Modeling and Simulation of the ECOSat-III Attitude Determination and Control System

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Abstract

Attitude determination and pointing control in cube satellites is challenging mainly due to volumetric constraints and lack of small attitude sensors. In addition, the attitude control systems in CubeSats typically employ magnetorquers as the main actuators, which become less effective at lower orbital inclinations and suffer from reduced pointing accuracy when compared to reaction wheels. As a result, most CubeSat control designs are characterized by considerable pointing errors. In the present work, an attitude determination and control system (ADCS) using low-cost sensors and actuators that allows for a pointing accuracy better than 4.78 deg is proposed. The development and comparison of four attitude estimation algorithms is performed using sun sensor, magnetometer, and micro-electromechanical systems (MEMS) gyro measurements. The multiplicative extended Kalman filter (MEKF) is implemented as a benchmark for the other three algorithms: the unscented quaternion Kalman filter (UQKF), the two-step optimal estimator (2STEP), and the constant gain geometric attitude observer (GEOB). For attitude control, an enhanced version of the B-dot control law using magnetorquers is implemented for the detumbling stage. A three-axis sliding mode controller is employed for the nominal, Earth-tracking phase, using a specially designed reaction wheel system (RWS). A momentum dumping system using the magnetic torquers is also devised to ensure that the RWS does not reach saturation. Simulations are performed in an environment for the ECOSat-III CubeSat. The proposed ADCS yields a pointing error lower than 2 deg and detumbles the satellite in 3.5 orbits.

Keywords: Attitude determination, attitude control, CubeSat, nonlinear estimation, nonlinear control, Kalman filtering

1. Introduction

A CubeSat (cube satellite) is a type of miniaturized satellite formed from the combination of 10 cm cubes, each with a mass of up to 1.33 kg. The CubeSat project was born in 1999 from a joint effort between Cal Poly and Stanford University to promote an inexpensive way for universities and students to develop skills and experience on the design, manufacturing and testing of small satellites. From this outline, it becomes evident that CubeSats are subjected to strict cost, power, mass and volume constraints. A typical ADCS, in particular, constitutes a very large percentage in terms of these four budgets for a regular satellite. Therefore, classical attitude determination and control strategies must not be expected to work on a CubeSat without some adaptation, which by itself is not an easy task. This leads to cube satellite designs characterized by poor pointing accuracies. This market gap has been seen as an opportunity for some, and everyday more commercial-off-the-shelf (COTS) attitude hardware is becoming available for the regular consumer. In addition, it is also possible to do an in-house customized manufacture of these hardware components. This work proposes an ADCS capable of delivering a pointing performance better than 4.78 deg. The system is designed in the framework of the ECOSat-III satellite (represented in Figure 1), the second satellite project of the University of Victoria within the Canadian Satellite Design Challenge (CSDC). During the course of its mission, ECOSat-III will need to fulfill certain pointing requirements. The initial task of the ADCS is to perform the detumbling of the satellite after separation. Then, it must be able to adjust its attitude from an arbitrary orientation in order to align its hyperspectral camera with nadir. This alignment is smoothly maintained throughout the orbit in what is called Earth, or nadir, tracking mode. During passes over Victoria, BC, the satellite may be required to enter targeting mode to point its antenna directly to the ground station. It may also be re-
required to enter safe mode, where the solar panels are aligned with the Sun’s direction to maximize power generation. The ADCS of ECOSat-III features a MEMS gyroscope, a magnetometer, a sun sensor, six magnetorquer rods and four reaction wheels in order to achieve the pointing requirements.

The most popular recursive algorithm for attitude estimation is the extended Kalman filter (EKF) [1]. For the past fifteen years, the EKF with the unit quaternion as the parameterization of rigid-body attitude has been the prime choice in ADCS designs for CubeSats, such as the AAU [2], the NCUBE [3], the SwissCube [4], the WPI [5] and the RAX [6] satellites. The linearization of the dynamics and measurement models inherent to the EKF, however, led to a growth in nonlinear attitude estimation methods in an attempt to obtain a better performance. Three of these alternatives are explored in this paper and compared to the EKF.

The first belongs to the realm of unscented filtering, which shows a performance improvement in terms of convergence properties. Unscented filtering has also been studied for CubeSat attitude estimation [7–9]. The second is part of a category termed two-step filtering, where the estimation process is divided into a first-step that uses an auxiliary state in which the measurement model is linear, and an iterative second-step to recover the desired attitude states. Two-step filtering has not been implemented on a CubeSat before. The third alternative approach belongs to the realm of nonlinear observers, which are formulated in terms of the attitude error dynamics, as opposed to the stochastic nature of attitude filters. Observers often feature global stability proofs, i.e. they can converge from any initial condition, which makes them quite attractive for the spacecraft attitude determination problem. Although they are a relatively new topic in attitude estimation, nonlinear observers have already been considered for CubeSats [10, 11].

As pointing requirements became more demanding and on-board computers became more capable, the design of spacecraft control systems has shifted towards active designs, namely three-axis stabilization. For CubeSats and other resource-limited spacecraft, magnetorquers—magnetic rods or coils that interact with the Earth’s magnetic field in order to generate a torque—are often the primary actuator for attitude control due to their simplicity and low cost. For a better pointing accuracy, some CubeSat designs are adopting reaction/momentum wheels as the primary actuator. This is made possible due to the rising advances in miniaturized satellite components, namely of the flywheels, which would often be characterized by a large diameter and mass to provide maximum momentum storage. The CanX-4 and CanX-5 nanosatellites (15 kg mass each) include three orthogonally-mounted reaction wheels, and three magnetorquers which manage the wheels’ momentum, achieving a pointing accuracy of roughly 1 deg [12]. The Delphi-3xt CubeSat uses the same actuator suite to achieve a mean pointing accuracy of 6 deg [13]. To the author’s knowledge, a RWS with four flywheels has not been used for attitude control in CubeSats before.

