(4')

1.

$2V$ — $200\,km/h$

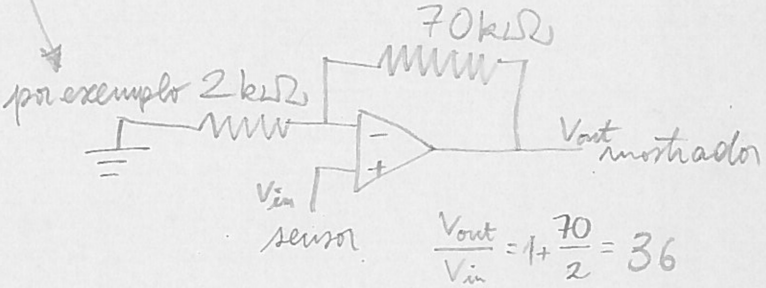
$1{,}8V$ — $180\,km/h$

$\underbrace{50\,m/s \text{ — } 180\,km/h \text{ — } 50\,mV}_{\text{sensor}}$ $\quad \underbrace{\times 36}_{\text{cond. sinal}}$

$0\,m/s$ — $0\,km/h$ — $0\,mV$ $\quad \underbrace{0V \text{ — } 0\,km/h}_{\text{mostrador}}$

$$\frac{1{,}8V}{50mV} = \frac{180}{5} = 36$$

por exemplo



$$\frac{V_{out}}{V_{in}} = 1 + \frac{70}{2} = 36$$

2. a) $v_{som} = \dfrac{2d}{t} \iff d = \dfrac{1}{2} v_{som}\, t$

b) $100\,mV$ — $1\,s$

$5V$ — x

$x = \dfrac{5}{0{,}1} = 50\,s$ logo $\max d = \dfrac{1}{2} \times 1500 \times 50 =$

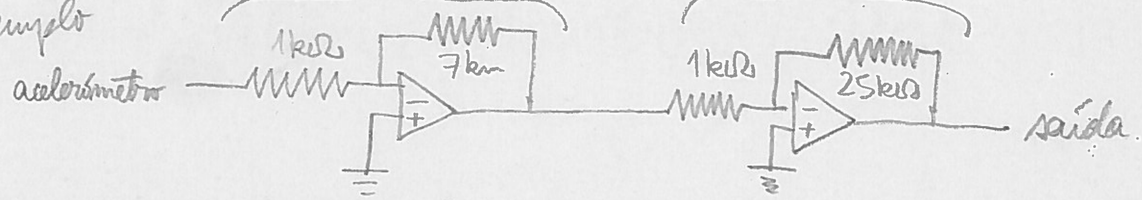
$= 37500\,m = 37{,}5\,km$

3. $14\,mV/g = \dfrac{0{,}014\,V}{9{,}8\,m/s^2} = 1{,}4286 \times 10^{-3}\, V\,s^2/m$

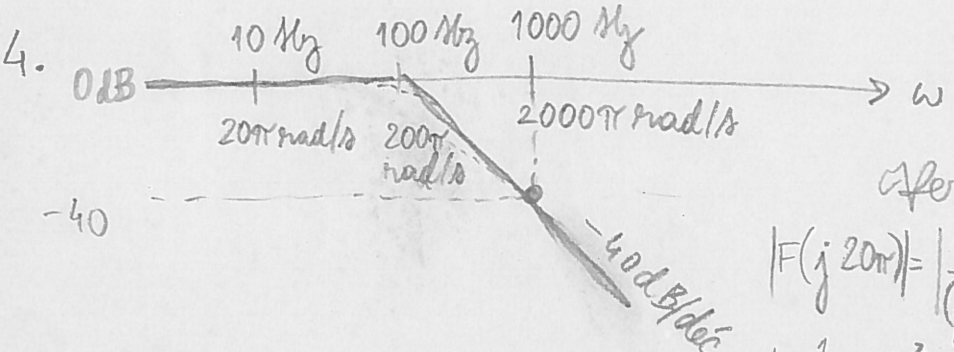
Temos de amplificar $\dfrac{0{,}25}{1{,}4286 \times 10^{-3}} = 175$ vezes

$-7 \quad \times \quad -25 \quad = 175$

Por exemplo



4.



$$F(s) = \frac{(200\pi)^2}{(s + 200\pi)^2} = \frac{394{,}8 \times 10^3}{(s + 628{,}32)^2}$$

Verificação:

