Distributed Predictive Control and Estimation

–File 5–

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2022

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Distributed MPC

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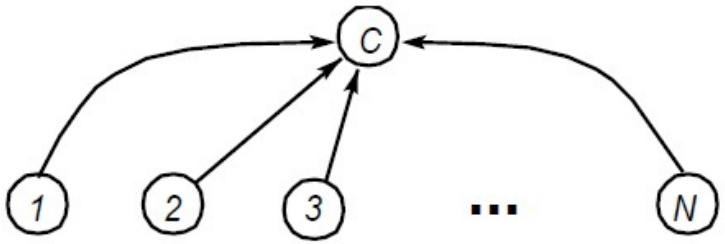
Example 1: centralized solution

Find the average of the money in the pockets of the members of the assistance in the first row.

Trivial using a centralized communication solution

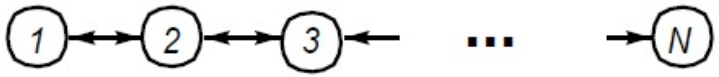


Each node (person) i communicates the amount of money y_i inside his pocket to the central node that simply computes the average as

$$S = \frac{1}{N}\sum_{i=1}^{N} y_i$$

Example 1: distributed solution

Assume now that there is no central node and that each person can only talk to their immediate neighbors. The communication structure is then



One possibility consists in an iterative procedure in which each node (person)

1. Tells his neighbors about how much money he has
2. Computes the average of the money in his pocket and in his neighbors pocket.
3. Repeats 1) and 2) iteratively, telling the average he has computed.

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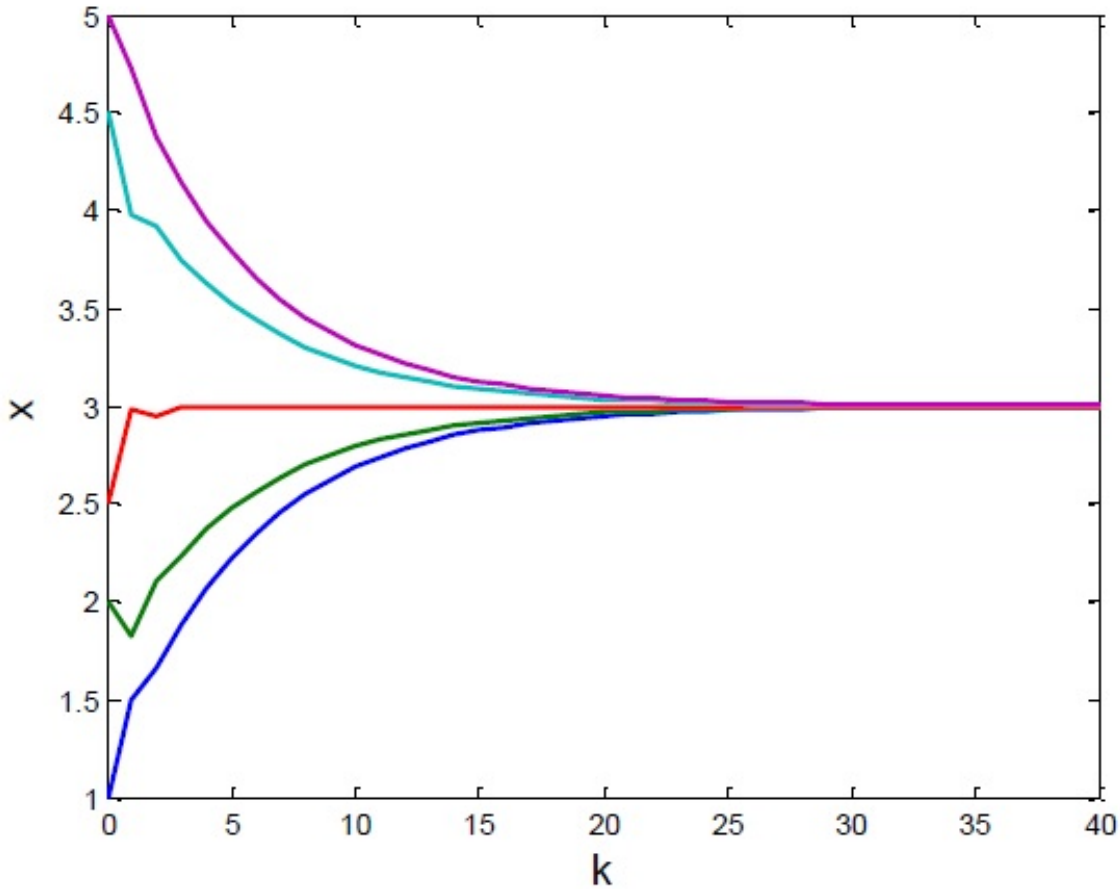
Example 1: distributed solution

$x(0) = [1 \quad 2 \quad 2.5 \quad 4.5 \quad 5]$ average = 3. It seems to work!?



Actually, in general, the estimate is not correct in all cases.

Example 1: what to retain

- Distributed processing: multiple "agents" cooperate locally to reach a common result.
- How to do the coordination?
- Does it converge?
- Performance evaluation: How close is the result from the centralized one?

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Example 2: bird formation fly

A classic example in distributed systems: Each bird decides its trajectory based on the position of its neighbors.

Example 3: wind energy harvesting

Seemingly isolated systems are actually coupled.

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Example 4: smart electric power systems

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Controller structures

- Centralized (multivariable) – Controller in a single central node;
- Decentralized – local controllers that act independently
- Distributed – local controllers coordinate with the neighbours

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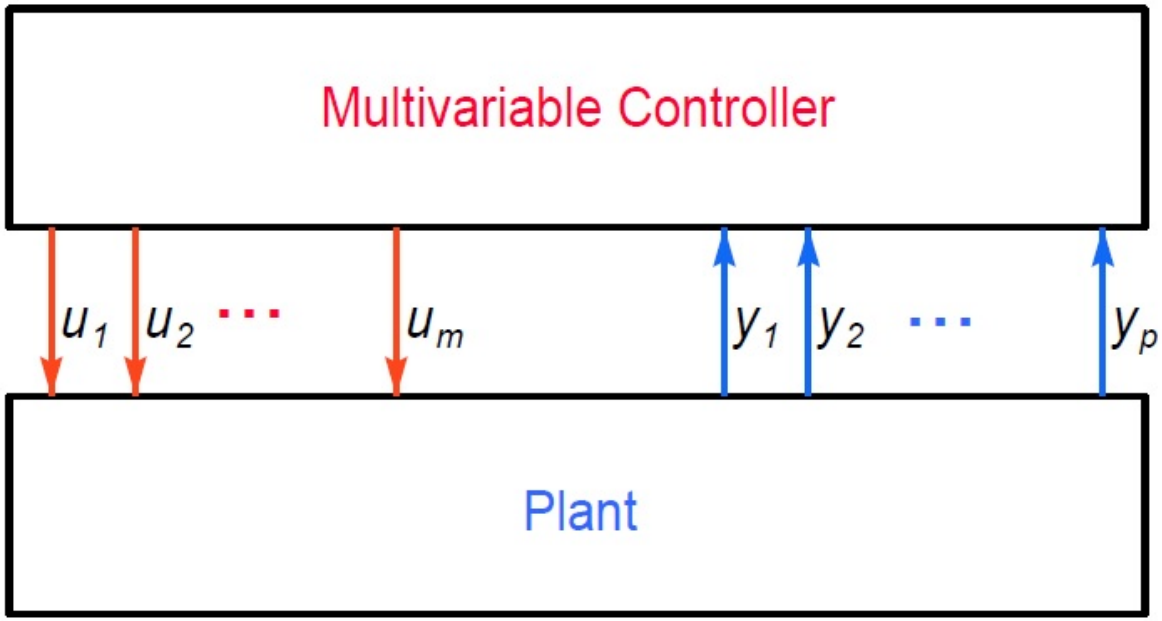
Controller structures: centralized (multivariable)



A single central controller receives all plant output signals and computes in a centralized way all the manipulated variables. Requires heavy communication.

