Signal-to-Noise Ratio Analysis and Noise Mitigation in Optical Coherent Access Networks

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\textbf{Abstract} — The drive for more bandwidth in the Optical Access Networks has renewed interest in coherent detection with polarization diversity. In this context, the digital signal processing has to handle with two different signals arrived from orthogonal polarizations. Coriant came up with a combined method for these two signals. This combined method is based on the signal power level. In this paper, we propose the symbol dispersion as better metric to combine polarizations, when compared to the signal power.

\textbf{Keywords} — Optical Communications, Access Optical Network, Coherent Detection.

I. INTRODUCTION

With the growing demand for higher data rates in telecommunications systems, the Full Service Access Network proposed a standard for the Next Generation Passive Optical Network, divided in two steps [1]. The first one, NGPON-1 allows the migration from GPON without interrupt the service. It preserves the key aspects of GPON while improves the data rate 4 times in downstream and 2 times in upstream. In the other hand, the NGPON-2, known as disruptive network, has no requirement in terms of coexisting with GPON [1], [2].

With this standard, Coriant has come up with Next Generation Optical Access networks (NGOA). These networks are based on UDWDM systems, heterodyne coherent detection with polarization diversity and DQPSK signal multiplexing. This architecture has the following goals: 100 subscribers, for maximum distance of 100 km and symmetric data-rates of 1 Gbps [2] [3].

Looking for the optical network unit (ONU) structure present in the NGOA, is provided a tunable laser that works as local oscillator in the coherent detection and also modulate the upstream signal. The downstream signal is 1 GHz far from the local oscillator, heterodyne detection. After coherent detection the digital signal processing (DSP) chooses the frequency allocated to that ONU. The Figure I.1 represents the ONU structure [4].

![Figure I.1: ONU-NGOA architecture.](image1)

In the Optical Line Terminal (OLT) the principle is the same, however is introduced the optical transmission group (OTG). The digital channels, already well organized in the spectrum in digital domain are modulated with the central frequency of the OTG, generating the downstream signal. In reception, once again the central frequency of OTG is used as local oscillator after that the DSP processes all the channels. The spectrum organization of OTG in OLT reception is presented in Figure I.2, [5].

![Figure I.2: Spectrum in OLT reception.](image2)

Usually the OTG has 8 up and down stream channels, the Figure I.2 only represents the upstream channels.

Attending on the DSP chain and the polarization diversity receiver, Coriant has been developing a combining algorithm for both polarizations. This algorithm among other aspects control the combining weight given to each polarization based on the signal power in each polarization. Between the two polarization signals, the signal with higher power has more relevance during combining. [5].

We easily understood that is possible to improve the combining algorithm. Sometimes we could have higher signal

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power (for a given polarization) but that signal is affected only by noise. When we are combining the two polarizations we are reducing the signal-to-noise ratio (SNR) instead of increasing the SNR. So, trying to avoid these situations we studied the symbol dispersion as new metric for combining signals.

To use the dispersion symbol in the combining algorithm we started to study the coherent detection systems and design a numerical simulation in order to obtain the relation between symbol dispersion and signal-to-noise ratio (SNR). After that and with some improvements on the numerical simulation of DSP implemented by Coriant, we evaluated the symbol dispersion according to bit error rate (BER) for laboratorial signal and synthetic signal.

II. OPTICAL COHERENT DETECTION SYSTEMS

A. Overall Coherent Detection System

Considering the receiving architecture present in NGOA, we will focus on coherent detection scheme, Figure II.1. This scheme has two polarization beam splitters (PBS), one coupler, balanced photodetectors (BPD) and the local oscillator laser (LO). The received signal (\(E_s(t)\)) and the local oscillator signal (\(E_{LO}(t)\)) are combining in the coupler and then split into two different polarizations. The components with the same polarization go to the BPD to be converted in electrical current. At the end we have two currents corresponding to two polarizations with the same information as the optical signal \(E_s(t)\) [6], [7].

