Física I

LEIC-T 2021-2022

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6ª Aula

Momento de Força e variação do Momento Angular;
A equação para o pêndulo simples e para o pêndulo físico;
Equilíbrio estático;
Forças centrais e Conservação do Momento Angular;
Energia potencial efetiva e órbitas de transferência;
Leis de Kepler do Movimento Planetário.

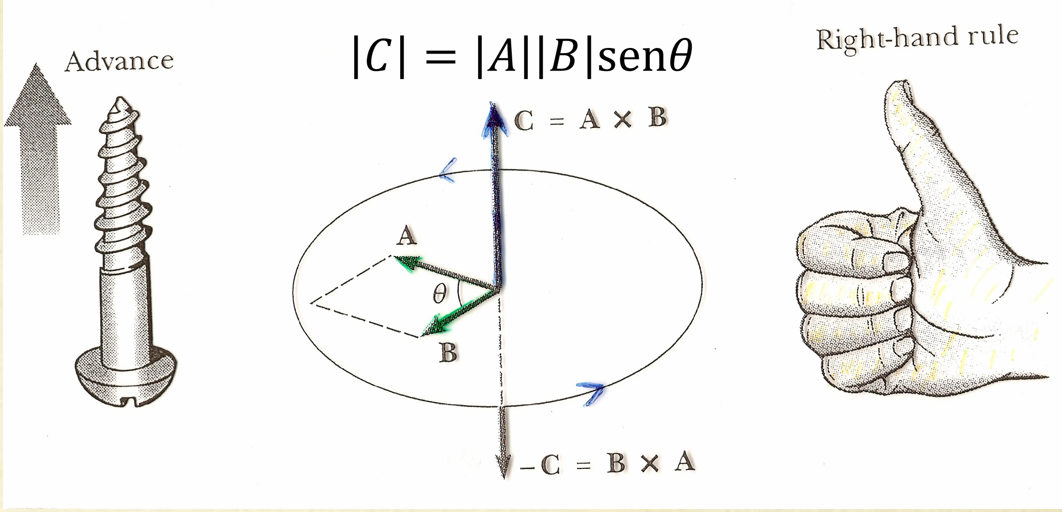
"A viagem real de descoberta não consiste em procurar novas paisagens mas em ter uma nova visão!"
Marcel Proust (1871-1922)

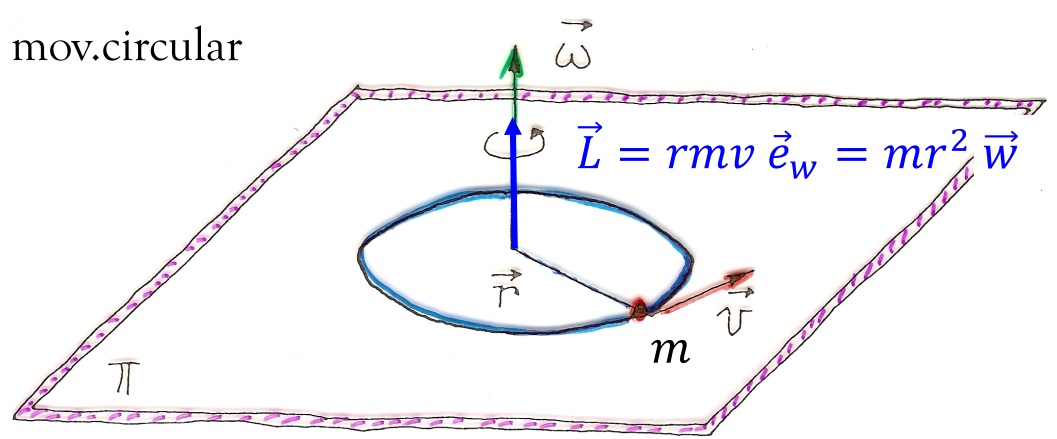
O Momento Angular (recapitulação)

$$\vec{L} = \vec{r} \times \vec{p} \quad [\text{kgm}^2/\text{s}]$$

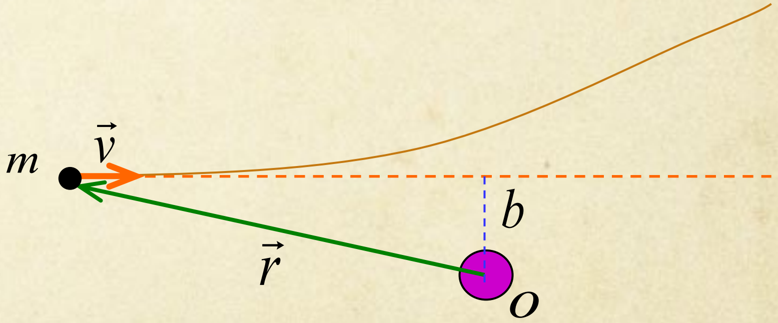
$$|\vec{L}| = |\vec{r}_i \times \vec{p}_i| \equiv r_i p_i \operatorname{sen}\theta$$

$$\vec{L} \perp \vec{r}, \vec{p}$$





$$\vec{L} = \vec{r} \times m\,\vec{v} = mvb\,\vec{e}_\perp$$



Rotação no plano (corpo pontual)

n massas pontuais:

$$\vec{L}_O = \sum \vec{r}_{i_O} \times \vec{p}_i$$

Variação do Momento Angular

$$\vec{L} = \sum \vec{r}_{\mathrm{i}} \times \vec{p}_i$$

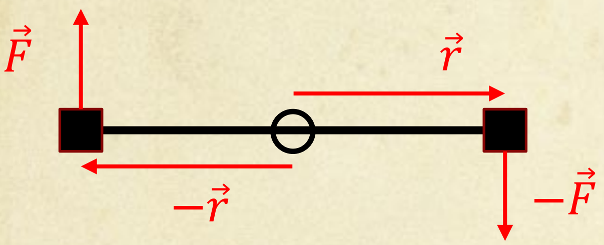
(Momento de Força, *torque*)

$$\frac{d\vec{L}}{dt} = \sum \vec{r}_{\mathrm{i}} \times \vec{F}_i = \vec{N} = \vec{\tau}$$

[N.m]

Binário:

(centro fixo)



