ARL-unbiased geometric control charts for high-yield processes

Manuel Cabral Morais

Department of Mathematics & CEMAT — IST, ULisboa, Portugal

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Agenda

Quality

- Fitness for use and conformance to requirements are the shortest and most consensual definitions of quality (Juran and Godfrey, 1999, p. 27; Crosby, 1979, p. 17).
- A curious fact that escapes most consumers nowadays: concerns about quality can be traced back to the Babylonian Empire (Gitlow *et al.*, 1989, pp. 8–9).

  Code of Hammurabi[1]

  Law 229: *If a builder builds a house for a man and does not make its construction firm, and the house which he has built collapse and cause the death of the owner of the house, that builder shall be put to death.*
- This *eye for an eye* approach to quality was also adopted by Phoenicians inspectors, who *eliminated any repeated violations of quality standards by chopping off the hand of the maker of the defective product* (Gitlow *et al.*, 1989, p. 9).

[1] A Babylonian law code, dating back to about 1772BC, named after the sixth Babylonian king, who enacted it. It consists of 282 laws dealing with matters of contracts, terms of transactions or addressing household and family relationships such as inheritance, divorce, paternity and sexual behavior.

The founder of statistical process control (SPC)

- We have to leap to the 20th. century to meet the father of modern quality control.

Walter A. Shewhart (1891–1967)

- Shewhart altered the course of industry by celebrating a perfect marriage between statistics, engineering, and economics.

- In a historic memorandum of May 16, 1924, to his superiors at Bell Laboratories, we can find the *lasting and tangible evidence* of that marriage, the quality control chart.

- Control charts are used to track process performance over time and detect changes in process parameters, by plotting the observed value of a statistic against time and comparing it with a pair of control limits.

  An obs. beyond the control limits indicate potential assignable causes responsible for changes in those parameters, thus, should be investigated and eliminated.

Nonconformity; defective item

- Each specification that is not satisfied by a unit of a product is considered a defect or nonconformity (Montgomery, 2009, p. 308).
- A unit with a least one defect is called a defective or nonconforming item.

Examples of high-yield industrial processes

Many processes produce defectives items at a rate less than 100 ppm.

- Wire bonding process in an integrated circuit assembly provides an electrical connection between a semiconductor die and the external leads (Chang and Gan, 2001).
- In the filling process in the manufacture of low voltage liquid crystal display units, an incompletely filled unit is regarded as a nonconforming item (Chan *et al.*, 2003).
- Exhaust valves seats are force fitted by insertion into the head of an engine; incorrect installation can lead to an engine failure; the target defective rate is less than 50 ppm (Steiner *et al.*, 2004).

Geometric (or CCC) chart with 3-σ limits

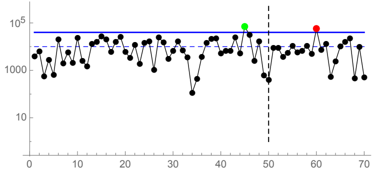
- The most popular procedure to monitor high-yield industrial processes can be traced back to Calvin (1983).
- **Control statistic**: cumulative count of conforming (CCC) items between the $(t-1)^{th}$ and t^{th} nonconforming units, X_t.
- **Distribution**: $X_t \overset{indep.}{\sim}$ geometric(p); $P_p(x) = (1-p)^x\, p,\ x \in \mathbb{N}_0$.
- **(Unknown) parameter**: p (fraction nonconforming).
- **Target value of p**: p_0.
- **True value of p**: $\rho \times p_0$ $(0 < \rho < 1/p_0;\ \rho$ magnitude of the shift$)$.
- $3-\sigma$ **control limits**:

$$LCL = (1-p_0)/p_0 - 3\sqrt{\frac{1-p_0}{p_0^2}}; \quad UCL = (1-p_0)/p_0 + 3\sqrt{\frac{1-p_0}{p_0^2}}.$$

- Triggers a signal and we **deem the process** (mean) **out-of-control** at sample t if $X_t \notin [LCL, UCL]$.
- Plot the number of conforming units (between consecutive defective units) on a **logarithmic scale** to accommodate large values of X_t.