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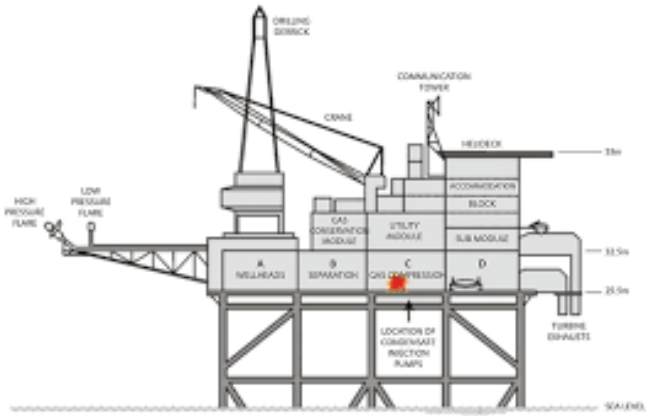
Centralized control: the Piper Alpha disaster

Piper Alpha was an oil drilling platform, in the North Sea, that exploded in 1988, killing 161 crew members.

The platform had a centralized computer controlled system that was considered to be able to contain any accident.

The first explosion occurred in a room that was separated from the computer by a thin metal wall. The computer was immediately destroyed, allowing the failures to propagate.



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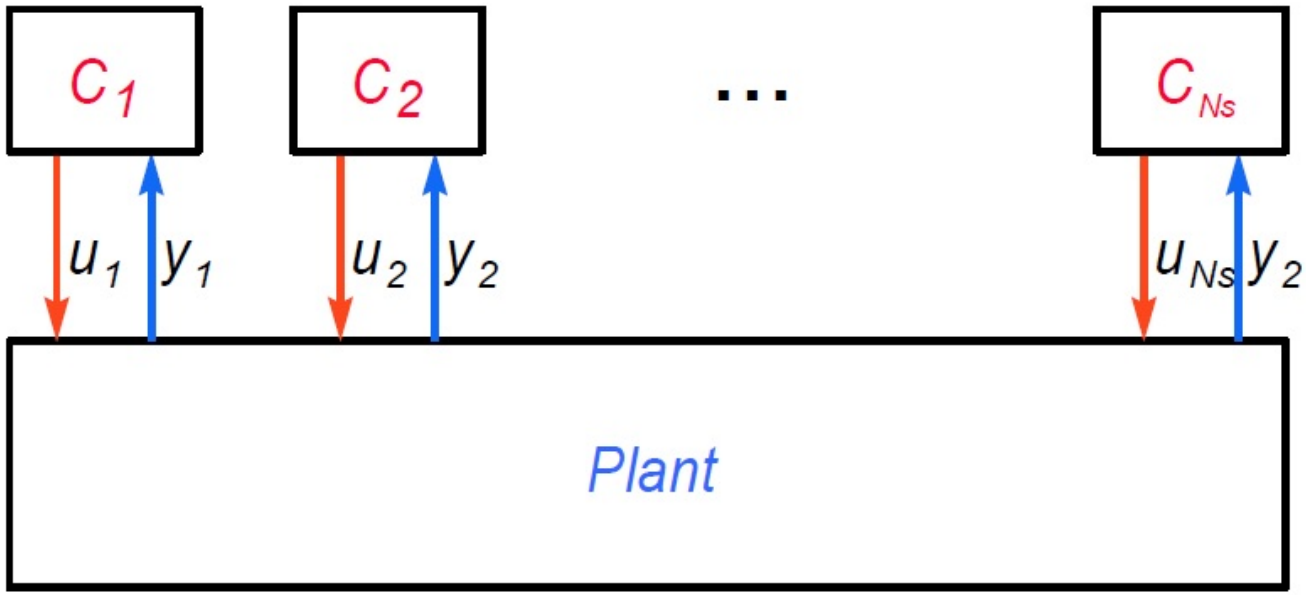
Controller structures: decentralized



A set of independent controllers, each one closing one feedback loop without any concern to the others. Stability problems.

Decentralized control: warning

Although each single loop has a good stability margin, the system controlled with decentralized control may be made unstable by small uncertainty terms.

See an example in

Doyle and Stein (1081). Multivariable Feedback Design: Concepts for a classical modern synthesis. IEEE TAC AC-26(1):4-16.

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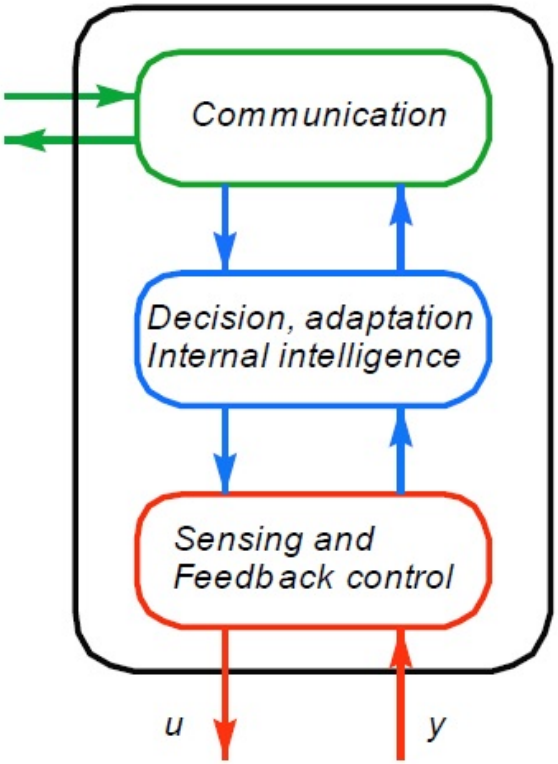
Control agents



A software entity capable of

- Communicating with other control agents
- Sensing and feedback control
- Changing the feedback control decisions depending on internal intelligence mechanism.

A step towards autonomous systems.

