State-of-Charge (SOC) Prediction of Lithium Iron Phosphate (LiFePO4) Batteries for Automotive Application Based on Intelligent Systems

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Abstract—This paper presents methods for the state-of-charge (SOC) estimation and for the prediction of future impedance profiles of LiFePO4 batteries. This study will use a recently dataset acquired from the unit Energy Technology at VITO Belgium company (http://www.vito.be/VITO/EN/HomepageAdmin/Home/WetenschappelijkOnderzoek/Energietechnologie/), which is currently making tests with the promising LiFePO4 batteries for automotive application.

The first method introduced in this paper, is based on a discrete SOC estimation algorithm. However, this method only allows the estimation of predefined SOCs. Afterwards a continuous SOC estimation algorithm is presented. The operation of the first method is performed applying an Adaptive Neuro Fuzzy Systems and, for the second method, a linear interpolation. Finally, the last Section describes an on-line method used for the prediction of the future Impedance Spectra data, i.e. the respective battery impedance profiles obtained by electrochemical impedance spectroscopy also with the use of fuzzy Systems.

Index Terms— LiFePO4, state of charge, open circuit voltage, adaptive neuro fuzzy inference system, electrochemical impedance spectroscopy, impedance profile prediction

I. INTRODUCTION

Battery usage has been growing, making them important sources of energy, therefore justifying their development. The batteries are currently used in various industry sectors, from small mobile devices as mobile phones or mp3s to electric vehicles. The batteries are also used for fixed devices such as UPS or as energy storage for buildings or even for the power grid.

The bigger the devices, the greater the number of batteries used. Each battery has a value of State of Charge “SOC” however, when used in groups, normally their SOC differ, which is not detected by any SOC measuring instruments. Although a set of the same batteries are theoretically identical, one of the causes for unbalanced SOCs of the batteries is that in practice they always show minor differences between each other, which could result in slightly different capacities. Even if they are physically identical, the exposure to different working conditions can cause SOC’s unbalance. For example in a pack of batteries, those located more in the center are usually hotter than at the periphery.

Unbalance of SOCs between batteries and the inaccuracy of the traditional SOC measuring instruments can result in two situations:

• A shorter life of the battery pack in question because some batteries of the pack can suffer over-charging or even under-charging, reducing their lifetime and therefore the life time of the pack;

• A second critical situation results in different usage times for each battery that can also result in diminished life times for the batteries with a higher usage. The most frequent use of some batteries in the pack can lead to rapid aging and thus result in the destruction of the batteries thereby decreasing the longevity of the pack.

These unbalances can be eliminated with the use of measuring devices and SOC balancing between batteries [1]. A problem with these tools is their lack of precision and tendency to be out of adjustment over time, i.e. the measured value of battery SOC becomes increasingly inaccurate [2].

This paper aims to provide a solution for these problems through the implementation and development of more accurate techniques for the measurement of the battery’s SOC and techniques that self-update over time maintaining the accuracy of the instruments.

1) The selections of the inputs that allow the best SOC estimation: analysis and testing.

2) Develop an intelligent system to predict the battery SOC’s value.

3) Development and testing of intelligent systems to predict the battery characteristics allowing an accurate SOC estimating.

A. The State of Charge (SOC) Estimating Methods

1) The state of charge
The SOC of a battery is a measure of the amount of its stored electrical energy. The prediction of the state of charge can be performed by invasive methods and non-invasive methods. Invasive methods are based on the chemical oxidation state of the active materials and are performed in laboratories. This method requires the batteries to be offline. Nowadays, the existence of mobile systems with batteries such as electric vehicles, battery operated power tools, or temporary storage systems for renewable energy sources, therefore sealed batteries technology, it requires the prediction/estimation of the state of charge with non-invasive methods. Thus, the research for non-invasive and instantaneous methods for SOC determination is becoming dominant.

2) Relationship between SOC and OCV
The OCV of a standard LiFePO₄ battery in function of SOC can be visualized in Figure 1.

