A Case Against a Few Traditional Quality Control Charts

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How this adventure began

- Lisbon, Nov. 1995: Prof. Ivette Gomes planted a UMPU chip in my brain.
  It lied (sort of) dormant for more than 16 years...
- Vienna, Sept. 2012: A chat with Prof. Sven Knoth sparked the chip.
  It is burning bright ever since...

What lies ahead

- Warm up
- Charts for counts of defects (joint work with S. Knoth & S. Paulino)
- Charts for the variance (joint work with S. Knoth)
- Final thoughts

Quality

- Fitness for use and conformance to requirements are the shortest and most consensual definitions of quality.

The founder of statistical process control (SPC)

- Concerns about quality can be traced back to the Babylonian Empire. However, we have to leap to the 20th. century to meet the father of modern quality control,

Walter Andrew Shewhart (1891–1967)

- In a historic memorandum of May 16, 1924, to his superiors at Bell Laboratories, we can find what is now known as a quality control chart.
- Control charts are used to track process performance over time and detect changes in process parameters, by plotting the observed value of a statistic against time and comparing it with a pair of control limits.

  An obs. beyond the control limits indicate potential assignable causes responsible for changes in those parameters, thus, should be investigated...

Defect

- Each specification that is not satisfied by a unit of a product is considered a defect or nonconformity (Montgomery, 2009, p. 308).

  E.g. flaw in the cabinet finish of a PC, broken rivet in an aircraft wing, etc.

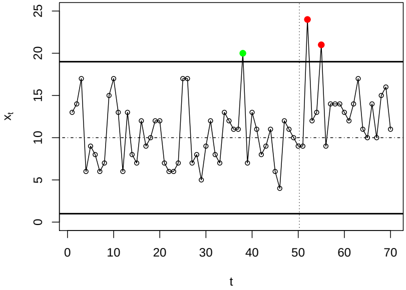
$c-$chart with 3-sigma limits

- The most popular procedure to control the mean count of defects (λ) in a constant-size sample.
- Control statistic: total count of defects in the t^{th} sample, X_t.
- Distribution: $X_t \overset{indep.}{\sim} Poisson(\lambda)$, $t \in \mathbb{N}$.
- Target mean: λ_0.
- Process mean: $\lambda = \lambda_0 + \delta$ (δ is the magnitude of the shift).
- $3 - \sigma$ **control limits**:

  $$LCL = \left\lceil \max\left\{0, \lambda_0 - 3\sqrt{\lambda_0}\right\}\right\rceil \qquad UCL = \left\lfloor \lambda_0 + 3\sqrt{\lambda_0} \right\rfloor.$$
- Triggers a signal and we **deem the process** (mean) **out-of-control** at sample t if $X_t \notin [LCL, UCL]$.

Example 1

- $\lambda_0 = 10$ (target mean count of nonconformities).
- Simulated data: first 50 samples — process known to be in-control; last 20 samples — process out-of-control ($\lambda = \lambda_0 + 2$).
- $LCL = \left\lceil \max\left\{0, \lambda_0 - 3\sqrt{\lambda_0}\right\} \right\rceil = 1$ $\quad UCL = \left\lceil \max\left\{0, \lambda_0 + 3\sqrt{\lambda_0}\right\} \right\rceil = 19$
- $c-$chart



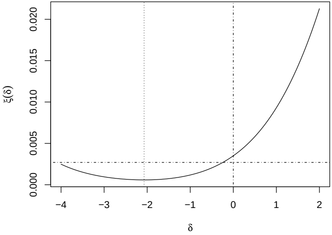
One false alarm: sample 38

Two valid signals: samples 52 and 55

Example 1 (cont'd)

Parallels with a repeated hypothesis test...

- $H_0 : \lambda = \lambda_0$ (process in-control); $H_1 : \lambda \neq \lambda_0$ (process out-of-control)
- Control statistic: $X \sim \text{Poisson}(\lambda),\ t \in \mathbb{N}$
- Rejection region: $W = \overline{[LCL, UCL]}$
- Exact power function: $\xi(\delta) = P_{\lambda_0+\delta}(X \in W), \quad \delta > -\lambda_0$

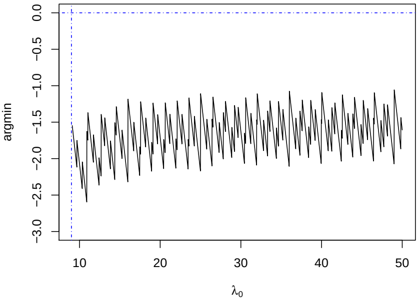


Problems

- significance level $\xi(0) \simeq 0.003500 \neq 0.0027 \simeq 1 - [\Phi(3) - \Phi(-3)]$;
- minimum of $\xi(\delta)$ not achieved at $\delta = 0 \quad \rightarrow$ biased test!

