Bridge Aerodynamic Stability
Instability Phenomena and Simplified Models

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Extended Abstract

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00. Introduction

With the increasing span length of cable-stayed bridges and suspension bridges, the effects of the service loads produced by roadway and railway traffic on the structure started to become less important than seismic and wind loads. The study of the effects of wind flow on these types of structures was born with the collapse of the Tacoma Narrows Bridge [1940]. In the field of Civil Engineering, very little of these effects was known. However, in Aeronautical Engineering the theories of Fluid Dynamics and Deformable Solid Mechanics were already being combined to study the dynamic behaviour of the shapes adopted for aircraft wings submitted to wind flow loads.

01. Analytical Aeroelasticity

01.1. Aeroelastic Forces

If a fixed section of a bridge deck is inserted in a uniform wind flow with average speed $U$ and angle of attack $\theta$, three kinds of aerodynamic forces are bound to appear (Drag, Lift and Pitching Moment):

\[
D = \frac{1}{2} \rho C_D B U^2 \quad (01a)
\]

\[
L = \frac{1}{2} \rho C_L B U^2 \quad (01b)
\]

\[
M = \frac{1}{2} \rho C_M B^2 U^2 \quad (01c)
\]

The variables $\rho$ and $B$ are the volumetric mass density of the fluid and the deck width, respectively, and the coefficients $C_D$, $C_L$ and $C_M$ can be called form or force coefficients. These coefficients are obtained for several angles of attack $\theta$ through wind tunnel testing.

The transformation to the horizontal and vertical axis of these forces can be made by:

\[
F_x = D \cos \theta - L \sin \theta \quad (02a)
\]

\[
F_y = -D \sin \theta - L \cos \theta \quad (02b)
\]

However, in this reference frame, $F_x$ and $F_y$ can also be rewritten in terms of their own form coefficient.

\[
F_x = \frac{1}{2} \rho C_{F_x} B U^2 \quad (03a)
\]
01.2. Movement equations

A deck cross-section immersed in a turbulent wind flow, with horizontal \((h)\) and vertical \((v)\) speed fluctuations, is bound to start vibrating. These vibrations cause displacements along the \(x\) and \(y\) axis as well as the rotation of the deck \((u_x, u_y, u_a)\). These displacements, along with their respective speeds \((\dot{u}_x, \dot{u}_y, \dot{u}_a)\), cause variations of the effective wind speed \(U\) and its angle of attack \(\theta\).

\[
\delta U = h - \dot{u}_x \quad (04a)
\]

\[
\delta \theta = -\frac{v}{U} + u_a + \frac{\dot{u}_y}{U} \quad (04b)
\]

Using equations (03) and (04), it is possible to represent the variation of the aerodynamic forces as:

\[
\delta F_x = -\rho U C_{F_x} u_x + \frac{1}{2} \rho U \frac{d C_{F_x}}{d \theta} \dot{u}_x + \frac{1}{2} \rho U \frac{d C_{F_x}}{d \theta} u_a + \rho U C_{F_x} \dot{h} - \frac{1}{2} \rho U \frac{d C_{F_x}}{d \theta} v \quad (05a)
\]

\[
\delta F_y = -\rho U C_{F_y} u_x + \frac{1}{2} \rho U \frac{d C_{F_y}}{d \theta} \dot{u}_y + \frac{1}{2} \rho U \frac{d C_{F_y}}{d \theta} u_a + \rho U C_{F_y} \dot{h} - \frac{1}{2} \rho U \frac{d C_{F_y}}{d \theta} v \quad (05b)
\]

\[
\delta M = -\rho U C_M u_x + \frac{1}{2} \rho U \frac{d C_M}{d \theta} \dot{u}_y + \frac{1}{2} \rho U \frac{d C_M}{d \theta} u_a + \rho U C_M \dot{h} - \frac{1}{2} \rho U \frac{d C_M}{d \theta} v \quad (05c)
\]

It is possible to rewrite equations (05) in their matrix form.

\[
\delta f = c_a \dot{u} + k_a u + w
\]

The matrix \(w\) has the parcels related to the wind turbulence and is not defined here. \(c_a\) and \(k_a\) are the aerodynamic damping and stiffness, respectively, and their components are:

\[
c_a = \begin{bmatrix}
-\rho U C_{F_x} & \frac{1}{2} \rho U \frac{d C_{F_x}}{d \theta} & 0 \\
-\rho U C_{F_y} & \frac{1}{2} \rho U \frac{d C_{F_y}}{d \theta} & 0 \\
-\rho U C_M & \frac{1}{2} \rho U \frac{d C_M}{d \theta} & 0
\end{bmatrix}
\]

\[
k_a = \begin{bmatrix}
0 & 0 & \frac{1}{2} \rho U \frac{d C_{F_x}}{d \theta} \\
0 & 0 & \frac{1}{2} \rho U \frac{d C_{F_y}}{d \theta} \\
0 & 0 & \frac{1}{2} \rho U \frac{d C_M}{d \theta}
\end{bmatrix}
\]

Including equation (06) in the equation that defines the oscillatory movement of a body subjected to an external force and moving the parcels on the right to the left side the following result is obtained:

\[
m \ddot{u} + (c - c_a) \dot{u} + (k - k_a) u = w
\]
where \( m \), \( c \) and \( k \) are the mass, damping and stiffness matrices per unit length. This means the forces exerted by the wind flow on the structure alter the global damping and stiffness matrices.

02. Instability Phenomena

02.1. Aerostatic Instability

02.1.1. Static Divergence

The occurrence of static divergence is triggered when the torsional parameter of the stiffness matrix is nulled by the effect of wind on the structure. This happens for high wind speeds relative to other aerodynamic instability phenomena.

Taking the torsional equation for the oscillatory movement of the deck cross-section subjected to the wind flow:

\[
I_g \ddot{u}_a + (c_a - c_{a_a}) \dot{u}_a + (k_a - k_{a_a})u_a = 0 \quad \text{with} \quad (k_a - k_{a_a}) < 0
\] (09)

02.2. Aerodynamic Instability

02.2.1. Vortex Shedding

Considering a body immersed in a uniform wind flow, it is easy to see this body creates a local disturbance in it. There is a separation of the flow that contours the body. This produces a pressure force, due to the encounter with the obstruction, on the upwind side and a suction force, due to the detachment of the flow, on the downwind side. This force results in the creation of vortices with alternating rotations, in the wake region, causing the body to vibrate.

