Effect of stairs on the seismic behaviour of columns

Cláudia Sá Nogueira Pereira
claudia.pereira@tecnico.ulisboa.pt

Instituto Superior Técnico
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Abstract

This paper focuses on the study of the nonlinear effects produced by imposed displacements due to seismic actions on columns of buildings that support stairs at their mid-height level. To this purpose, a single storey reinforced concrete sub-structure, consisting of two columns, two beams and stairs, will be analysed while making use of the finite element program SeismoStruct (2016). Two types of structural analyses will be executed — a static linear, with the objective of analysing the internal forces obtained, and a static pushover nonlinear, considering different quantities of longitudinal and transverse reinforcement as well as considering different levels of compressive forces being applied on the columns, to evaluate the structural deformation capacity.

Keywords: Stairs, Column, Imposed Displacements, Ductility, Confinement, Seismic Action

1. Introduction

These days, in the current design practice, stairs are usually not included in the global modelling of buildings. In fact, stair elements are often treated as being secondary elements whose consideration is made separately from the main structure with the intention of only withstanding vertical loadings. Nevertheless, when building structures are subjected to imposed horizontal displacements and stairs are connected to columns at an intermediate level between storeys, these stairs can produce negative effects in the local behaviour of the structure, namely, the emergence of the short column effect. This effect can significantly damage the columns that are so important to withstand the vertical loads. In this sense, it is important to analyse the effects that the stairs produce in columns of buildings, when subjected to horizontal displacements, in terms of ductility and through nonlinear analyses.

The first steps made in what is known today as seismic design, date back to the early 20th century, where a number of deadly earthquakes occurred in different countries, such as Italy, Japan, New Zealand and the United States of America (Lopes 2008; Calvi and Sullivan 2009; Priestley et al. 2007, cited in Camacho 2012, p.5). Across the globe, these earthquakes strongly contributed to the realisation that the structures should be designed to withstand horizontal solicitations caused by the seismic actions. At the time, these horizontal solicitations were often treated as horizontal forces, equivalent to a certain percentage of the structure weight, regardless of its period (Calvi and Sullivan 2009; Camacho 2012).

It was only after the Long Beach earthquake of 1933 that, according to Lopes (2008), the first concerns of ensuring the structures' ability to accommodate, horizontal forces as well as imposed displacements, triggered by the seismic solicitations, arisen. From that period onwards, the capacity design principles were developed in New Zealand by Park and Paulay (1976), cited in Priestley (2000), and ever since then, there has been a rising concern in evaluating the overall performance of buildings, especially considering that many structures had been able to endure inertia forces, calculated assuming an elastic behaviour, larger than their own structural strength (Priestley 2000; Calvi and Sullivan 2009; Camacho 2012).

Nonetheless, the current codes on seismic design of reinforced concrete structures, namely the Eurocode 8, still follow a force-based approach, usually involving a linear elastic analysis proceeded by the division of the internal forces obtained with an arbitrary behaviour coefficient to take into account the nonlinear effects. The levels of displacement achieved are only subsequently checked. In other words, the structure performance is not directly addressed at the initial stages of the design process (Priestley 2000).

The principal objective of this paper is to contribute to a better understanding of the structural behaviour of columns that support a slab of stairs at a mid-height level between storeys, under the effects of seismic actions. To this purpose, a single storey framed structure consisting of two columns, two beams and a slab of stairs is analysed while being subjected to an imposed horizontal displacement, which intends to simulate the effects resulting from an earthquake activity on a building.

It will be demonstrated that the overall structure performance significantly improves with the increase of the confinement level in the columns, through the use of transverse reinforcement, rather than improving through the increase of flexural reinforcement considered at the mid-height region of the column supporting the stairs. The measurement of the deformation capacity of the structure will provide the means to assess the overall structural performance and the deformation capacity will be measured by evaluating the values of bending moments, curvatures as well as displacements achieved, beyond the elastic phase, in the nonlinear range of the analyses.

2. Analytical Model

2.1. Material Inelasticity

The material inelasticity in the SeismoStruct (2016) program is modelled with distributed inelasticity framed elements. As pointed out by Calabrese et al. (2010), the entire member is modelled as an inelastic element and the source of inelasticity is defined at a sectional level, in the controlling sections known as integration sections. These sections are refined into fibres that are individually associated with a material uniaxial inelastic behaviour. The global inelasticity of the frame is obtained through the numerical integration of the response provided by each integration section (Calabrese et al. 2010). This numerical integration, however, is only approximate since it heavily depends on the number of controlling sections within a certain element. The distributed inelasticity modelling can be implemented by using a force-based approach where equilibrium is satisfied and a linear moment variation is enforced between the defined displacement increments.