- If $\lambda_0 \leq 9$ then $LCL = \left\lceil \max\left\{0, \lambda_0 - 3\sqrt{\lambda_0}\right\}\right\rceil = 0$ and the c-chart (with 3-sigma limits) triggers false alarms more frequently than valid signals in the presence of any decrease in λ.
- For $\lambda_0 > 9$, the power function of a $c-$chart with $3-\sigma$ control limits attains its minimum value at

$$\delta^\star(\lambda_0) = argmin_{\delta \in (-\lambda_\mathbf{0}, +\infty)}\ \xi(\delta) = \left[\frac{UCL!}{(LCL-1)!}\right]^{\frac{\mathbf{1}}{UCL-LCL+\mathbf{1}}} - \lambda_0 < 0.$$



Performance of c-charts

- **Run length** (RL) — number of samples inspected taken until a signal: $RL(\delta) \sim \text{geometric}(\xi(\delta))$.
- The performance is frequently assessed in terms of

$$ARL(\delta) = \frac{1}{\xi(\delta)}.$$

It is desirable that valid signals / false alarms are emitted as quickly as possible / rarely triggered, corresponding to a small out-of-control / large in-control ARL.

- It is crucial to swiftly detect not only increases but also decreases in λ.

Increases in λ mean process deterioration.

Decreases in λ represent process improvement (to be noted and possibly incorporated). It can also mean that a new inspector may not have been trained properly to inspect the process output.

Some variants of the $c-$chart (Aebtarm & Bouguila, 2011)

- transforming data
- standardizing data
- optimizing control limits

Best overall $c-$chart (optimal control limits)

Control limits are obtained by linear regression based on a table of the *best* $c-$chart limits for several values of λ_0 (Ryan & Schwertman, 1997):

- $LCL = 1.5307 + 1.0212\lambda_0 - 3.2197\sqrt{\lambda_0}$;
- $UCL = 0.6182 + 0.9996\lambda_0 + 3.0303\sqrt{\lambda_0}$.

Once more, dealing with a non-negative, discrete and asymmetrical distribution prevents us from:

- setting a chart with a pre-specified in-control ARL $(= 1/\alpha)$;
- defining an **ARL-unbiased** control chart (Pignatiello *et al.*, 1995; Acosta-Mejía, 1999) in the sense that it takes longer in average to trigger a false alarm than to detect any shift.

Basic facts

- A size α test for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_0 + \delta \neq \lambda_0$, with power function $\xi(\delta)$, is said to be **unbiased** if $\xi(0) \leq \alpha$ and $\xi(\delta) \geq \alpha$, for $\delta \neq 0$.
  The test is at least as likely to reject under any alternative as under H_0;

$$ARL(0) \geq \alpha^{-1} \qquad \text{and} \qquad ARL(\delta) \leq \alpha^{-1},\ \delta \neq 0.$$

- If we consider $\mathcal{C}$ a class of tests for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$, then a test in $\mathcal{C}$, with power function $\xi(\delta)$, is a **uniformly most powerful (UMP)** class $\mathcal{C}$ **test** if $\xi(\delta) \geq \xi'(\delta)$, for every $\lambda \neq \lambda_0$ and every $\xi'(\delta)$ that is a power function of a test in class $\mathcal{C}$.

- In this situation there is no UMP test, but there is a test which is UMP among the class of all unbiased tests — the **uniformly most powerful unbiased (UMPU) test**.

- The concept of an **ARL-unbiased** Shewhart-type **chart** is related to the notion of **UMPU test**.

