



**TÉCNICO**  
LISBOA

**Efeitos do Empenamento em Núcleos de Paredes  
Resistentes de Edifícios Altos de Betão**

**Effects of the Warping of the Walls of Central Cores in Tall  
Buildings made of Concrete**

**Diogo Martins Rufino Costa Caiano**

**Resumo Alargado em Inglês**

Orientadores: Prof. João Carlos de Oliveira Fernandes de Almeida  
Prof. José Joaquim Costa Branco de Oliveira Pedro

**Júri**

Presidente: Prof. Luís Manuel Coelho Guerreiro  
Orientador: Prof. João Carlos de Oliveira Fernandes de Almeida  
Vogal: Prof. Rui Vaz Rodrigues

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# 1 – Introduction

When facing projects related to tall buildings, the control of movement becomes a must. Most situations show us that it is impossible to use elements with big dimensions and rigidity all around a building, since with this many difficulties arise regarding torsional rigidity.

With this in mind, it's safe to say that the presence of well dimensioned cores in a building is of extreme importance. The correct functioning of these elements will provide the building with a good response towards any lateral load, including the ones that bring torsion to the mix.

This work's aim is to analyze 3 different modeling techniques of the main core in a tall building as well as to see the effects that warping torsion has on people, in terms of comfort, that inhabit that same structure.

## 2 – Warping Concept

### 2.1 – General Concept of Torsion

Torsion ( $T_z$ ), by itself, is formed by two parcels: St Venant torsion (uniform torsion -  $T_v(Z)$ ) and warping torsion (non-uniform torsion -  $T_w(Z)$ ). The expression that illustrates so is the following:

$$T_z = T_v(z) + T_w(z) \quad (2.1)$$

### 2.2 – St Venant Torsion

In this type of torsion all the points of a certain section remain on the same plane and no axial deformations are generated. Instead, what we have are shear tensions along that section. St. Venant torsion is given by:

$$T_v(z) = GJ_1 \frac{d\theta}{dz}(z) \quad (2.2)$$

Where  $J_1$  is a constant of torsion:

$$J_1 = \frac{b_1 t_1^3}{3} + \frac{b_2 t_2^3}{3} \quad (2.2)$$

And  $b_1$ ,  $b_2$ , and  $t_1$ ,  $t_2$  represent the dimensions of the flanges of a certain section, like an I section (Taranath, B. S., 1997).

**2.3 – Warping Torsion**

As opposed to St. Venant torsion, warping torsion causes axial deformations and with that all the points of a certain section do not belong to the same plane any longer (Figure 1).

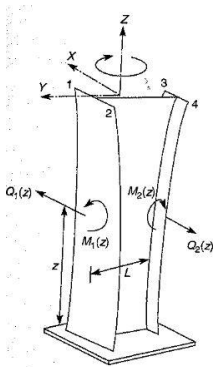


Figure 1 - Effects of Warping torsion by application of torsional force T (Taranath, B. S., 1997)

By applying a torsional force T, the edges 2 and 3 will come out and 1 and 4 will retract causing the existence of warping, which is given by the following expression:

$$T_{\omega} = -EI_{\omega} \frac{d^3 \theta}{dz^3} (z) \tag{2.3}$$

Where  $I_{\omega}$  is:

$$I_{\omega} = I_1 y_1^2 + I_2 y_2^2 \tag{2.4}$$

**2.4 – Bimoment and Sectorial Coordinate**

The tension of compression on a certain flange (I section of Figure 1 for example) is given by the expression:

$$\sigma_1(c_1, z) = \frac{M(z) L y_1 c_1}{I_1 y_1^2 + I_2 y_2^2} \tag{2.5}$$

or

$$\sigma_1(c_1, z) = \frac{B(z)\omega(c_1)}{I_\omega} \quad (2.6)$$

In equation 2.6, the bimoment,  $B(z)$ , is equivalent to  $B(z)=M(z)L$  and the parcel  $\omega(c_1)=y_1c_1$  is nothing more than a coordinate that goes by the name of sectorial coordinate or warping function.

