Final exam — February 12, 2022

Version F

Name: **Student n.:**

Instructions

- You have 120 minutes to complete the exam.

- Make sure that your test has a total of 14 pages and is not missing any sheets, then write your full name and student n. on this page (and your number in all others).

- The test has a total of 19 questions, with a maximum score of 100 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.

- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.

- The exam is open book and open notes. You may use a calculator, but any other type of electronic or communication equipment is not allowed.

- Good luck.

Part 1	Part 2	Part 3, Pr. 1	Part 3, Pr. 2	Total
32 points	18 points	25 points	25 points	100 points

Part 1: Multiple Choice Questions (32 points)

In each of the following questions, indicate your answer by *checking a single option.*

1. (4 points) An RNN-based sequence-to-sequence model with an attention mechanism translates an input sentence of M words into an output sentence with N words. How does the number of computational operations (algorithmic complexity) increase as a function of M and N?

    □ $O(M^N)$

    □ $O(\max(M, N)^2)$

    □ $O(MN)$

    □ $O(M + N)$

2. (4 points) A model is trained for 30 epochs with gradient descent and it leads to the following plot for its training and test losses:
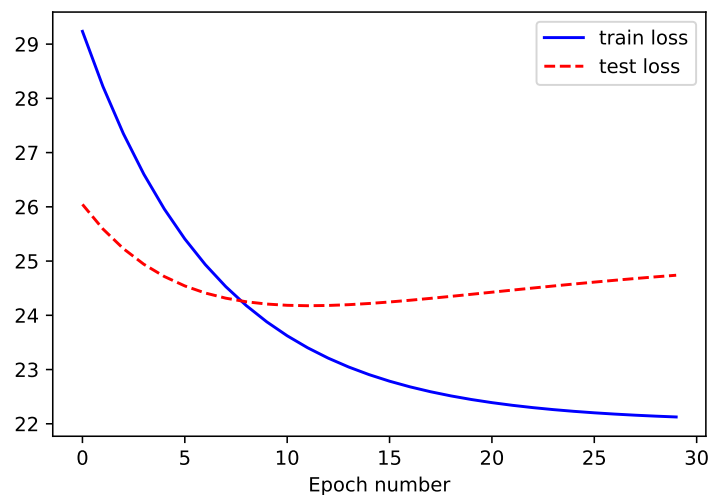
    Which of the following statements is a plausible explanation for what could be happening?

    □ The model is overfitting the training data.

    □ The model is underfitting the training data.

    □ The model generalizes well to unseen examples.

    □ None the above.

3. (4 points) A neural network is overfitting its training data. What strategies could mitigate this?

    □ Decrease the amount of training data.

    □ Increase the number of hidden units.

    □ Increase the dropout probability.

    □ All the above.

4. (4 points) Let $\vee, \wedge, \oplus$ denote respectively the OR, AND, and XOR Boolean logical operators, and $\neg$ denote Boolean negation. Assume Boolean values are represented as -1 (False) and $+1$ (True). Which of these logical functions cannot be learned by a single perceptron with inputs A and B?

☐ $A \vee B$

☐ $\neg A \wedge B$

☐ $(A \oplus B) \wedge A$

☐ $(A \wedge \neg B) \vee (\neg A \wedge B)$

5. (4 points) Let $L(\boldsymbol{w}) = \frac{1}{2} \sum_i (y_i - \boldsymbol{w}^\top \boldsymbol{\phi}(x_i))^2$ be the loss function corresponding to a linear regression problem. Which equation represents the stochastic gradient descent update for $\boldsymbol{w}$?