- The UMPU test derived by Lehmann (1986, pp. 135–136) for a real-valued parameter λ in the exponential family uses the critical function

$$\phi(x) = P(\text{Reject } H_0 \mid X = x) = \begin{cases} 1 & \text{if } x < LCL \text{ or } x > UCL \\ \gamma_{LCL} & \text{if } x = LCL \\ \gamma_{UCL} & \text{if } x = UCL \\ 0 & \text{if } LCL < x < UCL, \end{cases}$$

where LCL, UCL, γ_{LCL}, and γ_{UCL} are such that:

$$\begin{aligned} E_{\lambda_0}[\phi(X)] &= \alpha & &\text{(prob. of false alarm} = \alpha); \\ E_{\lambda_0}[X\,\phi(X)] &= \alpha\, E_{\lambda_0}(X) & &\text{(unbiased ARL).} \end{aligned}$$

- These equations are equivalent to

$$\gamma_{LCL} \times P_{\lambda_0}(LCL) + \gamma_{UCL} \times P_{\lambda_0}(UCL) = \alpha - \left[1 - \sum_{x=LCL}^{UCL} P_{\lambda_0}(x)\right] \quad (1)$$

$$\gamma_{LCL} \times LCL \times P_{\lambda_0}(LCL) + \gamma_{UCL} \times UCL \times P_{\lambda_0}(UCL) = \alpha \times E_{\lambda_0}(X) - \left[E_{\lambda_0}(X) - \sum_{x=LCL}^{UCL} x \times P_{\lambda_0}(x)\right]. \quad (2)$$

- However, (1) and (2) are not sufficient to define two control limits and two randomization probabilities.

Characterizing the ARL-unbiased c-chart

Inspired by this **UMPU test** and **randomized** tests (rarely used in SPC), we defined a $c-$chart that triggers a **signal** with:

- **probability one** if the sample **number of defects is in** $\overline{[LCL, UCL]}$;
- **probability** γ_{LCL} (resp. γ_{UCL}) if $x = LCL$ (resp. $x = UCL$).

Furthermore,

- the **randomization probabilities** are the solutions of the system of two linear equations (1) and (2):

$$\gamma_{LCL} = \frac{d\,e - b\,f}{a\,d - b\,c} \quad \text{and} \quad \gamma_{LCL} = \frac{a\,f - c\,e}{a\,d - b\,c},$$

where $a = P_{\lambda_0}(LCL)$, $b = P_{\lambda_0}(UCL)$, $c = LCL \times P_{\lambda_0}(LCL)$,
$d = UCL \times P_{\lambda_0}(UCL)$, $e = \alpha - 1 + \sum_{x=LCL}^{UCL} P_{\lambda_0}(x)$,
$f = \alpha \times E_{\lambda_0}(X) - E_{\lambda_0}(X) + \sum_{x=LCL}^{UCL} x \times P_{\lambda_0}(x)$.

Characterizing the ARL-unbiased c-chart (cont'd)

- To rule out pairs of control limits leading to $(\gamma_{LCL}, \gamma_{UCL}) \notin (0,1)^2$, the useful (LCL, UCL) are restricted to the following grid of non-negative integer numbers:
  - $\{(LCL, UCL) : L_{min} \leq LCL \leq L_{max},\ U_{min} \leq UCL \leq U_{max}\}$.

- The search for admissible values for $(\gamma_{LCL}, \gamma_{UCL})$ starts with $(LCL, UCL) = (L_{min}, U_{min})$ and stops as soon as an admissible solution is found.
  - $L_{min} = \max\left\{F^{-1}\left(\max\{0,\ F(U_{min}-1)-1+\alpha\}\right),\ G^{-1}\left(\max\{0,\ G(U_{min}-1)-1+\alpha\}\right)\right\}$
  - $L_{max} = \min\left\{\tilde{F}^{-1}(\alpha),\ \tilde{G}^{-1}(\alpha)\right\}$
  - $U_{min} = \max\left\{F^{-1}(1-\alpha),\ G^{-1}(1-\alpha)\right\}$
  - $U_{max} = \min\left\{\tilde{F}^{-1}\left(\min\{1,\ F(L_{max})+1-\alpha\}\right),\ \tilde{G}^{-1}\left(\min\{1,\ G(L_{max})+1-\alpha\}\right)\right\}$
  - $F(x) = P_{\lambda_0}(X \leq x)$ $\qquad G(x) = \frac{1}{E_{\lambda_0}(X)} \sum_{i=0}^{x} i \times P_{\lambda_0}(X = i)$
  - $F^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : F(x) \geq \alpha\}$ $\qquad \tilde{F}^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : F(x) > \alpha\}$
  - $G^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : G(x) \geq \alpha\}$ $\qquad \tilde{G}^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : G(x) > \alpha\}$

Rationale (Paulino *et al.*, 2016a, Appendix C)

Setting $\gamma_{LCL} = \gamma_{UCL} = 0$ (resp. $\gamma_{LCL} = \gamma_{UCL} = 1$) in eq. (1) and (2) leads to:

$\alpha \geq F(LCL-1)+1-F(UCL)$ and $\alpha \geq G(LCL-1)+1-G(UCL)$;

(resp. $\alpha \leq F(LCL)+1-F(UCL-1)$ and $\alpha \leq G(LCL)+1-G(UCL-1)$).

