Masters in Mechanical Engineering
Aerodynamics
1st Semester 2019/20

Exam 1st season, 10 January 2019 **Name :**
Time : 6:30PM **Number:**
Duration : 3 hours
1st Part : No textbooks/notes allowed
2nd Part : Notes and textbooks allowed

1st Part

Indicate if the sentences are true (T) or false (F) in the empty squares. For each theme, any combination of true and false is possible. The classification of each answer is the following:
Correct answer 0.25 marks.
Empty square 0 marks.
Incorrect answer -0.15 marks

1. In the mathematical models to simulate **turbulent** flows:

☐ In direct numerical simulation (DNS), the flow is always three dimensional but it can be steady.

☐ The dependent variables of Large Eddy Simulation (LES) are identical to those of the Reynolds-Averaged equations (RANS).

☐ DNS solutions are not affected by numerical errors.

☐ The flow is always steady in the solution of the Reynolds-Averaged equations (RANS).

2. Mass conservation and momentum balance for a two-dimensional boundary-layer of an incompressible fluid may be expressed as:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \ (1);$$

$$\rho \frac{\partial u_x u_x}{\partial x} + \rho \frac{\partial u_y u_x}{\partial y} = -\frac{dp}{dx} + \mu \frac{\partial^2 u_x}{\partial y^2} \ (2)$$

where x is a coordinate parallel to the wall, y is a coordinate perpendicular to the wall, u_x and u_y are the velocity components, p is the relative pressure, ρ is the density of the fluid and μ is dynamic viscosity of the fluid.

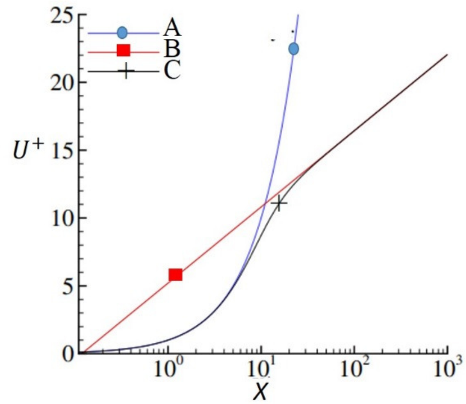
☐ The momentum balance in the y direction leads to $dp/dy \cong 0$.

☐ The pressure gradient (dp/dx) is determined by the ideal fluid solution of the external flow.

☐ The equations presented above are only valid for turbulent flow.

☐ The relative pressure p is measured with respect to the atmospheric pressure.

3. The figure below presents the mean horizontal velocity profile in the near-wall region of a turbulent flow.



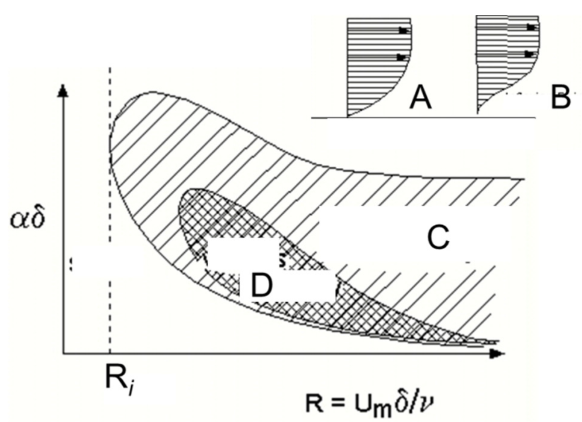
☐ Variable $X = y^+ = u_\tau y/\nu$.

☐ Line C is obtained from the definition of the total shear-stress in the buffer layer.

☐ Line B corresponds to the linear sub-layer.

☐ Line A is determined from the integration of $\tau_W = \mu \partial U/\partial y$, where y is the coordinate perpendicular to the wall, U is mean velocity component parallel to the wall, τ_W is the shear-stress at the wall and μ is the dynamic viscosity of the fluid.