2.2. Materials

The definition of materials was necessary to build the static pushover analyses. The strength class C30/37 was used for the concrete while the grade A500 was used for the reinforcing steel. The mechanical properties of the materials were specified according to Eurocode 2 Part 1-1. Regarding the materials' models in SeismoStruct (2016), the "Mander et al. nonlinear concrete model" and the "Bilinear steel model" were used.

According to SeismoStruct User Manual (2016), the first is a uniaxial nonlinear model in which the confining pressure is constant and the constitutive relationship is the one suggested by Mander et al. (1988). It is noteworthy to mention that the mean concrete tensile strength was not considered, in order to approximate the response of the monotonic models studied with the response of models with a cyclic behaviour, in which the occurrence of concrete cracking in a certain area during the first cycles disables that same area from resisting to tensile stresses in the following cycles.

The second is a uniaxial bilinear stress-strain model with a kinematic strain hardening rule assumed to have a linear variation with the increment of plastic strains. The value chosen for the strain hardening parameter was of 1% and ensured that the ultimate reinforcement stress was about 1.3 times higher than the yield reinforcement stress of 500MPa.

2.3. Sections

The different cross-sections of the inelastic force-based elements were defined in the sections module. The definition of these sections served the purpose to specify the various integration sections necessary to build the beams and the columns of the desired structural model.

A rectangular reinforced concrete cross-section was used to define the beams and the columns. In terms of dimensions, both the beams and the columns have 0.50 x 0.30 m^2. The concrete cover considered was of 0.025m in terms of thickness measured from the exterior of the stirrups.

The reinforcement chosen for the columns was between the minimum and maximum values allowed by Portuguese National Annex of Eurocode 2 Part 1-1. Both these values were solely dependent on the area of cross-section. The minimum value for the longitudinal reinforcement was $3cm^2$ while the maximum value was $60cm^2$.

The adopted longitudinal reinforcement of the beams was so that the resisting moment of the columns was 30% higher than the resisting moment of the beams in order to ensure the formation of plastic hinges on the beams. This avoids the development of a soft-storey type of failure and improves the structure performance under a strong seismic action. The importance of having a weak beam/strong column mechanism has been emphasised by Park and Paulay (1976), cited in Priestley (2000, p.1), Brito (2011) and the Eurocode 8 Part 1.

It was also in this module that the confinement factor was defined following every step suggested by Mander et al. (1988).

2.4. Elastic Frame Elements

The elastic frame elements were mainly used in the static analyses. Particularly, the elastic element representing the stairs was also used in the static pushover analyses. Table 1 displays various properties such as, the axial (EA) and flexural (EI) stiffnesses as well as the self-mass (PP), necessary to define the elastic beams, columns and stairs.

Table 1 – Properties of the elastic frame elements

Elements	Beams	Columns	Stairs
EA (kN)	4,93E+06	4,93E+06	5,90E+06
EI_1 (kN.m^2)	18471	18471	5500
EI_2 (kN.m^2)	51309	51309	355000
PP (ton/m)	0,38	0,40	0,46

It has been stated by the Eurocode 8 – Part 1, 4.3.1 (7) and 9.4 (3), that in order to take into account the effects of concrete cracking, it is possible to reduce by half the flexural stiffness considered in the non-cracked gross sections. As a result, all the different flexural stiffnesses considered in these elastic models were reduced by half. On the contrary, the axial and torsional stiffnesses obtained were those by considering the non-cracked gross sections.

2.5. Inelastic Force-Based Frame Elements

The inelastic force-based frame elements were used in the static pushover analyses to represent the beams and columns of the sub-structure being studied. The type of discretisation adopted is represented in Figure 1.

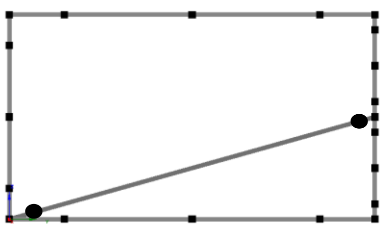

Figure 1 – Discretisation of the inelastic elements

The mesh was more refined in the areas where great variations of curvature were expected to occur. All of the discretised elements had three integration sections with 150 fibres each.