The electrical field input of the receiver signal and the local oscillator can be defined as:

\[
E_s(t) = a(t). e^{j\theta(t)}\sqrt{P_s} e^{j(\omega_s+t_\theta_s)} e^{j\theta_n(t)} \tilde{u}_s
\]
\[
E_{LO}(t) = \sqrt{P_{LO}} e^{j(\omega_{LO}+\theta_{LO})} e^{j\theta_{nLO}(t)} \tilde{u}_{LO}
\]

The parameters \(P_s\) and \(P_{LO}\) are the optical power of the signal and the local oscillator respectively. \(\omega_s, \theta_\theta\) and \(\omega_{LO}, \theta_{LO}\) are the frequency and phase of the optical signal and the local oscillator respectively. The \(a(t)\) and \(\theta(t)\) correspond to the amplitude and phase of the digital signal. Finally \(\theta_{ns}(t)\) and \(\theta_{nLO}(t)\) are the noise phase of optical signal and local oscillator respectively [6], [8], [9].

The resultant currents from each polarization in the coherent detection systems have the following expressions:

\[
I_x(t) = R \cos \varphi \cdot \sqrt{P_s} P_{LO} a(t) \cos[\theta(t) + \omega_{IF} t + \theta_n] + n_x(t)
\]
(3)

\[
I_y(t) = R \sin \varphi \cdot \sqrt{P_s} P_{LO} a(t) \cos[\theta(t) + \omega_{IF} t + \theta_n + \theta] + n_y(t)
\]
(4)

with a phase difference of \(\theta\). Where \(I_x(t)\) is the current from polarization \(x\) and \(I_y(t)\) is the orthogonal polarization represented by \(y\). \(\theta_n\) is the total phase noise. \(n_x(t)\) and \(n_y(t)\) are the noise in each polarized signal.

B. Coherent Detection system without polarization diversity

Despite the coherent detection system adapted in NGOA, presented in Figure II.1, for numerical simulation and considering the scope of this work the polarization diversity was not considered. In this sense the Figure II.2 represents a coherent detection system used in simulation.

\[
E_s(t) = [a(t). e^{j\theta(t)} + n_s(t)] \sqrt{P_s} e^{j(\omega_s+t_\theta_s)} \tilde{u}_x
\]
(5)

The coupler represented in Figure II.2 as the same transfer function as the coupler present in Figure II.1. The mainly difference inherent in these two diagrams is the lack of PBS in the second coherent detection system (Figure II.2), this means no polarization diversity.

The electrical field input of the receiver signal and the local oscillator can be defined as:

\[
E_s(t) = a(t). e^{j\theta(t)} \sqrt{P_s} e^{j(\omega_s+t_\theta_s)} \tilde{u}_x
\]
(5)
\[ E_{LO}(t) = \left[ \sqrt{P_{LO}} + n_{LO}(t) \right] e^{i(\omega_{LO} + \theta_{LO})} \hat{a}_x \]  

(6)

The resultant current of the coherent detection system presented in Figure II.2 have the following expressions:

\[ I(t) = 2R \sqrt{P_{LO} P_S} a(t) \cos[\omega_{IF} t + \theta(t)] + 2R \Re \left\{ \left[ \sqrt{P_{LO}} n_x(t) + \sqrt{P_S} a(t) n_{LO}(t) \right] e^{i\omega_{IF}} \right\}, \]  

(7)

where \( n_x(t) \) and \( n_{LO}(t) \) represents the signal and the Local Oscillator laser noise respectively. \( \omega_{IF} \) is the intermediated frequency defined \( \omega_{IF} = \omega_s - \omega_{LO} \).

C. Noise modelling

Considering the coherent detection system used in MATLAB simulation, to evaluate the relation between symbol dispersion and SNR is necessary to estimate the different types of noise during the reception. In the reception chain are presented three different noises: Thermal Noise, Signal Shot Noise and Amplified Spontaneous Emission Noise.