$\sum \vec{F}_{\mathrm{i}} = 0$ mas

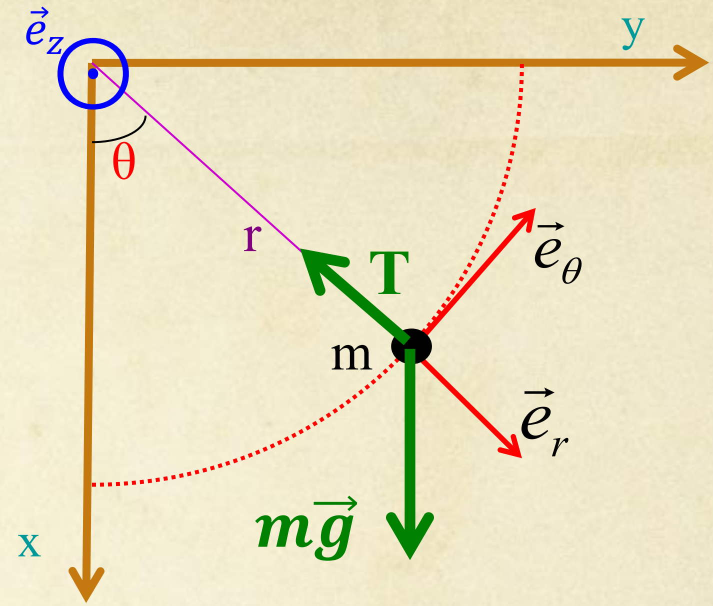
$$\sum \vec{r}_{\mathrm{i}} \times \vec{F}_i = rF\vec{e}_{\otimes} + rF\vec{e}_{\otimes} = 2rF\vec{e}_{\otimes}$$

$\vec{N} = \vec{\tau} = 2rF\vec{e}_{\otimes} \neq 0 \Rightarrow d\vec{L} = 2rFdt\vec{e}_{\otimes}$: Existe Rotação em torno do eixo (centro)

Momento do Binário *(ou simplesmente, Binário!)*

Pêndulo simples revisitado

$$\vec{N} \equiv \frac{d\vec{L}}{dt}$$



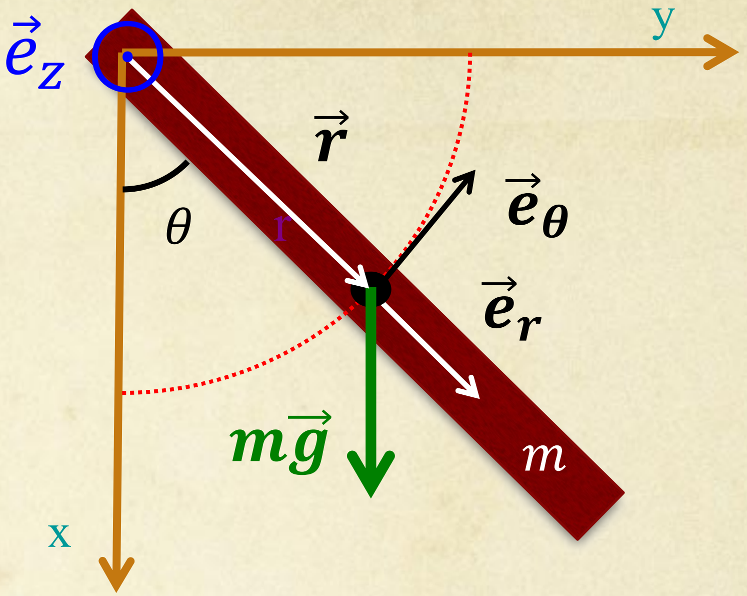
$$\vec{L} = \vec{r} \times \vec{p} = mr^2 \dot{\theta} \ \vec{e}_z$$

$$\vec{N} = \vec{r} \times (\vec{T} + m\vec{g}) = mr^2 \ddot{\theta} \ \vec{e}_z \Leftrightarrow$$

$$-r\, mg \operatorname{sen} \theta \ \vec{e}_z = mr^2 \ddot{\theta} \ \vec{e}_z \Leftrightarrow \boxed{\ddot{\theta} + \frac{g}{r} \operatorname{sen} \theta = 0}$$

Pêndulo Físico (barra presa num extremo)

$$\vec{N} \equiv \frac{d\vec{L}}{dt}$$

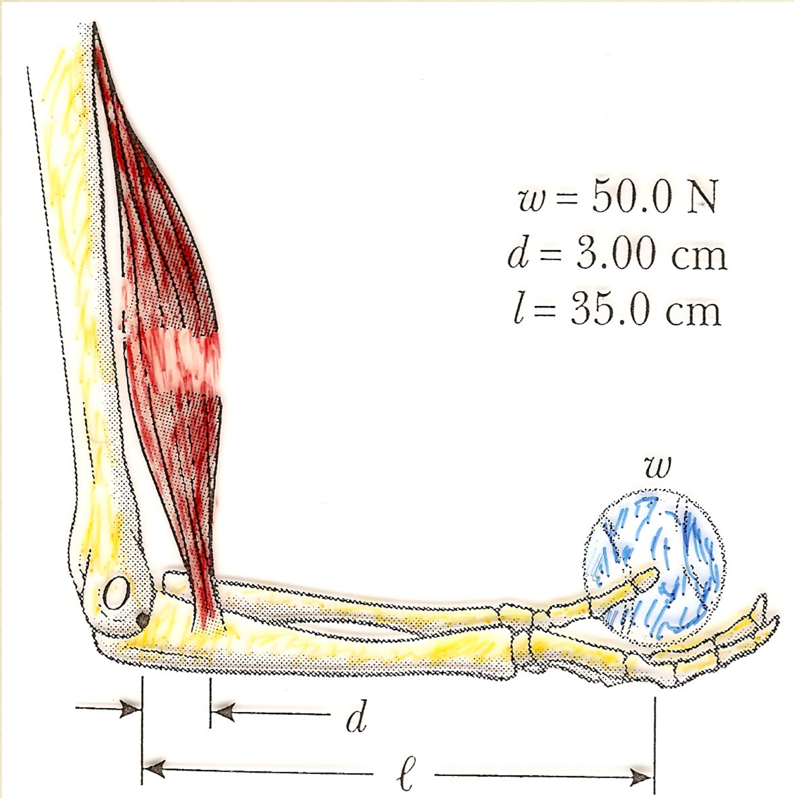


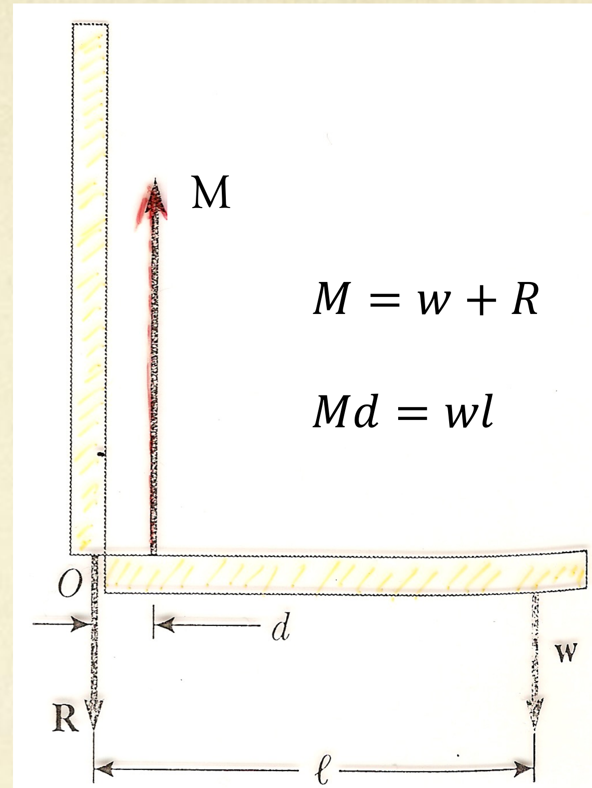
$$\vec{L} = I_O\vec{\omega} = \left(\frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2\right)\dot{\theta}\vec{e}_z$$