A bimoment, in its simplest form, consists of equal and opposite couples acting in parallel planes.

This coordinate is the parameter that expresses the axial response (in terms of tensions and displacements) of a certain point on the profile of a warping core, relative to other points around the section. (Taranath B. S., 1997)

### 3 – Structural Modeling of the Cores of Buildings

#### 3.1 – Types of core modeling

Throughout this work 3 different types of core modeling were used: bar elements, an isolated bar and shell-thin elements. On the first case, each bar corresponds to a fraction of one of the walls that constitutes each core. Each bar is 4 meters high (distance between slabs) and has a certain area section (Figure 2 - left). On the second case, an isolated bar will be the element that will define the entire core. An area section is defined and it will remain the same until a certain height (Figure 2 - center). As for the last type, shell-thin elements, it's the most commonly used for core modeling as well as other elements of a structure. Here, an area is divided into small squares and the more the area is divided the more accurate the results become (Figure 2 - right).

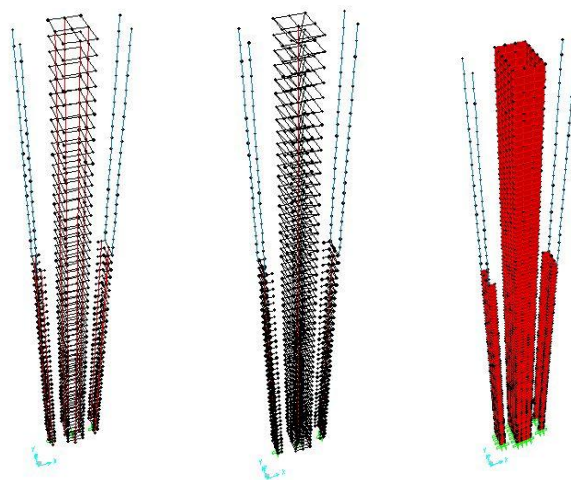


Figura 2 – Core modeling with bar elements (left), an isolated bar (center) and shell-thin elements (right)

### 3.2 – Characteristics of the modeled building

The model of the building, utilized to make the study, was a simplified version of the Tower Espacio in Madrid. It isn't an exact copy of that tower since it wouldn't be correct to study an existing building. The exterior form was transformed into a parallelepiped and all floors measure 43,8 by 28,4 meters. The building has a total height of 228 meters (57 floors each 4 meters high), there are cylindrical columns all around the building and there are a total of 3 cores (1 main core and 2 lateral cores).

However, there were a few variations regarding some measures that suffer a decrease as we climb towards the top of the model. Knowing this, it is possible to separate the model into 3 blocks. The first block corresponds to the first 35 floors, where the columns have 1,2 meters of diameter, two walls of the main core have 1,2 meters of thickness, while the other two have 0,80 meters, and all the vertical elements (columns and walls of the cores) are made of a high resistance concrete (C70/85). The second block goes from the floor 36 to the floor 47 and here the columns have 1,0 meters of diameter and the two walls of the main core that had 1,2 meters of thickness have instead just 1,0 meters. Moreover, the vertical elements of this block are made with a more common concrete (C40/50). Finally, the last block, which goes from the floor 48 to 57, shows columns with 0,8 meters of diameter and all the walls of the main core have 0,8 meters of thickness. Here, the concrete used was a C30/37.

Aside from these changes, it is also important to refer that, after floor 37, the lateral cores disappear. On its place appear 2 columns with the same diameter as the other columns of the same floor.

### 3.3 – Base Model

As said in the previous point, the Tower Espacio is on the basis of the model constructed using the program SAP2000. To ensure the case study model was accurately constructed, the vibration modes and frequencies of the Tower Espacio were also utilized to be compared to the ones to obtain through all the analysis in this work. Those vibration modes and frequencies can be seen at Table 1.

