Optimization of reinforcement's location of flat and cylindrical panels

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Abstract

The subject of this thesis considers the use of composite materials. These materials can be used in many industries, including aerospace, to construct different types of objects and structures.

This thesis deals with the development of a computational model which allows the optimization of the reinforcements of an OPENCELL structure, using the DMO's method (Discrete Material Optimization) to maximize the structure stiffness. The formulation is accomplished by defining the objective function, design variables, weight functions, constraints and applying loads. The software ANSYS is used to perform finite elements analyses and two optimizers are used, FAIPA and FMINCON (optimizer from MATLAB's toolbox). MATLAB is also used to create an interface between the optimizer and ANSYS.

The developed computational model allows to prove the effectiveness of this innovate way to use DMO's method, where pre defined configurations are used as candidates. Further, the computational model allows us to check which is the best way to make de reinforcement of the OPENCELL panel.

During this thesis several parameters and boundary conditions were used, in order to analyze the effectiveness of this new way to use DMO's method in different situations.

Key-words: *Composite Materials, OpenCell, Ansys, Finite elements, Matlab, structural optimization, DMO's method.*

1. Introduction

1.1 Motivation

Sandwich panels are widely used in mechanical and aerospace industry, therefore, any progress is extremely useful for this kind of engineering field. Being a new variant of sandwich panels, the OPENCELL structure's development assumes a greater importance.

The purpose of this thesis is to create a computational model which allows us to optimize the reinforcements of an OPENCELL structure (plates and cylindrical), taking into account certain requirements:

- Static loads
- Deformed shape
- Limited amount of material available for the reinforcement

The goal is to create a computational model for the structural analyses using ANSYS (finite elements software) and a computational model for optimization using MATLAB (numerical computing software)

The optimization of the structure is done by using the DMO method, in order to maximize the structural stiffness. Before the optimization, some possible configurations were defined. These configurations are obtained by cutting and bending one side of the OPENCELL painel. In the beginning, a flat structure will be studied. After that, the study will be done for a cylindrical panel.

1.2 State of art

The first steps in structural optimization were done in the end of the XIX century. In that period, the optimization was done by using analytic methods, but these methods allow us to solve only a small group of problems [1].

The process of optimization as we know it started in the 60's, with the development of new computers with large calculation capacity.

In 1981, Cheng and Olhoff [2] developed an efficient and general numerical algorithm able to obtain the optimal configuration of rectangular and axisymmetric annular plates, with the plate thickness as design variable.

Bendsoe and Kikuchi [3] treated the topology optimization problem for a plane disk structure as a material distribution problem, where the material is assumed to have a perforated microstructure with a continuously variable orientation and density of material. By the application of the perforated microstructure the authors obtained a formulation where the material within each of the elements of a finite element discretized structure is allowed to change in a continuous manner from a fully isotropic solid material, over a composite of variable density, to avoid. Each finite element hereby becomes a potential solid or void subdomain of the structure. One year later, Bendsoe created the SIMP method (Solid Isotropic Material with Penalization [4], which was implemented by Zhou and Rozvany, [5] in 1991.

While topology optimization problems were studied by Bendsoe and Kikuchi [3], new extensions of the problem appeared, with multiple loads [6], bi-material structures [7] and bending problems with shells and plates [8], [9] and [10]. For a general view of topology optimization problems see Bendsoe and Mota Soares [11], Rozvany and Bendsoe [12], Pedersen [13] and Olhoff and Rozvany [14].

In 1997, Chung and Lee [15] searched for the optimal shapes and locations of ribs that increase the stiffness of structures using the topology optimization technique. The optimization process consisted in finding the density of each element that minimizes the *compliance* of the entire structure.

Two years later, Krog and Olhoff [16] solved the stiffness optimization problem for different static loads and free vibration mode shapes, based on the homogenization method.

In 2005, Stegmann and Lund [17], [18] and [19] show a new parameterization method called DMO (Discrete Material Optimization). It is an extension of Sigmund and Torquato's work [20], which uses the three-phase optimization method to project the material distribution in composites with extreme thermal expansion coefficients.

There are many recent works which use the DMO method [21-23] due to its versatility and easy implementation. One of these works has been done by Wang and Wang who has used Sigmund's idea in a level-set framework for solving similar problems.

The DMO method is also used in this work in order to maximize the stiffness of the structure.

Recently, *PLY Engenharia* created a new group of sandwich panels which doesn't use an additional material to create the core, because it is obtained from external plates themselves. Its name is OPENCELL. This structure has two external plates resisting bending, as a typical sandwich configuration. From one of the plates some reinforcements are obtained by cutting and bending and then attaching them to the opposite plate.

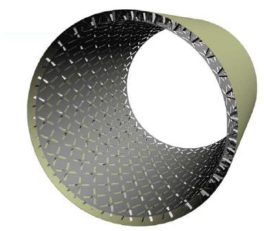


Figure 1: OpenCell cylindrical composite (from [24])

This kind of structures has many advantages:

- Lower production costs.
- Lower time and costs in recycling process.
- Lower weight in the structure.
- It is not necessary an additional material to create the core.

Thereby, it is possible to use a large number of reinforce configurations. Some of these configurations are in the figure below.

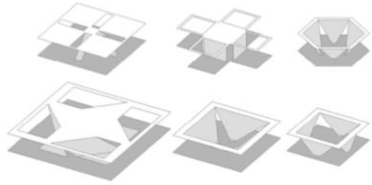


Figure 2: Some reinforce configurations in OpenCell structures (from [25])

2. Background

2.1 DMO method

The DMO method uses the same idea of parameterization that STO method has, but instead of choosing to use or not material, the goal is now to choose one from a group of pre defined materials, so that we can minimize the objective function. This is the main idea used in this work. However, it will be used in a different way.

