GNSS Positioning in Multi-Constellation Scenarios

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***Abstract*—A Global Navigation Satellite System, GNSS, can be defined as a system capable of providing the positioning with global coverage by means of a satellite constellation. The United States' GPS, the Russians GLONASS and the European's Galileo are some examples.**

These systems can be combined, resulting in an increase on the number of visible satellites, which provides better satellite geometry and consequently more precise and accurate position estimates. Given the dependence of the world in global navigation satellite systems, the improvements allow to overcome the limitations of each system which is of significant interest.

Even with many similarities, each system defines its own timescale, coordinate system and how the satellite position can be determined and therefore, they are not entirely compatible to each other. The first step on the combination process is to solve the interoperability problems.

Since Galileo is not fully operational, its constellation and satellite position can be simulated considering the envisioned space segment allowing its combination with other systems.

This work emphasizes the differences of the main global navigation satellite systems and explains how their combination is possible. The performance improvements in multi-constellation scenarios are presented in terms of availability, quality of satellite geometry, accuracy and precision. Specifically, the average gains of the combined $GPS + GLONASS + Galileo$ positioning solution in relation to the combined $GPS + GLONASS$ are performed, and both in relation to each system alone.

***Index Terms*—GNSS, GPS, GLONASS, Galileo, Multi-constellation.**

I. Introduction

A Global Navigation Satellite System can be defined as a system capable of providing the positioning with global coverage by means of a satellite constellation. GNSSs include the GPS, GLONASS, Galileo, Beidou and other regional systems.

The Global Positioning System GPS, which is the system developed by the United States, and the Globalnaya Navigatsionnaya Sputnikovaya Sistema GLONASS developed by the Russian Federation, are the current operational systems. The European's Galileo is in a development stage.

The Global Navigation Satellite Systems revolutionized the satellite navigation and the geodetic positioning, being used with scientific, commercial and military purposes. Combining multiple GNSS, there is a significant increase in number of visible satellites which improves the satellite geometry, resulting in a more precise solution estimates and in an increase in terms of availability. This combination not only improves many applications specially in poor sky visibility conditions but also these applications can take advantage of the different characteristics of each system.

In the next section the history and developments of GPS, GLONASS and Galileo are presented. The third section deals with the differences in the definition of each GNSS and explains how they can be overcome. In section IV, the algorithm to obtain the receiver position is discussed.

Since Galileo is not fully operational, the positioning or combination with other constellations will be possible using simulated orbital parameters. The section V presents an overview of the simulation of the Galileo's constellation and satellite position in orbital plane.

In sections VI and VII, the results and consequent conclusions are presented.

II. State-of-Art

The launch of the first satellite Sputnik 1 by U.S.S.R. in 1957 induced the development of satellite navigation systems. The U.S. Navy developed in 1960 the first satellite navigation system named Transit, with 5 satellites and in 1967, the U.S.S.R developed the Tsiklon with 31 satellites whose main objective was to provide an accurate positioning for submarines.

Although operational, these systems were far from perfect and the need of overcome their limitations imposed by the Cold War lead to the creation of the GPS and the GLONASS by the United States in 1973 and by the U.S.S.R. in 1976, respectively.

These new systems were considered globally operational in 1995 and an interest emerged in combining both systems in order to improve the performance since GPS public signals were intentionally degraded (selective availability).

The combination of these two systems was not always attractive due collapse of Russian economy and the consequent loss in satellites. Besides, the selective availability was turned off in 2000. Many positioning techniques were development for GPS during the Russian economic recession.

In the 1990's, the European Union predicted the need of its own navigation satellite system since GPS and GLONASS were designed for military purposes and their civil service could be switched off or the signals intentionally degraded. Given the dependence of the world in global navigation satellite systems, Galileo will provide highly accurate and guaranteed global positioning service under civilian control. The interoperability with other GNSS will be also possible.

Nowadays, the GLONASS is once again fully operational and modernized and the combination of this system with other GNSS was reconsidered. The Galileo, with 6 satellites in orbit at the moment, is not fully operational yet but studies can be done in order to improve the coverage and accuracy for subsequent development.

III. Combining GPS, GLONASS and Galileo

Taking into account the dependence of the world in global navigation satellite systems, is of significant interest that satellite navigation applications can take advantage of the benefits of the combination of different constellations, not only in terms of availability and accuracy of the estimates but also benefit of the characteristics of each constellation. One example is the pole coverage in case of Galileo which is not verified using only GPS.

However, the combination is not so easy because the global navigation satellite systems have differences that date back to the time of their initial development. The implementations differ in time-scales, coordinate frames and how the satellite position can be determined (which includes different ephemeris parameters).

If each difference is analyzed, solutions to overcome the combination problems will emerge, making the GNSS combination possible.

A. Space Segment

The Space Segment can be defined by a constellation of satellites that ensures the visibility of at least four satellites at any time and from anywhere on Earth whose main function is to transmit radio-navigation signals which will allow users estimate their position, velocity and time.

1) GPS space segment: The GPS constellation is composed by 24 satellites, although 32 are available, distributed in 6 orbital planes differing from plane to plane by 60 degrees in longitude of ascending node.

The satellites, operating in orbits at an altitude of 20 200 km and with an inclination of 55 degrees in relation to the equator, have a orbital period of 11 hours and 58 minutes.

2) GLONASS space segment: The GLONASS satellite constellation is composed by 24 satellites distributed over 3 orbital planes, differing from plane to plane by 120 degrees in longitude of ascending node.

In each plane there are 8 satellites, 45 degrees apart in argument of latitude. The value of argument of latitude for satellites in equivalent slots (seen as the location in the orbital plane) in 2 different orbital planes differs by 15 degrees.

The satellites operate in orbits at an altitude of 19 100 km and inclination of 64.8 degrees in relation to the equator. The orbital period is 11 hours and 15 minutes which is approximately 8/17 sidereal day meaning 17 orbital revolutions every 8 sidereal days.

3) Galileo space segment: The Galileo satellite constellation will be composed by 30 satellites distributed over 3 orbital planes at an inclination of 56 degrees in relation to the equator, differing from plane to plane by 120 degrees in longitude of ascending node.

Each plane will contain 9 operational satellites, 40 degrees apart in argument of latitude, plus 1 spare satellite to replace any of the operational satellites in case of failures.

