NONLINEAR SYSTEM IDENTIFICATION OF AUTONOMOUS MARINE VEHICLES

JOÃO GUILHERME FERREIRA

Abstract. The wealth of applications for autonomous marine vehicles is a great motivation for research into control and navigation strategies. The knowledge of a model of the system is at the heart of many of the methodologies employed in these areas.

The goal of this thesis is to develop methods for obtaining models for Autonomous Marine Vehicles. We will focus on models classed as "off-white", i.e., based on first principles physical modeling but with a stochastic component and parameters to be identified through experimentation.

In order to achieve the stated goal we will model not only the vehicle dynamics but also its sensors and actuators. We will also design and run the experiments necessary for the system identification procedure. Finally Maximum Likelihood Methods are applied in order to extract the parameters of the models from the experiments conducted.

With the objective of assessing the validity of the proposed estimators and methodologies, extensive simulation studies are also carried out.

Finally, even tough navigation or rather state estimation is not the chief concern of this thesis, we will also develop navigation filters as it is a required step for some of the methods here presented.

1. Introduction

System Identification dates its history as a field to the works of Ho and Kalman in deterministic realization theory[HO and Kálmán, 1966] and Åström and Bohlin Åström and Bohlin [1966] on the application of Maximum Likelihood techniques to the problem of estimation of linear dynamic systems. These two very distinct works spawned two paths Gevers et al. [2006] within the System Identification field, the *realization* path with the further works of Akaike on statistical realization theory and leading to the Subspace Identification methods Van Overschee and De Moor [1996] and the *prediction error methods* of which Ljung [1999] is an example.

Nonlinear models, whose identification is considered in [Ljung and Vicino, 2005] as the most active area in the field, are usually classified under different shades of gray. At one end of this spectrum white-box models rely on extensive physical modeling from first principles usually without recourse to experimentation. At the other end, black-box models which do not make use of any prior knowledge of the physical relation between input and output. Somewhere in the middle are different shades of gray which leverage differently the rigid structure associated with physical modeling and the flexible and adaptable one of black box models.

The goal of this thesis is to develop methodologies for obtaining models for Autonomous Marine Vehicles for the purpose of further enabling control and navigation efforts. With this in mind, the choice of directly modeling the nonlinear system was made as the current focus in control of AUVs seems to favor the nonlinear approach.

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2. System Identification

The dynamical systems we are concerned with in this thesis evolve in continuous time and as such are described by a non-linear stochastic differential equation (SDE) of the general form:

$$\dot{\mathbf{x}}(t) = f\left(t, \mathbf{x}(t), \mathbf{u}(t); \boldsymbol{\theta}\right) + \mathbf{w}(t; \boldsymbol{\theta}) \quad (2.1)$$

$$\mathbf{y}_k = h\left(\mathbf{x}(t_k), \mathbf{u}(t_k); \boldsymbol{\theta}\right) + \mathbf{v}_k \quad (2.2)$$

where:

- $x \in \mathcal{X}$ denotes the state of the system;
- $\mathbf{u} \in \mathbb{R}^m$ the exogenous input;
- $\boldsymbol{\theta}$ a set of model parameters we seek to estimate;
- f the vector field yielding the deterministic part of the system's dynamics;
- $\mathbf{w} \sim \mathcal{GP}\left(\mathbf{0}, Q(t; \boldsymbol{\theta})\, \delta(t-\tau)\right)$ a zero-mean Gaussian White Noise process whose covariance matrix may depend on the parameter $\boldsymbol{\theta}$
- h is the measurement function;
- $\mathbf{v}_k$ is measurement noise to be later described in the context of each sensor;
- t_k the time of the k-th sample.

This analysis will be divided according to the presence or absence of process noise. This is because while the exact likelihood function of a nonlinear OE model can be easily computed, for nonlinear PE models approximations have to be made owing to our inability to obtain the exact solution to the filtering problem described in the previous section.

In Output Error parametric models, the system is assumed to be deterministic with the exception of the addition of noise at the output. These models are useful when only the deterministic part of the system is of interest but their performance can suffer when process noise is in fact present.

