Stability analysis of thin-walled steel structures

Interaction between local and global buckling modes

Extended Abstract

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1 Introduction

This paper presents a study of the interaction between local and global buckling modes of thin-walled steel open sections (I-columns) and its consequences regarding the ultimate load capacity when compared to the design buckling load of compressed members suggested by the Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings (EN 1993-1-1:2005) (EC3).

This report was based on an I-column and it was developed by the use of finite element (FE) models with the help of Abaqus software from which the columns' behavior was analyzed and the ultimate load capacity obtained.

In order to validate the FE models and understand the interaction between local and global buckling modes, the theories regarding local buckling and global (flexural) buckling were studied and summarized. Then, the finite element modeling and its results were described, presented and discussed as well as compared to the EC3 prediction of the design buckling resistance for a compression member.

Notice that during this paper, the torsional buckling mode was not considered to the evaluation of the buckling interaction for the columns analyzed. Therefore, the global flexural buckling modes will be considered only as global buckling modes.

2 Theoretical formulations

Section 2 presents the theory concerning local and global buckling. First, local buckling will be explained followed by a brief summary of global buckling of columns.

Local buckling of thin-walled steel structures is directly related to stability of plates. Three different plates (loaded only in the longitudinal direction with uniform compression forces) were studied: simply supported plate; simply supported plate with one longitudinal edge free; I-section being formed by an assemblage of plates. The first two plates were used as boundaries to the results obtained for the I-section.

By using Kirchhoff's theory (Timoshenko & Woinowsky-Krieger, Theory of Plates and Shells, 1959) and equation (2.1) (with only compressive forces acting in one direction of the plate) derived by Saint-Venant at the end of the XIX century (Bulson & Allen, 1980), it was possible to derive the buckling load for the I-section column.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} * \left(\sigma_x t \frac{\partial^2 w}{\partial x^2}\right) \tag{2.1}$$

The two plates that assemble the I-section are characterized by its width: b_1 for the plate number one acting as the web and b_2 for the flange. Both plates have the same length defined by a. By using the double symmetry presented by the I-section, the I-section was reduced to Figure 2.1. Along with the boundary conditions, it was necessary to ensure equilibrium and compatibility between both plates, in order to evaluate the local buckling load.

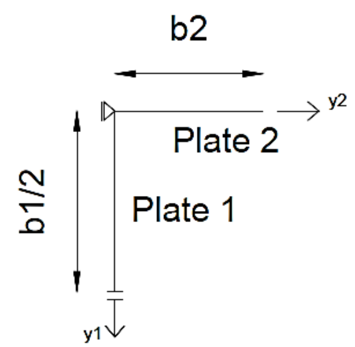

Figure 2.1: I section after applying the principles of symmetry.

It was considered that the plate buckles, along the loaded direction, into a certain number of sinusoidal half-waves. Equation (2.2) is the solution of equation (2.1), representing the function of the plates' displacements. Notice that both plates are assumed to have the same type of displacement.

$$w(x,y) = f(y)\sin\left(\frac{m\pi x}{a}\right) \tag{2.2}$$

By substituting the displacement's solution (2.2) into the differential equation (2.1) and integrating it, a new equation with four unknown constants arises. For the two plates, it turns out as being two equations with four different constants for each equation.

Using six boundary conditions plus two equations to restore equilibrium and compatibility between plate 1 and 2 (Figure 2.1), it will be possible to obtain the buckling load.

The critical buckling load can be written as follows:

$$\sigma_{cr} = \frac{K\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \tag{2.3}$$

Being σ_{cr} the lowest buckling stress, E the Young's modulus, υ the Poisson's ration, t the thickness of the plate, b the plate's width and K is a parameter that is directly proportional to the critical stress of the plate.

The six boundary conditions and the two equations in order to restore equilibrium and compatibility between the two plates can be written in a matrix format, allowing to define the bifurcation load by considering the corresponding determinant to be null. The buckling load can be written in order to the K parameter and the aspect ratio ϕ ($\phi = \frac{a}{b}$).

An example of the relation between K and ϕ for a flange with half of the web's width (with both plates having the same thickness) is presented in Figure 2.2.