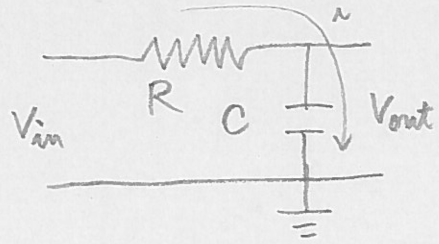
$$|F(j\,20\pi)| = \left| \frac{394{,}8 \times 10^3}{(j\,62{,}832 + 628{,}32)^2} \right| = \left| \frac{200^2 \pi^2}{(j20\pi + 200\pi)^2} \right| =$$

$$= \left| \frac{\frac{1}{(20\pi)^2}\, 20^2 \times 10^2 \pi^2}{\frac{1}{(20\pi)^2} (j20\pi + 200\pi)^2} \right| = \left| \frac{100}{(j + 10)^2} \right| = \frac{100}{|100 - 1 + 20j|} =$$

$$= \frac{100}{\sqrt{20^2 + 99^2}} = 0{,}990$$

β'

5. a)



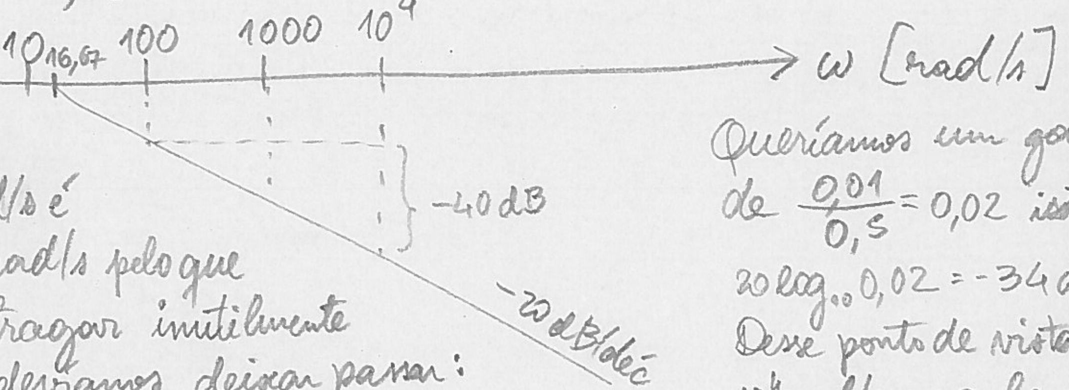
$$Z = \frac{U}{I} \Leftrightarrow I = \frac{U}{Z}$$

$$i = \frac{V_{in}}{R + \frac{1}{Cs}} = \frac{V_{out}}{\frac{1}{Cs}} \Rightarrow \frac{V_{in}}{\cancel{Cs}} = V_{out}\left(\frac{1+RCs}{\cancel{Cs}}\right) \Rightarrow$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1+RCs} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

Neste caso o filtro é

$$F(s) = \frac{\frac{1}{10\times10^{3}\times6\times10^{-6}}}{s + \frac{1}{0,06}} = \frac{16,67}{s+16,67}$$



Mas 16,67 rad/s é próximo de 10 rad/s pelo que estamos a estragar inutilmente o sinal que devíamos deixar passar:

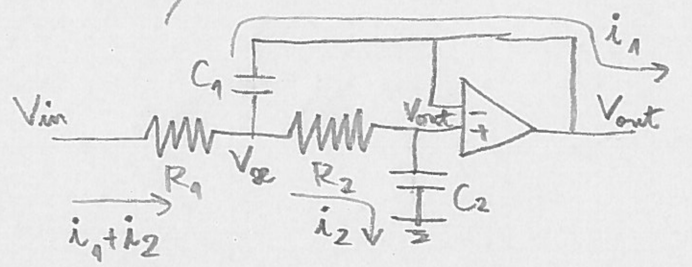
$$\left|\frac{16,67}{j10+16,67}\right| = \frac{16,67}{\sqrt{10^2+16,67^2}} = 0,857$$

Queríamos um ganho de $\frac{0,01}{0,5} = 0,02$ isto é $20\log_{10}0,02 = -34\,dB$. Desse ponto de vista, a 10^4 rad/s o ganho está abaixo de $-40\,dB$, não há problema.