![Figure 1 - Plot of OCV in function of SOC.][3]

This relationship between OCV and SOC makes it very simple to estimate a battery’s SOC. There are only a few factors that can change the value of OCV for each defined battery SOC, hence changing the initial OCV-SOC relation. The main factors are temperature and the number of cycles (charge/discharge) of the battery. When these parameters change, a small variation of OCV is observed. This variation is often neglected since the variation is very small.

The relationship between OCV and the previous factors is trivial since the variation is almost linear. The main disadvantage estimating SOC using OCV value

The main disadvantage of estimating SOC through the OCV is that when the battery is either discharging or charging, the operating voltage can be different from the OCV. Operating voltage depends on the characteristics of the respective battery and the load. Therefore, after disconnecting the battery from the load or charging source, it requires a period of time for the OCV to reestablish its normal values. This characteristic can be observed in Figure 2 where this period of time stayed around 10 minutes.

To estimate the SOC in function of OCV, the battery needs to be offline and requires a period of time for the voltage to reach a steady state condition after use, making impossible instantaneous SOC measuring. As observed in the Figure 2, the OCV of the battery after discharging (time: t1) and after charging (time: t3) requires a period of time to reach a steady state condition. This interval of time will depend on how the battery has been used and mainly on its capacity.

![Figure 2 - OCV after discharging and charging][4]

Given that disadvantage, a solution is developed in this paper to solve this problem. Instead of estimating the SOC directly with the measure of the OCV, which requires the battery to be disconnected during a pre-defined period of time, an impedance spectra database of the battery is created for certain SOC values.

For a number of values of OCV an EIS is performed and this data is saved on a “database of impedance spectra and the respective OCV”. Since the OCV relationship with SOC is clear as observed in Figure 1, this solution enables the calculation of SOC with the comparison of the current battery impedance characteristic and the impedance characteristic curves in the database. This pattern recognition technique allows SOC estimation without needing to disconnect the battery and wait for the OCV to reach a steady state condition. To measure the battery impedance through an EIS (explained next in Section II), the battery does not need to be unplugged. This can be done online while the battery is charging or discharging without modifying the respective battery’s characteristics while the EIS is performed. The only time it is required to unplug the battery is when the impedance database is created and when it needs to be updated since the battery impedance varies because of certain factors as, for example, number of cycles, as it will be explained and demonstrated further.

Concerning the creation of the database of impedance spectrums with their respective OCV, this procedure begins with the battery discharged and waiting a necessary time interval for the voltage to reach a state of rest. The OCV is then measured and an EIS measurement is performed. Through the battery charging for a fraction of time and repetition of the previous procedure until the battery is charged, one can obtain an initial database of impedance spectrums and respective OCV for a set of SOC values like 25%, 50%, 75% and 100%, for example. This type of procedure would only be done to create the initial impedance database and mainly when it needs to be updated. The increase of the number of impedance spectrum measurements during the charging process causes the battery to take longer to charge because of the time interval needed for its voltage to reach a state of rest. Taking into account that...
the battery impedance varies with increasing number of cycles, this database needs to be updated periodically. In our study, the database of impedance spectrums at the respective OCV (with corresponding SOCs) has been updated every month.

Regarding the selection of SOC values for the database, it is preferable to have them equally spaced and the criterion for choosing this number of selected SOCs has to be based in the analysis of the compromise between battery charging time and the number of outputs (method analyzed in Section II) or accuracy of SOC estimation (method analyzed in Section III). The greater the desired precision, the more impedance spectrum measurements are needed, subsequently longer battery charging, also taking into account the OCV relaxation time.

For example, from the data supplied by VITO, consider the four impedance spectrums shown in Figure 3 for 25%, 50%, 75% and 100% values of SOC.

Performing during a normal usage of the battery and in a regular basis an EIS experiment to acquire the current impedance spectrum of the battery, and matching it with those ones already in the database, the battery SOC can be estimated. In this paper, this matching between EIS with the database of impedances has been performed using fuzzy logic, which will be explained in Section II.

