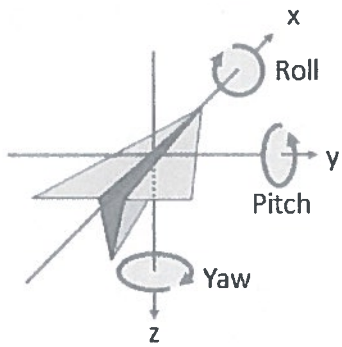
PROCESSAMENTO DE IMAGEM E VISÃO

Academic Year 2018-2019

Dept Electrical and Computer Engineering – Instituto Superior Técnico

Test: #1 Duration: 2 hours

---




1) [5pts] Consider and airplane that is monitoring the presence of drones over the Lisbon airport, and that the coordinates of the aircraft are defined according to the picture:

2 a) Suppose that the airplane rotates 30 degrees around the y axis, facing down, followed by a rotation of 45 degrees right, around the Z axis, calculate the rotation matrix, **R**.

2 b) Calculate the rotation axis, $\vec{n}$, and angle, $\theta$, corresponding to matrix ***R***.
(in the case you did not calculate $\vec{n}$, use $\vec{n} = (1 - 2\ \ 3)/\|\vec{n}\|)$

1 c) Prove that the obtained matrix is a rotation matrix.

2) [**5pts**] Consider the following perspective camera model that describes the projection of 3D points onto the corresponding 2D image points

$$\tilde{P} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 8 & 2 & 20 \\ 0 & -2 & 8 & 0 \end{bmatrix}$$

2 a) Explain how the structure of camera matrix depends on intrinsic and extrinsic parameters

2 b) Calculate the camera's optical centre and justify your approach and steps.

1 c) Determine the intrinsic parameters matrix, the coordinates of the principal point, and the rotation matrix between the world and the camera coordinate frames.

3) [**5pts**] A binocular camera system is observing table where a robot is picking and placing objects for assembling the control panel of an airplane. The position of the table is not calibrated with respect to the cameras.

2 a) Describe what is a *homography* and if it can be used to relate the coordinates of points between the two camera images and/or table plane.

2 b) Let *H* be a *homography* $\tilde{m}_1 \sim H\ \tilde{m}_2$. Explain how to calculate the parameters of *H* including the amount of data necessary and how to handle noisy data.

1 c) A conic (like a circle, ellipse, etc) in the projective plane can be expressed as $\tilde{m}^T \tilde{Q} \tilde{m} = 0$, with $\tilde{Q} = \tilde{Q}^T$. Explain how the points on a conic are mapped by a homographic transformation between two planes.

4) [**5pts**] Classify the following statements, as false or true, and justify your choices in detail.

1 a) Rigid transformations on the plane are linear mappings and, therefore, the parameters can be estimated linearly.

1 b) A Gaussian filter is used to highlight the contours of an image

1 c) The optic centre is the 3D reconstruction of the image principal point.

1 d) The gradient of an image is tangential to the image contours.

1 e) The motion field of a moving camera does not depend on the 3D structure of the scene.

---

Please read the questions carefully and derive the answers using mathematical models when appropriate.

TÉCNICO LISBOA

2018-19

Proc. Imagem & Visão

# TEST 1

a)

P1

$$R(y, -\pi/6) = \begin{bmatrix} \cos \pi/6 & 0 & \sin(-\pi/6) \\ 0 & 1 & 0 \\ -\sin(-\pi/6) & 0 & \cos \pi/6 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

$$R(z, \pi/4) = \begin{bmatrix} \cos \frac{\pi}{4} & -\sin \pi/4 & 0 \\ \sin \pi/4 & \cos \pi/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_z(\pi/4)\, R_y(-\pi/6) = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{6}/4 & -\sqrt{2}/2 & -\sqrt{2}/4 \\ \sqrt{6}/4 & \sqrt{2}/2 & -\sqrt{2}/4 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

b)

$$Rx = x \iff \begin{cases} \sqrt{6}/4\, x - \sqrt{2}/2\, y - \sqrt{2}/4\, z = x \\ \sqrt{6}/4\, x + \sqrt{2}/2\, y - \sqrt{2}/4\, z = y \\ 1/2\, x + \sqrt{3}/2\, z = z \quad \text{——} \quad z = \dfrac{x}{2-\sqrt{3}} \end{cases}$$

$$\iff \begin{cases} (2)-(1): \ \sqrt{2}\, y = y - x \\ \text{——} \\ z = x/(2-\sqrt{3}) \end{cases} \iff \begin{cases} y = x/(1-\sqrt{2}) \\ \text{——} \\ z = x/(2-\sqrt{3}) \end{cases}$$

$$M = [1 \quad -2.4142 \quad 3.7321] \,/\, 4.559$$

$$\vec{M} = \cancel{3.7321}[1 \quad -2.4142 \quad 3.7321]/4.559 = [0.2195 \quad -0.5299 \quad 0.8122]$$

$$[M]_x = \begin{bmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -0.8122 & -0.5299 \\ 0.8122 & 0 & -0.2195 \\ 0.5299 & 0.2195 & 0 \end{bmatrix}$$

$$[M]_x^2 = \begin{bmatrix} -0.9518 & -0.1163 & 0.1798 \\ -0.1163 & -0.7192 & -0.4341 \\ 0.1798 & -0.4341 & -0.3290 \end{bmatrix}$$

Rodrigues Formula

$$R = I + \sin(\theta)[M]_x + (1-\cos\theta)[M]_x^2$$

$$R(1,1) \Leftrightarrow \sqrt{6}/4 = 1 + \sin\theta \times \emptyset - (1-\cos\theta)\times 0.9518$$

$$\Leftrightarrow \cos\theta = (\sqrt{6}/4 - 1 + 0.9518)/0.9518 = 0.5927$$

$$\theta = 0.9364 \sim 53.6^\circ \sim 0.2981\pi$$

c) $R^T R = I$

$\det(R) = 1$. But since $R = R_z R_y$ we can simply prove that these properties hold for the original matrices.