In this technical note, a sliding mode controller developed in [14] is applied for the nominal, Earth-tracking phase. This implementation tackles the issue of unwinding commonly present in continuous feedback quaternion-based controllers, where unnecessary large distances are traveled to reach the desired attitude. A momentum dumping controller introduced in [15] is used, allowing the redundant set of flywheels to be desaturated in real-time during the nominal phase without interfering with the pointing performance. An improved version of the B-dot controller from [16] assures the spacecraft successfully detumbles from launch-induced condition asymptotically to null angular rates.

2. Background

2.1. Frames of Reference

Parameterizing the attitude involves determining the orientation of a certain frame of reference, the spacecraft body frame B, with respect to reference frame. In this work, the reference used is the Earth-centered inertial (ECI) frame. Defining additional frames of reference also proves useful in this task.

2.1.1 Spacecraft Body Frame

The spacecraft body frame is designated by \( B = \{\hat{b}_1, \hat{b}_2, \hat{b}_3\} \). The origin of the frame is typically fixed at the center of mass of the spacecraft and the axes rotate with it. The conventioned orientation depends on the spacecraft manufacturer. In the case of ECOSat-III, the \( \hat{b}_1 \) axis points in the direction of the hyperspectral camera’s field-of-view, the \( \hat{b}_3 \) is aligned with the normal of the bottom plate adjacent to the antenna, and \( \hat{b}_2 \) completes the
right-handed system.

2.1.2 Earth-Centered Inertial Frame
The ECI reference frame is designated by \( I = \{ \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3 \} \). The \( \mathbf{i}_1 \) axis is aligned with the vernal equinox direction, which is the intersection of the Earth’s equatorial plane with the plane of the Earth’s orbit around the Sun, in the direction of the Sun’s position relative to the Earth on the first day of spring. The \( \mathbf{i}_3 \) axis is aligned with the Earth’s North pole. Lastly, the \( \mathbf{i}_2 \) axis completes the right-handed triad. Since neither the polar axis nor the vernal equinox direction are inertially fixed, the ECI axes are defined to be mean orientations at a fixed epoch time. The used epoch is the current standard epoch, J2000.

2.1.3 Local-Vertical/Local-Horizontal Frame
The local-vertical/local-horizontal (LVLH) reference frame is designated by \( O = \{ \mathbf{o}_1, \mathbf{o}_2, \mathbf{o}_3 \} \). Also denoted by the orbital reference frame. It is especially convenient for Earth-pointing spacecraft, being attached to its orbit. The \( \mathbf{o}_3 \) axis points along the nadir vector towards the center of the Earth, the \( \mathbf{o}_2 \) axis is aligned with the negative orbit normal, and the \( \mathbf{o}_1 \) axis completes the right-handed system.

2.2. Attitude Kinematics and Dynamics
In the present study, the unit attitude quaternion is used as the orientation measure of the satellite, frame \( B \), relative to frame \( I \). The kinematic differential equation for quaternion propagation is given by [17]

\[
\dot{\mathbf{q}} = \frac{1}{2} \mathbf{\Omega}(\mathbf{w}) \mathbf{q}
\]

(1a)

\[
\mathbf{\Omega}(\mathbf{w}) = \begin{bmatrix} -[\mathbf{w} \times] & \mathbf{w} & 0 \\
\mathbf{w}^T & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix}
\]

(1b)

where \( \mathbf{q} \) is the attitude quaternion of frame \( B \) with respect to frame \( I \), \( \mathbf{w} \) is the angular velocity vector of frame \( B \) with respect to frame \( I \), resolved in frame \( B \), and \([\mathbf{w} \times]\) is the cross-product matrix for \( \mathbf{w} \) defined as

\[
[\mathbf{w} \times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\
\omega_3 & 0 & -\omega_1 \\
-\omega_2 & \omega_1 & 0 
\end{bmatrix}
\]

(2)

Occasionally the vector of modified Rodrigues parameters (MRPs) is used to represent the local attitude error. It is related to the quaternion as follows:

\[
\mathbf{p} = \frac{\mathbf{e}}{1 + q_4}
\]

(3)

where \( \mathbf{p} \) is the vector of MRPs, and \( \mathbf{e}, q_4 \) are the vector and scalar parts of the \( \mathbf{q} \), respectively.

The rotational motion of the spacecraft is governed by Euler’s second law of motion [18]:

\[
\mathbf{J} \ddot{\mathbf{w}} = -[\mathbf{w} \times] (\mathbf{J} \mathbf{w} + \mathbf{h}^w) - \tau^w + \tau^c
\]

(4)

where \( \mathbf{J} \) is the spacecraft inertia matrix, \( \mathbf{h}^w \) is the angular momentum of the RWS, \( \tau^w \) is the RWS torque, \( \tau^c \) is the external control torque, and \( \tau^p \) is the perturbative torque, all expressed in frame \( B \). The momentum of a single flywheel about is axis is given by

\[
\mathbf{h}_i^w = J_i^w \left[ \dot{\mathbf{w}}^w_i + (\dot{\mathbf{w}}^w_i)^T \mathbf{w} \right], \quad i = 1, \ldots, 4
\]

(5)

where \( J_i^w \) is the wheel spin inertia, \( \dot{\mathbf{w}}^w_i \) is the wheel spin rate with respect to frame \( B \) and \( \dot{\mathbf{w}}^w_i \) is the wheel spin axis expressed in frame \( B \). The momentum exchange between the RWS and the spacecraft is performed by controlling the motor torques, are equivalent to the time derivative of Eq. (5), \( \dot{\mathbf{h}}_i^w \). Then, the RWS torque in Eq. (4) acting on the spacecraft is computed as

\[
\tau^w = \mathbf{W}^w [\dot{\mathbf{h}}_1^w \ldots \dot{\mathbf{h}}_4^w]^T
\]

(6)

with

\[
\mathbf{W}^w = [\dot{\mathbf{w}}^w_1 \ldots \dot{\mathbf{w}}^w_4]^T
\]

(7)

3. Hardware
The sensor hardware consists of a three-axis MEMS gyro, a three-axis magnetometer and a two-axis sun sensor. The actuator hardware consists of the magnetorquers designed in-house for ECOSat-II, the predecessor of ECOSat-III, plus a specially designed RWS to improve attitude pointing accuracy.