3) Electrochemical impedance spectroscopy (EIS)

EIS is also an experimental technique for characterizing electrochemical systems. This technique acquires the impedance of a system over a wide range of frequencies, and thus the frequency response of the system. Often, data obtained by EIS is expressed graphically in a Bode plot or a Nyquist plot.

EIS is a very sensitive technique, since the perturbing AC signal is set very small, with the resultant polarization of the electrode staying in a linear potential region, so there is no destructive damage of the electrode.

4) Relationship between battery’s SOC and its impedance spectrum

In this paper, the impedance data analyzed and studied of LiFePO₄ batteries with a capacity of 7 Ah each one, was provided by VITO, the Flemish Institute for Technological Research. The frequency range used on all EIS is between 0.05 Hz and 10019.5 Hz, with each range performed twice for each SOC.

Figure 3 plots the impedance spectra measurements acquired by EIS technique for a battery with one month of use, 0°C of ambient temperature and four SOCs: 25% (red), 50% (yellow), 75% (cyan) and 100% (blue). It can be observed a relationship between the impedance curve profile and its SOC value.

According to VITO, the impedance spectra of the battery has been obtained with a periodic interval of one month. To show how the impedance changes over time, Figure 4 plots the impedance spectrum of the battery after two month of use. Comparing the impedance for 100% SOC (blue curve) at the first month of use (Fig. 3) with that one in the second month (Fig. 4) a significant change of impedance is observed.

As the impedance of the battery varies over time (real battery model), it is difficult to implement a fixed model, which simulates the battery’s behavior, that unequivocally allows the reading of the battery SOC as a function of the respective impedance profile. Notice that impedance variation is not linear, which presents a major pattern recognition problem. It is in this context that in further sections are presented a set of methodologies to estimate SOC in function of impedance. Those methodologies combine important techniques as: EIS, SOC estimation through OCV measuring and an adaptive neuro fuzzy inference system (Section II) or linear interpolation (Section III).

II. SOC ESTIMATION METHOD (IMPEDEANCE PROFILE COMPARATIVE ANALYSIS) APPLYING AN ADAPTIVE EURO-FUZZY SYSTEM

In this Section a first SOC estimation method is proposed. This estimation is based on a comparative analysis of impedance profiles.

The method is not online since it does not permit a continuous estimation of the battery’s SOC for the entire range of SOC (0% to 100%). This method allows the estimation of SOC but only for a set of predefined SOCs, for which was previously made an EIS experiment. The method compares the current battery impedance with previously impedances measures and, when a match exists, the output is the SOC of the respective matched impedance of the database. This match making is
performed with ANFISs, as it will be described in the following sections.

A. Construction of ANFISs with the EISs impedance data

The impedance spectrum for each SOC is obtained through EIS. This Section presents a methodology to estimate the battery’s SOC through comparative analysis of its actual impedance spectrum (current battery condition) with the impedance spectrum previously built with the EIS technique and stored in the database. This database will be composed of several impedance spectrum measurements and the respective SOC.

Assuming that the database has already been created with four EIS made for different random (In this paper, EIS data used in this method was provided and performed by VITO Institute for fixed SOC values, the proposed method operates equally for other SOC values.) battery SOC’s equally spaced, more precisely for 25%, 50%, 75% and 100%, thereby obtaining the impedance spectra, it is possible to construct the comparison system of impedances. This comparative analysis is done through a fuzzy system that uses only two input variables: the real part of complex impedance (resistance) and its imaginary part (reactance). The output of the fuzzy system will be the battery’s SOC.

Using the same experimental data of impedance as used before (EIS of the battery with a SOC of 25%, 50%, 75% and 100%, Section I - Figure 3), the domain of each input variable was divided into membership functions. Instead of triangular membership functions, Gaussian ones were used.