The likelihood function for such a model is:

$$L(\boldsymbol{\theta}; Y_T) = p(Y_T|\boldsymbol{\theta}) = p(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N|\boldsymbol{\theta})$$

When the output noise sequence$\{\mathbf{v}_k\}_{k=1,\ldots,N}$is zero-mean Gaussian and White (i.e. $\mathbf{v} \sim \mathcal{GP}\left(\mathbf{0}, R_k \delta_{k-l}\right)$):

$$\ln L(\boldsymbol{\theta}; Y_T) \propto -\frac{1}{2} \sum_{k=1}^{N} \left(\mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})\right)^T R_k^{-1} \left(\mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})\right)$$

where r is the dimension of $\mathbf{y}_k$, $\hat{\mathbf{y}}_k$ its predictor.

The gradient of the log-likelihood function (also called the score) can be computed as [Ljung, 1999]:

$$\nabla^T \ln L(\boldsymbol{\theta}; Y_T) = \sum_{k=1}^{N} \left(\mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})\right)^T R_k^{-1} \hat{\mathbf{y}}_{k,\boldsymbol{\theta}}(\boldsymbol{\theta})$$

where $\hat{\mathbf{y}}_{k,\boldsymbol{\theta}}(\boldsymbol{\theta})$ is the *output sensitivity* to the parameter. The Fisher Information Matrix is given

$$F = \sum_{k=1}^{N} \hat{\mathbf{y}}_{k,\boldsymbol{\theta}}^T(\boldsymbol{\theta}_\mathbf{0}) R_k^{-1} \hat{\mathbf{y}}_{k,\boldsymbol{\theta}}(\boldsymbol{\theta}_\mathbf{0})$$

and measures the sensitivity of the output to the parameters, the more sensitive the output response to a parameter is, the higher the information content for that parameter will be.

For linear-Gaussian prediction error (PE) models, the innovation sequence of the Kalman Filter is Gaussian and White and therefore the Likelihood function has the form:

$$\ln L(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{k=1}^{N}\left[\left(\mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})\right)^T S_k^{-1}(\boldsymbol{\theta})\left(\mathbf{y}_k - \hat{\mathbf{y}}_k(\boldsymbol{\theta})\right) + N\ln|S_k(\boldsymbol{\theta})| + Nr\ln(2\pi)\right] \tag{2.3}$$

with $\hat{\mathbf{y}}_k(\boldsymbol{\theta})$ the Kalman Filter Predictor. For nonlinear and/or non-gaussian models however computing the exact-likelihood function would require solving the exact filtering problem. One *ad-hoc* approach to approximating it would be using instead the popular Extended Kalman Filter (EKF) with Equation 2.3 as proposed in [Ljung, 1999, Mehra and Tyler, 1973, Jakoby and Pandit, 1987, Stepner and Mehra, 1973].

3. Problem Formulation

The general 6 Degrees of Freedom (DOF) rigid body equations of motion in the presence of current can be written as [Fossen, 1994]:

$$M\dot{\boldsymbol{\nu}} + C(\boldsymbol{\nu})\boldsymbol{\nu} + D(\boldsymbol{\nu}_r)(\boldsymbol{\nu}_r) + g(\boldsymbol{\eta}) = \boldsymbol{\tau} + \boldsymbol{\tau}_e \tag{3.1}$$

where $\boldsymbol{\nu}$ denotes the generalized velocity of the system and $\boldsymbol{\nu}_r$ that same velocity relative to the water, M the mass-matrix of the system, $C(\boldsymbol{\nu})\boldsymbol{\nu}$ the Coriolis and centripetal forces, $D(\boldsymbol{\nu}_r)(\boldsymbol{\nu}_r)$ the drag forces, $g(\boldsymbol{\eta})$ the restoring forces and $\boldsymbol{\tau}$ and $\boldsymbol{\tau}_e$ the generalized actuated forces and environmental forces respectively. Since in this thesis we are only concerned with the identification of the model of the vehicle at the surface, it is necessary to obtain the equations for the simplified 3 DOF model of the vehicle in the horizontal plane. We will thus use $\boldsymbol{\eta} = [x, y, \psi]^T$, $\boldsymbol{\nu} = [u, v, r]^T$ and $\boldsymbol{\tau} = [X, Y, N]$.