The idea is to choose one reinforce configuration (instead of choosing one type of material), from a pre defined group of possibilities.

As an example, we can observe the figure below. Using the cutting showed in the image, there are several ways of bending the reinforcements.

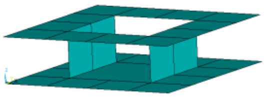


Figure 3: OpenCell structure – cutting type I

The next figure shows three different possibilities to bend the reinforcements.

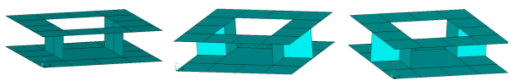


Figure 4: Possible bending cases

In this case, we will consider the bending used on the left image as "material 1", the bending on the middle figure as "material 2" and finally the last bending type as "material 3". Therefore, the goal is to choose the best "material" (reinforcement configuration) to minimize the *compliance* of the entire structure.

2.2 Structure Parameterization

The DMO's parameterization uses finite elements. The constitutive matrix, [D], is defined as the sum of all candidates constitutive matrices, $[D]_i$.

$$
\begin{aligned}
[D] = & \sum_{i=1}^{N^c} w_i [D]_i = \\
= & \, w_1[D]_1 + w_2[D]_2 + w_3[D]_3 + \\
& + \ldots + w_{N^c}[D]_{N^c} \;, \qquad 0 \leq w_i \leq 1
\end{aligned} \tag{1}
$$

Where N^c represents the candidates number.

The weight functions are represented by w_i and are directly connected to the design variables. The weight functions assume values between 0 and 1 to ensure that we don't obtain solutions without any physical meaning.

The goal of the DMO method is to ensure that the weight function values are 1 or 0 (use or not). As we have three different bending possibilities, in the previews example, only one $w_i = 1$ is allowed, the other two weight function must be zero, $w_i = 0$. Any element with values far away from 1 and 0 cannot be used because it doesn't have physical meaning.

It's easy to verify that in any optimization problem that uses mass constrains or natural frequencies as objective functions, the sum of the weight functions must be equal to one.

$$
\sum_{i=1}^{N^c} w_i = 1 \tag{2}
$$

It's important to ensure that the design variables initial values are uniform, in order not to favor any configuration.

Over the years, many interpolation schemes have been developed. From the general form of the DMO material interpolation, several interpolation schemes can be derived. In this work three of them have been tested. The most important expressions in the DMO schemes 1, 4 and 5 are presented below (see [17] for more details and information).

<u>DMO scheme 1</u>

For the general case scheme 1 is written as:

$$
\begin{aligned}
[D] = & \, (x_1)^p[D]_1 + (x_2)^p[D]_2 \\
& + (x_3)^p[D]_3 + \cdots + (x_{N^c})^p[D]_{N^c}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
[D] = & \sum_{i=1}^{N^c} w_i[D]_i = \sum_{i=1}^{N^c} (x_i)^p [D]_i \,, \\
& p \geq 1, \quad 0 \leq x_i \leq 1
\end{aligned} \tag{4}
$$

<u>DMO scheme 4</u>

$$
\begin{aligned}
[D] = & \sum_{i=1}^{N^c} w_i[D]_i = \\
= & \sum_{i=1}^{N^c} \left(((x_i)^p) \prod_{j=1;\, j\neq i}^{N^c} (1-(x_j)^p) \right) [D]_i, \\
& p \geq 1 \,, \quad 0 \leq x_i \leq 1
\end{aligned} \tag{5}
$$

The expression for three "materials" is written as:

$$
\begin{aligned}
[D] = & \, (x_1)^p(1-(x_2)^p)(1-(x_3)^p)[D]_1 \\
& + (x_2)^p(1-(x_1)^p)(1-(x_3)^p)[D]_2 \\
& + (x_3)^p(1-(x_1)^p)(1-(x_2)^p)[D]_3 \\
& p \geq 1 \,, \quad 0 \leq x_i \leq 1
\end{aligned} \tag{6}
$$

<u>DMO scheme 5</u>

For the general case scheme 5 is written as:

$$
[D] = \sum_{i=1}^{N^c} \frac{\hat{w}_i}{\sum_{k=1}^{N^c} \hat{w}_k} [D]_i \tag{7}
$$

where

$$
\begin{aligned}
\hat{w}_i = & \, (x_i)^p \prod_{j=1\,;\, j\neq i}^{N^c} \left(1-(x_j)^p\right), \\
& p \geq 1 \,, \quad 0 \leq x_i \leq 1
\end{aligned} \tag{8}
$$

2.3 Explicit Penalization Method

In some cases, the optimizer cannot choose between two or more candidates. It can happen, for example, due to symmetry effects of some structures or even some DMO's limitations (see[17]). This can cause some difficulties in pushing the design variables to their limit values.

In an attempt to improve the performance of these schemes, a penalty function method has been implemented, which explicitly penalizes the intermediate values as:

$$
\begin{aligned}
A = & \, k * \sum_{i=1}^{N^c} w_i{}^q (1-w_i)^q \,, \\
& q \geq 0 \,, \quad k \geq 0 \,, \quad 0 \leq w_i \leq 1
\end{aligned} \tag{9}
$$

This penalty function is designed to automatically decrease as the design variables tend towards 0 or 1. In the extreme values 0 and 1, it doesn't affect the objective function.

$$
\hat{C} = C + A \,, \qquad A \geq 0 \tag{10}
$$

In the following images it's possible to observe the penalization effect using $= 1$.