Each impedance spectroscopy is composed of 38 impedance measurements (each one with different frequencies), and for each SOC there are two impedance spectrums. Since there are 5 different SOCs, a total number of 380 data points have been obtained. A FIS is created with 50 membership functions equally spaced for each input variable.

Figure 5 shows, using a grid representation, how the fuzzy rules cover the space. Each rectangular area signifies the intercepted area of each membership function, in the interval [0.5, 1], between both inputs.

B. The Results (ANFIS rule-base: initial testing)

Figure 5 shows using a graphic representation the initial set of rules that will compose the fuzzy system. This is initially evaluated using the same data (usually called training data) used to construct the rule base, that is, the data points forming the impedance spectrum of the four SOC levels.

Let’s start inserting as input into the fuzzy system the training data of the battery, that is, the resistance and reactance data for 25%, 50%, 75% and 100% of SOC (each EIS consists of 38 impedance measurements for different frequencies, performed twice). The results are plotted in the Figure 6. The red asterisks represent the estimated SOCs from the fuzzy system, while the blue circles are the real SOC of the input data, i.e. the experimental data used to create de fuzzy system. The average and variance values of the results shown in Figure 6 can be seen in the table 1.

The average and variance values can be observed in the next table:

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Average</th>
<th>Variance</th>
<th>Mean Output Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIS of the battery with 25% SOC</td>
<td>28.9</td>
<td>64.6</td>
<td>3.9</td>
</tr>
<tr>
<td>EIS of the battery with 50% SOC</td>
<td>48.2</td>
<td>35</td>
<td>-1.8</td>
</tr>
<tr>
<td>EIS of the battery with 75% SOC</td>
<td>75.7</td>
<td>8.8</td>
<td>0.7</td>
</tr>
<tr>
<td>EIS of the battery with 100% SOC</td>
<td>97.2</td>
<td>83.3</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

Table 1 - Average and variance of the outputs of the Fuzzy system with 50 membership functions each variable.

1) Example of a possible algorithm scheme for estimating SOC (taking into account the presence of noise in the inputs).

As can be observed, there is some noise in the measurement of impedances through the EIS technique. For each SOC, two sweeps of measurements of impedances were made with the same array of frequencies one after the other. The two impedance profiles look slightly different due to measurement errors with the respective technique and its equipment. So, due to these errors, the fuzzy model has been analyzed in the presence of noise to simulate other possible incertitudes coming from the measurements of impedances.
From the profiles already available, other possible ones have been simulated, which have taken into account the difference of magnitudes observed between the two impedance profiles available for each SOC.

The simulation of those impedance profiles is based on adding random noise on the respective impedance profiles, in order to simulate the difference of magnitudes observed between the two impedance profiles available for each SOC. For example, considering the impedance profiles of the battery with 50% and 75% of SOC (two impedance profiles for each SOC), other profiles are simulated by adding noise to the respective impedance profiles and are used as inputs in the next example of a possible algorithm scheme for estimating SOC:

Depending on the amount of noise, which depends on the type of instruments used for measurement, the respective acceptance limits are defined. For example, for 50% the fuzzy model is tested and as result the output average and variance is obtained, now is possible to select limits of acceptance, in this case an acceptable limit for the average can be with a margin $\pm 2\%$ [48,52] and the variance $> 90.6$ (if 90.6 is a minimum value of the variance observed by several measurements of the battery with the $SOC = 50\%$). In table 2 the previous limits for 50% and for 75% are presented as an example.

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Output Average</th>
<th>Limits for the mean</th>
<th>Output Variance</th>
<th>Limits for the variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated EIS of the battery with 50% SOC</td>
<td>48.1</td>
<td>[48, 52]</td>
<td>90.6</td>
<td>[0, 91.1]</td>
</tr>
<tr>
<td>Simulated EIS of the battery with 75% SOC</td>
<td>75.1</td>
<td>[74, 76]</td>
<td>25.9</td>
<td>[0, 26.4]</td>
</tr>
</tbody>
</table>

Table 2 – Example of setting limits for the average and variance for the SOC estimation algorithm.