2.6. Loading Phases

In this module, the type of loading strategy employed in the pushover analysis was determined. The type of loading chosen consisted on controlling the response of a particular degree-of-freedom of the structure. Overall, the applied loads included permanent forces in the vertical direction and an incremental displacement in the transversal direction.

2.7. Performance Criteria

This SeismoStruct tab allowed the monitoring of the concrete core crushing, as well as, the reinforcement steel yielding and, respective, fracture. The types of criteria used for this monitoring were material strains. For instance, for the concrete core crushing, the ultimate compressive strains were introduced in the program. These strains are dependent on the quantity of reinforcement used, and therefore, they are also dependent on the confinement level of the section. The other strains considered were the value 2,5‰ for the steel yielding (ε_{sy}) and 75,0‰ for the ultimate reinforcement strain (ε_{su}), which was adopted in accordance with the Eurocode 2 Part 1-1, 3.2.2(2) for reinforcement bars of Class C.

3. Results and Discussion

3.1. Static Analyses

Two different elastic framed models were studied. Model A was a simple structure with two 5.00m long beams supported by two columns with 2.80m, as depicted in Figure 2, while Model B was a structure with two beams, two columns and a staircase at the mid-height level of the right column, as illustrated in Figure 3.

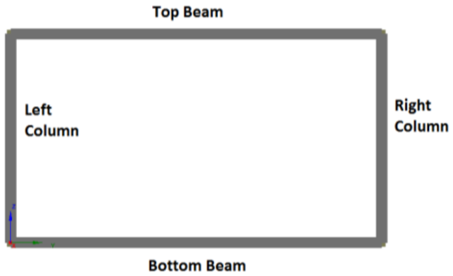

Figure 2 – Structural elements of Model A

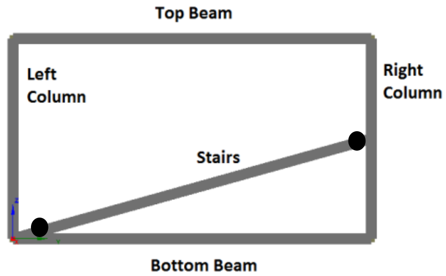

Figure 3 – Structural elements of Model B

It should be noted that the moments were released at the end supports of the stairs in Model B and each structural member of both models were subdivided into 100 elements to enable the drawing of the internal forces diagrams in Excel.

In terms of applied loadings, there was a 900kN compressive force above both columns, a 25kN/m distributed load on the beams, a 0,01m displacement on the top node of the right column, which compressed the stairs in Model B, as well as the self-weight of all the structural elements.

Figure 4 shows the internal forces diagrams of the right column.