The thermal noise is the minor contribution to the overall system. His physical meaning is the random fluctuations of thermally excited electrical carriers and his power is defined by expression (8).

\[ P_N = \langle i_R^2 \rangle \frac{B}{2} \left[ 1 + \frac{1}{12} \frac{B}{f_i} \right] \]  

(8)

In expression (8) the \( i_R \) is the resultant current from photodetector, \( B \) the electrical bandwidth and \( f_i \) is the electrical frequency [10].

The photodetection process of the incoming signal produces a corresponding electron flux and this flux has fluctuations. The fluctuation of the photocurrent is defined as the shot noise process. The shot noise is intimately related to the Poisson however for high-density, shot noise process is fairly approximated by Gaussian distribution [11],[10]. The spectral density of signal shot noise is in expression (9).

\[ S_{shot}(f) = 2q R P_R \]  

(9)

The parameter \( q \) is the elementary charge, \( R \) is the photodetector responsivity and \( P_R \) the power of the optical signal.

The amplified spontaneous emission (ASE) noise is responsible for main contribution to the optical link performance. Nowadays’ lightwave transmission systems benefit fully from extensive deployment of optical amplifiers.

These components operate directly in optical domain providing signal photo multiplication. During the signal photo multiplication occurs the spontaneous emission of photons. This physical process is the amplified spontaneous emission noise [11].

The power spectral density of amplified spontaneous emission noise is described by the following equation:

\[ S_{ase} = h v n_{sp} (G - 1), \]  

(10)

where \( h \) is Planck constant, \( v \) is the optical frequency, \( n_{sp} \) is the spontaneous emission factor and \( G \) is the gain of the amplifier.

For coherent systems this type of noise can be divided in two components, LO-Spontaneous beating noise and signal-LO beating noise, equation (11) and (12) [6].

\[ \sigma_{LO-sp}^2 = 2R^2 P_{LO} S_{ase} B_d \]  

(11)

\[ \sigma_{LO-LO}^2 = 2R^2 P_s S_{n_{LO}} B_d \]  

(12)

D. Theoretical SNR approach for Coherent Systems

The SNR was evaluated considering the coherent system present in Figure II.2, affected only by shot noise and amplified spontaneous emission noise previous described. The power of local oscillator also was considered bigger than 10 times the signal power (\( P_{LO} \gg 10 P_s \)) [6],[7],[12].

The signal-to-noise ratio can be approximated by the following expression:

\[ \text{SNR} = \frac{P_r}{S_{ase} B_d + \frac{P_r}{P_{LO}} S_{n_{LO}} B_d + \frac{h \omega_s}{\eta} \left( 1 + \frac{P_r}{P_{LO}} \right) B_d} \]  

(13)

where the \( P_r \) is the signal power, \( S_{ase} B_d \) is the LO-spontaneous beating noise, \( P_r / P_{LO} S_{n_{LO}} B_d \) is the signal-LO spontaneous beating noise and the \( h \omega_s / \eta (1 + P_r / P_{LO}) B_d \) is referent to the shot noise. In this approximation the thermal noise was neglected.

Take into account the modulation format used in this system (DQPSK) it’s also possible to compute the Bit-Error-Rate (BER) using SNR, expression (14) [6].

\[ BER = \frac{1}{2} e^{-\text{SNR}} \]  

(14)
III. RELATION BETWEEN SNR AND SYMBOL DISPERSION, MATLAB SIMULATION DESCRIPTION AND IMPLEMENTATION

To evaluate the relation between SNR and symbol dispersion was developed a simulation in MATLAB recreating a link from ONU to OLT with coherent technologies used in NGOA. Despite the modulation of optical emission expressions was neglected, the study goal befalls in the evaluation of relation between SNR and Symbol dispersion in reception.

The transmission diagram used in simulation is represented in Figure III.1.

![Figure III.1: Block diagram of transmission in MATLAB simulation](image)

In transmission is created random bit stream, then converted to electrical signal with phase modulation and imposes an intermediated frequency. The signal at an intermediated frequency is over-sampled to increase the signal resolution and filtered to cut the secondary frequency components inherent to over-sample process. In end the electrical signal is converted in optical domain at $\omega_c$ frequency.