3.1. Gyro: Silicon Sensing CRS09-01
The selected gyro for ECOSat-III is the Silicon Sensing CRS09-01 MEMS gyroscope. Its main characteristics are displayed in Table 1. It is one of the best commercially available MEMS gyros [19], selected from a trade-off with 3 others: the Systron Donner QRS116, the Honeywell HG1900 and the Analog Devices ADIS16135. Being a COTS equipment, it costs only 900 USD. This price is beaten by the ADIS16135, which costs 200 USD less; however, the noise figures in the ADIS16135 were deemed too high for this application, making it impossible to achieve the mission requirements. The other gyro considered in the trade-study had prices in the order of thousands of dollars, which is very much above the ADCS budget. The CRS09-01 was thus deemed the best choice for quality with respect to price.

3.2. Magnetometer: PNI RM3100
The three-axis magnetometer used in this work is the PNI RM3100. Its main characteristics are displayed in Table 2. The RM3100 is the successor of
the discontinued MicroMag3, a magnetometer developed also by PNI which has a CubeSat flight legacy aboard the RAX-2 satellite, built by the University of Michigan, and the Aeneas CubeSat, built by the University of Southern California. Other magnetometers considered in the trade study have their cost in the order of hundreds of dollars; the RM3100 costs only 61 USD and has a similar performance to the other magnetometers considered.

3.3. Sun Sensor: Solar MEMS Technologies NanoSSOC-A60

The two-axis sun sensor considered in this work is the Solar MEMS Technologies NanoSSOC-A60 analog sun sensor for nano-satellites. Its main characteristics are displayed in Table 3. Apart from less precise photocells and in-house developed sensors, the SSBV sun sensor is the typical choice for CubeSat sun sensing. However, Solar MEMS Technologies’ alternative NanoSSOC-A60 sensor costs 1000 USD less, with a similar accuracy and a wider field-of-view (FOV). The digital edition, NanoSSoc-D60, was also considered. However, it is 1500 USD more expensive than its analog counterpart. Adding a microcontroller with an ADC attached to the CAN bus to fit an analog sun sensor has essentially a negligible cost compared to the alternative.

3.4. Magnetorquer: Custom-Built Torque Rods

ECOSat-III’s three-axis magnetorquer is constituted by six orthogonally-oriented magnetic torque rods, two per axis. The magnetorquer main characteristics are displayed in Table 4. These torque rods were designed for ECOSat-II and optimized for the lowest supply voltage that yields the highest possible magnetic dipole moment, considering the size constraints and power budget of the satellite, and deemed fit for use in ECOSat-III. The torque core chosen for the design was the SMA 50, an alloy which consists of roughly 50% iron and 50% nickel. This material was selected because of its lightness and high permeability and saturation induction. There is not yet a cost estimate for the custom-built magnetorquers, but it is theorized that it will be negligible with respect to the total ADCS cost.

4. Attitude Determination

Since that the ECOSat-III sensor suite is conditioned with respect to the limitations mentioned in Section 1, a thorough study is conducted on attitude estimation algorithms. It is essential to develop an algorithm that allows for the robust determination of the attitude using the proposed sensors. To this end, four algorithms from different families of estimators are studied.

4.1. Multiplicative Extended Kalman Filter

The MEKF was developed in [24] using an EKF framework, which is considered the “workhorse of
is the estimated angular velocity and to counter the redundancy of the four-component quaternions $\delta q$ and $\delta \hat{q}$, while using the correctly normalized $\hat{q}$ to ensure a globally non-singular representation of the attitude.

It can be shown that $\alpha$ has components of roll, pitch and yaw error angles, giving a direct physical meaning to the state error covariance [27].

The continuous-time linearized MEKF error dynamics model is then given by

$$\Delta \dot{x}(t) = F(t)\Delta x(t) + G(t)w(t)$$

where the 6-dimensional error state $\Delta x = [\delta \alpha^T \Delta \beta^T]^T$ includes a term related to gyro bias estimation, the process noise $w \equiv [\eta_v^T \eta_n]^T$ has a spectral density

$$Q = \begin{bmatrix} \sigma_v^2 I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & \sigma_n^2 I_3 \end{bmatrix}$$

and the state dynamics and noise input matrices are given respectively by

$$F = \begin{bmatrix} -[\hat{\omega} \times] & -I_3 \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad G = \begin{bmatrix} -I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & I_3 \end{bmatrix}$$

The model of Eq. (13) is then used with the standard EKF formulation.

4.2. Unscented Quaternion Kalman Filter

The UQKF was developed in [28] directly in discrete-time as a spacecraft attitude determination technique based on unscented filtering. As in the MEKF, a generalized three-dimensional attitude representation is chosen to define the local attitude error, which is subjected to a multiplicative operation. However, the UQKF is based on calculating for each iteration a set of sample points, called the sigma points, that provide a better mapping of the probability distribution than the linearized model of the MEKF. The system model is given by

$$x_{k+1} = f(x_k, k) + w_k$$

$$\tilde{y}_k = h(x_k, k) + v_k$$

where $x_k$ is the discrete-time state at $t = t_k$, $\tilde{y}_k$ is the discrete measurement vector, $w_k, v_k$ are discrete process and measurement noise vectors with covariances $Q_k, R_k$, respectively, and $f_k, h_k$ are discrete differentiable, nonlinear functions. The propagation is done by calculating the set of sigma points, $\chi_k$:

$$\sigma_k \leftarrow 2n \text{ col. from } \pm \sqrt{(n + \lambda)} \left( P_k^+ + Q_k \right)$$