$$\vec{N} = \vec{r}\times m\vec{g} = \frac{ml^2}{3}\ddot{\theta}\ \vec{e}_z \Leftrightarrow$$

$$-\frac{l}{2}mg\,\mathrm{sen}\,\theta\ \vec{e}_z = \frac{ml^2}{3}\ddot{\theta}\ \vec{e}_z \Leftrightarrow \boxed{\ddot{\theta} + \frac{3g}{2l}\mathrm{sen}\,\theta = 0}$$

Equilíbrio estático





$$\sum \vec{F} = 0 \quad \text{e} \quad \sum \vec{N} = 0$$

Forças centrais

$$\vec{F} \parallel \vec{r}$$

$$\frac{d\vec{L}}{dt} = \vec{r}\times\vec{F} = 0$$

$$\vec{L} = \text{cte} = mr^2\dot{\theta}\vec{e}_w$$

$$E_C = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2\ (^*)$$

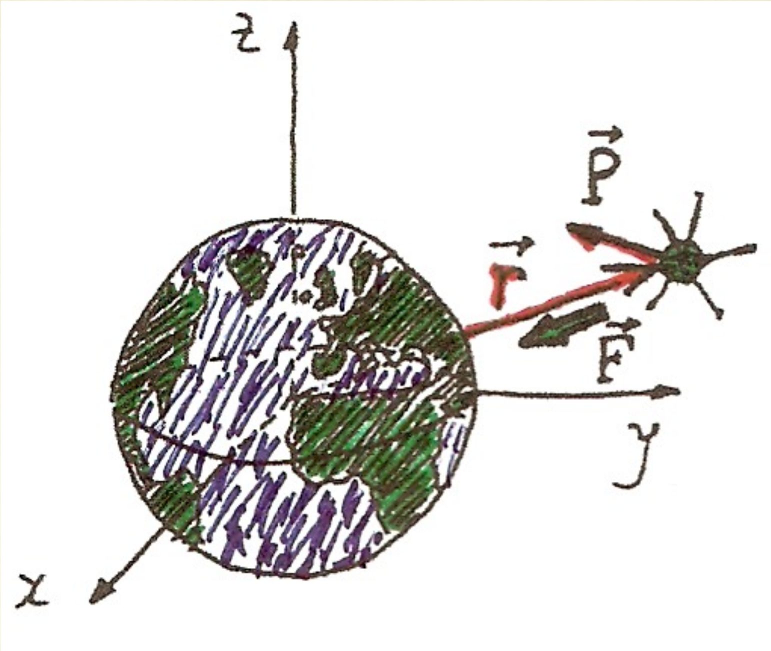
$$E = E_C + E_P$$

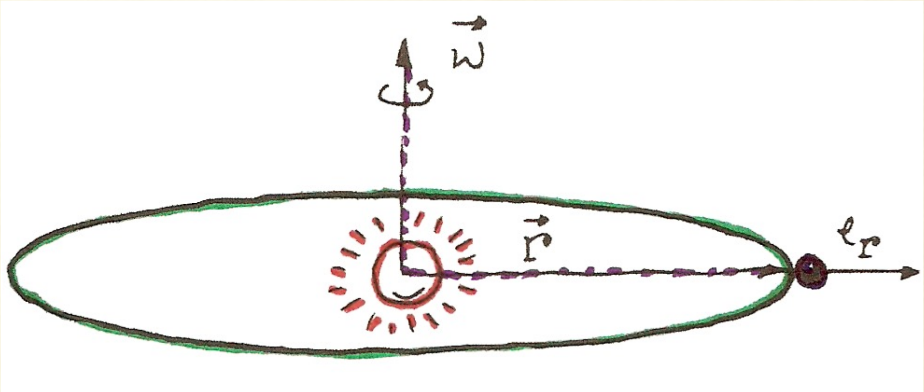
$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} - G_N\frac{Mm}{r}$$

$$E_{P_{\text{efectiva}}} = U(r)$$

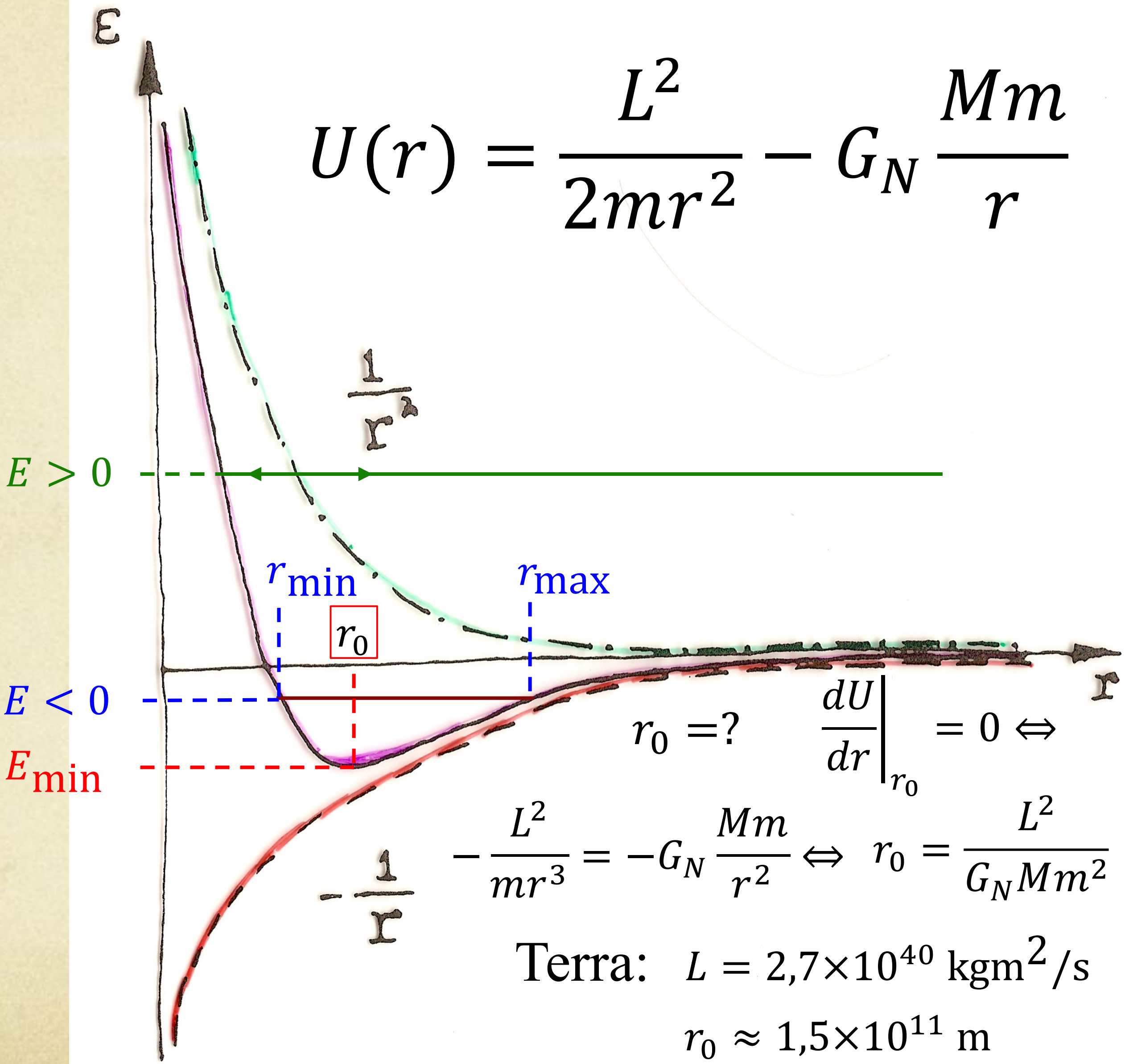
$(^*)\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta \Leftrightarrow$

