Optimal Lap Time for a Race Vehicle

Extended Abstract

Luís Henrique Vieira Brites Marques
Instituto Superior Técnico – Universidade de Lisboa

Abstract

The optimal lap time for a race vehicle in a race track represents the minimum possible time for a determined vehicle to negotiate a complete round about the race track. In this thesis, a 2D multibody dynamic analysis program is developed to allow modelling and simulating the vehicle and racetrack scenario by implementing all the necessary kinematic constraints, which includes a steering constraint for a 4-wheel vehicle with a front steering axle and the necessary force elements including traction and braking and the tyre-road contact.

A trajectory optimization, on a given track with a prescribed geometry, is performed using a mix of the shortest path and the least curvature criteria being the speed profile optimized by using maximum a minimum accelerations for the vehicle as it negotiates the curves.

A controller is developed in order to make the vehicle follow the predefined path and the optimal speed profile. This controller uses a preview distance, which allows for the vehicle to find its way even when it starts or goes off-track. The controller and the dynamic analysis program are demonstrated in a scenario in which the behavior of a Lancia Stratos in a real race track is analyzed. The same control methodology is then applied in a spatial vehicle model analyzed in the Dynamic Analysis Program (DAP3D) available for the spatial multibody dynamics.

Keywords: Race Vehicle, Lap Time Optimization, Multibody Dynamics, Vehicle Dynamics, Optimal Control

1 Introduction

The optimal lap time problem has great interest, not only in racing but also in the design of different transportation activities. Knowing what a vehicle is capable of, at its maximum level of performance, allows companies to understand the behavior of the vehicle negotiating any racetrack, even before it is built. Having reliable simulations permits saving resources by simulate testing scenarios, before doing real tests and thus being a tool for test design, vehicle design and road design.

The first step in this work consists in developing a 2D multibody dynamics analysis program able to analyze simple models of automobiles, by allowing using a limited number of bodies with revolute joints between the wheels and chassis. A steering constraint for the front wheels of the vehicle is developed and a tyre model is implemented to compute the lateral and longitudinal forces acting on each wheel of the vehicle.

In the second step, a controller is developed using a Linear Quadratic Regulator based on the work of Antos and Ambrósio [1], which is supported in a simplified vehicle to follow a predetermined trajectory the vehicle is acted upon exclusively by the steering angle and by the wheels torques. A preview distance for the control is implemented in this work in order not only to stabilize its action even when the car is off-track but also to better represent the realistic driver attitude. Furthermore, the controller is upgraded with the ability to follow a determined speed profile by acting on the traction and braking of the wheels.

Apart from the controlling methodology, an optimization of the trajectory and speed profile is carried based on Meier’s work [2]. A mixed shortest path and least curvature approach is used to find the optimal path while the speed profile is computed by using upper and lower bounds of the lateral and longitudinal accelerations that can be developed by the tyres due to the tyre contact forces.
2 Multibody dynamics overview

A multibody system is composed by several types of bodies, rigid and flexible, connected by mechanical joints, or kinematic joints, and acted upon by force elements, passive and active. For the simulation of specific scenarios other, non-standard, kinematic joints may have to be developed. The interaction forces, as for instance between tyres and road constitute, often, the most demanding task when developing multibody dynamics analysis tools.

2.1 Kinematic constraints

A kinematic joint constrains the relative motion between two bodies. They are holonomic algebraic constraints and also lower-pair category, which means that not only they are developed in the position space but also no information about the shape of the bodies is needed [3].

For the planar model used in this work only revolute and steering constraints are used, being revolute, spherical and translational constraints used in the spatial model.

The steering constraint is aimed to control the steering angle between each front wheel and the chassis. Note that both wheels, in Figure 2.1, are being steered equally, which is not what is required in vehicle applications where the Ackerman geometry needs to be taken into account [4]. The steering kinematic constraint equation is written as:

\[(\omega, x) \Phi = (\phi_w - \phi_c) - \phi_{str} = 0\]  

(2.1)

where \(\phi_w\) is the angular position of the wheel \(i\), \(\phi_c\) is the angular position of the chassis and \(\phi_{str}\) is the desired steering angle.

![Figure 2.1 - Simplified steering constraint representation](image)

2.2 Equations of motion

In order to perform the multibody dynamic analysis a set of equations of motion must be computed. For this purpose, the Jacobian matrix \(\Phi_q\) needs to be constructed as well as the \(\gamma\) vector. Furthermore, the mass matrix \(M\) and the external forces vector \(g\), need to be computed.