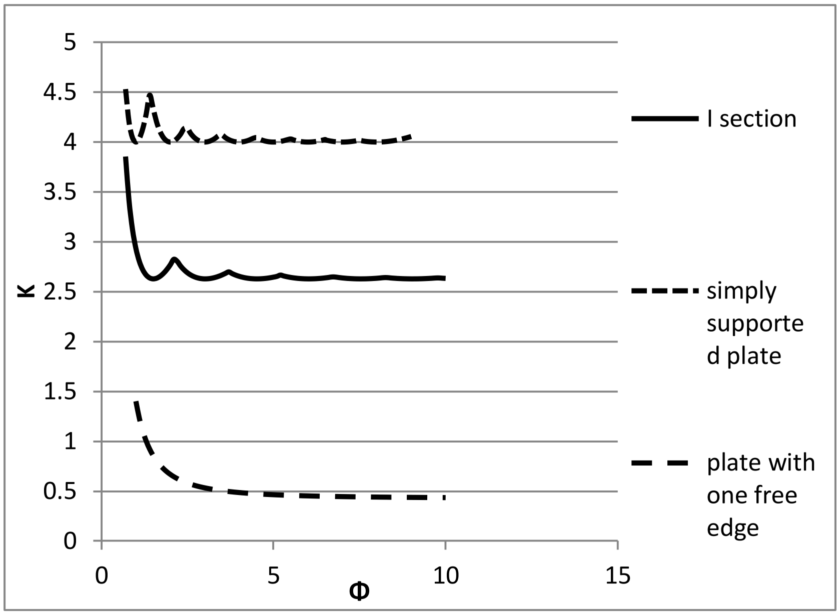

Figure 2.2: Curve K – Φ for I section with b2/b1=0. 5.

By applying the same procedure, it is possible to derive the critical buckling load for a simply supported plate and a simply supported plate with one longitudinal free edge. These two simple plates can be considered as boundaries for the I-section. In fact, in an extreme case, the flange's width could be so thin that the web would function as a simply supported plate. On the other hand, the web could be so small that in the limit, it would work as simple support for the flange. However, those boundaries weren't verified all the time as verified in Figure 2.3.

For a particular range of ratios between the flange's width and the web's, the K parameter was higher than the stress parameter defined from the upper limit (the simply supported plate). A possible explanation is that for a range of small widths of the flanges, the flanges may work as stiffeners of the web increasing its rigidity to rotation. For even smaller flanges, the critical stress of the I-section approaches the upper limit as it was expected.

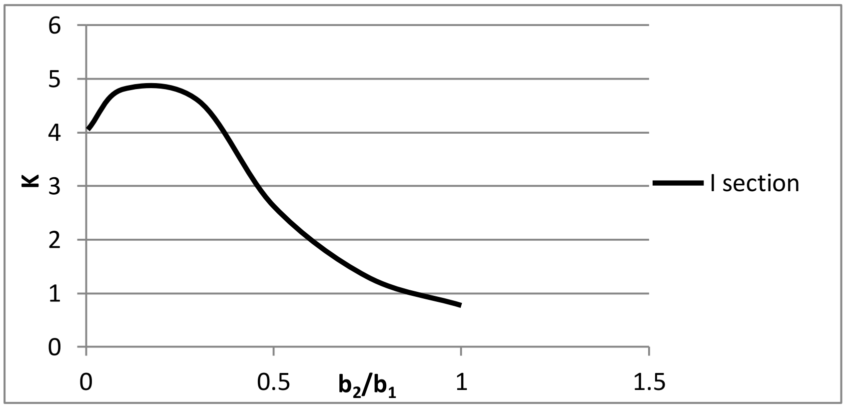

Figure 2.3: Minimum value of K for an I-section with variation of the relation between web and flange width.

Along with studying stability of plates and obtaining their local buckling loads, it was also important to study post-buckling of plates. von Kármán's theory (Bloom & Coffin, 1944) was studied as well as the concept of effective width (von Kármán, Sechler, & Donnell, 1932).

Regarding global buckling, the critical load for buckling of columns, derived by Euler in 1757 (Timoshenko & Gere, Theory of Elastic Stability, 1961), was studied. The same was done for the post-buckling of columns, being Koiter's theory (Koiter, 1945) briefly studied.

All these theories were studied in order to be possible to have a theoretical comparison and to validate the finite element (FE) model that was used to predict the behavior of several examples of columns and to compare the ultimate load capacity between the FE model and the EC3.

3 Finite element model

A finite element model was used to predict the ultimate load capacity of several thin-walled steel I-column examples. The modeling was performed by Abaqus commercial software.

It was used a perfectly reasonable mesh to model all the finite element models and its validation was performed by the comparison between the numerical results and the theoretical ones (Costa, Miguel, 2014).