The response of the structure depends on the frequency of vortex shedding. This frequency rises with the average wind speed. If the shedding frequency reaches the natural frequency, the cross-section starts vibrating with increased amplitude. Although this amplitude can be high enough to cause discomfort to pedestrians and even enhance material fatigue, it is not high enough to provoke the collapse of the bridge. Also, it is possible to control this amplitude of vibration by raising the damping of the structure. This phenomenon is denominated Lock-In.
02.2.2. Galloping

Galloping is a type of oscillating divergent phenomenon with vertical displacements with low frequencies. This type of phenomenon usually happens for wind speeds greater than the ones that cause flutter. This instability happens when aerodynamic damping reaches the same absolute value as structural damping for a vertical vibration mode, making the global damping null for this mode. This creates growing oscillation amplitudes leading to the eventual collapse of the structure.

\[ m\ddot{u}_y + \left(c_y - c_{ay}\right) \dot{u}_y + \left(k_y - k_{ay}\right) u_y = 0 \text{ with } \left(c_y - c_{ay}\right) < 0 \]  

(10)

02.2.3. Flutter

In the design of bridges with great span, flutter is the most relevant type of aerodynamic instability. This phenomenon is characterized by a divergent oscillatory movement of the bridge deck for increasing values of wind speed. Flutter can be multimodal, also called “classic flutter”, or it can be in a single vibration mode.

02.2.3.1. Theodorsen’s Theory applied to a flat plate

The original Theodorsen’s Theory is applied to an aerodynamic shape with cord length $2b$ which undergoes a horizontal displacement at speed $U$ and arrives at a generic position after a certain time $t$ has passed.

![Diagram of displacement of a flat plate](image)
However, this theory can be adapted for a flat plate with length of $2b$ acted by a wind flow of average speed $U$. This body undergoes a generic vertical displacement $h$ and a pitching rotation $\alpha$ around a localized axis at a generic distance $a \cdot b$ from the centre of the plate.

It is assumed that the plate movements are oscillatory in nature and can be expressed as $h = h_0 e^{i\omega t}$ and $\alpha = \alpha_0 e^{i\omega t}$. The variables $h_0$ and $\alpha_0$ represent the amplitude and phase angle of the movement and are complex in nature and $\omega$ represents the circular frequency. According to Theodorsen’s Theory, the aeroelastic forces along this type of plate, using the non-dimensionalized parameters $\tau = Ut/b$ as a time variable and $k = \omega b/U$ as a reduced frequency response, are lift $L$ and moment $M$ and can be defined as:

\[
L = 2\pi b \rho U^2 \left\{ C(k) \left[ a_0 \right. + \frac{i}{b} kh_0 + \left( \frac{1}{2} - a \right) ik\alpha_0 \right] - \frac{1}{2} k^2 \left( \frac{h_0}{b} - a\alpha_0 \right) + \frac{1}{2} ik\alpha_0 \right\} e^{ik\tau} \tag{11a}
\]

\[
M = 2\pi b^2 \rho U^2 \left\{ \left( \frac{1}{2} + a \right) C(k) \left[ a_0 \right. + \frac{i}{b} kh_0 + \left( \frac{1}{2} - a \right) ik\alpha_0 \right] - \frac{1}{2} k^2 \left( \frac{h_0}{b} - a\alpha_0 \right) + \left. \frac{1}{2} - a \right] ik\alpha_0 + \frac{k^2}{8} \alpha_0 \right\} e^{ik\tau} \tag{11b}
\]

These forces are expressed per unit of length. The parameter $\rho$ is the air density and $C(k)$ refers to the Theodorsen function ($C(k) = F(k) + iG(k)$).

### 02.2.3.2. Flutter Derivatives

**The Scanlan Model - two degrees of freedom**

The geometry of bridge deck sections makes it impossible to use Theodorsen’s theory for flat plates to analytically determine their oscillatory movement. Because of this, aeroelastic forces on these types of sections have a tendency to be expressed as a linear function of the same two degrees of freedom considered, affected by coefficients known as flutter derivatives. Flutter derivatives cannot be obtained analytically and have to be obtained through wind tunnel experimenting. The equations that define these aeroelastic forces are:

\[
L = \frac{1}{2} \rho U^2 B \left( KH_1 \ddot{u} + KH_2 \frac{B \ddot{u}}{U} + K^2 H_3 \dot{u} + K^2 H_4 \left( \frac{\dot{u}}{U} \right) \right) \tag{12a}
\]

\[
M = \frac{1}{2} \rho U^2 B^2 \left( KA_1 \ddot{u} + KA_2 \frac{B \ddot{u}}{U} + K^2 A_3 \dot{u} + K^2 A_4 \left( \frac{\dot{u}}{U} \right) \right) \tag{12b}
\]

The parameters $H_i'$ and $A_i'$ are called flutter derivatives. It is also noted that the term related to length $B$ is now defined as the total length of the section ($B = 2b$) and, consequently, the reduced frequency $K$ is now twice the value adopted for the flat plate ($K = \omega B/U = \omega 2b/U = 2k$).
Additionally, the coefficients $R_5^*$ and $S_6^*$ relate to vertical and torsional oscillation damping, respectively, whereas the coefficients $R_3^*$ and $S_5^*$ relate to the stiffness of those same oscillatory movements. Terms $R_6^*$, $R_3^*$, $A_1^*$ and $A_4^*$ are coupling coefficients between the two degrees of freedom.