b) É perfeitamente escusado isto, e fazendo antes $F(s) = \frac{100}{s+100}$ já deixamos passar 99% do sinal, tendo $-40\,dB$ de atenuação, no mínimo, para o ruído, o que continua a ser mais do que o exigido: $\frac{1}{RC} = 100$ consegue-se com, por exemplo,

$$C = 10\,\mu F = 10^{-5}\,F \quad e \quad R = 1\,k\Omega = 10^{3}\,\Omega \qquad \left(\frac{1}{RC} = \frac{1}{10^{-5}10^{3}} = \frac{1}{10^{-2}} = 10^{2}\right)$$

6. a)



$$\begin{cases} i_1 + i_2 = \dfrac{V_{in}-V_x}{R_1} & (1) \\ i_1 = \dfrac{V_x - V_{out}}{\frac{1}{C_1 s}} & (2) \\ i_2 = \dfrac{V_x - V_{out}}{R_2} & (3) \\ i_2 = \dfrac{V_{out}-0}{\frac{1}{C_2 s}} & (4) \end{cases}$$

p'

(4) $i_2 = V_{out} C_2 s$

(3) $V_{out} R_2 C_2 s = V_x - V_{out} \Leftrightarrow V_x = V_{out}(1+R_2 C_2 s)$

(1,2) $(V_x - V_{out}) C_1 s + V_{out} C_2 s = \dfrac{V_{in} - V_x}{R_1} \Rightarrow$

$$\Rightarrow V_{out} R_2 C_2 s R_1 C_1 s + V_{out} R_1 C_2 s = V_{in} - V_{out}(1+R_2 C_2 s)$$

$$\Leftrightarrow V_{out}(1 + R_1 C_2 s + R_2 C_2 s + R_1 R_2 C_1 C_2 s^2) = V_{in}$$

$$\Leftrightarrow \frac{V_{out}}{V_{in}} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1+R_2) C_2 s + 1}$$

b) Filtro passa-baixo de 2ª ordem.

c) $$G(s) = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{R_1+R_2}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$\rightarrow \omega_n^2$ logo $\sqrt{\dfrac{1}{R_1 R_2 C_1 C_2}} = 100 \times 2\pi$ rad/s

$\rightarrow 2\zeta\omega_n s$ logo $\dfrac{\frac{R_1+R_2}{R_1 R_2 C_1}}{2\sqrt{\frac{1}{R_1 R_2 C_1 C_2}}} = 0{,}7$

Queremos $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ com ganho -3dB na frequência $\omega = \omega_n$:

$$20\log_{10}\left|\frac{\omega_n^2}{(j\omega_n)^2 + 2\zeta\omega_n j\omega_n + \omega_n^2}\right| = -3 \Leftrightarrow \left|\frac{\omega_n^2}{-\omega_n^2 + 2\zeta\omega_n^2 j + \omega_n^2}\right| = 10^{-\frac{3}{20}} \Leftrightarrow$$

$$\Leftrightarrow \left|\frac{1}{2\zeta}\right| = 10^{-\frac{3}{20}} \Leftrightarrow 2\zeta = 10^{\frac{3}{20}} \Leftrightarrow \zeta = 0{,}7$$

Queremos $\begin{cases} R_1 R_2 C_1 C_2 = 2{,}533 \times 10^{-6} \\ \dfrac{R_1+R_2}{R_1 R_2 C_1} = 0{,}7 \times 400\pi = 879{,}6 \end{cases}$

Façamos, por exemplo, $C_2 = 100\,nF = 10^{-7} F$

Assim $R_1 R_2 C_1 = 25{,}33$ e portanto

$R_1 + R_2 = 22280$

Se $R_1 = R_2 = 11140\,\Omega = 11{,}14\,k\Omega$

$(11{,}14 \times 10^3)^2 C_1 = 25{,}33 \Leftrightarrow C_1 = 204 \times 10^{-9} F$