The reception block diagram uses the same optical coherent technic present in Figure II.2 yet it has extra signal processing requirements necessary to symbol recovery, Figure III.2.

![Figure III.2: Block diagram of reception used in MATLAB simulation](image)

The ASE noise is added to the optical signal in the optical noise box represented in Figure III.2 by ASE, this noise was computed by (10) and integrated along signal bandwidth. The ASE parameter was used to obtain different OSNRs for as given optical power. The OSNRs were in $[30, 60]$ dB range. Only the LO-spontaneous beating noise is considered in this simulation. For LO optical power higher than 10 times signal power this is a fairly approximation. The average optical power of the local oscillator is $16$ dBm.

During photo-detection the shot noise and thermal noise are enclosure in the electrical signal. The values used to compute each noise are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numeric Value</th>
<th>Unities</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responsivity</td>
<td>0.5</td>
<td>A/W (Ampére/watt)</td>
<td>$R$</td>
</tr>
<tr>
<td>Feedback Resistor</td>
<td>300</td>
<td>Ω (Ohms)</td>
<td>$R_f$</td>
</tr>
<tr>
<td>Temperature</td>
<td>300</td>
<td>K (Kelvin)</td>
<td>$T$</td>
</tr>
<tr>
<td>Equivalent Capacitance</td>
<td>0.5</td>
<td>pF (Farads)</td>
<td>$C_i$</td>
</tr>
<tr>
<td>Data rate</td>
<td>1</td>
<td>Gb/s</td>
<td>$B$</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>25</td>
<td>GHz</td>
<td>$B_n$</td>
</tr>
</tbody>
</table>

Table 1: Typical values used to compute Thermal and Shot noises [8], [9], [11].

In this simulation the optical noise spectral density had taken different values for different optical signal power in reception yet the LO power, shot noise and thermal noise were maintained fixed.

IV. RELATION BETWEEN SNR AND SYMBOL DISPERSION, MATLAB SIMULATION RESULTS

In this section will be described all important results obtained in the analysis of relation between SNR and Symbol dispersion.

First relevant observation is the accuracy between the theoretical SNR values computed by equation (13) for a given ASE noise spectral density and the simulation values obtained for the same ASE noise spectral density. Fall back on Figure IV.1 and Figure IV.2 it’s observed that the approximations applied during the reception chain are fairly acceptable.

In these two figures is also observed the bond of shot noise and high SNRs. This means the link performance limitation is also due to shot noise when the optical noise spectral density takes low values to empower high SNRs.
Figure IV.1: Theoretical computed SNRs for a given optical noise spectral density and five different values of signal power in reception, signal at intermediate frequency.

Figure IV.2: SNRs obtained in numerical simulation for a given optical noise spectral density and for five different values of signal power in reception, signal at intermediate frequency.

To introduce the major result obtained from this simulation is necessary describe the methodology used to extract the symbol dispersion according to SNR.

The dispersion symbol was interpreted as the variance of the normalized distance from the origin referential to symbol occurrences for each symbol decision region. The normalization was performed using the mean value of all occurrences in a given region. As was refer, the modulation used in NGOA and also in numerical simulation is DQPSK, so the resultant constellation has four decision regions, Figure IV.3. By way of illustrating the methodology used in this chapter, the figures here presented were obtained for an optical signal power of $-22.5 \, dBm$ and ASE noise power spectral density of $-147 \, dBm$.

Figure IV.3: DQPSK constellation obtained by numerical simulation.

For each region collects the normalized distance distribution and Gaussian approximation, Figure IV.4.

Figure IV.4: Normalized distance distribution and Gaussian approximation in the four decision regions.

Once implemented this method, the conditions to study the relation between the SNR and symbol dispersion are met.

Through numerical simulation running for different values of optical noise spectral density and reception optical signal power was possible to establish a mathematical relation between SNR and symbol dispersion.

For all values simulated was realized that, the distribution is always well approximated by a Gaussian curve with unitary mean, Figure IV.4.