Tower Espacio			
Modes	Main Displacement	Frequency (Hz)	Period (s)
1	Displacement X	0,126	7,94
2	Displacement Y	0,154	6,49
3	Torsion Z	0,323	3,10

Table 1 – Vibration Modes, frequencies and periods of Tower Espacio

After having a baseline and knowing what should be expectable it was possible to initiate the study.

### **3.4 – Analysis of the First 35 Floors of the Case Study Building**

Since the case study building has a significant amount of variations regarding the dimensions of some elements while we progress towards its top, it was thought to be a good idea to analyze just the first 35 floors, which have no variations.

To do this analysis, 6 different models were used. Two models were used for each of the core modeling types referred in point 3.1, one without openings in the main/central core and another one with openings originating a main core formed by 2 C's symmetrically opposed.

The analysis of these 35 floors was meant to study the main core in terms of its vibration frequencies, horizontal displacements as well as the vertical ones (to see the existence of warping and quantify it) and the longitudinal tensions at the most stressed section. This was achieved by applying 3 forces of 10000 kN at the top of the model (using the program SAP2000) at the shear center: Two horizontal forces, one pointing along X axis and the other along Y axis, and a torsional force which creates a moment along Z axis.

This study was done to be compared with an identical study of the complete building and understand the impact that all the variations in dimensions have regarding frequencies, displacements and tensions.

### **3.5 – Analysis of the Complete Case Study Building**

In this point, a similar study to the previous one was done. The same 6 types of models were used but this time the full 57 floors were used instead of just the first 35. With this, it became possible to compare all these 6 models with the previous 6 from the point 3.4 and reach conclusions.

From the comparison, it was possible to conclude that the models done with an isolated bar, to represent the main core, are the most rigid. Since it is a method that doesn't represent that well the correct position of the shear center, it isn't the most suited for tall buildings (like Tower Espacio).

Regarding the models done with bar elements and shell-thin elements, both are very good types of modeling since they represent, in a better way, the position of the shear center, thus the displacements and rotations as well as the tensions are more accurate. Between both, the finite elements come on top due to the nature of the method of dividing an area into small squares. This way, it is possible to obtain information as detailed as someone wants.

In terms of frequencies, it happened what was expected. The models with an isolated bar show the biggest frequencies, since they are the most rigid, and the other two types have lower frequencies and have very similar values, having the shell-thin elements models slightly lower frequencies.

Talking now about displacements and tensions, it was possible to conclude that for the first 35 floors the displacements and tensions wouldn't show any traces of warping, whereas the study of the

complete building showed small differences in terms of vertical displacements between the edges of the flanges of the main core with openings. Also, the tension values lead to the same conclusions as the vertical displacements, confirming the existence of warping.

## 4 – The addition of Outriggers in Tall Buildings

An outrigger is nothing but a set of walls that link the main core to the lateral cores and then to the columns, situated around the building normally, with the objective to make all these elements work together as a whole and decrease the horizontal displacements of the building (Figure 3). They occupy an entire floor, which might be used as a technical floor, and are normally placed at the height where the strongest lateral loads occur, to better control horizontal displacements.

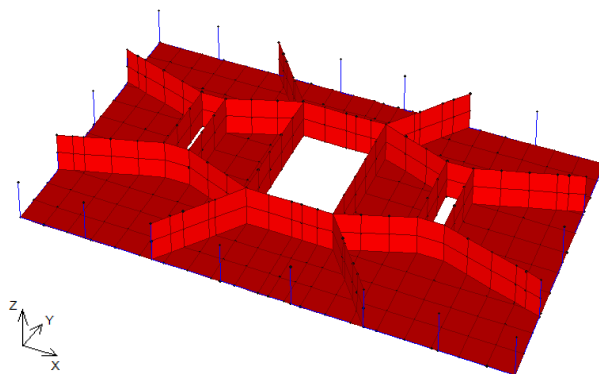


Figure 3 – Disposition of the walls that form an outrigger at a certain floor

To study the influence of the position and number of these elements, it was only used the model with the main core with openings and made with shell-thin elements, since it is the most accurate. Then, 3 different dispositions and number of outriggers were set to be compared: Just at the 57<sup>th</sup> floor, at the top and middle of the building and, for last, at each third, counting from the bottom (3 outriggers).