4. The figure below presents the lines of neutral stability of velocity profiles of laminar boundary-layers.



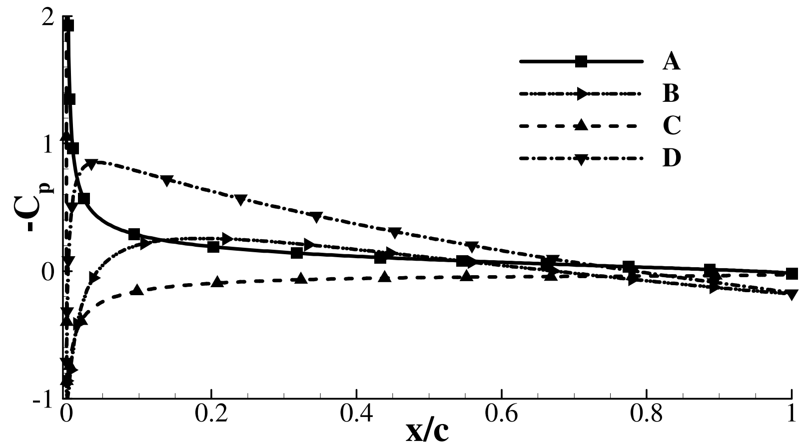
☐ R_i corresponds to the critical Reynolds number.

☐ The stable region of velocity profile A is region C.

☐ Region D is typical of flows with favourable pressure gradient.

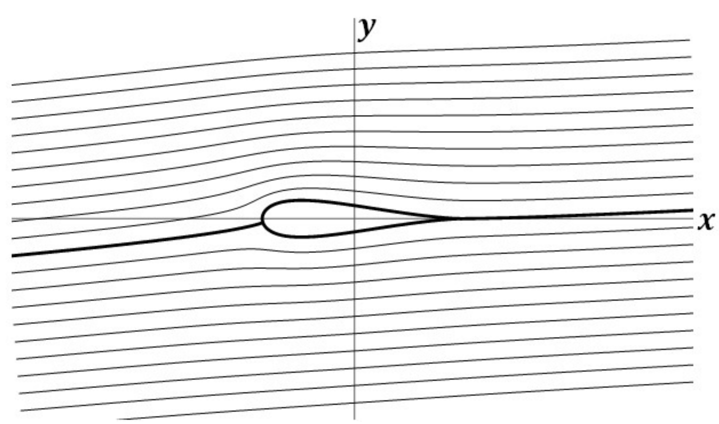
☐ This kind of stability analysis is valid for transition promoted by perturbations in the outer flows (by-pass transition).

5. The figure below presents the symmetric of the pressure coefficient –Cp along the chord of a thin and a thick symmetric airfoil. The positive angle of attack is the same for both airfoils. The pressure distribution was obtained for an ideal fluid using a conformal mapping technique.



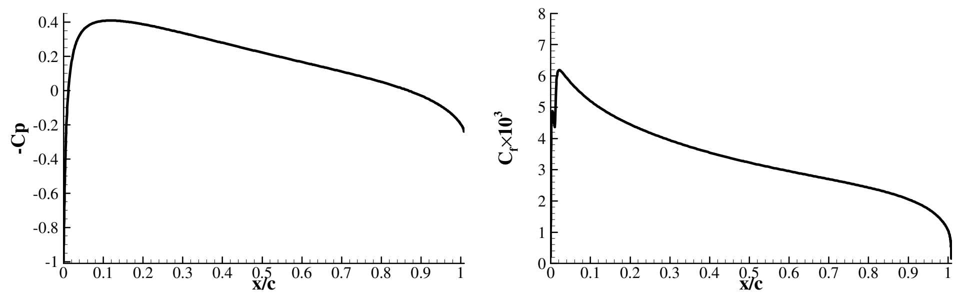
- [ ] Line **A** corresponds to the upper surface of the thick airfoil.
- [ ] The stagnation point is located at the same coordinate (*x/c*) for the two foils.
- [ ] Line **C** corresponds to the lower surface of the thin airfoil.
- [ ] The two airfoils exhibit the same resistance coefficient.

6. The figure below presents the flow around an airfoil obtained with a conformal map $z = f(\zeta)$ of the steady, two-dimensional, potential flow of an incompressible fluid around a circular cylinder of radius 1m **with the centre at the origin of the ζ reference frame.**



- [ ] The conformal map $z = f(\zeta)$ is the Joukowski map.
- [ ] To obtain the flow illustrated in the figure, a line vortex of negative intensity is required at the centre of the cylinder.
- [ ] The chord of the airfoil is larger than 4m.
- [ ] The pressure centre is located at the centre of the airfoil ($x=0$).

7. The figures below present **the distributions** of the (symmetric) of the pressure coefficient ($-C_p$) and skin friction coefficient ($C_f = \tau_w / \left(1/2 \rho U_\infty^2\right)$) along the chord (x/c) of an airfoil at an angle of attack of zero degrees ($\alpha = 0^o$) and Reynolds number based on the chord c and incoming velocity U_∞ of 6×10^6. The results were obtained with the Reynolds-Averaged (time-averaging) Navier-Stokes equations supplemented with the k-ω SST eddy-viscosity turbulence model.