It is possible to obtain the exact flutter derivatives of the flat plate by transforming equations (11), expressing them according to the new variables, and comparing with equations (12).

\[ L = \frac{1}{2} \rho U^2 B \left\{ 2\pi F \frac{\dot{u}_y}{U} + \left[ 1 + \frac{4G}{K} + 2 \left( \frac{1}{2} - a \right) F \right] \frac{\dot{u}_a}{U} + \pi \left[ 2F + \left( \frac{1}{2} - a \right) GK + \frac{K^2a}{4} \right] u_a \right\} \tag{13a} \]

\[ M = \frac{1}{2} \rho U^2 B^2 \left\{ \pi F \left( \frac{1}{2} + a \right) \frac{\dot{u}_y}{U} - \pi \left[ \frac{1}{2} \left( \frac{1}{2} + a \right) \right] - \frac{2G}{K} + \frac{a^2 - 1}{4} \right\} \frac{\dot{u}_a}{U} \]
\[ + \frac{\pi}{2} K^2 \left( a^2 + \frac{1}{8} \right) + 2G \left( \frac{1}{2} + a \right) + GK \left( a^2 - \frac{1}{4} \right) u_a - \frac{\pi}{2} \left[ \frac{K^2a}{2} + \left( \frac{1}{2} + a \right) 2GK \right] u_a \tag{13b} \]

For plates with symmetrical section ($a = 0$) the results are:

\[ KH_1^* = -2\pi F \]
\[ KH_2^* = -\pi \left[ 1 + \frac{4G}{K} + F \right] \]
\[ K^2H_3^* = -\pi \left[ 2F - \frac{GK}{2} \right] \]
\[ K^2H_4^* = \frac{\pi}{2} K^2 \left[ 1 + \frac{4G}{K} \right] \]

\[ KA_1^* = \frac{\pi}{2} F \]
\[ KA_2^* = -\pi \left( \frac{1}{2} - \frac{G}{K} - \frac{F}{4} \right) \]
\[ K^2A_3^* = \frac{\pi}{2} \left( \frac{K^2}{2} + F - \frac{KG}{4} \right) \]
\[ K^2A_4^* = -\frac{\pi}{2} KG \tag{14} \]

The functions, $F$ and $G$ continue to be in order of $k = K/2$.

**Aeroelastic Forces in Three Degrees of Freedom**

When adding a third degree of freedom to the model, the horizontal displacement following the direction of the wind appears. The associated aeroelastic force is called drag and it is here represented by the letter $D$. This horizontal displacement is important for the model's accuracy because these long span structures are usually very flexible in this direction and the first natural vibration mode of suspension bridges is often horizontal. In addition, equations (12) now have two extra parameters which relate lift and moment, respectively, to the horizontal displacement and speed.

\[ D = \frac{1}{2} \rho U^2 B \left[ K P_1 \frac{\dot{u}_y}{U} + K P_2 \frac{\dot{u}_a}{U} + K^2 P_3 u_a + K^2 P_4 \frac{u_x}{B} + K P_5 \frac{\dot{u}_y}{U} + K^2 P_6 \frac{\dot{u}_y}{B} \right] \tag{15a} \]
These flutter derivatives are all functions of the reduced frequency $\mathcal{V}$. There are total of 18 flutter derivatives, 6 for each force. $H^*_1$, $P^*_1$ and $A^*_3$ multiply the displacement, which means they are stiffness parameters, whereas $H^*_2$, $P^*_2$ and $A^*_2$ multiply the speeds, and are, therefore, related to damping. All other 12 parameters are related to coupling forces and illustrate the non-conservative nature of the aerodynamic forces.

02.2.3.3. The Flutter Phenomenon

It is also possible to group the three aeroelastic equations into a single matrix equation the same way it was done before for Galloping.

$$ F_a = C_a \dot{u} + K_a u $$

$C_a$ and $K_a$ are the aerodynamic damping and stiffness, respectively:


$$ K_a = \begin{bmatrix} pK^2P^*_4 & pK^2P^*_4 & pBK^2P^*_3 \\ pK^2H^*_5 & pK^2H^*_5 & pBK^2H^*_3 \\ pBK^2A^*_6 & pBK^2A^*_6 & pB^2K^2A^*_3 \end{bmatrix} $$

and $u$ and $F_a$ are defined as:

$$ u = \begin{bmatrix} u_x \\ u_y \\ u_a \end{bmatrix} \quad \quad F_a = \begin{bmatrix} D \\ L \\ M \end{bmatrix} $$

Following the same steps of equation (08):

$$ M \ddot{u} + C \dot{u} + K u = F_a = C_a \dot{u} + K_a u $$

$$ M \ddot{u} + (C - C_a) \dot{u} + (K - K_a) u = 0 $$

It is possible to see, in equations (19), that none of the terms has a direct correlation with the time variable. This means the aerodynamic instability does not have the same nature as the classic behaviour of resonance provoked by an external force. The occurrence of flutter, whether classic or in
a single vibration mode, happens when the parameter \((C - C_a)\) reaches zero. This can happen for multiple vibration modes at the same time, resulting in the first kind of flutter, or just for the torsional mode, resulting in the second kind.

02.2.3.4. Flutter in a single vibration mode

Considering a deck cross-section acted upon the above aeroelastic forces, if each degree of freedom is considered separately, the movement equations for the rotation becomes:

\[
I_g(\ddot{u}_a + 2\xi_a \omega_a \dot{u}_a + \omega_a^2 u_a) = \rho U^3 K A^2 \dot{u}_a + \rho U^2 B^2 K^2 A^2 u_a
\]

(20)

If all members on the right side are moved to the left side, it is possible to see the stiffness and damping of the system is affected:

\[
\ddot{u}_a + \left(2\xi_a \omega_a - \frac{\rho U^3}{I_g} K A^2\right) \dot{u}_a + \left(\omega_a^2 - \frac{\rho U^2 B^2}{I_g} K^2 A^2\right) u_a = 0
\]

(21)

The occurrence of flutter in this vibration mode corresponds to a null or negative global damping value:

\[
\left(2\xi_a \omega_a - \frac{\rho U^3}{I_g} K A^2\right) \leq 0
\]

(22)

which means an aerodynamic damping equal or greater that the structural damping:

\[
\frac{\rho B^4}{2I_g} A^4 \geq \xi_a
\]

(23)