The system which describes the multibody system constrained motion is written as [3]:

\[
\begin{bmatrix}
M & \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\lambda
\end{bmatrix}
=
\begin{bmatrix}
g \\
\gamma
\end{bmatrix}
\]

(2.2)
where $\lambda$ is the vector of Lagrange multipliers. The equations of motion, equation (2.2), have to be solved and integrated in time, which is prone to the accumulation of numerical errors in the positions and velocities. One of the most common methods to control the constraint errors is the one proposed by Baumgarte [5]. Note that this method does not correct the violation errors but simply control them within workable limits.

3 Vehicle dynamics

3.1 The automotive vehicle

In order to better understand how an automobile is built and runs, which is essential for this work, some concepts are presented, especially with respect to suspensions and steering systems. The suspension is one of the most important mechanisms, regarding the dynamics of an automobile, linking the wheels to the vehicle body while allowing their relative motion [6]. The wheels should be able to traction, brake and steer the vehicle using the suspensions linkages to withstand all the tie-ground contact forces involved.

Automotive suspension systems may be divided into three different groups: independent, dependent and semi-dependent suspensions [4]. Independent suspensions have importance particularly in race and sport vehicles.

To maneuver an automobile a steering system is needed in order to rotate the wheels and support the control of the vehicle heading according to the driver inputs. A steering system is usually composed by three main parts: steering mechanism, steering box and steering column [4].

There are some considerations to make regarding the steering kinematics. When an automobile is negotiating a corner at a slow speed, there is a relationship between the inner wheel angle and the outer one, as the arc of circle travelled by each wheel implies a different steering angle for each wheel in order to have to turn slip-free [6]. Hence, there is a geometric condition, called Ackerman steering or Ackerman geometry, which provides the relation between the steering angles of the two wheels. The Ackerman condition is written as [4]:

$$\tan(\delta_i) = \frac{w}{R_i - \frac{t}{2}} \quad ; \quad \tan(\delta_o) = \frac{w}{R_o + \frac{t}{2}}$$

(3.1)

where $w$ is the vehicle wheelbase, distance between the two axles, $t$ is the vehicle track, distance between wheel of the same axle, and $\delta_i$ and $\delta_o$ are the angle of the inner and outer wheels, respectively. $R_i$ is the distance between the turning center and the point of the rear axle at the vehicle half-track width.

There are other concepts which have importance in vehicle dynamics, and in particular in race dynamics, such as aerodynamics. However in this work all aerodynamic forces are neglected in order to maintain the complexity of the models at a reasonable level.

3.2 Tyre mechanics

When an automobile is designed it is known that the tyres are the only means to transfer forces between the vehicle and the road [6] being the objective to maximize the lateral forces that the tyre can produce. The tyre is a complex composite structure made up of many rubber components and reinforcement steel chords, as well. Radial and non-radial tyres are divided depending on the metallic reinforcement chords orientation [6]. Radial tyres are the most used, which usually have a better cornering ability, which means that they permit higher lateral forces.

Figure 3.1 shows the tyre coordinate system and the forces and moments applied in each wheel being the complete tyre mechanisms described in this referential. $F_x$ is the tyre longitudinal force, $F_y$ is the tyre lateral force and $F_z$ is the normal force, which is mostly dependent on vehicle weight and load transfers. $M_x$, $M_y$, and $M_z$ are the roll moment, rolling resistance moment and
self-aligning moment. The angles $\alpha$ and $\gamma$ are the side slip and camber angles, respectively and they have great influence in the tyre behavior, $u$ is the velocity of the wheel.

Both longitudinal and lateral forces are functions of the side slip angle, slip ratio, camber angle and the normal force. The self-aligning torque is also dependent on the same parameters.

Figure 3.1 - Tire coordinate system

The linearized forces and aligning moment, for a small level of slip are [7]:

$$F_x = C_s s$$

(3.2)

$$F_y = C_{\alpha} \alpha + C_{\gamma} \gamma$$

(3.3)

$$M_z = -C_{M_s} \alpha + C_{M_{\gamma}} \gamma$$

(3.4)

where $C_s$ is the longitudinal slip coefficient, $C_{\alpha}$ is the cornering stiffness and $C_{\gamma}$ is the camber stiffness. Regarding the self-aligning torque, $C_{M_s}$ is the aligning stiffness and $C_{M_{\gamma}}$ is the camber contribution for the aligning stiffness. All these parameters are specific properties of each tyre.