☐ $\boldsymbol{w}^{(k+1)} \leftarrow \boldsymbol{w}^{(k)} - \eta \sum_i (y_i - \boldsymbol{w}^\top \boldsymbol{\phi}(x_i)) \boldsymbol{\phi}(x_i)$

☐ $\boldsymbol{w}^{(k+1)} \leftarrow \boldsymbol{w}^{(k)} - \eta (y_i - \boldsymbol{w}^\top \boldsymbol{\phi}(\boldsymbol{x}_i)) \boldsymbol{\phi}(x_i)$

☐ $\boldsymbol{w}^{(k+1)} \leftarrow \boldsymbol{w}^{(k)} - \eta (y_i - \sigma(\boldsymbol{w}^\top \boldsymbol{\phi}(x_i))) \boldsymbol{\phi}(x_i)$, where $\sigma(z) = 1/(1+e^{-z})$ is the sigmoid function.

☐ $\boldsymbol{w}^{(k+1)} \leftarrow \boldsymbol{w}^{(k)} - \eta (y_i - \text{sign}(\boldsymbol{w}^\top \boldsymbol{\phi}(x_i))) \boldsymbol{\phi}(x_i)$, where $\text{sign}(\cdot)$ is the sign function

6. (4 points) Which one of the following statements is true?

☐ Auto-encoders with non-linear activations and a squared loss are equivalent to PCA.

☐ Neural networks with a single hidden layer with linear activations are universal approximators.

☐ Convolutional layers are equivariant to translations and rotations.

☐ None of the above.

7. (4 points) Consider the following computation graph, where $\sigma(z) = 1/(1+e^{-z})$ is the sigmoid function. What is the derivative of y with respect to x?
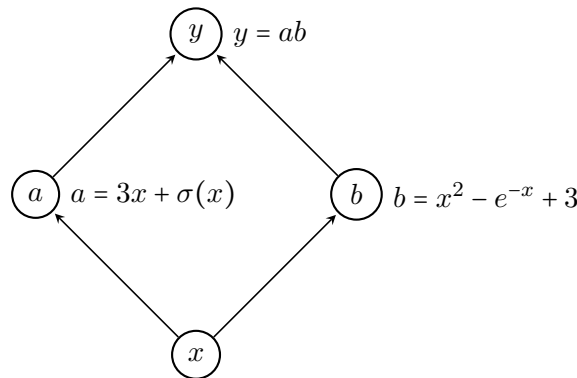
☐ $3 + \sigma(x)(1 - \sigma(x)) + 2x + e^{-x}$.

☐ $a(3 + \sigma(x)(1 - \sigma(x))) + b(2x + e^{-x})$.

☐ $b(3 + \sigma(x)(1 - \sigma(x))) + a(2x + e^{-x})$.

☐ 0.

8. (4 points) Which one of the following statements is **false**?

☐ Upper level layers (closer to the output) tend to learn more abstract representations (shapes, forms, objects) compared to bottom level layers.

☐ Distributed representations generally require fewer dimensions than local (one-hot) representations.

☐ Gradient clipping can prevent vanishing gradients.

☐ Transformer models can be used for computer vision applications.

You can use this space as draft.

Part 2: Short Answer Questions (18 points)

Please provide **brief** answers (1-2 sentences) to the following questions.

1. (6 points) Explain how dropout regularization works.

2. (6 points) Explain the role and need for positional encoding in transformers.

3. (6 points) Mention one advantage of contextualized word embeddings (e.g. BERT) over static word embeddings (e.g. word2vec or GloVe).

Part 3: Problems (50 points)

Problem 1: Convolutional Neural Networks (25 points)

In their retail store, Yolanda and Zach currently use a card punching system to register the entry and exit times of their 6 employees. However, they heard about recent advances in computer vision systems and decided to replace that system by face recognition using CNNs.