Assuming the vehicle is level ($\theta = \phi = 0$) the kinematic equations become:

$$\begin{aligned} \dot{u} &= \cos(\psi)u - \sin(\psi)v \\ \dot{v} &= \sin(\psi)u + \cos(\psi)v \\ \dot{\psi} &= r \end{aligned} \tag{3.2}$$

Assuming motion in z, θ and ϕ is nonexistent ($w = p = q = 0$), that the center of gravity of the vehicle coincides with the origin of the body frame and ignoring the environmental disturbances we have:

$$\begin{aligned} m_u\dot{u} - m_v vr + d_u u + d_{u|u|}(u - u_c)|u - u_c| &= X \\ m_u\dot{v} + m_u ur + d_v v + d_{v|v|}(u - v_c)|u - v_c| &= Y \\ m_r\dot{r} - (m_u - m_v)uv + d_r r + d_{r|r|}r|r| &= N \end{aligned} \tag{3.3}$$

where m_x denotes the diagonal element of the mass-inertia matrix relevant to the degree of freedom x and d_x and $d_{x|x|}$ its linear and quadratic drag coefficients. Following [Fossen, 1994] we will use a random walk model for the current in the inertial frame $\boldsymbol{\nu}_c^I = \mathbf{w}_c$ where $\mathbf{w}_c$ is a Gaussian White Noise process. If $\mathbf{w}_c = \mathbf{0}$ this model simply tells us that the current is constant in the inertial frame.

The vehicle used in this thesis, Medusa, was developed and constructed by the Dynamic Systems and Ocean Robotics Laboratory (DSOR) group from the Institute for Systems and Robotics (ISR) - IST.

It typically carries an Attitude and Heading Reference System (AHRS) and a Global Positioning System (GPS). It is actuated by 2 lateral thrusters standing roughly 30 cm apart.

Previous work on the Medusa vehicles [dos Santos Ribeiro, 2011] estimated its model parameters by normalization of the non-dimensional parameters of another vehicle and further hand tuning to match observed data. A further

i	m_i	d_i	$d_{i\|i\|}$
u	$37\,kg$	$0.2\,kg\,s^{-1}$	$25\,kg\,m^{-1}$
v	$47\,kg$	$50\,kg\,s^{-1}$	$0.01\,kg\,m^{-1}$
r	$9.69\,kg\,m^2$	$4.14\,kg\,m\,s^{-1}$	$6.23\,kg\,m$

TABLE 1. Nominal parameters of the Medusa's horizontal model.

iteration of these values presented in Table 3 will be taken as nominal values of the parameters and used in the simulation studies.

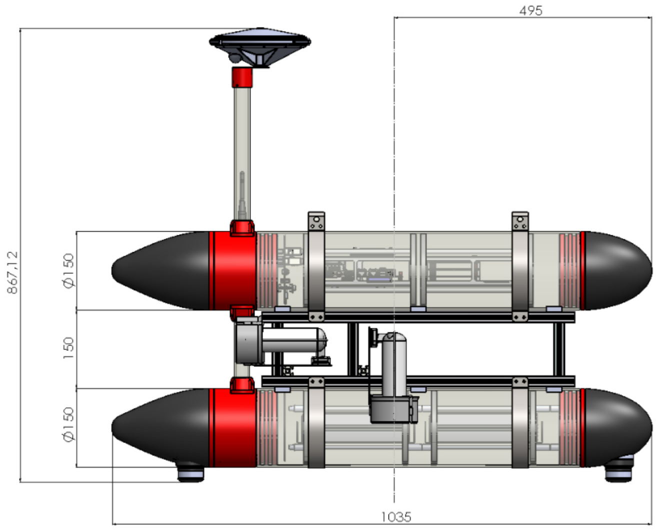

FIGURE 3.1. Profile view of the Medusa vehicle with dimensions in mm. Source [Maia, 2013].

4. SENSORS AND ACTUATORS

In order to design the identification experiments and carry them out, models for the sensors and actuators are required.

