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Advanced Plasma Physics

MEFT 2021/22

Problem Class 7

Clearly present your approximations and enclose all pertinent calculations. Try to solve the problems yourself. Follow the instructions of the Lecturer.

Problem 1. Axion electrodynamics. As discussed in class, the hypothetical particles called "axions" are pseudo-scalar fields that couple very weakly to the electromagnetic fields,

$$\mathcal{L}_{\text{int}} = -\frac{g}{4}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

where $F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor (an anti-simmetric tensor), $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual tensor and g is the coupling constant (it has units of the inverse of a frequency, eV^{-1}). In natural units ($\hbar = c = 1 = \epsilon_0 = \mu_0 = 1$), the modified electromagnetic Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu + \partial_\mu\varphi\partial^\mu\varphi^* - \frac{1}{2}m_\varphi|\varphi|^2 + \mathcal{L}_{\text{int}}.$$

a) Show that the interaction Lagrangian can be further simplified to

$$\mathcal{L}_{\text{int}} = g\mathbf{E}\cdot\mathbf{B}\varphi.$$

b) Show that the equation of motion of the axion field φ is given by

$$\left(\Box + m_\varphi^2\right)\varphi = g\mathbf{E}\cdot\mathbf{B},$$

where $\Box \equiv \partial_\mu\partial^\mu$.

c) We now investigate the mixing between axions and plasmons. For that, we use the modified Poisson equation (that you can derive, together with the remaining Maxwell's equations, by varying the Lagrangian with respect to A_ν),

$$\boldsymbol{\nabla}\cdot(\mathbf{E} + g\varphi\mathbf{B}) = e(n_0 - n_e).$$

Linearize the relevant equations for the problem (fluid equations for n_e and $\mathbf{u}_e$, the Poisson equation for $\mathbf{E}$ and the Klein-Gordon equation for φ) for the case of a magnetized plasma, $\mathbf{B} = B_0\mathbf{e}_x$. Neglect thermal effects in the plasma.

d) Apply a Fourier transform to show that the eigenvalue problem reads

$$\begin{bmatrix} \omega^2 - \omega_p^2 & -ig\dfrac{eB_0 n_0}{m_e}k \\ ig\dfrac{eB_0}{k} & \omega^2 - M_\varphi^2 - k^2 \end{bmatrix} \begin{bmatrix} \tilde{n}_1 \\ \tilde{\varphi}_1 \end{bmatrix} = 0,$$

where $M_\varphi = \sqrt{m_\varphi^2 + g^2 B_0^2}$ is the effective axion mass in the plasma.

e) Obtain the dispersion relation for the upper (U) and lower (L) polariton modes, as

$$\omega_{\mathrm{U,L}}^2 = \frac{1}{2}\left(\omega_\varphi^2 + \omega_p^2 \pm \sqrt{\left(\omega_\varphi^2 - \omega_p^2\right)^2 + 4\Omega^4}\right),$$

where $\Omega = \sqrt{gB_0\omega_p}$ is the coupling strength. Make a plot of the two dispersions, comparing with the bare dispersions $\omega = \omega_p$ and $\omega = \omega_\varphi \equiv \sqrt{M_\varphi^2 + k^2}$.