$$\chi_k(0) = \tilde{x}_k^+$$

$$\chi_k(i) = \sigma_k(i) + \tilde{x}_k^+, \quad i = 1, 2, \ldots, 2n$$

Table 5: RWS characteristics for individual flywheel/motor [23].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum rotational speed</td>
<td>rpm</td>
<td>36 800</td>
</tr>
<tr>
<td>Motor mass</td>
<td>g</td>
<td>1.1</td>
</tr>
<tr>
<td>Flywheel mass</td>
<td>g</td>
<td>3.85</td>
</tr>
<tr>
<td>Spin inertia</td>
<td>kg m(^2)</td>
<td>5.67 \times 10^{-7}</td>
</tr>
<tr>
<td>Maximum power</td>
<td>W</td>
<td>0.8</td>
</tr>
<tr>
<td>Cost</td>
<td>USD</td>
<td>280</td>
</tr>
</tbody>
</table>

real-time spacecraft attitude estimation” [25]. The MEKF follows a multiplicative approach for the error quaternion in the body frame, denoted by $\delta q$:

$$\delta q \equiv q \otimes \hat{q}^{-1}$$

where $\hat{q}$ is the estimated quaternion and $\otimes$ stands for quaternion multiplication. The MEKF uses a linearized model for the propagation of $\delta q$:

$$\delta \dot{e} = -[\hat{\omega} \times] \delta e + \frac{1}{2} \Delta \omega$$

$$\delta \hat{q}_4 = 0$$

where $\hat{\omega}$ is the estimated angular velocity and $\Delta \omega \equiv \omega - \hat{\omega}$. A continuous-time model for the measured angular velocity using a rate-integrating three-axis gyroscope is given by the first order Markov process [25]

$$\hat{\omega} = \omega + \beta + \eta_v$$

$$\hat{\beta} = \eta_n$$

where $\omega$ is the measured rate, $\beta$ is the gyro bias and $\eta_v, \eta_n$ are independent zero-mean Gaussian white noise processes with

$$E\{\eta_v(t)\eta_v^T(a)\} = \sigma_v^2\delta(t - a)I_3$$

$$E\{\eta_n(t)\eta_n^T(a)\} = \sigma_n^2\delta(t - a)I_3$$

where $I_3$ is the $3 \times 3$ identity matrix and $\sigma_v, \sigma_n$ are performance indicators of the gyro sensor and are called the angle random walk (ARW) and rate random walk (RRW), respectively. When a gyro is used to measure $\omega$, the state vector is augmented to include error components of such measurement, such as the bias $\beta$. Since $\delta q$ represents a small rotation, the following relation is valid [26]:

$$\delta q(\alpha) = I_q + \frac{1}{2} \bar{\alpha} + \mathcal{O}(||\alpha||^2)$$

where $\alpha$ is a three-component vector, $\mathcal{O}(||\alpha||^2)$ denotes higher order terms, $I_q \equiv [0_{1 \times 3} 1]^T$ is the identity quaternion, and $\bar{\alpha} \equiv [\alpha^T 0]^T$. The MEKF estimates the parameter $\alpha$ to counter the redundancy of the four-component quaternions $\delta q$ and $\delta \hat{q}$, while using the correctly normalized $\hat{q}$ to ensure a globally non-singular representation of the attitude.
where \( n = 6 \) is the dimensionality of the system and \( \mathbf{x}_k^+ \), \( \mathbf{P}_k^+ \) are the previous post-update state estimate and covariance matrix, respectively, and \( \lambda \) is a design parameter. The sigma points are then propagated according to the system dynamics and the measured angular rate:

\[
\mathbf{x}_{k+1}(i) = f(\mathbf{x}_k(i), k), \quad i = 0, 1, \ldots, 2n
\]  

(18)

The predicted state \( \hat{\mathbf{x}}_{k+1}^+ \) and covariance \( \mathbf{P}_{k+1}^- \) are given by a weighed sum using the previous transformation of the sigma points.

Considering that an update is performed at \( t = t_k \), the predicted observation \( \hat{\mathbf{y}}_k \) is given by a weighed sum of the reference directions subject to the measurement model, \( \mathbf{y}_k(i) = h(\mathbf{x}_k(i), k) \). The innovation covariance \( \mathbf{P}_{k}^{yy} \), and cross-correlation matrix \( \mathbf{P}_{k}^{xy} \) are computed through a weighed sum of \( \mathbf{y}_k, \hat{\mathbf{y}}_k, \hat{\mathbf{x}}_k^+ \), and \( \mathbf{x}_k \). The update stage is thus reworked as

\[
\mathbf{K}_k = \mathbf{P}_{k}^{xy} (\mathbf{P}_{k}^{yy})^{-1}
\]

(19a)

\[
\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^+ + \mathbf{K}_k (\mathbf{y}_k - \hat{\mathbf{y}}_k)
\]

(19b)

\[
\mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k \mathbf{P}_k^{xy} \mathbf{K}_k^T
\]

(19c)

where \( \hat{\mathbf{x}}_k^+, \mathbf{P}_k^+ \) are the updated state and covariance matrix, respectively, and \( \mathbf{K}_k \) is the Kalman gain. Using the same gyro model from Eqs. (10), the state vector is composed of the attitude error and bias vectors:

\[
\mathbf{x}_k(0) = \hat{\mathbf{x}}_k^+ \equiv \left[ (\delta \hat{\mathbf{p}}_k^+)^T \ (\hat{\mathbf{p}}_k^+)^T \right]^T
\]

(20)

Here, \( \delta \hat{\mathbf{p}}_k^+ \) is the MRPs three-dimensional parameterization of the error quaternion used to propagate and update the quaternion. Eq. (18) portends the following. The conversion from error MRPs to error quaternions \( \delta \mathbf{q}_k^+ (i) \) is done using the inverse transform of Eq. (3) and the corresponding sigma points. The following quaternions are generated:

\[
\hat{\mathbf{q}}_k^+(0) = \hat{\mathbf{q}}_k^+
\]

(21a)

\[
\hat{\mathbf{q}}_k^+(i) = \delta \mathbf{q}_k^+(i) \otimes \hat{\mathbf{q}}_k^+, \quad i = 1, 2, \ldots, 2n
\]

(21b)

These quaternions are propagated using the quaternion kinematic equation and the estimated angular velocity. Then, the propagated error quaternions are given as

\[
\delta \mathbf{q}_{k+1}(i) = \delta \mathbf{q}_{k+1}(i) \otimes \hat{\mathbf{q}}_{k+1}(0)^{-1}, \quad i = 0, 1, \ldots, 2n
\]

(22)

This result is then transformed back to error MRPs which integrate part of the propagated sigma points. Note that, for the propagation stage, \( \hat{\mathbf{q}}_{k+1}^+ = \hat{\mathbf{p}}_{k+1}^+ \), so the part of \( \mathbf{x}_{k+1}^+ \) corresponding to the bias is constant from the previous time-step. After the update stage at \( t = t_k \), as in the EKF, the attitude error \( \delta \hat{\mathbf{p}}_k^+ \) is reset to zero.