$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$





Energia potencial efetiva



$$U(r) = \frac{L^2}{2mr^2} - G_N \frac{Mm}{r}$$

$$r_0 = ? \qquad \left.\frac{dU}{dr}\right|_{r_0} = 0 \Leftrightarrow$$

$$-\frac{L^2}{mr^3} = -G_N \frac{Mm}{r^2} \Leftrightarrow r_0 = \frac{L^2}{G_N M m^2}$$

Terra: $L = 2{,}7\times 10^{40}\ \text{kgm}^2/\text{s}$

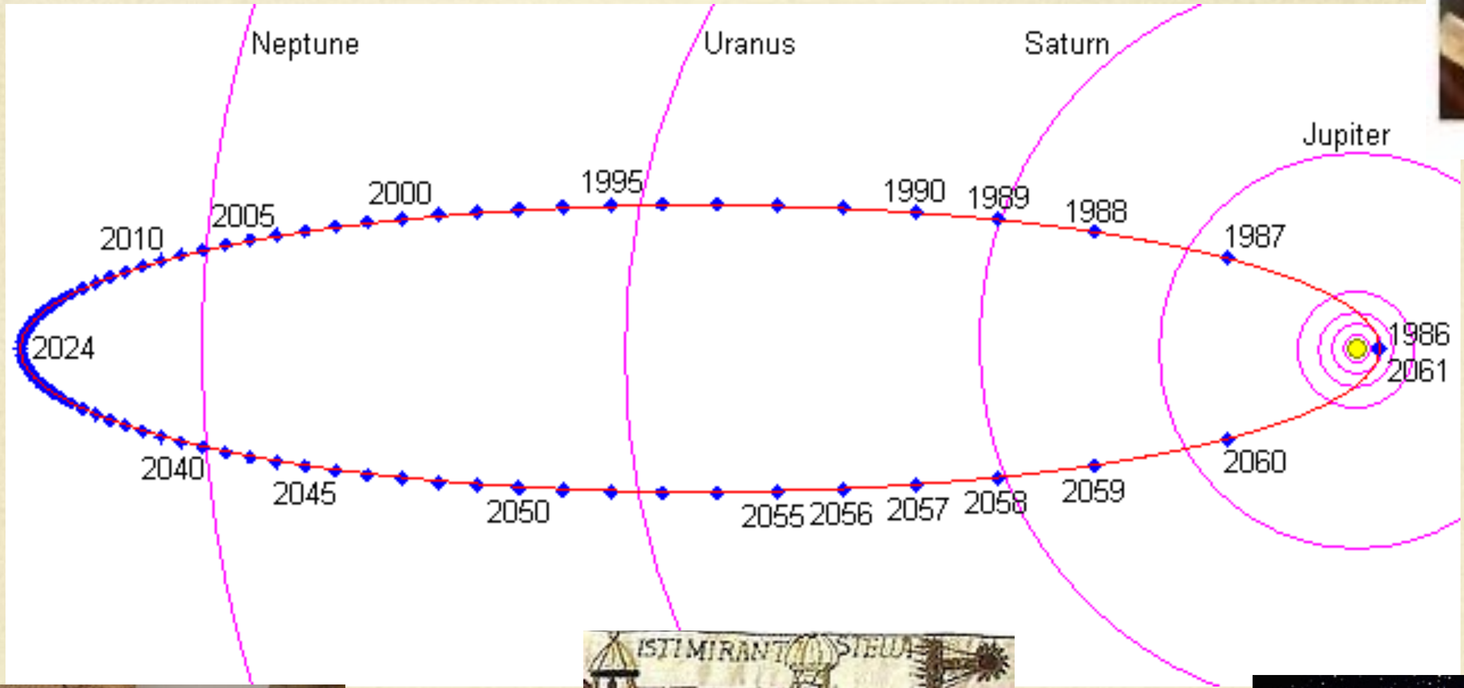
$r_0 \approx 1{,}5\times 10^{11}\ \text{m}$