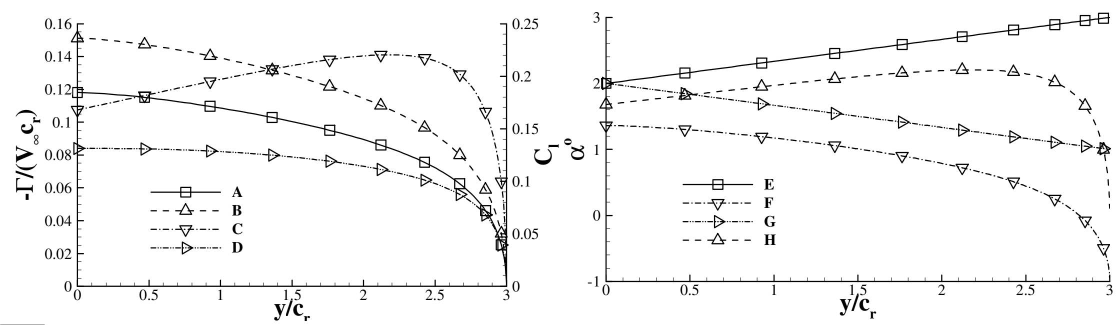
☐ The pitching moment around the aerodynamic centre is zero.

☐ Estimating the friction resistance coefficient from a zero pressure gradient boundary-layer flow will lead to a value smaller than that obtained in the simulation.

☐ The airfoil is a NACA airfoil of the 6 digits series.

☐ Transition from laminar to turbulent flow occurs at a smaller (x/c) coordinate than the one expected for natural transition.

8. The figure below presents the distributions of circulation Γ, lift coefficient C_l, geometric angle of attack α_{geom} and effective angle of attack α_e along the semi-span (wing root at y=0) of two finite wings at the same angle of attack. One wing has a symmetric section and the other has a section with positive camber. One wing is rectangular and the other one is tapered. c_r is the root chord.



☐ The wing with symmetric section is rectangular.

☐ Line **D** corresponds to the circulation of the wing that has negative twist.

☐ The two wings have the same aspect ratio Λ.

☐ The lift coefficient of the wing with positive twist corresponds to line **C**.

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2nd Part

1. A boundary-layer velocity profile is approximated by the following equation

$$\frac{U}{U_e} = C_1 \tanh\left(C_2 \frac{y}{\delta}\right) \Leftarrow y \leq \delta,$$

where U is the velocity component parallel to the wall, U_e is the velocity of the external flow, y is the coordinate perpendicular to the wall, δ is the boundary-layer thickness and C_1 and C_2 are constants.

a) Determine the relation between C_1 and C_2 or, if possible, the values of C_1 and C_2.

b) Is it possible to obtain an accurate solution of zero pressure gradient laminar boundary-layer using the proposed profile? **Give a quantitative justification for your answer**.

c) For $C_1 = 1.0045$ and $C_2 = 3.054$ the ratio $\theta/\delta = 7/72$. Is it possible to obtain a good solution of zero pressure gradient turbulent boundary-layer using only the proposed profile and the von Kármán integral equation? **Give a quantitative justification for your answer**.

$$\tanh(x) = \frac{\text{senh}(x)}{\cosh(x)} \; ; \; \text{senh}(x) = \frac{e^x - e^{-x}}{2} \; ; \; \cosh(x) = \frac{e^x + e^{-x}}{2} \; ;$$

$$\int_0^1 C_1 \tanh\left(C_2 \frac{y}{\delta}\right) \mathrm{d}\frac{y}{\delta} = \frac{C_1}{C_2} \ln[\cosh(C_2)] \; ; \; \int_0^1 C_1^2 \tanh^2\left(C_2 \frac{y}{\delta}\right) \mathrm{d}\frac{y}{\delta} = C_1^2 \left(1 - \frac{\tanh(C_2)}{C_2}\right) \; ;$$

$$\frac{\partial(U/U_e)}{\partial(y/\delta)} = \frac{C_1 C_2}{\cosh^2(C_2\, y/\delta)}$$