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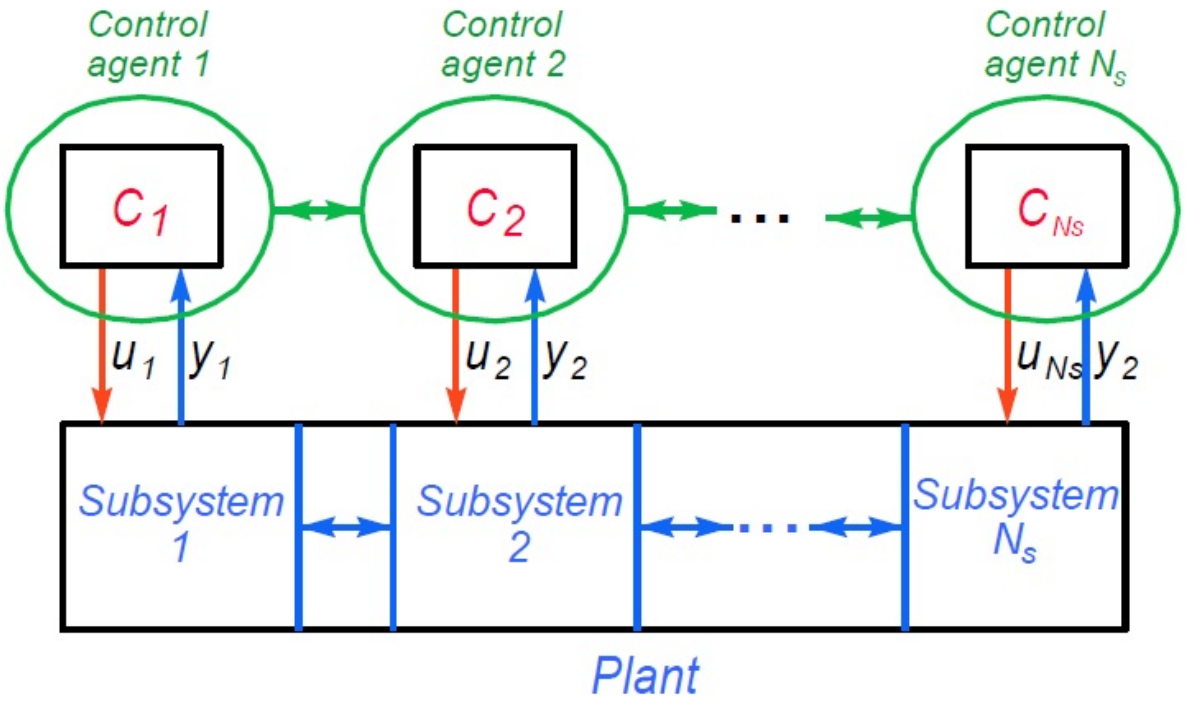
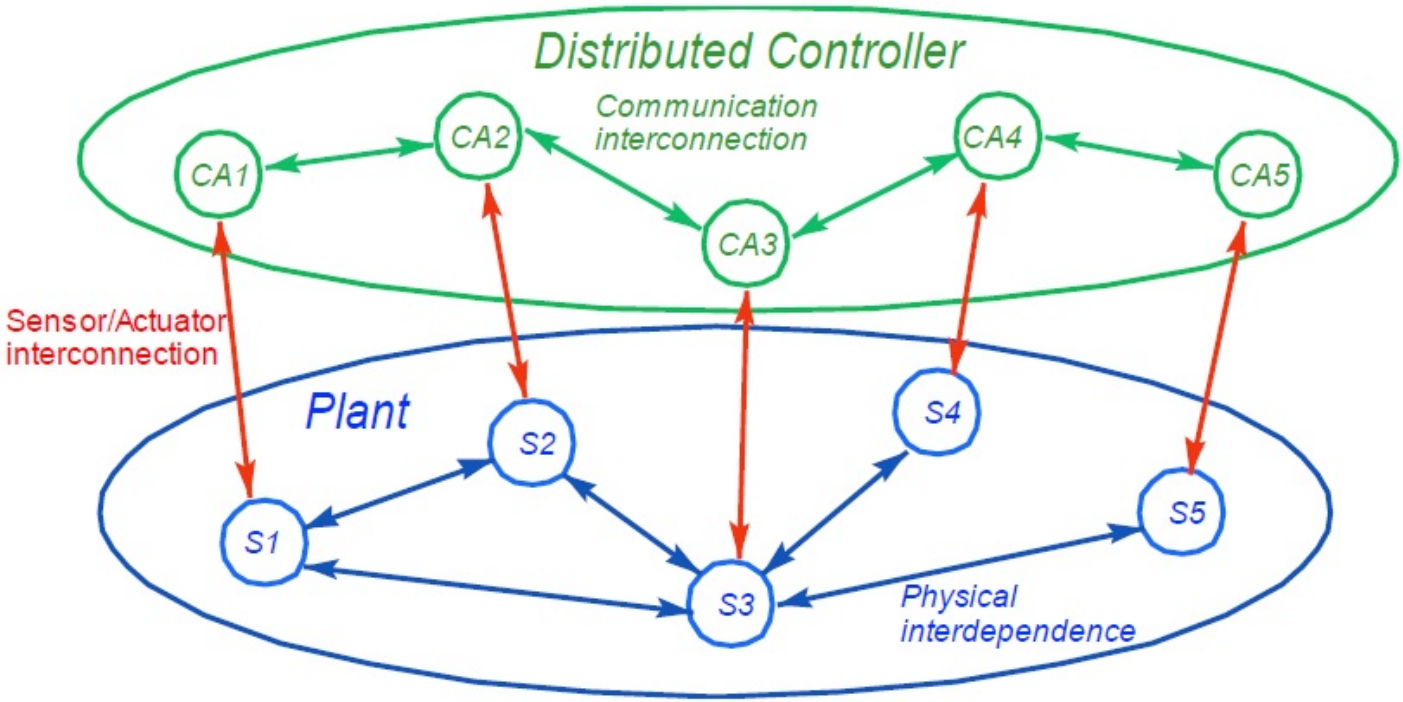
Controller structures: distributed



- Decompose the plant in subsystems.
- Encapsulate the local controllers in control agents.
- Create communication links.
- Define the algorithm that allows the control agents to act in a coordinated way.

Distributed control as an interconnection of control agents



Considering the plant/controller interconnection as a graph as a number of advantages: Results from graph theory; reconfiguration.

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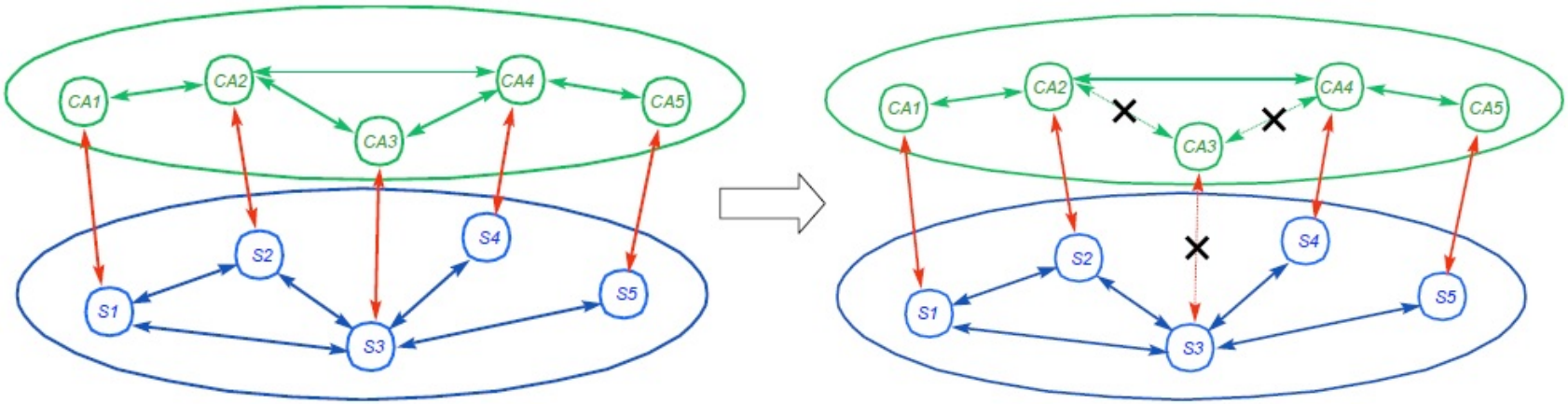
Controller reconfiguration



Controller reconfiguration in response to a partial fault.

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Centralized control: the Piper Alpha disaster

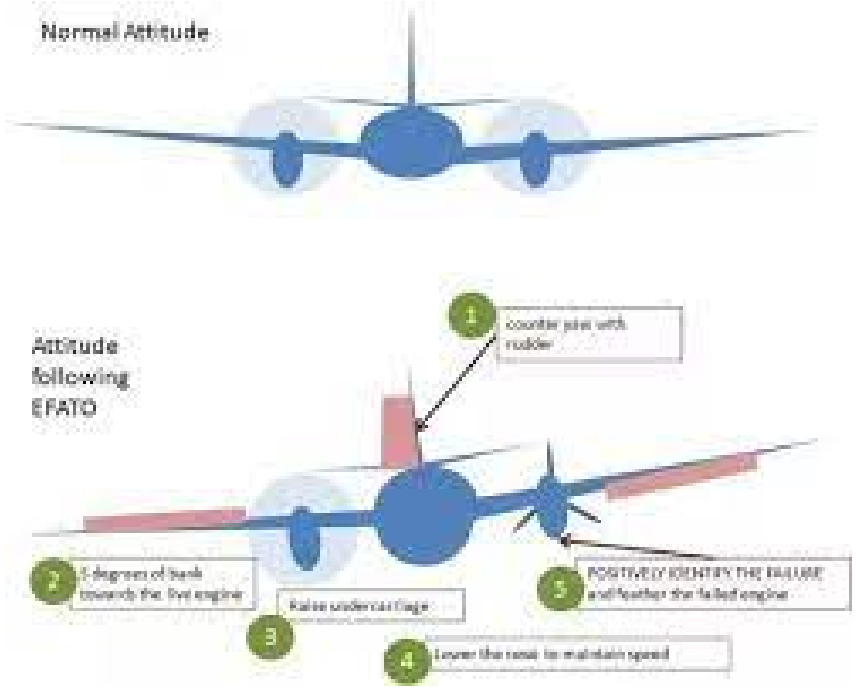
Change the pairing between actuators and sensors.