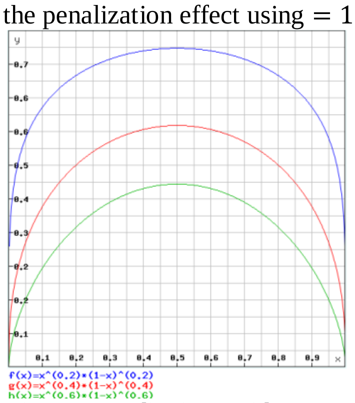


Figure 5: Penalization A with 0<q<1

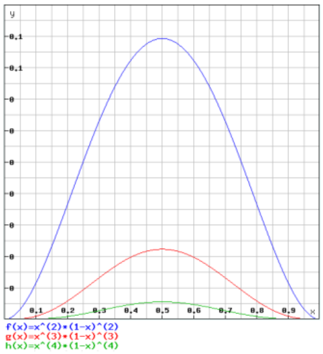


Figure 6: Penalization A with q>1

From figure 6 we can see that by using *q>1* the objective function is slightly penalized nearly to the limit values, which can lead to an unrealistic solution. On the other hand, when we use *q=0.2* the penalization has almost no variation nearly to 0.5, this can cause problems when initial values of 0.5 are used to the design variables.

Taking in account the difficulty of some schemes in ensuring that the sum of the weights is always one, another penalty function has been implemented:

$$B = \frac{c}{2} * \left(\sum_{i=1}^{N^c} w_i - 1 \right)^2 \qquad c \geq 0\,, \quad 0 \leq w_i \leq 1 \tag{11}$$

This function penalizes the objective when the sum of the weight functions is not 1.

We can use both penalizations as:

$$\hat{C} = C + A + B, \qquad A \geq 0\,, \quad B \geq 0 \tag{12}$$

2.4 Objective Funtion

The *compliance* is the work done by all applied loads in a structure, when it is in balance. Minimizing the *compliance*, *C*, means finding the maximum stiffness but also minimizing the displacement caused by the loads.

Through the finite element method [18, 19, 27] turns out that the global stiffness matrix, $[K]$, is the sum of all element stiffness matrices, $[K]^e$, and it is written as:

$$[K]^e = \int_V [B]^T[D][B]\, dV \tag{13}$$

$$[K] = \sum_{e=1}^{N^e} [K]^e \tag{14}$$

Where $[B]$ is the strain-displacement matrix and $[D]$ is the constitutive matrix. With the load vector, f, and the stiffness matrix we can obtain de displacements,

$$[K]u = f \tag{15}$$

We can also relate the compliance, the load vector and the displacements,

$$C = f^T u = u^T [K] u \tag{16}$$

Therefore, the *compliance* is two times the elastic strain energy of the structure, U.

$$C = 2U \tag{17}$$

In order to obtain the objective function and his gradient related with the design variables we can write,

$$C(u, x) = f^T u \tag{18}$$

$$\frac{dC}{dx}(u, x) = \frac{\partial C}{\partial x} + \frac{\partial C}{\partial u}\frac{\partial u}{\partial x} = f^T \frac{\partial u}{\partial x} \tag{19}$$

$$0 \leq x_i \leq 1$$

As we can see, the objective function doesn't depend explicitly on the design variables and that's the reason why the derivative $\frac{\partial C}{\partial x}$ is zero.

In spite of not being directly related to his gradient, the objective function can be easily determined through the elastic strain energy or even the displacements. In order to obtain the gradient the case isn't so easy because we don't know the relation $\frac{\partial u}{\partial x}$.

This problem can be solved by using analytical methods [24], where we can determine the gradient exact value.

2.5 Adjoint Method

The adjoint method is an analytical method that allows us to obtain the objective function derivative. The expression (15) can be written as,

$$[K]u = f \quad \leftrightarrow \quad [K]u - f = 0 \tag{20}$$

Now, it's possible to do the derivatives as,

$$\frac{\partial([K]u)}{\partial x} - \frac{\partial f}{\partial x} = 0 \tag{21}$$

$$\leftrightarrow \quad \frac{\partial [K]}{\partial x} u + [K]\frac{\partial u}{\partial x} - \frac{\partial f}{\partial x} = 0 \tag{22}$$

Multiplying the last expression by a nonzero vector, we obtain,

$$v^T \frac{\partial [K]}{\partial x} u + v^T [K] \frac{\partial u}{\partial x} - v^T \frac{\partial f}{\partial x} = 0 \tag{23}$$

$$\leftrightarrow \quad v^T [K] \frac{\partial u}{\partial x} = v^T \frac{\partial f}{\partial x} - v^T \frac{\partial [K]}{\partial x} u \tag{24}$$

This vector, v, is called adjoint displacement.

