Abstract— Considering the great economical and scientific value of space exploration, there is a high demand for reliable navigation systems. These systems must ideally be independent, low-cost and with real-time capabilities. It is this extended abstract goal to study, validate and propose a complete system for navigation using pulsar radio emissions. The proposed system is divided into two parts, the receiving and the navigation subsystems. The first part of the system is tasked with the signal reception and pre-processing, performing de-dispersion, signal folding and filtering, all to allow the use of wide bandwidth and reduction of the required antenna size. The navigation block performs nonlinear Kalman filtering, producing a state vector that describes both the position and velocity of the receiver. Two different navigation techniques are presented, a first one using a simple time-of-arrival model and second a one, more complex, combining the Doppler shift and the inherent pulsar period slowdown. The system performance is analyzed through multiple simulations, showing that the receiver is capable of acquiring and processing weak signals and that the navigation block can produce a quality output, even when there is significant noise in the received data.

Keywords—Pulsar Star; Pulsar Navigation; Dispersion; Kalman Filter; Interstellar Medium; Doppler.

I. INTRODUCTION

Since their discovery in 1967 pulsar stars have been the focus of a broad range of studies, particularly for navigation purposes. Pulsars have been used in the Pioneer plaques and the Voyager Golden Record, depicting the Sun’s position relative to 14 pulsars. Pulsars are one of the possible outcomes of a supernova explosion, possessing massive density while conserving most of the magnetic field and the angular velocity of the initial star. Having a much smaller size than the original start boosts the effect of the magnetic field and angular velocity, which in turn induces the emission of narrow beams of electromagnetic radiation through the poles. Also due to the high angular velocity of the star, this radiation takes the form of pulses at a receiver. Pulsars are extremely stable timing sources and, considering also that they are widely spread throughout the galaxy, this makes them useful navigating beacons, analogue to the GPS system satellites [1].

The proposed pulsar navigation system is composed by two parts, a receiver and a navigation block. The first block is tasked with receiving the pulse (pulsar signal), de-dispersing, folding and filtering it, in order to afterwards determine its time-of-arrival (signal peak instant). This time of arrival information is sent to the navigation block. Using the time-of-arrival data and the pulsar angular positions, the navigation subsystem can calculate the spacecraft velocity and position, relative to a chosen frame of reference. This new information must then be sent back to the receiver as it needs to know, for the very least, the pulsar identity and the spacecraft velocity. Therefore, to start the system, the initial location and velocity of the spacecraft must be known or estimated.

The main focus of this work will be the design of appropriate modules for signal de-dispersion, noise removal and time-of-arrival (TOA) determination, as well as the development of a suitable Kalman filter.
II. RECEIVER SUBSYSTEM

As previously mentioned, the receiver block processes the data received by the system’s antenna. The data is sampled and, with the pulsar signal information, de-dispersed and folded to increase the signal-to-noise ratio. The folding process, performed with the correct period, will cancel out signals that do not possess the pulsar period, clearing out most of the initial noise. It must be stated that the folding period must be a Doppler shifted version of the initial pulsar period, taking into account the spacecraft real or estimated velocity. When no a-priori velocity information is available, a search must be performed at multiple velocities. Finally the folded data is processed by a matched filter, further isolating the signal so that, with square-timing-recovery, the pulse arrival times can be determined and sent to the navigation block.

A. De-Dispersion Stage

During its propagation through the Interstellar Medium (ISM) the pulsar signal is scattered and dispersed, becoming broader and harder to receive and recognize. The dispersion can be thought as a filter, making different frequencies of the signal arrive at different time instants [1]. The time delay between the arrival of the pulse at frequencies \( f_1 \) and \( f_2 \) can be expressed in seconds as:

\[
\Delta t \approx 4.15 \times 10^6 ms \times \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) DM.
\]

were \( DM \) (dispersion measure) is the integrated column density of free electrons between an observer and a pulsar.