Change control objectives.

Reconfiguration is an important part of fault tolerant control.

MPC plays an important role in fault tolerant control (FTC).
Another important component of FTC: fault detection and isolation (FDI).

An example of reconfiguration in aircraft control



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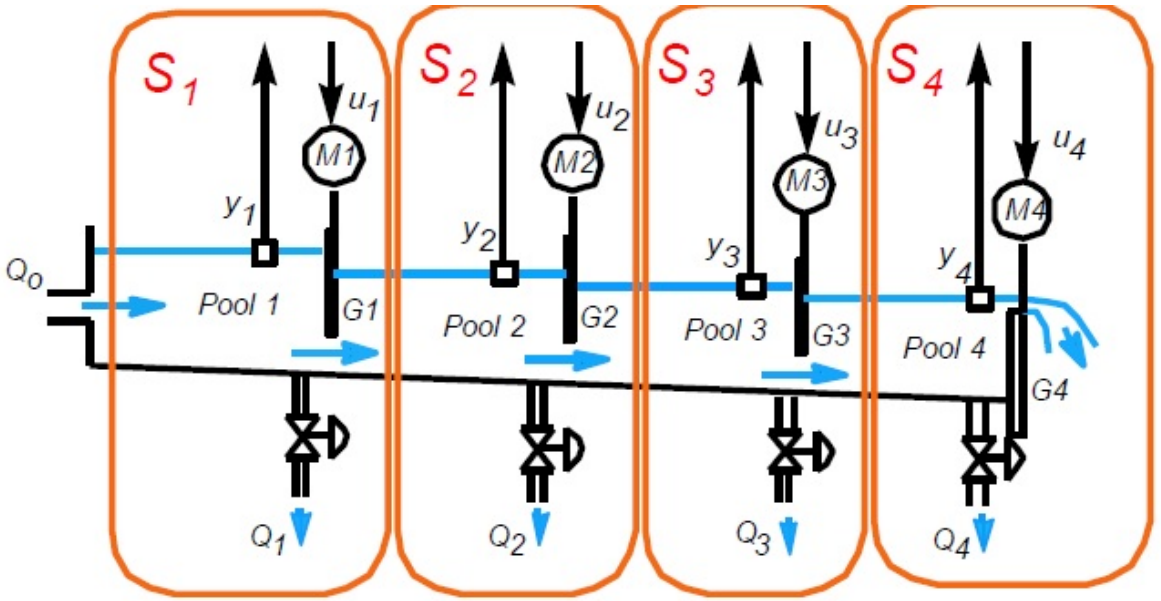
Example 5: decomposing the canal in subsystems



The canal is decomposed in a chain of subsystems.

Each subsystem comprises a pool, its downstream gate and an oftake.

Subsystems are physically coupled.

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Example 5: physical model of one stretch

The Saint-Venant equations - PDE embedding mass and momentum conserv.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l$$

$$\frac{\partial Q}{\partial t} + \frac{\partial Q^2/A}{\partial x} + gA\frac{\partial h}{\partial x} + gA(S_f - S_0) = kq_lV$$

- t time [s]
- x space [m]
- $A(x,t)$ flow cross sectional area [m^2]
- $Q(x,t)$ discharge [m^3s^{-1}]
- $q_l(x,t)$ lateral discharge per unit length [m^2s^{-1}]
- k constant
  inflow: $q_l > 0 \rightarrow k = 0$;
  outflow: $q_l < 0 \rightarrow k = 1$
- g gravitational acceleration [ms^{-2}]
- $h(x,t)$ water level [m]
- $S_f(x,t)$ friction slope
- $V(x,t)$ water velocity [ms^{-1}]

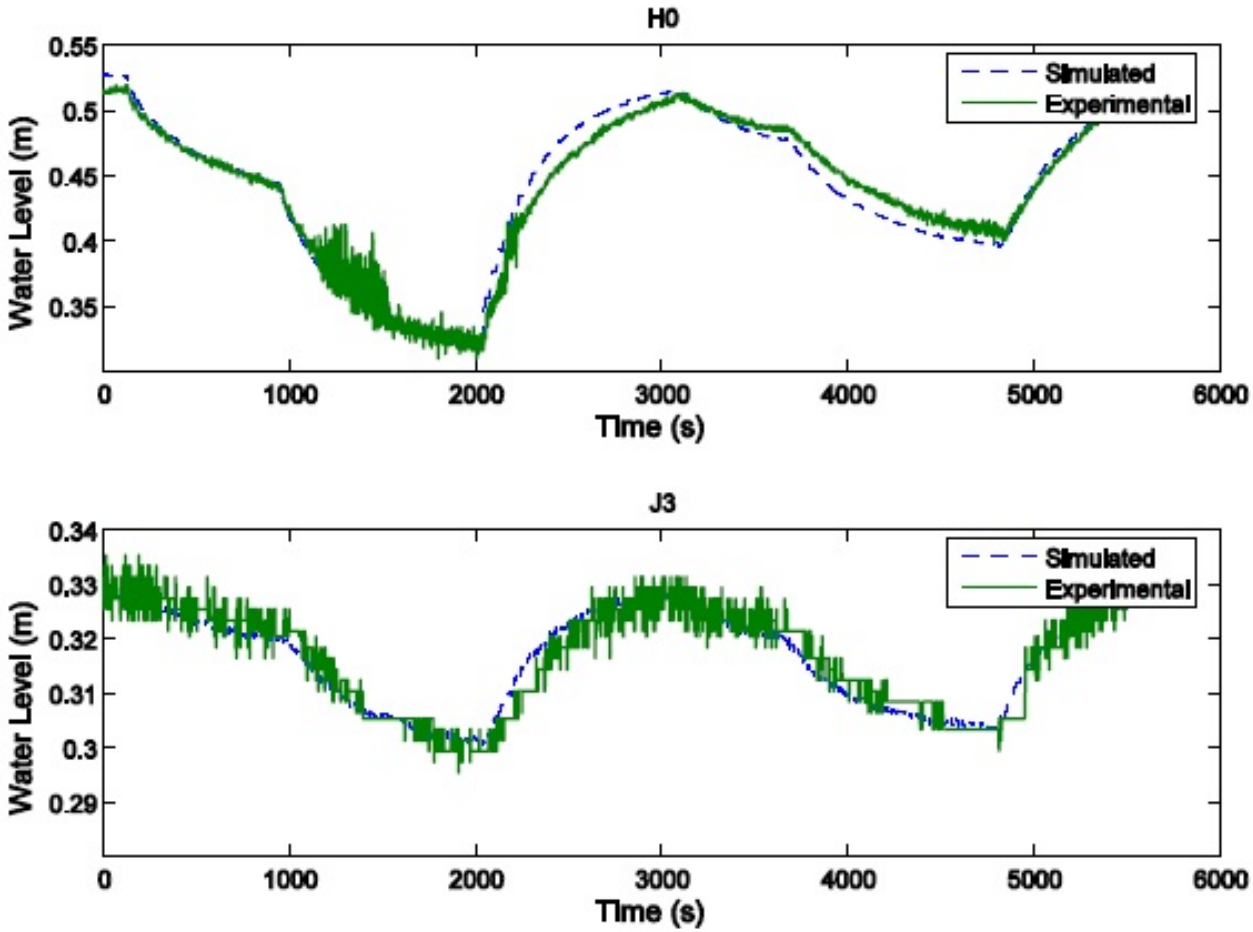
The friction is modeled using the Manning-Strickler formula:

$$S_f = \frac{Q|Q|n^2}{A^2R^{4/3}} \quad (3)$$

where:

- $R(x,t)$ hydraulic radius [m]
- n Manning coefficient [$m^{-1/3}s$]

Example 5: experimental results and nonlinear model



Numeric integration with Preissman method.