- **ARL function**

  A signal is triggered by the ARL-unbiased c-chart with probability

$$\xi_{unbiased}(\delta) = 1 - \sum_{x=LCL}^{UCL} P_{\lambda_0+\delta}(x) + \gamma_{LCL} \times P_{\lambda_0+\delta}(LCL) + \gamma_{UCL} \times P_{\lambda_0+\delta}(UCL).$$

  Its corresponding ARL function:

$$ARL_{unbiased}(\delta) = \frac{1}{\xi_{unbiased}(\delta)}.$$

- **Randomization of the emission of the signal**

  Can be done in practice by incorporating the generation of a pseudo-random number from a Bernoulli distribution with parameter γ_{LCL} (resp. γ_{LCL}) in the software used to monitor the data fed from the production line, whenever the observed number of defects is equal to LCL (resp. UCL).

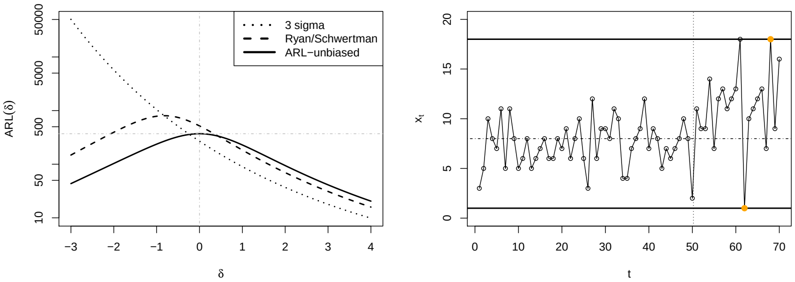
Example 2 — ARL-unbiased c-charts

- $\lambda_0 = 0.05, 0.1(0.1)1, 2 - 20$
- In-control ARL $= 370.4$ $\quad(\alpha = 0.0027)$
- Limits of the search grid, control limits and randomization probabilities

λ_0	L_{min}	L_{max}	LCL	U_{min}	U_{max}	UCL	γ_{LCL}	γ_{UCL}
0.05	0	0	0	2	2	2	0.002778	0.031347
0.1	0	0	0	3	3	3	0.002886	0.562609
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.9	0	0	0	5	5	5	0.005639	0.032019
1	0	0	0	6	6	6	0.006159	0.686904
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	1	1	1	18	18	18	0.482414	0.444451
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	4	9	8	34	37	35	0.566150	0.549842

Expectedly, the search grid/control limits may grow larger as λ_0 becomes larger.

Example 3 — *ARL-unbiased* vs. 3-σ limits and R&S c-charts ($\lambda_0 = 8$; $\delta = 3$)



As opposed to the c-chart with 3-sigma control limits and its variants...

- The ARL-unbiased c-chart can take a **pre-specified in-control ARL**.
- The associated **ARL curve attains a maximum when λ is on target**.
- It **tackles the curse of the null LCL** and detects decreases in λ in a timely fashion, by relying on the **randomization prob**.
- The two • correspond to obs. 62 and 68 which are equal to LCL and UCL and are responsible for valid signals due to randomization.

Related work (by submission date)

- Paulino, Morais and Knoth (2016a). An ARL-unbiased c-chart.
  Quality and Reliability Engineering International, 32: 2847–2858.
- Morais (2016). An ARL-unbiased np-chart.
  Economic Quality Control 31: 11–21.
  Monitoring the expected number of defective items in a sample of size n; improvement on a *nearly* ARL-unbiased p-chart by Acosta-Mejía (1999); limits and rand. prob. check with the critical function from *ump* R package (Geyer and Meeden, 2004 & 2005).
- Paulino, Morais and Knoth, S. (2016b). On ARL-unbiased c-charts for INAR(1) Poisson counts. *Statistical Papers*.
  Controlling the mean of first-order integer-valued autoregressive Poisson counts; dependent control statistics requiring another search algorithm; R program.
- Morais and Knoth (2016). On ARL-unbiased charts to monitor the traffic intensity of a single server queue.
  Proc. XIIth. Int'l. Workshop on Intelligent Statistical Quality Control, 217–242.
  3 ARL-unbiased charts; one with a mixed control statistic with an atom in zero.
- Morais (2017). ARL-unbiased geometric and CCC_G control charts.
  Sequential Analysis 36: 513–527.
  Monitoring the fraction conforming in high-yield processes; cumulative count of conforming chart under group inspection; improvement on *nearly* ARL-unbiased charts by L. Zhang *et al.* (2004) and C.W. Zhang *et al.* (2012).