The satellites will operate in orbits at an altitude of 23 222 km and the orbital period is about 14 hours .

B. Time Systems

GPS, GLONASS and Galileo define their own time-scale which are connected to different realizations of UTC allowing the transformation from one time-scale to another.

1) GPS time-scale: The GPS time-scale is established by the Control Segment and it is coincident with the UTC (USNO) time-scale maintained by U.S. Naval Observatory at midnight of January 5, 1980.

GPS time differs from UTC because is not corrected with an integer number of leap seconds[1] and therefore, it is a continuous time scale. The navigation message contains the requisite data to identify this time-scale: GPS Week number WN which is the number of weeks since the midnight of January 5, 1980, and GPS Time of Week TOW which is number of seconds since the transition from the previous week.

2) GLONASS time-scale: GLONASS time is generated on base of GLONASS Central Synchronizer (CS) and it is coincident with Moscow time UTC(SU) or UTC+3hours[2]. This time-scale is periodically corrected by an integer number of seconds (leap seconds correction) as well as by UTC corrections according to the Bureau International de l'Heure (BIH) notifications.

In the navigation message, the time scale is represented by the GLONASS day number, which is the number of days since last leap year and denoted by N_T, and by the GLONASS time of day, the number of seconds within the GLONASS day t.

3) Galileo time-scale: The Galileo System Time (GST) is maintained by the Galileo Central Segment and its start epoch is the midnight between 21 and 22 August, 1999. This time-scale is ahead of UTC by 15 leap seconds since January 1, 2009 [3], and therefore is also a continuous time-scale.

In the navigation message it is represented by the Week Number WN, a sequential week number from the origin of the Galileo time, and by the Time of week TOW which is the number of seconds that have occurred since the transition from the previous week.

Since the receiver clock offset must be estimated together the receiver position, in a combined positioning solution there will be a different clock offset referred to each time-scale which will be estimated separately. This brings the disadvantage of additional satellites are need but, taking into account that when combining different constellations there are a higher number of visible satellites, the required and additional satellite for each clock offset is not a problem.

[1] Since August 2012, the difference is 16 leap seconds.

[2] Nowadays, Moscow time is UTC+4hours but GLONASS still employs the definition UTC+3hours for compatibility reasons.

[3] At the start epoch, GST was ahead of UTC by 13 leap seconds.

C. Coordinate Systems

Like time-scales, GPS, GLONASS and Galileo use their own coordinate system to define the satellite orbits. GPS employs the World Geodetic System 1984 (WGS-84), GLONASS emloys the Parametry Zemli PZ-90.11 and Galileo will establish an independent realization of the International Terrestrial Reference System (ITRS) whose requirements on the three-dimensional differences of the position compared to the most recent ITRF are to not exceed 3 cm, named Galileo Terrestrial Reference Frame (GTRF).

Although defined similarly, each coordinate system uses a different set of reference stations in its realization and therefore, in a combined positioning solution, the coordinate differences must be taken into account, otherwise the coordinate frame of the receiver position is undefined. The solution is to transform the satellite coordinates at signal transmission time from one coordinate frame to another before the definition of design matrix.

Since WGS-84 is more common than PZ-90.11 or GTRF, the GLONASS satellite positions in PZ 90.11 will be transformed into WGS-84 and Galileo's as well. To achieve that it's necessary to know the transformation parameters.

Relative to the actual PZ-90.11, the parameters of transformation from WGS-84 into PZ-90.11 was made available [4] and is defined as:

- $\Delta x = -0.013\ m$, $\Delta y = +0.106\ m$, $\Delta z = +0.022\ m$
- $R_x = -2.30\ mas$, $R_y = +3.54\ mas$, $R_z = -4.21\ mas$
- $\delta s = -0.008 \cdot 10^{-6}\ m$

where $\Delta x, \Delta y, \Delta z$ are the origin translation between the coordinate systems, R_x, R_y, R_z are the rotations to establish parallelism between the coordinate systems and δs is the scale factor. mas stands for mili-arcseconds and must be converted to radians before being used.

However, it is not official how to do the transformation but, since the 7-parameters are known, the Helmert Transformation can be used.

Relatively to Galileo, the information related to the implemented coordinate system like geodetic parameters and transformation parameters are not in the Galileo official interface document (ICD) yet but the GTRF transformation parameters to ITRF2005 are known.

D. Satellite Clock Offset and Orbit Determination

When the objective is to calculate the user position, the first step is the determination of the signal transmission time and thereafter the satellite position at this instant. Like time-scales and coordinate systems, the algorithms to determine the satellite position in orbital plane and to correct the satellite clock offset are also different.

1) Satellite Clock Offset: A satellite clock error is an offset in satellite clock (from its GNSS time-scale) which appears as a deviation on the range measurements made by a receiver.

The GPS satellite clock offset can be determined adding two components, the satellite on board clock offset component $\delta\tilde{t}_{Sat}$ and a relativistic component Δt_{rel}. Each component can be calculated using the following equations [1]:

$$\delta\tilde{t}_{Sat} = a_0 + a_1 (t - t_{oc}) + a_2 (t - t_{oc})^2 \tag{1}$$

$$\Delta t_{rel} = -4.442807633 \cdot 10^{-10} e\sqrt{A} \sin(E) \tag{2}$$

The navigation message provides the polynomial coefficients a_0, a_1 and a_2 corresponding to the satellite clock offset, drift and drift rate, the clock data reference time t_{oc} and the orbit eccentricity e. The semi-major axis of the satellite orbit A and the orbital eccentric anomaly E are determined during the process of satellite position determination.

For L_1 and L_2 pseudorange measurements, the satellite clock offsets can be computed according to equation 3 and 4, since L_1 and L_2 signals may be transmitted by the satellite at different time instants due to different hardware delays in the equipment used on board the satellite [1].

$$\delta\tilde{t}_{Sat,L1} = \delta\tilde{t}_{Sat} + t_{gd} \tag{3}$$

$$\delta\tilde{t}_{Sat,L2} = \delta\tilde{t}_{Sat} + \gamma t_{gd} \tag{4}$$

where, t_{gd} is the satellite hardware group delay broadcast in navigation message and $\gamma = \left(\frac{f_{L1}}{f_{L2}}\right)^2$, with f_{L1} and f_{L2} denoting the nominal center frequencies.