$R_z^T R = I$ ; $R_y^T R = I$ $\quad \det(R) = \det(R_z)\det(R_y) = 1$

TÉCNICO LISBOA

## P2

$$P = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 8 & 2 & 20 \\ 0 & -2 & 8 & 0 \end{bmatrix} = K\left[R \mid -R\,O_c\right]$$

$O_c$ → Camera optic center in world coordinate

a) $P = K[R \mid \underbrace{-R\,O_c}_{T}]$  $R, T \rightarrow$ extrinsic parameters Rotation and Translation between the world and camera coordinate frame.

$K$ = intrinsics, 3x3 matrix with pixel height / width and principal point coordinate.

b) The optic center cannot be projected in the image plane. It is the only point in the entire Universe with this feature.

$$\emptyset = K[R \mid T]\begin{bmatrix} O_c \\ 1 \end{bmatrix} \Leftrightarrow KR\,O_c + KT = \emptyset$$

illegal result

$$\Leftrightarrow O_c = -(KR)^{-1}(KT) = -R^{-1}T$$

$$= -\begin{bmatrix} 10 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & -2 & 8 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 20 \\ 0 \end{bmatrix} = -\begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \times 0.5882 = -\begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} / 1.7$$

$$= \begin{bmatrix} 0 & -2.353 & -0.588 \end{bmatrix}^T$$

c) Considering the first 3 columns of P.

$$KR = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & -2 & 8 \end{bmatrix}$$

$$KR * (KR)^T = K \underbrace{R R^T}_{I} K^T = K K^T$$

$$= \begin{bmatrix} 100 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$$

$$\text{Hence } (\lambda K)^2 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & \underbrace{68}_{\lambda^2} \end{bmatrix}$$

scale factor

$$K = \begin{bmatrix} 10/\sqrt{68} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_x = \emptyset$

$c_y = \emptyset$

$S_y = 1$

$S_x = \cancel{1.4706}$ $1.2127$

$$K = \begin{bmatrix} \cancel{1.4706}\ 1.2127 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

TÉCNICO LISBOA

Problem P3

a) A Homography is a projective transformation in $\mathbb{P}^2$. It is defined by a $3\times3$ matrix $\underset{\sim}{H}$ up to a non-zero scale factor.

A Homography can map a point on a plane in the 3D space to its projection on an image plane. It can as well map the image projections in two cameras of a point lying on a 3D plane.

b) A homography is defined by a $3\times3$ matrix up to a non-zero scale factor. The equation $\underset{\sim}{m}_1 \sim \underset{\sim}{H}\, \underset{\sim}{m}_2$ is defined in projective coordinates and represents 2 equations.

We can then use the coordinates of 4 points (in the 3D plane and the corresponding image projection), to define 8 eqs to solve for the params of $\underset{\sim}{H}$.

In the presence of noisy data, one should use more points and use least-squares:

$$A\Theta = b \iff \hat{\Theta} = (A^TA)^{-1}A^Tb$$

$b \to 2n \times 1$

$\Theta \to 8$ params

$A \to 2n \times 8$

with $n$ being the number of points

With outliers, one should use an outlier rejection method such as RANSAC.

c) Conic equation

$$\underset{\sim}{m}^T \underset{\sim}{Q}\, \underset{\sim}{m} = \emptyset \qquad \text{and} \qquad \underset{\sim}{Q} = \underset{\sim}{Q}^T \qquad (1)$$

Homographic Transformation

$$\underset{\sim}{m} = \underset{\sim}{H}\, \underset{\sim}{m}_1 \longrightarrow \underset{\sim}{m}^T = \underset{\sim}{m}_1^T \underset{\sim}{H}^T \qquad (2)$$

Replacing (2) in (1) we have :

$$\underset{\sim}{m}_1^T \underset{\sim}{H}^T \underset{\sim}{Q}\, \underset{\sim}{H}\, \underset{\sim}{m}_1 = 0 \iff \underset{\sim}{m}_1^T \underset{\sim}{Q}_1\, \underset{\sim}{m}_1 = 0$$

with $\underset{\sim}{Q}_1 = \underset{\sim}{H}^T Q\, \underset{\sim}{H}$

The original conic becomes

$$\underset{\sim}{Q}_1 = \underset{\sim}{H}^T Q\, \underset{\sim}{H}$$

$$\underset{\sim}{m} \xrightarrow{\underset{\sim}{H}^{-1}} \underset{\sim}{m}_1$$

$$\underset{\sim}{m}^T \underset{\sim}{Q}\, \underset{\sim}{m} = \emptyset \longrightarrow \underset{\sim}{m}_1^T \underset{\sim}{Q}_1\, \underset{\sim}{m}_1 = 0 \; ; \; \underset{\sim}{Q}_1 = \underset{\sim}{H}^T \underset{\sim}{Q}\, \underset{\sim}{H}$$

$\boxed{P4}$

a) False Cannot be estimated linearly as it depends non-linearly on the parameters $\cos(\theta)$, $\sin(\theta)$

b) False. Gaussian filters are Low Pass and blur images.

c) FALSE. The optic center is the point common to all optic rays

d) FALSE The direction of the image gradient is perpendicular to the direction of the contour

e) FALSE. It ~~depends~~ is the sum of two terms, rotation and translation of the camera. The term associated to the cam translation depends on depth.