Example 1

- Wire bonding process in an integrated circuit
  $p_0 = 10^{-4}$ (target fraction nonconforming)
- Simulated data: first 50 obs. of X_t — process in-control; last 20 obs. of X_t — process out-of-control ($p = 2 \times p_0$).
- $$LCL = \left\lceil \max\left\{0,\ (1-p_0)/p_0 - 3\sqrt{(1-p_0)/p_0^2}\right\} \right\rceil = 0$$
  $$UCL = \left\lfloor (1-p_0)/p_0 + 3\sqrt{(1-p_0)/p_0^2} \right\rfloor = 39\,997$$
- Geometric chart

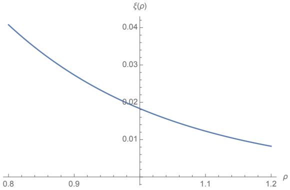


One false alarm: unit $506\,313 + 45$ $(x_{45} = 70\,728)$
One valid signal: unit $678\,446 + (50 + 10)$ $(x_{60} = 55\,600)$

Example 1 (cont'd)

Parallels with a repeated hypothesis test...

- $H_0 : p = p_0$ (process in-control); $H_1 : p \neq p_0$ (process out-of-control)
- Control statistic: $X \sim \text{geometric}(p),\ t \in \mathbb{N}$
- Rejection region: $W = (0, LCL) \cup (UCL, +\infty)$
- Exact power function: $\xi(\rho) = P_{\rho\, p_0}(X \in W)$, $\rho \in (0, 1/p_0)$



Problems

- minimum of $\xi(\rho)$ not achieved at $\rho = 1$; $\xi(\rho) < \xi(1),\ \rho > 1$
- significance level: $\xi(1) \simeq 0.018316 \neq 0.0027 \simeq 1 - [\Phi(3) - \Phi(-3)]$.

Performance

- **Run length** (RL) — number of units inspected taken until a signal is triggered: $RL(\rho) \sim \text{Geometric}(\xi(\rho))$.
- The performance is frequently assessed in terms of $ARL(\rho) = 1/\xi(\rho)$.
  It is desirable that false alarms (resp. valid signals) are rarely triggered (resp. emitted as quickly as possible), corresponding to a large in-control (resp. small out-of-control) ARL.
- It is crucial to swiftly detect not only increases but also decreases in p.
  Increases in p mean process deterioration.
  Decreases in p represent process improvement (to be noted and possibly incorporated). It can also mean that a new inspector may not have been trained properly to inspect the process output.
- Keep in mind that $LCL = \left\lceil (1-p_0)/p_0 - k\sqrt{(1-p_0)/p_0^2} \right\rceil = 0 \Leftrightarrow \sqrt{1-p_0} < k$ and holds as long as $k \geq 1$.
  Thus, $ARL(\rho) > ARL(1)$, for some ρ, i.e., the chart triggers false alarms more frequently than some valid signals.

Variants to mitigate the poor performance of the geometric chart

- Xie and Goh (1997) recommended the use of *exact probability limits*:

$$LCL_\alpha = \ln(1-\alpha/2)/\ln(1-p_0), \quad \text{and} \quad UCL_\alpha = \ln(\alpha/2)/\ln(1-p_0),$$

where α represents the acceptable risk of false alarm.

- Zhang *et al.* (2004) suggested:
  - taking $L \in A = \{1, \dots, LCL_{max}\}$, with $LCL_{max} = \lfloor \ln(1-\alpha)/\ln(1-p_0) \rfloor$;
  - finding $U : P_{\rho p_0}(X < L) + P_{\rho p_0}(X > U) \approx \alpha$, for each $L \in A$;
  - defining the set C of all such pairs of control limits (L, U);
  - choosing $(L^*, U^*) \in C$ that *most nearly equalizes the tail probab.*, i.e.,

$$|P_{p_0}(X < L^*) - P_{p_0}(X > U^*)| = \min_{(L,U)\in C} |P_{p_0}(X < L) - P_{p_0}(X > U)|.$$

$$ARL^*(\rho) = [P_{\rho\, p_0}(X < L^*) + P_{\rho\, p_0}(X > U^*)]^{-1}, \quad 0 < \rho < 1/p_0.$$

Both charts are ARL-biased: ARL does not attain a maximum at $\rho = 1$.
Moreover, the in-control ARL does not coincide with α^{-1}, a pre-specified value.