Now we must impose v as solution,

$$v^T [K] = \frac{\partial C}{\partial u} \tag{25}$$

Through the lastest expressions,

$$\frac{dC}{dx}(u, x) = \frac{\partial C}{\partial x} + \frac{\partial C}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial C}{\partial x} + v^T [K] \frac{\partial u}{\partial x} \tag{26}$$

$$\leftrightarrow \quad \frac{dC}{dx}(u, x) = \frac{\partial C}{\partial x} + v^T \frac{\partial f}{\partial x} - v^T \frac{\partial [K]}{\partial x} u \tag{27}$$

Considering that $\frac{\partial C}{\partial u} = f^T$, (see (18)) we obtain,

$$v^T[K] = f^T \tag{28}$$

Being the stiffness matrix, $[K]$, symmetric, we can write,

$$v = u \tag{29}$$

Once again, f is independent of the design variables, so $\frac{\partial f}{\partial x} = 0$. With $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial C}{\partial x} = 0$, replacing v by u in the expression (27), we finally obtain,

$$\frac{dC}{dx} = -u^T \frac{\partial [K]}{\partial x} u \tag{30}$$

In order to have the solution for all design variables, the final expression is,

$$\frac{dC}{dx_i} = -u^T \frac{\partial [K]}{\partial x_i} u \ , \qquad i = 1,2,3, \dots, N^x \tag{31}$$

Knowing the displacement vector, u, it's only missing $\frac{\partial [K]}{\partial x}$. The way this derivative is obtained will be explained in the next section (2.6). Thus, we can have the solution of the objective function derivative.

2.6 Objective Function Gradient

In order to obtain the objective function derivative, it is essential to have the solution of the global stiffness matrix gradient, $\frac{\partial [K]}{\partial x}$.

The constitutive matrix is obtained through the sum of all candidates constitutive matrices. In case of three candidates it can be written as,

$$[D] = \sum_{i=1}^{3} w_i [D]_i = w_1[D]_1 + w_2[D]_2 + w_3[D]_3 \tag{32}$$

Thus, the number of design variable is equal to candidate's number. The design variables vector is,

$$x = \{x_1, x_2, x_3\}^T \tag{33}$$

Using the DMO scheme 5, we can write the weight functions as,

$$w_1 = \frac{\hat{w}_1}{\hat{w}_1 + \hat{w}_2 + \hat{w}_3} \tag{34}$$

$$w_2 = \frac{\hat{w}_2}{\hat{w}_1 + \hat{w}_2 + \hat{w}_3} \tag{35}$$

$$w_3 = \frac{\hat{w}_3}{\hat{w}_1 + \hat{w}_2 + \hat{w}_3} \tag{36}$$

As we can see the weights depend on the design variables,

$$\hat{w}_1 = (x_1)^p[1 - (x_2)^p][1 - (x_3)^p] \tag{37}$$

$$\hat{w}_2 = (x_2)^p[1 - (x_1)^p][1 - (x_3)^p] \tag{38}$$

$$\hat{w}_3 = (x_3)^p[1 - (x_1)^p][1 - (x_2)^p] \tag{39}$$

The global stiffness matrix, $[K]$, is also obtained by the sum of all element stiffness matrices, $[K]^e$.

$$[K] = \sum_{e=1}^{N^e} \int_V [B]^T[D][B] \, dV \tag{40}$$

If we consider three candidates,

$$[K] = \sum_{e=1}^{N^e} \sum_{i=1}^{3} \int_V [B]^T w_i [D]_i [B] \, dV \tag{41}$$

$$= w_1 \sum_{e=1}^{N^e} \int_V [B]^T[D]_1[B] \, dV + w_2 \sum_{e=1}^{N^e} \int_V [B]^T[D]_2[B] \, dV + w_3 \sum_{e=1}^{N^e} \int_V [B]^T[D]_3[B] \, dV \tag{42}$$

Simplifying,

$$[K] = w_1[K]^1 + w_2[K]^2 + w_3[K]^3 \tag{43}$$

Where,

$$[K]^1 = \sum_{e=1}^{N^e} \int_V [B]^T[D]_1[B] \, dV \tag{44}$$

$$[K]^2 = \sum_{e=1}^{N^e} \int_V [B]^T[D]_2[B] \, dV \tag{45}$$

$$[K]^3 = \sum_{e=1}^{N^e} \int_V [B]^T[D]_3[B] \, dV \tag{46}$$

In this work, both global and element stiffness matrices will be obtained by ANSYS software.

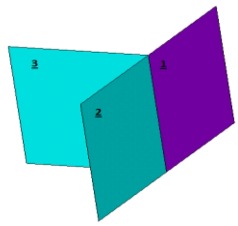


Figure 7: Reinforcements numbers

In the three candidates case (figure 4) the configuration 1 consists in using the reinforcements <u>1</u> and <u>2</u>, the configuration 2 uses the reinforcements <u>1</u> and <u>3</u> and finally the configuration 3 consists in using the reinforcements <u>2</u> and <u>3</u>.

Knowing that w_1 is directly related to configuration 1, w_2 with configuration 2 and w_3 with configuration 3, we have the following expressions for the element stiffness matrices,

$$[K] \text{ Placa } \underline{1} \leftrightarrow w_1 [K]^+ + w_2 [K]^+ + w_3 [K]^- \tag{47}$$

$$[K] \text{ Placa } \underline{2} \leftrightarrow w_1 [K]^+ + w_2 [K]^- + w_3 [K]^+ \tag{48}$$

$$[K] \text{ Placa } \underline{3} \leftrightarrow w_1 [K]^- + w_2 [K]^+ + w_3 [K]^+ \tag{49}$$

The matrices $[K]^+$ and $[K]^-$ represent the stiffness matrix of each plate when it is used ($[K]^+$) or not used ($[K]^-$) in the problem. For example, in case of using configuration 1 we have $[K]^{+}$ in the ribs 1 and 2 and $[K]^{-}$ in the plate 3. It will be used the value $E = 200x10^{9}$ for each used rib and $E = 200x10^{3}$ for unused ribs.

