1b) $\mathcal{L}\left[1000 - e^{-6t}\right] = \dfrac{1000}{s} - \dfrac{1}{s+6}$ (linhas 2 e 5, tabela 2.1)

1c) $\mathcal{L}\left[97{,}5\times10^{-3}\sin(0{,}2785t) + 546{,}9\times10^{-3}e^{0{,}9575t}\cos(0{,}9649t)\right]$

$$= 97{,}5\times10^{-3}\,\frac{0{,}2785}{s^2+0{,}2785^2} + 546{,}9\times10^{-3}\,\frac{s-0{,}9575}{(s-0{,}9575)^2+0{,}9649^2}$$

(linhas 9 e 12, tabela 2.1)

2a) $\mathcal{L}^{-1}\left[\dfrac{1}{3s^2+15s+18}\right] = \dfrac{1}{3}\mathcal{L}^{-1}\left[\dfrac{A}{s+3}\right] + \dfrac{1}{3}\mathcal{L}^{-1}\left[\dfrac{B}{s+2}\right] =$

raízes: $3(s^2+5s+6)=0 \Leftrightarrow$

$\Leftrightarrow s = \dfrac{-5\pm\sqrt{25-24}}{2} = \dfrac{-5\pm1}{2} < \begin{matrix}-3\\-2\end{matrix}$

juntando fica

$\dfrac{As+2A+Bs+3B}{(s+3)(s+2)}$ logo queremos

$\begin{cases}A+B=0\\2A+3B=1\end{cases}$ $\begin{cases}B=-A\\-A=1\end{cases}$ $\begin{cases}B=1\\A=-1\end{cases}$

$= \dfrac{1}{3}\left(-e^{-3t}\right) + \dfrac{1}{3}e^{-2t}$

2b) $\mathcal{L}^{-1}\left[\dfrac{1}{5s^2+6s+5}\right] = \dfrac{1}{5}\mathcal{L}^{-1}\left[\dfrac{1^2}{s^2+2\times\frac{3}{5}\times1\,s+1^2}\right] =$

raízes $s = \dfrac{-6\pm\sqrt{36-100}}{10} \in \mathbb{C}\setminus\mathbb{R}$

$= \dfrac{1}{\frac{4}{5}}e^{-\frac{3}{5}t}\sin\left(\frac{4}{5}t\right) =$

$= \dfrac{5}{4}e^{-\frac{3t}{5}}\sin\dfrac{4t}{5}$

Logo é melhor fazer

$s^2 + \dfrac{6}{5}s + 1$

$\omega^2 = 1 \Rightarrow \omega = 1$

$2\xi\omega = \dfrac{6}{5} \Rightarrow \xi = \dfrac{3}{5} \Rightarrow \square = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \dfrac{4}{5}$

2c) $\mathcal{L}^{-1}\left[\dfrac{8s^2+34s-2}{s^3+3s^2-4s}\right] = \mathcal{L}^{-1}\left[\dfrac{A}{s} + \dfrac{B}{s+4} + \dfrac{C}{s-1}\right] =$

juntando fica

$\dfrac{A(s^2+3s-4)+B(s^2-s)+C(s^2+4s)}{s(s+4)(s-1)}$

raízes: $s = \dfrac{-3\pm\sqrt{9+16}}{2} = \dfrac{-3\pm5}{2} < \begin{matrix}-4\\1\end{matrix}$

e ainda 0

O numerador é

$s^2(A+B+C)+s(3A-B+4C)+(-4A)$

logo $\begin{cases}A+B+C=8\\3A-B+4C=34\\-4A=-2\end{cases}$ $\begin{cases}B+C=15/2\\-B+4C=65/2\\A=1/2\end{cases}$ $\begin{cases}B=15/2-C\\5C=80/2\\—\end{cases}$ $\begin{cases}B=-1/2\\C=8\\A=1/2\end{cases}$

$= \dfrac{1}{2} - \dfrac{1}{2}e^{-4t} + 8e^{t}$

2d) $\mathcal{L}^{-1}\left[\frac{s^2+2s+8}{2s+4}\right] = \mathcal{L}^{-1}\left[\frac{1}{2}s + \frac{8}{2s+4}\right] = \frac{1}{2}\frac{d}{dt}\mathcal{L}^{-1}[1] + 4\mathcal{L}^{-1}\left[\frac{1}{s+2}\right]$

$$\begin{array}{l|l} s^2+2s+8 & 2s+4 \\ \hline s^2+2s & \frac{1}{2}s \\ \hline 0+0+8 & \end{array}$$

$= \frac{1}{2}\frac{d\delta(t)}{dt} + 4e^{-2t}$

2e) $\mathcal{L}^{-1}\left[\frac{-s^2+5s-2}{s^3-2s^2-4s+8}\right] = \mathcal{L}^{-1}\left[\frac{A}{s+2} + \frac{B}{s-2} + \frac{C}{(s-2)^2}\right] =$

aqui é usar Matlab:
roots([1 -2 -4 8])
dá $-2, +2, +2$

$\frac{A(s^2-4s+4)+B(s^2-4)+C(s+2)}{(s+2)(s-2)^2} =$

$= \frac{s^2(A+B)+s(-4A+C)+(4A-4B+2C)}{(s+2)(s-2)^2}$

$\begin{cases} A+B=-1 \\ -4A+C=5 \\ 4A-4B+2C=-2 \end{cases}$ $\begin{cases} B=-1-A \\ C=5+4A \\ 4A+4+4A+10+8A=-2 \end{cases}$ $\begin{cases} \\ \\ 16A=-16 \end{cases}$ $\begin{cases} B=0 \\ C=1 \\ A=-1 \end{cases}$