4.3. Two-Step Optimal Estimator

The 2STEP algorithm was first developed by N.J. Kasdin and applied to spacecraft attitude estimation in [29]. It divides the measurement update stage into two steps: a linear first step through a nonlinear transformation of the desired states and a second step minimization that recovers these desired states.

The second step state \( \mathbf{x} \) is selected to be the desired quantities of interest, i.e. the attitude quaternion and gyro bias. Next, a first-step state \( \mathbf{y} \) is defined as a nonlinear mapping of \( \mathbf{x} \) and in which the measurement model is linear, that is

\[
\mathbf{y}_k = \mathbf{F}_k(\mathbf{x}_k)
\]

(23a)

\[
\hat{\mathbf{y}}_k = \mathbf{H}_k \mathbf{y}_k + \mathbf{v}_k
\]

(23b)

where \( \mathbf{F}_k \) is a nonlinear mapping function, \( \mathbf{H}_k \) is a linear measurement matrix. The measurement model used throughout this work, despite being quadratic in the quaternion, is linear in the attitude matrix \( \mathbf{A} \). The first-step state can therefore be chosen to include the elements \( A_{ij}, i, j = 1, 2, 3 \) of \( \mathbf{A} \) as

\[
\mathbf{y}_k = \begin{bmatrix} \dot{\mathbf{y}}_k^T \\ \mathbf{y}_k \end{bmatrix}, \quad \mathbf{y}_k = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \ldots \\ \beta_{2n} \end{bmatrix}
\]

(24)

Again, \( \omega \) is assumed to be obtained with a rate integrating gyro using the previous model. Rather than assuming a constant angular velocity during each time interval, 2STEP uses a first-order hold for the angular velocity between samples. The time propagation equations are modified accordingly. Then, a measurement update can be performed using the standard Kalman filter equations.

The second-step update involves finding the estimate \( \hat{\mathbf{y}}_k^+ \) from the first-step estimate \( \mathbf{y}_k^+ \) and its corresponding covariance matrix \( \mathbf{P}_{k}^+ \) by minimizing the cost function

\[
\mathcal{J} = \frac{1}{2} \left[ \mathbf{y}_k^+ - \mathbf{F}(\mathbf{x}_k) \right]^T \mathbf{P}_k^{-1} \left[ \mathbf{y}_k^+ - \mathbf{F}(\mathbf{x}_k) \right] + \frac{\lambda_L}{2} \left( \mathbf{x}_k^T \mathbf{B} \mathbf{x}_k - 1 \right)
\]

(25)

where \( \lambda_L \) is a Lagrange multiplier introduced to include the quaternion unit norm constraint and \( \mathbf{B} \) is a matrix that maps \( \mathbf{x} \mapsto \mathbf{q} \). The minimization of Eq. (25) may be done using a numerical least squares algorithm, such as the Gauss-Newton method. The initial first-step state and covariance is done through a Monte Carlo initialization using the initial second-step state and covariance.

4.4. Constant Gain Geometric Attitude Observer

GEOB was developed in [30] for UAV applications as an explicit complementary filter with bias correction. It was later applied in [31] to estimate the attitude using a single vector measurement and gyro,
assuming unbiased angular velocity measurements. It is formulated in discrete time as

\[
\begin{align*}
\hat{q}_{k+1} &= \Omega_k (\hat{\omega}_{k+1} + k_p \mu_{k+1}) \hat{q}_k \\
\hat{\beta}_{k+1} &= \hat{\beta}_k - k_f \mu_{k+1} \Delta t \\
\mu_{k+1} &= \sum_{i=1}^{N} k_i \hat{y}_{ik} \times \hat{y}_{ik}, \quad i = 2, 3, \ldots, N
\end{align*}
\]

(26a)

(26b)

(26c)

where \( k_p, k_f, k_i \) are positive gains, \( \hat{y}_{ik}, \hat{y}_{ik} \) are the i-th measurement and predicted measurement, respectively, \( N \) is the number of measurements, and \( \Delta t \) is the gyro sampling time. GEoB has proven stability properties when either the reference directions are stationary, or the reference vectors are time-varying but the gyro measurements are bias-free. In this work, the case when the reference directions are time-varying and the gyro measurements are biased is tested with simulations for the case of ECOSat-III.

5. Attitude Control

The nominal, momentum dumping, and detumbling control algorithms are presented in this section.

5.1. Nominal Mode

The sliding-mode controller for the nominal control of ECOSat-III was developed in [14]. An optimal control law for reaction wheels for asymptotic tracking of spacecraft maneuvers using sliding mode control is used, producing a maneuver to the reference attitude trajectory in the shortest distance. Let \( \delta q = q \otimes (q^*)^{-1} \) denote the error between the true quaternion \( q \) and the commanded quaternion \( q^* \). The controlled angular velocity is denoted by \( \omega^c \). The sliding manifod is given by

\[
\nu = \left( \omega - \omega^c \right) + k_c \text{sign}(\delta q) \Xi^T(q^*)q
\]

(27)

where \( \delta q \) is the scalar part of \( \delta q, k_c \) is a scalar gain and

\[
\Xi(q) \equiv \begin{bmatrix} q_4 I_3 + [\mathbf{e} \times] & -\mathbf{e}^T \\
-\mathbf{e} & 0 \end{bmatrix}
\]

(28)

The control law is given by

\[
\tau^w = J \left\{ \frac{1}{2} k_c \text{sign}(\delta q) \left[ \Xi^T(q^*) \Xi(q) \omega - \Xi^T(q^*) \omega^c - 2\mathbf{G}\hat{\omega} \right] - [\omega \times](J\omega + h^w) \right\}
\]

(29)

where \( \mathbf{G} \) is a 3 x 3 positive definite, diagonal matrix, and \( \hat{\theta} \) is a saturation function dependent on the sliding manifold \( \nu \) and the boundary layer thickness \( \varepsilon_i \). The discontinuous term \( \mathbf{G}\hat{\omega} \) is added to account for model uncertainties. The saturation function replaces the typical sign function in order to minimize chattering in the control torques.