The expression that allows us to obtain the global stiffness matrix gradient is,

$$\frac{\partial[K]}{\partial x} = \sum_{e=1}^{N^e}\sum_{i=1}^{N^c}\int_V [B]^T \frac{\partial w_i}{\partial x}[D]_i[B]\, dV \tag{50}$$

For the three candidates case we have,

$$\frac{\partial[K]}{\partial x} = \sum_{i=1}^{N^c}\frac{\partial w_i}{\partial x}[K]^i \quad \leftrightarrow \tag{51}$$

$$\leftrightarrow \quad \frac{\partial[K]}{\partial x} = \frac{\partial w_1}{\partial x}[K]^1 + \frac{\partial w_2}{\partial x}[K]^2 + \frac{\partial w_3}{\partial x}[K]^3 \tag{52}$$

Once again, in the case of three candidates we have,

$$\frac{\partial[K]}{\partial x}\text{Placa1} \leftrightarrow \frac{\partial w_1}{\partial x_i}[K]^+ + \frac{\partial w_2}{\partial x_i}[K]^+ + \frac{\partial w_3}{\partial x_i}[K]^- \tag{53}$$

$$\frac{\partial[K]}{\partial x}\text{Placa2} \leftrightarrow \frac{\partial w_1}{\partial x_i}[K]^+ + \frac{\partial w_2}{\partial x_i}[K]^- + \frac{\partial w_3}{\partial x_i}[K]^+ \tag{54}$$

$$\frac{\partial[K]}{\partial x}\text{Placa3} \leftrightarrow \frac{\partial w_1}{\partial x_i}[K]^- + \frac{\partial w_2}{\partial x_i}[K]^+ + \frac{\partial w_3}{\partial x_i}[K]^+ \tag{55}$$

2.7 Constraints

In this thesis, only one type of constraints is used, its name is unilateral constraints. These constraints define limit values for the design variables. In this work the upper limit is one and the lower limit is zero.

$$0 < x_i < 1 \tag{56}$$

The optimizer must guarantee that these limits aren't exceeded.

3. Implementation

3.1 Ansys

With the purpose of obtaining the elastic strain energy, a finite element analysis will be done using ANSYS. This energy will be written in a *.txt* file and used by the optimizer in order to estimate the optimum objective function value.

In order to run ANSYS software it's used an APDL which receives, from MATLAB, the Young modulus values of each reinforcement. The ANSYS software runs without direct intervention. Using MATLAB, the program allows us to run ANSYS in *batch mode*. Both *input* and *output* files are *.txt* files.

From ANSYS we obtain the following data,

- Element stiffness matrices
- Elastic strain energy
- Element nodes
- Nodes DOF (degree of freedom)

Interface ANSYS/MATLAB

The next figure allows us to understand how these two softwares interact.

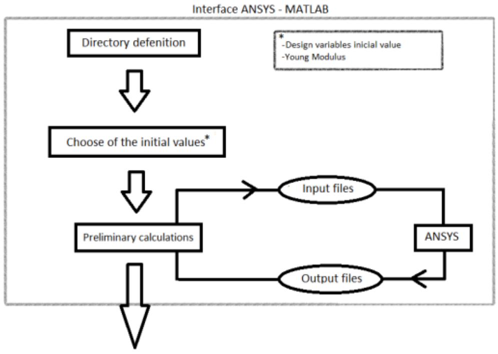


Figure 8: Interface ANSYS/MATLAB

3.2 Optimization algorithm

The optimization algorithm implemented in this work uses DMO method and adjoint method to compute the gradient in order to find which is the best way of cutting and bending the plate and maximize the structure's stiffness.

Two optimizers are used in this work, FAIPA [28] (Feasible Arc Interior Point Algorithm) and FMINCON (from MATLAB toolbox). FAIPA is an interior point algorithm for the minimization of a nonlinear function with equality and inequality constraints. These consist on fixed point iterations to solve the KKT (karush-Kuhn_Tucker) conditions. FMINCON [29] is a MATLAB's function that also allows us to solve nonlinear constraint problems. In both optimizers the user must define certain parameters as the stop criterion, optimization algorithm, and others.

In the figure below we have the representation of the process; in (a) we have the optimization algorithm and (b) represents the DMO and the adjoint methods implementation.

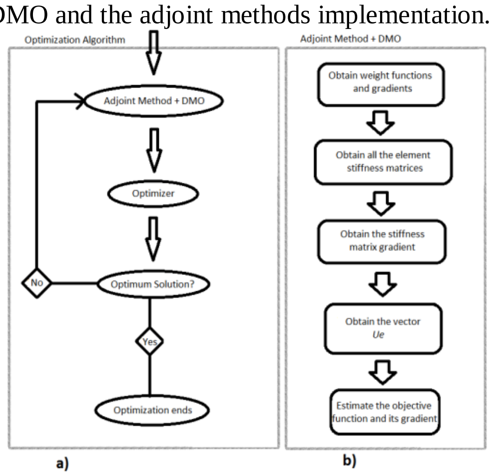


Figure 9: a) Optimization algorithm b) Adjoint method + DMO

4. Results

4.1 Flat structure

In this section the most relevant results of this work will be presented. The element *SHELL181* was the chosen element for the analyses. It is an element with four nodes and six DOF per node.

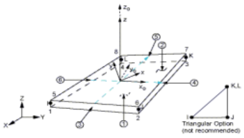


Figure 1: SHELL181

All studies presented below were performed in a fixed structure, as shown in the image,

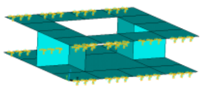


Figure 2: Fixed structure

The first study has been done by using FAIPA and with DMO scheme 4 and 5, without penalization functions. In this case, six design variables were used. Thus, we have three possible configurations on each side,

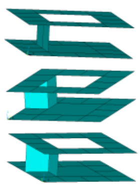
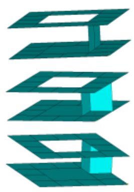
Left side		***Right side***	
	w_1 w_2 w_3		w_4 w_5 w_6

Table 1: Possible configurations

In the next table it's possible to see that using 0.5 as initial value for the design variable, we obtain intermediate solutions for both schemes.