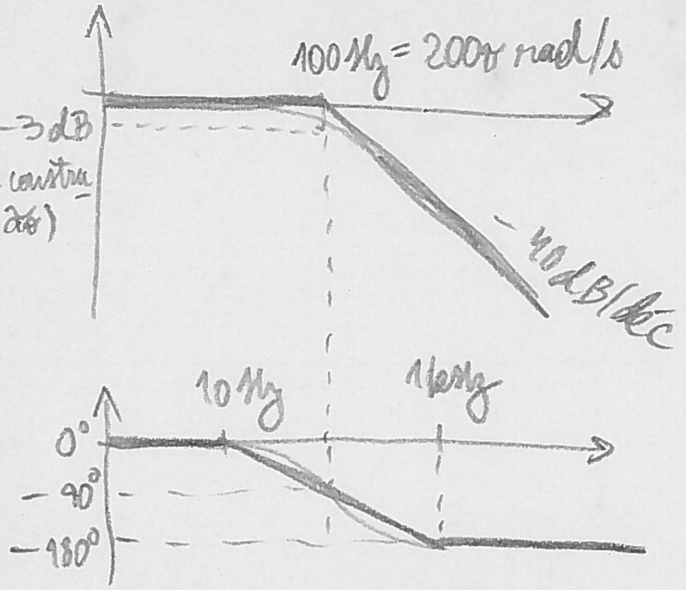
$= 204\,nF$

Em resumo:

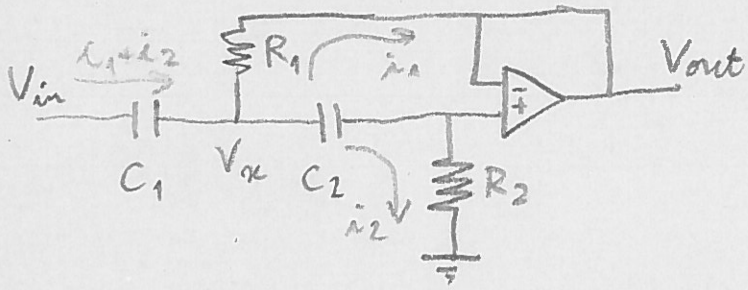
$R_1 = R_2 = 11{,}14\,k\Omega$

$C_1 = 204\,nF$ $\quad C_2 = 100\,nF$

d)



e)



$$\begin{cases} i_1 + i_2 = \dfrac{V_{in}-V_x}{\frac{1}{C_1 s}} & (1) \\ i_1 = \dfrac{V_x - V_{out}}{R_1} & (2) \\ i_2 = \dfrac{V_x - V_{out}}{\frac{1}{C_2 s}} & (3) \\ i_2 = \dfrac{V_{out} - 0}{R_2} & (4) \end{cases}$$

(4) $i_2 = \dfrac{V_{out}}{R_2}$

(3) $$\frac{V_{out}}{R_2} = V_x C_2 s - V_{out} C_2 s \Leftrightarrow V_{out}\left(\frac{1}{R_2} + C_2 s\right) = V_x C_2 s \Leftrightarrow$$

$$\Leftrightarrow V_x = V_{out}\left(1 + \frac{1}{R_2 C_2 s}\right)$$

(1,2) $$\frac{V_x - V_{out}}{R_1} + \frac{V_{out}}{R_2} = V_{in} C_1 s - V_x C_1 s \Rightarrow$$

$$\Rightarrow V_{out}\left(\cancel{\frac{1}{R_1}} + \frac{1}{R_1 R_2 C_2 s}\right) - V_{out}\cancel{\frac{1}{R_1}} + V_{out}\frac{1}{R_2} = C_1 s\left(V_{in} - V_{out}\left(1 + \frac{1}{R_2 C_2 s}\right)\right)$$

$$\Leftrightarrow V_{out}\left(\frac{1}{R_1 R_2 C_1 C_2 s^2} + \frac{1}{R_2 C_1 s}\right) = V_{in} - V_{out}\left(1 + \frac{1}{R C_2 s}\right) \Leftrightarrow$$

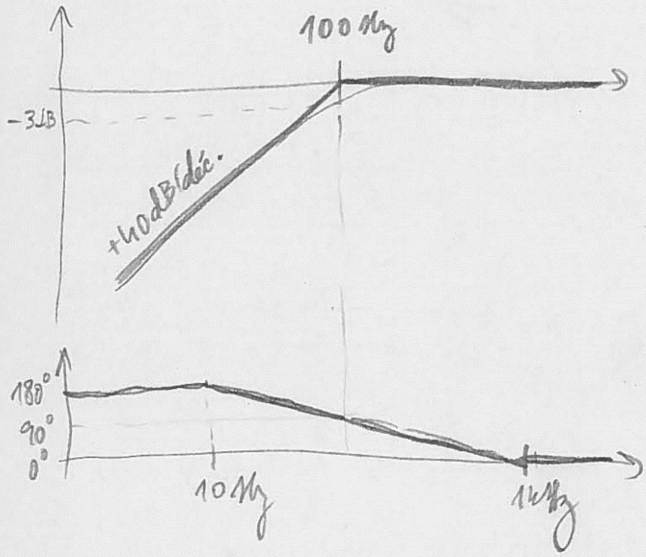
$$\Leftrightarrow V_{out}\left(\frac{1}{R_1 R_2 C_1 C_2 s^2} + \frac{1}{R_2 C_1 s} + \frac{1}{R_2 C_2 s} + 1\right) = V_{in} \Leftrightarrow$$