4.1. **Sensor Noise.** Sensor noise is a stochastic process that is often non-white. It can contain different components with different power laws such as (see for instance [IEEE, 1998, 1999]). In this thesis we will make use of a simple sum of white and random walk Gaussian components:

$$\mathbf{n}_k = \mathbf{w}_k + \frac{1}{1-q^{-1}}\mathbf{r}_k$$

where $\mathbf{w}_k$ and $\mathbf{r}_k$ are mutually uncorrelated white noise processes. Note that by taking the covariance of $\mathbf{r}_k$ to be 0,

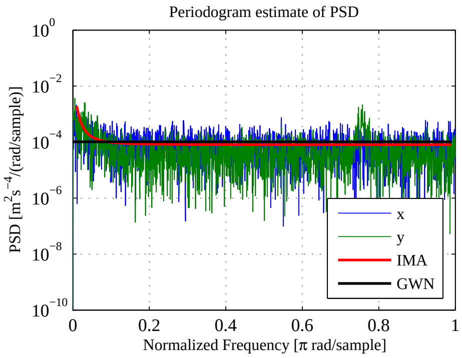

FIGURE 4.1. Periodogram estimate of the accelerometer horizontal data compared with the PSD of the identified models.

we get a simple white noise with constant bias model (the initial value of the random walk component), otherwise the "bias" may change in time. We will further assume that the component along each axis is uncorrelated with the others (i.e., That the covariance matrices of $\mathbf{w}_k$ and $\mathbf{r}_k$ are diagonal). This model in the equivalent innovation form given by the following Integrated Moving Average (IMA(1,1)) model:

$$\mathbf{n}_k = diag\left[\frac{1+c_x q^{-1}}{1-q^{-1}}, \frac{1+c_y q^{-1}}{1-q^{-1}}, \frac{1+c_z q^{-1}}{1-q^{-1}}\right]\mathbf{e}_k \tag{4.1}$$

where $\mathbf{e}_k \sim \mathcal{GP}\left(\mathbf{0}, R_k\delta_{k-l}\right)$ and R_k is diagonal. The parameters of this model are therefore the initial value $\mathbf{n}_0$ (can be regarded as the initial bias), the diagonal elements of R_k and the three c coefficients.

This model can be fit to static sensor data by Maximum Likelihood Methods Ljung [1999]. Depicted in Figures 4.1 and 4.1 are the PSD for the identified IMA models for accelerometer ad GPS.

4.2. **Actuators.** The following simple model was used to describe the relationship between the reference RPM command n_{ref} and the thruster RPMs:

$$\dot{n}\left(t\right) = a\left(n_{ref} - n\right)$$

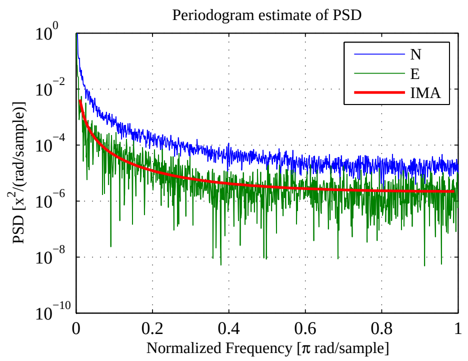

FIGURE 4.2. Periodogram estimates for the x, y and z axis of the magnetometer along with the PSD of the identified models.

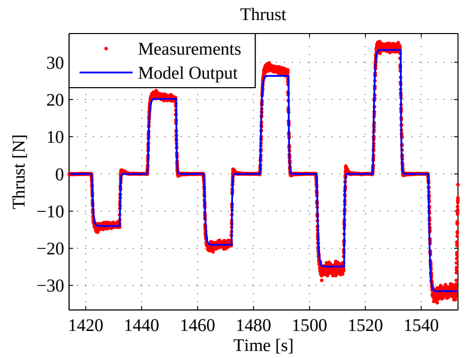

FIGURE 4.3. Identified RPM response to the step sequence (left) and a zoom of the thrust response for the last few steps (right).