To train the system, they collected a large dataset of pictures from their 6 employees, Alice, Berta, Chad, Diane, Eric, and Frank. Each picture in the dataset is a 192×256 grayscale picture, similar to those depicted in Fig. 1, and is labeled according to the corresponding employee.
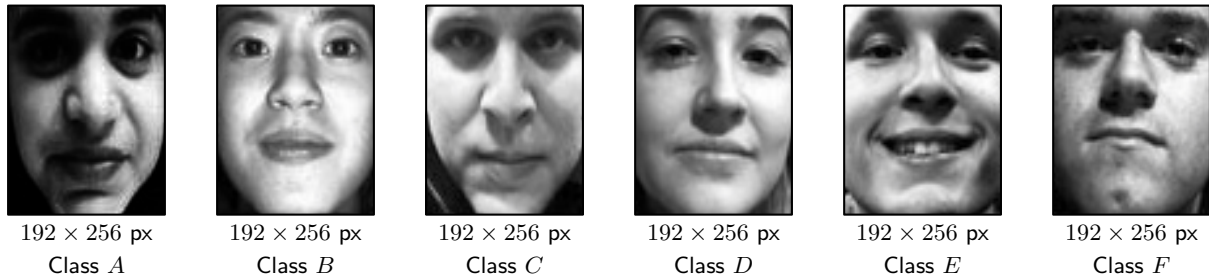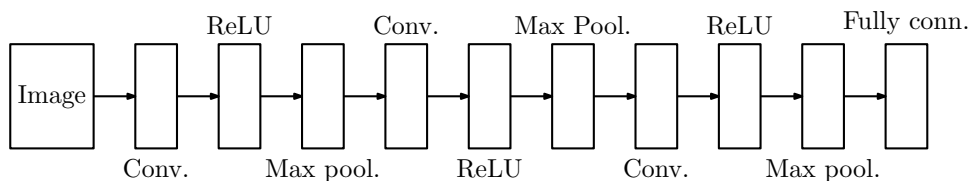

| 192 × 256 px | 192 × 256 px | 192 × 256 px | 192 × 256 px | 192 × 256 px | 192 × 256 px |
| Class A | Class B | Class C | Class D | Class E | Class F |

Figure 1: Sample pictures from the 6 classes that the CNN must recognize. Alice corresponds to class A, Berta to class B, etc.

1. (4 points) Briefly explain in 1-2 sentences why a CNN is an adequate choice of architecture for Yolanda and Zach's task (image classification).

2. (7 points) Suppose that, in their classifier, their use the following architecture:

Image → Conv. → ReLU → Max pool. → Conv. → ReLU → Max Pool. → Conv. → ReLU → Max pool. → Fully conn.

which is specified using the following Pytorch code snippet:

```
nn.Sequential(
    nn.Conv2d(1, 5, kernel_size=3, stride=1, padding=1),
    nn.ReLU(),
    nn.MaxPool2d(kernel_size=2, stride=2),
    nn.Conv2d(5, 10, kernel_size=5, stride=1, padding=0),
    nn.ReLU(),
    nn.MaxPool2d(kernel_size=2, stride=2),
    nn.Conv2d(10, 20, kernel_size=2, stride=2, padding=0),
    nn.ReLU(),
    nn.MaxPool2d(kernel_size=5, stride=2),
    nn.Flatten(),
    nn.Linear(2800, 6))
```

Fill in the following table with the adequate values.

Layer	Output size	N. weights	N. biases
Input	$192 \times 256 \times 1$	0	0
1st conv. layer			
1st pooling layer			
2nd conv. layer			
2nd pooling layer			
3rd conv. layer	$23 \times 31 \times 20$	800	20
3rd pooling layer	$10 \times 14 \times 20$	0	0
Output layer			

3. (7 points) Consider the diagram in Fig. 2, containing the brightness values for the first window of pixels in one of the images in the dataset.
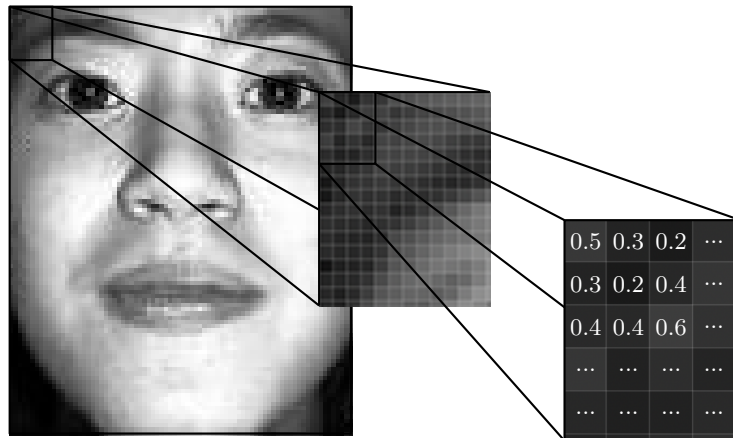

Figure 2: Brightness values for one of the images in the dataset.