The GLONASS navigation message contains parameters to determine the offset of the time frame of the transmitting satellite to system time. These parameters are the difference between the equipment delays in L_1 and L_2, $\Delta\tau_n$, the time-scale offset to system time, $\tau_n(t_b)$, and the relative deviation of predicted carrier frequency from nominal value, γ_n, at reference time t_b. The parameter γ_n contains not only the effects of the satellite clock drift, but also the gravitational and relativistic effects [3], unlike GPS.

The satellite clock offset can be computed using the relation:

$$\delta t_{Sat} = -\tau_n(t_b) + \gamma_n (t - t_b) \tag{5}$$

For a L_1 pseudorange measurement, the satellite clock offset is still computed according to equation 5. For a L_2 pseudorange measurement, the inter-frequency bias must be considered.

$$\delta t_{Sat,L_2} = \delta t_{Sat} - \Delta\tau_n \tag{6}$$

Although in relation to Galileo System Time, Galileo navigation message broadcasts the same parameters presented for GPS and therefore, the same equations are applicable [2].

2) Satellite Orbit Determination: Each GNSS broadcasts different parameters to characterize the satellite orbit (ephemeris and almanac) and employs different algorithms to compute the satellite position. In the following, the GPS and GLONASS algorithms when using the ephemeris parameters are described.

The GPS ephemeris contain a set of Keplerian orbital elements and their perturbation factors that describes the satellite orbit at a reference time, denoted t_{oe}, usually updated every 2 to 4 hours. To determine the satellite position coordinates in WGS-84 coordinate system at a specified time t within the GPS Week, it is required to use the following algorithm [1].

1) Time from ephemeris reference epoch:

$$t_k = \begin{cases} t - t_{oe} - 604800, & \text{if } (t - t_{oe}) > 302400 \\ t - t_{oe} + 604800, & \text{if } (t - t_{oe}) < -302400 \\ t - t_{oe}, & \text{otherwise} \end{cases} \tag{7}$$

2) Orbit semi-major axis:

$$A = \left(\sqrt{A}\right)^2 \tag{8}$$

3) Corrected satellite mean angular velocity:

$$\eta = \sqrt{\frac{\mu}{A^3}} + \Delta\eta \tag{9}$$

4) Mean anomaly:

$$M = M_0 + \eta t_k \tag{10}$$

5) Eccentricity (solved by an iterative method):

$$E = M + e\sin(E) \tag{11}$$

6) True anomaly:

$$\theta = \arctan\left(\frac{\sqrt{1-e^2}\sin(E)}{\cos(E) - e}\right) \tag{12}$$

7) Argument of latitude:

$$\phi = \theta + \omega \tag{13}$$

8) Corrected argument of latitude:

$$u = \phi + \delta u \tag{14}$$

$$\delta u = C_{uc} \cdot \cos(2\phi) + C_{us} \cdot \sin(2\phi) \tag{15}$$

9) Orbit radius:

$$r_0 = A\left(1 - e\cos(E)\right) \tag{16}$$

10) Corrected orbit radius:

$$r = r_0 + \delta r \tag{17}$$

$$\delta r = C_{rc} \cdot \cos(2\phi) + C_{rs} \cdot \sin(2\phi) \tag{18}$$

11) Corrected angle of inclination:

$$i = i_0 + \delta i + IDOT \cdot t_k \tag{19}$$

$$\delta i = C_{ic} \cdot \cos(2\phi) + C_{is} \cdot \sin(2\phi) \tag{20}$$

12) Corrected longitude of the ascending node:

$$\Omega = \Omega_0 - \dot{\Omega}_e t + \dot{\Omega} t_k \tag{21}$$

13) WGS-84 satellite cartesian coordinates at time t:

$$x = r\left(\cos(u)\cos(\Omega) - \sin(u)\cos(i)\sin(\Omega)\right) \tag{22}$$

$$y = r\sin(u)\sin(\Omega)\left(1 + \cos(i)\right) \tag{23}$$

$$z = r\sin(u)\sin(i) \tag{24}$$

where M_0 is the mean anomaly at reference time, Δn is the mean motion difference from computed value, e is the eccentricity, $\sqrt{A}$ is the square root of the semi-major axis, Ω_0 is the longitude of ascending node of orbital plane at weekly epoch, i_0 is the inclination angle at reference time, ω is the argument of perigee, $\dot{\Omega}$ is the rate of right ascension, $IDOT$ is the rate of inclination angle, C_{uc}, C_{us} are the amplitude of the Cosine and Sine Harmonic correction terms to the argument of latitude, C_{rc}, C_{rs} are the amplitude of the Cosine and Sine Harmonic correction terms to the orbit radius, C_{ic}, C_{is} are the amplitude of the Cosine and Sine Harmonic correction terms to the angle of inclination and t_{oe} is the ephemeris reference epoch, in seconds of GPS week.

Unlike GPS, GLONASS ephemeris contain the satellite position in PZ-90.11 at a reference time t_b (x, y, z), the satellite velocity (v_x, v_y, v_z) and its acceleration due to luni-solar attraction (X'', Y'', Z''). These parameters are usually updated every 30 minutes and the value of t_b refers to the center of the 30 minutes interval [3]. To determine the satellite position at a specified time, it is required to integrate six orbital differential equations describing the satellite motion in a Earth-Centered Inertial (ECI) referential and taking into account the Coriolis force, published in GLONASS Interface Control Document [3]. The ephemeris parameters are then used as initial values in integration.

$$\frac{dx}{dt} = v_x \tag{25}$$

$$\frac{dy}{dt} = v_y \tag{26}$$

$$\frac{dz}{dt} = v_z \tag{27}$$

$$\frac{dv_x}{dt} = -\frac{\mu}{r^3}x - \frac{3}{2}J_2^0\frac{\mu a_e^2}{r^5}x(1 - 5\frac{z^2}{r^2}) + \omega_e^2 x + 2\omega_e v_y + X'' \tag{28}$$

$$\frac{dv_y}{dt} = -\frac{\mu}{r^3}y - \frac{3}{2}J_2^0\frac{\mu a_e^2}{r^5}y(1 - 5\frac{z^2}{r^2}) + \omega_e^2 y - 2\omega_e v_x + Y'' \tag{29}$$

$$\frac{dv_z}{dt} = -\frac{\mu}{r^3}z - \frac{3}{2}J_2^0\frac{\mu a_e^2}{r^5}z(3 - 5\frac{z^2}{r^2}) + Z'' \tag{30}$$

where r is the orbital radius and can be calculated with $r = \sqrt{x^2 + y^2 + z^2}$. The accelerations X'', Y'', Z'' due to luni-solar attraction are constant in the integration interval $t_b \pm 15$ minutes [3].