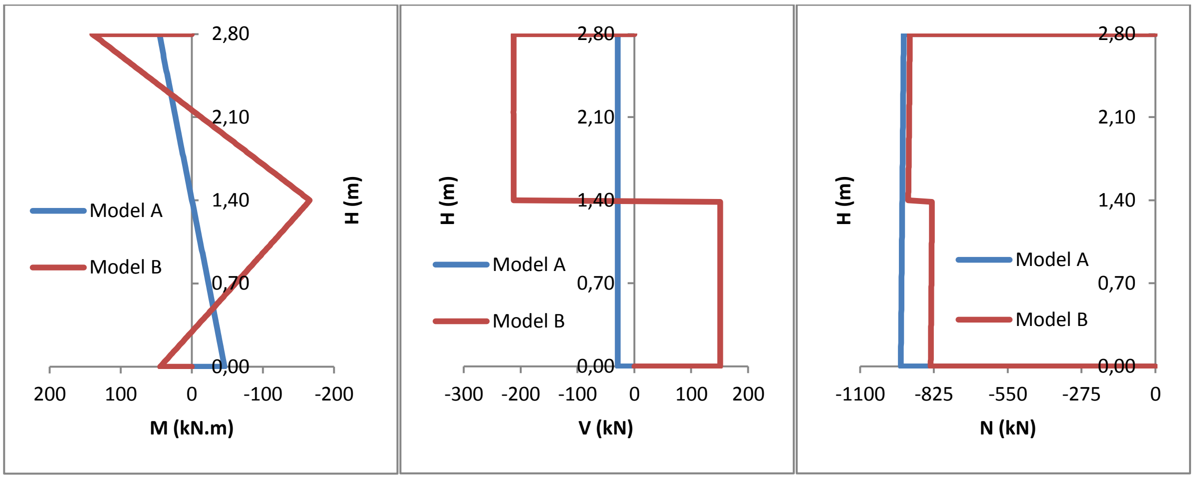

Figure 4 – Internal forces diagrams on the right column

The differences between Model A and Model B are quite noticeable for the same level of displacement. In Model B, the presence of the stairs at the mid-height level of the right column significantly alters the level of internal forces obtained along the column. The bending moments and shear forces obtained are significantly stronger. The level of axial and shear forces achieved at the top and bottom halves of the column are different, which was expected considering that the stairs transmit an axial force that has to be absorbed by the right column. The results obtained in Model B derive from a well-known occurrence, commonly referred as being the short column effect. As pointed out by Lopes (2008), a landing of stairs, at a mid-height level of a column, restrains the horizontal displacements at that same level. The bottom half of the column is almost unable to deform, which forces the top half to solely absorb an applied horizontal displacement. In this case, since the top half of the right column is obviously shorter than the left column, the deformations and internal forces imposed on the right side are ultimately greater. Special care should be taken with the levels of shear forces obtained because of the looming possibility of shear failure occurring.

Taking into account the level of internal forces of these elastic analyses, with the imposed horizontal displacement, according to current design procedures, there is a tendency to increase in Model B the amount of flexural reinforcement of the right column to counteract the high bending moments experienced due to the presence of the stairs.

3.2. Static Pushover Analyses

For the static pushover analyses use was made of the structure represented in Figure 3. The columns and beams were represented with inelastic force-based elements and the stairs were represented with an elastic framed element. Overall, three different types of structural analyses were created: Model C, Model D and Model E. The difference between each of these non-linear models lied in the reinforcement used in the right column. When comparing to Model C, Model D had an increase in the amount of longitudinal reinforcement used while Model E had an increase in the amount of transverse reinforcement around the mid-height level of the right column, where a plastic hinge was expected to form. Table 2 shows the reinforcement adopted in the right column along the different integration sections and across the different models created.

In what concerns the reinforcement of the remaining structural elements, the 5.00m beams had 4ϕ20+2ϕ16 for the longitudinal reinforcement distributed along 1.50m measured from both end supports and the rest of the beam span had 4ϕ20. The transverse reinforcement of the beams was constant and equal to ϕ6//0.125. The left column had in terms of longitudinal bars 10ϕ25 along its height, ϕ8//0.10 for the transverse reinforcement over 0.45m away from the extremities and ϕ8//0.20 for the rest of the column.

Table 2 – Right Column Reinforcement

<table>
<tr><th>Element Label</th><th>H (m)</th><th>Integration Section</th><th>Model C</th><th>Model D</th><th>Model E</th></tr>
<tr><td rowspan="3">CR1</td><td>0,00</td><td>CR1(a)</td><td rowspan="4">10ϕ25; ϕ8//0,10</td><td rowspan="4">12ϕ25; ϕ8//0,10</td><td rowspan="4">10ϕ25; ϕ8//0,10</td></tr>
<tr><td>0,11</td><td>CR1(b)</td></tr>
<tr><td>0,21</td><td>CR1(c)</td></tr>
<tr><td rowspan="3">CR2</td><td>0,21</td><td>CR2(a)</td></tr>
<tr><td>0,46</td><td>CR2(b)</td><td rowspan="16">10ϕ25; ϕ8//0,20</td><td rowspan="16">12ϕ25; ϕ8//0,20</td><td rowspan="4">10ϕ25; ϕ8//0,20</td></tr>
<tr><td>0,70</td><td>CR2(c)</td></tr>
<tr><td rowspan="3">CR3</td><td>0,70</td><td>CR3(a)</td></tr>
<tr><td>0,95</td><td>CR3(b)</td></tr>
<tr><td>1,19</td><td>CR3(c)</td><td rowspan="8">10ϕ25; ϕ8//0,10</td></tr>
<tr><td rowspan="3">CR4</td><td>1,19</td><td>CR4(a)</td></tr>
<tr><td>1,30</td><td>CR4(b)</td></tr>
<tr><td>1,40</td><td>CR4(c)</td></tr>
<tr><td rowspan="3">CR5</td><td>1,40</td><td>CR5(a)</td></tr>
<tr><td>1,51</td><td>CR5(b)</td></tr>
<tr><td>1,61</td><td>CR5(c)</td></tr>
<tr><td rowspan="3">CR6</td><td>1,61</td><td>CR6(a)</td></tr>
<tr><td>1,86</td><td>CR6(b)</td><td rowspan="4">10ϕ25; ϕ8//0,20</td></tr>
<tr><td>2,10</td><td>CR6(c)</td></tr>
<tr><td rowspan="3">CR7</td><td>2,10</td><td>CR7(a)</td></tr>
<tr><td>2,35</td><td>CR7(b)</td></tr>
<tr><td>2,59</td><td>CR7(c)</td><td rowspan="4">10ϕ25; ϕ8//0,10</td><td rowspan="4">12ϕ25; ϕ8//0,10</td><td rowspan="4">10ϕ25; ϕ8//0,10</td></tr>
<tr><td rowspan="3">CR8</td><td>2,59</td><td>CR8(a)</td></tr>
<tr><td>2,70</td><td>CR8(b)</td></tr>
<tr><td>2,80</td><td>CR8(c)</td></tr>
</table>