Suppose that, after training, one of the filters in the first convolutional layer is defined by the parameters

$$\boldsymbol{K} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad b = 0.5.$$

For the filter provided, compute the the top-left-most value after the pooling layer. Do not forget that the first convolutional layer includes a padding of size 1 (use zeros as the padding value).

4. (7 points) Suppose that Yolanda and Zach decide to add some skip connections to their network, as indicated in the diagram:
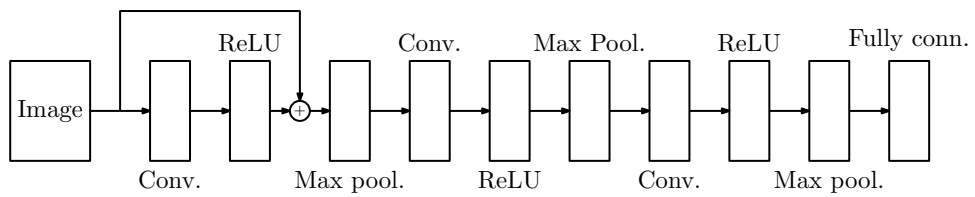
Repeat Question 3, but now considering the skip connection indicated in the diagram above.
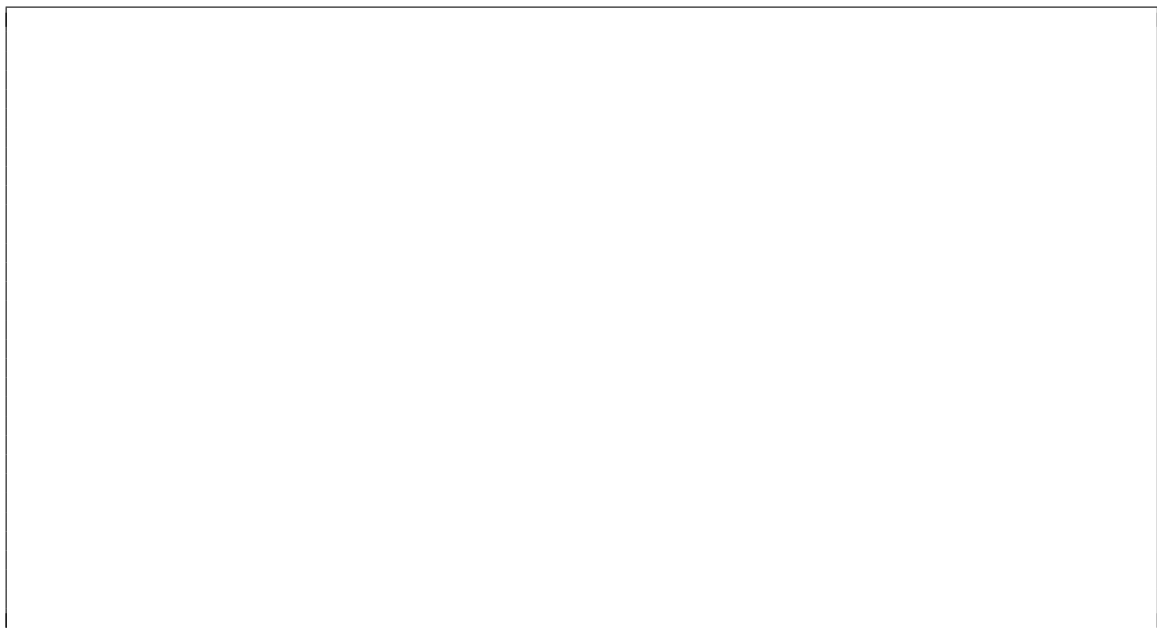
Problem 2: Sequence-to-Sequence Models (25 points)

Bartholomew (known to his friends as Bart) had an idea for a project: building a system to summarize news articles into a short sentence (e.g., a tweet). He collected a dataset with news documents and their corresponding tweets, which he will use to train a summarization model.