These differential equations are too complex to solve analytically and therefore, integration is performed numerically using the recommended Fourth-order Runge-Kutta method [3]. Thus, the accuracy of obtained satellite position and velocity will be dependent of the integration step width h.

To evaluate the influence of the integration step width on accuracy, adjacent ephemeris were integrated backward and forward from reference point (center point of the interval of validity) during 7 days and the deviations in their endpoints positions and velocities were analyzed. Table I presents the position deviations.

These results show that until a step width of 30 seconds, the integration error does not depend on step width h and although, it diminishes with the increasing step width around 50-225 seconds, the improvement is in the order of millimeters. The remaining error is also caused by approximations in the orbital force model and by the

TABLE I
POSITION INTEGRATION ERROR.

Step$[s]$	$\overline{\lvert\Delta x\rvert}[m]$	$\overline{\lvert\Delta y\rvert}[m]$	$\overline{\lvert\Delta z\rvert}[m]$	σ_x^2	σ_y^2	σ_z^2
1	0.48090	0.48164	0.51443	0.17972	0.16109	0.16525
5	0.48090	0.48164	0.51443	0.17972	0.16109	0.16525
10	0.48090	0.48164	0.51443	0.17972	0.16109	0.16525
30	0.48089	0.48163	0.51443	0.17972	0.16109	0.16525
45	0.48085	0.48159	0.51439	0.17970	0.16107	0.16523
50	0.48083	0.48157	0.51437	0.17969	0.16106	0.16522
60	0.48075	0.48150	0.51431	0.17965	0.16102	0.16520
90	0.48012	0.48097	0.51379	0.17937	0.16075	0.16497
100	0.47972	0.48062	0.51345	0.17919	0.16058	0.16483
150	0.47531	0.47679	0.50954	0.17718	0.15868	0.16315
180	0.47011	0.47262	0.50465	0.17506	0.15638	0.16098
225	0.46035	0.46546	0.49289	0.17237	0.15280	0.15562
300	0.49728	0.49640	0.47175	0.19281	0.17498	0.14839
450	1.62869	1.59929	0.98673	1.31210	1.08819	0.36930
900	27.22225	26.86325	18.26381	335.97314	262.32343	45.44828

simplifications in Runge-Kutta method. A dynamic step width can be used, decreasing the computational load.

Galileo uses the ephemeris parameters presented for GPS. The satellite antenna phase center position at GST time t can be computed using the same algorithm, although the final result is in GTRF coordinates [2].

IV. Position Estimation

A. Estimation Algorithm: Weighted Least Squares

The Weighted Least Squares is a special case of General Least Squares whose approach is to find a solution of an overdetermined system by means of the minimization of the cost function [6].

$$\begin{aligned} \hat{x} &= argmin_x \left\{ (Z - H \cdot x)^T W (Z - H \cdot x) \right\} = \\ &= \left(H^T W H\right)^{-1} H^T W Z \end{aligned} \quad (31)$$

where $\hat{x}$ is the best fit for the vector of the unknowns to be estimated x, Z is the vector containing the measurements,H is a matrix called design matrix which maps the state space into the observations space and W is the Weighting matrix.

B. Position Estimation: Standard Point Positioning

Standard Point Positioning uses a positioning principle based on solving a geometric problem from the pseudorange measurements to the satellites.

When combining measurements of more than one constellation, will be necessary an additional satellite for each unknown added (receiver clock offset with respect to each system time). In particular case of GPS, GLONASS and Galileo combined positioning solution, it will be required at least 6 visible satellites and the problem becomes:

$$x = \begin{bmatrix} x_R & y_R & z_R & c \cdot \delta t^{GPS} & c \cdot \delta t^{GLO} & c \cdot \delta t^{Gal} \end{bmatrix}^T \quad (32)$$

$$H_{GPS} = \begin{bmatrix} \frac{x_R - x^{GPS}}{\rho} & \frac{y_R - y^{GPS}}{\rho} & \frac{z_R - z^{GPS}}{\rho} & 1 & 0 & 0 \end{bmatrix} \quad (33)$$

$$H_{GLO} = \begin{bmatrix} \frac{x_R - x^{GLO}}{\rho} & \frac{y_R - y^{GLO}}{\rho} & \frac{z_R - z^{GLO}}{\rho} & 0 & 1 & 0 \end{bmatrix} \quad (34)$$

$$H_{gal} = \begin{bmatrix} \frac{x_R - x^{Gal}}{\rho} & \frac{y_R - y^{Gal}}{\rho} & \frac{z_R - z^{Gal}}{\rho} & 0 & 0 & 1 \end{bmatrix} \quad (35)$$

where ρ is the geometric range from receiver to satellite.

As presented in subsection of coordinate systems, the satellite positions $\vec{x}^{GPS}$,$\vec{x}^{GLO}$ and $\vec{x}^{Gal}$ must be given in the same coordinate frame to obtain a valid receiver position x_R, y_R, z_R. Then, the coordinates of the receiver position are expressed in the frame used for the satellite positions.

V. Galileo Simulation

Since Galileo constellation is not fully operational, the positioning or combination with other constellations will be possible using simulated orbital parameters.

A. Keplerian Orbital Parameters

The classic orbital elements, a set of six Keplerian elements, specify the satellite movement through the determination of the its orbit and respective orientation in space [8].

The orbital orientation in space is defined by the inclination i, the longitude of the ascending node Ω and the perigee argument ω.