3.3 Pacejka tyre model

One of the most known approaches to model the tyre mechanics is the Pacejka Magic Formula [7]. This method uses a formula which describes the tyre contact forces as function of the side slip angle, slip ratio, camber angle and normal load. Pacejka magic formula has several parameters which are obtained using experimental data and each tyre has its own parameters.

The general form of the Magic Formula is given by [7]:

$$Y = D \sin[C \arctan\{B(X + S_H) - E(B(X + S_H) - \arctan B(X + S_H))\}] + S_v$$

(3.5)

where $Y$ may assume the value for $F_x$, $F_y$ or even for $M_z$. Furthermore the input variable $X$ may be the slip angle $\alpha$ or the slip ratio $s$, depending on which force is being computed. $B$, $C$, $D$ and $E$ are the Pacejka magic formula coefficients. For a detailed description of the model the reader is directed to the work of Pacejka [7].
3.4 Computational implementation

The computational algorithm is implemented as shown in Figure 3.2, with the tyre model already implemented in the 2D dynamics analysis program as a force that develops between the rigid bodies to which the tyre is attached and the road. A tyre model function is created, which computes the longitudinal and lateral forces and the aligning moment for each tyre during each time step. The tyre coefficients are obtained in the literature, being those for the Hoosier 13x8 slick racing tyre used here for demonstration purposes.

In the planar multibody dynamics code the normal load $F_z$ is considered constant for all 4 wheels, which means that no load transfer is considered here being the normal load in each wheel one fourth of the total weight of the car.

The automobile model is steered either by input files or by the controller, as it is shown in Figure 3.2. The steering angles are used in the steering kinematic constraint where the Ackerman condition is neglected. In the case where the steering angles are controlled by an input file, this is written as a text file, .txt, where each line provides an incremental value. In each line the initial time, the end time and the steering angle variation during this time interval is provided.

The steering angle variation during the input time interval is not linear so that the time derivatives of the steering angle are such that angle variation is smooth. In order to do that, a polynomial function is computed for each steering input [8]:

$$
\delta_i(t) = \delta_i + 6 \frac{\Delta \delta}{\Delta t} (t - t_i)^2 - 8 \frac{\Delta \delta}{\Delta t^3} (t - t_i)^3 + 3 \frac{\Delta \delta}{\Delta t^4} (t - t_i)^4
$$

(3.6)

where $\delta_i$ is the steering angle, $\delta_i$ is the input’s initial steering angle, $\Delta \delta$ is the steering angle variation provided in the input file, $\Delta t$ is the input time interval in which the complete variation must occur and $t_i$ is the input initial time.
3.5 **Lancia Stratos model**

The purpose of this thesis is not to evaluate any particular car model being Lancia Stratos chosen because all the necessary data to describe the model is available in the work of Ambrósio and Gonçalves [9]. The simplified multibody system for the Lancia Stratos used in the 2D dynamics analysis program is adapted from its spatial counterpart [8]. The multibody system is composed by only 3 bodies as shown in Figure 3.3.

![Figure 3.3 - Simplified planar Lancia Stratos multibody system](image)

3.6 **Demonstration case**

With the objective of demonstrating the features of the program developed here and to explore the performance of the vehicle model for a lane changing maneuver, a demonstration case is presented. Table 3.1 and Table 3.2 show the input values, for the traction/braking torque and for the steering, given to the model in this demonstration case.

**Table 3.1 - Wheel torques inputs for the demonstration case**

<table>
<thead>
<tr>
<th>Input number</th>
<th>Initial time, [s]</th>
<th>End time, [s]</th>
<th>Wheel torques, [N.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>3.0</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>6.0</td>
<td>-100</td>
</tr>
</tbody>
</table>

As Table 3.1 presents the initial time is the moment when the torques start to be applied to the wheels and it goes on until the end time is reached. Positive torque values are used for traction, while braking torques assume negative values. When the vehicle is being accelerated the input torque is applied exclusively on the rear wheels, since the vehicle is rear wheel driven.

**Table 3.2 - Steering angle inputs for the demonstration case**

<table>
<thead>
<tr>
<th>Input number</th>
<th>Initial time, [s]</th>
<th>End time, [s]</th>
<th>Steering angle, [º]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.0</td>
<td>7.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>9.0</td>
<td>9.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>3</td>
<td>11.0</td>
<td>11.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>4</td>
<td>13.0</td>
<td>13.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

In the steering angle input the time interval between initial and end times is for the steering angle to reach the final steering angle value, which means that when the end time is reached, the steering angle value will remain with the same until new input is given.