Observing table 2 is possible to create an algorithm for the estimation of SOC. For example, the steps of the algorithm can be the following schematized in Figure 7.

III. ONLINE SOC ESTIMATION METHOD

In the previous section, a method that estimates the SOC through impedance comparative analysis has been presented. The biggest drawback of that method is that it does not allow a continuous estimation of the battery’s SOC for its entire range of 0-100%, only matching impedance spectrums with those ones previously included in the fuzzy rule base. Therefore, a method that allows the estimation for the entire range of SOC is presented as solution. This method, since it incorporates the entire range of SOCs as output, is an online method that continuously provides an estimation of the battery’s SOC, as result. As already mentioned, the impedance profile of the battery varies with its SOC and also with the number of discharge/charge cycles already experienced by the battery. This presents a major difficulty for an online SOC estimation through the impedance comparative analysis.

As mentioned before, the shorter the time interval between measurements of impedances, EISs, the smaller the impedance profile variation between these time intervals. In our case, the available laboratorial measurements from VITO were done with a time interval of 1 month. Given these characteristics, follow an online SOC estimation algorithm is proposed which fuzzy rules are updated when new impedance spectrums are obtained.

A. On-line SOC estimation Algorithm

The battery impedance profiles changes with its SOC, using a more compacted form, each impedance profile has been represented by its centroid. Figure 8 shows the centroid associated with each SOC using the same color of its impedance curve.

Assuming that the impedance spectrums evolve accordingly with a certain order, one can represent this evolution looking the “path” taken by centroids run as SOC changes. Figure 9 at right contains a green line connecting the centroids to indicate their evolution from 25% red to 100% blue.
Interpolating the previous data not only for a 2D “path” but for a 3D region, it is possible to fit a surface using only the impedance centroids. This fitting method is named bilinear interpolation [5].

As a result of the linear interpolation of the centroids of the impedances, a 3D function with 3 coordinates as SOC, Re[Z] and -Im[Z] is constructed. Figure 10 displays this function where each light blue dot represents the impedance centroids from Figure 9. This method of interpolation allows the construction of new data points within the range of a discrete set of known data points. A model was built with this method, which calculates the battery’s SOC based on the values of the centroids of the known impedance profile.

For the estimation of SOC using this model, some centroids of the actual impedance profile of the battery may be located outside the surface covered by interpolation and as solution, an algorithm was implemented. The algorithm computes the closest point of the surface covered by interpolation of the respective centroid.

**B. The Results (Testing the algorithm)**

The first test for the fuzzy model built using the previous interpolation technique will be to use intermediate impedance profiles as input data, and to analyze the output of the model, verifying if it corresponds to the expected SOC. Assuming that the impedance profile variation is linear between the known impedances, intermediate impedances between 25%-50%, 50%-75%, and 75%-100% are projected (the mean between each impedance spectrum), 37.5%, 62.5% and 87.5% respectively. Those will be the expected SOC to be estimated by the fuzzy model when test data, the intermediate impedance profile centroids, are inserted as inputs. Figure 11 (a) to (c) displays the output of the interpolation model with the estimated impedance centroids. Estimation error for each case is listed in Table 3.

![Figure 9](image1.png)

*Figure 9 - Assumed path of the evolution of the centers of gravity represented with a green line, 25% (red dots), 50% (yellow dots), 75% (cyan dots), 100% (blue dots) (2nd month).*

![Figure 10](image2.png)

*Figure 10 - Linear Interpolation model computed with 4 impedance centroids of the battery with: 25%, 50%, 75% and 100% of SOC respectively, (2nd month).*

![Figure 11](image3.png)

*Figure 11 - Representation of the output of the interpolation model for the intermediate centroids: (a) 37.5%, (b) 62.5% and (c) 87.5% of SOC.*