$x_0 = [0.5 \dots 0.5]$	$C\ [J]$	Iterations
DMO 4	0.0261831643937	21
DMO 5	0.0147606978855	22

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
3.32e-7	0.25	0.25	3.32e-7	0.25	0.25
9.38e-9	0.50	0.50	9.38e-9	0.50	0.50

Table 2: DMO Scheme 4 and 5

Due to the symmetry of the structure, the program cannot choose only one reinforcement, thus the solution is unrealistic. However, when we use different initial points, for the design variables the results change.

$x_0 = [0.7\ 0.6\ 0.4\ 0.7\ 0.6\ 0.4]$	$C\ [J]$	Iterations
DMO 4	0.0182767566535	35
DMO 5	0.0147606985908	36

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
6.53e-5	**0.96**	7.21e-5	6.53e-5	**0.96**	7.21e-5
1.20e-7	0.50	0.50	1.20e-7	0.50	0.50

Table 3: DMO Scheme 4 and 5

In this case, scheme 4 seems to be better. While scheme 5 continues to give us intermediate values, scheme 4 has already chosen the configurations $w2$ and $w5$.

In order to solve the DMO scheme 5 problem, the penalization function A, presented in 2.3, has been used. Once again, using 0.5 as initial value the program obtains intermediate solutions, so the results below were obtained by using 0.7 as initial value for the design variables.

$x_0 = [0.7 \dots 0.7]$	$C\ [J]$	Iterations
DMO 5	0.09368349487977	65

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
0.999	5.91e-4	3.73e-4	**0.999**	3.73e-4	5.91e-4

Table 4: DMO5 + Penalization A

The use of penalization function solves the problem due to the symmetry. The program chooses the configurations one and four as optimum solutions, with $w_i = 1$, and for all other candidates the program obtains $w_i = 0$. If we run all possible solutions directly in ANSYS, we will see that the optimum solution isn't this one. As the energy values in both cases are close to each other, the existence of local minimums or even the low refinement of the mesh can justify the choice of the candidates one and four.

In order to compare optimizers (FAIPA and FMINCON) and understand which gives us the best solution, some analyses have been done using FMINCON. The table below presents the differences in the solutions using 0.5 as initial value of the design variables for FMINCON and 0.7 for FAIPA. In both cases DMO scheme 5 was used.

DMO5	$C\ [J]$	Iterations	Time
FAIPA $x_0 = [0.7 \dots]$	0.09368349487977	65	300.9 seg
FMINCON $x_0 = [0.5 \dots]$	0.1323060416045	161	529.3 seg

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
0.999	5.91e-4	3.73e-4	**0.999**	3.73e-4	5.91e-4
0.999	5.27e-4	5.61e-4	**0.999**	5.61e-4	5.28e-4

Table 5: FAIPA vs FMINCON

Solutions

FAIPA	
FMINCON	

Table 6: FAIPA and FMINCON solutions

As we can see, FMINCON solves the problem by using 0.5 as initial value for the design variables, but once again it gives us a solution that may not be the optimum solution. If we compare to FAIPA's solution the objective function value is higher in FMINCON's case. As we will see, it can be a mesh problem, because we are using one element per area mesh, due to the large computation time. Since the goal is to demonstrate that this new way of using DMO method works, for now, the mesh isn't the most important.

In order to better understand this, an analysis using a mesh with four elements per area is showed in table 8. Table 7 shows the possible results obtained directly from ANSYS.

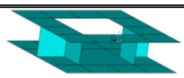
 $w1 = 1$ and $w4 = 1$	***C = 1.42712508 J***
 $w1 = 1$ $w5 = 1$// $w1 = 1$ $w6 = 1$ $w2 = 1$ $w4 = 1$ // $w3 = 1$ $w4 = 1$	***C = 1.56634909J***
 $w2 = 1$ $w5 = 1$ // $w3 = 1$ $w6 = 1$	***C = 1.62963454J***
 $w2 = 1$ $w6 = 1$ // $w3 = 1$ $w5 = 1$	***C = 1.73830807 J***

Table 7: ANSYS solutions - 4 elements mesh

As we can see, the optimum solution, using four elements per area mesh, is when $w1 = 1$ and $w4 = 1$. This solution is different from the one obtained when one element mesh is used. That's why the program chooses the reinforcement one and four on table 4 and 5.

On the tables below it's possible to analyze the optimum solution obtained in a four elements per area mesh.

DMO 5	$C\ [J]$	Iterations	Time
FMINCON $x_0 = [0.5 \ldots 0.5]$	1.2493492603407	133	454.9 seg

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
0.991	1.13e-12	8.77e-3	**0.991**	8.83e-3	4.10e-10

Table 8: 4 elements mesh

Once again, the results confirm configurations $w1$ and $w4$ as optimum solution.

Then, the results from a structure with nine reinforcements will be presented, in order to prove that this idea works in bigger structures. In this case all the reinforcements are equal and still being used six design variables.

DMO 5	$C\ [J]$	Iterations	Time
FMINCON $x_0 = [0.5 \ldots]$	48.651831168447	112	404.3 seg

$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
0.999	6.04e-4	6.00e-4	**0.999**	6.00e-4	6.04e-4

Table 9: Structure with 9 reinforcements

The solution presents all the reinforcements in the same direction, perpendicular to the fixed sides of the structure.