R and S charts with 3-sigma limits

- The most popular procedures to control the standard deviation (σ) in a constant-size sample.
- Control statistic: range and standard deviation of t^{th} sample, R_t and S_t.
- Quality characteristic: $X \overset{indep.}{\sim} Normal(\mu, \sigma^2)$.
- In-control standard deviation: σ_0.
- Process standard deviation: $\sigma = \theta\,\sigma_0$ (θ is the magnitude of the shift).
- $3-\sigma$ **control limits**: QC textbooks[a] and QC practitioners tend to adopt symmetric limits (in-control $\sigma_0 = 1$)

$$R \text{ chart:} \quad d_2 \pm 3d_3 \quad \text{and} \quad S \text{ chart:} \quad c_4 \pm 3\sqrt{1-c_4^2}.$$

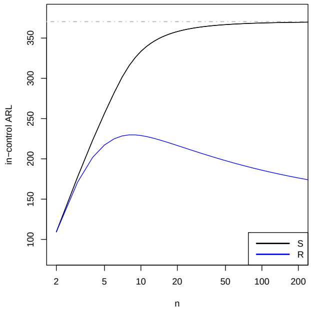
(In-control: $E(S) = c_4\,\sigma_0$, $V(S) = \sigma_0^2 - c_4\,\sigma_0^2$, $E(R) = d_2\,\sigma_0$, $Var(R) = d_3^2\,\sigma_0^2$.)

- A signal is triggered and **the process is deemed out-of-control** at sample t if the control statistic is beyond the control limits.

[a] As far as we know, there is an honourable exception: Uhlmann (1982, pp. 212-215).

R and S charts with 3-sigma limits

- For small sample sizes n (≤ 5 and ≤ 6 for $S-$ and $R-$charts, resp.) the LCL is negative.
- The in-control ARL differs considerably from the nominal value 370.4.



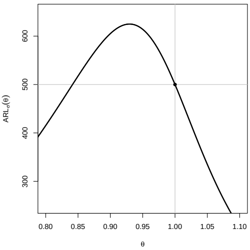
- Krumbholz (1992): ARL-unbiased R chart.
  Pignatiello *et al.* (1995): ARL-unbiased R and S charts derived with a pre-specified in-control ARL (=1000)

$$\text{R} : d_2 - k_L d_3,\ d_2 + k_U d_3; \quad \text{S} : c_4 - k_L'\sqrt{1 - c_4^2},\ c_4 + k_U'\sqrt{1 - c_4^2}.$$

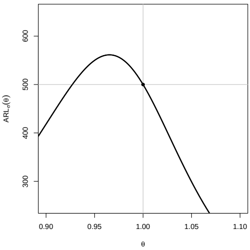
A closer look at the ARL-biased S^2−chart

- $LCL = \frac{\sigma_0^2 \, a(\alpha, n)}{n-1}$, $UCL = \frac{\sigma_0^2 \, b(\alpha, n)}{n-1}$, $a(\alpha, n) = F^{-1}_{\chi^2_{n-1}}(\alpha/2)$, $b(\alpha, n) = F^{-1}_{\chi^2_{n-1}}(1-\alpha/2)$.
- It surfaces pointedly that $ARL_\sigma(1) = 500 < ARL_\sigma(\theta)$, for some $\theta \in (0, 1)$.

$n = 5$



$n = 10$



- $argmax_{\theta \in \mathbb{R}^+} ARL_\sigma(\theta) = \theta^*(\alpha, n) = \sqrt{\frac{b(\alpha, n) - a(\alpha, n)}{(n-1)\{\ln[b(\alpha, n)] - \ln[a(\alpha, n)]\}}}$.