03. Wind Tunnel Testing

By simulating the effect of the wind flow around the structure, it is possible to evaluate the effects of these types of actions on it. One example of these effects is the aeroelastic forces that arise within the structure. The usage of wind tunnel testing helps determine the force coefficients \(C_D\), \(C_L\) and \(C_M\), defined in section 01.1, and the flutter derivatives, both in order of the angle of attack and mean wind speed.
04. Approximated Formulas

04.1. The Eurocode (EN)

04.1.1. Vortex Shedding

According to Annex E: Vortex shedding and aeroelastic instabilities of the EN1991-1-4, the effect of vortex shedding on a structure should be investigated "when the ratio of the largest to the smallest crosswind dimension of the structure, both taken in the plane perpendicular to the wind, exceeds 6". However, even if this condition is verified, the verification for vortex shedding can be avoided if the wind speed on the site of the structure is low enough. The EN1991-1-4 defines this "low enough" as:

$$U_{\text{crit},i} > 1.25v_m$$

(24)

The parameter on the left ($U_{\text{crit},i}$) is the critical wind speed for Lock-In for vibration mode $i$, whereas the parameter of the right ($v_m$) is defined in the normative as "the characteristic 10 minutes mean wind velocity" and is specified in section 4.3.1 (1) of EN1991-1-4.

The critical wind speed for vibration mode $i$, $U_{\text{crit},i}$, is the wind speed at which the frequency of vortex shedding equals the natural frequency for the vibration mode $i$. This mode is characterized for being a bending vibration mode.

As explained before, for the Lock-In phenomenon to occur, the frequency $f_s$ must be the same as one of the natural frequencies for a bending vibration mode $\omega_i$.

$$U_{\text{crit},i} = \frac{b\omega_i}{S_i}$$

(25)

$b$ is the dimension of the object (the height for bridge decks) and $S_i$ is the Strouhal number for that section. This is the expression applied in the EN to find the wind speed at which resonant vortex shedding occurs.

04.1.2. Galloping

According to Annex E: Vortex shedding and aeroelastic instabilities of the EN1991-1-4, the wind speed at which the structure starts to show this divergent phenomenon called galloping is defined by the following simple expression:

$$U_{\text{crit}} = \frac{2S_e}{a_e} \omega_{1,y} b$$

(26)

Again, the width $b$ of the body corresponds to the height in bridge decks. $S_e$ is the Scruton number and $\omega_{1,y}$ is the natural frequency for the first cross-wind vibration mode.
The only missing coefficient is $a_c$. This is the factor of galloping instability and it is dependent on the configuration of the cross-section. The *EN* provides a table with several cross-sections and the corresponding value of the factor of galloping instability. However, if this coefficient is not known, the *EN* also predicts the use of $a_c = 10$ which produces a much lower value of the critical wind speed for galloping.

If the following condition is true:

$$U_{c,G} > 1,25v_m$$ (27)

where $v_m$ is the mean wind velocity at the point where galloping process is expected, then the structure is considered safe for this kind of phenomenon.

However, if the critical wind speed for galloping $U_{c,G}$ is close enough to the critical wind speed for vortex shedding $U_{crit}$:

$$0,7 < \frac{U_{c,G}}{U_{crit}} < 1,5$$ (28)

then it is extremely likely the interaction between these two phenomena occurs. In this case, the *EN* advises the use of external specialized bibliography to verify the safety of the structure.

04.1.3. Flutter in a single vibration mode

In the case of flutter, the *EN* provides conditions based on simple structure criteria. If these conditions are verified, the structure is considered safe for this type of phenomenon. However, if this is not the case, the *EN* advises the consultation of specialists.

These criteria established by the *EN* determine if the structure is prone to flutter instability. They are to be verified in the order given and if at least one of them fails, the structure is considered as not prone to this phenomenon. These criteria are:

1) The cross-section of the structure as an elongated shape with $b/d < 0,25$;

```
   d
  /\  
 /   
/     
```

Figure 03 - Illustration of the dimensions adopted for the first criteria for the verification of flutter instability.

2) The torsional axis is parallel to the plane of the plate and normal to the wind direction and the distance between the windward edge of the plate and the torsional centre is at least $d/4$;
3) The lowest vibration mode is a torsional mode. If this is not the case, then the lowest translational vibration mode must have a natural frequency which is at least 2 times bigger than the lowest torsional natural frequency.

If all of these criteria are met, the EN provides a simple expression for the calculation of the critical wind speed on the onset of flutter divergence:

\[ U_{div} = \left[ \frac{2k_a}{\rho d^2 \frac{dC_M}{d\theta}} \right]^{\frac{1}{2}} \]  

\(k_a\) is the torsional stiffness and \(\rho\) is the air density. The length \(d\) is the one defined in Figure 03 and \(dC_M/d\theta\) is the derivative of the moment force coefficient, defined in section 01.1 of this report, in order of the angle of attack.

Finally, this verification is considered successful if the critical wind speed from expression (29) is bigger than two times the mean wind speed \(v_m\).

\[ U_{div} > 2v_m \]  

04.2. Flutter in a single torsional mode - the CECM formula

Much like the EN, the CECM also provides a simple formula to solve the critical wind speed for torsional flutter. This formula very simple, based on the deck geometry and the fundamental natural torsional frequency:

\[ U_{cr} = \Gamma B f_a \]  

\(B\) and \(f_a\) have already been defined as the deck’s width and the fundamental natural torsional frequency. The parameter \(\Gamma\) is based on the section geometry and ranges from 2 to \(\infty\) (which is the case of the aerofoil – a stable structure for this type of instability). Annex E contains the table from the CECM with the values of \(\Gamma\).

04.3. Flutter in a torsional and bending mode - The Selberg Formula

In 1961, A. Selberg proposed a simple formula for calculating the critical wind speed for flutter instability:

\[ U_{cr} = 3,71 f_a B \sqrt{1 - \left( \frac{f_v}{f_a} \right)^2 \frac{\sqrt{mI_y}}{\rho B^2}} \]  

Even though this formula can only be applied to thin plates, it is possible to adapt it to deck cross-sections by applying a factor \(\eta\), provided in the CECM, which depends on the geometry of the deck. This factor ranges from 0,10 to 1,00 (a table with this factor for several geometrical shapes is provided.
in Annex F). However, this formula only produces reliable result under certain conditions. The conditions are:

1) The ratio between the fundamental torsional natural frequency and the fundamental bending natural frequency should be at least 1.5;

\[ \frac{\omega_T}{\omega_Y} > 1.5 \]  

(59)

2) The product \( 2\pi \xi \), which represents the logarithmic decrease, should be around 0.05.