Example 5: model decomposition

Sub-systems interact only through their manipulated inputs

$$x(k+1) = Ax(k) + Bu(k) + \Gamma d(k) \qquad y(k) = Cx(k)$$

Matrix A is imposed to be block diagonal:

$$A = \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 & 0 \\ B_{21} & B_{22} & B_{23} & 0 \\ 0 & B_{32} & B_{33} & B_{34} \\ 0 & 0 & B_{43} & B_{44} \end{bmatrix}$$

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Local subsystem dynamics with interaction only through the input

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \Gamma_i \delta_i(k)$$

$$\delta_i(k) = \begin{bmatrix} d_i(k) & u_{i-1}(k) & u_{i+1}(k) \end{bmatrix}^T$$

$$y_i(k) = C_i x_i(k)$$

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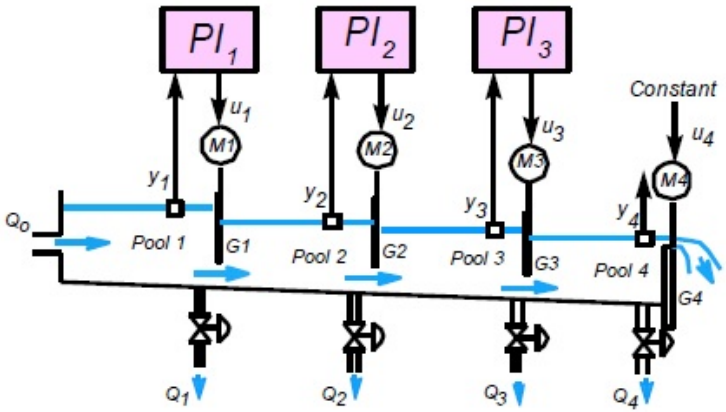
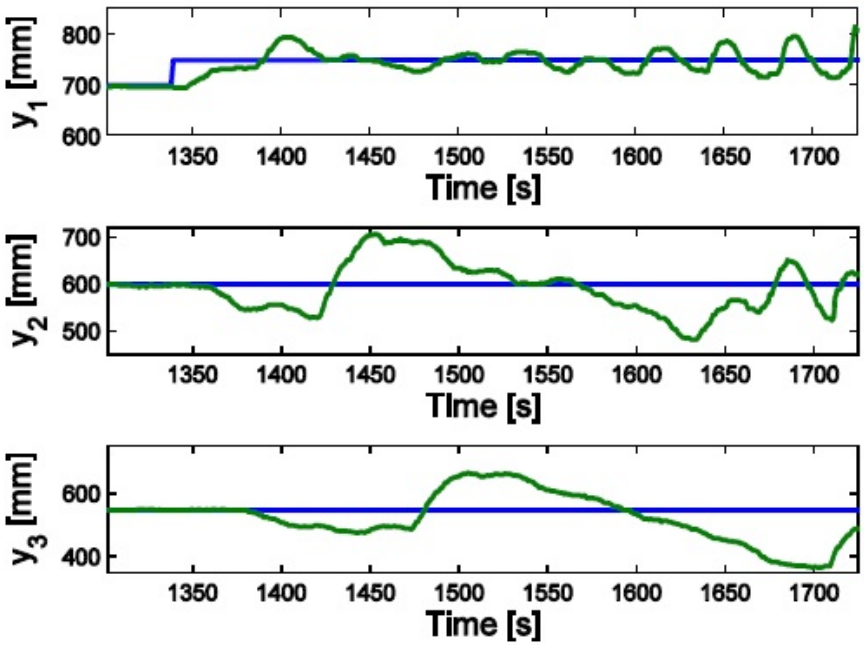
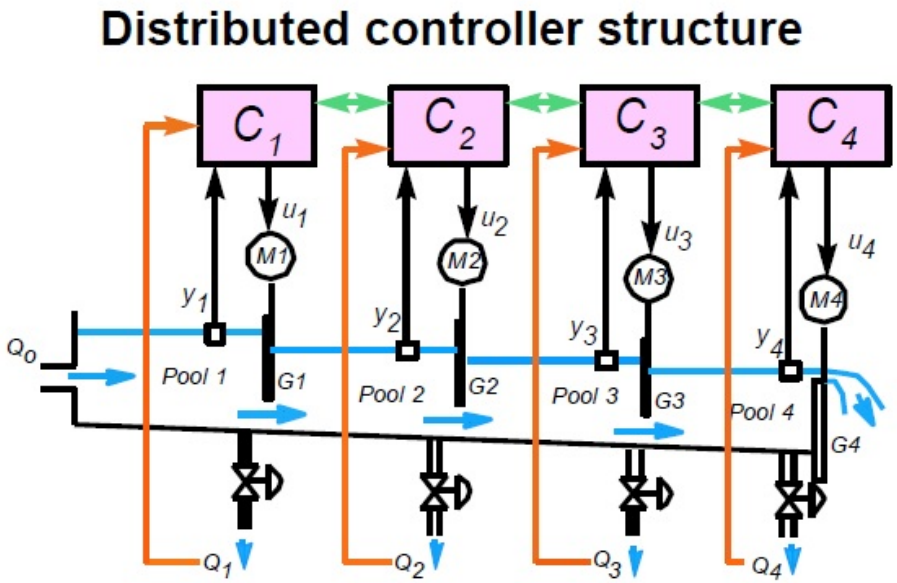
Does it work with decentralized PI controllers?





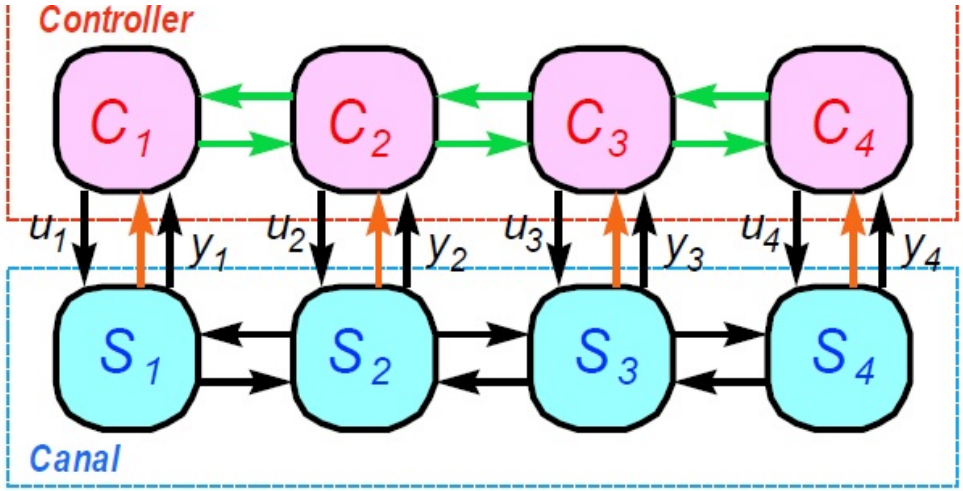
May easily become unstable!

Distributed control structure



A chained network of upstream local level controllers, each one communicating only with their neighbors, exchanging information about their decisions on the manipulated and output variables.

Graph of the distributed control structure



How to design:

- The controllers?
- The coordination algorithm?