In order to figure out which DMO scheme is the best to use in large analyses, with more than six design variables, the next table is presented. It compares the number of iterations and computation time using three different DMO schemes with penalization functions. Six design variables were used in all cases. While scheme 5 only uses penalizations A, scheme 4 and 1 use penalizations A and B (see section 2.3).

	Iterations	Time (seg)
DMO 5	161	529.3
DMO 4	112	339.0
DMO 1	112	199.5

Table 10: Iterations and computational time

It's obvious that the best option, in case of large analyses, is DMO scheme 1 with penalization functions A and B, because it is much faster than the others.

4.2 Cylindrical structure

After confirming that this new method works in flat structures, it's time to verify if we can use it in cylindrical structures. The next analyses were done by using nine elements per area mesh and one side fixed structure. In this case, 12 candidates will be used and a pressure of 150N/m^2 will be applied. FMINCON is the function used to optimize the problem.

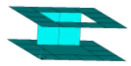
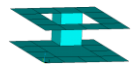
Side <u>1</u>		***Side <u>2</u>***	
	w_1		w_4
	w_2		w_5
	w_3		w_6

Table 10: Possible configurations – side <u>1</u> and <u>2</u>

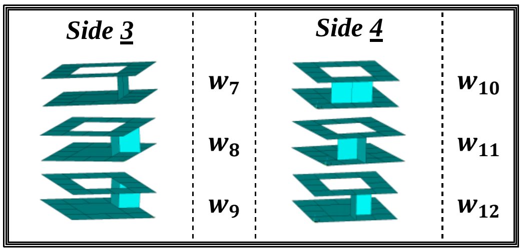


Table 11: Possible configurations – side 3 and 4

$$\sum_{i=1}^{6} w_i = 1 \quad ; \quad \sum_{i=7}^{12} w_i = 1 \qquad (57)$$

Only one of the first six candidates and one of the last six candidates are chosen.

The results of the first analysis are in the next table.

$DMO\,5$		$C\ [J]$	Iterations	Time	
FMINCON $x_0 = [0.5 \ldots 0.5]$		0.15475996779229	325	9824 seg	
$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
1.00	1.94e-4	8.46e-5	0.00	0.00	0.00
$w7$	$w8$	$w9$	$w10$	$w11$	$w12$
0.00	0.00	0.00	0.00	0.00	**1.00**

Table 12: Cylindrical Structure with pressure

Solution

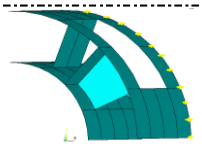


Figure 12: Optimum Solution

As we can see, this method also works in a cylindrical structure and the program chooses a perpendicular reinforcement on the left side and type L reinforcement on the front side.

After that, it's time to verify if the program changes its solution if a different load is applied. In this case, we decided to apply a one point load on the left side. Taking into account this load, it's expected a change in the perpendicular reinforcement, on the left, to type L reinforcement. The obtained results are presented below.

$DMO\,5$		$C\ [J]$	Iterations	Time	
FMINCON $x_0 = [0.5 \ldots 0.5]$		0.12521829931845	338	6413 seg	
$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
0.00	**1.00**	0.00	0.00	0.00	0.00
$w7$	$w8$	$w9$	$w10$	$w11$	$w12$
0.00	**1.00**	0.00	0.00	0.00	0.00

Table 13: Cylindrical Structure with one point load

Solution

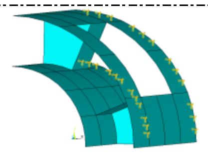


Figure 13: Optimum solution

As expected the solution changes. In this case the program chose a type L reinforcement on the left and right sides.

Finally, the last analysis was done by using a cylindrical structure with four reinforcements. Due to the large optimization time, all reinforcements will be equal and the same twelve candidates will be used. A pressure of 1500N/m^2 will be applied. The results are presented in the next table.

$DMO\,5$		$C\ [J]$	Iterations	Time	
FMINCON $x_0 = [0.5 \ldots 0.5]$		2.29714982586376	1066	28854 seg	
$w1$	$w2$	$w3$	$w4$	$w5$	$w6$
2.47e-4	1.83e-4	**0.999**	1.42e-4	1.46e-4	1.44e-4
$w7$	$w8$	$w9$	$w10$	$w11$	$w12$
9.40e-7	1.13e-6	1.47e-7	6.42e-4	**0.999**	2.25e-5

Table 14: Cylindrical Structure with 9 reinforcements

Solution

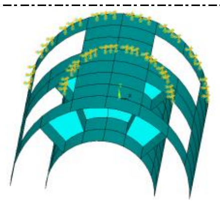


Figure 14: Optimum Solution

Once again the results show that this method works in cylindrical structures with one or multiple reinforcements and different kinds of loads.

5. Conclusions and future work

This Study makes it possible to demonstrate that DMO method can be used with different configurations instead of different materials, as design variables.

No DMO scheme shows to be efficient without penalty functions because they all present intermediate solutions. According to the results, when a penalty function is used, scheme 5 proved to be the best DMO scheme. However, for large analyses it's better to use scheme 1 with two penalizations due to his shorter computation time.

Finally, it has been proved that this new way of using DMO method is feasible for flat or cylindrical structures, with one or more reinforcements and different kinds of loads or boundary conditions.