S4R elements were used to model the columns. It is a standard large-strain shell element with "4-node general-purpose shell, finite membrane strains" (Abaqus 6.13 Documentation). It has 6 degrees of freedom per node: three translations and three rotations. It also has a uniformly reduced integration to avoid shear and membrane locking. It also converges to shear flexible theory for thick shells and to the classical theory for thin shells. With all these characteristics, the element chosen is a suitable element to analyze stability of thin-walled structures.

The material of the FE models was defined as being the S355 steel and the material was considered as being elastic perfectly plastic. The ultimate loads were obtained accordingly to the EC3 guidance which states that the load is the maximum load before the column starts to have an unstable behavior.

In order to study the interaction between buckling modes, it was necessary to introduce initial geometric imperfections (local and global) into the perfect columns. By the use of the *IMPERFECTION Abaqus command, local and global imperfections were introduced. It was performed by using the shapes of the buckling modes with a certain amplitude defined by the EC3. The elastic buckling modes from the perfect columns were obtained from the Abaqus *BUCKLE command. After introducing the geometric imperfections into the columns, the nonlinear analysis to determine the column behavior and its ultimate load was performed by means of the *STATIC command based on the Riks method.

The applied loads were always uniform compression loads applied at the edges of the column as nodal loads.

Concerning the boundary conditions, the columns used were simply supported along the major bending axis. Around the minor axis, the columns were prevented from rotate at the ends in order to have similar global buckling loads. About torsion, the in-plane (cross-sectional plane) rotations were prevented from occurring.

4 Results

The objective of the present paper was to study the interaction between local and global buckling modes of an I-column. In order to achieve the proposed objective, an I-column was defined in order to have similar local and global buckling loads leading to unusual cross-section. To study how the buckling modes interact, three cases (with three different lengths) were analyzed: one with similar buckling loads; one with the global buckling loads lower than the local buckling and the other with the local buckling load lower than the global ones.

The results from the three cases revealed that there was interaction between local and global (flexural) buckling modes and that interaction changed the behavior of the column from a stable behavior to an unstable one. Notice that when local and global buckling modes are analyzed in separate, the column’s behavior is stable.

It was also verified that the level of instability was higher when the critical buckling load was closer to the local buckling load. Therefore, it was concluded that the local buckling mode has a significant effect on the instability of the columns.

Not only local and global buckling modes interact between each other. The results from the same three cases showed that interaction between global modes was also observed, concluding that, two types of interaction was present in the analysis of the I-column: interaction between local and global buckling modes and interaction between global modes.