The curves obtained for different optical signal powers in reception are almost equal between them, Figure IV.5. Herewith conclude that the relation between symbol dispersion and SNR are independent from optical signal power.
V. NUMERICAL ANALYSIS FOR NGOA-DSP RECEPTION SYSTEM USED IN OLT

Through first numerical simulation that recreates a NGOA coherent receiver, was noticeable the dispersion symbol metric represented by variance of normalized distance distribution expresses the SNR of the signal and this means the overall performance. So is easily perceptible that, instead of using a combining algorithm based on polarization component power to perform the signal mixing, symbol dispersion method gives better results. It is needful corroborate the previous evidence in real scenarios. This section introduces a numerical analysis of NGOA-DSP MATLAB simulation, provided by Coriant R&D department, to evaluate the real improvement of symbol dispersion when compared to signal power decision method. Coriant also contributed with numerical simulation to generate synthetic signal.

The NGOA-DSP numerical simulation is responsible to test two orthogonal polarizations independently and also the resultant combined signal, giving performance parameters as BER, signal power and other algorithm performance parameters.

To study the symbol dispersion with NGOA-DSP numerical simulation was required to develop the same analysis method that was implemented in the numerical simulation of section III, to obtain the normalized variance of distance distribution in each decision region.

This analysis consisted in obtaining the variance of distance distribution for each decision region and BER for each polarization signal and also for the combined resultant signal performed by a combining method developed by Coriant. The combining method due to confidentiality reasons will not be described in this article.

After that, BER was compared with the reciprocal variance and signal power. These sets of parameters were also compared with the equivalent obtained for the other polarization and the combined signal.

At the beginning the analysis started with just 10 sets generated by synthetic signal generator. These signals were only affected by Gaussian noise. With this first test, was confirmed that the signal component with minor variance value was also the component with minor BER, this means major SNR.

Advancing in analysis was decided to introduce an offset between two orthogonal signals besides Gaussian noise. So using the synthetic signal generator, was executed the previous breakdown method. The coherent between combined signal and two polarizations observed in the first test was not observed in this case. The random delay introduced in two orthogonal components reveal some issues during delay adjustment in the combining algorithm. These rectifications bring some adjustments in the combined signal constellation leading to an improvement of variance although this improvement was not observed in BER.

After this preliminary study, the analysis proceeds to real signal obtained from experimental NGOA tests and provided by Coriant. The Analysis of all 47 real signal sets has some issues. Some sets were just noise, forcing extremely high values of BER, or in other hand, with low noise leading BERs near 0. Consequently these cases were removed from analysis. Additionally the convergence time required aligning the phase and frequency of both components in combining algorithm was not taken into account. This can lead a higher BER because some extra errors can occur during this process.

The final analysis was compound by 21 sets of vertical polarization, horizontal polarization and combined component. In these 21 real signal sets the same conclusions as the synthetic signal with offset could be made. The relation between symbol dispersion (variance) and BER is only consistent in vertical and horizontal components. In combined signal was no pattern between symbol dispersion and BER. This is shown in Figure V.1 and Figure V.2.
Figure V.2: BER for the three components (horizontal, vertical and combined signals) for 10 representative tests.

It also possible to conclude that is only profitable combines the two polarizations when their variances were close to each other. However this principle is based on threshold, a value defining close variance. This value should be settled to decide when polarizations should be combine or not. To realize the close mean variance value that best fit this criterion, some values were tested. The result is present in Figure V.3.

Figure V.3: Evaluation of threshold value to be used in close variance criterion.

The interval [0.015; 0.018] has the values that optimize this criterion. This interval was also verified in the analysis of the 47 sets.

Furthermore the Pearson Coefficient statistical metric was also included. This metric measures the correlation among two given distributions. It is evaluation was incited by the fact that every obtained correlation (section III) between the distance distribution and his approximation to a Gaussian distribution is upper to 0.7. In other words from the first numerical simulation was observed that every single distribution was fairly approximated to a Gaussian curve.