The orbit can be defined in the plane by the semi-major axis a and the eccentricity e which characterize the ellipse, and by the time of perigee

B. Satellite Coordinates in the Orbital Plane

The satellite position in the orbital plane can be expressed in Cartesian coordinates as follows:

$$x_0 = r\cos\theta \ , \ y_0 = r\sin\theta \ , \ z_0 = 0 \quad (36)$$

where θ is the angle measured from the perigee, also known as True Anomaly, and r is the distance between the Earth and determined by:

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\theta}. \quad (37)$$

Besides the True Anomaly θ, defined as the angle from the perigee, there are other anomalies like Eccentric Anomaly E and Mean Anomaly M. The True Anomaly and the Mean anomaly can be obtained from the Eccentric Anomaly using the equations 38 and 39, respectively:

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right) \quad (38)$$

$$M = E - e\sin E. \quad (39)$$

C. Orbit Simulation

Considering the information available in the European GNSS Service Centre webpage (*www.gsc-europa.eu*) relative to 4 Galileo satellites in orbit, some approximations can be made in order to simulate the Galileo orbit and the satellite orbital positions.

The orbit eccentricity is almost zero, so a circular orbit $(e = 0)$ can be considered. The true and mean anomalies will be equal to the eccentric anomaly and therefore the satellite travels equal areas in the same time and consequently there is no velocity differential $(\Delta\eta = 0)$. The mean angular velocity is, in this particular case, the angular velocity.

Taking this into account and knowing the initial satellite position (the true anomaly θ_0 at instant t_0), the satellite position can be inferred from:

$$\theta(t) = \theta_0 + \eta(t - t_0) \quad (40)$$

Different orbital planes are 120 degrees apart in Longitude of ascending node. One can use as initial condition in the simulation the following values, at weekly epoch:
$(\Omega_0^{plane1} = 113.6°,\ \Omega_0^{plane2} = 233.6°,\ \Omega_0^{plane3} = 353.6°)$.

The inclination is 56 degrees and the semi-major axis is 29599.8 kilometers.

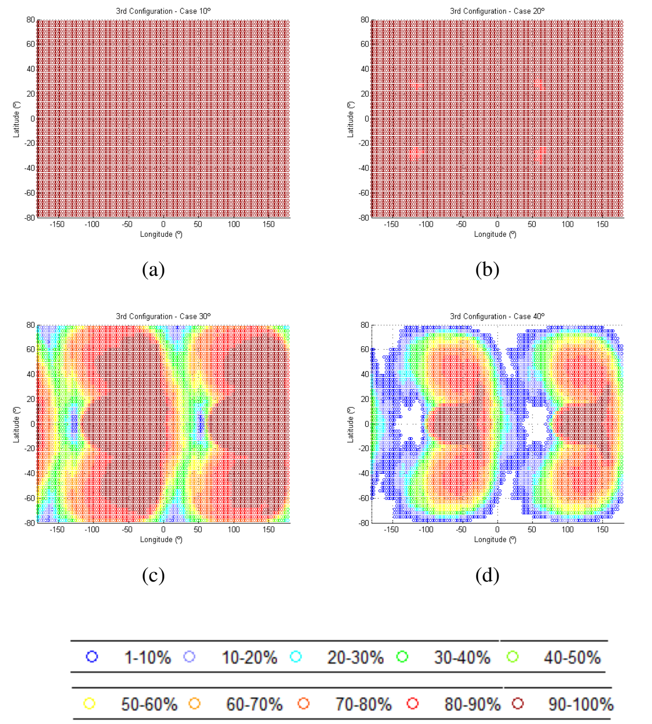
Consecutive slots in an orbital plane are 40 degrees apart in true anomaly.

The satellites which are in the same slot of two consecutive planes (120 degrees apart in longitude of ascending node) have an offset in the true anomaly value. In order to discover the offset which ensures high value of availability, 5 configurations were considered: with no offset, an offset of 5 degrees, an offset of 10 degrees, an offset of 15 degrees and an offset of 20 degrees. For each of these 5 configurations, the availability was presented in terms of the satellites at an elevation equal or higher to 10, 20, 30 or 40 degrees.

Regardless the configuration, for the cutoff angle of 10 degrees the availability is equal or higher than 95 % worldwide, providing also a good coverage even at latitudes up to 75 degrees north unlike GPS.

Analogously, for the cutoff angle of 20 degrees the availability varies from 85 to 100 %, but more often higher than 95 % and still providing a good coverage even at latitudes up to 75 degrees north.

Although the results are quite close, the configurations 3 and 4 introduce better world coverage in terms of availability higher than 90 % when the cutoff angle is 20 and 30 degrees. Taking into account that the configurations which ensure better results are with offsets of 10 (configuration 3) and 15 (configuration 4) degrees (figures 1 and 2), the chosen value is 13 degrees.



(e) Label

Fig. 1. Availability with the third configuration and (a) Cutoff angle of 10 degrees, (b) Cutoff angle of 20 degrees, (c) Cutoff angle of 30 degrees and (d) Cutoff angle of 40 degrees.

D. Pseudorange Simulation

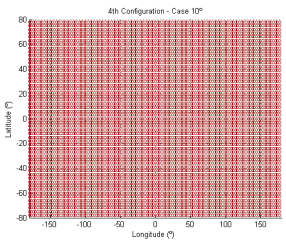
Analogously to the orbit, the apparent distance to the satellites is also simulated. This distance is calculated knowing the satellite S instant of transmission position and receiver R instant of signal arrival position in which is introduced the tropospheric and satellite clock offset residuals and the ionospheric error component not corrected when the NecQuick G model is used (about 30% of the ionospheric error). The satellite clock offset was assume as zero.

According to IGS, the satellite clock offset estimates produced by different ACs agree with standard deviations of 0.02 - 0.06 nanoseconds [7]. The satellite clock residual in seconds can be represented assuming a Normal distribution with zero mean and standard deviation of 0.04 nanoseconds .

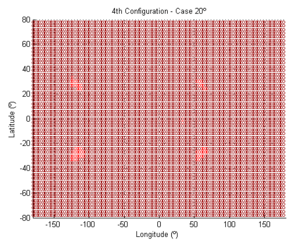
Typically, the tropospheric delay varies between 2.5 and 25 meters depending upon the satellite elevation angle el_R^S [6]. The respective residual, which is also dependent on the satellite elevation, varies from 5 to 10 centimeters at the zenith and can reach an error of 0.5-1 meter at 5 degrees of elevation [6].

Taking these values into account, the tropospheric residual can be represented assuming a the Normal distribution representation with zero mean and standard deviation of $\frac{0.08}{\sin(el)}$ meters.