The force exerted by the thruster is then related to its rpm by a static quadratic quadratic relation with odd and even terms:

$$f\left(t\right)=c_{1}n\left(t\right)\left|n\left(t\right)\right|+c_{2}n^{2}\left(t\right)$$

To identify the thruster model, the thruster was set up inside the pool connected through a rod to a support structure where a load cell measured the force exerted. A simple OE model was then fitted to both this force output and the thruster's rotation speed measurements for a sequence of progressively increasing step commands (depicted in figure 4.2) and two chirp command sequences (one of each sign).

a	$\left[s^{-1}\right]$	2.794
c_1	$[N]$	40.05
c_2	$[N]$	1.11

TABLE 2. Parameters of the identified thruster model.

5. DESIGN OF EXPERIMENTS

Given that the sway degree of freedom is unactuated, it is difficult to design an experiment that will excite it sufficiently. As such we turned to an optimal design approach where we seek to maximize the determinant of the FIM within a certain class of parametrized inputs. The chosen class was that of a sequence of steps with different amplitudes and durations as depicted in Figure 5. The variables chosen to characterize each input sequence are the amplitudes of the command for the left and right thrusters, U_{Lk} and U_{Rk}, and the duration of each step ΔT_k. Furthermore, the number of steps and total duration of the experiment was fixed. The optimal design of the input sequence can thus be formulated as:

$$\left(\mathbf{U}_{L}^{*},\mathbf{U}_{R}^{*},\mathbf{T}^{*}\right)=\arg\max\left|F\left(\mathbf{U}_{L},\mathbf{U}_{R},\mathbf{T}\right)\right|$$

In the design of the identification experiments we will assume an OE model to compute the Fisher Information Matrix. The reason for this is twofold:

- While we may have a reasonable guess for the values of the dynamics parameters, the value of the process noise's covariance matrix is rather more difficult to know a priori;
- The FIM for a PE model is considerably more time consuming to compute.

Unfortunately this problem is very hard to solve, the cost function has multiple local minima and too many variables while being somewhat costly to evaluate. We will make no claim of being able to obtain the optimal solution

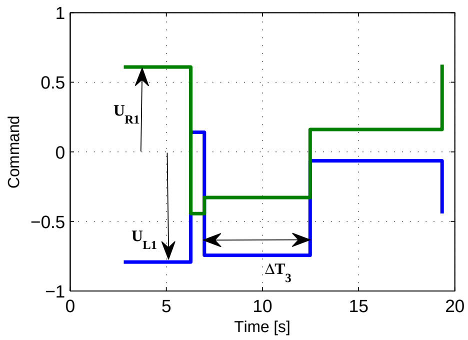

FIGURE 5.1. Parametrization of the Input Sequence.

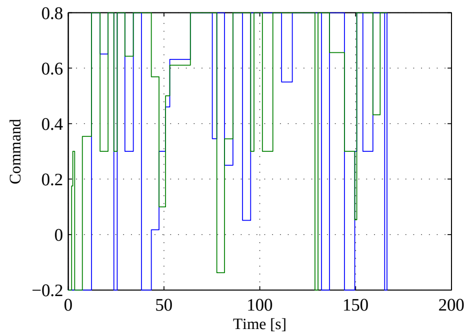

FIGURE 5.2. Designed input for the 3DOF model identification.

merely settling for a good enough one. The philosophy approaching this problem was simply of improving the solution until it represented a significant improvement over a random experience and given reasonable time-constraints. The optimization algorithm chosen for this was the *Pattern Search* algorithm [Hooke and Jeeves, 1961, Audet and Dennis Jr, 2002], a derivative free global optimization algorithm in the same vein of the Nelder-Mead's simplex method.

An example of a designed experiment meant to estimate the entire parameter set is depicted in figure 5.

