PROCESSAMENTO DE IMAGEM E VISÃO
Academic Year 2016-2017
Dept Electrical and Computer Engineering – Instituto Superior Técnico

Test: #1 Duration: 2 hours

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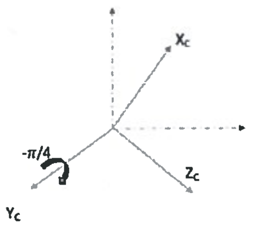
1) [5pts] Consider that a cartesian coordinate frame {A} rotated by 90º around the Z axis followed by a rotation of 90º around the Y axis.

2 a) Calculate the rotation matrix, ***R***

2 b) Calculate the rotation axis, $\vec{n}$

1 c) Calculate the rotation angle, θ, using the Rodrigues formula.

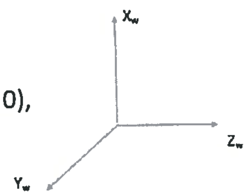
2) **[5pts]** Suppose that you have a perspective camera, with the optical centre located at the coordinates $O_C = [10 \quad 0 \quad 0]^T$ and rotated -45º around the Y axis, according to the diagram.



2 a) Calculate the rotation matrix relating the camera and world coordinate frames

2 b) Assuming that the camera intrinsic parameters are $K = I$, calculate the camera projection matrix

1 c) Calculate the coordinates of the point on the ground (world plane where X = 0), that projects onto the image principal point.



3) **[5pts]** Consider a stereo camera, i.e. a device that integrates two optical cameras, and that the stereo camera is pointing to a patch of the Mars surface, that is approximately planar, and that the autonomous spacecraft has to verify if there are obstacles or holes in the area.

3 a) Describe an algorithm that can be used to solve this problem, even when the cameras are not calibrated (because it is difficult to place a calibration grind on Mars)

2 b) Assuming that the spacecraft has eventually landed and that we can make use of a calibration object, explain the procedure required to calibrate the cameras including: (i) the design of the calibration object, (ii) number of parameters to calibrate, (iii) number of points required for the calibration. Include the main equations and steps required for calibrate the two cameras.

4) **[5pts]** Classify the following 3 statements, as false or true, and justify your choice. Answer the final question and justify your response.

1 a) A Gaussian filter can be used to remove image noise and it is more effective when the standard deviation, σ, is small.

1 b) The magnitude of the gradient of an image, calculated on the surroundings of a contour, is related to the orientation of the contour,

1 c) In the same condition as before, the orientation of the image gradient is parallel to the contour direction.

2 d) Consider a 1D image $I = [1 \;\; 4 \;\; 4 \;\; 2 \;\; 4 \;\; 4\, 8 \;\; 5 \;\; 4 \;\; 4 \;\; 5 \;\; 20 \;\; 4]$. Calculate the output of a 5x1 median filter applied to this image. Calculate only those points for which the median filter fits completely within the image. Repeat the process with an average filter 5x1. Compare and discuss the differences and when one should choose use the median or the average filter.

Please read the questions carefully and derive the answers using mathematical models when appropriate.

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PIV - Test 1 Resolution
2015 - 16

P1)

a) $$RYZ = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}}_{R_y(\pi/2)} \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_z(\pi/2)} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

b) To find a point x in the rotation axis:

$$Rx = x \Leftrightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} z = x \\ x = y \\ y = z \end{cases} \Leftrightarrow x = y = z \qquad \boxed{\vec{n} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T / \sqrt{3}}$$

(note that $\vec{n}$ has unit norm)

c) Rodrigues Formula

$$R = I + \sin\theta\, K + (1 - \cos\theta) K^2$$

$$\Leftrightarrow \sin\theta\, K - \cos\theta\, K^2 = R - I - K^2$$

$$K = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} / \sqrt{3}$$

Using first line/column:

$$\sin\theta \times \frac{0}{3} + \frac{2}{3} \cos\theta = 0 - 1 + \frac{2}{3}$$

$$\Leftrightarrow \cos\theta = -1/2 \Leftrightarrow \boxed{\theta = 2\pi/3}$$

$$K^2 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} / 3$$

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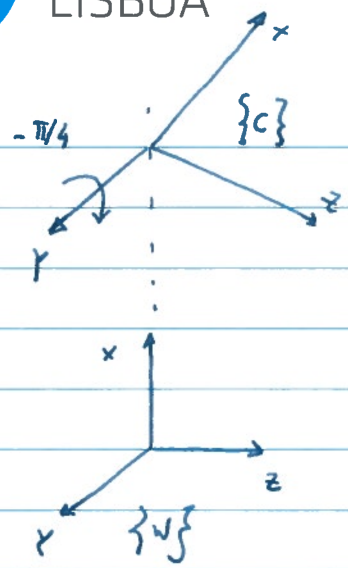
P2



a) $R = \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$

Optic center: $O_c = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix}^T$

b) Projection matrix: $P = K[R \mid T]$ → T: world origin expressed in the camera frame