The trajectory for the vehicle center of mass as a result of the traction and steering inputs for a motion that lasts 15 seconds is shown in Figure 4.1. It is obvious that the vehicle successfully completes the lane change maneuver.
Vehicle control

The controller developed and implemented here is based on the work of Antos and Ambrósio [1]. The objective in their work was the development of a virtual driver to enable a vehicle to follow a provided parameterized trajectory, without previewing or traction or braking.

The preview distance used to control the vehicle steering is not only a natural way of driving for a human driver but also a way to smooth the vehicle steering. A controller with previewing is generally more stable, especially when the vehicle is far from the ideal position. Instead of trying to find the closest point in the trajectory, the controller looks for a point further ahead, \( O_1 \), using a predefined preview distance. Using that point and the current vehicle position \( O_0 \) and orientation, the controller computes a new trajectory by using a cubic polynomial.

For the preview implementation the controller design used is the same as the one developed by Antos and Ambrósio [1]. However, there are some changes in the trajectory to follow. Instead of following the trajectory provided by the input file, new trajectories are computed and as the car moves on, new trajectories are computed using a cubic polynomial.

Besides the control of the trajectory it is also important to control the vehicle velocity, called here as the speed profile following feature. Unlike the previewing method, the speed profile controller changes de design of the controller by Antos and Ambrósio [1], even if it continues to be based that. To perform the best time in a race track there is the need to accelerate and brake de car, which leads to the controller which ensures that the vehicle follows a predefined speed profile for the longitudinal velocity of the vehicle.
4.1 Demonstration cases

The first demonstration case is meant to show how the controller behaves when using the preview distance. Firstly a simulation with the car starting off-track is presented to show the ability to get back to the track. In this first demonstration case the vehicle center of mass initial position is set off-track, i.e., instead of being placed at the origin, the car is moved 2 meters away in the $y$ direction, as shown in Figure 4.3. The car initial velocity is 7.5 m/s.

![Figure 4.3 - Demonstration case for the vehicle control with previewing. The vehicle starts off-track.](image)

The demonstrative case for the vehicle performing an entire lap with the previewing controller implemented is now analyzed. The vehicle trajectory is very close to the predefined trajectory, as observed in Figure 4.4. It is clear that the vehicle runs the entire track following close to the reference trajectory. The race track used here is fictional and is created for demonstration purposes.

![Figure 4.4 - Vehicle trajectory for the entire lap demonstration case with preview](image)

The relative lateral distance between the vehicle and the reference track are shown in Figure 4.5. The change of curvature induces some oscillations in the system, which are damped in a reasonable short period of time. This is due to an overshooting of the steering angle when starting a segment with a different curvature. The controller tries to minimize the distance error, but the
continuous curvature change, concerning the new preview trajectories that are continuously being created, makes the task difficult. Nevertheless, the maximum lateral error is 0.2138 meter.

![Lateral error graph](image)

Figure 4.5 - Lateral relative position between the vehicle and the reference trajectory using preview

In the second demonstration case the controller is used with the speed control function, which ensures that the vehicle follows not only the trajectory but also a predefined speed profile. Figure 4.6 shows the speed profile entered as input together with the actual vehicle longitudinal velocity. It is evident that the controller correctly tracks the vehicle speed required, with a relative error when the speed profile changes velocity. Also a small delay in achieving the target velocity is observed throughout the simulation. Regardless, the results are satisfactory because the controller is able to follow a speed profile, even while cornering.

![Vehicle velocity graph](image)

Figure 4.6 - Vehicle longitudinal velocity and predefined speed profile

Figure 4.7 shows the relative error between the vehicle velocity and the speed profile required, being the controller able to maintain the error close to zero while the speed is constant. When the speed changes the controller needs some time to return the error to zero again. The maximum value of the velocity error is 0.2533 m/s.

![Velocity error graph](image)

Figure 4.7 - Error of the vehicle longitudinal velocity with respect to the speed profile

5 The optimal lap time problem

Many works have been developed in this field being the approach taken by Meier [2] the basis of the development presented here. Meier work addresses two different phases: the trajectory optimization and the speed profile optimization.
The optimal trajectory consists in finding the best compromise between the shortest trajectory and the least curvature trajectory, which allows the vehicle to have higher velocities while turning. Finally the best combination between trajectory and speed profile, which gives the minimal lap time, is found by an iterative process.

The shortest trajectory is the shortest possible line that can be described within the track limits, i.e. between the right and left limits of the track. This seems to be a good manner of minimizing the lap time, since the travelling distance is minimized. However, it may not allow the vehicle to reach high speeds because it usually creates segments of the trajectory where the curvature is too high, which induces higher lateral accelerations on the car.