O cometa Halley

T ~ 75 anos

Edmond Halley

1656-1742



164 AC

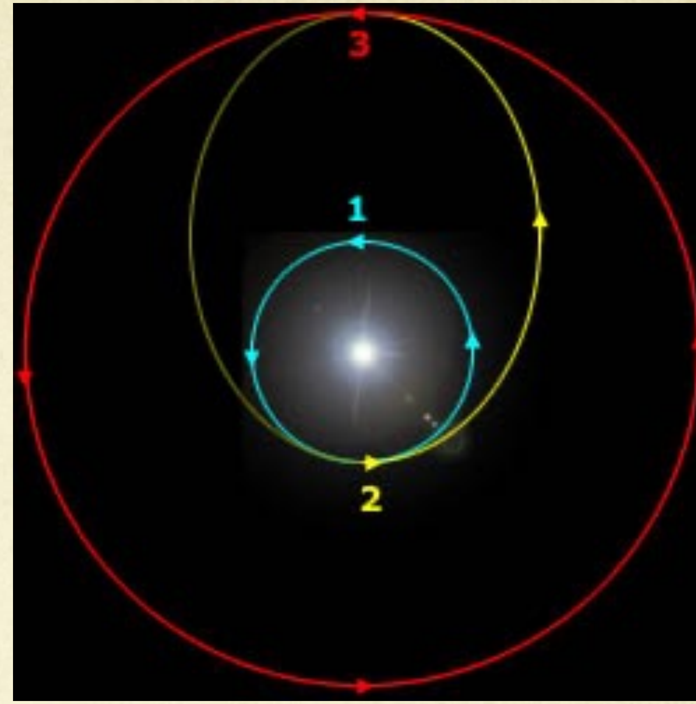
1066

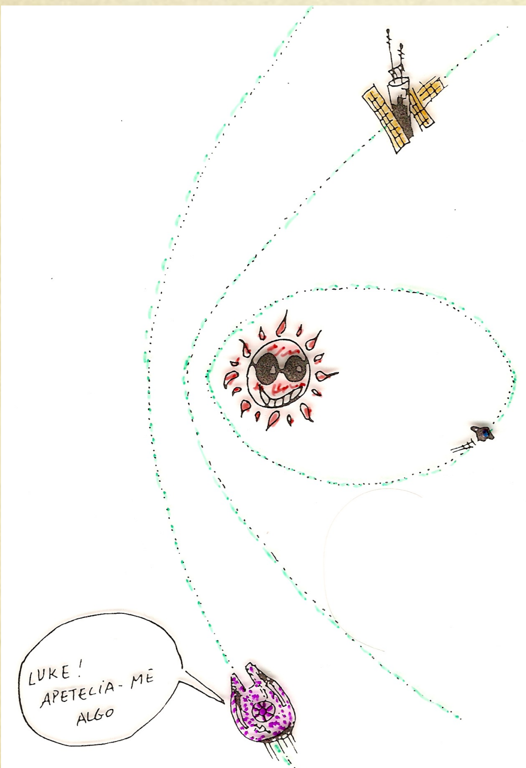
1986

Órbitas de transferência $E, \vec{L} = \text{cte}$

$1 \to 3$

$$\Delta E = \frac{1}{2} G_N M m \left(\frac{1}{r_1} - \frac{1}{r_3}\right) = \frac{G_N M m}{2 r_1 r_3}(r_3 - r_1)$$





$1 \to 2 \quad + \quad 2 \to 3$

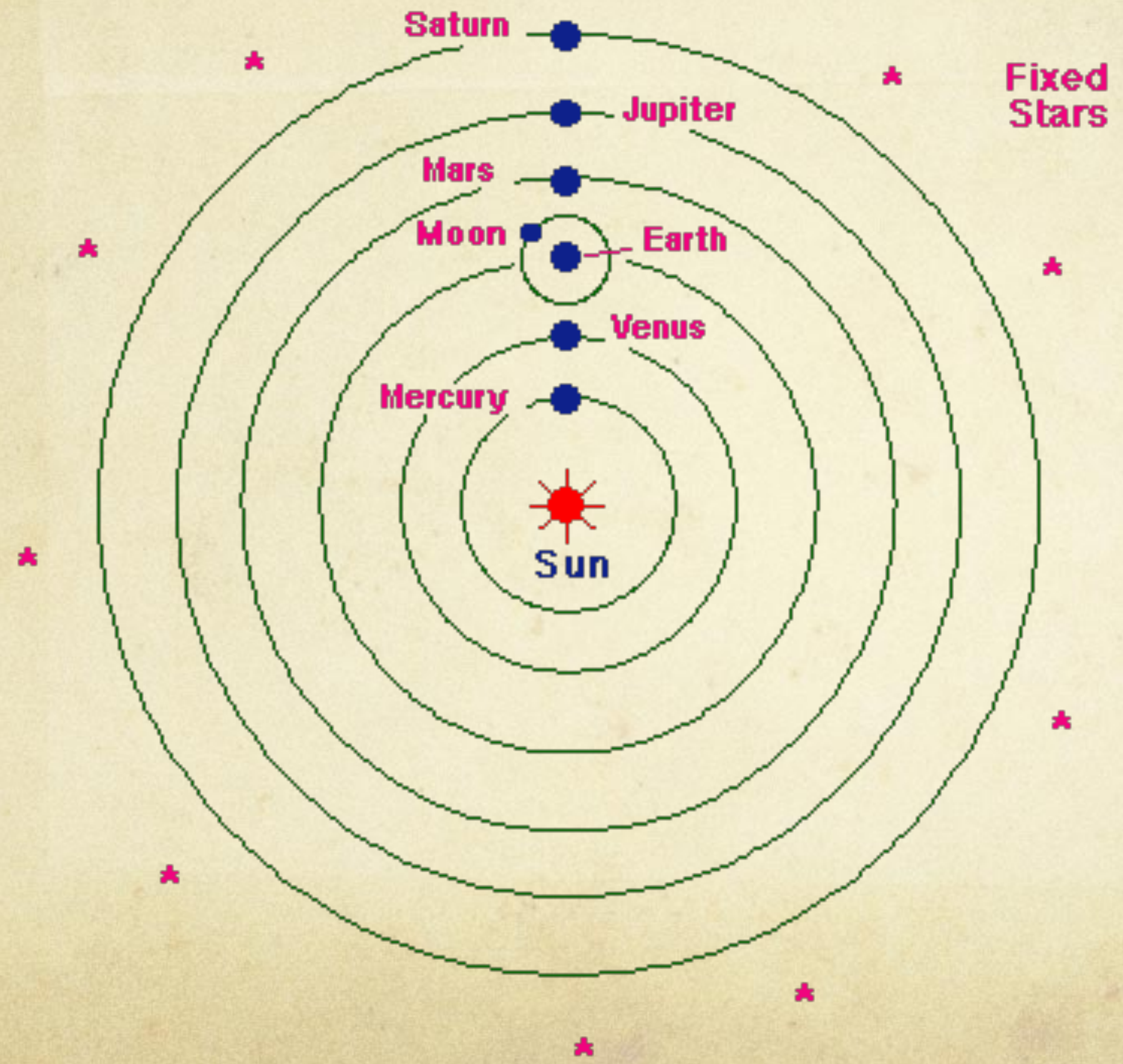
$$\Delta v_{12} = v_{2i} - v_1 = \frac{L_2 - L_1}{m r_1}$$

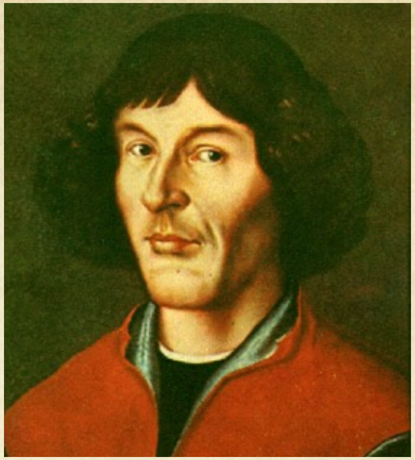
$$\Delta v_{23} = v_3 - v_{2f} = \frac{L_3 - L_2}{m r_3}$$

$$L_2 = L_{\text{transfer}}$$

com $$L^2_{\text{transfer}} = 2 m^2 G_N M \left(\frac{r_1 r_3}{r_1 + r_3}\right)$$