After applying the same loads as the previous study at the top of the building and getting the values for frequencies, horizontal and vertical displacements and longitudinal tensions, it was possible to understand the importance of the outriggers for tall buildings.

As previously said, the control of movement is of extreme importance when tall buildings come to the equation and the outriggers permit just that. By adding an outrigger at the top, the horizontal displacements decreased about 15%, with one at the top and another in the middle a decrease of an additional 5% was visible. However, when adding a third outrigger there was just a slight decrease of the lateral displacements which permitted to conclude that, unless in a very specific case, it is not worth having more than 2 outriggers.

Furthermore, the differences at certain points, in terms of longitudinal tensions, showed once again that the warping was present.



# 5 – Example of Application – Global analysis of Wind Effects

## 5.1 - Introduction

Finally, in this chapter, a real lateral load was used to do the analysis: The Wind. On one hand, the goal was to study the building in terms of frequencies, horizontal and vertical displacements and longitudinal tensions as it was done in previous chapters. On the other hand, it was supposed to measure the accelerations (with the help of EC1-1-4) at the top of the building and convert those into accelerations rms (root mean square), to understand the effect of those accelerations on the inhabitants of the building in terms of comfort.

To achieve the first goal, the same study was done twice but with different load cases. One had the wind as a base variable in a combination with the vertical loads and the second case had just the wind as a load. This would permit to understand the actual effect of the wind over the case study building. As for the second goal, after obtaining the accelerations on the top of the building and convert them into accelerations rms, those accelerations were compared with the curve of the norm ISO 6897. This way, it would be possible to reach a conclusion regarding how bad the comfort of the inhabitants of the case study building would be affected.

For both goals in this chapter, 4 different models were used and all of them had openings on 2 of the walls of the main/central core: The first one had its main core constructed with bar elements, the second was done using an isolated bar and the third and fourth using shell-thin elements. The last 2 models have, however, a little twist, since the lateral load (wind) is applied on the façade of the building with 5% of eccentricity (Shell-thin Elements (1)) and the fourth model, on top of that, also has an outrigger implemented at two thirds of its height (Shell-thin Elements (2)).

## 5.2 – Analysis of Frequencies, Displacements and Tensions

From the first analysis, using a load combination and the wind as the base variable in it, the results showed that, in terms of modeling, the conclusions about rigidity and information availability maintained on par with the previous studies. Aside from this, the study showed little to no signs of warping existence when considering just the vertical displacements. However, the longitudinal tensions came to confirm a subtle existence of warping.

Regarding the other analysis, using just the wind as the only load and not being affected by any coefficients, the results were on the same page as the ones of the study above. Moreover, it was possible to understand the importance of the existence of aerodynamic studies for tall buildings, since the gathered values of the horizontal displacements (most of all) came to show that the effects of the wind on tall buildings it's all but passive.

### 5.3 – Analysis in Terms of Comfort

This point of the work represents the pinnacle of the whole study. Here, it is finally possible to understand the impact that a real lateral load (the wind) has on a tall building and to what extent that affects the inhabitants in terms of comfort.