Because of the use of wide receiver bandwidths, the dispersion of the signal is not negligible, significantly distorting the pulse for some pulsars. In order to reverse this phenomenon, two different methods of de-dispersion were considered. For the first, incoherent de-dispersion, the signal is divided into multiple small bands, little enough that within each the phenomenon of dispersion is practically inexistent. To each band a time delay is considered, according to equation (1), as a mean of properly aligning each signal. Finally, the resulting signals are added, creating a slightly imperfect de-dispersed signal whose resolution is dependent on the width of the channels.

Alternatively to this method coherent de-dispersion can be used, providing more accurate results yet being computationally more demanding. In this method a time convolution (or multiplication in the frequency domain) of pulsar signal by the following chirp function is performed [2]:

\[
H(f_0 + f) = e^{\frac{j2\pi f_0}{T_f + a}}f^2.
\]

Coherent de-dispersion is mostly performed in the frequency-domain, as it is more efficient. First the signal is converted to the frequency-domain with a Fast Fourier Transform (FFT) and afterwards multiplied by equation (2). Finally an Inverse Fast Fourier Transform (IFFT) is performed, yielding a time-domain de-dispersed signal, used in the following stages.

B. Overlap-add Convolution

Being the received signal by nature a continuous stream of samples, it becomes a necessity to addresses only small portions of it at any given time. Therefore, coherent de-dispersion should be performed with overlap-add convolution [3]. Considering an initial signal \( x[n] \) that is partitioned into smaller non-overlapping sequences, these are converted into the frequency-domain and multiplied with the FFT of \( h[n] \) (the filter function, i.e., de-dispersion function), resulting in the signal \( y_k[n] \). Finally the \( y_k[n] \) sequences are taken back to the time-domain and superimposed, building up the final output.

\[
\begin{align*}
\text{Overlap} & \quad \text{add method.} \\
\end{align*}
\]

C. Folding Stage

The received signal is significantly affected by noise, thus making it a necessity to employ adequate filtering techniques. Taking into account that the signal is periodic, with a known period, a folding stage can be used, in which new information is added to previously existing one, combining multiple pulses together in order to build up a usable signal. As for the noise, Gaussian by nature, it will cancel out. Defining the received signal as:

\[
y(t) = x(t) + n(t)
\]

where \( x(t) \) is the pulsar signal and \( n(t) \) is the Gaussian noise, the folding operation can be described as [4]:

---

Fig. 2. Schematic of an incoherent de-dispersion module.

Fig. 3. The overlap–add method.
\[
Z(t) = \frac{1}{K} \sum_{k=0}^{K-1} y(t + kP) \tag{4}
\]

In which \(K\) is the number of pulses folded and \(P\) the pulse period. The expected outcome of the folding procedure stage is:

\[
E[Z(t)] = E\left[ \frac{1}{K} \sum_{k=0}^{K-1} y(t + kP) \right] = E\left[ \frac{1}{K} \sum_{k=0}^{K-1} (x(t + kP) + n(t + kP)) \right] = E[x(t)] + \frac{1}{K} \sum_{k=0}^{K-1} E[n(t + kP)] = E[x(t)]. \tag{5}
\]

In the previous equation, \(E\) is the expected value operator.

As aforementioned, in order for the folding operation to be successful, it must be performed with an estimated period that takes into account the speed of the spacecraft. The degree of accuracy on assumed pulse period can be observed in the time drift of the maximum’s location. If a time drift of \(\Delta t\) occurs, during an observation time \(t_{\text{obs}}\), the folding period can be corrected as [1]:

\[
P_{\text{corrected}} = P \left(1 + \frac{\Delta t}{t_{\text{obs}}} \right). \tag{6}
\]

D. Matched filter

Knowing the average pulse shape, a matched filter can be used in order to further isolate the pulsar signal from the noise. The matched filter is described as [4]:

\[
h(t) = a P_n^* (t - t) \tag{7}
\]

where \(a\) is an arbitrary constant and \(P_n\) is the average pulse shape. Assuming that the input is again \(K\) pulses of period \(P\) each, the output of the matched filter will be:

\[
z(t) = \frac{1}{K} \int_0^{KP} y(s) h(t - s) ds = \frac{1}{K} \sum_{k=0}^{K-1} j_{kP} y(s) h(t - s) ds. \tag{8}
\]