Copérnico (1473-1543)





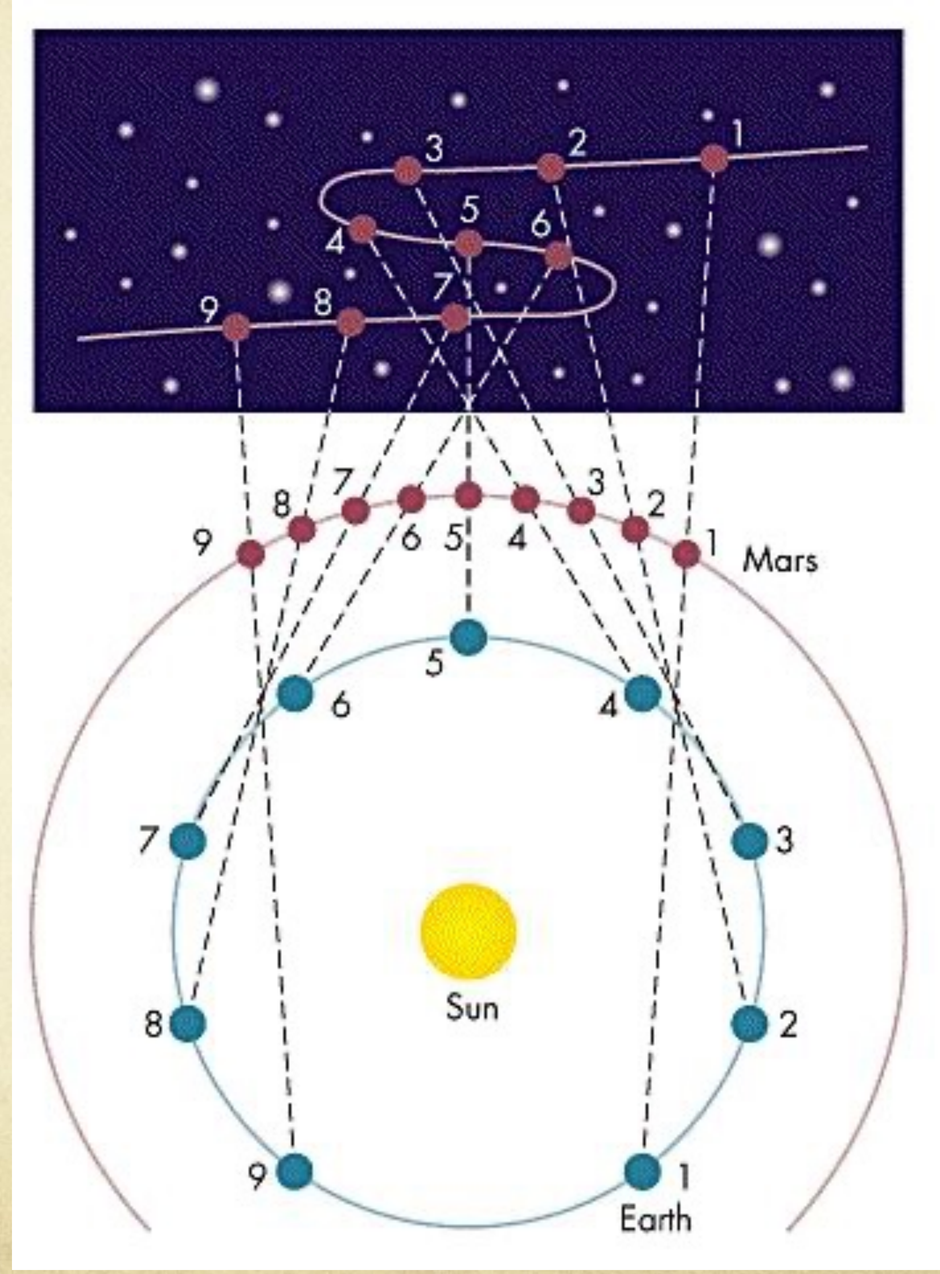

NICOLAI
COPERNICI TO-
RINENSIS DE REVOLVTIONI-
bus orbium cœlestium,
Libri VI.

IN QVIBVS STELLARVM ET FI-

BASILEAE, EX OFFICINA
HENRICPETRINA.