In relation to the ionospheric delay, which is also dependent on the satellite elevation angle el_R^S, the common error values are up to 15 meters at the zenith and 9-45 meters at 5 degrees of elevation. The NecQuick G, model, used in single frequency



(a)



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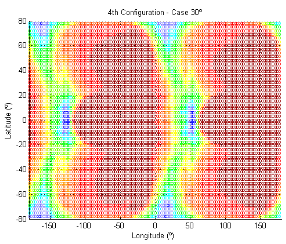
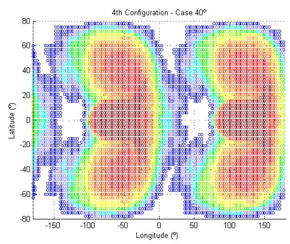


(c)



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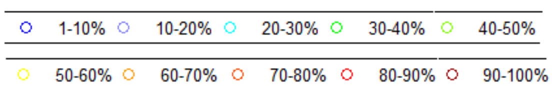


(e) Label

Fig. 2. Availability with the fourth configuration and (a) Cutoff angle of 10 degrees, (b) Cutoff angle of 20 degrees, (c) Cutoff angle of 30 degrees and (d) Cutoff angle of 40 degrees.

measurements, only corrects 70 % of the ionospheric error and therefore one third of the error should be introduced into the simulated distance. Taking into account the typical ionospheric errors, the ionospheric component of the simulated distance can be modulated assuming a Normal distribution with zero mean and standard deviation $\frac{0.3 \cdot 1}{\sin(el_R^S)}$ meters.

Both tropospheric residual and ionospheric error have positive magnitudes and this must be taken into account when a negative number is obtained from the Normal distribution.

Assuming the presented error and residuals as independent Gaussian variables, using the Central Limit Theorem its sum is also normal distributed with:

$$\sigma^2 = \sigma^2_{clock\ residual} + \sigma^2_{tropo\ residual} + \sigma^2_{iono} \tag{41}$$

VI. Results

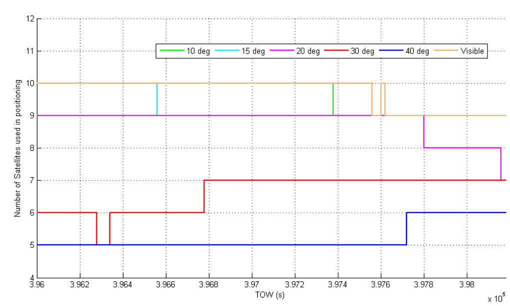
This section presents the results and the evaluation of the performance of combined $GPS + GLONASS$ and $GPS + GLONASS + Galileo$ solutions against the GPS-only, GLONASS-only and Galileo-only solutions. The improvements in performance are analyzed in terms of solution availability, quality of the geometry and precision and accuracy of the position estimates.

In the cases where Galileo was not considered, the solution was obtained using real data, otherwise was a combination of real (GPS and GLONASS) and simulated data (Galileo). The real data refers to October 9^{th} 2014 with start at 14:00 UTC and was acquired for the GPS and GLONASS positioning using a ProFlex 500 from Ashthec, a receiver designed for high precision surveys which provides real time position estimation [10], and the L1/L2 $GPS+GLONASS$ ProFlex 500 survey antenna (AT1675-7M).

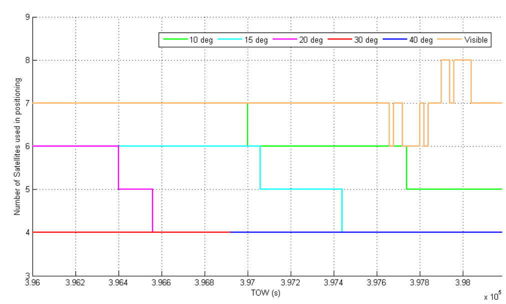
A. Solution Availability

In the satellite navigation problem there are 4 unknowns: the three coordinates of the receiver position and the receiver clock offset with respect to GNSS time-scale and therefore at least 4 satellites are needed. In particular case of combined positioning solutions like GPS and GLONASS, and GPS, GLONASS and Galileo there is a receiver clock offset with respect to each GNSS time-scale, and consequently at least 5 and 6 satellites are need, respectively.

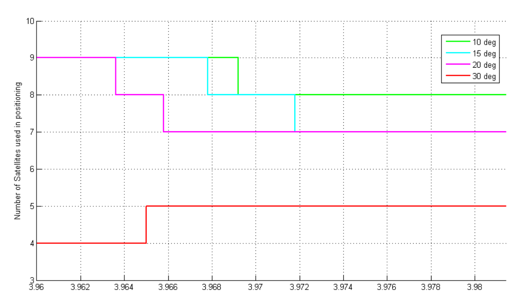
In order to evaluate the solution availability for different angles of elevation as criteria on satellite selection, the number of satellites for the GPS-only, GLONASS-only, Galileo-only, GPS and GLONASS, GPS and GLONASS and Galileo were analyzed.



(a) GPS



(b) GLONASS



(c) Galileo

Fig. 3. Number of satellites used in positioning.

For cutoff angles between 10 and 20 degrees, all GNSS alone perform well and are capable of providing a solution. GPS and Galileo have a higher number of satellites to provide a solution in this case. Between 20 and 30 degrees there is a notorious loss in number of satellites for all GNSS and GLONASS and Galileo are the constellations more affected. This is not so visible in GPS taking into account the initial number of available satellites higher than each other.

The cutoff angles of 30 and 40 degrees were a severe criteria because limits the number of satellites which can be used in positioning solution. These angles simulate big or even mega-cities where the multipath effect consists in a problem. Using these severe masks this effect can be reduced improving the solution. However, GLONASS and Galileo start to present outages or even unable to satisfy the requirements for positioning.

In relation to GLONASS constellation with cutoff angle 40 degrees, for almost 40 % of the time there were less than 4 satellites and therefore not plotted in figure 3 (b). At other time, there was just the enough number of satellites to calculate the receiver position. That was expected since that there were only 4 satellites when the cutoff angle value criteria was 30 degrees.

Considering the same cutoff angle of 40 degrees, in Galileo particular case there were no enough satellites required to positioning solution and therefore is not represented.

In these special cases, the combined positioning solutions is better because not also increase the number of satellites but also is capable of providing a solution.

The table II summarizes the position availability of the time interval considered.

TABLE II
SOLUTION AVAILABILITY.