  The range of the interval where $ARL(\theta) > ARL(1)$ decreases with n because the interquantile range $[b(\alpha, n) - a(\alpha, n)]$ (resp. $b(\alpha, n)/a(\alpha, n)$) increases (resp. decreases) with n, for fixed α, thus, $[\theta^*(\alpha, n)]^{\frac{n-1}{2}}$ increases with n.

Pignatiello *et al.* (1995), certainly inspired by Ramachandran (1958), proposed

$$LCL = \frac{\sigma_0^2}{n-1} \times \tilde{a}(\alpha, n) \qquad UCL = \frac{\sigma_0^2}{n-1} \times \tilde{b}(\alpha, n).$$

- Power function and ARL function

$$\begin{aligned} \tilde{\xi}_\sigma(\theta) &= 1 - \left\{ F_{\chi^2_{n-1}}\left[\tilde{b}(\alpha, n)/\theta^2\right] - F_{\chi^2_{n-1}}\left[\tilde{a}(\alpha, n)/\theta^2\right] \right\}, \ \theta \in \mathbb{R}^+ \\ \widetilde{ARL}_\sigma(\theta) &= 1/\tilde{\xi}_\sigma(\theta) \end{aligned}$$

- Critical values

$$\begin{aligned} F_{\chi^2_{n-1}}\left[\tilde{b}(\alpha, n)\right] - F_{\chi^2_{n-1}}\left[\tilde{a}(\alpha, n)\right] &= 1 - \alpha \quad (1) \\ \tilde{b}(\alpha, n) \times f_{\chi^2_{n-1}}\left[\tilde{b}(\alpha, n)\right] - \tilde{a}(\alpha, n) \times f_{\chi^2_{n-1}}\left[\tilde{a}(\alpha, n)\right] &= 0. \quad (2) \end{aligned}$$

  (1) $\rightarrow \widetilde{ARL}_\sigma(1) = 1/\alpha$; (2) $\rightarrow$ condition for unbiasedness.

- (2) $\Leftrightarrow \left[\tilde{b}(\alpha, n)/\tilde{a}(\alpha, n)\right]^{\frac{n-1}{2}} = \exp\left\{[\tilde{b}(\alpha, n) - \tilde{a}(\alpha, n)]/(2\theta^2)\right\}$, as mentioned by Fertig and Proehl (1937), Ramachandran (1958), Kendall and Stuart (1979) and Pignatiello *et al.* (1995), or put in equivalent equations by Uhlmann (1982) and Krumbholz and Zoeller (1995).

- $\tilde{a}(\alpha, n)$ and $\tilde{b}(\alpha, n)$ can be found in:

  Ramachandran (1958) — 2 decimal places (dp), $\alpha = 0.05$, $n - 1 = 2(1)8(2)24, 30, 40, 60$;

  Tate and Klett (1959) — 4 dp, $\alpha = 0.001, 0.005, 0.01, 0.05, 0.1$, $n - 1 = 2(1)29$;

  Pachares (1961) — 5 dp, $\alpha = 0.01, 0.05, 0.1$, $n - 1 = 1(1)20, 24, 30, 40, 60, 120$;

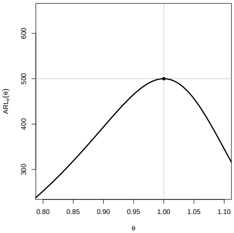
  Kendall and Stuart (1979) — 2 dp, $\alpha = 0.05$, $n - 1 = 2, 5, 10, 20, 30, 40, 60$;

  Pignatiello *et al.* (1995) — 4 dp, $\alpha = 0.005, 0.00286, 0.0020$, $n = 3(2)15(10)55$;

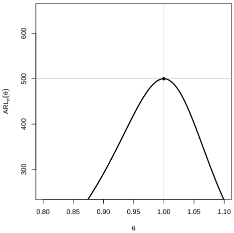
  Knoth and Morais (2013) — 4 dp, $\alpha = 0.001, 0.002, 1/370.4, 0.003, 0.004, 0.005, 0.010$, $n = 2, 3, 4, 5, 7, 10, 15, 100$.

- ARL-unbiased S^2−chart offers more protection against ↑ & ↓ in σ.

$n = 5$



$n = 10$



Soon in the *spc* R package! *Wait for an update [...] 0.6.0 on the way.* (SK)

Surprisingly, the ARL-unbiased designs of more sophisticated charts for σ have already been implemented.