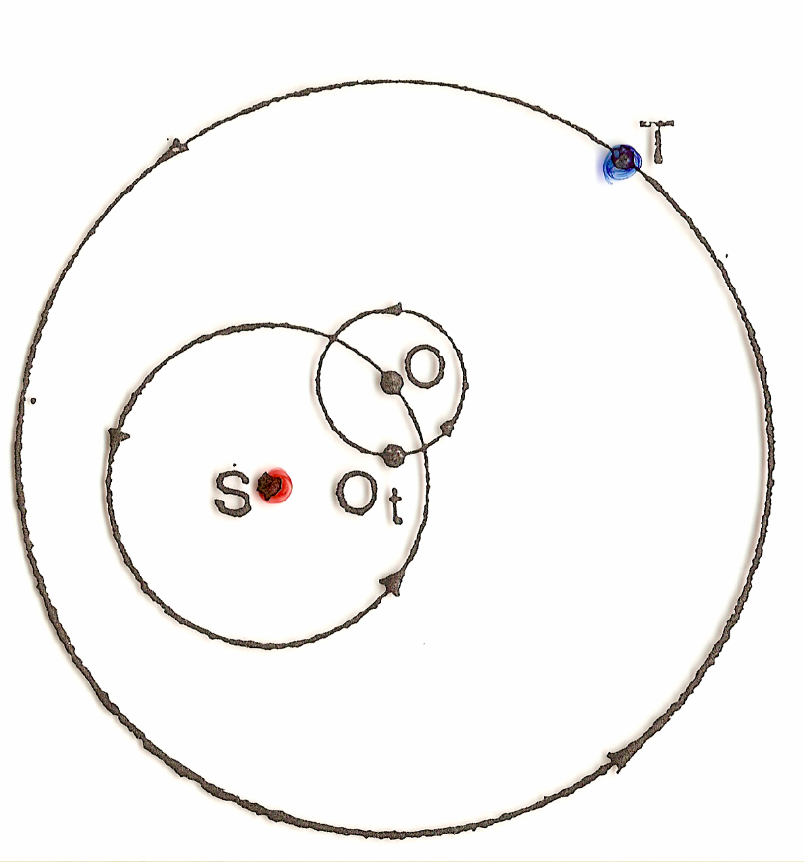
De Revolutionibus
Orbium Caelestium

O Movimento retrógrado

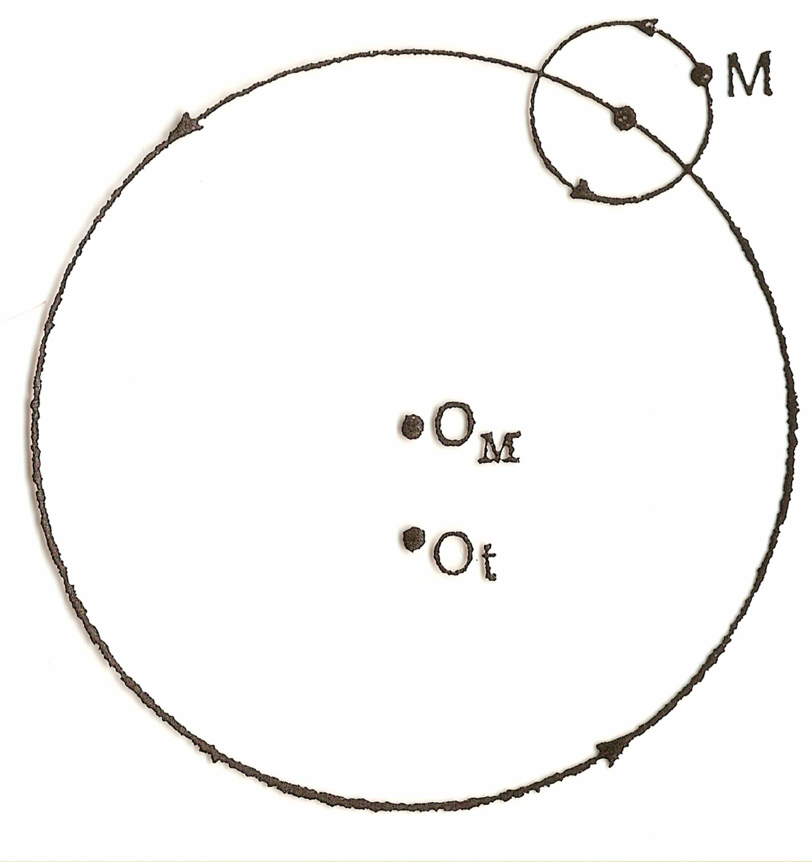


Mas não chega !

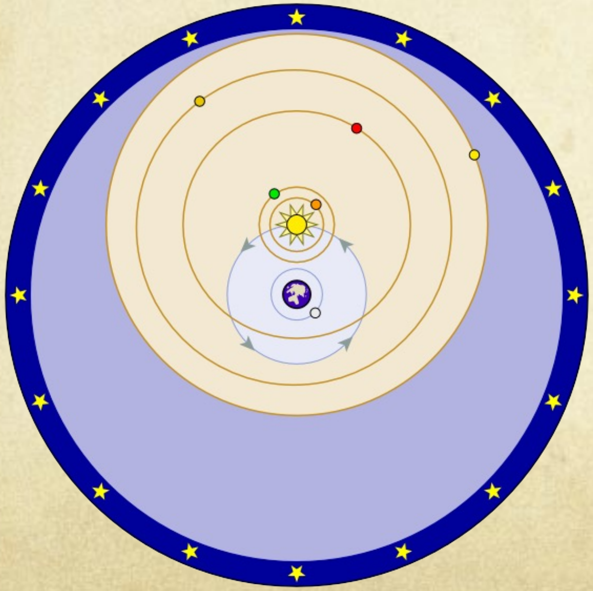
Órbita da Terra



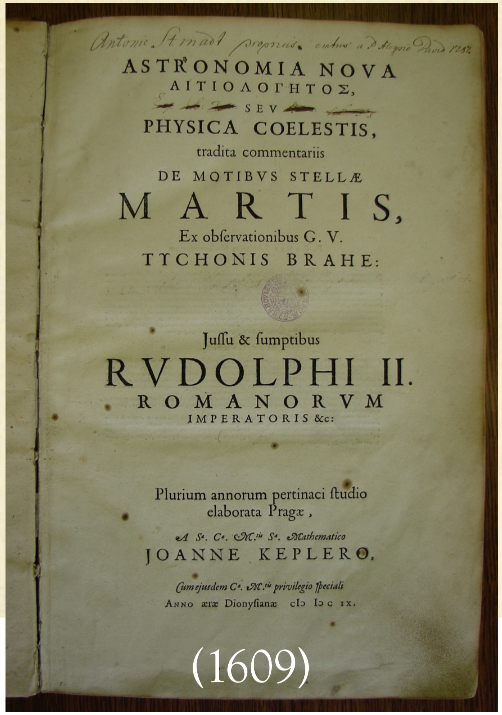
Órbita de Marte



Tycho Brahe
(1546-1601)



Johannes Kepler (1571-1630)



ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,
SEV
PHYSICA COELESTIS,
tradita commentariis
DE MOTIBVS STELLÆ
MARTIS,
Ex obſervationibus G. V.
TYCHONIS BRAHE:

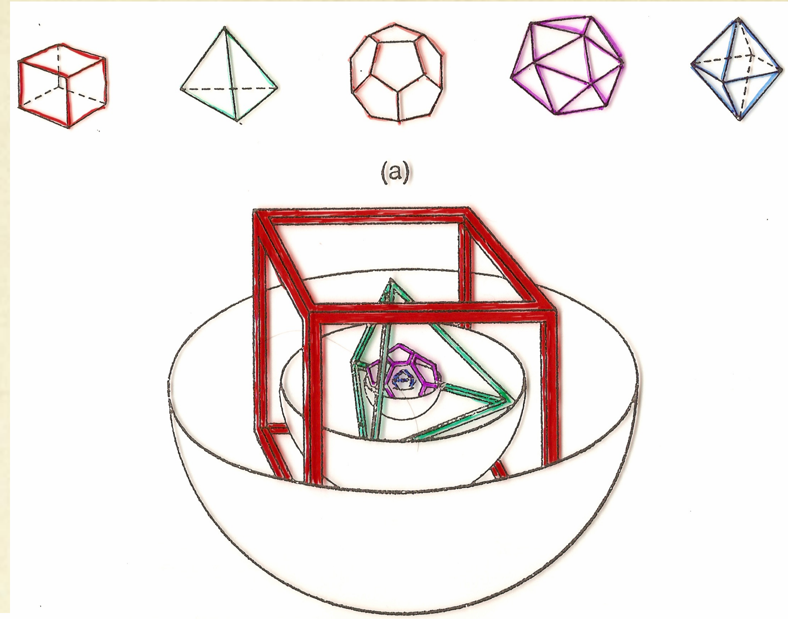
Juſſu & ſumptibus
RVDOLPHI II.
ROMANORVM
IMPERATORIS &c:

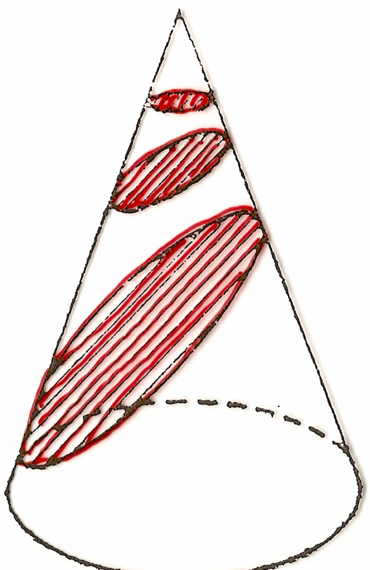
Plurium annorum pertinaci ſtudio
elaborata Pragæ,

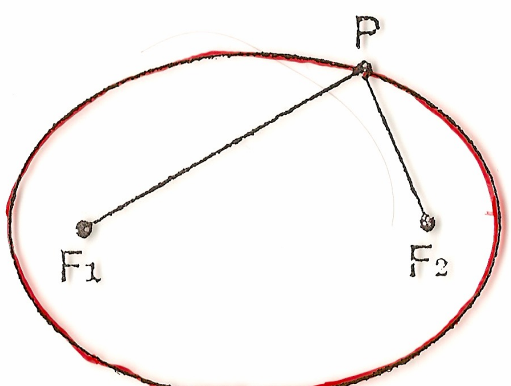
A S^x. C^x. M.^tis S^x. Mathematico
JOANNE KEPLERO,

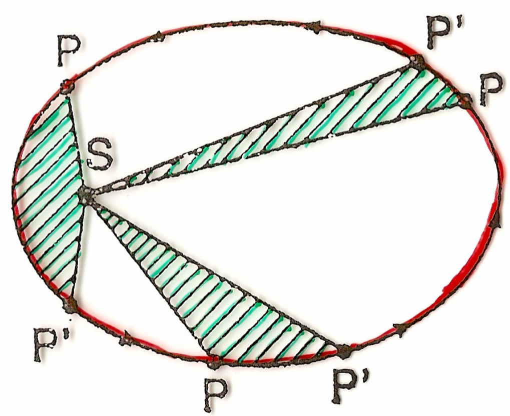
Cum ejusdem C^x. M.^tis privilegio ſpeciali
ANNO æræ Dionyſianæ cIↄ Iↄ c IX.

(1609)





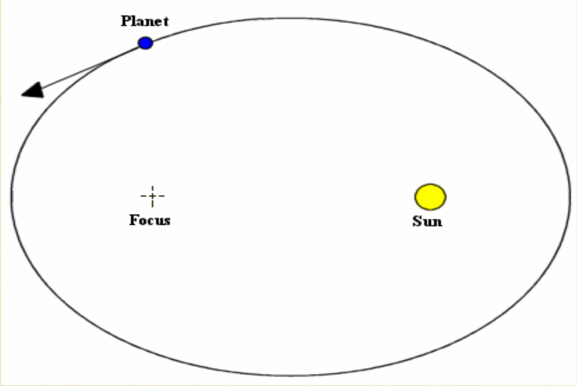




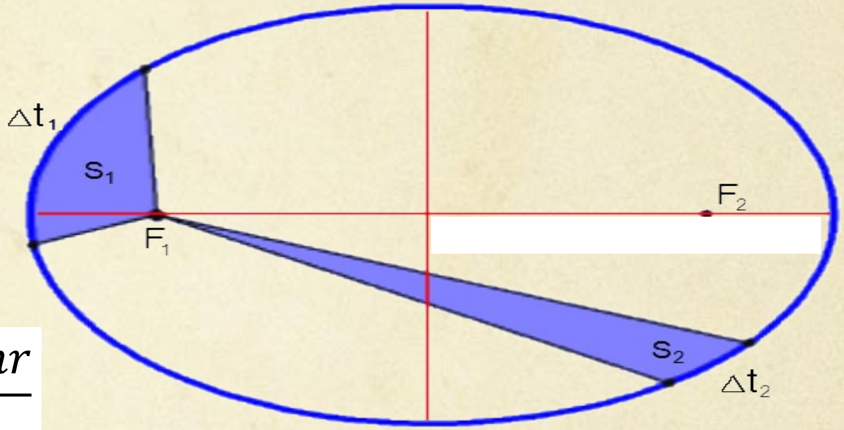
Leis de Kepler

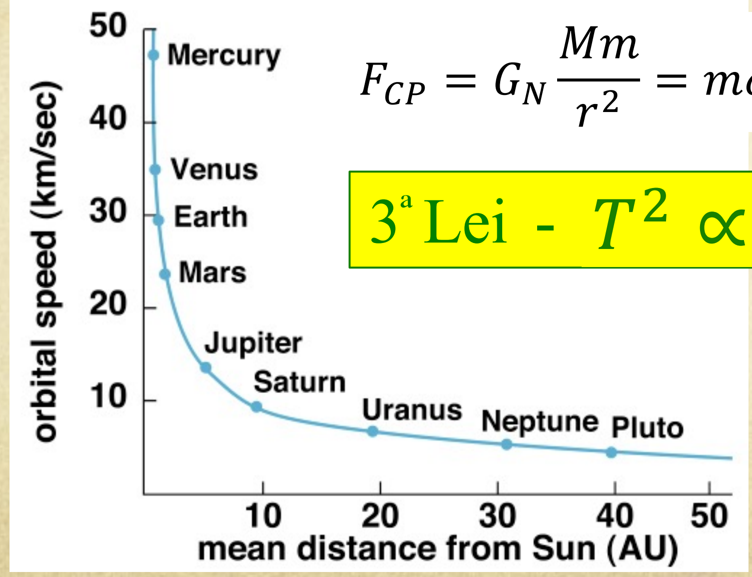
$\vec{F} \parallel -\vec{r}$ $\quad\quad$ $\vec{r}\times m\vec{v} = \text{cte}$ $\quad\quad$ $\vec{v} \perp \vec{r}$

1ª Lei – Movimento num plano; Órbitas elípticas (Sol no foco)



2ª Lei – Áreas iguais em tempos iguais





$$F_{CP} = G_N \frac{Mm}{r^2} = m\omega^2 r = \frac{4\pi^2 m r}{T^2}$$

(para órbitas aprox. circulares temos:)

3ª Lei - $T^2 \propto r^3$

$$v = \omega r \qquad L = \text{cte} = mvr = mr^2 \frac{d\theta}{dt}$$

$$\frac{L}{m} dt = r.r d\theta = 2\ \text{área}$$

Sist.solar:

$$T_T = \frac{2\pi}{\sqrt{G_N M_\odot}} r_T^{3/2} = 5{,}44\times 10^{-10} r_T^{3/2}$$