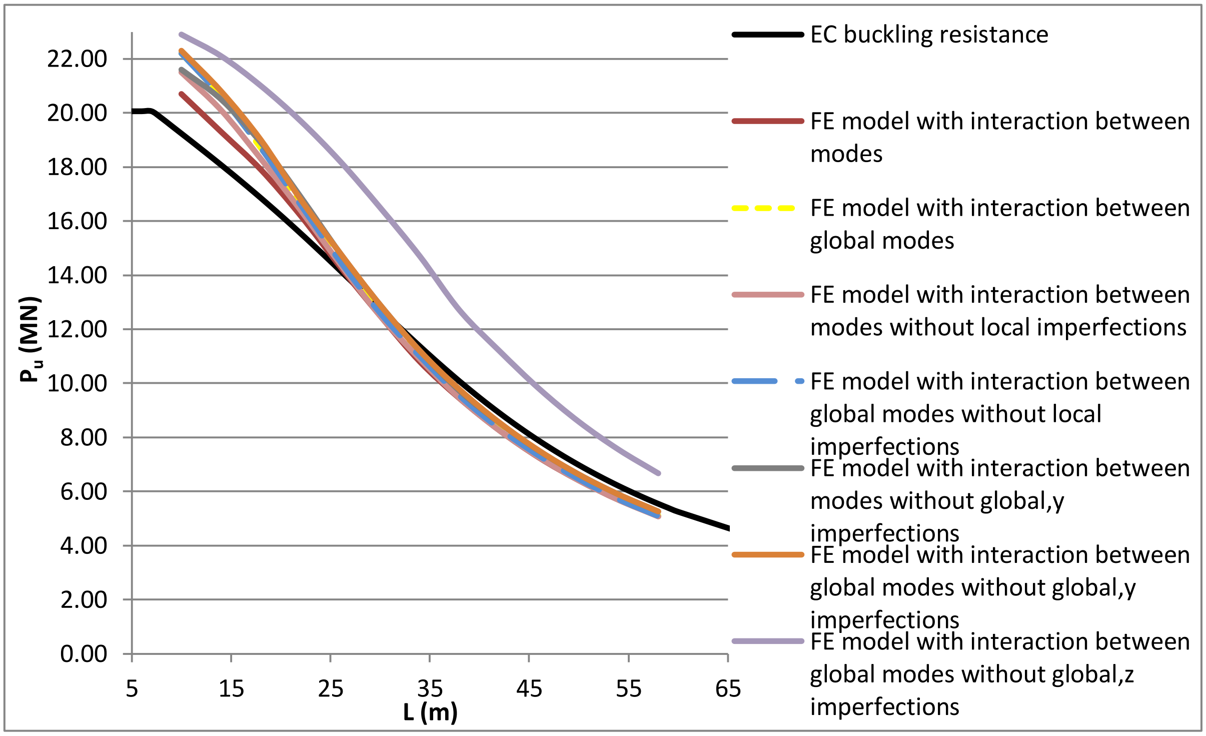

Figure 4.1: Parametric study performed with the FE models of the I-column.

The results in Figure 4.1 present the ultimate load capacity for several different columns' length along with the EC3 design buckling resistance for a compressed member. These results can be compared to the investigation realized by (Becque & Rasmussen, 2009).

The investigation developed by Becque & Rasmussen (Becque & Rasmussen, 2009), studied the interaction between local and global buckling in a stainless steel (with a non-linear behavior, anisotropy and increased corner properties due to cold working) column. A FE analysis was performed for back-to-back channels in compression, in order to predict the ultimate load capacity of a simply supported column. Several parametric studies for an extensive range of column's slenderness were conducted by Becque & Rasmussen (Becque & Rasmussen, 2009) considering different types of stainless steels. The ultimate load capacity results were compared to several design codes including the European code. Becque & Rasmussen concluded that although the EC3 achieve good predictions of the ultimate load capacity for small values of slendernee, the EC3 failed to predict safely the interaction between modes for high values of the column's slenderness.

The work performerd by Becque & Rasmussen (Becque & Rasmussen, 2009) though it has different type of material and cross-section, the results and conclusions from (Becque & Rasmussen, 2009) can be qualitatively compared to the results obtained from the FE analysis presented by Figure 4.1.

Figure 4.1 shows that the EC3, for lower values of the slenderness ratio, can provide conservative predictions of the ultimate load. But for higher values of the slenderness ratio, the EC3 may fail to predict and to account the interaction between buckling modes for the I-columns.

For small values of the slenderness ratio, the EC3 takes into account the effect of the local buckling mode by using the effective width concept. Notice that the effective width is also used for the evaluation of the resistance for higher slenderness ratio. Besides the use of the effective width, the buckling resistance for a compressed member is obtained by considering a yielding criterion.

On the other hand, the guidance from the Eurocode 3 to perform a finite element nonlinear analysis defines the ultimate load as the maximum load before the columns starts to be unstable. This can cause big differences between the two approaches (EC3 and finite element analyses) because for small lengths the ultimate load can have a significant increase of strength after yielding.

By analyzing higher values of the slenderness ratio, the impact of the local buckling mode and the local geometric imperfection by in-plane deformations, was verified due to the instability of the columns. Although, by the results given by Figure 4.1, the in-plane deformations were not the main cause of the ultimate loads being lower than the EC3 buckling resistance.

In opposition for the smaller lengths of the column, where the curve “interaction between modes” (the column is allowed to have in-plane deformations as well as global flexural deformations) has lower ultimate load capacity compared to the curve “interaction between global modes” (the column does not have in-plane deformations), for higher lengths, the difference between the ultimate loads are not that significant, indicating that the impact of the local buckling (in-plane deformations) decreased for longer columns. Notice that even for higher lengths, the interaction between local and global buckling modes induces instability in the column.

Since, local imperfections and in-plane deformations have a minor effect (for high values of the column’s slenderness) on the ultimate load capacity despite the changing of the column’s behavior, a possible reason for the difference between the ultimate load capacity obtained from the FE analysis and the buckling resistance evaluated by the EC3 may be due to the interaction between global modes.

The curve “FE model with interaction between global modes without global,y imperfections” presented in Figure 4.1 is almost similar to the buckling resistance obtained from the EC3. This model does not have global geometric imperfection over the major buckling plane.

On the other hand, the curve “FE model with interaction between global modes” has both global geometric imperfections the results show that the difference between this curve and the one given by the EC3 is higher (by 8%) compared to the difference from the curve “FE model with interaction between global modes without global,y imperfections” and the EC3. Therefore, a possible reason is the fact that the EC3 does not take into consideration the global geometric imperfection over both buckling axis and therefore it does not consider interaction between global buckling modes.