\(z\) can still be expressed as:

\[
z(t) = \frac{1}{K} \sum_{k=0}^{K-1} \int_0^P y(s + kP) h(t - s - kP) ds
\]

\[
z(t) = f_0^P \frac{1}{K} \sum_{k=0}^{K-1} y(s + kP) h(t - s - kP) ds. \tag{9}
\]

being \(h(\cdot)\) periodic with period \(P\) the filter output becomes:

\[
z(t) = f_0^P Z_e(t) h(t - s) ds = a \int_0^P z_e(t) x^*(\tau - t + s) ds \tag{10}
\]

were \(Z_e\) is the output of the folding module. In discrete time \((t = mT_s, m = 0, \ldots, N-1)\) equation (10) becomes:

\[
z(m) = a \frac{f_s}{f_0} \sum_{n=0}^{N-1} Z_e(nT_s)x^*(\tau - mT_s + nT_s) \tag{11}
\]

were \(T_s = \frac{1}{f_s}\) is the sampling period, \(f_s\) is the sampling rate and \(N = f_0P\). As the signal and the noise are uncorrelated, when \(N \rightarrow \infty\) the filter output becomes:

\[
\lim_{N \rightarrow \infty} z(m) = aPR_{XX}(\tau - mT_s) \tag{12}
\]

were \(R_{XX}\) is the autocorrelation of \(x(t)\).

E. TOA Determination

The determination of the peak time was based on an algorithm of square-timing-recovery. In this algorithm a DFT (Discrete Fourier Transform) of the signal is performed, in order to determine the amplitude and phase of the sinusoid with period equal to signal’s period. Taking into account that a time offset becomes a phase shift in the frequency domain, the signal maximum instant can be determined.

The algorithm, applied to this problem, becomes [5]:

\[
\hat{t} = -\frac{P}{2\pi} \text{arg} \left\{ \sum_{k=0}^{N-1} e^{i2\pi k\hat{f}_0 T_s} \right\} \tag{13}
\]

where \(\hat{t}\) is the estimated maximum time, \(N\) is the number of samples per period and \(T_s\) is the sampling period. With this algorithm it’s possible to accurately determine both the signal maximum time and consequently the pulse period, essential information for the navigation subsystem.

III. RECEIVER SUBSYSTEM SIMULATION

In order to successfully simulate the previously described techniques, a realistic model of the pulsar signal was developed. Due to the star’s rotation the pulsar signal can be seen as a cyclostationary process [6]:

\[
x(t) = p(t) \cdot a(t) \tag{14}
\]

where:

- \(p(t)\) is a periodic repetition of a base pulse;
- \(a(t)\) is a zero mean, stochastic process.

\(a(t)\) is assumed to be a Gaussian correlated, stationary process.

Being \(p(t)\) a periodic signal, it can be expressed as:

\[
p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nT_0 t}, -\infty < t < \infty. \tag{15}
\]

The coefficients \(c_n\) are determined from the average pulse shape (fig. 4):

\[
P_m(t) = Ra(0) \cdot \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nT_0} \cdot \sum_{m=-\infty}^{\infty} c_m e^{j2\pi mT_0} \tag{16}
\]

were \(Ra(t)\) is the autocorrelation of process \(a(t)\). Knowing the \(c_n\) coefficients and the pulsar signal emission spectrum, the autocorrelation of process \(a(t)\) can be calculated:

\[
Ra(t) = \frac{\sum_{n=0}^{N-1} c_n e^{j2\pi nT_0} e^{-j2\pi nT_0 t}}{\sum_{n=-\infty}^{\infty} c_n e^{j2\pi nT_0}}. \tag{17}
\]

With knowledge of \(p(t)\) and \(a(t)\), the pulsar signal becomes fully described, allowing the simulation of multiples pulses, as can be seen in fig. 5.
The average pulse for pulsar J1012+5307

Fig. 4. The average pulse for pulsar J1012+5307

A sequence of 5 simulated pulses, from J1012+5307.