1. (7 points) Bart's sister (Lisa) is taking a course on *deep learning* and she recommended using a sequence-to-sequence architecture based on a *recurrent neural network* (RNN) for this problem. Bart tested a simple RNN-based sequence-to-sequence model (without any attention mechanism) on a small-scale experiment. He is using a very small vocabulary (7 words, including the <STOP> symbol), shared between the source and target, and using the same embedding vectors for both sides. The embedding vectors are

$$\boldsymbol{x}_{\text{Deep}} = [0, 1]^\top, \quad \boldsymbol{x}_{\text{learning}} = [1, 0]^\top, \quad \boldsymbol{x}_{\text{works}} = [-1, -1]^\top,$$
$$\boldsymbol{x}_{!!!} = [2, -1]^\top, \quad \boldsymbol{x}_{\#\text{deep}} = [-1, 2]^\top, \quad \boldsymbol{x}_{\text{lol}} = [0, -1]^\top, \quad \boldsymbol{x}_{\text{<stop>}} = [1, 1]^\top.$$

The initial hidden state of the RNN, $\boldsymbol{h}_0$, is all-zeros. The input-to-hidden matrix is

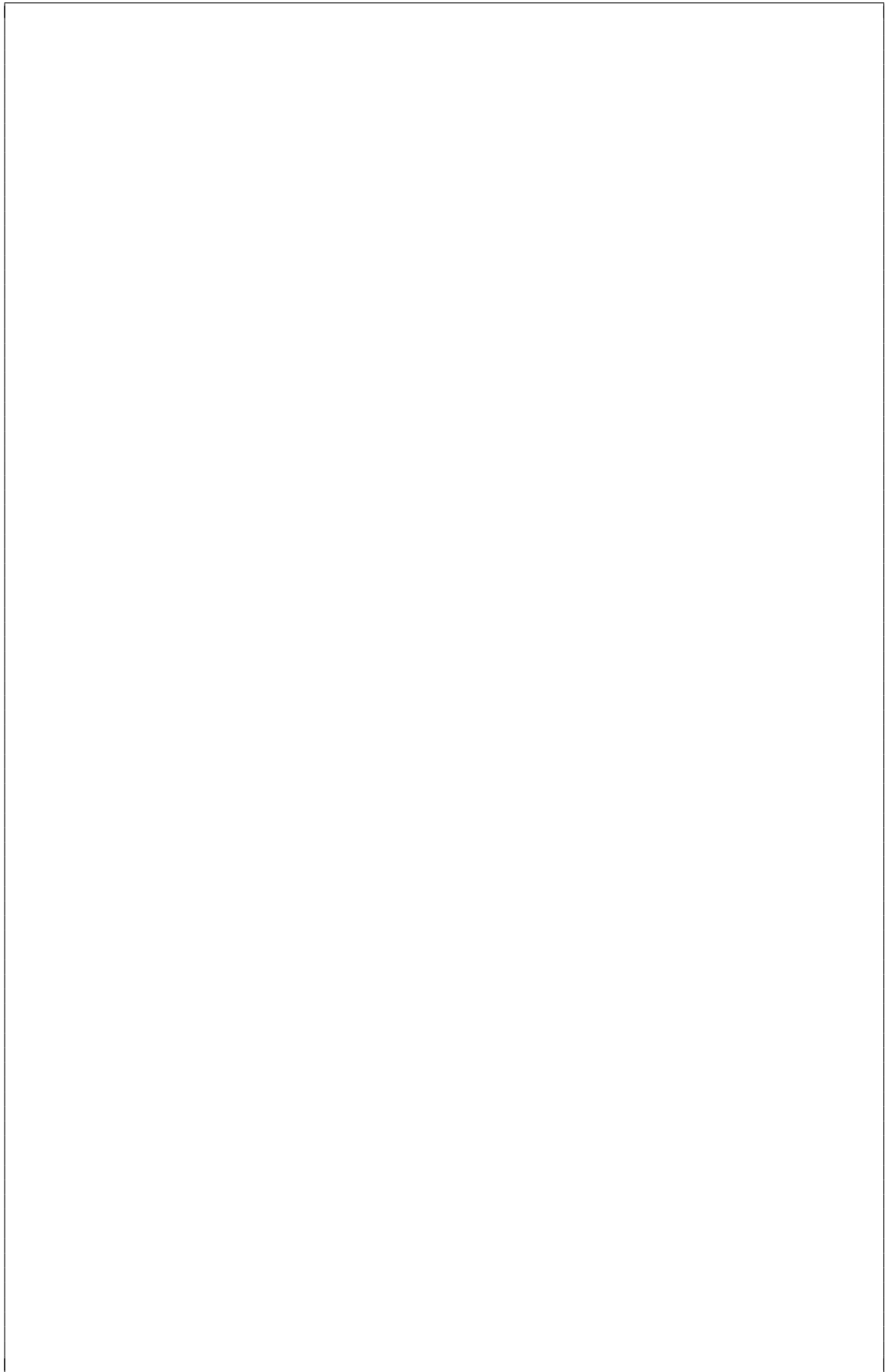$$\boldsymbol{W}_{hx} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}.$$