5.2. Momentum Damping

A momentum management technique with redundant reaction wheels is implemented in this work following the strategy from [15]. To manage the momentum of the reaction wheels, a de-spin torque is imposed on each wheel using a feedback on the spin rates as

\[
\tau_{w}^f = -k_f J^w_w \left( \omega^w_c - \hat{\omega}^w \right)
\]

(30)

where \( J^w_w = \text{diag}[J_1^w \ldots J_4^w] \) is the diagonal matrix of the wheels’ spin inertia, \( \omega^w_c = [\omega^w_1 \ldots \omega^w_4]^T \) is the vector of wheel spin rates, \( \hat{\omega}^w \) is the vector of nominal wheel spin rates, and \( k_f \) is a scalar gain.

The use of this feedback law results in an excess torque that is compensated by the magnetorquers as

\[
\mathbf{m}_w^c = -([\zeta \times] \mathbf{W}^m) \mathbf{J}^w_w \tau_{w}^f
\]

(31)

where \( \mathbf{m}_w^c \) is the vector of the commanded dipole moment for each reaction wheel, \( \mathbf{W}^m \) is analogous to Eq. (7) for the six magnetorquer axes, \( \zeta \) is the Earth’s magnetic field flux density expressed in frame \( \mathcal{B} \) and \( \zeta^T \) denotes the pseudoinverse. Since the magnetorquers cannot generate a torque in the direction of \( \zeta \), this command may cause a perturbative drift on the spacecraft which is corrected by adding a corrective motor torque term:

\[
\Delta \tau_{w} = (\mathbf{W}^w)^\dagger (\mathbf{r}^c - \mathbf{W}^w \tau_{w}^f)
\]

(32)

where \( \mathbf{r}^c = -([\zeta \times] \mathbf{W}^m) \mathbf{m}_w^c \) is the torque applied on the spacecraft by \( \mathbf{m}_w^c \). If \( \tau_{w} = (\mathbf{W}^w)^\dagger \tau_{w}^f \) is the requested nominal control torque for each wheel from Subsection 5.1, then the total commanded reaction wheels motor torque is given by

\[
\tau_{w} = \tau_{w} + \tau_{w}^f + \Delta \tau_{w}
\]

(33)

5.3. Detumbling Mode

After its launch, the spacecraft is said to be tumbling, as it is characterized by very large angular rates which cannot be effectively controlled by RWS. The ADCS therefore has the responsibility of driving its angular velocity to zero so that it may enter the nominal control mode. An improvement of the B-dot control law follows from [16] and is given by

\[
\mathbf{m}^c = -\frac{k_w}{\|\zeta\|} \left[ \frac{\hat{\zeta}}{\zeta} \right] \omega
\]

(34)

where \( \mathbf{m}^c \) is the magnetorquer dipole expressed in frame \( \mathcal{B} \), \( \hat{\zeta} \) is the normalization of \( \zeta \) and \( k_w \) is a scalar gain. The commanded dipole of each torque rod is obtained through \( \mathbf{m}^c = (\mathbf{W}^m)^\dagger \mathbf{m}^c \). Reference [16] provides a way to compute the optimal gain \( k_w \) that ensures Eq. (34) leads the angular rates asymptotically to zero.
Figure 3: Performance of attitude estimators in Case 2 with $\Delta \varphi_0 = 180$ deg, $\| \Delta \hat{\beta}_0 \| = 2400$ deg h$^{-1}$. The grey vertical bars represent the eclipse condition.

Figure 4: Performance indicators Case 3 with $\Delta \varphi_0 = 180$ deg, $\| \Delta \hat{\beta}_0 \| = 2400$ deg h$^{-1}$. The grey vertical bars represent the eclipse condition.

Figure 5: Norm of the angular velocity in Case 4 with $\| \omega_0 \| = 57.3$ deg s$^{-1}$.

6. Results

Test results for the implemented estimation and control methods are presented in this chapter. Simulations are performed using a Simulink® model developed for ECOSat-III. This model includes spacecraft (S/C) orbit and attitude perturbations from the following sources: aerodynamic drag, solar radiation pressure, non-spherical Earth mass distribution, spacecraft gravity gradient and magnetic field interference. The general model parameters used are shown in Table 6. The state variables are updated using a Dormand-Prince Runge-Kutta 8(7) integrator with a 0.1 s fixed time-step. All sensor
measurements are updated at a rate of 10 Hz. The control variables are updated at a rate of 0.5 Hz for the nominal and momentum dumping control; the delayed time is a result of the dynamics of the RWS and magnetorquers, which require 2 s to settle on a steady-state value. For the detumbling control, the magnetometer control variables are updated at 10 Hz due to very high angular rates. The tests are run on an Intel® CoreTM i7-5700HQ CPU @ 2.70 GHz, 4 cores.

Four test cases are devised to emulate realistic scenarios for ECOSat-III.