In order to assess how sensitive the Fisher Information matrix is to the assumed nominal parameters, Monte Carlo simulations were carried out. A uniform distribution with support inside a box with vertices given by $0.75\boldsymbol{\theta}$ and $1.25\boldsymbol{\theta}$ was assumed as a prior over the true parameter. The FIM was computed for 1000 independent samples of

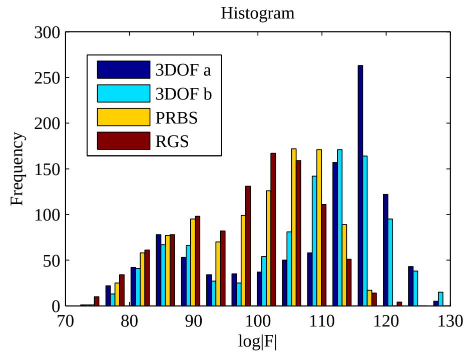

FIGURE 5.3. Histogram of the log-determinant of the FIM for the 3DOF experiment.

the parameter from this distribution, for both a the designed input sequences and a pseudo-random binary sequence (PRBS) and a random Gaussian sequence (RGS) whose bandwidth was optimized for the nominal parameters. A histogram of the logarithm of the determinant of the FIM is depicted in figure 5 where it is evident that the designed experiments outperform these classical inputs even when the true parameter of the system deviates from the value used to optimize the FIM:

6. System Identification

A set of three experiments was conducted in order to separately identify the a sets of mass, linear drag and quadratic drag coefficients for both Yaw and Surge DOF. For the PE model the process noise covariance is also estimated but not reported. The fit of an OE and a PE models to the Gyroscope Data in the Yaw experiment and to the GPS Data in the Surge experiment are depicted in figures 6 and 6 respectively. The identified parameters for each method and experiment are presented in tables 6 and 6.

i	m_r $[kg\,m^2]$	d_r $[kg\,m\,s^{-1}]$	$d_{r\|r\|} = [kg\,m]$
OE	6.0708	1.4452	18.4697
PE	6.6458	2.1412	17.0322

TABLE 3. Estimated Parameters for both the OE and PE MLE estimators.

i	m_u $[kg]$	d_u $\left[kg\,s^{-1}\right]$	$d_{u\|u\|}$ $\left[kg\,m^{-1}\right]$
OE	39.8539	0.0003	31.4743
PE	39.8006	0.0002	31.5035

TABLE 4. Estimated Parameters for both the OE and PE MLE estimators.

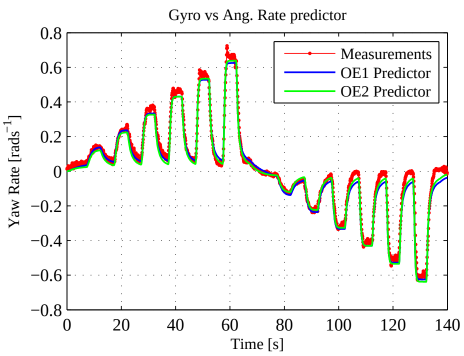

FIGURE 6.1. Gyro Measurements and both OE model's yaw rate predictors.

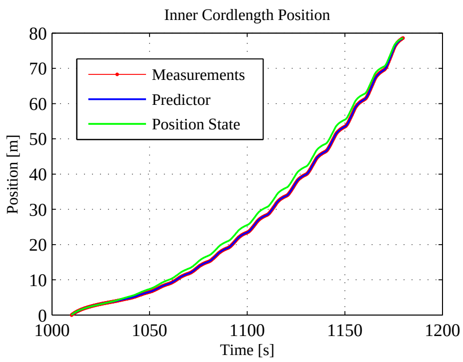

FIGURE 6.2. Position measurements, position predictor and the position state of the OE model (left). On the right are the first differences of these same signals.

Both a straightforward 3 DOF horizontal model and an augmented model including only the horizontal dynamics but the full kinematics (substituting gyro measurements for their dynamic equations) were considered in attempting to fit the data of the designed experiments. Only PE models were considered as the output error assumption can not be counted on for an experiment involving motion in three degrees of freedom. The full dynamic parameters identified are reported in table 6. As we can see, the values obtained differ slightly from those identified in the previous two experiments. This can be attributed to a number of factors from the thrusters behaving differently (less efficiently) in these experiments than in the straightforward, pure common mode or differential mode experiments previously carried out, to the excitation of unmodeled dynamics like the 3 ignored degrees of freedom or sloshing dynamics.