The least curvature trajectory is an alternative approach that allows the vehicle to reach higher velocities while cornering. Assuming that the vehicle has a maximum lateral acceleration, limited by the grip produced by the tyre-road contact, it is clear that when the curvature is lower, the velocity can reach higher values. Hence, with higher velocities lower lap times may be achieved. However, the total trajectory length is longer than the shortest trajectory one.

As none of the two presented trajectories is optimal, a combination of the two may have good results and reduce the lap time. Meier suggests to combine the two approaches.

Having the optimal trajectory computed, it is necessary to get the maximum velocity that the vehicle can sustain at each point of the track, by optimizing the speed profile. The method implemented by Meier [2] tries to minimize the lap time, by relating the distance and the vehicle velocity at each track segment. This method assumes that the maximum and minimum longitudinal and lateral accelerations of the vehicle are known and constant. As a consequence, if the values of the accelerations are not accurate enough, the results may be too conservative. A demonstration case for the optimal trajectory and speed profile is shown in Figure 5.1.

![Figure 5.1 - Optimal trajectory and speed profile for the demonstration case](image)

### 6 Spatial analysis

Some advantages can be obtained by using the spatial analysis. It allows a much more detailed vehicle modelling, where the most important bodies and kinematic joints can be included in the model, instead of having a simplified planar vehicle model. The fact of having 3 dimensions assures the representations of several mechanical components, starting by the suspension behavior, which is not included in the planar analysis and can be represented with detail here. Hence, it is possible to simulate the roll angle of the chassis as well as the camber angle variation in the wheels. Moreover, mass transfers are computed in the dynamic analysis and have great impact in the variation of the normal load on the tyres, which affects all the tyre forces.

The tyre model used in the DAP3D program is the U/A Tyre Model [10][11][12], which different from the one used in the planar dynamic analysis program (Pacejka’s Magic Formula [7]). Furthermore, the tyre properties used are also different.
In what concerns to the road modelling, some advantages are taken from the use of the 3D analysis because it allows the track model to have climbs and downs and also lateral inclination. Hence, real track geometries can be modelled and more accurate results can be obtained.

Regarding the control strategy, the same approach used in the planar analysis is applied here, which means that the controller acts the same way either if the road is planar or not. Furthermore, concerning the lap time optimization the same method is also considered, being the road considered planar for the trajectory optimization and also for the speed profile optimization.

The multibody model of the Lancia Stratos for the spatial analysis is based on the model used by Ambrósio and Gonçalves [9].

A demonstration case for the spatial analysis is presented, being the track used the same as for the planar analysis. The methodology applied for this demonstration case uses the standard controller, without preview and speed profile, as the computational effort needed for the spatial analysis is much greater than for the planar case. Furthermore, the simulation performed does not make the vehicle complete an entire lap, but only the first curve. However, it permits to conclude that the controller also have a good behavior for the spatial case. The vehicle initial velocity is 7.5 m/s and it starts at the origin, as shown in Figure 6.1, which depicts the vehicle trajectory along the track.

![Vehicle trajectory for the spatial demonstration case](image)

**Figure 6.1** Vehicle trajectory for the spatial demonstration case

7 Conclusions

A 2D multibody dynamic analysis program was successfully developed with all the necessary features needed to simulate the behavior of a car in a race track. The second part of this work, with respect to the controller, is implemented with the update of a previously developed controller by adding new features to it. First of all, it is ensured that the controller has the ability to lead the vehicle to complete a complete round to any track.

A preview distance is then added to the controller, which smooths the trajectory of the vehicle and better corresponds to the way that a human drives a car. The controller is updated
with the ability to follow a prescribed speed profile by applying torques to the wheels of the car, which consequently leads the dynamic analysis program to produce tyre-ground contact forces.

Another of this work consists in finding the optimal lap time for a race track, which is found by optimizing the trajectory first and by computing the optimal speed profile for the trajectory. The combination of this two different methodologies provides the optimal lap time.

The last part of the work was to adapt all the developments to the spatial case, using a detailed Lancia Stratos multibody model, which was achieved in part. The fact that the 3D case needs much greater computational effort did not allow all the developments to be adapted from the planar analysis, as the only results obtained were for a small segment of the track with the standard controller. The comparisons between the planar and spatial case do not bring up great conclusions and the fact that the tyre model used is different for the two cases also makes the results harder to compare. However, it became obvious that the controller also work well for the spatial analysis.

8 References