Fig. 5. A sequence of 5 simulated pulses, from J1012+5307.

The generated pulses can then be dispersed and subjected to additive Gaussian noise, according to the supposed pulsar signal-to-noise ratio (SNR). For an antenna with an effective area of 10 m², the best pulsars display an SNR ranging from −40 dB to −60 dB [6]. On Fig. 6, even for a relatively high SNR value, the simulated pulses almost disappear below the noise level. However, with proper processing most of the noise can be removed, allowing the detection of the signal maximum. Fig. 7 shows the final output of matched filter for the pulsar J1012+5307, after 3600 seconds of folding at a sampling rate of 100 MHz, for an initial signal SNR = −40 dB.

Fig. 6. The pulses in Fig. 5, after dispersion and addition of Gaussian noise (SNR = −5 dB).

Fig. 7. The processed signal, after de-dispersion, folding and matched filter.

For the millisecond pulsar J1012+5307, the achieved timing error, \( e = \hat{\tau} - \tau \), is presented in the following table, where for each SNR and folding time, \( T_{int} \), combination, 100 trials were performed.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>( T_{int} ) [min]</th>
<th>Average value (( \mu_e )) [( \mu s )]</th>
<th>Standard deviation (( \sigma_e )) [( \mu s )]</th>
<th>Equivalent precision [Km] (( \sigma_e \cdot c ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>−40</td>
<td>1</td>
<td>−7.7</td>
<td>1.3 \times 10^{-4}</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>−5.7</td>
<td>6.7 \times 10^{-5}</td>
<td>20</td>
</tr>
<tr>
<td>−50</td>
<td>1</td>
<td>−42</td>
<td>1.8 \times 10^{-4}</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>−51</td>
<td>1.1 \times 10^{-4}</td>
<td>33</td>
</tr>
<tr>
<td>−60</td>
<td>1</td>
<td>−27</td>
<td>1.0 \times 10^{-4}</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>−25</td>
<td>1.6 \times 10^{-4}</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 1. The receiver’s achieved timing error, for J1012+5307.

From the previous table it can be inferred that, for the supposed SNR range and the employment of a millisecond pulsar, the system’s equivalent precision will range from a few dozen to a few hundreds of kilometers. This value can however be greatly improved by usage of proper signal processing (filtering) in the navigation subsystem.

IV. NAVIGATION SUBSYSTEM

At the navigation block, the peak times coming from the receiver subsystem are processed, in order to estimate the system location and velocity. As stated before, two different navigation principles are considered. The first method, TOA, compares the arrival time of a pulse on the spacecraft, \( t_{asc} \), with the predicted arrival time of the same pulse, at a reference location, \( t_{asref} \). Based on the resulting time difference, and considering that the signal propagates at approximately the speed of light in vacuum, it is possible to estimate the distance between the two points. As for the second method, alternative, the inherent pulsar period slowdown and the received signal Doppler shift are jointly exploited, in order to determine the spacecraft location and velocity.
For both the TOA and the alternative methods, the chosen main frame of reference was the ICRF (centered at the solar system barycenter, SSB), even though any point in space can be chosen, as long as enough pulsar information is available, referred to that point.

A. TOA method

With an estimated spacecraft location, \( \tilde{r}_{sc} \) (relative to a chosen main frame of reference), it is possible to transfer the pulse arrival time (in the spacecraft) to the reference location, \( \tilde{t}_{a_{\text{ref}}} \). Taking into account only the geometric signal delay (ignoring clock error and relativistic effects), an error in the supposed location of spacecraft, translates into a time of arrival difference (or equivalently a phase difference) between the transferred pulse and the predicted one, i.e., \( t_{a_{\text{ref}}} \neq \tilde{t}_{a_{\text{ref}}} \).

Based on this time difference, both the assumed position and velocity of the spacecraft can be adjusted, in an iterative process until this phase difference is very close to zero and the correct state vector (position and velocity) is determined.