2. Consider the steady, bi-dimensional, potential and incompressible flow around a circular cylinder. The radius of the cylinder is 1m and its centre is located at $\zeta_o = \xi_o + \boldsymbol{i}\eta_o$. The uniform incoming flow makes an angle α with the real axe ξ and the magnitude of the velocity is $\boldsymbol{U}_\infty$. At the centre of the cylinder, there is a line vortex with the required intensity to guarantee that there is a stagnation point at the intersection of the cylinder with the positive real axe. The airfoil illustrated in figure 1 was obtained from the Kármán-Treftz conformal map

$$\boldsymbol{z} = \boldsymbol{k}\frac{(\boldsymbol{\zeta}+\mathbf{1})^{\boldsymbol{k}}+(\boldsymbol{\zeta}-\mathbf{1})^{\boldsymbol{k}}}{(\boldsymbol{\zeta}+\mathbf{1})^{\boldsymbol{k}}-(\boldsymbol{\zeta}-\mathbf{1})^{\boldsymbol{k}}},$$

and it has an internal angle at the trailing edge of $\tau = 14.4^o$.

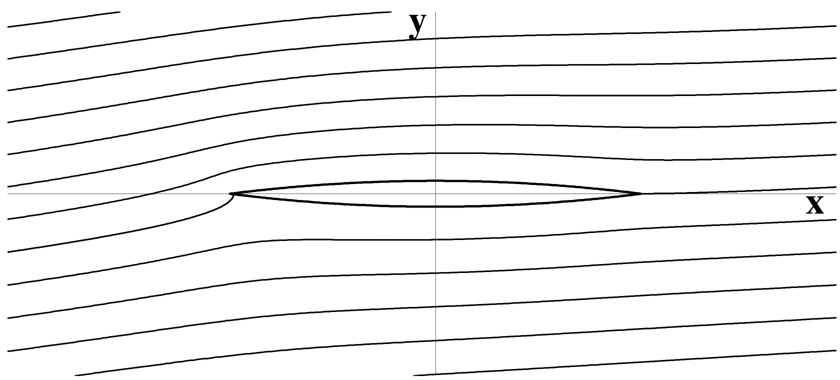

Figure 1

a) Determine the geometric characteristics of the airfoil and write the complex potential that represents the flow in the plane of the cylinder as a function of the angle of attack α. State clearly the selected reference frame.

b) Determine the lift coefficient C_l of the airfoil of figure 1 at small angles of attack. **Discuss the result**.

c) For the angle(s) of attack that satisfy $C_p > -3$ $(C_p = (p - p_\infty)/(1/2\,\rho U_\infty^2))$, determine the number of locations on the airfoil surface that exhibit $C_p = 0$. **Give a clear justification to your answer**.

3. Figure 2 presents the distributions of the (symmetric) of the pressure coefficient ($-C_p$) and the skin friction coefficient ($C_{fo} = \tau_w / \left(1/2 \rho V_\infty^2\right)$) along the chord (x/c) of the Eppler 374 (positive camber) foil at an angle of attack of seven degrees ($\alpha = 7^o$). The Reynolds number based on the chord c, velocity of the uniform incoming flow V_∞ and kinematic viscosity of the fluid is equal to $\boldsymbol{Re} = \mathbf{3 \times 10^5}$. The results were obtained with the Reynolds-Averaged Navier-Stokes equations (time averaging) supplemented by the k-ω SST eddy viscosity model with a transition model. Figure 2b) presents also the C_f evolutions for zero pressure gradient boundary-layers in laminar and turbulent regimes.

   a) Identify which are the results obtained for the upper and lower surfaces of the airfoil (S1 and S2) in figure 2a). Identify for the legend of figure 2b) (L1, L2, L3 and L4) which are the lines that correspond to the upper and lower surface of the airfoil and to the laminar and turbulent zero pressure gradient boundary-layers. **Give a clear justification to your answer**.

   b) The lift, drag, friction drag and pressure drag coefficients are given in table 1. Justifity **quantitatively** the values of each force coefficient.