	3DOF	6DOF K
m_u $[kg]$	43.0775	40.6120
m_v $[kg]$	49.8655	46.3097
m_r $\left[kg\,m^2\right]$	13.5756	11.5091
d_u $\left[kg\,s^{-1}\right]$	8.4193	9.4546
d_v $\left[kg\,s^{-1}\right]$	81.3337	83.7277
d_r $\left[kg\,m\,s^{-1}\right]$	6.4794	5.7557
$d_{u\|u\|}$ $\left[kg\,m^{-1}\right]$	27.8441	29.1166
$d_{v\|v\|}$ $\left[kg\,m^{-1}\right]$	11.7343	11.5098
$d_{r\|r\|}$ $[kg\,m]$	19.9464	18.3117

TABLE 5. Identified parameters

In figures 6,66 and 6 we can see that the innovation sequences of the EKF predictors are far from the zero-mean white noise ideal and appear to be correlated to the inputs. This hints at the existence of unmodeled dynamics that are significant for the experiments carried out.

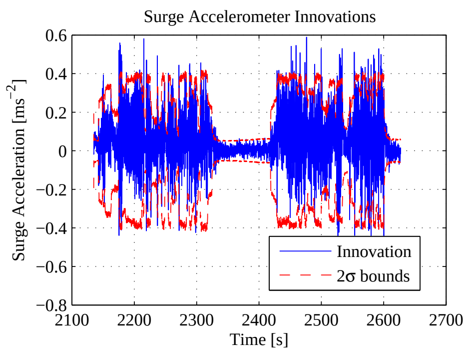

FIGURE 6.3. Surge acceleration innovations for the 3 DOF model.

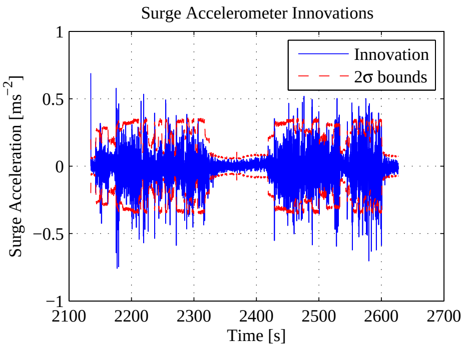

FIGURE 6.4. Surge acceleration innovations for the 6 DOF model.

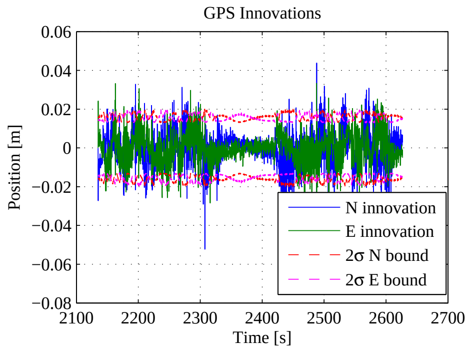

FIGURE 6.5. Position innovations for the 3 DOF model.

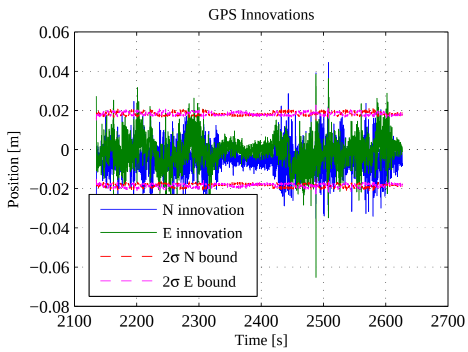

FIGURE 6.6. Position innovations for the 6 DOF model.

7. Conclusion

This thesis addressed the problem of system identification for Autonomous Marine Vehicles. A full methodology for obtaining a model for the system was developed, starting with the modeling of sensors and actuators, the designing of the experiments to extract the maximum amount of information out of the system in study and culminating with application of the developed estimators to both real and simulated data. The methods proposed to deal with the problem of system identification in itself were shown to have good performance in computer simulations where the assumptions that they are built on are by and large verified. Unfortunately this did not translate so well to practice. Out of the two classes of models explored, output error models were only applicable in practice to the two very simple 1 DOF experiments conducted and the PE models explored still left relevant unmodeled dynamics unaccounted for.

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