In the NGOA-DSP numerical simulation, was observed the obtained distributions for the three possible cases in two antagonists’ scenarios: when the combined component is the best effort and when it isn’t. With this analysis was possible to conclude for the second case that the normalized distance distribution obtained for one of the polarizations signal component had no correlation to a Gaussian distribution, Figure V.4.

Figure V.4: Normalized distance distributions for the three cases (horizontal polarization, vertical polarization, combined component) and the respective Gaussian approximation when the two polarization mix is required and when it is worthless.

Despite the structural improvements made in the analysis, the conclusions were the same as for the case with 47 real signal tests. It will be presented the obtained values for Pearson Coefficient for the three cases, Figure V.5.

Figure V.5: Pearson Coefficient for the three cases (Blue – Combined component; Green – Horizontal Polarization; Navy Blue- Vertical Polarization, Red - Threshold)

Nevertheless the effort spent on Pearson Coefficient allowed developing a decision method for these twenty one tests that works perfectly. The decision method for combined polarizations will be presented:
Figure V.6: Decision method for two polarizations.

From the diagram displayed in Figure V.6 is observed the two decision metrics used were: the variance of normalized distance distribution in a specific symbol decision region and the Pearson correlation coefficient between the normalized distance distribution and the approximation to a Gaussian distribution. The values used in this two decision parameters were experimentally optimized. In the case of the variance the optimized value study was presented in Figure V.3. For the correlation coefficient the criteria to choose the correct values are the following: first in this case the correlation factor used is the sum of the two correlations of the two polarizations inside a given test; the range of this value is [0; 2]; the value 1.8 represents the case when one polarization has a perfect correlation and the other has to be bigger than 0.7. The 0.7 value statistically speaking is the minimum value to translate a good correlation. The $S_H$ and $S_V$ represent the signals from Horizontal Polarization and Vertical polarization respectively. $\bar{\text{Var}}_H$ and $\bar{\text{Var}}_V$ are the mean variance of normalized distance distribution for the four decision regions in horizontal and vertical polarization accordingly. The variables $\rho_H$ and $\rho_V$ are the Pearson Correlation coefficient for each polarization.

The results explained here were based on numerical simulations on MATLAB; they require some experimental test to assess their potential.

VI. CONCLUSIONS

This work was divided in three major topics. The first is the theoretical study of NGOA and his reception method. After that was provided an analysis between SNR and Symbol dispersion to validate or not the potential of using some symbol dispersion metric instead of signal power to perform the mixing of two polarizations signals. When the goal is to improve the overall SNR during signal mixing this analysis confirmed the symbol dispersion is a better metric when compared with signal power condition.

After the first overall conclusion it was necessary to test and evaluate those evidences. To do that was necessary to exploit the NGOA-DSP numerical simulation with real and synthetic signal sets, to understand the full potential of symbol dispersion metric and also the Coriant algorithm behavior. Due this analysis was possible to improve the combined algorithm. However this improvement was based in thresholds, this constrain leads a non-perfect combined algorithm. To do so, it will be necessary to understand the reason why and how, during the combining process, the algorithm changes the constellation in order to avoid that effect in constellation. With this improvement it will be possible predict the minor symbol dispersion value in a set of the three possibilities leads to the higher SNR, no longer be necessary the threshold values studied here.

The analysis also gives some clues about the changes on constellation obtained for combined component when compared to the obtained constellations for horizontal and vertical polarizations. These differences are related to the offset between the two polarizations. The phase correction applied in the two polarizations is considering erroneous this offset and probably it leads some invalid corrections. Sometimes a given occurrence after phase correlation gets closer to the optimal decision point, but in the wrong region, this means wrong phase correction. The phase correction in this algorithm is based in the imaginary part of two complex exponentials corresponding to the two polarization signals.

Will be also important implementing the decision metric developed and his evaluation. In a different point of view the development of a mathematically simulation recreating the algorithm chain to answer the why and the how the constellation changes during signal mixing is also important.

REFERENCES