Ɛ

$$\Leftrightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1+\frac{1}{R_2C_1s}+\frac{1}{R_2C_2s}+\frac{1}{R_1R_2C_1C_2s^2}} =$$

$$= \frac{s^2}{s^2+\underbrace{\left(\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right)}s+\underbrace{\frac{1}{R_1R_2C_1C_2}}_{\omega_n^2}}$$

É um passa-alto de segunda ordem.



$$\begin{cases} \sqrt{\frac{1}{R_1R_2C_1C_2}} = 200\pi \\ \frac{C_1+C_2}{R_2C_1C_2} = 0{,}7\times 2\times 200\pi \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} R_1R_2C_1C_2 = 2{,}533\times10^{-6} \\ \frac{C_1+C_2}{R_1R_2C_1} = 879{,}6 \end{cases}$$

Façamos por exemplo $C_2 = 10\,mF = 10^{-2}\,F$

Então $R_1R_2C_1 = 2{,}533\times10^{-4}$ e portanto $C_1+C_2 = 0{,}2228 \Rightarrow C_1 = 212{,}8\,mF$

Logo $R_1R_2 = \frac{2{,}533\times10^{-6}}{212{,}8\times10^{-3}\times10\times10^{-3}} = \frac{2{,}533}{2128} = 1{,}2\times10^{-3}$ Podemos ter $R_1 = R_2 = 35\,m\Omega$

O problema é que, em c), os valores eram razoáveis. Aqui temos condensadores enormes e resistências minúsculas, a ponto de não fazerem sentido.

7. 8 bits: 256 valores, de 0 a 255

d) resolução: 10 V / 256 = 39,1 mV

a) 10101001_2 = 1+8+32+128 = 169

169 x resolução = 6,602 V

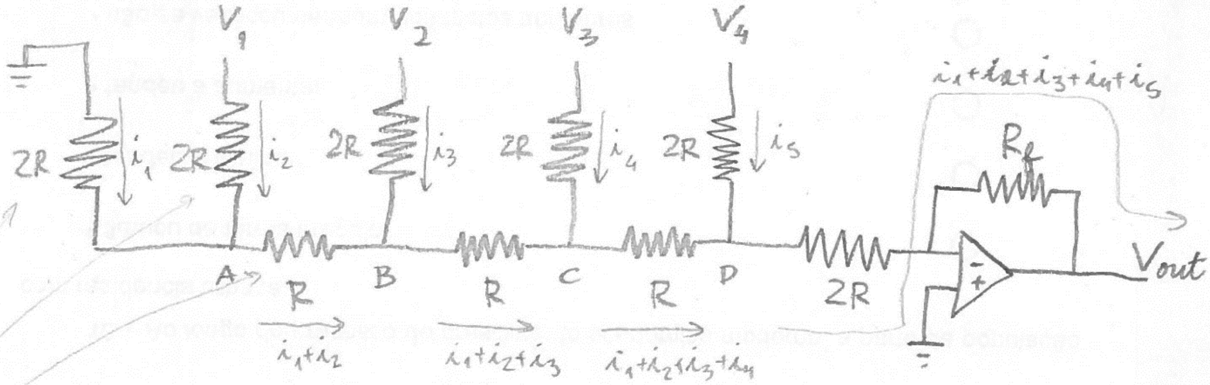
b) 01010111_2 = 1+2+4+16+64 = 87

87 x resolução = 3,398 V

c) 3 / resolução = 76,8

arredondando para baixo, 76 = 4+8+64 = 01001100_2

8.