Yet Zhang *et al.* (2004) calls it a *nearly ARL-unbiased geometric chart.*

Basic facts

- A size α test for $H_0 : p = p_0$ against $H_1 : p = \rho\, p_0 \neq p_0$, with power function $\xi(\rho)$, is said to be **unbiased** if $\xi(1) \leq \alpha$ and $\xi(\rho) \geq \alpha$, for $\rho \neq 1$. The test is at least as likely to reject under any alternative as under H_0;

$$ARL(1) \geq \alpha^{-1} \qquad \text{and} \qquad ARL(\rho) \leq \alpha^{-1},\ \rho \neq 1.$$

- If we consider $\mathcal{C}$ a class of tests for $H_0 : p = p_0$ against $H_1 : p \neq p_0$, then a test in $\mathcal{C}$, with power function $\xi(\rho)$, is a **uniformly most powerful (UMP)** class $\mathcal{C}$ **test** if $\xi(\rho) \geq \xi'(\rho)$, for every $p \neq p_0$ and every $\xi'(\rho)$ that is a power function of a test in class $\mathcal{C}$.

- In this situation there is no UMP test, but there is a test which is UMP among the class of all unbiased tests — the **uniformly most powerful unbiased (UMPU) test**.

- The concept of an ARL-unbiased Shewhart-type chart is related to the notion of UMP test.

- The UMPU test derived by Lehmann (1986, pp. 135–136) for a real-valued parameter p in the exponential family uses the critical function

$$\phi(x) = P(\text{Reject } H_0 : p = p_0 \mid X = x) = \begin{cases} 1 & \text{if } x < LCL \text{ or } x > UCL \\ \gamma_{LCL} & \text{if } x = LCL \\ \gamma_{UCL} & \text{if } x = UCL \\ 0 & \text{if } LCL < x < UCL \end{cases}$$

LCL, UCL, γ_{LCL}, and γ_{UCL} are such that:

$$\begin{aligned} E_{p_0}[\phi(X)] &= \alpha && \text{(prob. of false alarm} = \alpha) \\ E_{p_0}[X\,\phi(X)] &= \alpha\, E_{p_0}(X) && \text{(unbiased ARL).} \end{aligned}$$

- In the geometric case, these eq. are equivalent to

$$\gamma_{LCL} \times P_{p_0}(LCL) + \gamma_{UCL} \times P_{p_0}(UCL) = \alpha - \left[1 - \sum_{x=LCL}^{UCL} P_{p_0}(x)\right] \quad (1)$$

$$\begin{aligned} \gamma_{LCL} \times LCL \times P_{p_0}(LCL) &+ \gamma_{UCL} \times UCL \times P_{p_0}(UCL) \\ &= \alpha \times E_{p_0}(X) - \left[E_{p_0}(X) - \sum_{x=LCL}^{UCL} x \times P_{p_0}(x)\right]. \quad (2) \end{aligned}$$

- However, equations (1) and (2) are not sufficient to define two control limits and two randomization probabilities.

Characterizing the ARL-unbiased geometric chart

Inspired by this UMPU test, we defined a **geometric chart that triggers a signal** with:

- **probability one** if the obs. number of conforming units between two consecutive nonconforming units, x, **is below LCL or above UCL**;
- **probability** γ_{LCL} (resp. γ_{UCL}) if $x = LCL$ (resp. $x = UCL$).

Furthermore,

- the **randomization probabilities** are the solutions of the system of two linear equations (1) and (2):

$$\gamma_{LCL} = \frac{d\,e - b\,f}{a\,d - b\,c} \qquad \text{and} \qquad \gamma_{LCL} = \frac{a\,f - c\,e}{a\,d - b\,c},$$

where $\quad a = P_{p_0}(LCL), \quad b = P_{p_0}(UCL), \quad c = LCL \times P_{p_0}(LCL),$
$d = UCL\times\, P_{p_0}(UCL), \quad e = \alpha - 1 + \sum_{x=LCL}^{UCL} P_{p_0}(x),$
$f = \alpha \times E_{p_0}(X) - E_{p_0}(X) + \sum_{x=LCL}^{UCL} x \times P_{p_0}(x).$

Characterizing the ARL-unbiased geometric chart (cont'd)