![Fig. 8. Iterative determination of position and velocity for the TOA method.](image)

Fig. 8 displays this iterative cycle. An initial assumption of position and velocity is given, from the planned trajectory parameters of the spacecraft \( \{1\} \). The iteration starts with a pulsar observation, during which the received pulses are recorded \( \{2\} \). The signals are processed, in order to remove most of the noise \( \{3\} \). The time of arrival (peak time) is determined \( \{4\} \) and afterwards corrected for the proper motion of the spacecraft, by transferring the peak time to the inertial reference location \( \{5\} \). Next the phase difference between the measured and the predicted peak time (both in reference to the inertial frame of reference) is calculated \( \{6\} \). If the phase difference is nonzero then the velocity and the position considered in \( \{1\} \) must be corrected and a new iteration is initiated \( \{7\} \). Alternatively, if the phase difference is already zero or very small, then the estimated values of position and velocity are considered correct.

If \( \tilde{r}_{sc} \) is a good estimation of \( r_{sc} \), along the pulsar line of sight, no phase ambiguity will occur and therefore:

\[
\tilde{r}_{sc} \cdot \tilde{u} = r_{sc} \cdot \tilde{u} + \left( t_{a_{\text{ref}}} - \tilde{t}_{a_{\text{ref}}} \right) \cdot c
\]

(18)

were \( \tilde{u} \) is the unit vector for the pulsar and \( c \) is the speed of light in vacuum.

Performing this process for three carefully chosen pulsars, it becomes possible to determine the three-dimensional position of the spacecraft.

It must stated that, due to the periodic nature of the signals, multiple ambiguous solutions may exist. In order to avoid this problem, the domain of possible solutions can be restricted to a finite space around an initial assumed position, or additional pulsars can be processed, as displayed in Fig. 8.

![Fig. 9. Solving the ambiguity problem by observing four pulsars \( \{7\} \). Straight lines represent planes of constant pulse phase; black dots indicate intersections of planes.](image)

It should be noted that it is not possible to directly determine the spacecraft velocity with the time of arrival measures. However an approximate velocity is calculated following:

\[
\tilde{v}_{i} = \frac{(\tilde{r}_{sc_{i}} - \tilde{r}_{sc_{i-1}})}{t_{i} - t_{i-1}}
\]

were \( \tilde{r}_{sc_{i}} \) is the spacecraft location at time \( t_{i} \) and \( \tilde{r}_{sc_{i-1}} \) is the spacecraft location at time \( t_{i-1} \). Both this locations are determined as previously described.

In order to achieve better accuracy in the phase shift measure, it must be used a mathematical model for the time transfer between the spacecraft and the reference location that includes relativistic effects and the clock error should be used \( \{8\} \):

\[
\tilde{t}_{\text{new}} - \tilde{t}_{\text{ref}} = \frac{1}{c} \tilde{u} \cdot (\tilde{r}_{sc}) - \sum_{i} \frac{GM_{i}}{c^{2}} \ln \left| \frac{\tilde{u} \cdot \tilde{r}_{sc_{i}} + r_{sc_{i}}}{\tilde{u} \cdot \tilde{r}_{ps_{i}} + r_{ps_{i}}} \right|
\]

\[+ \int \left( \sum_{i} \frac{GM_{i}}{c^{2}r_{sc_{i}}} + \frac{\tilde{v}_{sc}^{2}}{2c^{2}} - \text{const} \right) dt + \Delta t_{\text{clock}}. \]  

(19)

In the previous equation, \( \tilde{r}_{sc} \) and \( r_{sc} \) are the spacecraft location and velocity (relative to a chosen main frame of reference), \( G \) is universal gravitational constant, \( M_{i} \) is the mass of the
celestial body $i$, $\bar{r}_{SC}$ is the spacecraft location relative to the celestial body $i$, $\bar{r}_p$ is the pulsar position, also relative to $i$, and finally $\Delta t_{clock}$ is the clock error.

**B. Alternative method**

In this method the spacecraft location and velocity are determined by analyzing the received signal period, $P_r$. This period is calculated based on the signal peak time (pulse time of arrival) measured by the receiver subsystem.