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output (%)</th>
<th>Output Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance spectrum centroid of the battery with 37.5% of expected SOC</td>
<td>37.50</td>
<td>0</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 62.5% of expected SOC</td>
<td>60.16</td>
<td>-2.34</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 87.5% of expected SOC</td>
<td>85.78</td>
<td>-1.72</td>
</tr>
</tbody>
</table>

*Table 3 – Output and error of the algorithm for impedance spectrum centroids of the battery with a SOC of 37.5%, 62.5%, and 87.5% as inputs (For 25%, 50%, 75% and 100% the output error = 0)*
The highest observable absolute error for these entries was 2.34%, which represents a very good result, in the error boundaries of SOC estimation.

1) Noise Analysis
The model has been tested with the presence of noise at the inputs. As shown in Figure 12, ± 10% white noise added to each variable (different noise vectors for each variable), real and imaginary parts of the impedance respectively, and the output of the model was obtained as shown in table 4.

Since the noise does not significantly alter the position of the centroids, the output of the model will vary slightly as observed by comparing table 3 and 4.

IV. IMPEDANCE PROFILES PREDICTION
In the previous sections, it has been proposed two main methodologies to estimate the battery’s SOC with the creation of an impedance profiles database for various SOC values. The impedance profiles were obtained using the EIS technique at VITO. Using that data, two models have been established to SOC estimation: a fuzzy rule-based model and an offline SOC estimation method based on a linear interpolation model. In these two models, the current impedance profile of the battery, or the centroid of the current impedance profile of the battery, were used as input data of the model to result as output the respective estimated SOC.

Battery’s impedance profile changes with time and, related with these changes, its SOC also changes. So the accuracy of models will depend on the time needed to update the impedance profiles database. The shorter it is, more accurate are the models. However, increasing precision by decreasing the update’s time interval, will cause a lengthier charging, as mentioned in Section I, which can be seen as a problem, depending on how the battery will operate.

As a solution, in this Section, a technique is proposed to predict the impedance profile of the battery for several SOC values. The technique will allow the prediction of the impedance profiles instead of effectuating the impedance profiles measure during charging, which could be a lengthy process.

In terms of battery data supplied by VITO, the time interval considered for performing EIS experiments of the battery for different SOCs (25%, 50% and 100%) was 1 month for a period of 10 months of tests. Our goal will be to analyze this "past information" and to predict the next month’s impedance profile that, without any estimating algorithm could have been obtained through EIS experiments for each SOC.

The prediction of the impedances profile will be obtained applying a fuzzy rule-based system as before. For the fuzzy rules construction, with the goal to predict future impedance profiles, all the information available from VITO obtained through EIS was used. The fuzzy model has 3 input variables: the real and imaginary impedance values, and their corresponding frequency. It has to be noticed that the first two inputs are variations of impedance and not only absolute values.

A. Impedance profiles prediction method
In this section, only the impedance profiles for SOCs of 25%, 50% and 100% have been used to create the model, since for 75% the measurements from VITO are incomplete, i.e., for this SOC the impedance profiles were not obtained all months. The ANFIS for impedance profile prediction will use as input the difference between the impedance of the previous and current month. The output will be the difference between the impedance of the current month and the next one. To isolate every measure of impedances of different frequency another

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output (%)</th>
<th>Output Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance spectrum centroid of the battery with 25%</td>
<td>25.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 37.5% of expected SOC</td>
<td>38.42</td>
<td>0.92</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 50%</td>
<td>49.44</td>
<td>-0.56</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 62.5% of expected SOC</td>
<td>62.16</td>
<td>-0.34</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 75%</td>
<td>74.58</td>
<td>-0.42</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 87.5% of expected SOC</td>
<td>87.57</td>
<td>0.07</td>
</tr>
<tr>
<td>Impedance spectrum centroid of the battery with 100%</td>
<td>99.33</td>
<td>-0.67</td>
</tr>
</tbody>
</table>