Figure 5 identifies the labels used on the discretised elements representative of the beams and columns of the substructure.

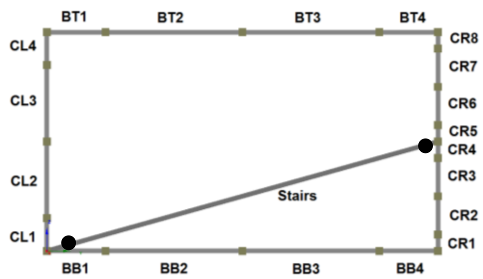

Figure 5 –Elements labels on the structure

Regarding the loadings applied in these non-linear analyses, the self-weight of all the elements as well as the beams' distributed loadings of 25kN/m due to the slabs' weight were considered. The self-weight of the beams and the columns were specified separately in the materials module as 24kN/m^3 for the concrete class C30/37 and as 78.5kN/m^3 for the reinforcing steel A500. The stairs' specific weight was defined in the elements class module as being 0.46ton/m. In addition, three different compressive loadings were applied on top of the columns: 450kN, 900kN and 1500kN, which corresponded to the following normalised axial forces values of 15%, 30% and 50% and two types of displacement were applied on top of the right column. One from right to left, causing the stairs to be under compression (Type I) and another from left to right, causing the stairs to be under tension (Type II). Overall, with the different reinforcements and applied loadings considered, there were in total 18 different structural models.

It was assumed, throughout the analyses, that the shear strength corresponding to the transverse reinforcement of the right column was high enough to support the shear corresponding to the flexural strength.

In terms of results, it is important to discuss some global response parameters such as the maximum structural displacements (δ_u) achieved on the 18 different models. Table 3 summarises these maximum applied displacements at the top of the right column.

Table 3 – Maximum applied displacements (cm)

N (kN)	δ_u (cm)					
	Analyses Type I			Analyses Type II		
	Model C	Model D	Model E	Model C	Model D	Model E
450	-6,77	-4,99	-6,77	3,60	3,10	4,07
900	-4,65	-3,86	-6,46	2,78	2,70	3,31
1500	-3,29	-3,37	-3,70	2,46	2,49	2,75

It is possible to observe that, in general, the maximum displacements decrease with the increase of the axial forces applied on top of the columns. Within the same levels of axial forces, the Models E analyses present higher displacement values in comparison with Models C and Models D analyses. Moreover, the level of displacements achieved on the analyses Type I are higher than those achieved on the analyses Type II.

Considering that the structure followed the capacity design principles where the deformation was assumed to be mainly by flexure, the structure collapse was expected to occur by the concrete failure under compression or the reinforcement failure under tensile strains. In the analyses Type I, Model C (450kN) and Model E (450kN; 900kN) suffered convergence problems on the BT1(a) section of the top beam where reinforcement failure was on the verge of occurring. The rest of the analyses suffered concrete failure at the CR4(c) section of the right column. In the analyses Type II, all of the models suffered concrete failure at the CR5(a) section of the right column except for Model E(450kN) that suffered reinforcement failure at the BT4(c) section of the top beam. It is interesting to notice that the reinforcement failure occurs, near the top beam support sections, when the level of axial forces being applied on the columns is relatively low, 450kN or 900kN as opposed to 1500kN. Therefore, the right column mid-height sections are able to resist longer to the imposed displacements. Furthermore, in the Models E analyses, the confinement level at the mid-height section is greater than the confinement level of Models C and Models D, which positively delays the occurrence of concrete failure at that section.