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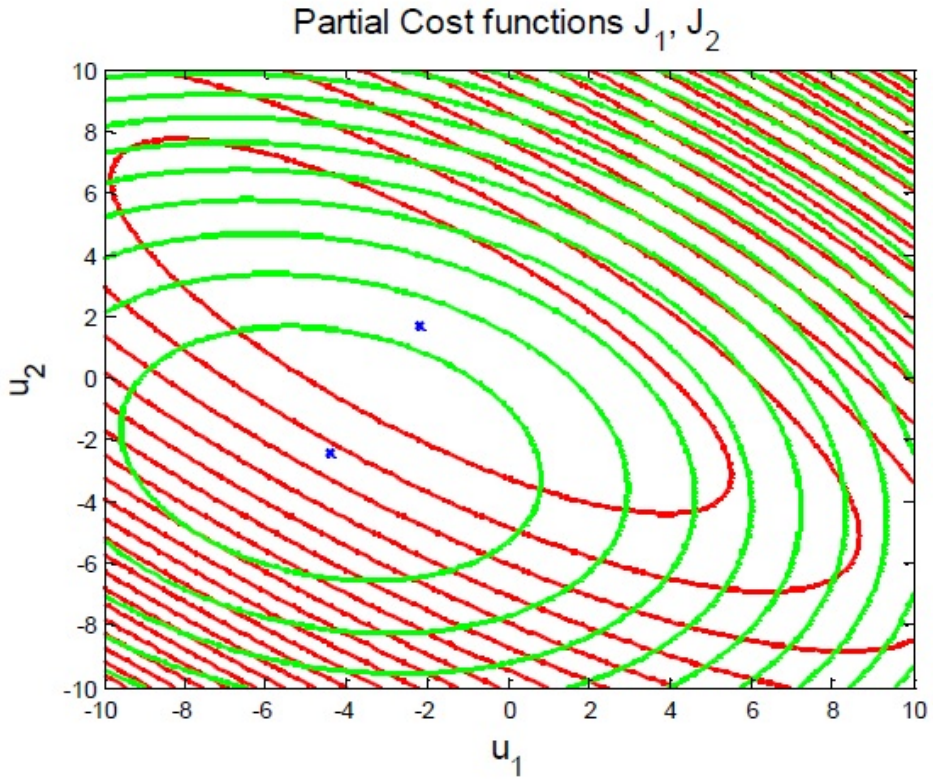
Multiobjective optimization



Find the minimum of a global cost function made by the addition of local cost functions:

$$J(u_1,u_2) = J_1(u_1,u_2) + J_2(u_1,u_2)$$

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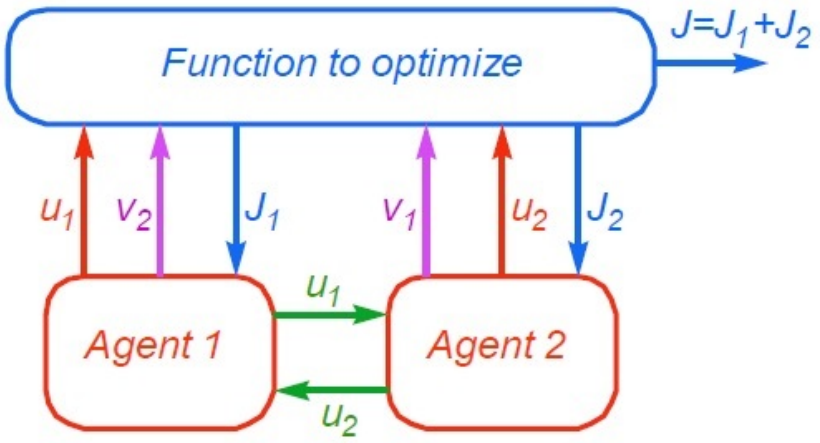
Distributed optimization



$$\min_{u_1,u_2}\left[J_1(u_1,u_2)+J_2(u_1,u_2)\right]$$

Replicate the independent variables for each agent.

Agent 1 minimizes $J_1(u_1,v_2)$ where v_2 is a replica of u_2.

Agent 2 minimizes $J_2(v_1,u_2)$ where v_1 is a replica of u_1.

A mechanism to reconcile v_1 with u_1 and v_2 with u_2 is needed.

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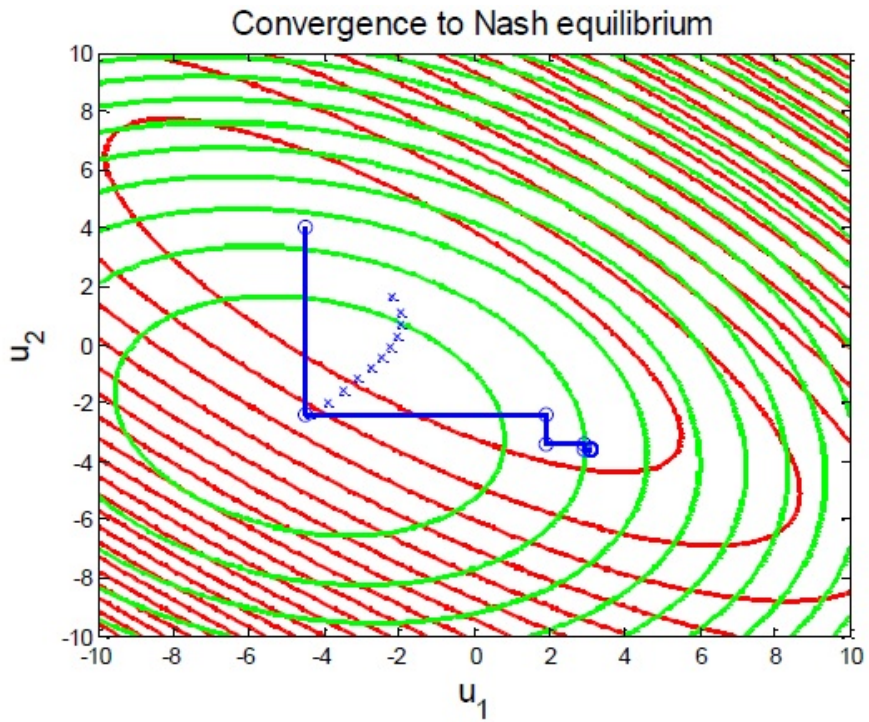
Distributed optimization: game approach

Convergence to the Nash equilibrium of alternating variable minimization.



Iterate the steps:

1) Optimize $J_1(u_1,u_2)$ with respect to u_1, keeping u_2 constant;

2) Optimize $J_2(u_1,u_2)$ with respect to u_2, keeping u_1 constant.

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Who invented the Nash equilibrium?

The movie-picture *A beautiful mind* tells the story of John Forbes Nash, Jr., winner of the Nobel Prize in Economics of 1994, who introduced the Nash equilibrium in the context of Economics.

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The dual problem

Primal problem: $\min_u J(u)$ subject to $Au - b = 0$

Lagrangian: $L(u, \lambda) = J(u) + \lambda^T (Au - b)$

Dual function: $g(\lambda) = \inf_u L(u, \lambda)$

Dual problem: $\lambda^* = \arg\max g(\lambda)$

$$u^* = \arg\min_u L(u, \lambda^*)$$

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Dual approach to distributed optimization

$$\min_{u_1,u_2}\left[J_1(u_1,u_2)+J_2(u_1,u_2)\right]$$

Decomposition

Agent 1

$$\min_{u_1,v_2} J_1(u_1,v_2)$$

Subject to $v_2 = u_2$

Agent 2

$$\min_{v_1,u_2} J_2(v_1,u_2)$$

Subject to $v_1 = u_1$

$$\max_{\lambda_i}\min_{v_i,u_i}\left[J_1(u_1,v_2)+J_2(v_1,u_2)+\lambda_1(u_2-v_2)+\lambda_2(u_1-v_1)\right]$$

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Dual approach to distributed optimization (cont.)

$$\max_{\lambda_i} \min_{v_i, u_i} \left[J_1(u_1, v_2) + J_2(v_1, u_2) + \lambda_1 (u_2 - v_2) + \lambda_2 (u_1 - v_1) \right]$$

Agent 1

$$\min_{u_1, v_2} \left[J_1(u_1, v_2) - \lambda_1 v_2 + \lambda_2 u_1 \right]$$

$$\max_{\lambda_1} \left[\lambda_1 (u_2 - v_2) \right]$$

Agent 2

$$\min_{v_1, z_2} \left[J_2(v_1, u_2) - \lambda_{21} v_1 + \lambda_1 u_2 \right]$$

$$\max_{\lambda_2} \left[\lambda_2 (u_1 - v_1) \right]$$

Adjustment of multipliers ensure coordination.