Table 4 - Output and error of the algorithm for impedance spectrum centroids of the battery with a SOC of 25%, 37.5%, 50%, 62.5%, 75%, 87.5% and 100%, as inputs with noise added.
input was considered to the model, corresponding to the frequencies of the performed impedance profiles. The inputs, real and imaginary part of the complex impedance, were both normalized with the highest value of each one. The difference value between impedance profiles between two consecutive months, represents the input data to produce the antecedent of the fuzzy rules. On the other hand, the difference of impedance between months two and three will form the consequents of the rules. Performing the same procedure for all months, except the last one, the inputs and outputs to be used as rule antecedents and consequents are listed in Table 5.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIS 1ºmonth - EIS 2ºmonth</td>
<td>EIS 2ºmonth - EIS 3ºmonth</td>
</tr>
<tr>
<td>EIS 2ºmonth - EIS 3ºmonth</td>
<td>EIS 3ºmonth - EIS 4ºmonth</td>
</tr>
<tr>
<td>EIS 3ºmonth - EIS 4ºmonth</td>
<td>EIS 4ºmonth - EIS 5ºmonth</td>
</tr>
<tr>
<td>EIS 4ºmonth - EIS 5ºmonth</td>
<td>EIS 5ºmonth - EIS 6ºmonth</td>
</tr>
<tr>
<td>EIS 5ºmonth - EIS 6ºmonth</td>
<td>EIS 6ºmonth - EIS 7ºmonth</td>
</tr>
<tr>
<td>EIS 6ºmonth - EIS 7ºmonth</td>
<td>EIS 7ºmonth - EIS 8ºmonth</td>
</tr>
<tr>
<td>EIS 7ºmonth - EIS 8ºmonth</td>
<td>EIS 8ºmonth - EIS 9ºmonth</td>
</tr>
</tbody>
</table>

Table 5 - Inputs and the respective outputs of the ANFIS.

Since there is only one output per fuzzy model, one FIS was built for the real part of the impedance (FIS_Real) and another for the imaginary part (FIS_Img). The fuzzy model named FIS_Real has 3 input variables and one output, as shown in Table 6 (second row).

The second fuzzy model, FIS_Img, is used to predict the imaginary part of the impedance profile. Its input variables are the same ones used for the previous fuzzy model, FIS_Real, as shown Table 6 (third row).

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Output variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIS_Real</td>
<td>Re(Δzi)</td>
</tr>
<tr>
<td>FIS_Img</td>
<td>Im(Δzi)</td>
</tr>
</tbody>
</table>

Table 6 - FIS_Real and FIS_Img inputs and output.

Each input variable was divided by five triangular membership functions, symmetric and equally spaced, as illustrated in Figure 13.

To build the fuzzy model, the impedance profiles measured of months 1 to 9 were used as training data.

1) The Results (A)

To test the operation of this fuzzy model, the EIS data composing the impedance profile of month 9 was inserted as input and the prediction of the real part of the EIS one month ahead (month 10) was obtained as output, as illustrated in Figure 14.

Inserting the same input, the EIS data of month 9, the imaginary part of the impedance profile of month 10 is expected as output, as it can be observed in the Figure 15.
Observing the prediction of the real and imaginary part of the impedance profile of next month (10th month), Figure 14 and 15 respectively, a prediction error is noticed, i.e. a difference between the predictions values (red dots) and the actual impedance values (blue dots). This prediction error can cause the inaccuracy of SOC estimation methods, for example the methods presented in Section II and Section III. The mean absolute error of FIS_Real and FIS:Img models are listed on Table 7.