If these two conditions are met, the usage of the Selberg formula produces an acceptable value of the critical wind speed.

### 05. Approximated Models (Multimodal Flutter Analysis)

#### 05.1. Modal Analysis

With the objective of finding the critical wind speed for flutter a modal analysis of the bridge deck can be carried out. For this analysis, a previous calculation of the structural natural frequencies \( (\omega_i) \) and vibration modes \( (\phi_i) \) is needed. This is done by solving equation (33).

\[ (K - \omega_i^2 M) \phi_i = 0 \]  

(33)

Using the modal matrix the movement of the deck can be rewritten as a combination of vibration modes:

\[ u = \Phi q \]  

(34)

Replacing equation (34) in equation (19b) and multiplying by \( \Phi^T \) on the left:

\[ \Phi^T M \Phi \ddot{q} + \Phi^T (C - C_a) \Phi \dot{q} + \Phi^T (K - K_a) \Phi q = 0 \]  

(35)

It is possible to simplify this equation by defining the following matrices:

\[ M_R = \Phi^T M \Phi \quad C_R = \Phi^T (C - C_a) \Phi \quad K_R = \Phi^T (K - K_a) \Phi \]  

(36)

The matrices \( C_R \) and \( K_R \) are the reduced global damping and stiffness matrices, respectively. Equation (35) becomes:

\[ I \ddot{q} + C_R \dot{q} + K_R q = 0 \]  

(37)
There are many solutions for this equation. However, since the goal of this procedure is to find the wind speed at the onset of flutter instability, all solutions that are oscillatory in nature can be discarded and the solution can be assumed to be exponential in nature (characteristic of divergent behaviour).

\[ q = we^{\lambda t} \]  

(38)

and so, it’s first and second derivative are:

\[ \dot{q} = \lambda we^{\lambda t} \]
\[ \ddot{q} = \lambda^2 we^{\lambda t} \]  

(39)

Using the above solution and another trivial equation (adopted to help the process of solving equation (37)) a system of two equations appears.

\[
\begin{cases}
(-\lambda Iw + \lambda Iw)e^{\lambda t} = 0 \\
(\lambda^2 Iw + \lambda C_Rw + K_Rw)e^{\lambda t} = 0
\end{cases}
\]  

(40)

Changing it into its expanded matrix format:

\[
\begin{bmatrix}
\lambda I & 0 \\
0 & \lambda w
\end{bmatrix}
+ 
\begin{bmatrix}
C_R & K_R \\
-I & 0
\end{bmatrix}
\begin{bmatrix}
\lambda w \\
w
\end{bmatrix}
e^{\lambda t} = 0
\]  

(41)

This equation can be written in a more appealing and clean way by defining the variables \( w_\lambda \) and \( A \) as:

\[ w_\lambda = \begin{bmatrix} \lambda w \\ w \end{bmatrix} \]
\[ A = \begin{bmatrix} -C_R & -K_R \\ I & 0 \end{bmatrix} \]  

(42a)

\[ (A - \lambda I)w_\lambda e^{\lambda t} = 0 \]  

(42b)

The solution for this equation has to be found through an iterative procedure. This is because some of the prerequisite variables, the flutter derivatives employed to build the matrix \( A \), are dependent on the reduced frequency \( K \). In addition, this variable is also dependent on the damping frequency \( \omega_d \) \((K = B \omega_d/U)\) which is one of the outputs of this procedure, as demonstrated further ahead.

This equation is a non-linear problem of eigenvalues and eigenvectors in which the solutions are only dependent on the average wind speed \( U \). The solutions are complex in nature and can, therefore, be grouped in \( m \) conjugate pairs where to each pair of conjugate eigenvalues corresponds a pair of conjugate eigenvectors:

\[ \lambda = \lambda_R + i\lambda_I \]
\[ \bar{\lambda} = \lambda_R - i\lambda_I \]
\[ w_\lambda = w_{\lambda_R} + iw_{\lambda_I} \]
\[ w_{\bar{\lambda}} = w_{\lambda_R} - iw_{\lambda_I} \]  

(43)
With the $w$ vector and its corresponding eigenvalue $\lambda$, and using expressions (34) and (38), the damped oscillatory movement of the deck becomes defined.

$$u = \Phi w e^{\lambda_f t} = \Phi (w_R + iw_I) e^{(\lambda_R + i\lambda_I)t}$$

(44)

The real and imaginary parts of this equation can be separated to better correlate it with other oscillatory movement equations.

$$u = \Phi \left[w_R \cos \lambda_I t - w_I \sin \lambda_I t\right] e^{\lambda_R t} + i\Phi \left[w_R \sin \lambda_I t + iw_I \cos \lambda_I t\right] e^{\lambda_R t}$$

(45)

If the expression of the free reign oscillations of a linear oscillator with one degree of freedom is taken into account:

$$u = [A \cos \omega_a t - B \sin \omega_a t] e^{-\xi_a \omega_a t}$$

(46)

it is possible to see the real part of equation (72) is of the same form. Comparing both equations, it is evident that “the imaginary part of the eigenvalues $\lambda_I$ takes role of the damping frequency $\omega_a$ of a linear oscillator, and the real part of the eigenvalues $\lambda_R$ is associated with the product $-\xi_a \omega_a$.”

$$\omega_{aj} = \lambda_{ij}$$

$$\xi_{aj} = \frac{-\lambda_Rj}{\sqrt{\lambda_R^2 + \lambda_I^2}}$$

(47)

Much like Galloping, this divergent instability phenomenon called flutter can only happen when the global damping of the structure reaches a negative value, leading to an oscillatory movement with ever increasing amplitude until the eventual collapse. From equation (47), it is possible to see that the damping coefficient only has this nature when the real part $\lambda_R$ of the eigenvalue solutions obtained for equation (42b) has a positive value. If the wind speed is low enough, the value of $\lambda_R$ has a positive nature, leading to an oscillatory movement damped over time. However, as the wind speed rises, $\lambda_R$ becomes smaller and smaller until it eventually becomes null. This circumstance identifies the critical condition for flutter and the wind speed for which it occurs is called critical wind speed. If the speed of the flow raises even more, the damping coefficient becomes negative, leading to the instability phenomenon.