For a pulsar signal, the Doppler shift phenomenon can be observed on the received pulse, lengthening or shortening its duration. The experienced Doppler shift is mostly due to the receiver subsystem.

If the receiver is stationary, the pulse decay at the reference location (determined with equation (22)) and the received pulse length calculated with the signal peak time measures from the receiver subsystem.

In a more realistic situation, where the spacecraft is in motion, it must be taken into account both its velocity $\vec{v}_{SC}$ (from equation (21)) and acceleration $\vec{a}_{SC}$ (from equation (28)), therefore [9]:

$$
\frac{dP}{dt} = \frac{\dot{P}}{P_0}(1 + \dot{\varphi}_{SC} \vec{v} \cdot \vec{c} + \ddot{P}_{SC}) \left(1 + \frac{\dot{P}}{P_0}(t - t_0)\right) \left(1 + \ddot{\varphi}_{SC} \vec{v} \cdot \vec{c} + \dddot{P}_{SC}\right).
$$

**C. Employed filtering and integration**

In the navigation subsystem, advanced filtering is performed, through the employment of an Unscented Kalman Filter (UKF) [8] [9]. The UKF is a recursive algorithm, receiving an input of noisy data streams, and returning a statistically optimal estimate of the system state. The filter requires the development of accurate state, control and measurement models.

Due to the complexity of the spacecraft dynamics, the propagation of the state variables is performed by a Runge-Kutta-Nystrom (RKN) integrator [9].

**V. NAVIGATION SUBSYSTEM SIMULATION**

Concerning the validation of the navigation subsystem, several simulations of an interplanetary journey between Earth and Mars were performed. The parameters for the trajectory were chosen from the Matlab script `prop_e2m` [10]. As for the pulsars used, in order to achieve a good system performance, they were selected based on their spatial location and signal quality.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>Period [s]</th>
<th>$\dot{P}$ [ss$^{-1}$]</th>
<th>RA [hr]</th>
<th>Decl. [degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1642-03</td>
<td>0.388</td>
<td>$1.780 \times 10^{-15}$</td>
<td>16:45:0.2</td>
<td>-3.300</td>
</tr>
<tr>
<td>B0950+08</td>
<td>0.253</td>
<td>$2.229 \times 10^{-16}$</td>
<td>9:53:9.3</td>
<td>7.297</td>
</tr>
<tr>
<td>B1451-68</td>
<td>0.263</td>
<td>$9.826 \times 10^{-17}$</td>
<td>14:56:0.0</td>
<td>-68.728</td>
</tr>
</tbody>
</table>

Table 2. The chosen pulsars and their pulse period, pulse decay, right ascension and declination [11].

![Fig. 10. The orbits of Earth (blue) and Mars (black), and the trajectory of the spacecraft (red).](image-url)
Two measurement models were used, each for one of the navigation methods. It must be stated that in both measurement models all relativistic effects and clock errors were ignored.

For the TOA method the model was:

$$h(x_k) = \begin{bmatrix} \Phi_{ref_i}(t_k) + \frac{v_i}{c} \cdot \frac{\bar{r}_{sc}}{c} \\
\Phi_{ref_i}(t_k) + \frac{v_i}{c} \cdot \frac{\bar{r}_{sc}}{c} \\
\Phi_{ref_i}(t_k) + \frac{v_i}{c} \cdot \frac{\bar{r}_{sc}}{c} \end{bmatrix}$$

were $x_k$ is the state vector at time $t_k$ and the notation $i$ in $\Phi_{ref_i}$, $\bar{r}_{sc}$ and $P_{0_i}$ means that they are relative to the $i^{th}$ pulsar, where $i = 1, \ldots, 3$. In this equation $\Phi_{ref_i}(t_k)$ is the pulse phase in the reference location at time $t_k$, $\bar{r}_{sc}$ is the spacecraft location at time $t_k$ and $[\ ]_w$ is the phase wrapping operation. For this measured model each set of three rows calculates the received pulse phase in location $\bar{r}_{sc}$, at the time $t_k$, for each pulsar.

