{\mathbb{Q}}[S_T|\mathcal{F}_t] \xrightarrow{change\ to\ an\ equivalent\ martigale\ \mathbb{P}} = e^{\int_t^T \lambda_s ds} E^{\mathbb{P}}[S_t^{model}|\mathcal{F}_t]$$

Via discretization one can obtain the deterministic function that drives the risk λ_t as:

$$\lambda_{t-1} = \frac{\ln\left(\frac{F_t}{E_t}\right)}{\Delta t} - \sum_{s=t}^{T-2} \lambda_s$$

To recover the spot price process under the risk neutral measure is simply to (on our process):

$$X_t = \alpha\Delta t + \emptyset X_{t-1} - \boldsymbol{\sigma\lambda_{t-1}\Delta t} + \sigma\xi + \mu_j + \sigma_j\xi_j$$

$$X_t = \alpha\Delta t + \emptyset X_{t-1} - \boldsymbol{\sigma\lambda_{t-1}\Delta t} + \sigma\xi$$

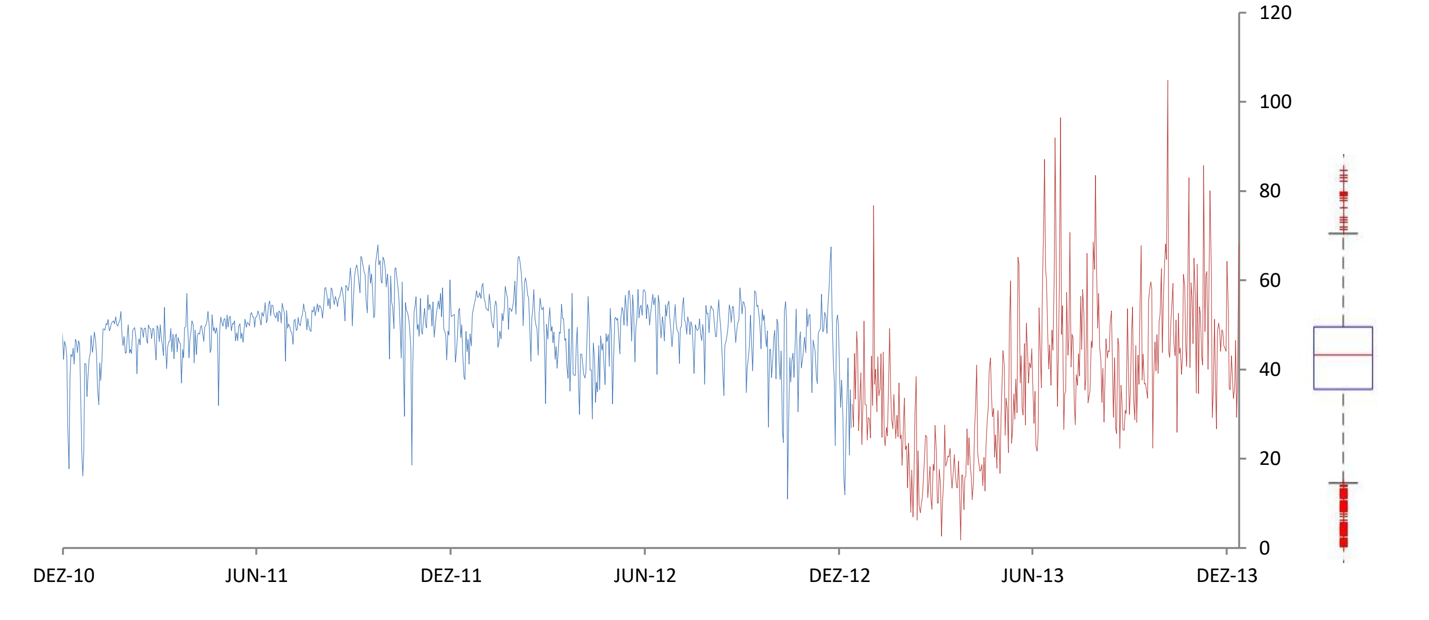
Allowing for the risk to be driven by the same Brownian volatility as the one obtained for the model.

Appendix F – Black Scholes Calculator

Table 15 – Automated calculator for pricing options with the BSM model. The chart in blue provides "historical data" and the red the path simulated. The boxplot on the right represents the confidence level from registered spot prices from the market.

Black-Scholes-Merton Model

Parameters			Price (€)	Delta	Gamma	Theta	Vega	Rho
Underlying Price (€)	42,56	**Call Option**	3,06	0,5428	0,0527	-0,0247	0,0707	0,0351
Strike Price (€)	42,56	**Put Option**	2,92	-0,4572	0,0527	-0,0224	0,0707	-0,0392
Volatility (p.a.[1])	42,20%							
Interest Rate (p.a.)	1,939%							
Dividend Yield (p.a.)	0,00%							
Time to Expiration								
Pricing date	28-12-2012							
Pricing Time								
Expiration Date	02-03-2013							
Expiration Time	64,00							
Time to expiration (p.a.)	17,53%							
Trading days[2]	360,00							



[1]percent annualized

[2] electricity is considered to have 360 buisness days, unlike the usual 252

Although the Greeks do not interfere with the pricing of options, they are extremely useful in reflecting what could happen in pricing changes given an asset price movement, change in volatility, time, etc. they are forward looking indicators that can be useful for traders. In our case, assume that each contract is written on 100MWh:

Delta: the amount the option price is expected to move based on a 1€ (unitary move) move **up** in the underlying asset. If the electricity price in the gross market moves from 42,56 to 43,56 (all other things equal) then the value of our option contract is expected to move up by 0,54 (as a fraction of the contract size). Using the average unit size for contracts (typically 100 shares) one extra euro per megawatt hour in electricity price would imply an extra 54€ worth in our contract (100MWh).

Gamma: if the delta is the speed, gamma can be seen as the acceleration. I.e. it represents the rate of change in Delta given a 1€ (unitary move) change in the underlying asset. From the example provided we can see that any additional unitary (upward) movement implies an additional 0,053 profit. Meaning that the second unitary move will profit with the same delta (0,54) plus 0,053, which given our hypothetical contract size would be equivalent to 59,3€, allowing one to profit from a compounding effect.

Theta: a measure of how will time impact the option worth. I.e. it represents the time decay in the sense that as we approach maturity the time value of the option becomes worthless and the contract will only be as valuable as its intrinsic value. That is why this indicator is **always** negative. Again in our case the theta tells us that every day our contract loses 2,5€ (-0,0247) worth to time decay. It is important to note that as we close in on maturity this decay rate increases in absolute value. For this reason time is always working against an option buyer, since he/she is betting on a given outcome and as time goes by the less time there is for that outcome to take place. On the other hand shorting these contracts has a positive effect for the trader.

Vega: represents quantitatively the change on an option contract for every 1% change in implied volatility (IV). Since IV is an unknown component, this indicator can be very useful for traders. I.e. we never know how volatile a given asset will be in the future and this can only be measured by market participants' trading activities. Since our market is only hypothetical (and hence no liquidity) there is not much to say about this indicator for our example. We would point out however that before any weather event we would expect an increase in this indicator, since market participants would try to anticipate them and profit from an increase in implied volatility. Given that it is not strange to think that high volatile markets tend to have higher values for vega, which is our case. A 1% change in implied volatility would mean a 7€ (!!!) move in the option worth.

Rho: it is only meaningful to address this indicator when maturities are longer than a year, for the impact of interest rates on option's worth are only relevant for large time frames. From our example for a one year contract a 1% change in interest rates would indicate a change of 3,5€ in the call option's worth.