<table>
<thead>
<tr>
<th>SOC</th>
<th>Mean absolute error between prediction and the EIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIS_Real</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.017617</td>
</tr>
<tr>
<td>50%</td>
<td>0.003091</td>
</tr>
<tr>
<td>100%</td>
<td>0.015790</td>
</tr>
<tr>
<td>FIS_Img</td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.012822</td>
</tr>
<tr>
<td>50%</td>
<td>0.008527</td>
</tr>
<tr>
<td>100%</td>
<td>0.009796</td>
</tr>
</tbody>
</table>

Through the analysis of the relative error of the prediction of the real and imaginary impedance, it is possible to observe that the prediction of the fuzzy models are not sufficiently accurate. The main two causes of this inaccuracy are:

- Insufficient experimental data in the construction of the fuzzy model: EISs were only done for 25%, 50% and 100% of battery’s SOC, and only for nine periods of time during 10 months.
- Experimental data was obtained through various EIS experiments with one month of interval, which in this case is too extensive, since the variation of impedance profile is too drastic as seen during monthly intervals. In this case, causing a loss of information about the evolution of the battery impedance profiles over time, that is, only with this impedance data, the recognition of a pattern is extremely difficult and inaccurate.

Introducing these reasons for the lack of precision of the fuzzy models, a method is proposed not as a solution for:

- the lack of data available for the construction of the system,
- the time interval being too long between EIS measurements, or
- the lack of EIS measurements for more SOCs.

The proposed method presented below takes into account that more data is needed to construct a fuzzy model that presents an impedance profiles prediction with an admissible level of accuracy.

B. Impedance profiles prediction method using data interpolation

The available data for this study was provided by VITO. However, this data has revealed itself incomplete to ensure our goal of an accurate prediction of battery impedance profiles. To improve the functionality of the fuzzy models, a new training data set was considered. Through computing intermediate impedance profiles between months (the mean between impedance profiles), is had been able to duplicate the training set. Let’s assume than that this procedure could be regarded as actual experimental data from the battery in which the EIS experiments were made every half month. For example, using the first and second EIS to compute an intermediate EIS as shown in Figures 16 and 17 for the real and imaginary data, respectively.
The two fuzzy models FIS_Real and FIS_Img are recalculated using the new training set (EISs of the battery with 25%, 50% and 100% SOC but made with a half month interval). Now, the input variables now, instead being the difference between EISs that were made with an interval of one month, it is calculated the difference between EISs with an interval of half a month. The domain for each input was again divided in five triangular membership functions as before.

Two fuzzy models were constructed, as before, one with the real part of the prediction of the impedance profile as output and the other one with the imaginary part. The input and output variables used in the fuzzy models are listed in Table 8.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIS_real</td>
<td>Real(Z_{\text{month}}(i) - Z_{\text{month}}(i-1)) \over \text{Real}(\Delta Z_i) = \text{Real}(Z_{\text{zo}})</td>
</tr>
<tr>
<td>FIS_Img</td>
<td>Real(Z_{\text{month}}(i) - Z_{\text{month}}(i-1)) \over \text{Real}(\Delta Z_i) = \text{Real}(Z_{\text{zo}})</td>
</tr>
</tbody>
</table>

Table 8 - ANFIS inputs and outputs (Real and imaginary part of the complex impedance profile).

1) The Results (B)

Entering as input in the fuzzy models the intermediate EIS between month 9th and 10th of the battery with 25%, 50% and 100% of SOC, the prediction of the EIS of month 10th (which was excluded in the computation of both models) is expected as output.

Figures 18 and 19 show the experimental and the predicted EIS profiles, the real and the imaginary part respectively.

Comparing Figures 17 and 18 with Figures 14 and 15 a clear increase of precision is observed between those predictions, the error is decreased, as can be observed comparing Table 9 with 7 (Error of the real and imaginary part of the impedance prediction).

<table>
<thead>
<tr>
<th>SOC</th>
<th>Mean absolute error between prediction and the EIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIS_Real</td>
<td>25%: 0.003408, 50%: 0.001678, 100%: 0.004260</td>
</tr>
<tr>
<td>FIS_Img</td>
<td>25%: 0.002402, 50%: 0.003300, 100%: 0.003664</td>
</tr>
</tbody>
</table>

Table 9 - Mean absolute error of the real part of the impedance profile.

REFERENCES