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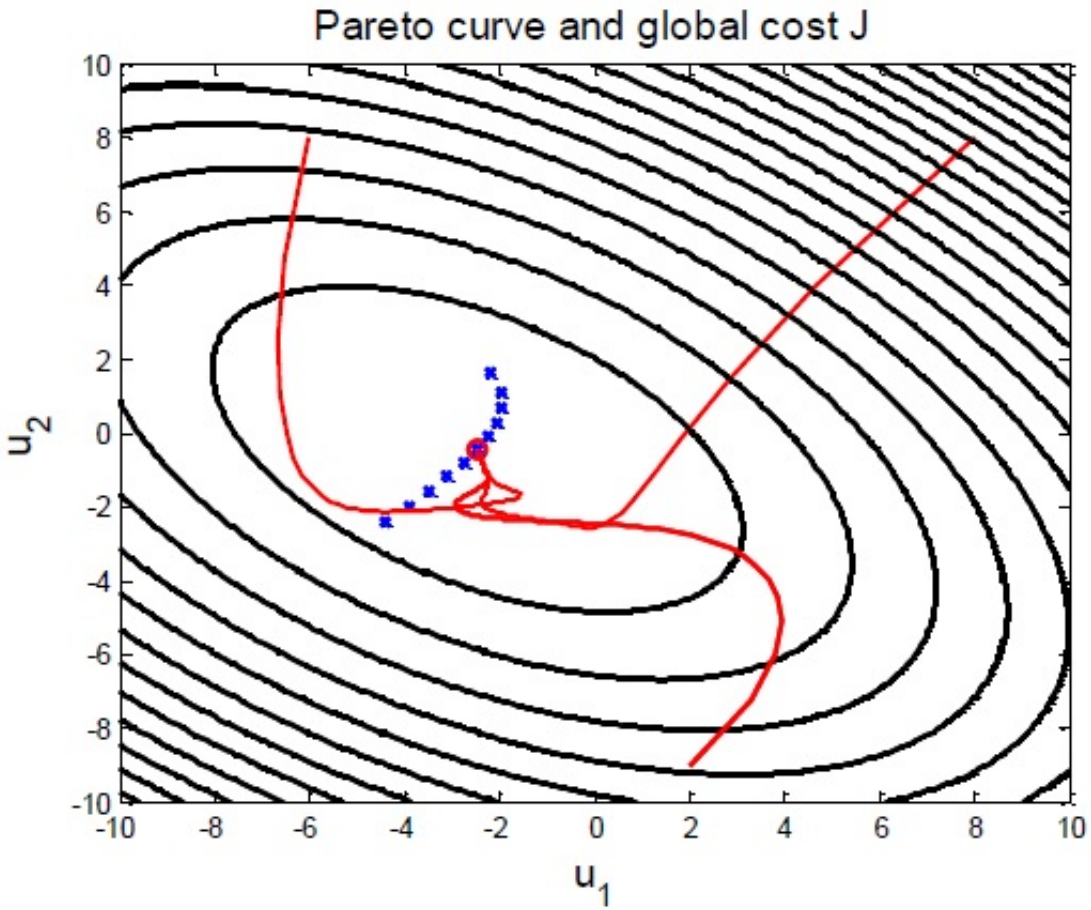
Dual approach to distributed optimization



ADMM – Alternating directions method of multipliers

Minimize $f(x)+g(z)$ subject to $x=z$

Augmented Lagrangian $L(x,z,\lambda)=f(x)+g(x)+\lambda^T(x-z)+\frac{\rho}{2}\|x-z\|^2$

Recursively execute the steps:

1) $x(k+1)=\arg\min_x L(x,\lambda(k),z(k))$

2) $\lambda(k+1)=\lambda(k)+\rho(x(k+1)-z(k))$

3) $z(k+1)=\arg\min_z L(x(k+1),\lambda(k+1),z)$

Distributed MPC with an altruistic cost

Each local control agent minimizes a quadratic cost that considers not only its tracking error but also the tracking errors of the neighbors:

$$J_1 = \sum_{i=1}^{N} \left[e_1^2(k+i) + e_2^2(k+i) + \rho_1 \left(\Delta u_1(k+i-1) \right)^2 \right]$$

$$J_2 = \sum_{i=1}^{N} \left[e_1^2(k+i) + e_2^2(k+i) + e_3^2(k+i) + \rho_2 \left(\Delta u_2(k+i-1) \right)^2 \right]$$

$$J_2 = \sum_{i=1}^{N} \left[e_2^2(k+i) + e_3^2(k+i) + \rho_1 \left(\Delta u_3(k+i-1) \right)^2 \right]$$

$e_i(k) = r_i(k) - y_i(k)$ is the tracking error for the level in pool i at time k.

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Stability constraints

Each local cost *i* is optimized under the following terminal constraints:

$$y_i(k+N+j) = r_i(k+N+j) \quad \text{for} \quad j = 1, \ldots, P$$

$N+P$ is the prediction horizon and P is the number of coincidence points.

These constraints ensure stability of the local control loops taken in isolation if $P \geq \dim(x)$ but do not ensure per se the stability of the overall system.

With additional assumptions it is possible to ensure stability of the overall system.

Selecting N and P determines the type of response, as will be shown.

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Back to the game approach: coordination procedure

At the beginning of each sampling interval execute the coordination recursive procedure

1. Initialize the manipulated variable for each control agent;
2. For each control agent optimize its local cost, given knowledge of the manipulated variable of its neighbors in the preceding iteration, that appear as feedforward variables;
3. If a number N_c of iterations that ensures convergence is performed, then stop. Otherwise, go to step 1.

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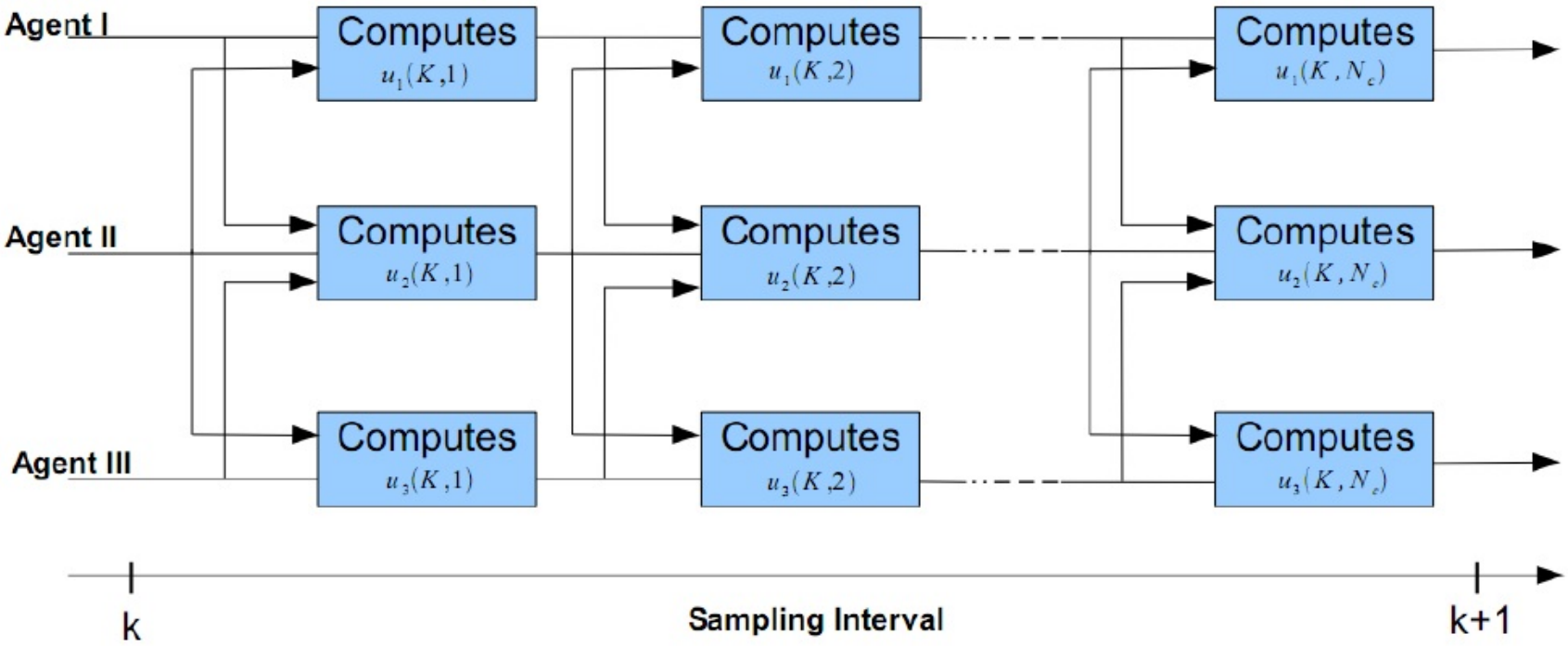
Time evolution of the coordination procedure