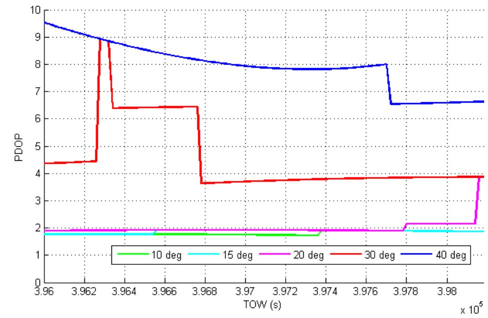
Cutoff Angle (deg)	GPS	GLONASS	Galileo	GPS GLONASS	GPS GLONASS Galileo
10	100	100%	100%	100%	100%
15	100%	100%	100%	100%	100%
20	100%	100%	100%	100%	100%
30	100%	100%	100%	100%	100%
40	100%	57%	0%	100%	100%

One important point is that Galileo ensures at least 7 satellites for cutoff angles between 10 and 15 degrees (or 20 degrees) proving that Galileo's space segment ensures 6-8 satellites all times and anywhere on Earth for the referred cutoff angles, as it was envisioned.

VII. PDOP

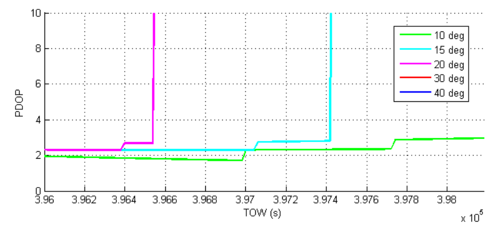
The Dilution of Precision DOP is a parameter which specifies the effect of the satellite geometry on solution precision. An ideal satellite geometry should have the satellites dispersed throughout the sky corresponding to a low DOP value, and not concentrated in one single region.

Specifically, the $PDOP$ is a measure of how precisely the receiver can compute its three dimensional position given the constellation geometry. Figures 4 to 7 show the PDOP values

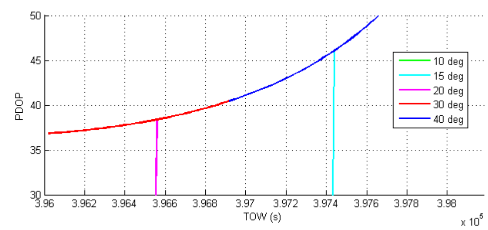


(a) GPS

Fig. 4. PDOP values for GPS-only positioning solution.



(a) GLONASS



(b) GLONASS (zoom in)

Fig. 5. PDOP values for GLONASS-only positioning solution.

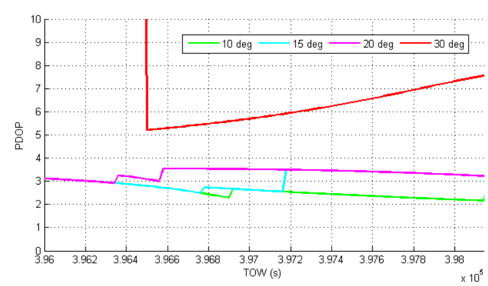


Fig. 6. PDOP values for Galileo-only positioning solution.

in the time interval considered for the different constellations and for the special cases of combining constellations.

Considering the cutoff angle of 10 degrees, the PDOP value of each constellation is always bellow four which ensures a good quality on the single constellation positioning case.

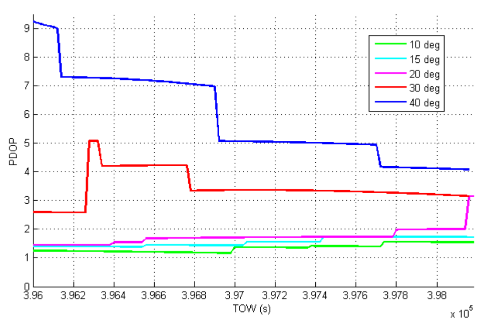
For cutoff angles between 10 and 20 degrees, it can be concluded that GLONASS starts to get its geometry severally degraded. Its PDOP value reach a value higher than 40 when the number of satellites diminishes until 4. GPS alone and Galileo still present PDOP values bellow 4.

Considering the cutoff angle of 30 degrees, GPS is capable of providing a PDOP value bellow 9 all the time and bellow 4 almost half of the time interval what it is possible with the increase of the number of satellites (from 6 to 7). Galileo alone starts to lose quality and presents a DOP value higher than 5 all the time interval providing rough positioning estimates or meeting the quality requirements for most applications.

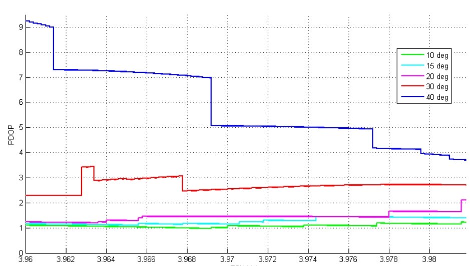
In this case, GLONASS alone provides estimates that must be discarded. Its geometry gets severally discarded and the PDOP value is higher than 35 during the time interval considered.

For a cutoff angle of 40 degrees, Galileo alone is unable to provide a solution and GLONASS presents service outages (almost 40 % of the time) but is unable to satisfy the requirements for most GNSS applications since the satellite geometry quality is evaluated with a value higher than 40 on PDOP (a geometry unfavorable). GPS starts to lose quality (PDOP value higher than 6 all the time) but is capable of providing a solution that meet the requirements of most applications.

Combining GPS and GLONASS constellations, among other advantages of this combination, improves the satellite geometry in relation to GPS or GLONASS alone. Regardless the cutoff angle considered, it can be easily seen that the PDOP values of the combined GPS and GLONASS constellation are always lower that the PDOP values of each constellation alone.



(a) GPS and GLONASS



(b) GPS, GLONASS and Galileo

Fig. 7. PDOP values for combined positioning solutions.

Until a cutoff angle of 10 degrees the PDOP of the combined GPS and GLONASS positioning solution is always bellow 2 which corresponds to an average gain of 25% and 38% in relation to GPS and GLONASS alone, respectively.

These gains are considerably larger when higher cutoff angles are considered, 30 and 40 degrees are examples.