$= -\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] + \mathcal{L}^{-1}\left[\frac{1}{(s-2)^2}\right] = -e^{-2t} + te^{2t}$

3a) $y''(t) + y(t) = te^{-t}$ $\qquad y(0)=y'(0)=0$

$\mathcal{L}$: $s^2Y(s) + Y(s) = \frac{1}{(s+1)^2} \Leftrightarrow Y(s)(s^2+1) = \frac{1}{(s+1)^2} \Leftrightarrow Y(s) = \frac{1}{(s+1)^2(s^2+1)} =$

$= \frac{As+B}{s^2+1} + \frac{C}{s+1} + \frac{D}{(s+1)^2} = \frac{(As+B)(s^2+2s+1)+C(s+1)(s^2+1)+D(s^2+1)}{(s+1)^2(s^2+1)} =$

$= \frac{As^3+2As^2+As+Bs^2+2Bs+B+Cs^3+Cs^2+Cs+C+Ds^2+D}{} =$

$= \frac{s^3(A+C)+s^2(2A+B+C+D)+s(A+2B+C)+(B+C+D)}{}$

$\begin{cases} A+C=0 \\ 2A+B+C+D=0 \\ A+2B+C=0 \\ B+C+D=1 \end{cases}$ $\begin{cases} C=-A \\ 2A=-1 \\ B=\frac{-A-C}{2} \\ D=1-B-C \end{cases}$ $\begin{cases} C=1/2 \\ A=-1/2 \\ B=\frac{\frac{1}{2}-\frac{1}{2}}{2}=0 \\ D=1-0-\frac{1}{2}=\frac{1}{2} \end{cases}$

$\mathcal{L}^{-1}$

$y(t) = \mathcal{L}^{-1}\left[\frac{-\frac{1}{2}s}{s^2+1} + 0 + \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{(s+1)^2}\right] = -\frac{1}{2}\cos t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t}$

3 b) $y''(t) + y(t) = t e^{-t}$, $y(0) = \frac{1}{2}$, $y'(0) = -\frac{1}{2}$

$\mathcal{L}$

$$s^2 Y(s) - \frac{1}{2}s + \frac{1}{2} + Y(s) = \frac{1}{(s+1)^2} \Leftrightarrow Y(s)(s^2+1) = \frac{1}{(s+1)^2} + \frac{1}{2}(s-1) \Leftrightarrow$$

$$\Leftrightarrow Y(s) = \frac{1}{(s+1)^2(s^2+1)} + \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{1}{s^2+1}$$

$\mathcal{L}^{-1}$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2(s^2+1)}\right] + \frac{1}{2}\mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right] - \frac{1}{2}\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] =$$

feito em 3a)

$$= -\frac{1}{2}\cos t + \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t =$$

$$= \frac{1}{2}e^{-t} + \frac{1}{2}te^{-t} - \frac{1}{2}\sin t$$

Já agora, verifiquemos:

$\frac{d}{dt}$

$$y'(t) = \cancel{-\frac{1}{2}e^{-t}} + \cancel{\frac{1}{2}e^{-t}} + \frac{1}{2}t(-e^{-t}) - \frac{1}{2}\cos t$$

$$y''(t) = \cancel{\frac{1}{2}e^{-t}} - \cancel{\frac{1}{2}e^{-t}} - \frac{1}{2}e^{-t} - \frac{1}{2}t(-e^{-t}) + \frac{1}{2}\sin t$$

$$y''(t) + y(t) = \cancel{-\frac{1}{2}e^{-t}} + \frac{1}{2}te^{-t} + \cancel{\frac{1}{2}\sin t} + \cancel{\frac{1}{2}e^{-t}} + \frac{1}{2}te^{-t} - \cancel{\frac{1}{2}\sin t} =$$

$= te^{-t}$ como queríamos, e

$y(0) = \frac{1}{2} + 0 - 0$ e $y'(0) = 0 - \frac{1}{2}$

4 d) $$\left.\begin{aligned} \lim_{t\to 0} \mathcal{L}^{-1}[F(s)] &= \lim_{s\to+\infty} \frac{s^3+2s^2+8s}{2s+4} = +\infty \\ \lim_{t\to+\infty} \mathcal{L}^{-1}[F(s)] &= \lim_{s\to 0} \frac{s^3+2s^2+8s}{2s+4} = 0 \end{aligned}\right\} \text{confirma-se pelo resultado de 2d)}$$

5.2a) $$F(s) = \frac{1}{3s^2+15s+18} \Rightarrow F(j\omega) = \frac{1}{-3\omega^2+15j\omega+18} =$$

$$= \frac{(18-3\omega^2) - j15\omega}{[(18-3\omega^2)+j15\omega][(18-3\omega^2)-j15\omega]} =$$

$$= \frac{18-3\omega^2}{(18-3\omega^2)^2+(15\omega)^2} + j\frac{-15\omega}{(18-3\omega^2)^2+(15\omega)^2}$$