05.2. Bidimensional Approach

Even though this is a generic model to be adopted with any amount of degrees of freedom, vibration modes and deck nodes, the modal analysis is often adopted as an approximated method using only two degrees of freedom. These are the vertical displacement and the pitching rotation.

The first step is to find the fundamental natural frequencies and vibration modes for the torsional and bending modes. If the deck is divided into $N$ nodes, these can be expressed in their vector format by:
\[ \omega_y \quad \phi_y = (\phi_{y,1} \quad \phi_{y,2} \quad \cdots \quad \phi_{y,N})^T \]  

\[ \omega_a \quad \phi_a = (\phi_{a,1} \quad \phi_{a,2} \quad \cdots \quad \phi_{a,N})^T \]  

Building the modal matrix and using it to transform the movement variables, the following is obtained:

\[
\begin{bmatrix}
\{u_y\} \\
\{u_a\}
\end{bmatrix} =
\begin{bmatrix}
\phi_{y,1} & \phi_{a,1} \\
\phi_{y,2} & \phi_{a,2} \\
\vdots & \vdots \\
\phi_{y,N} & \phi_{a,N}
\end{bmatrix}
\begin{bmatrix}
\{q_y\} \\
\{q_a\}
\end{bmatrix}
\]

(50)

The mass, damping and stiffness matrices have 2 x 2 components. These are represented in equations (51).

\[
M = \begin{bmatrix}
m & 0 \\
0 & I_g
\end{bmatrix} \quad C = \begin{bmatrix}
c_y & 0 \\
0 & c_a
\end{bmatrix} \quad K = \begin{bmatrix}
k_y & 0 \\
0 & k_a
\end{bmatrix}
\]

(51)

To solve equation (42b) there is only one more step that needs to be taken. This step is another transformation of the movement variables so they represent a divergent type of movement. This is done through expression (52).

\[
\begin{bmatrix}
\{q_y\} \\
\{q_a\}
\end{bmatrix} = \begin{bmatrix}
w_y \\
w_a
\end{bmatrix} e^{\lambda t}
\]

(52)

05.2.1. Non-dimensional simplification

The above method is sufficient to calculate the critical flutter speed. However, a small error in the input can lead to imperceptible variations in the output which can be carried on. This model is prone to those input error because every variable has to be in the appropriate dimensions. To overcome this liability, it is possible to simplify the model so that every variable in the input has a non-dimensional nature.

Stating with the equation of the oscillatory movement of the deck subjected to an aerodynamic force:

\[ M \ddot{u} + C \dot{u} + Ku = F_a \]  

(53)

and expanding all its components:

\[
\begin{bmatrix}
m & 0 \\
0 & I_g
\end{bmatrix} \begin{bmatrix}
\ddot{u}_y \\
\ddot{u}_a
\end{bmatrix} + \begin{bmatrix}
c_y & 0 \\
0 & c_a
\end{bmatrix} \begin{bmatrix}
\dot{u}_y \\
\dot{u}_a
\end{bmatrix} + \begin{bmatrix}
k_y & 0 \\
0 & k_a
\end{bmatrix} \begin{bmatrix}
\ddot{u}_y \\
\ddot{u}_a
\end{bmatrix} = \begin{bmatrix}
L \\
M
\end{bmatrix}
\]

(54)
The parameters $c_y$ and $c_a$ are the damping coefficient for the vertical and rotation displacements, respectively, $k_y$ and $k_a$ are the stiffness parameters for the vertical and rotation displacements, respectively, and $m$ and $I_\theta$ are the mass coefficients for the vertical and rotation displacements, the mass and the mass moment of inertia, respectively. All of these coefficients are expressed per unit length. Using equations (12a) and (12b), it is possible to further develop the variables on the right side of equation (54) and rewrite them in their matrix format:

$$\begin{bmatrix} L \\ M \end{bmatrix} = \frac{1}{2} \rho UBK \begin{bmatrix} H_1^2 & B^2 A_1^2 \\ H_2^2 & B^2 A_2^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix} + \frac{1}{2} \rho U^2 K^2 \begin{bmatrix} H_4^2 & B^2 H_3^2 \\ H_5^2 & B^2 H_3^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix}$$

(55)

Replacing equation (55) in equation (54):

$$\begin{bmatrix} m & 0 \\ 0 & I_\theta \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix} + \begin{bmatrix} c_y - \frac{1}{2} \rho UBKH_1^2 & - \frac{1}{2} \rho UB^2 K^2 H_4^2 \\ - \frac{1}{2} \rho UB^2 K A_1^2 & c_a - \frac{1}{2} \rho UB^3 K A_2^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix} + \begin{bmatrix} k_y - \frac{1}{2} \rho U^2 K^2 H_5^2 & - \frac{1}{2} \rho U^2 B K^2 H_3^2 \\ - \frac{1}{2} \rho U^2 B^2 K A_4^2 & k_a - \frac{1}{2} \rho U^2 B^2 K^2 A_3^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix} = 0$$

(56)

Re-arranging equation (56) and using non-dimensionalized variables, the following is obtained:

$$\begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix} \begin{bmatrix} (\ddot{u}_y)'' \\ (\ddot{u}_a)'' \end{bmatrix} + \begin{bmatrix} 2\xi_y - \mu \omega K H_1^2 & - \mu \omega K H_1^2 \\ - \mu \omega K A_1^2 & 2\xi_a r^2 \omega - \mu \omega K A_2^2 \end{bmatrix} \begin{bmatrix} (\ddot{u}_y)'' \\ (\ddot{u}_a)'' \end{bmatrix} + \begin{bmatrix} 1 - \mu \omega^2 K^2 H_4^2 & - \mu \omega^2 K^2 H_3^2 \\ - \mu \omega^2 K^2 A_4^2 & r^2 \omega^2 - \mu \omega^2 K^2 A_3^2 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_a \end{bmatrix} = 0$$