Considering the work developed and the results obtained in this thesis, some proposals for future work could be to:

- Analyze different ways of cutting (ex: triangular shape) and bending (ex: 45º instead of 90º)
- Study and develop a new cutting way that makes it possible to reinforce the four sides of the hole.
- Study and develop new and better weight functions to this problem, instead of DMO schemes.
- Study the influence of different loads (pressure, moments…) and different boundary conditions.
- Consider multiple load cases.

References

[1] Coutinho, K.D. (2006), "Método de Otimização Topolígica em Estruturas Tridimensionais". Tese de Mestrado em Engenharia Mecânica, Universidade Federal do Rio Grande do Norte.

[2] Cheng K.T. and Olhoff N., (1981), "An investigation concerning optimum design of solid elastic plates", International Journal of Solids and Structures 17, 305-323.

[3] Bendsoe M.P., Kikuchi N., (1988), "Generating optimal topologies in structural design using a homogenization method", Computer Methods in Applied Mechanics and Engineering 71, 197-224.

[4] Bendsoe M.P., (1989), "Optimal shape design as a material distribution problem", Structural Optimization 1, 193-202.

[5] Zhou, M. and G.I.N. Rozvany (1991), "The COC algorithm - Part II. Topological, geometrical and generalized shape optimization", Computer Methods in Applied Mechanics and Engineering 89, 309-336.

[6] Diaz A., Bendsoe M.P., (1992), "Shape optimization of structures for multiple loading situations using a homogenization method", Structural Optimization 4, 17-22.

[7] Olhoff N., Thomsen J., Rasmussen J., (1993), "Topology optimization of bi-material structures", In: Pedersen P, editor. Optimal Design with Advanced Materials. Amsterdam: Elsevier, p. 191-206.

[8] Suzuki K., Kikuchi H., (1991), Layout optimization using the homogenization method: generalized layout design of three-dimensional shells for car bodies. In: Rozvany G.I.N., editor. Optimization of large structural systems, p. 26-110 Nato-ASI, Lecture Notes.

[9] Soto C.A., Diaz A.R., (1993), "Optimum layout of plate structures using homogenization", In: Bendsùe M.P., Mota Soares C.A., editors, Topology design of structures, Dordrecht: Kluwer, p. 387-407 Nato-ASI.

[10] Soto C.A., Diaz A.R., (1993), "A model for layout optimization of plate structures", In: Pedersen P., editor, Optimal design with advanced materials, Amsterdam: Elsevier, p. 285-337.

[11] Bendsoe M.P., Mota Soares C.A., (1993) editors. "Topology design of structures", Dordrecht: Kluwer.

[12] Rozvany G.I.N., Bendsoe M.P., Kirsch U., (Rev. 1995), "Layout optimization of structures", Applied Mechanics Reviews 48, 41-119.

[13] Pedersen P., (1993), "Optimal design with advanced materials", Elsevier, Amsterdam.

[14] Olhoff N, Rozvany G.I.N., (1995), editors. WCSMO-1 "Proceedings of First World Congress of Structural and Multidisciplinary Optimization", Goslar, Germany. Oxford: Pergamon.

[15] Chung J. and Lee K., (1997), "Optimal design of rib structures using the topology optimization technique", Proceedings of Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science 211, 425–437.

[16] Krog L.A. and Olhoff N., (1999), "Optimum topology and reinforcement design of disk and plate structures with multiple stiffness and eigenfrequency objectives", Computers & Structures 72, 535–563.

[17] Stegmann, J. (2005). "Analysis and optimization of laminated composite shell structures", PhD Thesis, Institute of Mechanical Engineering, Aalborg University.

[18] Lund, E. and Stegmann, J. (2005), "Discrete material optimization of general composite shell structures", Topological Design Optimization of Strutures, Machines and Material: Status and Perspective, 89-98.

[19] Lund, E. and Stegmann, J. (2005), "On structural optimization of composite shell structures using a discrete constitutive parameterization", Wind Energy 8, 109-124.

[20] Sigmund, O. and Torquato, S. (1997), "Design of material with extreme thermal expansion using a three-phase topology optimization method", Journal of the Mechanics and Physics of Solids 45, 1037-1067.

[21] Ferreira, R. T. L. and Rodrigues, H.C, Guedes, J. M., Hernandes, J. A. (2013), "Hierarchical optimization of laminated fiber reinforced composites", Preprint submitted to Composite Structures, January 23.

[22] Ferreira, R.T.L. and Hernandes, J.A. (2013), "New approximations for sequential optimization with discrete material interpolations", in: Proceedings of the 10th World Congress on Structural and Multidisciplinary Optimization, May 19-24, Florida, USA.

[23] Vatanabe, S.L., Paulino, G.H., Silva, E.C.N. (2013), "Design of functionally graded piezocomposites using topology optimization and homogenization – Toward effective energy harvesting materials", Computer Methods in Applied Mechanics and Engineering,

[24] Valente, António. OpenCell Technology (Tech & Applications Perspective).

[25] Allen H., (1993), "Analysis and Design of Structural Sandwich Panels", Pergmon Press, UK.

[26] Bendsoe, M.P. and O. Sigmund (2003), "Topology optimization - theory, methods and Applications", Second Edition, Springer, USA.

[27] Reddy, J. N. (2006), "An Introduction to Finite Element Method", Third Edition, McGran-Hill, USA.

[28] Herskovits, J. and Santos, G. (1998), "Feasible Arc Interior Point Algorithms for Nonlinear Optimization", in Fourth World Congress on Computational Mechanics, Buenos Aires.

[29] Coleman, T., Branch, M.A., and Grace, A., "Optimization toolbox for use with MATLAB", version 2, The Math Works.