Case 1

The S/C is assumed to be in a state after detumbling mode and before nominal mode in which the acquisition of the attitude estimate is desired. No attitude control is active. At the start of the simulation run, the rotational rate has a magnitude of four times the orbital rate, i.e. \( \| \omega \| = 4 n_{orb} \approx 0.24 \text{ deg} s^{-1} \), and along the direction \( \{1/\sqrt{3} \ 1/\sqrt{3} \ 1/\sqrt{3}\}^T \). The true initial attitude quaternion is assumed to be \( q_0 = [1 \ 0 \ 0 \ 0]^T \). A combination of four conditions to initialize the estimators is considered: a moderate initial attitude estimation error with \( \hat{q}_0 = [\sqrt{2}/2 \ 0 \ \sqrt{2}/2]^T \); an extreme initial attitude estimation error with \( \hat{q}_0 = [0 \ 0 \ 0 \ 1]^T \); a moderate initial bias estimation error with \( \hat{\beta}_0 = [6.8784 \times 10^{-3} \ 7.2609 \times 10^{-4} \ 1.1594 \times 10^{-2}]^T \); and an extreme initial bias estimation error with \( \hat{\beta}_0 = [1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2}]^T \). These conditions correspond to angular errors of \( \delta \varphi = 90 \text{ deg} \) and \( \delta \varphi = 180 \text{ deg} \) described about the axis \( \hat{n} = [1 \ 0 \ 0]^T \) and to bias estimation errors of \( \| \Delta \hat{\beta}_0 \| = 100 \text{ deg h}^{-1} \) and \( \| \Delta \hat{\beta}_0 \| = 2400 \text{ deg h}^{-1} \) in magnitude, respectively. The initial covariance matrices for MEKF, UQKF and 2STEP were fine-tuned in accordance to each initial condition. The parameters of the attitude estimation algorithms are shown in Table 7 (also for subsequent cases). The four algorithms are run in parallel for each of the four different conditions. The resulting angular estimation error over time is plotted in Figure 2.

Case 2

In a more realistic scenario, the initial conditions are difficult to estimate. Particularly, after the detumbling stage is completed, the attitude quaternion is random. This makes it impossible to perform an initial covariance matrix tuning with an a priori knowledge of the initial state. Given GEOB’s convergence properties from Case 1 (see Figure 2), it is used for this case to initialize the three other estimators since it does not make use of covariance data. The circumstances of Case 2 are the same as those for Case 1. Only one initial condition is considered, with \( \hat{q}_0 = [0 \ 0 \ 0 \ 1]^T \) and \( \hat{\beta}_0 = [1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2}]^T \) rad s\(^{-1} \), corresponding to \( \delta \varphi_0 = 180 \text{ deg} \) and \( \| \Delta \hat{\beta}_0 \| = 2400 \text{ deg h}^{-1} \). Three instances are run in parallel: GEOB is run initially and at \( t = 700 \text{ s} \), approximately 12 min after the start of the simulation, the switch to MEKF, UQKF or 2STEP is performed. The filters are initialized with the last state computed by GEOB and their initial covariance matrices were fine-tuned assuming convergence. The resulting angular estimation error over time is plotted in Figure 3.

Case 3

For Case 3, the performance of the nominal mode and momentum dumping controllers is evaluated. Given the analysis done for Case 2 (see Figure 3), the attitude is estimated using UQKF initialized with GEOB. In order to stress the system, the attitude estimator and controller are activated simultaneously, i.e. no time is given for the attitude estimate to converge before the control begins. The attitude is controlled using the RWS while the momentum dumping is performed with the magnetorquers. The momentum dumping controller is continuously active. As in Case 1 and 2, the S/C is considered detumbled and the initial rotational rate is the same as Cases 1 and 2. The true initial attitude quaternion is assumed to be \( q_0 = [-0.4950 \ -0.1122 \ -0.7274 \ -0.4619]^T \), yielding an initial angular error between the true and commanded attitudes equal to \( \delta \varphi_0 = 180 \text{ deg} \) described about the axis \( \hat{n} = [1 \ 0 \ 0]^T \) and an initial error between the true and commanded angular rates equal in magnitude to \( \| \Delta \omega \| = 0.2766 \text{ deg s}^{-1} \). The initial quaternion and bias estimates are taken to be \( \hat{q}_0 = [0.1122 \ -0.4950 \ -0.4619 \ 0.7274]^T \) and \( \hat{\beta}_0 = [1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2} \ 1.1712 \times 10^{-2}]^T \), resulting in an initial angular error between the true and estimated attitudes equal to \( \delta \varphi_0 = 180 \text{ deg} \) described about the axis \( \hat{n} = [0 \ 0 \ 1]^T \) and to an initial bias estimation error of \( \| \Delta \hat{\beta}_0 \| = 2400 \text{ deg h}^{-1} \) in magnitude. The switch from GEOB to UQKF occurs at \( t = 700 \text{ s} \). The parameters of the attitude control algorithms are shown in Table 8 (also for case 4). The resulting pointing angular error and the RWS angular speed tracking error are plotted in Figure 4.

Case 4

For the last case, the objective is to detumble the S/C. The initial angular rate is set to a very high value of \( \| \omega \| = 57.3 \text{ deg s}^{-1} \) along the direction \( \{1/\sqrt{3} \ 1/\sqrt{3} \ 1/\sqrt{3}\}^T \). The true initial attitude quaternion is assumed to be \( q_0 = [-0.4950 \ -
0.1122 −0.7274 −0.4619\(^T\). No attitude estimation algorithm is active. The detumbling is performed using the magnetorquers only; the RWS is inactive. The evolution of the S/C body angular rate norm is plotted in Figure 5.

6.1. Discussion

Case 1 compares the performance of the four attitude estimators in the presence of four combinations of adverse initial conditions. From Figure 2, all four algorithms are found to converge. For the case of the stochastic estimators, MEKF, UQKF and 2STEP, a similar steady state response is observed, with a maximum error at roughly 1.3 deg and a minimum below a hundredth of a degree. In the case of MEKF and UQKF, the initial errors cause delayed convergence times, where more sensitivity to errors in the initial attitude estimation rather than in the bias is shown and UQKF outperforms MEKF in every situation. The convergence time of 2STEP is approximately constant for all conditions and shorter than those of the previous two filters. Note that it was the most sensitive one to variations in the initial covariance matrix. The observer, GEOB, shows the shortest convergence time, but a poor steady state performance, averaging 10 deg and even reaching 68 deg in error.