(57)

This equation can be contracted to its matrix format:

$$M^*(\dddot{u}^*)'' + C^*(\dddot{u}^*)' + K^*\dddot{u}^* = 0$$

(58)

Re-doing the same kind of steps done from equation (37) to (42b) it is possible to transform the equation above in the same kind of system to be solved iteratively. Multiplying by $M^{-1}$ on the left:

$$I(\dddot{u}^*)'' + C_R(\dddot{u}^*)' + K_R\dddot{u}^* = 0$$

(59)

Doing the same steps as in section 05.2., the eigenvalue problem becomes:
\[ w_{x^*} = \begin{bmatrix} \lambda^* w \end{bmatrix} \quad A^* = \begin{bmatrix} -C^*_R & -K^*_R \\ I & 0 \end{bmatrix} \] (60a)

\[(A^* - \lambda^* I)w_{x^*}e^{\lambda^* \tau} = 0 \quad (60b)\]

Equation (60b) is once more a non-linear problem of eigenvalues and eigenvectors, which can only be solved through an iterative procedure. The solution for this problem continues to be a pair of pair of conjugate eigenvalues and eigenvectors.

Since the variable \( \lambda^* \) is non-dimensional and using the same deduction adopted from equation (44) until equation (47), the vibration frequencies on the onset of flutter instability and the damping coefficient associated to it can be obtain from equations (61).

\[ \frac{\omega_{aj}}{\omega_y} = \lambda_{ij} \]
\[ \xi_{aj} = \frac{-\lambda_{Rj}}{\sqrt{\lambda_{Rj}^2 + \lambda_{ij}^2}} \] (61)

06. Cable-stayed Bridge of the highway in Funchal

This bridge was opened in September 2000. It belongs to a highway that surrounds the city of Funchal in Madeira, Portugal. Its cable-stayed design was picked due to the high density urban occupation and the railway that goes under it.

The bridge has a total of 200 m in length and a main span of 92 m, which is quite small compared with some well-known bridges of the same design. Two deck cross sections were studied both with an unusual shape, which resembles a triangle, with 21.5 m width and a height of just 2 m. The first one was fully made of reinforced concrete whereas the second one was a joint steel-reinforced concrete cross section. These can be seen in Figure 04.

![Figure 04 - Bridge spans for the viaduct in Funchal](image)
06.1. The *Eurocode* formula

With all criteria for the use of the formula fulfilled, it is possible to use equation (29) to find the critical wind speed for flutter according to the *Eurocode*. The derivative of the moment force coefficient is the one corresponding to a null angle of attack ($dC_M/d\theta = 0,04$).

<table>
<thead>
<tr>
<th>Reinforced Concrete</th>
<th>Composite Box Girder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(EN)$</td>
<td></td>
</tr>
<tr>
<td>$[dC_M/d\theta = 1.51]$</td>
<td>$394,4 \text{ m/s}$</td>
</tr>
<tr>
<td>$U_{cr}(EN)$</td>
<td>$[dC_M/d\theta = 0.04]$</td>
</tr>
</tbody>
</table>

06.2. The CECM formula for “torsional flutter”

To use the CECM formula for flutter in a single mode (torsional), there is only the need to define the \( \Gamma \) factor. Even though the shape of the deck cross-section is unusual, it is possible to define a lower limit for this factor (\( \Gamma = 9 \)).

<table>
<thead>
<tr>
<th>Reinforced Concrete</th>
<th>Composite Box Girder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(CECM)$</td>
<td></td>
</tr>
<tr>
<td>[torsional]</td>
<td>$315,4 \text{ m/s}$</td>
</tr>
</tbody>
</table>

06.3. The CECM formula for “classical flutter”

Even though some of the criteria for the use of the Selberg formula could not be met, its use was done for academic purposes.

<table>
<thead>
<tr>
<th>Reinforced Concrete</th>
<th>Composite Box Girder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(\text{Selberg})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$359,5 \text{ m/s}$</td>
</tr>
<tr>
<td>$U_{cr}(\text{CECM})$</td>
<td>$[\text{classical}]$</td>
</tr>
</tbody>
</table>
06.4. Bidimensional Models

The results for using the non-dimensionalized bidimensional models present before are expressed in the table below:

<table>
<thead>
<tr>
<th>Reinforced Concrete</th>
<th>Composite Box Girder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_r^{(Bidimensional Model)}$</td>
<td>$(372 \text{ m/s})$</td>
</tr>
</tbody>
</table>

07. The Third crossing over the Tagus River

To help with the ever increasing traffic over the Tagus river (both roadway and railway), a third bridge was designed to add to the 25 de Abril Bridge and the Vasco da Gama Bridge. However, due to the lack of funding, this project never made it to the construction phase but the aerodynamic stability of this solution was tested.

The TTT (*Terceira Travessia do Tejo*) has a total of 1140 m in length and a main span of 540 m. The deck is held 50 m above the water level by pairs of cables, with 15 m of spacing between each other, connected to two main pillars (cable-stayed solution). The cross section consists of two levels, with the top level, allocated for roadway traffic, having just over 30 m width and the bottom level, allocated for railway traffic, having just below 21 m. This is illustrated in Figure 03.8.
07.1. The Eurocode formula

Even though some of the criteria were not met, equation (55) was used to find the critical wind speed for flutter according to the Eurocode. In this case, the derivative of the moment force coefficient, according to the EN, has the value of 0.61 whereas the derivative obtained by experimental testing has the value of 0.06. Once again, the EN values are more conservative, leading to lower critical wind speeds.

Table 06 - Critical wind speed for torsional flutter (EN formula) for the TTT

<table>
<thead>
<tr>
<th>$U_{cr}(EN)$</th>
<th>$[dC_m/d\theta = 0.61]$</th>
<th>1056.9 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(EN)$</td>
<td>$[dC_m/d\theta = 0.06]$</td>
<td>3511.1 m/s</td>
</tr>
</tbody>
</table>

Analysing the results, it is possible to confirm what was predicted. This formula provides very unreliable results for this type of deck (a truss at two levels). However, if an equivalent rectangular shape (with the same drag force) could be used for more reliable results. This shape should be adapted for each procedure.