→ R: Rotation between the world and camera frames

→ K: intrinsic parameters ($K = I$)

From the optic center property:

$$\underbrace{\emptyset}_{\text{cannot be projected}} = K[R \mid T] \begin{bmatrix} O_c \\ 1 \end{bmatrix} \iff T = -R O_c = -\begin{bmatrix} 10/\sqrt{2} & 0 & 10/\sqrt{2} \end{bmatrix}$$

$$P = \left[\begin{array}{ccc|c} 1/\sqrt{2} & 0 & -1/\sqrt{2} & -10/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & -10/\sqrt{2} \end{array}\right]$$

c) $\underset{\sim}{m} = \begin{pmatrix} 0 \\ 0 \\ \lambda \end{pmatrix} = \underset{\sim}{P}\,\underset{\sim}{M}$, $\underset{\sim}{M} = \begin{bmatrix} 0 \\ Y \\ Z \\ 1 \end{bmatrix}$

$$\begin{cases} Y = 0 \\ -1/\sqrt{2}\, Z - 10/\sqrt{2} = 0 \end{cases} \iff \boxed{\begin{cases} X = Y = 0 \\ Z = -10 \end{cases}}$$

principal point $x = y = \emptyset$!

$$M = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 1 \end{bmatrix}$$

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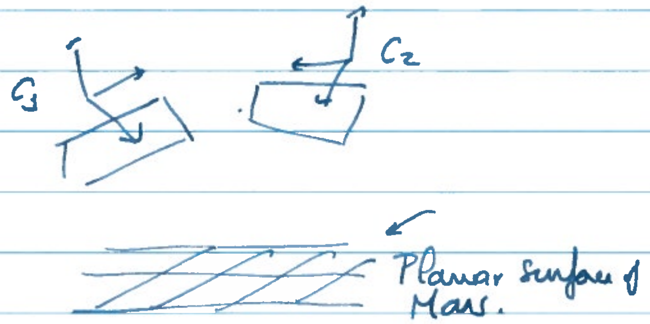
P3



a) Since the surface of Mars can be approximated by a plane, there will be a homography that maps the projections of these points on the plane, directly from I_1 to I_2 (images of C_1, C_2)

$$\underset{\sim}{x}_2^i = H_{12}\, \underset{\sim}{x}_1^i$$

$\underset{\sim}{H}$: 3×3 homography matrix, defined up to scale. There are 8 parameters to estimate and 4 point correspondences are needed to do that.

Obstacles can be detected (and holes) because they will violate the planarity assumption and, thus, the homography constraint.

b) $\underset{\sim}{m} = \underset{\sim}{P}\, \underset{\sim}{M}$

$\underset{\sim}{P}$ – 3×4 camera matrix, defined up to scale and having 11 unknowns. Each point known in 3D/2D provides 2 equations, hence 6 points are needed.

$$\lambda \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} p_1^T \\ p_2^T \\ p_3^T \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow x = \frac{p_1^T M}{p_3^T M}, \quad y = \frac{p_2^T M}{p_3^T M}$$

$$\begin{cases} x\, p_3^T M = p_1^T M \\ y\, p_3^T M = p_2^T M \end{cases} \Leftrightarrow \begin{bmatrix} M^T & 0_{4\times1} & -x M^T \\ 0_{4\times1} & M^T & -y M^T \\ - & - & - \\ & \underbrace{}_{2\text{ eqs/point}} & \\ - & - & - \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \emptyset \qquad A \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = 0$$

→ least squares solution yields the eigenvector with the smallest eigenvalue

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Problem 4

a) **FALSE** When σ decreases, the low-pass characteristic of the filter is attenuated. When $\sigma = 0$, the Gaussian filter would have no effect.

b) **FALSE** The magnitude of the gradient depends on the contrast across the contour.

c) **FALSE** The orientation of the gradient is perpendicular to the edge orientation

d) Data [1 4 4 2 4 4 8 5 4 4 5 20 4]

1 2 4 4 4

2 4 4 4 4

2 4 4 4 8

2 4 4 5 8

4 4 4 5 8

4 4 4 5 8

4 4 5 5 8

4 4 5 5 20

4 4 4 5 20

Image :	1	4	4	2	4	4	8	5	4	4	5	20	4
Median	*	*	4	4	4	4	4	4	5	5	4	*	*
average	*	*	3	$\frac{18}{5}$	$\frac{22}{5}$	$\frac{23}{5}$	5	5	$\frac{26}{5}$	$\frac{38}{5}$	$\frac{34}{5}$	*	*