- To rule out pairs of control limits leading to $(\gamma_{LCL}, \gamma_{UCL}) \notin (0,1)^2$, the useful (LCL, UCL) are restricted to the following grid of non-negative integer numbers:
  - $\{(LCL, UCL): \; L_{min} \leq LCL \leq L_{max}, \; U_{min} \leq UCL \leq U_{max}\}$.
- The search for admissible values for $(\gamma_{LCL}, \gamma_{UCL})$ starts with $(LCL, UCL) = (L_{min}, U_{min})$ and stops as soon as an admissible solution is found.
  - $L_{min} = \max\left\{F^{-1}\left(\max\{0, \; F(U_{min}-1) - 1 + \alpha\}\right), \; G^{-1}\left(\max\{0, \; G(U_{min}-1) - 1 + \alpha\}\right)\right\}$
  - $L_{max} = \min\left\{\tilde{F}^{-1}(\alpha), \; \tilde{G}^{-1}(\alpha)\right\}$
  - $U_{min} = \max\left\{F^{-1}(1-\alpha), \; G^{-1}(1-\alpha)\right\}$
  - $U_{max} = \min\left\{\tilde{F}^{-1}\left(\min\{1, \; F(L_{max}) + 1 - \alpha\}\right), \; \tilde{G}^{-1}\left(\min\{1, \; G(L_{max}) + 1 - \alpha\}\right)\right\}$
  - $F(x) = P_{p_0}(X \leq x)$
  - $G(x) = \dfrac{1}{E_{p_0}(X)} \sum_{i=0}^{x} i \times P_{p_0}(X = i)$
  - $F^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : F(x) \geq \alpha\}$
  - $\tilde{F}^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : F(x) > \alpha\}$
  - $G^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : G(x) \geq \alpha\}$
  - $\tilde{G}^{-1}(\alpha) = \min\{x \in \mathbb{N}_0 : G(x) > \alpha\}$

(For the rationale and detailed derivation of this grid see Paulino *et al.*, 2016a, Appendix C.)

- **ARL function**

  A signal is triggered by the ARL-unbiased geometric chart with probability

$$\xi_{unbiased}(\rho) = \left[1 - \sum_{x=LCL}^{UCL} P_{\rho\, p_0}(x)\right] + \gamma_{LCL} \times P_{\rho\, p_0}(LCL) + \gamma_{UCL} \times P_{\rho\, p_0}(UCL).$$

  Its corresponding ARL function:

$$ARL_{unbiased}(\rho) = 1/\xi_{unbiased}(\rho).$$

- **Randomization of the emission of the signal**

  Can be done in practice by incorporating the generation of a pseudo-random number from a Bernoulli distribution with parameter γ_{LCL} (resp. γ_{LCL}) in the software used to monitor the data fed from the production line, whenever the observed number of nonconforming items is equal to LCL (resp. UCL).

Example 2 — ARL-unbiased geometric charts

- $p_0 = 10^{-i}$, $i = 5, 4, 3, 2$
- in-control ARL $= 200, 370.4$ $\quad (\alpha = 0.005, 0.0027)$
- Limits of the search grid, control limits and randomization probabilities

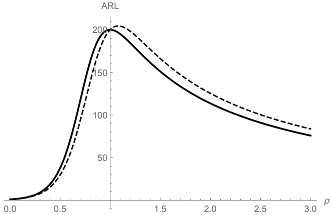
	in-control ARL = 200							
p_0	L_{min}	L_{max}	LCL	U_{min}	U_{max}	UCL	γ_{LCL}	γ_{UCL}
0.00001	441	501	441	743009	743294	743230	0.792137	0.754626
0.0001	44	50	44	74298	74326	74319	0.177234	0.318435
0.001	4	5	4	7426	7430	7428	0.415872	0.349557
0.01	0	0	0	739	739	739	0.440987	0.207035
	in-control ARL = 370.4							
p_0	L_{min}	L_{max}	LCL	U_{min}	U_{max}	UCL	γ_{LCL}	γ_{UCL}
0.00001	240	270	240	812554	812706	812674	0.736799	0.103324
0.0001	24	27	24	81252	81267	81263	0.072600	0.166090
0.001	2	2	2	8122	8123	8122	0.406312	0.224264
0.01	0	0	0	808	808	808	0.240561	0.010422

Expectedly, the search grid/control limits grow comparatively larger, as we deal with smaller values of the target fraction nonconforming.

Example 3 — *Nearly ARL-unbiased* vs. ARL-unbiased geometric charts

- $p_0 = 10^{-3}$, $\alpha = 0.005$
- Relative gain in the ARL when we replace the *nearly* ARL-unbiased design with the ARL-unbiased chart: $[1 - ARL(\rho, \gamma_{LCL}, \gamma_{UCL})/ARL^*(\rho)] \times 100\%$.