For cutoff angles between 30 and 40 degrees, the gain relative to GLONASS alone is higher than 85 % due to severe degradation of the GLONASS constellation geometry taking into account the loss of the satellites. In these particular cases there are only 4 satellites.

Relative to GPS, the gains are higher than 10 % and even higher than 20% when severe angles are used as a criteria. This can be explained by the fact that GPS satellite geometry is not so affected unlike GLONASS. Besides, GLONASS contributes with only 4 satellites.

Only for 40 degrees there is a notorious lose of the quality of the geometry and in this case the gain shows that.

TABLE III
AVERAGE GAIN OF THE COMBINED GPS AND GLONASS POSITIONING SOLUTION.

In relation to	GPS	GLONASS
10 deg	25.905%	38.364%
15 deg	17.361%	59.176%
20 deg	13.142%	81.339%
30 deg	21.451%	92.067%
40 deg	26.973%	89.251%

The average gains presented are summarized in table III.

Attending in figure 7, it can be easily seen that the combined GPS, GLONASS and Galileo positioning presents better PDOP values comparing to the combined GPS and GLONASS positioning solution and therefore better than each constellation alone since the last combined constellation perform better than each GNSS alone and now there are more satellites involved on the positioning.

The average gains of the combination GPS, GLONASS and Galileo in relation to each GNSS alone and to the combination GPS and GLONASS are summarized in table IV.

TABLE IV
AVERAGE GAIN OF THE COMBINED GPS, GLONASS AND GALILEO POSITIONING SOLUTION.

In relation to	GPS	GLONASS	Galileo GLONASS	GPS
10 deg	40.106%	49.897%	57.276%	18.928%
15 deg	32.397%	67.075%	58.548%	18.324%
20 deg	27.265%	84.447%	57.204%	16.063%
30 deg	37.886%	93.874%	64.278%	21.071%
40 deg	27.518%	89.886%	–	0.633%

Considering the cutoff angle of 40 degrees, the low average gain of 0.633% in relation to the GPS and GLONASS combination is initially due to the contribution of only one satellite from Galileo constellation. The number of satellites of this constellation increases only at the end of the time interval considered and therefore the improvement is not so significant.

The combined geometry is better than the geometry of each constellation alone. This improvement is a significant one when severe cutoff angles are considered.

VIII. ERRORS, PRECISION AND ACCURACY

The HDOP and the VDOP are the horizontal and vertical dilution of precision, and decompositions of the PDOP. These parameters construe how precisely the receiver can compute its own position horizontally and vertically given the constellation geometry which can be also evaluated in terms of errors and subsequent quality of the solution: precision and accuracy.

The precision refers to how close are the position estimates and can be inferred from the standard deviation of that estimates.

The accuracy refers to how close are the positions estimates from the true receiver position. It is evaluated using the mean of the position estimates or considering the mean deviation from the know receiver position and the position estimate.

Regardless the cutoff angle, either GPS or GLONASS performed well in relation to the horizontal component, being GPS better than GLONASS. Relatively to the vertical component, the GLONASS solutions present values one order of magnitude higher than GPS solutions. This is more noticeable for higher cutoff angles due the loss of satellites and subsequent degradation of satellite geometry.

When comparing with the combined GPS and GLONASS and the combined GPS,GLONASS and Galileo, the combined solutions presents improved results which are specially noticeable in the vertical component of the error.

The combined GPS, GLONASS and Galileo positioning solution also presents improvements in relation to the combined GPS and GLONASS positioning solution.

TABLE V
ERROR RESULTS FOR CUTOFF ANGLE OF 10 DEGREES.

Error [m]	GPS	GLONASS	GPS GLONASS	GPS GLONASS Galileo
$\mu_{horizontal}$	3.359	2.642	2.505	0.4387
$\sigma_{horizontal}$	2.028	1.543	0.876	0.259
$\mu_{vertical}$	-3.820	2.921	-2.305	0.252
$\sigma_{vertical}$	6.157	5.191	4.414	1.113

Taking into account table V as an example, it is shown that the combined GPS, GLONASS and Galileo positioning solution is closer to the true position of the receiver and therefore provides more accurate position estimates (the mean values of the vertical and horizontal errors are the lowest one) than GPS, GLONASS alone and even than the combined GPS and GLONASS solution. The combined GPS and GLONASS positioning solution is also more accurate than the solution provided by GPS or GLONASS alone.

The standard deviation of the horizontal and vertical components of the error are also lower than those presented by each GNSS alone when the combined solution is considered. It can be said that the combined positioning solutions provide more precise solutions what was expected taking into account the favorable satellite geometry provided by the increase in number of the satellites.

In addiction, the combined GPS, GLONASS and Galileo presents more precise estimates than any other positioning solution analyzed.

The conclusions for the other different cutoff angles are analogous.

IX. CONCLUSION

GPS, GLONASS and Galileo are three systems that even with many similarities define their own coordinate system,time-scale and the algorithm to calculate the satellite position in orbital plane. Taking into account these differences, the systems can be used in applications which take advantage of a more accurate solution.

The described time scales are connected to different realizations of UTC which allow the transformation from one time-scale to another.

Analogously, if the transformation parameters from one coordinate frame to another are known it is possible to combine constellations. The solution is to transform the obtained satellite coordinates at signal transmission time from one coordinate frame to another before the definition of design matrix. The receiver position will be obtained in the final coordinate frame, otherwise it would be undefined.

The navigation message of the different GNSS contains parameters that, when combined with algorithms described in the official documentation, allow users to correct the satellite clock offset and to define the satellite orbit and thus the determination of the satellite position.

Since Galileo constellation is not fully operational, the positioning or combination with other constellations will be possible using simulated orbital parameters.

Using real data, it was shown that the combination $GPS + GLONASS$ not only increase the number of visible satellites but also improve the position estimates when compared to GPS and GLONASS alone. The satellite geometry is better which corresponds to a lower PDOP value. Combining real and simulated data it was also possible prove that $GPS + GLONASS + Galileo$ perform better than the combined $GPS + GLONASS$ or GPS, GLONASS and Galileo -only solution. These improvements are noticeable for GLONASS-only or future Galileo-only users.

The availability, the precision and the accuracy of a combined solution is significantly higher than the solution provided by each system alone. This is mainly noticeable for severe cutoff angles as 30 or 40 degrees and therefore important for applications whose requirements are the performance under poor visible conditions.

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