EWMA (exponentially weighted moving average)

- Control statistic

$$Z_i = \begin{cases} z_0, & i = 0 \\ (1-\lambda)\, Z_{i-1} + \lambda\, S_i^2, & i \in \mathbb{N}. \end{cases}$$

z_0 initial value, often σ_0^2; λ constant chosen from $(0,1]$; small (resp. large) values of λ should be used to efficiently detect small (resp. large) shifts.

- (Asymptotic) control limits

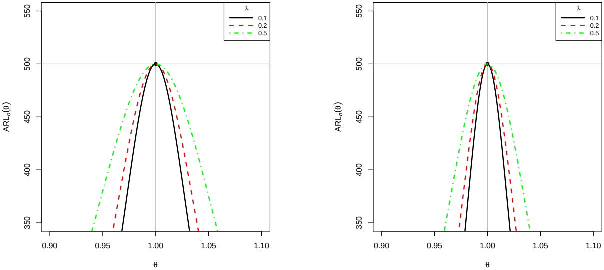
$$LCL = \sigma_0^2 - \tilde{a}_E(\alpha, n)\sqrt{\frac{2\lambda\sigma_0^2}{(2-\lambda)(n-1)}}, \quad UCL = \sigma_0^2 + \tilde{b}_E(\alpha, n)\sqrt{\frac{2\lambda\sigma_0^2}{(2-\lambda)(n-1)}}$$

The critical values $\tilde{a}_E(\alpha, n)$ and $\tilde{b}_E(\alpha, n)$:

  - are chosen in such way that $ARL(1) = 1/\alpha > ARL(\theta)$, $\theta \neq 1$;
  - can be found in Knoth and Morais (2013) — 4dp, $\lambda = 0.1$, $ARL(1) = \alpha^{-1}$, $\alpha = 0.001, 0.002, 1/370.4, 0.003, 0.004, 0.005, 0.010$, $n = 2, 3, 4, 5, 7, 10, 15, 100$.

- We can use the *spc* R package to:
  - derive ARL-unbiased EWMA$-S^2$ and CUSUM (cumulative sum) charts with a given in-control ARL;
  - plot the ARL profiles, determine the RL quantiles, etc.
- The ARL values decrease as we move away from $\theta = 1$, but also decrease more rapidly around $\theta = 1$ than in the ARL-unbiased S^2 chart.

$n = 5$ $\qquad$ $n = 10$

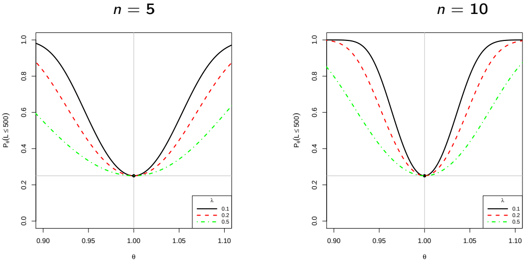


- Since the control statistics of EWMA charts are dependent, we fail to relate *ARL-unbiased* EWMA$-S^2-$ charts to unbiased or UMPU tests. However, we can explore other criteria to set up EWMA$-S^2-$charts to monitor dispersion, e.g., find control limits such that

$$P[RL_\sigma(1) \leq RL^{(0)}] \leq \alpha \quad \text{and} \quad P[RL_\sigma(\theta) \leq RL^{(0)}] \geq \alpha, \forall \theta \neq 1,$$

  where $RL^{(0)}$ is a pre-specified planned monitoring horizon, α is the prob. of at least one false alarm in this monitoring horizon.

- *unbiasedProb* EWMA$-S^2-$chart $(RL^{(0)} = 500, \alpha = 0.25)$



Nearly symmetrical profiles of $P[RL_\sigma(\theta) \leq 500]$ in the vicinity of $\theta = 1$.

By relying on the long and successful history of control charting, we believe that the ARL-unbiased charts have the potential to play a major role in the timely detection of the deterioration and improvement of real (industrial) processes.

On going and future work

Considerable attention has been given to *ARL-unbiased* charts in the continuous case; however, the literature is scarcer when for the discrete case.

We are planning to derive (or have already derived) ARL-unbiased:

- CUSUM charts to speed up the detection of shifts of the process mean of i.i.d., INAR(1) and Binomial INAR(1) counts;
- thinning-based EWMA to swiftly detect shifts in i.i.d. and INAR(1) Poisson counts.

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