The recurrent matrix $\boldsymbol{W}_{hh}$ is the identity matrix. All biases are vectors of zeros. The RNN uses `relu` activations.

Compute the last state of the encoder RNN, $\boldsymbol{h}_4$, for the input document "Deep learning works". Show all your calculations.
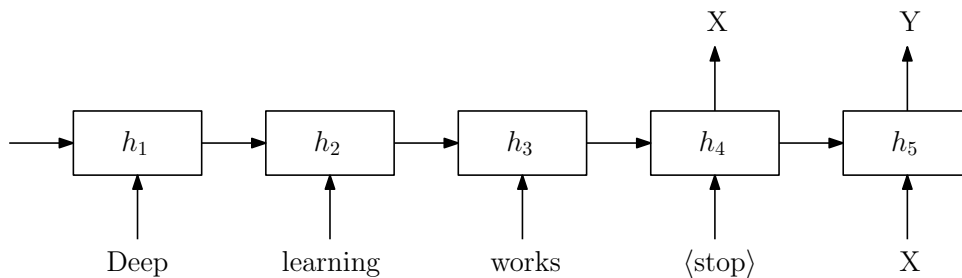
(you can continue in the next page)

2. (8 points) Assume that in the previous question we obtained $\boldsymbol{h}_4 = [0, 2, 2]^\top$. Assume that the decoder RNN has the same parameters as the encoder RNN, and the hidden-to-output matrix is

$$
\boldsymbol{W}_{yh} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ -2 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 1 \end{bmatrix}.
$$

The target word probabilities at time step t are given by softmax($\boldsymbol{W}_{yh}\boldsymbol{h}_t$), where $\boldsymbol{h}_t$ is the corresponding state of the decoder RNN.

Compute the first two words of the generated tweet using **greedy decoding**.

3. (6 points) After playing with this network for a while, Bart realized that it didn't work well for long documents and therefore decided to add an attention mechanism. In the first decoding step, using **scaled dot-product attention** with h_4 as the query vector and h_1, h_2, h_3 as the key and value vectors, compute the attention probabilities and the resulting context vector (use $h_1 = [0, 0, 1]^\top$, $h_2 = [0, 2, 2]^\top$, $h_3 = [1, 0, 0]^\top$, $h_4 = [0, 2, 2]^\top$).

4. (4 points) Lisa's friend, Allison, who is also knowledgeable about deep learning, told Bart about transformers and large pretrained models. Give one example of a pretrained model that Bart could use for this task and the necessary steps to use it.

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You can use this space as draft.