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Convergence of the coordination procedure

From iteration $l-1$ to iteration l, in each sampling interval k, the coordination procedure progresses as the state of a linear system:

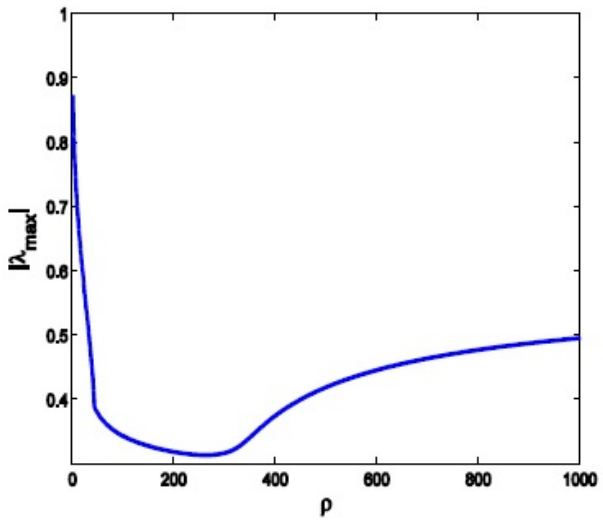
$$u(k,l) = \Xi u(k,l-1) + \Psi$$

where Ξ is a matrix and Ψ is a vector.

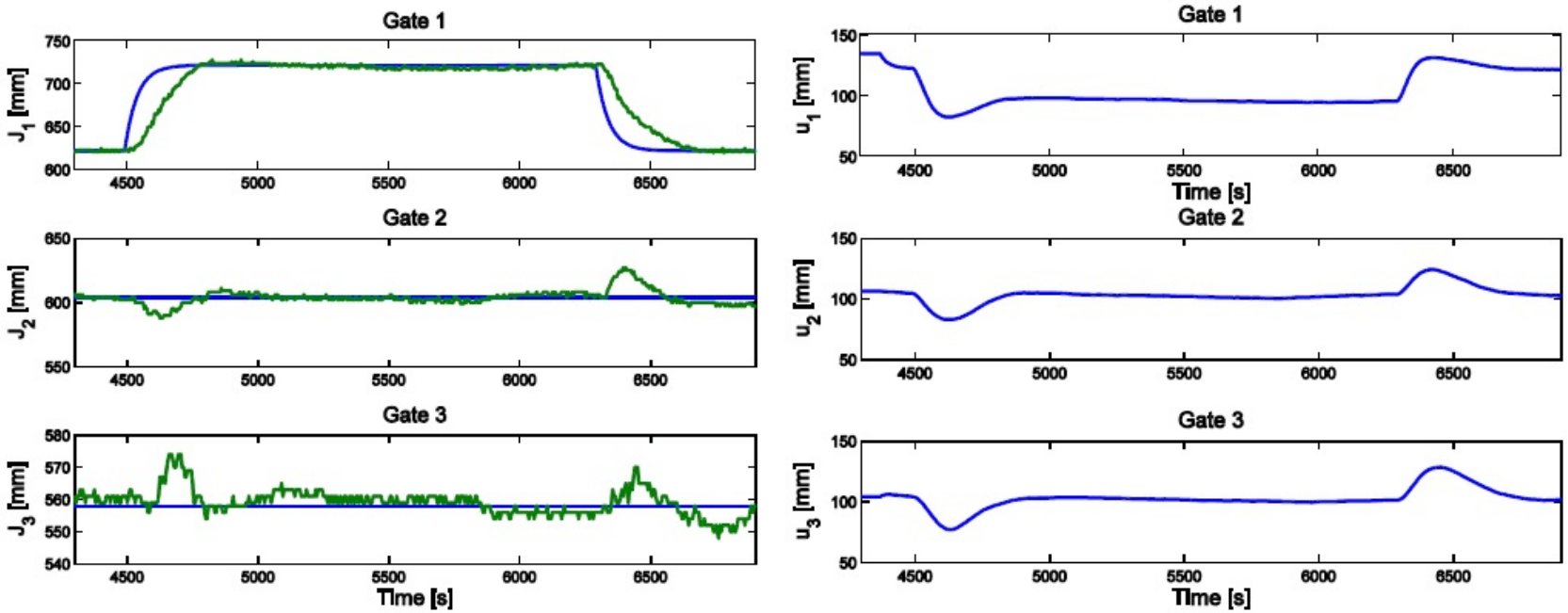
Converges if $\max\left(eig\left(\Xi\right)\right) < 1$.

This spectral radius is affected by the control penalty weights ρ.

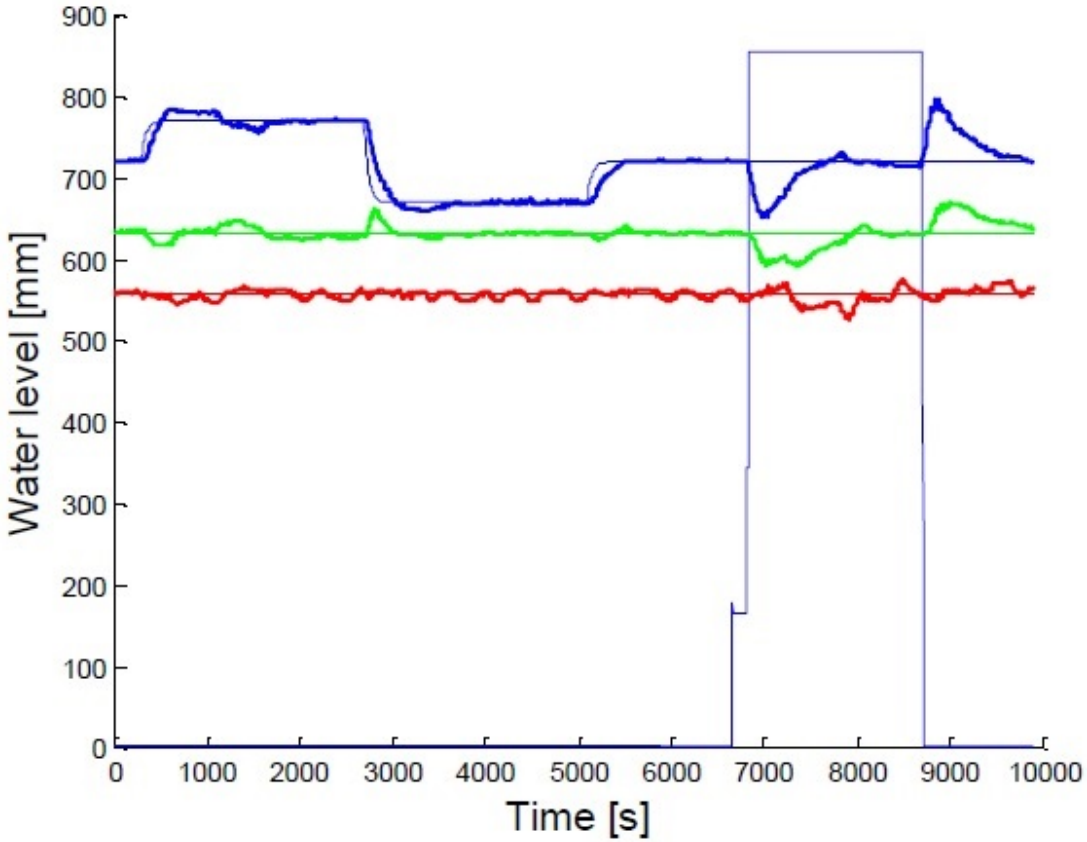
If all the ρ are equal:



Experimental results (1)



Experimental results (2)



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