As for the second navigation method, the measurement model was simply derived from the equations (17), (21) and (22), where all relativistic effects were ignored [9]:

$$h_{alt}(x_k) = \begin{bmatrix} \frac{\bar{r}_{sc}}{c} \left( \frac{1 + v_i}{c} \right) \left( 1 + \frac{v_i}{c} (t_k - t_i) \right) \frac{\Phi_{ref_i}(t_k)}{c} \\
\frac{\bar{r}_{sc}}{c} \left( \frac{1 + v_i}{c} \right) \left( 1 + \frac{v_i}{c} (t_k - t_i) \right) \frac{\Phi_{ref_i}(t_k)}{c} \\
\frac{\bar{r}_{sc}}{c} \left( \frac{1 + v_i}{c} \right) \left( 1 + \frac{v_i}{c} (t_k - t_i) \right) \frac{\Phi_{ref_i}(t_k)}{c} \end{bmatrix}$$

Similarly to the TOA measurement’s model, the notation $i$ in $P_{0_i}$, $\bar{r}_{sc}$ and $\bar{P}_i$ means that they are relative to the $i^{th}$ pulsar, where $i$ takes the values 1, 2 and 3.

In this equation the first three rows calculate the pulse period decay of the received pulses at the spacecraft, for each one of the pulsars, and the final three rows determine the received pulse duration, at the spacecraft, again for each pulsar.

Lastly, as a mean of fastening the simulations, it was assumed that the spacecraft would not need to correct its path and so no control model was developed or tested.

### B. Simulation results

In order to test the navigation subsystem capabilities, four scenarios were analyzed, emphasizing low-accuracy situations and the influence of integration time. These simulations were performed for the expected receiver subsystem capabilities, based on the information presented in Table 1.
For the most usual noise levels in the received signal, the TOA navigation method displays good results, faithfully replicating the spacecraft’s trajectory and velocity. It must be stated that this is largely due to the use of appropriate filtering procedures (UKF).

For the alternative navigation method, it was found that the concept of navigation based on pulse decay is unable to achieve acceptable results. It is however expectable that, with a more sensitive and accurate receiver, better results can be attained. The velocity determination using the Doppler effect appears to be relatively precise, and may be a viable option.

VI. CONCLUSION

A pulsar based navigation system was studied, proposed and analyzed in this document. As evidenced by the results of the multiple simulations performed, its composing subsystems displayed good theoretical results. It was shown that the receiver’s de-dispersion, folding and matched filter blocks are able of effectively recover the signal, from the noise noise, and that the pulse peak time can be accurately determined with by the square-timing-recovery module. The navigation subsystem performed as expected, accurately determining the location of the spacecraft, even with severe noise in the received pulsar signal. With the use of appropriate filtering, the information acquired by the receiver can be used successfully for spacecraft navigation purposes.

Table 3. Simulations results.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR [dB]</td>
<td>−40</td>
<td>−50</td>
<td>−60</td>
<td>−60</td>
</tr>
<tr>
<td>Folding time [s]</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>3600</td>
</tr>
<tr>
<td>$\sigma_x$ [s]</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$1.8 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>TOA method Max. error [m]</td>
<td>0.5</td>
<td>1.1</td>
<td>1.8</td>
<td>0.2</td>
</tr>
<tr>
<td>TOA method Max. error [m/s]</td>
<td>$5.9 \times 10^{-8}$</td>
<td>$1.5 \times 10^{-7}$</td>
<td>$2.8 \times 10^{-7}$</td>
<td>$2.7 \times 10^{-8}$</td>
</tr>
<tr>
<td>Alternative method Max. error [m]</td>
<td>$8.3 \times 10^7$</td>
<td>$2.0 \times 10^8$</td>
<td>$2.0 \times 10^8$</td>
<td>$3.1 \times 10^7$</td>
</tr>
<tr>
<td>Alternative method Max. error [m/s]</td>
<td>9.9</td>
<td>20.8</td>
<td>20.1</td>
<td>3.8</td>
</tr>
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REFERENCES