To obtain the accelerations at the top of the case study building, the same 4 models were used. By relying on the equations of the EC1-1-4, the dimensions and materials of the building as well as the direction of the wind (X or Y), it was possible to calculate the wind action and reach the desired accelerations. After that, it was also needed to convert them into accelerations rms and change the period of return from 50 to 5 years, after which the following values were obtained (Table 2):

Accelerations due to Wind Load along X and Y axis				
Acceleration rms ( $accel_{rms}$ )	Bar Elements	Isolated Bar	Shell-thin Elements (1)	Shell-thin Elements (2)
$accel_{rms,X}$	0,025	0,020	0,025	0,023
$accel_{rms,Y}$	0,046	0,036	0,042	0,038

Table 2 – Accelerations rms due to Wind Load along X and Y axis

Finally, it was possible to compare all these values with the curve from the norm ISO 6897 and reach conclusions (Figures 4 and 5).

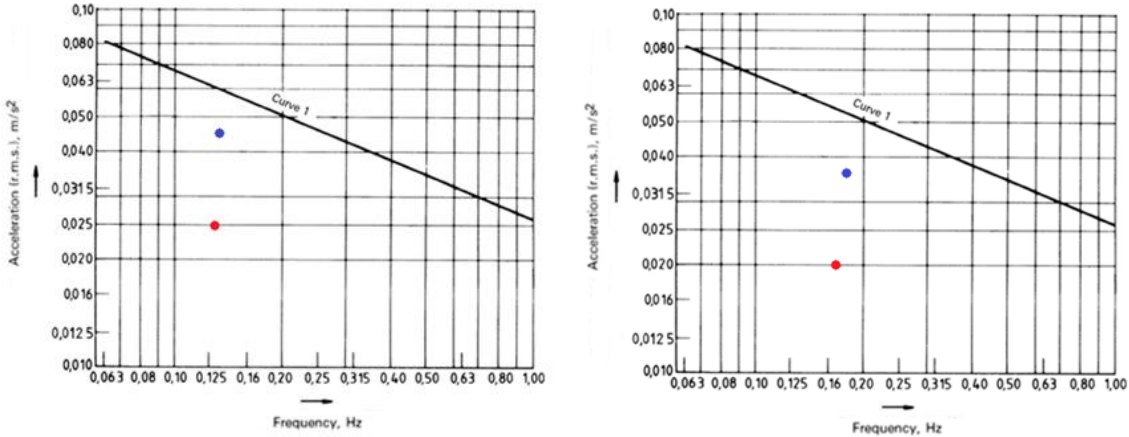


Figure 4 – Accelerations rms for Bar Elements model (left) and for Isolated Bar model (right)

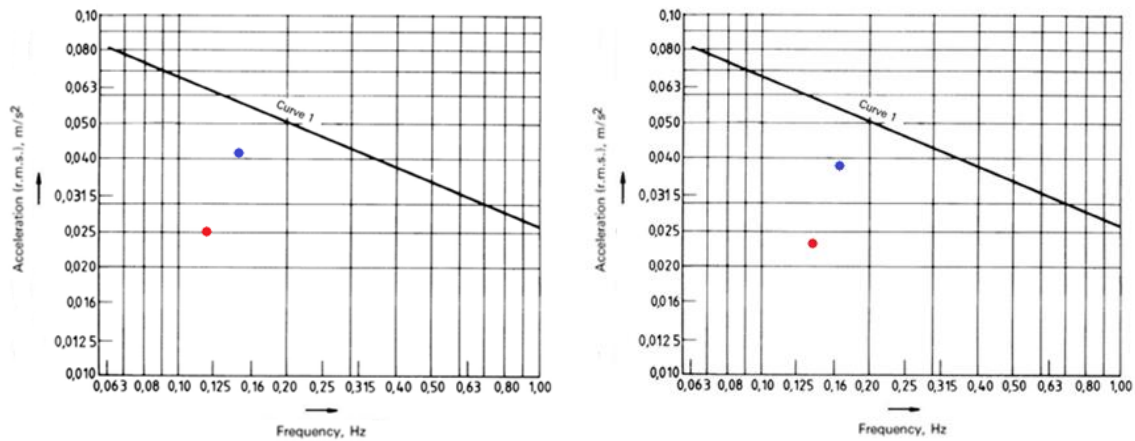


Figure 5 – Accelerations rms for Shell-thin Elements (1) model (left) and for Shell-thin Elements (2) model (right)

Having all the elements needed and judging by the comparisons of the colored dots (red for acceleration along X axis and blue along Y axis) and the curve 1 on Figures 4 and 5, it is clearly visible that the values are below the admitted limit and by a comfortable margin. In other words, it was possible to understand right away that, for a wind of this intensity, the inhabitants at the top of the case study building could only feel uncomfortable once every 5 years.