C_1	C_2	C_3	C_4
0.014	1.12	0.0045	0.0095

Table 1

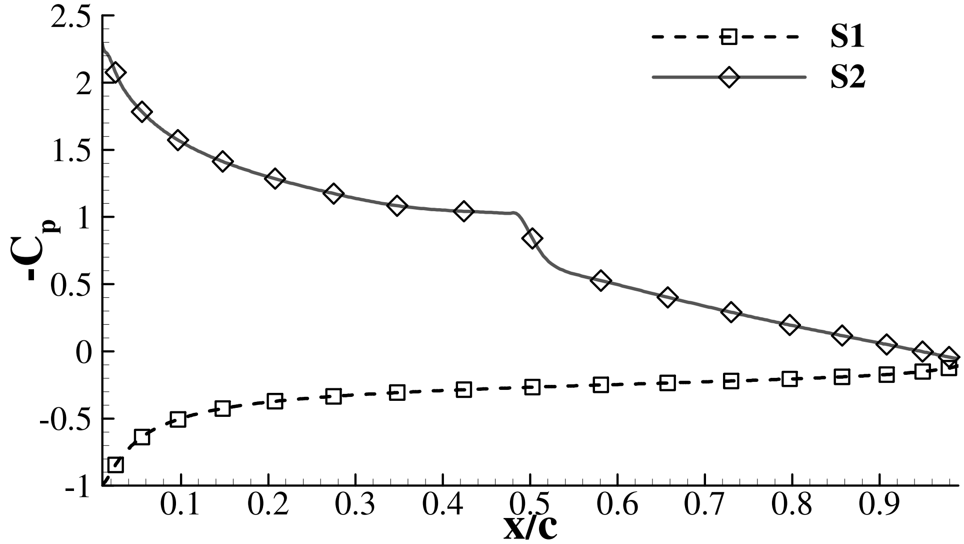

Figura 2a)

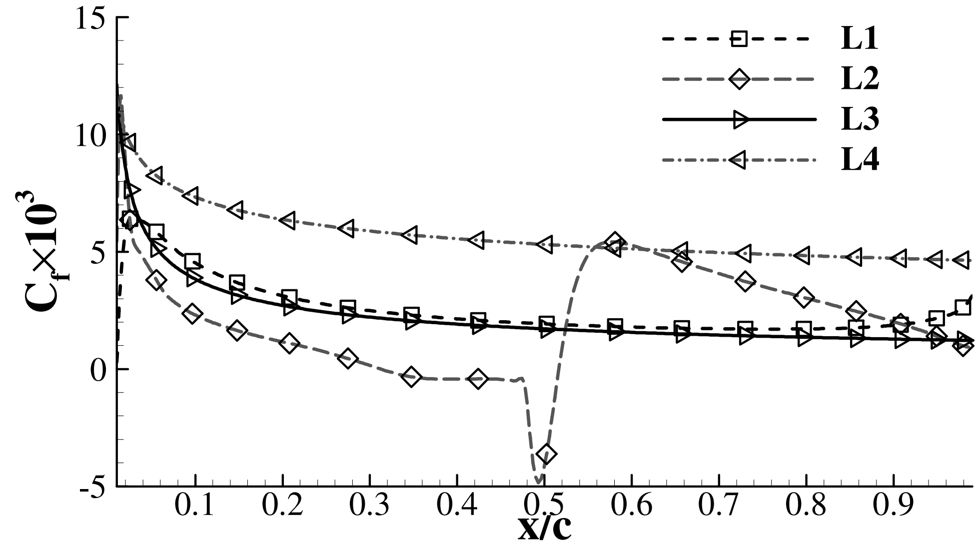

Figura 2b)

c) Assuming that all near-wall cells have the same height h, determine the value of h/c knowing that the numerical results gave $y^+_{\max} = 0.14$, $y^+_{\mathrm{med}} = 0.02$ and $y^+_{\min} = 5.4 \times 10^{-4}$, where y^+ corresponds to the near-wall cells height in wall coordinates.

4. A small aircraft has a wing with the same section along the span, no sweep, no dihedral and no twist. The wing has an area of $S = 8\mathrm{m}^2$ and aspect ratio $\Lambda = 8$. At small angles of attack, the aerodynamic coefficients of **the wing** are:

   $$C_L = 5.03(\alpha + 0.035) \quad \text{with } (\alpha \text{ in radians})$$
   $$C_D = 0.006 + 0.0523 C_L^2.$$

   Assume that the drag coefficient of the aircraft is equal to the drag coefficient of the wing and that the circulation distribution along the span is elliptic.
   $\nu_{air} = 1.51 \times 10^{-5}\mathrm{m}^2/\mathrm{s}$, $\rho_{air} = 1.2\mathrm{Kg/m}^3$.

a) Determine the aerodynamic coefficients of the **wing section** and indicate which type of airfoil is the wing section.

b) Determine the weight of the aircraft knowing that it was designed to obtain the minimum trust force when flying at 144km/h in a region without wind.

c) If the aircraft is flying with head wind at 10km/h (against the aircraft movement), which trajectory should it take to keep on flying at 144km/h (climb, keep altitude, or descend)? Determine the trust force for those conditions.