In what regards the variation of the bending-moment diagrams, Figure 6 and Figure 7 show the general shape of those diagrams at failure for respectively the analyses Type I and Type II.

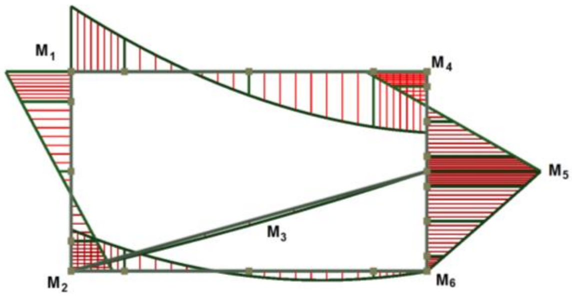

Figure 6 – Bending moment diagram (analyses Type I)

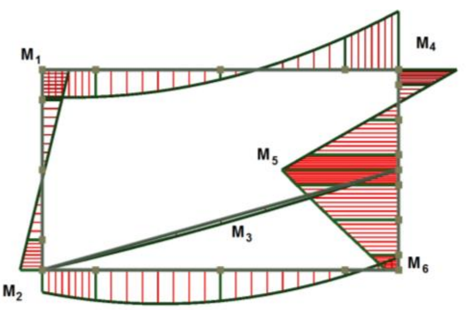

Figure 7 – Bending moment diagram (analyses Type II)

The bending moments are in accordance with expected structural deformed shape. The bending moment values at failure of the right column are particularly greater when comparing to the other structural elements because of the stairs' presence that restricts the deformation of the right column with the imposed displacement being applied. At last, the variation of the right column curvatures for the analyses Type I (900kN) near failure is represented in Figure 8.

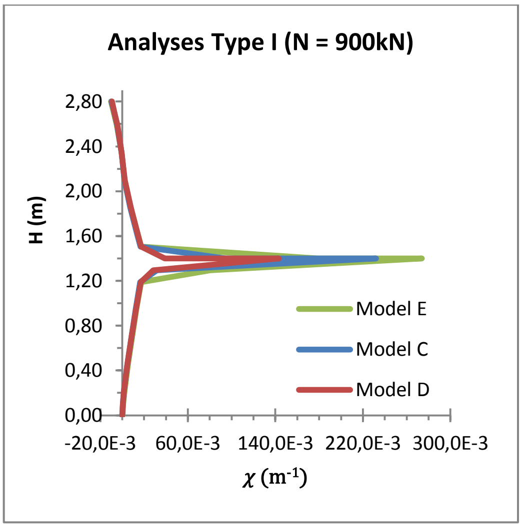

Figure 8 – Right column curvatures diagrams

The diagrams of the other analyses are analogous to this one. The data in Table 4 indicates the yield and ultimate curvatures of all the analyses performed. It is possible to notice the negative effects of the compressive force on the columns in all of the analyses. As the compressive force increases, the yield curvature also increases whilst the ultimate curvature decreases, similarly to the displacements trending. Brito (2011) studied with detail the negative influence of the axial forces on the ductility of reinforced concrete structures, the higher the compressive force, the higher is the extension of the compressed zone at the cross-section level, and consequently, the concrete failure of the cross-section occurs under a smaller curvature value.

Table 4 – Yield and ultimate curvatures (‰/m) at right column mid-height section

N (kN)	χ_y (‰/m)					
	Analyses Type I–CR4(c)			Analyses Type II–CR5(a)		
	Model C	Model D	Model E	Model C	Model D	Model E
450	16,28	17,06	16,81	-17,33	-17,92	-17,66
900	18,19	18,37	18,29	-19,19	-19,57	-19,34
1500	20,22	20,28	20,15	-21,19	-21,30	-21,23
N (kN)	χ_u (‰/m)					
	Analyses Type I–CR4(c)			Analyses Type II–CR5(a)		
	Model C	Model D	Model E	Model C	Model D	Model E
450	250,71	220,81	249,76	-252,46	-154,14	266,85
900	231,96	142,97	273,96	-160,49	-101,05	-236,00
1500	116,04	96,02	169,79	-98,41	-89,14	-141,47

It can also be observed that the variations in terms of the flexural and transverse reinforcement have little influence in the values of the yield curvatures. The same cannot be said about the ultimate curvatures. The increase in the flexural reinforcement, from Model C to Model D, considerably decreases the ultimate curvatures achieved. Conversely, the increase in the transverse reinforcement, from Model C to Model E, increases the ultimate curvatures reached because of the higher level of confinement.