With respect to run times, an average run of UQKF is roughly 4 times more computationally expensive than one run of MEKF. However, both still take less than one thousandth of a second to run each iteration on average. 2STEP is the most expensive algorithm, being 8 times slower than MEKF and 2 times slower than UQKF, on average. GEOB is the fastest algorithm: it is 4 times faster than MEKF, 17 times faster than UQKF and 34 times faster than 2STEP, on average.

The three filters were tested with an initialization done with GEOB with extreme initial attitude and bias errors, as the observer showed good convergence properties for every initial condition without changing the gain tuning. The resulting performance was comparable for the three filters. According to the analysis done so far, UQKF is faster than 2STEP and less sensitive to the initial covariance tuning corresponding to the GEOB-provided initialization than either 2STEP or MEKF. Regarding the estimator to be selected for the mission, it is therefore reasonable to dismiss 2STEP since the marginal gain in estimation performance that occurs for some segments does not justify the extra time it takes to run. As no limitation on the computational burden for this mission was imposed, an attitude estimation algorithm using GEOB for the initialization and UQKF for the steady state phase, as it is more likely to converge, is considered.

For Case 3, the nominal mission phase is simulated, where the devised estimation algorithm is used alongside the Earth-tracking and momentum dumping controllers to command the RWS and the magnetorquers to point the satellite at the Earth and maintain that orientation. Starting from extreme initial errors, Figure 4a shows the pointing error converges before a quarter of orbit is even completed, roughly 20 min after the start of the simulation. The desired attitude is tracked with a precision better than 2 deg, showing an average error of roughly 1 deg during eclipse and 0.7 deg during daylight. Figure 4b shows the momentum of the reaction wheels is successfully managed for the whole simulation, as the wheel spin rates track the desired biases and do not saturate. The small oscillations observed are due to the compensation of the external disturbance torques, which are not modeled by the controllers.

Lastly, Case 4 tests the detumbling algorithm using magnetorquers. Considering the S/C is detumbled when the norm of the angular velocity reaches \(||\omega|| \approx 4v_{orb}\), as mentioned in Case 1, then it can be seen from Figure 5 that the process is completed after 3.5 orbits. The figure window shows the angular velocity norm decreasing asymptotically towards zero; the S/C is thus prevented from becoming “stuck” in a pure spin state, i.e., when all but one component of the angular velocity are reduced to zero.

7. Conclusions

A fully functional ADCS for CubeSats was designed. The hardware selected consisted in COTS low-cost, small sensors and specially designed in-house actuators. The system was tested in a Simulink\textsuperscript{®} environment designed for ECOSat-III. Four different attitude estimators were compared. The convergence of MEKF and UQKF was found to be influenced by errors in the initial state estimates, with UQKF converging faster. 2STEP achieved fast convergence in every situation, but was the slowest and the most sensitive to the tuning of the initial covariance matrix. The steady-state performance of the three filters was comparable. GEOB was the fastest algorithm, both in terms of run time and convergence, but had the worst steady state error. An attitude estimator comprised of GEOB for the initialization and UQKF for the steady-state was tested with the nominal and momentum dumping controllers. Convergence was obtained 20 min after the start of the simulation and the desired attitude was shown to be tracked with a precision better than 2 deg. This represents a convergence obtained in 11.2\% of ECOSat-II’s time and a 58\% improvement on the imposed pointing error limit. The S/C was successfully detumbled in 3.5 orbits, the equivalent to 92.7\% of the detumbling time achieved previously for ECOSat-II.
Acknowledgements
The author would like to thank Dr. Daniel Choukroun of the Space Engineering department at the Delft University of Technology, and Mr. Alireza Khosravian of the College of Engineering & Computer Science at the Australian National University for their valuable participation and input to this research.

Appendix

Table 6: General simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Epoch</td>
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<td>2018-01-01, 00:00:00</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>a [km]</td>
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</tr>
<tr>
<td>Eccentricity</td>
<td>ϵ [-]</td>
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<tr>
<td>Inclination</td>
<td>i [deg]</td>
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</tr>
<tr>
<td>Period</td>
<td>Torb [h]</td>
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</tr>
<tr>
<td>S/C inertia</td>
<td>Ji [kg m^2]</td>
<td>(0.0069, 0.0357, 0.0361)</td>
</tr>
<tr>
<td>S/C mass</td>
<td>M [kg]</td>
<td>3.66</td>
</tr>
<tr>
<td>S/C residual dipole</td>
<td>m_{res} [A m^2]</td>
<td>0.0280</td>
</tr>
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Table 7: Estimator parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Gyro ARW</td>
<td>σ_v [rad s^{-1/2}]</td>
<td>2.90888 \times 10^{-5}</td>
</tr>
<tr>
<td>Gyro RRW</td>
<td>σ_u [rad s^{-3/2}]</td>
<td>3.49308 \times 10^{-8}</td>
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<tr>
<td>Mag. std. dev.</td>
<td>σ_{tam} [T]</td>
<td>1.5 \times 10^{-5}</td>
</tr>
<tr>
<td>Sun s. std. dev.</td>
<td>σ_{ss} [rad]</td>
<td>2.9 \times 10^{-3}</td>
</tr>
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<td>UQKF tuning params</td>
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</tr>
<tr>
<td></td>
<td>γ [-]</td>
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</tr>
<tr>
<td></td>
<td>κ [-]</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>λ [-]</td>
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</tr>
<tr>
<td></td>
<td>e [-]</td>
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<tr>
<td></td>
<td>f [-]</td>
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<tr>
<td>Mag. gain</td>
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<td>Sun s. gain</td>
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<td>Bias gain</td>
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<td>Quaternion gain</td>
<td>k_P [rad s^{-1}]</td>
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Table 8: Control gains.

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<tr>
<td>Nominal</td>
<td>k_c [rad s^{-1}]</td>
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<tr>
<td></td>
<td>G [rad s^{-1}]</td>
</tr>
<tr>
<td></td>
<td>ε_i [rad s^{-1}]</td>
</tr>
<tr>
<td>Momentum dumping</td>
<td>k_f [rad s^{-1}]</td>
</tr>
<tr>
<td>Detumbling</td>
<td>k_w [kg m^2 s^{-1}]</td>
</tr>
</tbody>
</table>

References