	$ARL(\rho)$		
ρ	*nearly* unbiased	unbiased	% gain
0.5	29.6309	37.6573	−27.0878%
0.8	138.8971	162.7097	−17.1440%
0.9	178.4955	191.8332	−7.4723%
1.0	199.9869	200.0000	−0.0066%
1.1	204.1971	194.9502	4.5285%
1.2	198.2454	184.4424	6.9626%
1.5	166.1584	151.0359	9.1013%



- The randomization of the emission of a signal allows the ARL-unbiased geometric chart to take less time to detect decreases in p than to trigger a false alarm, even if we are dealing with a null LCL.
- The out-of-control ARL performance of the *nearly ARL unbiased* and the ARL-unbiased geometric charts differ markedly when $p_0 \geq 10^{-3}$.
- The ARL-unbiased geometric chart is more (resp. less) sensitive to proc. deterioration (resp. improvement) than the *nearly ARL-unbiased* geom. chart.

- **ARL-unbiased geometric chart**

  As opposed to the geometric chart with 3-sigma control limits and its variants...

  - The ARL-unbiased geometric chart can take a **pre-specified in-control ARL**.
  - The associated **ARL curve attain a maximum when p is on target**.
  - It **tackles the curse of the null LCL** and detects decreases in p in a timely fashion, by relying on the **randomization probababilities**.
  - The ARL-unbiased geometric chart is also **RL-unbiased**:
    all out-of-control RL quantile curves are pointwise below the corresponding in-control RL quantile curve, i.e., all out-of-control RL are stochastically smaller (in the usual sense) than the in-control RL.

- **ARL-unbiased CCC_G chart**

  The use of the geometric chart is limited to situations where units of product are inspected sequentially...

  The cumulative count conforming (CCC) chart under group (G) inspection uses as its control statistic the cumulative number of samples of size n inspected until a nonconforming sample is encountered (Y).

$$P(Y = y) = P_{p,n}(y) = [(1-p)^n]^{y-1}\,[1-(1-p)^n], \quad y \in \mathbb{N}.$$

  Y is geometric r.v., taking values in $\mathbb{N}$ and with param. $p_n = 1-(1-p)^n$.

  To define an ARL-unbiased CCC_G chart, we need to:

  - obtain an ARL-unbiased geometric chart with false alarm rate α and a target value $1-(1-p_0)^n$;
  - add one unit to the control limits of the ARL-unbiased geometric chart and consider the same randomization probabilities.

- By relying on the long and successful history of control charting, we believe that the ARL-unbiased geometric and CCC_G charts have the potential to play a major role in the timely detection of the deterioration and improvement of real high-yield processes.

- **Future work**

  Derive...

  - ARL-unbiased versions of the existing $CCC-r$ charts to monitor the cumulative count of items until the r^{th} nonconforming item,[2] suchlike the ones discussed by Xie *et al.* (1998), ..., Albers (2010).

  - ARL-unbiased designs of the CUSUM (cumulative sum) and EWMA (exponentially weighted moving average) charts proposed by Chang and Gan (2001) and Yeh *et al.* (2008) for geometric output.

[2] X has a negative binomial distribution with parameters r and p.

Related SPC papers by submission date

- Paulino, S., Morais, M.C. and Knoth, S. (2016a). An ARL-unbiased c-chart. *Quality and Reliability Engineering International*, 32: 2847–2858. http://onlinelibrary.wiley.com/doi/10.1002/qre.1969/epdf
- Morais, M.C. (2016a). An ARL-unbiased np-chart. *Economic Quality Control* 31: 11–21. (Different search algorithm; *Mathematica* program)
- Paulino, S., Morais, M.C., and Knoth, S. (2016b). On ARL-unbiased c-charts for INAR(1) Poisson counts. *Statistical Papers*. http://rdcu.be/nGs8.
- Morais, M.C. and Knoth, S. (2016). On ARL-unbiased charts to monitor the traffic intensity of a single server queue. Proceedings of the XIIth. International Workshop on Intelligent Statistical Quality Control, 217–242. http://www.hsu-hh.de/compstat/index_8sVJz3C3s0oQzk3M.html (Complex search algorithm; R program)

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