07.2. The CECM formula for “torsional flutter”

In the case of the TTT deck cross-section, due to its height-width ratio, the CECM predicts a $\Gamma$ factor always greater than 9. The upper bound does not exist because the deck is considered to be “stable”.

Table 07 - Critical wind speed for torsional flutter (CECM formula) for the TTT

| $U_{cr}(CECM)$ | $[\text{torsional}]$ | $> 267.3$ m/s |

07.3. The CECM formula for “classical flutter”

The $\eta$ factor is not defined for this case in the CECM table. However, a lower bound can be defined if the section is considered as a box section. The $\eta$ factor takes the value of 0.6 in this case.

Table 08 - Critical wind speed for classical flutter (Selberg formula) for the TTT

| $U_{cr}(CECM)$ | $[\text{classical}]$ | $> 328.9$ m/s |
| $U_{cr}(Selberg)$ | 548.2 m/s |
07.4. Bidimensional Models

The same considerations done for the Funchal Bridge were done for this case. The results of the bidimensional model algorithm are expressed in Table 09.

Table 09 - Critical wind speed for classical flutter (Bidimensional Model) for the TTT

<table>
<thead>
<tr>
<th>Third crossing over the Tagus River</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(\text{Bidimensional Model})$</td>
</tr>
</tbody>
</table>

08. Result analysis

08.1. Cable-stayed Bridge of the highway in Funchal

The following table contains the results obtained for the critical wind speed for flutter for both deck cross sections. These were obtained through the use of the $EN$ formula, the $CECM$ formula, the $CECM$ approach based on the Selberg formula and the non-dimensionalized bidimensional model.

Table 10 - Critical wind speed for flutter for both cross sections of the bridge in Funchal

<table>
<thead>
<tr>
<th></th>
<th>Reinforced Concrete</th>
<th>Composite Box Girder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(EN)$ $[dC_U/d\theta = 1.51]$</td>
<td>394.4 m/s</td>
<td>229.0 m/s</td>
</tr>
<tr>
<td>$U_{cr}(EN)$ $[dC_U/d\theta = 0.04]$</td>
<td>2423.4 m/s</td>
<td>1406.9 m/s</td>
</tr>
<tr>
<td>$U_{cr}(CECM)$ $[\text{torsional}]$</td>
<td>315.4 m/s</td>
<td>371.5 m/s</td>
</tr>
<tr>
<td>$U_{cr}(\text{Selberg})$</td>
<td>359.5 m/s</td>
<td>239.1 m/s</td>
</tr>
<tr>
<td>$U_{cr}(CECM)$ $[\text{classical}]$</td>
<td>215.7 m/s</td>
<td>143.5 m/s</td>
</tr>
<tr>
<td>$U_{cr}(\text{Bidimensional Model})$</td>
<td>($372$ m/s)</td>
<td>($205$ m/s)</td>
</tr>
</tbody>
</table>

Looking at Table 04.1, it is possible to see the results from each route are fairly discrepant. A list of all

It is possible to see that even the most conservative methods provide values of the critical wind speed for flutter instability (whether classical or torsional) far greater than the mean wind speed $v_m$. However, special care has to be taken into account when using the bidimensional models. Using the flutter derivatives for an aerofoil-like structure, the structure was evaluated as stable, with a small range of wind speeds (around $372$ m/s for the reinforced concrete solution and $205$ m/s for the composite box girder) where the damping becomes relatively small. To better simulate the actual structure, the flutter derivatives used for this model should be obtained through wind tunnel testing.
08.2. The Third crossing over the Tagus River

The following table contains the results obtained for the critical wind speed for flutter for the TTT. These were obtained through the use of the EN formula, the CECM formula, the CECM approach based on the Selberg formula and the non-dimensionalized bidimensional model.

Table 11 - Critical wind speed for flutter for the Third crossing over the Tagus River

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{cr}(EN)$</td>
</tr>
<tr>
<td>$U_{cr}(EN)$</td>
</tr>
<tr>
<td>$U_{cr}(CECM)$</td>
</tr>
<tr>
<td>$U_{cr}(Selberg)$</td>
</tr>
<tr>
<td>$U_{cr}(CECM)$</td>
</tr>
<tr>
<td>$U_{cr}(Bidimensional Model)$</td>
</tr>
</tbody>
</table>

In addition to all the consideration made for the cable-stayed Bridge of the highway in Funchal, there is also the fact that this is not a usual deck shape. Because of the cross section’s height and it having two decks, the use of the approximated formulas, especially the EN formula, produces unreliable results. Once again, the bidimensional model evaluates the deck as stable, with the same decrease in the damping coefficient within a small range around 651 m/s. More reliable results could have been obtained for this method if the flutter derivatives would have been found for this specific cross-section.

09. Conclusions

The instability phenomena a bridge deck can be subjected when immerse in a wind flow can have different natures:

1) they can be aerostatic like the case of Static Divergence.
2) they can be aerodynamic. Such is the case of Vortex Shedding, Galloping and Flutter.

There are methods, which have evolved in time, that have been proven to produce sufficiently approximate results to determine the critical wind speeds for most instability phenomena. The usage of wind tunnel testing as also improved, with larger and more powerful tunnel being deployed which can be adapted for several types of testing goals.

As said above, the usages of wind tunnels can have several goals. They can be employed to test complete models for finding the actual critical wind speed for all different phenomena or they can be employed to find some of the deck’s properties needed for the use of analytical methods. One
example of this latter scenario is the finding of flutter derivatives adopted to calculate the critical wind speed for multi-modal flutter using the Scalan Model.

There is also the need to know how to efficiently verify the stability of a bridge deck without recurring to processes that consume a lot of time. With that in mind, some normatives suggest the use of approximated formulas that mostly use parameters related to the deck cross section. Nevertheless, the use of these formulas is conditioned by certain criteria which establish their reliability.

The employed methods’ goal is to find the critical wind speed for flutter, whether torsional or classical. The analysis of the results is done in the section below.

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