$$2R = \frac{0-A}{i_1} \Rightarrow i_1 = -\frac{A}{2R}$$

$$2R = \frac{V_1 - A}{i_2} \Rightarrow i_2 = \frac{V_1-A}{2R}$$

$$R = \frac{A-B}{i_1+i_2} \Rightarrow i_1+i_2 = \frac{A-B}{R} = \frac{V_1-A}{2R} - \frac{A}{2R} \Rightarrow 2A-2B = V_1-2A \Leftrightarrow 4A = V_1+2B$$

$$\Leftrightarrow A = \frac{1}{4}V_1 + \frac{1}{2}B$$

etc, resistência por resistência

$$2R = \frac{V_2-B}{i_3} \Rightarrow i_3 = \frac{V_2-B}{2R}$$

$$R = \frac{B-C}{i_1+i_2+i_3} \Rightarrow i_1+i_2+i_3 = \frac{B-C}{R} = \frac{A-B}{R} + \frac{V_2-B}{2R} \Rightarrow 2B-2C = 2A-2B+V_2-B$$

$$\Rightarrow 5B-2C = 2A+V_2 \Rightarrow 5B-2C = \frac{1}{2}V_1 + B + V_2 \Rightarrow$$

5'

$$\Rightarrow 4B-2C = \frac{1}{2}V_1 + V_2 \Rightarrow B = \frac{1}{2}C + \frac{1}{4}V_2 + \frac{1}{8}V_1$$

$$2R = \frac{V_3-C}{i_4} \Rightarrow i_4 = \frac{V_3-C}{2R}$$

$$R = \frac{C-D}{i_1+i_2+i_3+i_4} \Rightarrow i_1+i_2+i_3+i_4 = \frac{C-D}{R} = \frac{B-C}{R} + \frac{V_3-C}{2R} \Rightarrow$$

$$\Rightarrow 2C-2D = 2B-2C+V_3-C \Rightarrow 5C-2D = 2B+V_3 \Rightarrow$$

$$\Rightarrow 5C-2D = C + \frac{1}{2}V_2 + \frac{1}{4}V_1 + V_3 \Rightarrow 4C-2D = V_3 + \frac{1}{2}V_2 + \frac{1}{4}V_1 \Rightarrow$$

$$\Rightarrow C = \frac{1}{2}D + \frac{1}{4}V_3 + \frac{1}{8}V_2 + \frac{1}{16}V_1$$

$$2R = \frac{V_4-D}{i_5} \Rightarrow i_5 = \frac{V_4-D}{2R}$$

$$2R = \frac{D-0}{i_1+i_2+i_3+i_4+i_5} \Rightarrow i_1+i_2+i_3+i_4+i_5 = \frac{D}{2R} = \frac{C-D}{R} + \frac{V_4-D}{2R} \Rightarrow$$

$$\Rightarrow D = 2C-2D+V_4-D \Rightarrow 4D + 2C + V_4 \Rightarrow$$

$$\Rightarrow 4D = D + \frac{1}{2}V_3 + \frac{1}{4}V_2 + \frac{1}{8}V_1 + V_4 \Rightarrow 3D = V_4 + \frac{1}{2}V_3 + \frac{1}{4}V_2 + \frac{1}{8}V_1$$

$$R_f = \frac{0-V_{out}}{i_1+i_2+i_3+i_4+i_5} \Rightarrow i_1+i_2+i_3+i_4+i_5 = \frac{-V_{out}}{R_f} \Rightarrow$$

$$\Rightarrow \frac{D}{2R} = -\frac{V_{out}}{R_f} \Rightarrow V_{out} = -\frac{R_f}{2R}D = -\frac{R_f}{6R}\left(V_4 + \frac{1}{2}V_3 + \frac{1}{4}V_2 + \frac{1}{8}V_1\right) =$$

$$= \left(-\frac{R_f}{48R}\right)\left(\underbrace{V_1}_{LSB} + 2V_2 + 4V_3 + 8V_4\right)$$

a) $\begin{matrix}1 & 0 & 0 & 1\\ V_4 & V_3 & V_2 & V_1\end{matrix}$ $\qquad 1001_{(2)} = 9_{(10)}$ $\qquad V_{out} = \left(-\frac{R_f}{48R}\right)(5+40) = 45\left(-\frac{R_f}{48R}\right) =$

$$= 9 \times 5 \times \left(-\frac{R_f}{48R}\right)$$

b) $\begin{matrix}0 & 1 & 0 & 1\\ V_4 & V_3 & V_2 & V_1\end{matrix}$ $\qquad 0101_{(2)} = 5_{(10)}$ $\qquad V_{out} = 5 \times 5 \times \left(-\frac{R_f}{48R}\right)$