4. Conclusions and Recommendations

The present paper focused on the study of the nonlinear effects produced by imposed displacements on a column supporting a slab of stairs at middle height between floors. It has been shown that performing an elastic analysis on a reinforced concrete framed structure subjected to a horizontal displacement, wherein one of the columns supports a slab of stairs, leads to the obtainment of high levels of bending moments and shear forces at that same column. The stairs have a considerable axial stiffness that restrains the deformation capacity of the column, causing a short-column effect. The increase of the internal forces felt in the column would indicate that the quantity of flexural reinforcement calculated without considering the effect of the stairs was insufficient. As a result, there would be a need to increase the strength, in order to withstand those high levels of internal forces experienced due to the stairs. Nevertheless, increasing the flexural reinforcement would start a cycle that not only it would increase the bending moments but it would also increase the levels of shear force experienced. It is noteworthy to mention that an increase in the levels of shear forces would have the inconvenience of potentially escalating the occurrence of shear induced type of failures.

Following the elastic analysis, a nonlinear analysis was executed on the structure being studied. Emphasis was laid on the examination of the available ductility, while considering different types of longitudinal and transverse reinforcement. It was noticed that increasing the column flexural reinforcement to resist an imposed displacement did not improve the structure’s performance. Despite the greater flexural stiffness for deformations at or higher than yielding, the available curvatures ductility was considerably smaller. Furthermore, it was observed that variations in the flexural reinforcement did not produce significant variations in the values of yield curvatures achieved. This proves that these values are independent of strength, which completely opposes to what is assumed in the current seismic design codes. By this means, increasing the quantity of flexural reinforcement to withstand imposed displacement does not avoid or delay the structural yielding. Conversely, increasing the transverse reinforcement in the plastic hinges regions of the column ensured a better performance and increased the deformation capacity, as the displacements and curvatures ductility obtained were higher. The level of confinement and, subsequently, the ultimate compressive concrete strain were greater, which enabled a higher ductility availability to withstand the applied horizontal displacement. In addition, it has been demonstrated that the larger the intensity of the axial forces being applied on the column, the smaller were the overall displacements and curvatures ductility because there was a significant increase in the depths of the neutral axis on the cross-sections.

In terms of general recommendations, it can be said that it is vital to include the consideration of stairs on the analytical models of building structures under seismic actions. In the current design practice, stairs are usually not included in the global modelling of buildings. As a consequence, this procedure severely underestimates the effects on the columns that support stairs. Therefore, those columns may not be adequately reinforced to withstand a seismic movement and the structural collapse may be potentially induced.

In addition, it is very important to take into account the ductility demand on the region where the stairs connect to the columns, which in this case was the mid-height level of the column. It was demonstrated that in the mid-height region, a plastic hinge was formed and for this situation, Eurocode 8 – Part 1 already states in section 5.2.3.4 (2) a) that a sufficient curvature ductility shall be provided in all critical regions of primary seismic elements, including column ends. Eurocode 8 – Part 1 further states in section 5.4.3.2.2 (4) the definition of the critical regions, offering the possibility to calculate the critical length, measured from both end sections of a primary seismic column. Nevertheless, if Eurocode 8 guidelines are followed and the analytical model does not include stairs, then the mid-height region of columns supporting stairs is not considered to be critical and, thus, it does not need to be reinforced accordingly. So, in order to avoid misinterpretations, Eurocode 8 should explicitly include guidelines to design columns that support stairs at an intermediate level, such as the necessity to have an adequate amount of confinement especially at the regions where plastic hinges are expected to be formed, namely the mid-height level and the top and bottom ends of the column. It should also be included the fact that the longitudinal reinforcement does not need to be increased due to the short column effect. As it was demonstrated, increasing the longitudinal reinforcement, does not improve the overall behaviour of the structure. On the other hand, increasing the amount of transverse reinforcement counteracts the aggravated effect of the higher levels of shear forces and increases the concrete confinement to overcome the increase of the ductility demand.

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