For a particular length (42 meters), the results from Figure 4.1 were studied further in detail. By evaluating the normal stresses due to the axial force and the normal stresses due to bending on both directions based on equation (4.1), it was possible to compare several FE models and the consequences of having in-plane deformations interaction with the global modes and the interaction between only global buckling modes.

From the FE analysis, the transverse displacements are known as well as the applied load. Therefore it was possible to obtain the maximum normal stress at the cross-section in the middle of the column by applying equation (4.1).

$$\sigma_N = \frac{P}{A} + \frac{P * v}{W_z} + \frac{P * w}{W_y} \tag{4.1}$$

Being P the axial force, v and w the displacement with the initial geometric imperfection in the y-direction and z-direction respectively and finally W_z and W_y are the section modulus.

The stresses were obtained for three different models: the FE model with interaction between local and global buckling modes without local imperfections; the FE model with interaction between global modes without local imperfections; and the normal stresses were also obtained from the FE model with interaction between global buckling modes without local imperfections and global imperfections on the major buckling axis.

Based on the results presented in Table 4.1, Table 4.2 and Table 4.3, it is possible to realize that for the model without local imperfections, the yielding load is the lowest even without local imperfections, in-plane reductions has an impact on the column. That impact is observed by comparing the yielding loads from Table 4.1, Table 4.2. If there is no global geometric imperfections in the major buckling plane, the yielding load will be significant higher (Table 4.3) than the two first models (Table 4.1 and Table 4.2).

Table 4.1: Normal stresses due to the axial force and bending for a FE model with interaction between local and global buckling modes without local imperfections.

P (MN)	7.17
σ_N (MPa)	104.63
σ_{My} (MPa)	208.52
σ_{Mz} (MPa)	61.24
σ_{total}(MPa)	374.39

Table 4.2: Normal stresses due to the axial force for a FE model with interaction between global buckling modes without local imperfections.

P (MN)	7.20
σ_N (MPa)	105.13
σ_{My} (MPa)	209.56
σ_{Mz} (MPa)	61.18
σ_{total}(MPa)	375.86

Table 4.3: Normal stresses due to the axial force for a FE model with interaction between global buckling modes without local imperfections and globa,y imperfections.

P (MN)	8.03
σ_N (MPa)	117.28
σ_{My} (MPa)	263.56
σ_{Mz} (MPa)	0.00
σ_{total}(MPa)	380.84

5 Conclusions

Since the beginning of this paper the main objective was to study the interaction between local and global buckling modes of thin-walled steel structures. Therefore, finite element models were modeled to study the behavior of a particular I-column defined for the purpose of the study.

With the use of the finite element models, it was verified that the interaction between local and global buckling modes altered the column's behavior to an unstable state. The impact of the interaction was greater for lower values of slenderness because local buckling loads were similar or even lower than the global buckling loads. The comparison between the ultimate loads obtained from the FE model and the design buckling loads evaluated by the EC3 also showed that the use of the effective width concept and the yielding criterion by the EC3 was conservative despite the differences of the methodologies used to obtain both loads (FE analysis and EC3).

For higher values of the slenderness ratio, the impact of local buckling and local geometric imperfections was reduced. But it was verified that the ultimate load obtained from the FE analysis was lower than the buckling resistance evaluated by the EC3, though the difference was not substantial. A possible explanation may be the interaction between global modes and the consideration of global geometric imperfections in the two buckling planes.

It appears that the EC3 does not consider global imperfections on both buckling planes and by doing a non-linear analysis with one global imperfection, the ultimate load obtained from the FE analysis was similar to the resistance of the EC3. However, by performing an analysis with global imperfections on both planes, it was verified interaction between the two global buckling modes and the ultimate load capacity of the column suffered some reduction. This result leads to the conclusion that the EC3 may not consider the interaction between global buckling modes.

The conclusion regarding the study of the interaction between local and buckling modes and the comparison of the ultimate capacity load of the column to the buckling resistance evaluated by the EC3 is that the EC3 gives good predictions for the buckling resistance when interaction local buckling mode is governing. On the other hand, when the column's slenderness increases and global buckling starts to be more relevant than the local buckling, interaction between global modes (with some impact of in-plane deformations) may result in a slightly difference from the EC3, being the ultimate buckling load resultant from the interaction between global modes lower than the design buckling resistance.

6 References

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