Furthermore, having into account the small vertical displacements, it can be said that the warping that results from those displacements isn't enough to affect the inhabitants of the building in terms of comfort.

Finally, with this last study, it was possible to conclude that any of the modeling types used for the cores is viable. For sure that the model that uses an isolated bar has its limitations in terms of the information a person can assess, but even so, the final accelerations were not that far away from the ones of the other models (see Table 2). When facing tall buildings, the model with the isolated bar might not be the most suited for the job as it was explained in previous chapters. However, it can be used.

## 6 – Conclusions

To sum up the most important points, in terms of the modeling types used, they all have its place as plausible options for core modeling. Opting for an isolated bar model for the core will result in less information and not so accurate, but it's a very simple way to construct the cores of a model of a building. As for the models using bar elements or shell-thin elements, those are the most common types to use and provide much more accurate information, being the last one the most precise.

Regarding the outriggers, those elements are without a doubt very important to control and restrain the horizontal displacements of tall buildings. As it was seen, real horizontal loads (like the wind) generate displacements that aren't that friendly to the structure of a tall building. That said, having other ways to restrain the movement beyond the increase of the rigidity of the cores it's always good.

Finally, by comparing the accelerations rms with the curve of the norm ISO 6897, it was possible to conclude that the warping generated by the action of the wind on the case study building, with the intensity of the one used for this work, was not enough for the inhabitants of the top of the building to feel uncomfortable but once every 5 years.

## References

Aguirre Gallego, M. (2008). Torre Espacio. Aspectos Constructivos de Ejecución de la Estructura. Hormigón y Acero, 59, 249, 45 – 56.

Alarcón Lopez de la Manzanara, J. (2010). La Torre Espacio en Madrid. Revista Técnica CEMENTO HORMIGÓN, 36.

Camarinha, R. (2008). Acção e Efeitos do Vento em Edifícios Altos. Dissertação para obtenção do grau de Mestre em Engenharia Civil. Instituto Superior Técnico, Universidade Técnica de Lisboa.

Choi, H. S., Ho, G., Joseph, L. and Mathias, N. (2012). Outrigger Design for High-Rise Buildings, CTBUH Technical Guides.

Cook, N. (2007). Designers' Guide to EN1991-1-4. Eurocode 1: Actions on Structures - Wind Actions. Thomas Telford.

Eurocode 1: Actions on Structures – Part 1-4: General Actions – Wind Actions. LNEC, 2010

Montalvão, M. T. (2009). Vibrações Induzidas pelo Vento em Edifícios Altos. Dissertação para obtenção do grau de Mestre em Engenharia Civil. Instituto Superior Técnico, Universidade Técnica de Lisboa.

Norm ISO 6897. (1984). Obtained in February 2015, from Web site ISO: [http://www.iso.org/iso/catalogue\\_detail.htm?csnumber=13419](http://www.iso.org/iso/catalogue_detail.htm?csnumber=13419)

Ramilo, N. (2009). Modelação de Núcleos de Edifícios. Dissertação para obtenção do grau de Mestre em Engenharia Civil. Instituto Superior Técnico, Universidade Técnica de Lisboa.

Taranath, B. S. (1997). Steel, Concrete, and Composite Design of Tall Building. McGraw-Hill.

Timoshenko, S. (1941). Strength of Materials: Part II: Advanced Theory and Problems. D. Van Nostrand Company, Inc., Second Edition.